INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

# *Project-Team Magique-3D*

# *Modélisation avancée en Géophysique 3D*

*Bordeaux - Sud-Ouest*

Theme : Observation and Modeling for Environmental Sciences

Activity Report

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*MAGIQUE-3D is an INRIA Project-Team joint with University of Pau and Pays de l'Adour and CNRS (LMA, UMR 5142 and MIGP, UMR 5212)*

# 1. Team

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# 2. Overall Objectives

## 2.1. General setting

The MAGIQUE-3D project is associated to two laboratories of University of Pau (Department of Applied Mathematics - LMA associated with CNRS, and Department of Modeling and Imaging in Geosciences - MIGP, also associated with CNRS). Gathering several researchers of different backgrounds in geophysics, physics, mathematics and scientific computing, the main purpose of this project is to establish a link between progress in high-resolution 3D scientific computing and various fields of geophysics. We wish on the one hand to develop sophisticated modeling tools (by integrating physical aspects of the phenomena under study) and to validate them in a rigorous way, and on the other hand to apply them to real cases of geophysical interest.

A first strength of this project is its intrinsically multi-disciplinary character. Moreover, the topics studied can lead to applications in fields other than seismology or seismic studies in the oil industry, for instance medical tomography, non destructive testing of materials or underwater acoustics.

A second strength of this project is that it is strongly related to the regional and national industrial environment, in particular regarding how to go from theoretical studies of relatively complex media to real applications of the methods developed for real cases encountered in the field. Our main industrial partner is TOTAL, whose main research center is located in Pau. We develop strong collaborations with this petroleum company. The project could also establish links with the industrial valorization unit of University of Pau (ValUPPA), which would ensure close contacts between the researchers of the project and the PME/PMI (small and medium-size companies) of the region of Pau.

Since the MAGIQUE-3D team has been created (January 2005), its overall objectives have been reorganized to answer to numerous questions from its main industrial partner TOTAL. MAGIQUE-3D is now a team-project (from july 2007) and its research program is composed of two main topics that structure the original parts of the activities of the group. The first topic, entitled ‘Depth Imaging’, is related to modeling of seismic wave propagation in complex geological structures, taking into account underlying physical phenomena. The main part of our research program has been defined jointly by working groups composed of members of MAGIQUE-3D and of our main industrial partner TOTAL. The second topic, that could be given the general title ‘Advanced modeling in wave propagation’, is related to the realistic numerical simulation of complex three-dimensional geophysical phenomena and its comparison with real data recorded in the field. We also participate in different research programs that allow us to work on other aspects of scientific computing in the context of external collaborations.

## 2.2. Highlights of the year

The MAGIQUE-3D group organized the 9th international congress on Mathematical and Numerical Aspects of Waves Propagation, the so-called WAVES09 conference which was held from 15 till 19 June 2009 in the Palais Beaumont in Pau. Waves 2009 gathered more than 250 participants coming from more than 30 different countries.
4 Ph.D students defended their thesis: C. Blitz in April, C. Baldassari, M. Grigoroscuta and R. Madec in December. R. Martin defended his Habilitation à Diriger les Recherches in June.
The team has hired T.M. Laleg-Kirati as a Research Accociate (CR).

# 3. Scientific Foundations

## 3.1. Modeling

The main activities of Magique-3D in modeling are the derivation and the analysis of models that are based on mathematical physics and are suggested by geophysical problems. In particular, Magique-3D considers equations of interest for the oil industry and focus on the development and the analysis of numerical models which are well-adapted to solve quickly and accurately problems set in very large or unbounded domains as it is generally the case in geophysics.

## 3.2. Depth Imaging

The research program of MAGIQUE-3D for depth imaging is divided into two topics that both deal with the wave equation solution in a complex 3D medium but that differ in terms of the used methods. The first topic is entitled "One-way and two-way models" and is based on numerical micro-local analysis. From a practical point of view, such investigations are supposed to lead to the solution of approximate wave equations but they have the advantage of giving rise to fast solution methods which, if not accurate in general situation, can provide approximate solutions that can subsequently be used as data to initialize implicit schemes for solving full wave equations. The second topic is entitled "High-performance methods for solving wave equations". It is more classical and deals with the development of innovative resolution methods and based on finite element methods. We essentially focus on the development of mixed hybrid methods which are based on the combination of Lagrange polynomials and plane waves. By using finite element discretization, we can take the topography into account which allows us to outperform existing Reverse Time Migration methods based on finite difference methods.

## 3.3. Wave propagation in porous media

The propagation of waves in porous media is of interest in many geophysics applications. MAGIQUE-3D develops different axes of research on this subject. By using numerical methods like finite differences, finite elements or meshless techniques like the various boundary integral methods (Boundary Integral Method, Indirect Boundary Element Method, Meshless Galerkin Method...) we aim at solving the equations describing porous media. MAGIQUE-3D has started to work on Biot models. The first studies that we have undertaken were devoted to the mathematical analysis of a linear and nonlinear model coupling an elastic wave equation written in the solid structure and a diffusion equation written in the fluid. We currently work on numerical modeling of Biot's model. We develop a numerical method based on the one hand on the use of a three-dimensional spectral-element method for space discretization, which is a finite-element method with an exactly diagonal mass matrix, and on the other hand on an explicit scheme for time discretization.

## 3.4. High Performance methods for solving wave equations

Nowadays the wave equation can be solved with very good accuracy using finite-element methods but the main difficulty that remains is related to the very large amount of computer memory storage that is required, in particular in 3D. Moreover, the solution of the time dependent wave equation with classical finite elements requires the inversion of a very large mass matrix at each time step. To avoid this problem, we develop space-discretization methods such as spectral element methods or Discontinuous Galerkin methods which turn the mass matrix onto an easily invertible (block)-diagonal matrix. We also try to improve the time-discretization methods by using local time stepping schemes which enable us to use a small time step only on the part of the mesh where it is needed and to use a coarse time step everywhere else. From a computational point of view, we dedicated a large part of our effort on parallel computing and we have established since the beginning of 2006 a collaboration with the Barcelona Supercomputing Center (BSC, Spain).

## 3.5. Absorbing Boundary Conditions and Perfectly Matched Layers

In most of the problems of wave propagation we have to deal with unbounded domains. It is then helpful to define (artificial) boundaries defining the numerical model under study in order to reduce the computational costs.

Since the innovative work of [63], this question was addressed by using Absorbing Boundary Conditions (ABC) [54] or damping zones (sponge layers) [52]. In both cases, results are often not very satisfactory because spurious phases are reflected inside the computational domain, in particular at grazing incidence in the case of paraxial conditions, and at low frequency in the case of sponge layers. However these conditions are easy to include in numerical schemes and can be constructed such that they preserve the sparsity of the discrete matrix. This justifies why MAGIQUE-3D works on the improvement of ABCs.

