INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Mathfi

Financial Mathematics

Paris - Rocquencourt

Theme : Stochastic Methods and Models

Activity Report

2009

Table of contents

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2. Overall Objectives

2.1. Highlights of the year

Concerning research, a lot of effort has been put again this year in the study of Lévy processes in financial modelling.

The project Credinext, supported by Pôle Finance Innovation has started.

2.2. Introduction

MathFi is a joint INRIA project-team with ENPC (CERMICS) and the University Paris-est Marne la Vallée, located in Rocquencourt and Marne la Vallée.

The development of increasingly complex financial products requires the use of advanced stochastic and numerical analysis techniques. The scientific skills of the MathFi research team are focused on probabilistic and deterministic numerical methods and their implementation, stochastic analysis, stochastic control. Main applications concern evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, calibration of financial models, risk management. Special attention is paid to models with jumps, stochastic volatility models, asymmetry of information. The MathFi project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. Premia web site: http://www.premia.fr .

3. Scientific Foundations

3.1. Simulation of Stochastic Differential Equations

Participants: B. Jourdain, A. Alfonsi, D. Lamberton, M. Sbai.

Most financial models are described by SDEs. Except in very special cases, no closed-form solution is available for such equations and one has to approximate the solution via time-discretization schemes in order to compute options prices and hedges by Monte Carlo simulations. Usually this is done by using the standard explicit Euler scheme since schemes with higher order of strong convergence involve multiple stochastic integrals which are difficult to simulate. In addition, the weak order of convergence of the explicit Euler scheme can be improved by using Romberg-Richardson's extrapolations. Nethertheless, some schemes with weak order of convergence two or more have been designed recently. The idea is either to replace the multiple Brownian integrals by discrete random variables which share their moments up to a given order or to integrate Ordinary Differential Equations associated with the vector fields giving the coefficients of the Stochastic Differential Equation up to well-chosen random time-horizons. Another interesting new direction of investigation is the design of exact simulation schemes.

Three directions of research have been investigated in the Mathfi project. First, fine properties of the Euler scheme have been studied [72], [6], [74]. Secondly, concerning SDEs for which the Euler scheme is not feasible, A. Alfonsi and V. Lemaire [1] have proposed and analysed new schemes respectively for Cox-Ingersoll-Ross processes and for equations with locally but not globally Lipschitz continuous coefficients. Last, the team has contributed to the new directions of research described above. For CIR processes, A. Alfonsi has designed a scheme with weak order two even for large values of the volatility parameter. Adapting exact simulation ideas, B. Jourdain and M. Sbai [7] have proposed an unbiased Monte Carlo estimator for the price of arithmetic average Asian options in the Black-Scholes model.

3.2. Numerical methods for option pricing and hedging and model calibration

Participants: B. Jourdain, A. Alfonsi, D. Lamberton, M. Sbai, V. Bally, B. Lapeyre, A. Sulem, A. Kebaier, C. Labart, J. Lelong, D. Pommier, L. Abbas-Turki, A. Ahida, A. Zanette, E. Dia.

Efficient computations of prices and hedges for derivative products is a major issue for financial institutions (see [76]) .

This is done by using either Monte-Carlo methods or partial differential equations techniques. Monte-Carlo simulations are widely used because of their implementation simplicity and because closed formulas are usually not available. Speeding up the algorithms is a constant preoccupation in the developement of Monte-Carlo simulations. The team is mainly concerned with *adaptive versions* which improve the Monte-Carlo estimator by relying only on stochastic simulations.

The team has also been active on numerical methods in models with jumps and large dimensional problems.

This activity in the MathFi team is strongly related to the development of the Premia software.

Model calibration: The modeling of the so called *implied volatility smile* which indicates that the Black-Scholes model with constant volatility does not provide a satisfactory explanation of the prices observed in the market has led to the appearance of a large variety of extensions of this model as the local volatility models (where the stock price volatility is a deterministic function of price level and time), stochastic volatility models, models with jump, and so on. An essential step in using any such approach is the *model calibration*, that is, the reconstruction of model parameters from the prices of traded options. This is an inverse problem to that of option pricing and as such, typically ill-posed.

The calibration problem is yet more complex in the interest rate markets since in this case the empirical data that can be used includes a wider variety of financial products from standard obligations to swaptions (options on swaps). The underlying model may belong to the class of short rate models like Hull-White [73], [66], CIR [69], Vasicek [84] etc. or to the popular class of LIBOR (London Interbank Offered Rates) market models like BGM [67].

The choice of a particular model depends on the financial products available for calibration as well as on the problems in which the result of the calibration will be used.

The calibration problem is of particular interest for MathFi project because due to its high numerical complexity, it is one of the domains of mathematical finance where efficient computational algorithms are most needed.

3.3. Malliavin calculus and applications in finance

Participants: V. Bally, A. Kohatsu-Higa, A. Sulem, A. Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [77]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [79], D. Bell [65] D. Ocone [81], B. Øksendal [85]) give expositions of the subject. See also V. Bally [64] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [80].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [78], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods. We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be "smooth" in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequenses of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M}E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [71] and [70] and the papers by Bally et al., Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [75].

L. Caramellino, A. Zanette and V. Bally have been concerned with the computation of conditional expectations using Integration by Parts formulas and applications to the numerical computation of the price and the Greeks (sensitivities) of American or Bermudean options. The aim of this research was to extend a paper of Reigner and Lions who treated the problem in dimension one to higher dimension - which represent the real challenge in this field. Significant results have been obtained up to dimension 5 [5] and the coresponding algorithms have been implemented in the Premia software.

Moreover, there is an increasing interest in considering jump components in the financial models, especially motivated by calibration reasons. Algorithms based on the integration by parts formulas have been developped in order to compute Greeks for options with discontinuous payoff (e.g. digital options). Several papers and two theses (M. Messaoud and M. Bavouzet defended in 2006) have been published on this topic and the corresponding algorithms have been implemented in Premia. Malliavin Calculus for jump type diffusions - and more general for random variables with localy smooth law - represents a promising field of research, also for applications to credit risk problems.

More recently the Malliavin calculus has been used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P.Imkeller in [75]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [75]).

