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Project-Team POEMS

Wave propagation: Mathematical Analysis and Simulation

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Theme : Computational models and simulation

Activity Report

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2. Overall Objectives

2.1. Overall Objectives

The propagation of waves is one of the most common physical phenomena one can meet in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) and to the scale of the universe (electromagnetic waves, gravity waves), to the scale of the atom (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us.

The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in the various domains of physics and engineering science.

The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas justify together a research project in Scientific Computing entirely devoted to this theme.

The project POEMS is an UMR (Unité Mixte de Recherche) between CNRS, ENSTA and INRIA (UMR 2706). The general activity of the project is oriented toward the conception, the analysis, the numerical approximation, and the control of mathematical models for the description of wave propagation in mechanics, physics, and engineering sciences.

Beyond the general objective of contributing to the progress of the scientific knowledge, four goals can be ascribed to the project:

- the development of an expertise relative to various types of waves (acoustic, elastic, electromagnetic, gravity waves, ...) and in particular for their numerical simulation,
- the treatment of complex problems whose simulation is close enough to real life situations and industrial applications,
- the development of original mathematical and numerical techniques,
- the development of computational codes, in particular in collaboration with external partners (scientists from other disciplines, industry, state companies...)

3. Scientific Foundations

3.1. Scientific Foundations

Our activity relies on the existence of mathematical models established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation (or a system of partial differential equations) of the hyperbolic type that are often (but not always) linear for most of the applications we are interested in. The prototype equation is the wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0,$$

which can be directly applied to acoustic waves but which also constitutes a simplified scalar model for other types of waves (This is why the development of new numerical methods often begins by their application to the wave equation). Of course, taking into account more realistic physics will enrich and complexify the basic models (presence of sources, boundary conditions, coupling of models, integro-differential or non linear terms,...)

It is classical to distinguish between two types of problems associated with these models: the time domain problems and the frequency domain (or time harmonic) problems. In the first case, the time is one of the variables of which the unkonwn solution depends and one has to face an evolution problem. In the second case (which rigorously makes sense only for linear problems), the dependence with respect to time is imposed a priori (via the source term for instance): the solution is supposed to be harmonic in time, proportional to $e^{i\omega t}$, where $\omega > 0$ denotes the pulsation (also commonly, but improperly, called the frequency). Therefore, the time dependence occurs only through this pulsation which is given a priori and plays the rôle of a parameter: the unknown is only a function of space variables. For instance, the wave equation leads to the Helmholtz wave equation (also called the reduced wave equation) :

$$-c^2 \Delta u - \omega^2 u = 0.$$

These two types of problems, although deduced from the same physical modelization, have very different mathematical properties and require the development of adapted numerical methods.

However, there is generally one common feature between the two problems: the existence of a dimension characteristic of the physical phenomenon: the wavelength. Intuitively, this dimension is the length along which the searched solution varies substantially. In the case of the propagation of a wave in an heterogeneous medium, it is necessary to speak of several wavelengthes (the wavelength can vary from one medium to another). This quantity has a fundamental influence on the behaviour of the solution and its knowledge will have a great influence on the choice of a numerical method.

Nowadays, the numerical techniques for solving the basic academic and industrial problems are well mastered. A lot of companies have at their disposal computational codes whose limits (in particular in terms of accuracy or robustness) are well known. However, the resolution of complex wave propagation problems close to real applications still poses (essentially open) problems which constitute a real challenge for applied mathematicians. A large part of research in mathematics applied to wave propagation problems is oriented towards the following goals:

- the conception of new numerical methods, more and more accurate and high performing.
- the treatment of more and more complex problems (non local models, non linear models, coupled systems, ...)
- the study of specific phenomena or features such as guided waves, resonances,...
- the development of approximate models in various situations,
- imaging techniques and inverse problems related to wave propagation.

These areas constitute the main fields of interest for the Project POEMS.

4. Application Domains

4.1. Introduction

We are concerned with all application domains where linear wave problems arise: acoustics and elastodynamics (including fluid-structure interactions), electromagnetism and optics, and gravity water waves. We give in the sequel some details on each domain, pointing out our main motivations and collaborations.

4.2. Acoustics

As the acoustic propagation in a fluid at rest can be described by a scalar equation, it is generally considered by applied mathematicians as a simple preliminary step for more complicated (vectorial) models. However, several difficult questions concerning coupling problems have occupied our attention recently.

Aeroacoustics, or more precisely, acoustic propagation in a moving compressible fluid, is for our team a new and very challenging topic, which gives rise to a lot of open questions, from the modelling until the numerical approximation of existing models. Our works in this area are partially supported by EADS (and Airbus). The final objective is to reduce the noise radiated by Airbus planes.