More recently, Bérenger [47] introduced an innovative condition for Maxwell's equations, which has the property of being perfectly adapted to the model, in the sense that no spurious phase is produced in the domain before discretization and use of a numerical scheme. The resulting model is called a Perfectly Matched Layer (PML). Because of its efficiency, the PML method quickly became very popular in electromagnetics. Next, based on an analogy between Maxwell's equations and linear elasticity written as a first order system in velocity and stress, several authors adapted the PML approach to the propagation of elastic waves in infinite domains (see e.g. [53], [55]). However, for instance in the case of elastodynamics (and even more for porous media) and for shallow-water equations, the classical PMLs are known to be unstable. Moreover the layers do not properly handle grazing rays which gives rise to spurious reflections. This is why MAGIQUE-3D develops new PML models both for elastodynamics, porous media and geophysical fluids.

# 4. Application Domains

## 4.1. Seismic Imaging

The main objective of modern seismic processing is to find the best representation of the subsurface that can fit the data recorded during the seismic acquisition survey. In this context, the seismic wave equation is the most appropriate mathematical model. Numerous research programs and related publications have been devoted to this equation. An acoustic representation is suitable if the waves propagate in a fluid. But the subsurface does not contain fluids only and the acoustic representation is not sufficient in the general case. Indeed the acoustic wave equation does not take some waves into account, for instance shear waves, turning waves or the multiples that are generated after several reflections at the interfaces between the different layers of the geological model. It is then necessary to consider a mathematical model that is more complex and resolution techniques that can model such waves. The elastic or viscoelastic wave equations are then reference models, but they are much more difficult to solve, in particular in the 3D case. Hence, we need to develop new high-performance approximation methods.

Reflection seismics is an indirect measurement technique that consists in recording echoes produced by the propagation of a seismic wave in a geological model. This wave is created artificially during seismic acquisition surveys. These echoes (i.e., reflections) are generated by the heterogeneities of the model. For instance, if the seismic wave propagates from a clay layer to sand, one will observe a sharp reflected signal in the seismic data recorded in the field. One then talks about reflection seismics if the wave is reflected at the interface between the two media, or talks about seismic refraction if the wave is transmitted along the interface. The arrival time of the echo enables one to locate the position of this transition, and the amplitude of the echo gives information on some physical parameters of the two geological media that are in contact. The first petroleum exploration surveys were performed at the beginning of the 1920's and for instance, the Orchard Salt Dome in Texas (USA) was discovered in 1924 by the seismic-reflection method.

## 4.2. Seismology

We already applied our techniques to the study of strong ground motion and associated seismic risk in the Los Angeles basin area. This region consists of a basin of great dimension (more than 100 km $\times$ 100 km) which is one of the deepest sedimentary basins in the world (the sedimentary layer has a maximum thickness of 8.5 km underneath Downtown Los Angeles), and also one of the most dangerous in the world because of the amplification and trapping of seismic waves. In the case of a small earthquake in Hollywood (September 9, 2001), well recorded by more than 140 stations of the Southern California seismic network TriNet, we managed for the first time to fit the three components of the displacement vector, most of the previous studies focusing on the vertical component only, and to obtain a good agreement until relatively short periods (2 seconds).

We wish to improve these studies of seismic risk in densely populated areas by considering other regions of the world, for example the Tokyo basin, the area of Kobe or the Mexico City region. We also plan to generalize this type of calculations to the knowledge and modeling of site effects, i.e. of the local amplification of the response of the ground to seismic excitation. The study of such effects is an important observation in urban areas to be able to anticipate the damage to constructions and, if necessary, to plan the organization of search and rescue operations. It is also a significant element of the definition of paraseismic standards. Site effects can be determined experimentally, but that requires the installation of stations for a sufficient period of time to record a few tens of seismic events. Numerical modeling makes it possible to avoid this often long and difficult experimentation, assuming of course that one has good knowledge of the geological structure of the subsurface in the studied area. We thus propose in the MAGIQUE-3Dproject to use the numerical techniques mentioned above for instance to quantify the effects of topographic variations in the structure.

## 4.3. Non destructive testing, Medical Imaging

The problems of seismic imaging can be related to non destructive testing, in particular medical imaging. For instance, the rheumatologist are now trying to use the propagation of ultrasounds in the body as a noninvasive way to diagnose osteoporosis. Then, the bones can be regarded as elastodynamic or poroelastic media while the muscles and the marrow can be regarded as acoustic media. Hence the computational codes we use for seismic imaging could be applied to such a problem.

## 4.4. Submarine acoustics

The SPECFEM software package is able to simulate the propagation of waves in the context of time domain fluid-structure interaction. We have actually started to consider underwater acoustics in collaboration with P. Cristini (MIGP, UPPA) and F. Sturm (Laboratoire de Mécanique des Fluides et d'Acoustique, École Centrale de Lyon, France).

# 5. Software

## 5.1. SPECFEM3D

The MAGIQUE-3D project is based (in part) on existing software packages, which are already validated, portable and robust. The SPECFEM3D software package, developed by Dimitri Komatitsch and his colleagues in collaboration with Jeroen Tromp and his colleagues at the California Institute of Technology and at Princeton University (USA), and which is still actively maintained by Dimitri Komatitsch and his colleagues, allows the precise modeling of seismic wave propagation in complex three-dimensional geological models. Phenomena such as anisotropy, attenuation (i.e., anelasticity), fluid-solid interfaces, rotation, self-gravitation, as well as crustal and mantle models can be taken into account. The software is written in Fortran95 with MPI message-passing on parallel machines. It won the Gordon Bell Prize for best performance of the Supercomputing'2003 conference. In 2006, Dimitri Komatitsch established a new collaboration with the Barcelona Supercomputing Center (Spain) to work on further optimizing the source code to prepare it for very large runs on future petaflops machines to solve either direct or inverse problems in seismology. Optimizations have focused on improving load balancing, reducing the number of cache misses and switching from blocking to non-blocking MPI communications to improve performance on very large systems. Because of its flexibility and portability, the code has been run successfully on a large number of platforms and is used by more than 150 academic institutions around the world. In November 2008 this software package was again among the six finalists of the pretigious Gordon Bell Prize of the SuperComputing'2008 conference in the USA [51] for a calculation performed in parallel on 150,000 processor cores, reaching a sustained performance level of 0.16 petaflops.