A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. These SDEs can be interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals, as introduced by Russo and Valois [83]. This methodology has been applied for utility maximization with insiders.

3.4. Optimal stopping

Participants: A. Alfonsi, B. Jourdain, D. Lamberton.

The theory of American option pricing has been an incite for a number of research articles about optimal stopping. Our recent contributions in this field concern optimal stopping for one dimensional diffusions and American options in exponential Lévy models.

In the context of general one-dimensional diffusions, we have studied optimal stopping problems with bounded measurable payoff functions. We have obtained results on the continuity of the value function and its characterization as the unique solution of a variational inequality in the sense of distributions, both in finite and infinite horizon problems (collaboration between D. Lamberton and Michail Zervos, London School of Economics).

We have explained how to calibrate a continuous and time-homogeneous local volatility function from the prices of perpetual American Call and Put options (A. Alfonsi and B. Jourdain).

The use of jump diffusions in financial models goes back to Merton (1976). More recently, there has been a growing interest for more sophisticated models, involving Lévy processes with no diffusion part and infinite activity (see, in particular, papers by Carr, Geman, Madan and Yor). One of our PhD students (M. Mikou) works on the qualitative properties of American option prices in exponential Lévy models. A number of results on the exercice boundary and on the so called smooth fit property have been established.

3.5. Stochastic Control and Backward Stochastic Differential equations (BSDEs)

Participants: V. Bally, J.-Ph. Chancelier, M.C. Kammerer-Quenez, A. Sulem.

B. Øksendal (Oslo University) and A.Sulem have written a book on Stochastic control of Jump diffusions [11]). The types of control problems covered include classical stochastic control, optimal stopping, impulse control and singular control. Both the dynamic programming method and the maximum principle method are discussed, as well as the relation between them. Corresponding verification theorems involving the Hamilton-Jacobi Bellman equation and/or (quasi-)variational inequalities are formulated. There are also chapters on the viscosity solution formulation and numerical methods. In the second edition (2007), a chapter on optimal control of stochastic partial differential equations driven by Lévy processes and a section on optimal stopping with delayed information have been added. Applications to portfolio optimization problems and insurance problems have been studied.

In the context of risk measures, M.C. Quenez (assistant professor at UPEMLV until 2007, now Prof Paris VII) has shown how some dynamic measures of risk can be induced by Backward Stochastic Differential Equations and A. Sulem and B. Øksendal in [35] have studied risk-indifference pricing in incomplete markets with jumps using stochastic control theory and PDE methods.

4. Application Domains

4.1. Application domains

- Option pricing and hedging
- Calibration of financial models
- Portfolio optimization
- Risk management
- Insurance-reinsurance optimization policy
- Insider modeling, asymmetry of information

5. Software

5.1. Development of the software PREMIA for financial option computations

Participants: A. Alfonsi, V. Bally, J-Ph. Chancelier, A. De la Vaissière, B. Jourdain, A. Kebaier, O Kudryavtsev, I. Laachir, C. Labart, A. Kolotaev, B. Lapeyre, J. Lelong, D. Pommier, A. Sulem, X. Wei, A. Zanette, V. Zherder.

Figure 1.

5.1.1. Description of Premia

PREMIA is a platform dedicated to the development of algorithms and scientific documentation for *option pricing*, *hedging* and *model calibration* (http://www.premia.fr).

This project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is:

1. to be a powerful testing platform for comparing different numerical methods between each other;
2. to build a link between professional financial teams and academic researchers;
3. to provide a useful teaching support for Master and PhD students in mathematical finance.

The development of Premia started in 1999 and 11 are released up to now and registered at the APP agency.

5.1.2. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. Equity derivatives:

   The following models are considered:

   Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods Malliavin calculus based methods.

Dynamic Hedging for Black-Scholes and jump models is available.

Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. Interest rate derivatives

   The following models are considered:

   HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

   Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. Credit derivatives: CDS, CDO

   Reduced form models and copula models are considered.

   Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. Hybrid products:

   PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. Energy derivatives: swing options

   Mean reverting and jump models ar considered.

   Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

5.1.3. Latest features

Premia 11 has been delivered to the consortium members in February 2009. This year we have developped and implemented new algorithms for pricing equity derivatives in stochastic volatility models and models with jumps, further developped routines for pricing American options in the LIBOR interest rate market and for credit derivatives with dynamic models. J. Lelong has performed modification of the testing procedure to run on parallel architectures.

New algorithms for the release 12 of Premia to be delivered in March 2010 to the Consortium:

- **Interest Rate Derivatives**
  - A stochastic volatility Libor model and its robust calibration *Working paper Belomestny Mathew Schoenmakers (2007)*
  - True upper bounds for Bermudean products via Non-Nested Monte Carlo. D. Belostomeny, C. Bender, J. Schoenmakers. *Mathematical Finance Volume 191 January 2009*
  - Pricing and calibration in HW2D model.
- **Credit Risk Derivatives**(A. Kebaier, C. Labart, J. Lelong)
  - Default Contagion in Large Homogeneous Portfolios *Alexander Herbertsson, No 272, Working Papers in Economics from Göteborg University, Department of Economics*
  - Advanced credit portfolio modeling and CDO pricing. *Eberlein R.Frey E. A. von Hammerstein in Mathematics: Key Technology for the Future, W. Jager, and H.-J. Krebs, (Eds.), Springer (2008), p.p. 253-280*

- Hedging default risks of CDOs in Markovian contagion models *J.-P. Laurent, A. Cousin, J-D. Fermanian* Dynamic hedging of synthetic CDO-tranches with spread-and contagion risk *Frey, R. and Backhaus, J., preprint, department of mathematics, Universitat Leipzig*
- Pricing Credit from the top down with affine point processes, *Errais, Giesecke, Goldberg (2006).*
- A.Alfonsi J.Lelong A Closed-form extension to Black-Cox formula.