Vibroacoustics, which concerns the interaction between sound propagation ans vibrations of thin structures, also raises up a lot of relevant research subjects. Our collaboration with EADS on this subject, with application to the confort of the cockpits of airplanes, allowed us to develop a new research direction about time domain integral equations.

A particularly attractive application concerns the simulation of musical instruments, whose objectives are both a better understanding of the behavior of existing instruments and an aid for the manufactoring of new instruments. The modeling and simulation of the timpani and of the guitar have been carried out in collaboration with A. Chaigne of ENSTA. We intend to initiate a new collaboration on the piano.

4.3. Electromagnetism

This is a particularly important domain, first because of the very important technological applications but also because the treatment of Maxwell's equations poses new and challenging mathematical questions.

Applied mathematics for electromagnetism during the last ten years have mainly concerned stealth technology, electromagnetic compatibility, design of optoelectronic micro-components or smart materials.

Stealth technology relies in particular on the conception and simulation of new absorbing materials (anisotropic, chiral, non-linear...). The simulation of antennas raises delicate questions related to the complexity of the geometry (in particular the presence of edges and corners). Finally micro and nano optics have seen recently fantastic technological developments, and there is a real need for tools for the numerical simulation in these areas.

Our team has taken a large part in this research in the past few years. In the beginning, our activity was essentially concerned with radar furtivity (supported by the French Army and Aeronautic Companies). Now, it is evolving in new directions thanks to new external (academic and industrial) contacts:

- We have been developing since 2001 a collaboration with ONERA on EM modeling by higher order methods (theses of S. Pernet and M. Duruflé).
- As partners of ONERA, we have been selected by the CEG (a research organism of the French Army) to contribute to the development of a general computational code in electromagnetism. The emphasis is on the hybridization of methods and the possibility of incorporating specific models for slits, screens, wires,...
- Optics is becoming again a major application topic. In the past our contribution to this suject was quite important but remained at a rather academic level. Our recent contacts with the Institut d'Electronique Fondamentale (Orsay) (we have initiated with them a research program about the simulation of micro and nano opto-components) are motivating new research in this field.
- Multiscale modelling is becoming a more and more important issue in this domain. In particular, in collaboration with the LETI(CEA) in Grenoble, we are interested in simulated devices whise some of the geometric characterictics are much smaller than the wavelength.

4.4. Elastodynamics

Wave propagation in solids is with no doubt, among the three fundamental domains that are acoustics, electromagnetism and elastodynamics, the one that poses the most significant difficulties from mathematical and numerical points of view. Our activity on this topic, which unfortunately has been forced to slow down in the middle of the 90's due to the disengagement of French oil companies in matter of research, has seen a most welcomed rebound through new academic and industrial contacts.

The two major application areas of elastodynamics are geophysics and non destructive testing. A more recent interest has also been brought to fluid-stucture interaction problems.

- In geophysics, one is interested in the propagation of elastic waves under ground. Such waves appear as natural phenomena in seisms but they are also used as a tool for the investigation of the subterrain, mainly by the petroleum industry for oil prospecting (seismic methods). This constitutes an important field of application for numerical methods. Our more recent works in this area have been motivated by various research contracts with IFP (French Institute of Petroleum), IFREMER (French Research Institute for the Sea) or SHELL.
- Another important application of elastic waves is non-destructive testing: the principle is typically to use ultra-sounds to detect the presence of a defect (a crack for instance) inside a metallic piece. This topic is the object of an important cooperation with EDF (French Company of Electricity) and CEA Saclay in view on the application to the control of nuclear reactors.

  At a more academic level, we have been interested in other problems in the domain of elastic waves in plates (in view of the application to non-destructive testing) through our participation to the GDR Ultrasons. In this framework, we have developped our resarches on multi-modal methods, exact transparent conditions or shape reconstruction of plates of variable cross section.
- Finally, we have recently been led to the study of fluid-solid interaction problems (coupling of acoustic and elastic waves through interfaces) as they appear in underwater seismics (IFREMER) and stemming from ultra-sound propagation in bones (in contact with the Laboratoire d'Imagerie Paramétrique of Paris VI University).

4.5. Gravity waves

These waves are related to the propagation of the ocean swell. The relevant models are derived from fluid mechanics equations for incompressible and irrotational flows. The applications concern in large part the maritime industry, if particular the questions of the stability of ships, sea keeping problems, wave resistance,... The application we have recently worked on concerns the stabilization of ships and off-shore platforms (contract with DGA).