## 5.2. Gar6more2D and Gar6more3D

The softwares Gar6more2D and Gar6more3D compute the solution to the problem of wave propagation in infinite bilayered acoustic/acoustic, acoustic/elastodynamic, acoustic/porous or porous/porous media, respectively in two and three dimensions. They are both written in FORTRAN90 and Gar6more3D has been parallelized using MPI. Gar6more2D has already been used for the validation of poroelastic codes. Both softwares are distributed under a CeCILL license at the url http://www.spice-rtn.org/library/software/Gar6more2D/ and http://www.spice-rtn.org/library/software/Gar6more3D/.

# 6. New Results

## 6.1. Inverse acoustic problems

**Participants:** Chokri Bekkey, Hélène Barucq, Rabia Djellouli.

The determination of the shape of an obstacle from its effects on known acoustic or electromagnetic waves is an important problem in many technologies such as sonar, radar, geophysical exploration, medical imaging and nondestructive testing. This inverse obstacle problem (IOP) is difficult to solve, especially from a numerical viewpoint, because it is ill-posed and nonlinear [56]. Moreover, the precision in the reconstruction of the shape on an obstacle strongly depends on the quality of the given far-field pattern (FFP) measurements: the range of the measurements set and the level of noise in the data. Indeed, the numerical experiments performed in the resonance region (for example [80], [87], [72], [73]), that is for a wavelength that is approximately equal to the diameter of the obstacle, tend to indicate that in practise, and at least for simple shapes, a unique and reasonably good solution of the IOP can be often computed using only one incident wave and *full aperture* far-field data (FFP measured only at a limited range of angles), as long as the aperture is larger than $\pi$. For smaller apertures the reconstruction of the shape of an obstacle becomes more difficult and nearly impossible for apertures smaller than $\pi/4$.
Given that, and the fact that from a mathematical viewpoint, the FFP can be determined on the entire $S1$ from its knowledge on a subset of $S1$ because it is an *analytic* function, we propose a solution methodology to extend the range of FFP data when measured in a limited aperture and not on the entire sphere $S1$. Therefore it would be possible to solve numerically the IOP when only limited aperture measurements are available. However, due to the analyticity nature of the FFP, the reconstruction or the extension of the far-field pattern from limited measurements is an inverse problem that is *severely ill-posed* and therefore very challenging from a numerical viewpoint. Indeed preliminary numerical results indicate that the reconstruction of the FFP using the discrete $L2$ minimization with the standard Tikhonov regularization is very sensitive to the noise level in the data. The procedure is successful only when the range of measurements is very large which is not realistic for most applications.
We propose a multi-step procedure for extending/reconstructing the FFP from the knowledge of limited measurements. The proposed solution methodology addresses the ill-posedness nature of this inverse problem using a *total variation* of the FFP coefficients as a penalty term. Consequently the new cost function is no longer differentiable. We restore the differentiability to the cost function using a perturbation technique [81] which allows us to apply the Newton algorithm for computing the minimum. The multi-step feature of the proposed method consists in extending the FFP at each step by an $n$ degrees increment.
We investigate the effect of the frequency regime and the noise level of the performance of the proposed solution methodology. Preliminary results obtained in the case of two-dimensional sound-soft disk-shaped scatterer have been performed. They illustrate the potential of the solution methodology for enriching the FFP measurements for various frequencies and levels of noise. The solution methodology and the numerical results have been presented in a Research Report [39] and a paper has been submitted in November.

## 6.2. A new modified equation approach for solving the wave equation

**Participants:** Cyril Agut, Hélène Barucq, Julien Diaz, Abdelaâziz Ezziani.

In this work, we construct new fourth order schemes in space and time for the wave equation by applying the modified equation technique in an original way. Indeed, most of the works devoted to the solution of the wave equation consider first the space discretization of the system before addressing the question of the time discretization. The idea is here to invert the discretization process by applying first the time discretization thanks to the modified equation technique and after to consider the space discretization. After the time discretization, an additional bilaplacian operator, which can not be discretized by the classical finite elements appears. If the acoustic medium is homogeneous or has smooth heterogeneities, the solution is C1 and therefore we have to consider C1 finite elements (such as the Hermite ones) or Discontinuous Galerkin finite elements (DGFE) whose C1 continuity is enforced through an appropriate penalty term. In a strongly heterogeneous media, the solution is no longer C1 because of the discontinuities of the physical parameters and Hermite elements are not adapted to this problem. DGFE can however be used by imposing the continuity of a suitable physical quantity (corresponding to the classical transmission conditions) instead of the C1 continuity. Moreover, this technique can be extended to obtained arbitrarily even high-order schemes by considering p-laplacian operators and we have also considered the sixth order scheme. We have compared the solution obtained by the new method with DGFE to the one obtained with a classical second order method (using also DGFE) in 1D and 2D first in a simple homogeneous medium then in a heterogeneous bilayered medium. Numerical results showed that the scheme with the bilaplacian (resp. 3-laplacian) operator is actually fourth (resp. sixth) order accurate. Moreover it appeared that the stability condition is not penalized, in the sense that the stability conditions of the 4th and the 6th order schemes are very close to the one of the classical Leap-Frog scheme (actually it is even a little higher). This means that the new schemes provide high accuracy for the same cost than the classical second-order scheme.

Now, we are considering the adaptation of the order of the scheme in the various cells of the mesh. To do so, we consider for instance a scheme with bilaplacian on the coarse cells of a given mesh and a classical leap-frog scheme on the fine cells. To perform the space discretization, we have to take into account appropriate transmission conditions at the interface between the fine and the coarse mesh. Premilinary results in 1D shows that the accuracy of the global scheme is not hampered provided that the fine cells are small enough compared to the coarse mesh.
Our results have been presented to the peer-reviewed WAVES 2009 (Pau, June 2009) and ICTCA (Dresden, September 2009) conferences.

## 6.3. An unsplit convolutional Perfectly Matched Layer improved at grazing incidence for the seismic wave equation

**Participants:** Dimitri Komatitsch, Roland Martin, Abdelaâziz Ezziani.

The Perfectly Matched Layer (PML) absorbing technique has become popular in numerical modeling in elastic or poroelastic media because of its efficiency to absorb waves at non-grazing incidence. However, after numerical discretization, at grazing incidence large spurious oscillations are sent back from the PML into the main domain. The PML then becomes less efficient in the case of sources located close to an edge of the truncated physical domain under study, in the case of thin slices or for receivers located at large offset. In [71] we developed a PML improved at grazing incidence for the elastic wave equation based on an unsplit convolutional formulation for the seismic wave equation written as a first-order system in velocity and stress. This so-called Convolution-PML (CPML) has a cost that is similar in terms of memory storage to that of the classical PML. In [77] we introduced a similar technique for the two-dimensional Biot poroelastic equations and show its efficiency for both non dissipative and dissipative Biot porous models based on a fourth-order staggered finite-difference method used in a thin mesh slice. The results obtained are significantly improved compared to the classical PML. In [34] we applied our unsplit CPML to viscoelastic media, and in [78] we developed a variational formulation of the CPML.