- **Insurance**
  - A bivariate model for evaluating fair premiums of equity-linked policies with maturity guarantee and surrender option. Costabile, M., Gaudenzi, M., Massabo, I., Zanette, A. 2009 *Insurance: Mathematics and Economics 45 (2), pp. 286-295*
- **Equity**
  - A. Achdou, T. Arnarson and D. Pommier: Calibration of American options in Levy models. Finite Difference. Option Pricing Using Fourier Transforms: A Numerically Efficient Simplification. *M. Attari*
  - D. Pommier: Sparse grid pricing of basket options.
  - T. Lelièvre and D. Pommier: Low rank approximation method for basket options.
  - E. Dia and D. Lamberton: Montecarlo methods for infinite activity Levy models for Lookback options.
  - Pricing Variance Swap: Pricing options on realized variance in the Heston model with jumps in returns and volatility *Artur Sepp: Journal of Computational Finance, Volume 11 / Number 4, Fall 2008*
  - O. Kudryavtsev: Pricing in Regime Switching models.
  - A Multinomial Approximation for American Option Price In Levy Process Models(Maller-Solomon-Szymaier Mathematical Finance October 2006 - Vol. 16 Issue 4 Page 589-694)
  - A generalization of the Hull and White formula with applications to option pricing approximation *E. Alos: Finance and Stochastics 10 (3), 2006, p. 353-365*
  - Polynomial Processes and their applications to mathematical Finance.*C.Cuchiero, M. Keller-Ressel, J.Teichmann: preprint arXiv/0812.4740*
  - Adaptive control variates for pricing multi-dimensional American options *Samuel M. T. Ehrlichman and Shane G. Henderson Journal of Computational Finance, Volume 11 / Number 1, Fall 2007*
  - A Tree-based Method to price American Options in the Heston Model, Vellekoop, M.H. and Nieuwenhuis, J.W. *to appear Journal of Computational Finance*
  - Pricing American Options under Stochastic Volatility and Stochastic Interest Rates Alexey N. Medvedev O. Scaillet
  - A Stochastic Volatility Alternative to SABR*L.C.G. Rogers L.A.M. Veraart J. Appl. Probab. Volume 45, Number 4 (2008), 1071-1085.*
  - Multi-level Monte Carlo path simulation. *M.B. Giles, Operations Research, 56(3):607-617, 2008.*
  - A. Alfonsi, A. Ahdida: High order discretization of Wishart process.
  - Time dependent Heston model *E. Benhamou, E. Gobet and M. Miri, Time Dependent Heston Model(March 24, 2009).*
  - E. Voltchkova: Localization of the Black-Scholes equation using transparent boundary conditions.
  - M. Gaudenzi, M.A. Lepellere, A. Zanette: The Singular Points method for Pricing American Path-Dependent Options.*to appear Journal of Computational Finance*

5.1.4. Software organization

The software Premia provides a collection of C/C++ routines and scientific documentation in PDF and HTML. More precisely, Premia is composed of :

- a library designed to describe derivative products, models, pricing methods and which provides basic input/output functionalities.
- a collection of pricing routines: in this way, the routines of Premia can easily be plugged into other financial softwares.
- a scientific documentation system. It is created from hyperlinked PDF files which describe the pricing routines and the general numerical methods involved like Monte Carlo methods, lattice methods, etc.

Premia is available for Windows and Linux operating systems. It provides Excel and Scilab/Nsp interfaces. Reports in PDF can be automatically generated at the end of each computation session.

The size of Premia is around 500 Mb, with 12 Mb of source code in C.

J. Lelong has continued to improve and maintain the Nsp interface for Premia. The manual, which he started to write a year ago, now documents almost all functionalities provided by the interface. J. Lelong and D. Pommier are developing a numerical library for Premia to give the contributors a unified scientific library (PNL). The version 1.0 (November 2009) (around 27000 lignes of code C) will be registered at the APP agency. Here are the major contributions of J. Lelong in the development of the PNL:

- Interface for the Amos library which provides approximations for the complex Bessel functions.
- Complete reorganisation of the source code.
- Linear algebra : matrix exponential, complex matrices, efficient matrix vector operations (inspired from *Blas*, D. Pommier also worked on that point), resolution of linear systems, interfaces for some *Lapack* functions (LU decomposition, matrix logarithm, computation of eigenvalues and eigenvectors).
- Root finding functions.
- Interface for the Fast Fourier Transform library *FFTPack*.
- Laplace inversion functions.
- Sorting functions.
- Numerical integration methods for one and two variate functions.

5.1.5. Consortium Premia

Premia is developed in interaction with a consortium of financial institutions or departments presently composed of: CALYON, Société Générale, Natixis, Bank Austria and RZB (Raiffeisen Zentralbank Österreich AG). The participants of the consortium contribute to finance the development of Premia and help to determine the directions in which the project evolves. They have access to the complete software with the source and the documentation. Every year, a new release is delivered to the Consortium members.

Moreover, a restricted version of Premia is available on Premia web site http://www.premia.fr and can be downloaded with a special license for academic and evaluation purposes.

6. New Results

6.1. Discretization of stochastic volatility models

Participants: M. Sbai, B. Jourdain, A. Alfonsi, A. Ahida.

In usual stochastic volatility models, the process driving the volatility of the asset price evolves according to an autonomous one-dimensional stochastic differential equation. B. Jourdain and M. Sbai assume that the coefficients of this equation are smooth. Using Itô's formula, they get rid, in the asset price dynamics, of the stochastic integral with respect to the Brownian motion driving this SDE. Taking advantage of this structure, they propose

- a scheme, based on the Milstein discretization of this SDE, with order one of weak trajectorial convergence for the asset price,
- a scheme, based on the Ninomiya-Victoir discretization of this SDE, with order two of weak convergence for the asset price [61].

They also propose a specific scheme with improved convergence properties when the volatility of the asset price is driven by an Orsteim-Uhlenbeck process.

A. Alfonsi and A. Ahida are working on the simulation of Wishart processes. Wishart matrices process are increasingly used in finance to model a multiname volatility. A. Alfonsi and A. Ahida have obtained an exact simulation procedure, and a second-order scheme for general affine diffusions on symmetric positive matrices.

6.2. Monte Carlo simulations and stochastic algorithms for option pricing

Participants: E.H.A. Dia, B. Jourdain, C. Labart, D. Lamberton, J. Lelong.

D. Lamberton and El Hadj Aly DIA develop Monte-Carlo methods for exotic options in models with jumps. Some results concerning lookback and barrier options in jump-diffusion models have been obtained, in particular, estimates for expectations involving discrete vs continuous maxima of the sample paths of a Lévy process. They also have results on the effect of truncating small jumps and replacing them by Brownian motion. They also study the valuation of lookback and digital barrier options in exponential Lévy models without positive jump.