## 6.4. Approximation of one-way equations

**Participants:** Hélène Barucq, Julien Diaz, Taous-Meriem Laleg-Kirati.

Seismic migration techniques used in petroleum field are based on the resolution of the wave equation. We are interested in this study in a one way formulation of this equation. The numerical resolution of this problem is difficult and requires the approximation of a Fourier Integral Operator (FIO). Computing FIO is very heavy (long time computation and big storage space). An algorithm based on a Fourier transform representation of FIO was proposed in [82] where the symbol of the FIO was approximated by separation variables functions. Although this approach reduces the computational cost, it does not give good results in heterogenous media. The objective of this study consists in developing a fast and precise algorithm for FIO computation. So first we have been interested in the application of *curvelets*, which are recent multiscale tools for data representation, analysis, and synthesis. The curvelets suggest a new form of multiscale analysis combining ideas of geometry and multiscale analysis [49], [50]. As shown in [48], curvelets provides a parsimonious representation of FIO but unfortunately they are not as good numerically as described by the theory. Then many other algorithms for FIO computation have been developed for example in [58] where a method based on the discrete symbol calculus was introduced. We have chosen to apply an algorithm proposed in [57]. Unlike the curvelets which works in the phase space, the product of the frequency and spatial spaces, this algorithm decomposes the FIO in the frequency domain. The FIO kernel is decomposed into two terms: a diffeomorphism which can be computed with a nonuniform Fast Fourier Transform FFT and a residual factor computed with a numerical separation of the spatial and frequency variables. The computational cost and the storage space in this case are sensitively reduced. A review article is under preparation.

## 6.5. Higher Order Absorbing Boundary Conditions for the Wave Equation in Discontinuous Galerkin Schemes

**Participants:** Hélène Barucq, Julien Diaz, Véronique Duprat.

The numerical simulation of waves propagation generally involves boundary conditions which both represent the behavior of waves at infinity and provide a mathematical tool to define a bounded computational domain in which a finite element method can be applied. Most of these conditions are derived from the approximation of the Dirichlet-to-Neumann operator and when they both preserve the sparsity of the finite element matrix and enforce dissipation into the system, they are called absorbing boundary conditions. Most of the approximation procedures are justified into the hyperbolic region which implies that only the propagative waves are absorbed. If the exterior boundary is localized far enough from the source field, the approximation is accurate and the absorbing boundary condition is efficient. However, the objective is to use a computational domain whose size is optimized since the solution of waves problems requires to invert matrices whose order is very large and is proportional to the distance between the source field and the exterior boundary. Hence it is a big deal to derive absorbing boundary conditions which are efficient when the exterior boundary is close to the source field and it is necessary to construct conditions which are efficient not only for propagative waves but both for evanescent and glancing waves. In a recent work [43], new conditions have been derived from the modal analysis of the wave equation set in the neighborrhood of a prolate spheroidal boundary. From the numerical analysis of the error, it has been proven [44] that these conditions are efficient for each type of waves and then, they outperform the current absorbing conditions. However, the derivation procedure in [43] is based on the representation of the Helmholtz equation in an elliptic coordinate system reproducing the geometry of the exterior boundary and it is not obvious how to generalize the conditions to an arbitrarily-shaped boundary. Recently, a new condition has been derived [65] from an approximation of the Dirichlet-to-Neumann operator which is valid both for propagative and evanescent waves and it extends the condition which was formerly proposed by Higdon [67]. By using a classical finite element scheme, Hagstrom et al. [65] have shown the improvements induced by the new condition. We have then addressed the issue of integrating the new condition into a DG scheme which is more appropriate to reproduce the propagation of waves into heterogeneous media and we have observed numerical instabilities [36]. We have next considered optimized ABCs adapted to arbitrarily-shaped regular boundaries and we have constructed a transparent condition based on the decomposition of the exact solution into a propagating field, an evanescent field and a grazing field. Then, a new condition is obtained by combining the approximations of the transparent condition in the three corresponding regions (hyperbolic, elliptic and glancing regions). It is not classical in the sense that it involves

a fractional derivative arising from the grazing part of the solution. However, the condition is easily included into a finite element scheme and we have implemented it into an Interior Penalty Discontinuous Galerkin formulation. Numerical experiments have been performed and the results have shown that it does not modify the CFL condition. Furthermore, the absorption rate is improved when compared to classical ABCs.
Recently, we have enriched the ABC by including a condition for grazing waves. The condition involves a fractional derivative of the curvilinear abscissa in the sense of Caputo. It is discretized by a quadrature formula where the weights can be neglected outside a small neighborhood of each point of the surface. The discretized condition is thus pseudo-local. A new ABC has then be constructed by applying it to the initial one. Since it is pseudo-local, it does not break down the local property of the initial condition and it does not modify the long-time behavior.
Our results have been presented to the peer-reviewed WAVES 2009 (Pau, June 2009) and ICTCA (Dresden, September 2009) conferences. A paper has been submitted

## 6.6. Simulation of seismic wave propagation in an asteroid based upon a non-blocking MPI spectral-element method

**Participants:** Dimitri Komatitsch, Roland Martin, Céline Blitz, Nicolas Le Goff, Philippe Lognonné.

In order to better understand the internal structure of asteroids orbiting in the Solar system, and then the response of such objects to impacts, seismic wave propagation in asteroid 433-Eros is performed numerically based on a spectral-element method at seismic frequencies from 0 Hz to 5 Hz. In the year 2000, the NEAR Shoemaker mission to Eros has provided images of the asteroid surface, which contains numerous fractures that likely extend to its interior. Our goal is to be able to propagate seismic waves resulting from an impact in such models. For that purpose we create and mesh both homogeneous and fractured models with a highly-dispersive regolith layer at the surface using the CUBIT mesh generator developed at Sandia National Laboratories (USA). The unstructured meshes are partitioned using the METIS software package in order to minimize edge-cuts and therefore optimize load balancing in our parallel non-blocking MPI implementation. In [76] and [23] we perform actual simulations and show that we can obtain good performance levels and good scaling when we implement overlapping of communications with calculations. Calculations were performed at CINES in Montpellier, France.

## 6.7. Numerical methods combining local time stepping and mixed hybrid elements for the terrestrial migration

**Participants:** Caroline Baldassari, Hélène Barucq, Henri Calandra [Expert Engineer, TOTAL], Bertrand Denel [Research Engineer, TOTAL], Julien Diaz.