D. Lamberton and S. M. Ould Aly study exotic options and stochastic volatility models. They have obtained results on the effective computation of option prices in a stochastic volatility model, in the context of variance swap modelling.

6.3. Optimal stopping and American Options

Participants: D. Lamberton, M. Mikou, B. Jourdain, M. Vellekoop, D. Pommier, T. Arnarson.

- **Optimal stopping of one-dimensional diffusions:** In continuation of joint work with Mihail Zervos (London School of Economics), D. Lamberton has been working on an example of optimal stopping with irregular payoff (the staircase option).

- **American options in exponential Lévy models:** D. Lamberton and M. Mikou [12] have obtained results on the regularity of the American put price in the case of general exponential Lévy models. They also derive the asymptotic behavior of the early exercise boundary near maturity in non-classical cases.

- **American Put option with discrete dividends :** B. Jourdain and M. Vellekoop (Twente University) are interested in the regularity of the optimal exercise boundary for the American Put option when the underlying asset pays a discrete dividend at a known time during the lifetime of the option. The ex-dividend asset price process is assumed to follow Black-Scholes dynamics and the dividend amount is a deterministic function of the ex-dividend asset price just before the dividend date. The solution to the associated optimal stopping problem can be characterised in terms of an optimal exercise boundary which, in contrast to the case when there are no dividends, is no longer monotone. They prove monotonicity, continuity and a high contact principle at times in a left-hand neighbourhood of the dividend date for different dividend payment functions.

- **Calibration of American options in Lévy models.** D. Pommier, T. Arnarson and Y. Achdou (Paris 7 University) are investigating a numerical procedure for the calibration of American option in Lévy models, based on optimality conditions on the parameters for minimization of a least square problem involving the observed option prices.

6.4. Risk

- *Credit Risk*: A. Alfonsi J. Lelong have studied a very simple extension of the Black-Cox model for a single default. Namely, they have considered the case where the default intensity can take different levels according to the firm value. In this model, they know the Laplace transform of the default time, and are able to calibrate the model rather quickly to the CDS prices [55].
- *Liquidity risk*: A. Alfonsi, A. Schied and A. Slynko have continued to work on a simple limit order book model [57]. First they have obtained the optimal strategy for buying a large amount of shared with a given number of trades, when the market resilience is exponential. This problem is closely related to the absence of Price Manipulation Strategies. Then, the case of more general resilence kernels has been considered. The model is no longer Markovian, and a new kind of manipulation strategies have been identified.

6.5. Stochastic control of jump diffusions, Stochastic Maximum principles and BSDEs

Participants: A. Cretarola, J.J.A Hosking, F. Russo, A. Sulem.

Robust control. A. Sulem and B. Øksendal study robust (worst case scenario) optimal stochastic control of jump diffusions and equivalent martingale measures.

Singular stochastic control. A. Sulem in a recent joint work with B. Øksendal study general singular control problems of Lévy processes, in which the controller has only partial information and the system is not necessarily Markovian. Two different approaches are considered: (i) by using Malliavin calculus, leading to generalized variational inequalities for partial information singular control (of possibly non-Markovian systems) (ii) by introducing a singular control version of the Hamiltonian and using backward stochastic differential equations (BSDEs) to obtain a partial information maximum principle for such problems. They show that the two methods are related, and find a connection between them. They then study the relation between the generalized variational inequalities found in (i) and general reflected backward stochastic differential equations (RBSDEs) for Lévy processes. These are again shown to be equivalent to general optimal stopping problems for such processes. Combining this, a connection between singular control and optimal stopping is obtained.

Optimal control of forward-backward stochastic differential equations with jumps. In [34], A. Sulem and B. Øksendal present various versions of the maximum principle for optimal control of forward-backward SDEs (FBSDEs) with jumps. This study is motivated by risk minimization via g-expectations. They first prove a general sufficient maximum principle for optimal control with partial information of a stochastic system consisting of a forward and a backward SDE driven by Lévy processes. They then present a Malliavin calculus approach which allows them to handle non-Markovian systems. Finally they give examples of applications in finance, namely they study the risk minimizing portfolio problem and a utility optimization problem under risk constraint.

Risk-Minimizing Stopping and Backward Stochastic Differential Equations Following [34], A. Cretarola and A. Sulem consider a coupled system of FBSDEs with jumps and study an optimal stopping problem in which the decision maker uses a dynamic convex risk measure to evaluate the risk of the financial standing.

Backward stochastic differential equations under partial information
A. Cretarola and Francesco Russo are studying the existence and the uniqueness of a solution to BSDEs of the form

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)\mathrm{d}\langle M\rangle_s - \int_t^T Z_s \mathrm{d}M_s - (O_T - O_t), \quad 0 \le t \le T$$

driven by the general martingale M and investigate connections to dynamic risk measures and applications to hedging of derivatives or insider trading. As a first step, the linear case has been studied. Then, they have considered the nonlinear case, by assuming the continuity of the martingale M and they have shown existence and uniqueness results by using similar arguments as in [82]. Now they are focusing on the jump case. They have already proved existence by using some results that can be found in [68].

Maximum principles for stochastic differential games. John Joseph Absalon is starting to look with Agnès Sulem at maximum principles for games with spike perturbation method.

6.6. Pricing and hedging in incomplete markets

Participants: F. Russo, A. Sulem.

Risk indifference pricing in jump diffusion markets. In [35], A. Sulem and B. Øksendal study the risk indifference pricing principle in incomplete markets: The (seller's) *risk indifference price* $p_{\text{risk}}^{\text{seller}}$ is the initial payment that makes the *risk* involved for the seller of a contract equal to the risk involved if the contract is not sold, with no initial payment. We use stochastic control theory and PDE methods to find a formula for $p_{\text{risk}}^{\text{seller}}$ and similarly for $p_{\text{risk}}^{\text{buyer}}$. In particular, we prove that

$$p_{\text{low}} \leq p_{\text{risk}}^{\text{buyer}} \leq p_{\text{risk}}^{\text{seller}} \leq p_{\text{up}},$$

where p_{low} and p_{up} are the lower and upper hedging prices, respectively.

Variance Optimal Hedging. In [53], F. Russo, with S. Goutte and N. Oudjane address pricing and hedging issues in incomplete markets and consider applications to the electricity markets. The case when the asset price is a process with independent increments or an exponential of those processes is studied.

An anticipative stochastic calculus approach to pricing in markets driven by Lévy processes. In [54], A. Sulem and B. Øksendal use the Itô-Ventzell formula for forward integrals and Malliavin calculus to study the stochastic control problem associated to utility indifference pricing in a market driven by Lévy processes. This approach allows them to consider general possibly non-Markovian systems, general utility functions and possibly partial information based portfolios. In the case of exponential utility function $U_\alpha = -\exp(-\alpha x)$; $\alpha > 0$, asymptotics properties for vanishing α are obtained. In the special case of full information based portfolios and no jumps a recursive formula for the optimal portfolio in a non-Markovian setting is given.

6.7. Stochastic analysis and Malliavin calculus

Participants: V. Bally, F. Russo, A. Sulem.

The classical Malliavin Calculus for jump type processes as it was developped in the book of Bichteler Gravereu and Jacod 1985 concerns Poisson point measure in which the jump amplitudes represent independent random variables. V. Bally considers equations driven by Poisson random measures in which the law of the jumps depends on the position of the solution of the stochastic equation. This corresponds to physical models as the Bolzmann equation for example. The two preprints of V. Bally in colaboration with E. Clément and N. Fournier [59], [60] concern this topic. This is also a continuation of the work done with M.P. Bavouzet and M. Marouen [42] concerning the sensitivity computations in jump type models in finance.

V. Bally is studying the problem of lower bounds for the density of a functionalthe since several years. The last two contributions in this area are papers in collaboration with A. Meda and B. Fernandez from the University of Mexico (tubes estimates) [18] and another one in collaboration with A. Kohatsu-Higa [19].

Following an idea of Malliavin and Thalmaier, V. Bally and L. Caramellino (Tor Vergata University) use the Riesz transform in order to give regularity criterions for the law of a random variable under week hypothesis [58].

F. Russo studies modelling of vortex filaments via stochastic integrals via regularisation and the probabilistic representation of a non-linear PDE of porous medium type and discontinuous coefficient. Both the non-degenerate case and the degenerate case are considered.

7. Contracts and Grants with Industry

7.1. Consortium Premia

The consortium Premia is centered on the development of the pricer software Premia. It is presently composed of the following financial institutions: CALYON, Société Générale, Natixis, Bank Austria, Raiffeisen Zentralbank Österreich AG.

8. Other Grants and Activities

8.1. ANR programs

- ANR program GCPMF "Grid Computation for Financial Mathematics" February 2006-June 2009. Partners : Calyon, Centrale, EDF, ENPC, INRIA, Ixis, Paris 6, Pricing Partner, Summit, Supelec.

  Global coordinator: B. Lapeyre.
- ANR program BigMC (Monte Carlo Methods in large dimension).

  Partners ENST, ENPC, University Paris-Dauphine.

  ENPC coordinator: B. Jourdain

8.2. Pôle compétitivité

The project “Credinext” on credit risk derivatives has started in the “Pôle Finance Innovation”.

Partners: Euronext Paris, Lunalogic, Pricing Partners, CMAP (Ecole Polytechnique), CERMICS/ENPC, Université Paris-Est Marne la Vallée (Laboratoire de Mathématiques Appliqués), INRIA (projet Mathfi).

8.3. International cooperations

- Part of the European network "Advanced Mathematical Methods for Finance" (AMaMef). This network is supported by the European Science Foundation (ESF).
- Collaborations with the Universities of Oslo, Bath, Chicago, Mexico, Osaka, Rome II and III, Tokyo Institute of Technology

9. Dissemination

9.1. Conference and Seminar organisation

- A. Alfonsi

  - Co-organizer of the working group seminar of MATHFI “Stochastic methods and finance”.
- A. Alfonsi, J.-F. Delmas, B. Jourdain, B. Lapeyre:

  - 3rd conference on numerical methods in finance, Ecole des Ponts ParisTech, 15-17 april 2009.
- B. Lapeyre

  - Member of the program committee of the "Parallel and Distributed Computing in Finance" May 25-29, 2009, Roma (Italy).

- F. Russo

  - Coorganisation (with E. Valkeila, J.M. Corcuera and F. Biagini) of an AMAMEF workshop on “No semimartingale models in finance”, May 2009, Helsinki, Finland.

  - Member of the Scientidic Committee of the Conference **Défis actuels de la finance**, University Paris 13, November 2009.

- A. Sulem

  - Member of the Scientific Committee of the Optimal stopping with applications symposium, Turku Finland 15-19 June 2009. http://web.abo.fi/fak/mnf/mate/gradschool/optimalstopping2009/

9.2. Editorship

- D. Lamberton

  - Associate Editor of *Mathematical Finance*

  - Co-Editor of *ESAIM P&S*.

- A. Sulem

  - Associate editor of SIAM Journal on Financial mathematics *SIFIN*

  - Associate editor of Journal of Applied Mathematics and Stochastic Analysis *JAMSA*

9.3. Services to the scientific community

- B. Jourdain

  - Deputy head of the doctoral school ICMS, University Paris-Est

- C. Labart

  - Reviewer for Mathematical Reviews and Referee for Stochastic Processes and Their Applications, Mathematical Finance and Mathematics of computation.

- D. Lamberton

  - in charge of the master program “Mathématiques et Aplications" (Universities of Marne-la-Vallée, Créteil and Evry, and Ecole Nationale des Ponts et Chaussées).

  - Director of the Mathematical department (UFR de mathématiques), Université Paris-Est Marne-la-Vallée.

  - Member of the Steering Committee of the ESF European Network "Amamef" (http://www.iac.rm.cnr.it/amamef/); in charge of the GDR "Méthodes Mathématiques pour la finance", which is the national CNRS group related to the network(until the end of 2009).

- B. Lapeyre

  - President of the Doctoral Department at Ecole des Ponts

- F. Russo

  - Member of the Professors council of the graduate program of LUISS University (Rome). Title: Mathematical methods for finance, economics and insurance.