We work on the development of a software for the Reverse Time Migration of acoustic waves which combines mixed hybrid elements in space and a local time stepping scheme. The discretization in space allows us to take the topography into account, which usually outperforms finite difference schemes. In order to illustrate this point, we have compared the solution obtained with a Dicontinuous Galerkin Finite Element Method (DGFEM) to the one obtained with finite difference methods or with the GSP method and we have shown that DGFEM gives much more accurate results. We have also compared the solution obtained with DGFEM to the one obtained with spectral element methods, which requires the use of quadrilateral (in 2D) of hexaedral (in 3D) cells and is therefore less convenient to deal with a topography. Both solution present the same order of accuracy, which encourage us to use DGFEM to deal with a topography.

Near the topography (or in very thin layers), the cells of the mesh have to be very small compared to the cells far from the topography. In such configuration, it is not useful to user high order elements in the whole domain and we propose to use second order elements in the fine part of the mesh and high order elements in the coarse part. The numerical experiments we have performed show actually that there is no difference between the solutions obtained with meshes composed only of high-order cells and with meshes composed of second order cells near the topography. Moreover the use of second order cells reduces dramatically the computational burden.

Since we use explicit time-schemes such as Leap-Frog scheme (for second order time-discretization) or Modified Equation scheme (for higher-order time discretization), the time step is contrained by the so-called CFL (Courant-Friedrichs-Lewy) condition, which is dictated by the smallest cell of the mesh. If the mesh contains both very small and very large cells, it is necessary to use a local-time stepping strategy as the one proposed by [27]. Moreover, if the mesh contains cells of various order, it is useful to adapt the order of the time-discretization to the space discrization. We have then extended the local time-stepping method to handle different order of time-discretization, and we have tested the new method in 1D and in 2D. The 1D tests showed that this method does not hamper the accuracy of the space discretization while it allows to decrease the computational burden. The 2D experiments showed that this method could be implemented in the Reverse Time Migration algorithm. The results of the year have been presented in 4 peer-reveiewed conferences which are listed below.

## 6.8. Geological horizon propagation into large seismic volumes

**Participants:** Jonathan Gallon, Bruno Jobard, Hélène Barucq.

In the petroleum industry, seismic data is the starting point of many processes for the exploration for oil reservoirs. The dimensions of these *seismic cubes* keep increasing as the regions undergoing exploration keep growing and have increased spatial resolution. We are interested in finding efficient algorithms of *geological horizon propagation* that can handle large 3D and 4D (pre-stack) seismic volumes. Such horizons are used for constructing structural sub-surface models. In order to process larger volumes, our strategy consists in to partition them and simultaneously propagate an initial horizon seed in adjacent sub-blocs. To validate this approach, we take special care to ensure that the individual surfaces connect at the bloc limits and that the overall quality is close to the single-threaded propagation. Further performance improvements are planned by organizing the data along a special layout on disk and using cache hierarchies.

## 6.9. Discontinuous Galerkin methods with plane waves basis for Helmholtz's equation in 3D

**Participants:** Mohamed Amara, Rabia Djellouli, Magdalena Grigoroscuta.

We have designed a new mixed-hybrid-type solution methodology to be applied for solving high-frequency Helmholtz problems. The proposed approach distinguishes itself from similar methods by a local approximation of the solution with oscillated finite elements polynomials satisfying the wave equation. A weak continuity of the solution across the element interfaces is enforced using Lagrange multipliers. Note that the discontinuous nature of the approximation at the element-level allows to apply static condensation of primal variable prior to assembly. Therefore, the computational cost of the proposed method is reduced, and is mainly dependent on the total number of Lagrange multipliers degrees of freedom, and by the sparsity pattern of the resulting matrix. Hence, the proposed approach combines the features of standard Galerkin finite elements techniques in terms of implementation complexities, and the oscillating aspect of the shape functions needed for approximating waves in the high frequency regime. Preliminary numerical results obtained in the case of two-dimensional wave guides and scattering problems using lower order element (OP41, OP82) clearly illustrate the computational efficiency of the proposed solution methodology. We have also analyzed mathematically the convergence of the proposed method in the case of OP41 element and have established a priori and a posteriori error estimates. We propose here a new discontinuous Galerkin formulation, based on a local approximation of the solution by plane waves that satisfy the wave equation. In order to enforce a weak continuity across the element interfaces, we introduce Lagrange multipliers. The method is built in a variational formulation framework that leads to a linear system associated with a positive definite Hermitian matrix. This matrix results from using a stabilized-like technique. Therefore, we use a preconditioned conjugate gradient algorithm to solve the system without computing the resulting matrix. We have recently applied a regularized-type procedure to the proposed method to address the loss of the stability due to the violation of the inf/sup condition in the case of higher order elements. Consequently the modified mixed hybrid formulation leads to a linear system associated with a positive definite Hermitian matrix. The results are available in a Research Report [38] and they have been presented at the two peer-reviewed conferences Waves 2009 and COMPDYN 2009.

## 6.10. Improved numerical approaches for seismic wave equation at a fluid-solid interface in the oil industry

**Participants:** Ronan Madec, Dimitri Komatitsch, Julien Diaz, Pierre Thore.

We have studied in [29] how to implement plane wave sources with any incidence angle for the spectral-element time-domain method. We have also implemented absorbing boundary conditions following the ideas of [46], which is crucial to ensure that no significant spurious waves propagate back into the main domain. We have validated the method and checked its accuracy for the numerical modeling of seismic wave propagation, in particular in order to compute the response of a free surface with topography under the incidence of a plane wave with different angles by comparing the results obtained to the Method of Fundamental Solutions (MFS), which is a new method to solve the wave equation in the frequency domain.

In the oil industry, many important oil reservoirs are located offshore and it is therefore of interest to be able to simulate seismic wave propagation in deep offshore geological media and it is crucial to reduce the cost of the calculations in the thick but homogeneous and therefore simple water layer. Explicit numerical methods used to model wave propagation must satisfy a stability condition called the Courant-Friedrichs-Lewy condition (CFL) that depends on the maximum velocity of the medium and on the ratio between the time step and the size of a grid cell. In the case of a deep offshore model, the velocity of P waves in the thick homogeneous water layer located upon the ocean bottom is generally slower than in the solid part of the model located underneath the ocean flow and therefore the local CFL is different within each medium. But to ensure numerical stability, one has to take a small global time step which is imposed by the solid part of the medium, and therefore waste some calculation time in the simpler fluid part of the model. An idea that can be used to overcome this is to implement substepping in time to reduce the computation time by taking a local time step and still honor the CFL condition by increasing the time step in the layer of slower velocity (the water layer). In [33] we therefore implemented in a spectral-element method an idea developed in [61] that ensures the conservation of energy along the interface.

## 6.11. New absorbing boundary conditions for the numerical simulation of the scattering by elongated obstacles

**Participants:** Hélène Barucq, Rabia Djellouli, Anne-Gaëlle Saint-Guirons.

Recently, a new class of absorbing boundary conditions called local approximate DtN absorbing boundary conditions (DtN) has been proposed to be applied on exterior artificial prolate spheroidal-shaped boundaries [21]. Unlike the standard approximate local DtN boundary conditions that are restricted to circular- or spherical-shaped boundaries (see [64], [66]), the proposed conditions are applicable to exterior elliptical- or prolate spheroidal-shaped boundaries that are more suitable for surrounding elongated scatterers because they yield to smaller computational domains. These absorbing boundary conditions are designed to be exact for the first modes. They can be easily incorporated in any finite element parallel code while preserving the local structure of the algebraic system. Moreover, the analysis of the performance of these conditions in the *low* frequency regime, when using an On-Surface radiation condition formulation [74], revealed that these conditions are very accurate regardless of the slenderness of the boundary [21], [85]. In addition, it has been demonstrated that the *second-order* local DtN condition (DtN2) outperforms the widely-used second-order absorbing boundary conditions (BGT2) [45] when expressed in prolate spheroidal coordinates [84], [83].

We have extended the investigation of the performance of the local approximate DtN2 absorbing boundary condition to the case of the *high* frequency regime. More specifically, we have performed an analytical and numerical study to assess the effect of the slenderness of the exterior boundary on the accuracy and the efficiency of the proposed absorbing boundary condition. We have conducted this study using a domain-based formulation, that is the artificial boundary is located at some distance from the surface of the scatterer, since the OSRC approach is not adapted for such an analysis, as previously observed in [42]. We have proven that, in the high frequency regime, the reflected waves at the artificial boundary decay faster than $1/(ka)^{15/8}$ where $k$ is the wavenumber and $a$ is the semi-major axis of this boundary. Numerical results illustrate the accuracy

and the efficiency of the proposed absorbing boundary condition when used for solving acoustic scattering problems in domain-based formulation.
The results of the year are available in a Research Report [40] and they have been presented in 2 peer-reviewed conferences Waves 2009 and Enumath 2009. This work has also been accepted for a regular lecture at the National Congress in Numerical Analysis (CANUM 2009).

## 6.12. High Performance Computing for the seismic wave equation at very high resolution

**Participants:** Dimitri Komatitsch, Roland Martin, Pieyre Le Loher.

With the very rapid evolution of personal computers, computer clusters, and supercomputers, nowadays the seismic wave equation can be solved with very good accuracy using very precise techniques implemented based on parallel computing in the context of so-called High-Performance Computing (HPC). This has been a central part of our research activity in the last few years and increasingly more in 2008 and 2009. In particular with some colleagues from the CINES supercomputing center in Montpellier (France) we have performed some very large scale calculations that are currently being published.

Using the high-order finite-element method implemented in our SPECFEM3D software package [13], we for instance studied the influence of topography modeled at very high resolution on seismic wave propagation in the region of Taipei in Taiwan [75] [31], [32].

We also applied the technique to model seismic wave propagation at very high frequency in the whole Earth [70], [86]. In November 2008 our SPECFEM3D software package was again among the six finalists of the pretigious Gordon Bell Prize of the SuperComputing'2008 conference in the USA [51] for a calculation performed in parallel on 150,000 processor cores, reaching a sustained performance level of 0.16 petaflops. And in June 2009 Dimitri Komatitsch won the third BULL Joseph Fourier Prize with it.

In the context of a collaboration with Gordon Erlebacher from Florida State University (USA) who visited us for a month in May-June 2008 we ported our modeling algorithm to a NVIDIA graphics video card (Graphical Processing Units – GPU) using the CUDA language on top of a C implementation of our code. This technique is known as General-purpose Processing on Graphical Processing Units (GPGPU) and had never been used before for a high-order finite-element technique, which induces significant technical problems in particular regarding memory accesses. In [30] we used it to improve the speed of our code by a factor of 25.

Finally, in [62] we used a finite-element code to model stress redistribution in Island following a large earthquake that occurred there in June 2000 and study how changes in the stress field could have had an influence on the triggering of a second earthquake that occurred in the same region a few days later.

## 6.13. Perfectly Matched Layers for the Shallow Water equations

**Participants:** Hélène Barucq, Julien Diaz, Roland Martin, Carlos Couder, Mounir Tlemcani [Assistant Professor, University of Oran, Algeria].

In the past few years ago, substantial progress has been made in the development of the PML technique for the Euler equations, starting with the studies for cases with constant mean flows, followed by extensions to cases with non-uniform mean flows. Most recently, applications of PML to linearized Navier-Stokes equations and non-linear Navier-Stokes equations have been discussed in [69], [68], [79]. Although the PML technique itself is relatively simple when it is viewed as a complex change of variables in the frequency domain, it is important to note that, for the PML technique to yield stable absorbing boundary conditions, the phase and group velocities of the physical waves supported by the governing equations must be consistent and in the same direction.

For applications of full non-linear Navier-Stokes equations, we use an optimized perfectly matched layer (PML) technique that has proven to be efficient in elastodynamics to absorb surface waves as well as body waves with non grazing incidence [71]. It is possible to use this unsplit convolutional PML (CPML) for staggered and finite difference integration schemes to improve the computational efficiency reducing the number of computational arrays and therefore the memory storage for the flux domain. We applied this first to non-linear Shallow water equations in presence or not of Coriolis forces and friction forces, and also to subsonic and supersonic directed flows for industrial contexts of interest.

Possible applications of the Shallow water equations with PML are for massive avalanches like turbidites, pollutant contaminants travelling in coastal regions (Gulf of Mexico) with many chemical species, avalanches and tsunamis, ocean-earth interactions etc.... We restricted successfully our results obtained for Navier-Stokes to directed chanellized shallow water flows.

During this year, we studied air critical ejector diffuser simulations. The ejectors are commonly used to extract gases in the petroleum industry where it is not possible to use an electric bomb due the explosion risk because the gases are flammable. In this work we develop a numerical code to investigate the unsteady supersonic flow in the ejector diffuser to have an efficient tool that allows modeling different diffusers designs. The model is developed using curvilinear coordinates transformation to adapt the ejector design to a regular computational plane where a finite differences scheme could be applied, and for control the outflow conditions we use a convolutional perfectly matched layer (CPML) technique. The CPML has demonstrated its convenience because the time evolutions of damping mechanisms do not need to be split and only the space derivatives of velocities need to be stored at each time step reducing the number of computational array used in the numerical code. In this context of a directed flow, the results obtained shows that the perfectly matched layer (CPML) technique can absorb efficiently the out-going supersonic flux at the outlet condition and absorbing zone is reflection less.