  - Advisor (Commiassario) for teaching of mathematics at the “Liceo cantonale Lugano” (Switzerland).

- A. Sulem

  - Member of the evaluation committee of the University Paris-Dauphine.

  - Member of the doctoral board of the the University Paris-Dauphine.

9.4. Teaching

- A. Alfonsi

  - Probabilités et Statistiques”, first year course at the Ecole des Ponts.

  - Modéliser, Programmer et Simuler”, second year course at the Ecole des Ponts.

  - “Calibration, Volatilité Locale et Stochastique”, third-year course at ENSTA (Master with Paris I).
- V. Bally

  - Malliavin Calculus and applications in finance (30h) Master II, UMLV-ENPC, Finance.

  - Interest rates. (20h) Master II, UMLV-ENPC, Finance.

  - Risk analysis. Master II, IMIS UMLV.

  - Probability Master I, UMLV.
- DIA El Hadj Aly

  - Tutor of a Master Thesis in Applied Mathematics preparing to a PhD’s program, August-September 2009.
- B. Jourdain

  - course "Probability theory and statistics", first year ENPC

  - course "Introduction to probability theory", 1st year, Ecole Polytechnique

  - course "Stochastic numerical methods", 3rd year, Ecole Polytechnique

  - projects in finance and numerical methods, 3rd year, Ecole Polytechnique
- B. Jourdain, B. Lapeyre

  - course "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée
- J.-F. Delmas, B. Jourdain

  - course "Jump processes with applications to energy markets", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée
- C. Labart

  - Lectures on “Discrete time models for finance” at ENSTA (2nd year course), 18 hours and at University Pierre et Marie Curie (Master “Probabilités et Finance”) 12 hours.

  - Lectures on “Stochastic Calculus : applications of Itô’s formula and stochastic differential equations” at University Pierre et Marie Curie (Master IFMA) 12 hours.

  - Lectures on “Random Models: Markov chains and Markov Processes”, at University Pierre et Marie Curie (Master IFMA), 28 hours.

  - Practicals on “Probability”, 12 hours, Polytech’Paris.

  - Practicals on “Numerical methods for differential equations”, 48 hours at University Pierre et Marie Curie (Licence 3).
- D. Lamberton

  - Third year of Licence de mathématiques (differential calculus, differential equations), Université Paris-Est Marne-la-Vallée.

  -Préparation à l’agrégation interne de mathématiques, Université Paris-Est Marne-la-Vallée.

  - Master course “Calcul stochastique et applications en finance", Université Paris-Est Marne-la-Vallée.
- B. Lapeyre

  - Ecole des Ponts, 2nd year, "Introduction to mathematical methods for finance", 2009

- J. Lelong

  - Lectures on “Parallel programming in financial mathematics” at Ensimag (third year course)

  - Lectures on “Numerical Methods in Finance” at Ecole Nationale Supérieure de Techniques Avancées (second year course)

  - Supervision of financial engineering projects at Ecole Nationale Supérieure de Techniques Avancées (second year course)

  - Lectures on “Discrete time martingales” at Ecole Nationale Supérieure de Techniques Avancées (second year course)

  - Lectures on “Monte Carlo methods for American option pricing” at Collège de l’Ecole Polytechnique.

- A. Sulem

  - Course on numerical methods in finance, Master II MASEF University Paris-Dauphine (21 hours)

9.5. Internship advising

- B. Jourdain:

  - Pierre-Yves Lagrave : Implementation into Premia of approximation formulas for options in stochastic volatility models given by Elisa Alos in "A generalisation of the Hull and White formula with applications to option pricing approximation", Finance and Stochastics, 10(3) 353–365, 2006

- A. Kebaier:

  - Kaouther Hajji, Ecole Polytechnique de Tunis (November 2009 to April 2010) months internship) on Interacting particle systems for the computation of rare credit portfolio losses

- C. Labart:

  - Cédric Allali, student of Master 2 “Statistique et Modèles aléatoires en finance”, University Paris-Diderot for 5 months on the hedging of CDOs based on the following papers

    – Hedging Default Risks of CDOs in Markovian Contagion Models. by J.P. Laurent A.Cousin J.D.Fermanian

    – Dynamic hedging of synthetic CDO-tranches with spread risk and default contagion, by R. Frey and J. Backhaus, 2007

    – Pricing and Hedging of Portfolio Credit Derivatives with Interacting Default intensities, by R. Frey and J. Backhaus, 2007

    – Portfolio Credit Risk Models with Interacting Default Intensities: a Markovian Approach, by R. Frey and J. Backhaus, 2004

  leading to two algorithms implemented in PREMIA.

  - Yisheng Wang, student of ENSTA (2nd year) for 3 months on the pricing of parisian options in local volatility models. PDE and Monte Carlo approaches have been implemented in Premia.

9.6. PhD defences

- Mohamed Mikou:

  Options américaines dans le modèle exponentiel de Lévy,

  Defended December 2th at Université Paris-Est Marne la Vallée, Adviser: D. Lamberton

- Mohamed Sbai:

  Dependence modelling and simulation of stochastic processes in finance,

  Defended November 25th at Ecole des Ponts, Adviser: B. Jourdain.

9.7. PhD advising

- A. Alfonsi and T. Lelièvre

  - José Infante Acevedo (1st year). Half of this thesis will be denoted to liquidity risk.
- V. Bally

  - Stephano di Marco, 3rd year, student in Scola Normale Superiore di Pisa, in collaboration with M. Pratelli from the University of Pisa.

  - Victor Rabiet, 1st year. Grant of ENS Cachant.
- B. Jourdain

  - Mohamed Sbai,

  "Dependence modelling and simulation of stochastic processes in finance", Mohamed Sbai, defended November 25th 2009, ENPC

  - Maxence Jeunesse, 1st year

  "Study of some numerical methods in finance",
- D. Lamberton

  - Mohamed Mikou. American options in models with jumps. ATER at Université Paris-Est Marne-la-Vallée. Thesis defense on December 2nd, 2009

  - El Hadj Aly Dia (4th year). Monte-Carlo methods for exotic options in models with jumps. Ingénieur-expert at INRIA.