In the same time, we have worked on the stabilization of a PML for the linearized Shallow Water equations.Indeed, the PML technique induces some instabilities when applied to Euler equations [69] or to shallow water equations. Much works have been devoted to the stabilization of the PML for linearized Euler equations [68], [79] and we have developed a stable PML for the linearized shallow water equations with a Coriolis term and a uniform mean flow. Our method follows an idea proposed by Nataf [79] for linearized Euler equations and is based on the splitting of the vorticity waves from the advective and entropy waves. Then since the propagation of vorticity and entropy waves is governed by a classical transport equation, these waves can be easily absorbed by a transparent and exact condition at the end of the layer and the PML condition is applied to the advective waves only. We have used a transformation proposed by Hu [68] before applying a classical PML technique to avoid an exponential growth of the waves and numerical experiments [20] show both the accuracy of the condition and its long-time stability. The main results have been presented at the peer-reviewed conference Waves 2009.

## 6.14. Analytical computation of the Green's function of bilayered porous media

**Participants:** Julien Diaz, Abdelaâziz Ezziani, Nicolas Le Goff.

The Cagniard-de Hoop is particularly well known in the physics and engineering communities for calculating analytical solutions of time-dependent wave propagation problems in stratified media, especially in seismology. However, it had never been applied to the wave propagation in heterogeneous porous media. The computation of analytical solutions in this kind of media is particularly interesting to obtain reference solutions for the validation of numerical codes, but it is also useful for a for a better understanding of the reflexion/transmission properties of the media. Using the Cagniard-de Hoop technique, we have provided the solution to the problem of wave propagation in bilayered acoustic/poroelastic [25], [26] and poroelastic/poroelastic [59], [60] media both in two and three dimensions. We have implemented these solutions in two computational codes, Gar6more2D (for the 2D problem) and Gar6more3D (for the 3D problem).

# 7. Contracts and Grants with Industry

## 7.1. Contracts with TOTAL

- Modélisation et simulation numérique pour la migration terrestre par équations d'ondes tridimensionnelles.

  Period: 2007 January - 2009 december, Management: INRIA Bordeaux Sud-Ouest, Amount: 145000 euros.
- Résolution de l'équation d'Helmholtz 3D par une méthode de Galerkin discontinue DGM utilisant des bases d'ondes planes.

  Period: 2007 January - 2009 december, Management: INRIA Bordeaux Sud-Ouest, Amount: 139000 euros.
- Propagation automatique de Surface nD Filtrage et traitement de la sismique avant stack Period: 2008 January - 2010 december, Management: INRIA Bordeaux Sud-Ouest, Amount: 45000 euros.
- Analyse méthodologique pour la génération de maillages irréguliers et de leur décomposition en sous-domaines sur calculateurs parallèles pour la propagation d'ondes sismiques en milieu géologiques complexes Period: 2009 May - 2010 April, Management: INRIA Bordeaux Sud-Ouest, Amount: 50000 euros

## 7.2. Contract with CSUN

In the context of the Associate Team MAGIC.

Period: 2009 January - 2011 December, Total Amount: 15000 USD

# 8. Other Grants and Activities

## 8.1. Depth Imaging Partnership

MAGIQUE-3D maintains active collaborations with TOTAL. In the context of depth imaging and with the collaboration of Henri Calandra from TOTAL, MAGIQUE-3D coordinates research activities dealing with the development of high-performance numerical methods for solving wave equations in complex media. This project involves French academic researchers in mathematics, computing and in geophysics, and is funded by TOTAL. At the end of 2009, two PhD students working in MAGIQUE-3Dfrom 2007, have defended their PhD dealing with new numerical imaging methods that are based on the solution of the full wave equation. Two Ph.D students advised by J. Roman and S. Petiton respectively started to work in january 2008 on computing aspects for optimizing the computational performances of our numerical methods.

The different partners have to work jointly on topics dealing with solving wave equations. A kick-off meeting will be organized in january 2010 to consider new subjects proposed by different INRIA teams.

To our knowledge, this network is the first in the French research community to establish links between industrial and academic researchers in the context of a long-term research program managed by an INRIA team.

## 8.2. ANR Project AHPI

The endeavour of this project is to develop some methodology for modelling and solving certain inverse problems using tools from harmonic and complex analysis. These problems pertain to deconvolution issues, identification of fractal dimension for Gaussian fields, and free boundary problems for propagation and diffusion phenomena. The target applications concern radar detection, clinical investigation of the human body (e.g. to diagnose osteoporosis from X-rays or epileptic foci from electro/magneto encephalography), seismology, and the computation of free boundaries of plasmas subject to magnetic confinement in a tokamak. Such applications share as a common feature that they can be modeled through measurements of some transform (Fourier, Fourier-Wigner, Riesz) of an initial signal. Its non-local character generates various uncertainty principles that make all of these problems ill-posed. The techniques of harmonic analysis, as developed in each case below, form the thread and the mathematical core of the proposal. They are intended, by and large, to regularize the inverse issues under consideration and to set up constructive algorithms on structured models. These should be used to initialize numerical techniques based on optimization, which are more flexible for modelling but computationally heavy and whose convergence often require a good initial guess. In this context, the development of wavelet analysis in electrical engineering, as well as signal and image processing or singularity detection, during the last twenty years, may serve as an example. However, many other aspects of Fourier analysis are at work in various scientific fields. We believe there is a strong need to develop this interaction that will enrich both Fourier analysis itself and its fields of application, all the more than in France the scientific communities may be more separate than in some other countries.

The project was created in july 2007. A first meeting took place to Pau in october 2007 and a second one to Orléans in september 2008. Collaborations have began with the Bordeaux team on the use of bandelet formalism for the seismic inversion and a post-doc, hired in october 2008, had in charge to analyze with us the feasibility of this apporach. We have worked on the approximation of seismic propagators involving Fourier integral operators by considering different approaches. This work has led to a review article that should be submitted soon. Moreover the post-doc has been hired as an INRIA research fellow in september 2009.

## 8.3. International collaborations

### *8.3.1. Visits*

- Dimitri Komatitsch and Roland Martin spent one month at the Barcelona Supercomputing Center in May-June 2009;
- Mounir Tlemcani spent one month in may 2008 in MAGIQUE-3D.
- Magdalena Grigoroscuta spent two months at CSUN (California State University at Northridge in april-may 2009.
- Chokri Bekkey spent two weeks in MAGIQUE-3Din june 2009.
- Rabia Djellouli spent 6 months in MAGIQUE-3Dfrom july to december 2009.
- Keddour Lemrabet (Professor, Alger University, Algeria) spent two months in MAGIQUE-3Din november and december 2009.

### *8.3.2. Associate team MAGIC*

Since january 2006, the team is associated to a team located at CSUN (California State University at Northridge) which is managed by R. Djellouli. Our common programm research takes part of the activities we develop in modeling essentially.

### *8.3.3. New collaborations*

- A collaboration with Erwan Faou (CR, Inria Rennes) started on the improvement of local time-stepping method for the wave equation.
- A new collaboration has began with David Pardo (Basque Center of Applied Mathematics, Bilbao, Spain). We work on a joint program concerning discontinuous approximations of wave equations. We applied to the program ARC in 2009 and submitted a proposal entitled "‘ARGIA: AbsoRbing conditions involving asymptotIc Analysis"’
- We have initiated a collaboration with S. Tordeux on the analysis of Absorbing Boundary Conditions and.Perfectly Matched Layers for wave equations. By applying asymptotic analysis, we would like to get information on the possible relation between ABCs and PMLs. By this way, ABCs could be optimized in the same sense than PMLs. We have included this work in the research program of ARGIA.

# 9. Dissemination

## 9.1. Scientific animation

- We organized the 9th international congress on Mathematical and Numerical Aspects of Waves Propagation, the so-called WAVES09 conference which was held from 15 till 19 June 2009 in the Palais Beaumont in Pau. Waves 2009 gathered more than 250 participants coming from more than 30 different countries.

## 9.2. Teaching

### *9.2.1. Lecture*

- Lecture/course to Master students (64 hours) at University of Pau, France, on "Calcul Parallèle et Modélisation en Géophysique" ("Parallel computing and geophysical modeling")
- Lecture/course to Master students (46 hours) at University of Pau, France, on "Propagation d’ondes et imagerie" ("Waves propagation and Imaging")

## 9.3. Participation in Conferences, Workshops and Seminar

Cyril Agut

- C. Agut, J. Diaz and A. Ezziani *High-order method in space and time for solving the wave equation*, WAVES 2009 (Pau, France, 15-19 june 2009, http://waves-2009.bordeaux.inria.fr/index.php?lang=en)
- C. Agut, J. Diaz and A. Ezziani *Fast high-order method for solving the acoustic wave equation in heterogeneous media*, ICTCA 2009 (Dresden, Germany, 7-11 september 2009, http://www.ictca2009.com/index.html)

Hélène Barucq

- H. Barucq, H. Calandra, B. Denel, C. Baldassari and J. Diaz *Numerical simulation of waves propagation applied to seismic imaging*,Invited talk in the context of the CMFT1 - Premier colloque franco-tunisien, 16-20 March 2009, Djerba, Tunisia (http://smf.emath.fr/VieSociete/Rencontres/France-Tunisie-2009)
- H. Barucq, C. Baldassari, H. Calandra, J. Diaz, *Finite Element Methods for the Reverse Time Migration*, Rencontres Industriels-Chercheurs, December, 15, organized by"’ le Pôle AVENIA et INNOVALIS Aquitaine"’.

Caroline Baldassari

- H. Barucq, H. Calandra, B. Denel, C. Baldassari and J. Diaz *The Reverse Time Migration technique coupled with interior Penalty Discontinuous Galerkin Method* , European Geosciences Union, EGU, 20th 25th April 2009, Vienna (Austria) , (http://meetings.copernicus.org/egu2009/index.html)
- *Using the Interior Penalty Discontinuous Galerkin method for the Reverse Time Migration* , 9th International conference on mathematical and numerical aspects of waves, WAVES'09, 15th 19th June 2009, Pau (France), (http://waves-2009.bordeaux.inria.fr)
- H. Barucq, H. Calandra, B. Denel, C. Baldassari and J. Diaz *Comparison of the Performance of Spectral Finite Elements and Interior Penalty Discontinous Galerkin Methods for the Reverse Time Migration* , International Conference on Spectral and High Order Methods, ICOSAHOM09, 22th 26th June 2009, Trondheim (Norway), (http://www.math.ntnu.no/icosahom)

Julien Diaz

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- H. Barucq, C. Baldassari and J. Diaz *Une méthode de Galerkine discontinue couplée à la technique de Reverse Time Migration*, Séminaire d'Analyse Numérique de l'IRMAR, Université de Rennes, 22 janvier 2009.
- H. Barucq, C. Baldassari and J. Diaz *An appropriate space discretization scheme for the Reverse Time Migration*, Seminar für Analysis und Numerik, Université de Bâle, Suisse, 25 septembre 2009.
- H. Barucq, C. Baldassari and J. Diaz *Space and time discretization of the wave equation for the reverse time migration* , Séminaire d'Analyse Numérique de l'EPFL, Lausanne, Suisse, 2 décembre 2009.

Véronique Duprat

- H. Barucq, J. Diaz and V. Duprat *IPDG formulation of the acoustic wave equation incorporating absorbing boundary conditions*, WAVES 2009 (Pau, France, 15-19 June 2009, http://waves-2009.bordeaux.inria.fr/)
- H. Barucq, J. Diaz and V. Duprat *New absorbing boundary conditions for the acoustic wave equation approximated by an IPDG formulation*, ICTCA 2009 (Dresden, Germany, 7-11 September 2009, http://www.ictca2009.com/)

Magdalena Grigoroscuta

- M. Grigoroscuta-Strugaru, M. Amara, H. Calandra and R. Djellouli *A modified discontinuous Galerkin method for Helmholtz problems*, Waves 2009, Pau, 15th-19th june 2009, http://waves-2009.bordeaux.inria.fr/
- M. Grigoroscuta-Strugaru, M. Amara, H. Calandra and R. Djellouli *On a modified discontinuous Galerkin method for solving efficiently Helmholtz problems*, COMPDYN 2009, Rhodes, 22nd-24th june 2009, http://www.compdyn2009.org/

Taous-Meriem LALEG-KIRATI

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Anne-Gaëlle Saint-Guirons

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- H. Barucq, R. Djellouli and A.-G. Saint-Guirons *Performance assesment of new local DtN ABC for prolate spheroid boundaries in the high frequency regime*, 9th International conference on mathematical and numerical aspects of waves, WAVES'09, 15th 19th June 2009, Pau (France), (http://waves-2009.bordeaux.inria.fr)

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