  - Sidi Mohamed Ould Aly (3rd year). Exotic options and stochastic volatility models. Allocataire de recherche, Université Paris-Est, doctorant-conseil at Natixis.

  - Lokmane Abbasturkki (1st year, started in March 2009). Modelling of correlation in high dimensions. This thesis is funded by Credinext.

  - Ayech Bouselmi (1st year, started in October 2009). Allocataire de recherche, Université Paris-Est. Lévy processes and multi-dimensional models in finance.
- B. Lapeyre and A. Alfonsi

  - Abdelkoddousse Ahdida (2nd year, Funding "Pôle Finance Innovation" - Créditnext Project) Current work on high order schemes and exact simulation of Wishart processes. Further work on the application of these processes to finance.
- F. Russo

  - Nadia Belaribi. Probabilistic representation of PDE's of porous media type equations: probabilistic and deterministic numerical simulations.

  - Cristina Di Girolami (in collaboration with Luiss University). Calculus via regularisation in infinite dimension and applications to mathematical finance.

  - Stéphane Goutte (in collaboration with Luiss University). Mean-variance hedgine in incomplet markets and applications to the electricity markets.

  - Ida Kruk. Malliavin calculus for general Gaussian processes.

  - Stefano Mega (in collaboration with Luiss University). Pricing of services related to telecommunications network.
- A. Sulem and R. Cont (University Paris 6)

  - Andrea Minca (1st year), funded by the Natixis Foundation for Quantitative Research. "Contagion in Financial Markets"

9.8. PhD reports

- B. Jourdain

  - Referee for the PhD thesis of Romain Deguest, "Model uncertainty in finance : risk measure and model calibration", Ecole Polytechnique, 12 november 2009

- F. Russo

  - Referee of the PhD or Habilitation theses of

  – Eulalia Nualart (I presented her habilitation).

  – Mohammed SBAI, Ecole Des Ponts, Paristech.

  – Makrem Sghairi, Paris 5.

- A. Sulem

  - Referee for the PhD thesis of D. Andersson: *Contributions to the Stochastic maximum principle*, KTH, Stockholm, Sweden, October 2009.

9.9. Participation to workshops, conferences and invitations

- A. Alfonsi

  Conferences:

  - “A closed-form extension to the Black-Cox model”, Nice 30th September, Recent Advancements in the Theory and Practice of Credit Derivatives.

  Talks:

  - “High order discretization schemes for the CIR process: application to Heston and Affine models” (in January at Paris XIII, in February at Oxford, and in May at Le Mans).

  - “Optimal execution and price manipulations in limit order book models”, November, Ecole Polytechnique.

  Invitations: - Oxford University, by Jan Obloj (18-02 to 20-02)

- V. Bally

  -16-25 April and 23 September - 10 October: visits to the University Tor Vergata for scientific collaboration with Lucia Caramellino.

  - 25-26 June: talk on "Lower Bounds for the Densities of Asian Type Stochastic Differential Equations." in the workshop organized by Denis Talay in INRIA Sophia Antipolis.

  - 13-17 July: invited to the 7th International ISSAC Congress at Imperial College London. Talk on "Integration by parts formulas and applications to equations with jumps.".

  - 27-31 July: Invited to the SPA Congress in Berlin. Talk on "Integration by parts formulas and applications to equations with jumps.".

- El Hadj Aly Dia

  Conferences:

  - Fourth General Conference on Advances Mathematical Methods in Finance, University of Oslo, Alesund (Norway), 4-10 May 2009. - Third conference on Numerical Methods in Finance, Ecole Nationale des Ponts et Chaussées, Paris, 15-17 April 2009.

  Workshops:

  - Journée des Doctorants du Séminaire Bachelier, Institut Henri-Poincaré, Paris, 06 Novemeber 2009. - Second SMAI Summer School in Financial Mathematics, Ecole Polytechnique, Paris, 24-29 August 2009. - Premia 11 meeting, Institut Louis Bachelier, Paris, 11 February 2009.

- B. Jourdain

  - BigMC seminar, 29 january, Exact simulation of stochastic differential equations, according to Beskos, Papaspiliopoulos and Roberts.

  - workshop computational Finance, RIMS Kyoto university, 10-12 August, High order discretization schemes for stochastic volatility models.

  - seminar of the chair "financial risks", 21 september, High order discretization schemes for stochastic volatility models.

  - petit déjeuner de la finance, 23 september, Coupling index and stocks.

  - BigMC seminar, 24 september, Robust adaptive importance sampling for Normal random vectors.

  - workshop on Optimal stopping and singular stochastic control problems in finance, National University of Singapore, 9-15 december : Exercise boundary of the American put option in the black-Scholes model with discrete dividends.

- A. Kebaier

  - congress on Non-Semimartingale Techniques in Mathematical Finance May 26-28, 2009, Helsinki University of Technology

- C. Labart

  - Seminar at Univesity Paris 13, January 2009.

- D. Lamberton

  Invitations:

  - 8th Winter School on Mathematical Finance in the Netherlands, Lunteren, January 2009. Special invited lecture "Some option pricing problems in exponential Lévy models".

  - AMaMeF 4th General Conference, Aalesund, Norway, May 2009, . "Discrete vs continuous supremum of Lévy processes".

  - Summer School in Financial Mathematics, Ljubljana, September 2009. A short course on optimal stopping and American options.

  - Finance and Stochastics Seminar, Imperial College, London October 2009. "On the approximation of the supremum of a Lévy process"

- B. Lapeyre

- J. Lelong

  - Workshop on stochastic algorithms, Dijon (University of Bourgogne), November 2009, invited speaker.

  - Seminar of Probability, LJK University of Grenoble, September 2009.

  - IEEE International Symposium on Parallel and Distributed Processing, Roma, May 2009.

  - Third Conference on Numerical Methods in Finance, Marne-la-Vallée (Ecole de Ponts), April 2009.

  - Seminar of Université du Maine, May 2009.

  - SMAI 2009 conference, La Colle sour Loup, May 2009.

- A. Minca

  Attended Schools:

  - 16-20 March 2009, CIRM, Luminy, Journées ALEA 2009 (Thematic School CNRS): Combinatorics, Random Graphs

- 6-17 July 2009, Ithaca, NY, 5th Cornell Probability Summer School: Mixing time of Markov chains, matching and point processes

Conferences:

- 18 September, London, Standard and Poor's Credit Risk Summit, invited talk on "Credit Default Swaps and Systemic Risk"

- 16-19 December, Sydney, Quantitative Methods in Finance, contributed talk on "Contagious Defaults and Systemic Risk in Financial Networks"

Seminars:

- 16 October, Groupe de travail Methodes Stochastiques en Finance, ENPC-INRIA-UPEMLV, talk on joint article with Rama Cont, "Recovering portfolio default intensities implied by CDO quotes"

- 29 October, Groupe de travail Probabilités et Finance, LPMA, Paris 6, "Recovering portfolio default intensities implied by CDO quotes"

- D. Pommier

  - 3rd Workshop on High-Dimensional Approximation, Sydney (Australia), February 2009, talk on High Dimensional PDE's methods applied to option pricing

- F. Russo

  Presented Conferences and Seminars:

  - Sydney (Australie), february 2009. University Seminar.

  - Koblenz (Allemagne), april 2009. Presentation to Debeka Versicherung (Insurance company)

  - Marne-la-Vallée, may 2009.

  - Helsinki (Fin), May 2009. No-semimartingale models in mathematical finance (Workshop).

  - Tianjin (China), June 2009. Workshop on Random dynamical systems and related topics.

  - Manchester (UK), August 2009. Conference on SDEs, SPDEs and related topics.

  - Purdue (USA), September 2009. Workshop Stochastic Analysis at Purdue 09.

  - Princeton (USA), October 2009. Stochastic analysis seminar, Bendheim center for finance.

  - New York (USA), October 2009 Stochastic analysis and mathematical physics seminar.

  - Oxford (UK), November 2009. Stochastic analysis seminar of the Man Institute.

  - Seoul (Corea), December 2009. Invitation at the Department of Mathematical Sciences, Seoul National University Presentation of a conference.

  Longer invitation stays abroad

  - Sydney (Australia), february 2009, 2 weeks.

  - Bielefeld (Allemagne), jul, aug. 2009, 6 weeks.

  - Purdue (USA), oct; 2009, 1 week.

  - Cambridge (UK), Newton Institute, march-april 2010, 6 weeks.

- A. Sulem

  - 29 Avril: seminar “Analysis on the Wiener space”, IHP, Paris talk on : Singular stochastic control with partial information of jump diffusions

- A. Zanette

  - Evaluating fair premiums of equity-linked policies with surrender option in a bivariate model, Third Conference on Numerical Methods in Finance ENPC Paris 2009,

  - International Congress on Insurance: Mathematics and Economics (IME2009) Istanbul 2009,

  - AMASES 2009 Parma

10. Bibliography

Major publications by the team in recent years

[1] A. ALFONSI. *On the discretization schemes for the CIR (and Bessel squared) processes*, in "Monte Carlo Methods and Applications", vol. 11, n^o 4, 2005, p. 355–384.

[2] B. AROUNA. *Adaptative Monte Carlo Method, A Variance Reduction technique*, in "Monte Carlo Methods and Applications", vol. 10, n^o 1, 2004.

[3] V. BALLY. *An elementary introduction to Malliavin calculus*, n^o 4718, Inria, Rocquencourt, February 2003, http://hal.inria.fr/inria-00071868, Research Report.

[4] V. BALLY, M. BAVOUZET, M. MESSAOUD. *Computations of Greeks using Malliavin Calculus in jump type market models*, in "Annals of Applied Probability", vol. 17, 2007, p. 33-66.

[5] V. BALLY, L. CARAMELLINO, A. ZANETTE. *Pricing American options by a Monte Carlo method using a Malliavin calculus approach*, in "Monte Carlo methods and applications", vol. 11, n^o 2, 2005, p. 97–133.

[6] E. CLÉMENT, D. LAMBERTON, A. KOHATSU-HIGA. *A duality approach for the weak approximation of stochastic differential equations*, in "Annals of Applied Probability", vol. 16, n^o 3, August 2006, p. 1124-1154.

[7] B. JOURDAIN, M. SBAI. *Exact retrospective Monte Carlo computation of arithmetic average Asian options*, in "Monte Carlo methods and Applications", vol. 13, n^o 2, 2007, p. 135–171.

[8] A. KOHATSU-HIGA, A. SULEM. *Utility maximization in an insider influenced market*, in "Mathematical Finance", vol. 16, n^o 1, 2006, p. 153–179.

[9] D. LAMBERTON, M. MIKOU. *The critical exercise price for the American put in an exponential Lévy model*, in "Finance & Stochastics", vol. 12, 2008, p. 561-581.

[10] M. N'ZI, Y. OUKNINE, A. SULEM. *Regularity and representation of viscosity solutions of Partial differential equations via backward stochastic differential equations*, in "Stochastic processes and their applications", vol. 116, n^o 9, 2006, p. 1319–1339.

[11] B. ØKSENDAL, A. SULEM. *Applied Stochastic Control of Jump Diffusions*, Universitext, Second Edition, Springer, Berlin, Heidelberg, New York, 257 pages 2007.

Year Publications

Doctoral Dissertations and Habilitation Theses

[12] M. MIKOU. *Options américaines dans le modèle exponentiel de Lévy*, Université Paris Est Marne la Vallée, 2009, Ph. D. Thesis.

[13] M. SBAI. *Modélisation de la dépendance et simulation de processus en finance*, Ecole des Ponts, 2009, Ph. D. Thesis.

Articles in International Peer-Reviewed Journal

[14] Y. ACHDOU, F. HECHT, D. POMMIER. *A posteriori error estimates for parabolic variational inequalities*, in "Journal of Scientific Computing", 2009, accepted for publication.

[15] A. ALFONSI. *High-order discretization scheme for the CIR process: application to the Heston model*, in "Mathematics of Computation", vol. 79, n^{o} 269, 2010, p. 209-237.

[16] A. ALFONSI, A. FRUTH, A. SCHIED. *Optimal execution strategies in limit order books with general shape functions*, in "Quantitative Finance", vol. 9, 2009, DOI:10.1080/14697680802595700.

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