5. Software

5.1. Advanced software

We are led to develop two types of software. The first category is prototype softwares : various softwares are developed in the framework of specific research contracts (and sometimes sold to the contractor) or during PhD theses. They may be also contributions to already existing softwares developed by other institutions such as CEA, ONERA or EDF. The second category is advanced software which are intended to be developed, enriched and maintained over longer periods. Such sofwares are devoted to help us for our research and/or promote our research. We have chosen to present here only our advanced softwares.

5.2. MELINA

This software has been developed under the leadership of D. Martin for several years in order to offer to the researchers a very efficient tool (in Fortran 77 and object oriented) for easily implementing finite element based original numerical methods for solving partial differential equations. It has specific and original potential in the domain of time harmonic wave problems (integral representations, spectral DtN conditions,...). Nowadays, it is fully functional in various application areas (acoustics and aeroacoustics, elastodynamics, electromagnetism, water waves). It is an open source software with on line documentation available at

http://perso.univ-rennes1.fr/daniel.martin/melina/

The software is regularly used in about 10 research laboratories (in France and abroad) and number of research papers have published results obtained with MELINA (see the Web site). Moreover, every 2 years, a meeting is organized which combines a workshop which teaches new users with presentations by existing users.

During the last four years, apart from various local improvements of the code, new functionalities have been developed:

- Higher order finite elements (up to 10^{th} order),
- Higher order quadrature formulae,
- DtN boundary conditions in 3D.

A new C++ version of the software is under development. We will take advantage of this evolution for extending the class of finite elements (mixed elements, tensor valued elements, ...).

This year, a beta version of MELINA++ has been achieved. It has been presented to the users at the Melina's days (12 -15 May 2009, Dinard, France).

5.3. MONTJOIE

This is a software for the efficient and accurate wave propagation numerical modeling in both time dependent or time harmonic regimes in various domains of application : acoustics, aeroacoustics, elastodynamics and electromagnetism . It is based essentially on the use of quadrilateral/hexaedric conforming meshes and continuous or discontinuous Galerkin approximations, The use of tensor product basis finctions coupled to judicious numerical quadrature techniques leads to important gains in both computing time and memory storage. Various techniques for treating unbounded domains have been incorporated : DtN maps, local absorbing conditions, integral representations and PML's.

We have written an interface for the use of other libraries : SELDON, a C++ linear algebra library (interfaced with BLAS and LAPACK) used for iterative linear solvers, MUMPS and UMFPACK for direct linear solvers, ARPACK for eigenvalue computations. The mesh generation is not part of the code. It can be done with Modulef, Gmsh, Ghs3D or Cubit.

This code has been developed by Marc Duruflé during his PhD thesis. Some other contributors have brought more specific enrichments to the code. The on line documentation is available at

http://montjoie.gforge.inria.fr/

In the framework of M. Bergot thesis, the simulation on hybrid meshes is now possible in Montjoie. Furthermore, the parallilization has been extended to time-harmonic equations.

6. New Results

6.1. Numerical methods for time domain wave propagation

6.1.1. Higher order time discretization of second order hyperbolic problems

Participants: Sébastien Impériale, Patrick Joly, Jerónimo Rodríguez.

Following the construction of optimal (namely maximizing the allowed CFL number) higher order schemes for second order hyperbolic problems of the form

$$\frac{d^2u}{dt^2} + \mathcal{A}u = 0, \tag{1}$$

we have extended in the case of first order system of the form

$$\frac{du}{dt} + \mathcal{A}u = 0, \tag{2}$$

where, here, $\mathcal{A}$ is an antisymmetric operator. The corresponding results are very similar.

6.1.2. Energy preserving schemes for hamiltonian systems

Participants: Patrick Joly, Juliette Chabassier.

A classical way of building numerical schemes for PDEs is to preserve, on a discrete level, a continuous conserved quantity, like an energy for instance. In 2008 we were interested in the so called “nonlinear hamiltonian systems of wave equations”, which can be written under the form $\partial_t^2\mathbf{u} - \partial_x \nabla U(\partial_x \mathbf{u}) = 0$, where the unknown $\mathbf{u}$ is valued in $\mathbb{R}^m$, and the function U maps $\mathbb{R}^m$ to itself. These systems preserve the continuous energy $\mathcal{E}(t) = \int \frac{1}{2}(\partial_t \mathbf{u})^2 + U(\partial_x \mathbf{u})$. This property leads, if the function U satisfies a sort of coercivity property, to a stability result for the solution of the PDE, hence the will to keep an analogous quantity conserved step to step in a discrete way.

We developed in 2008 an implicit, energy conservative, unconditionally stable, second order accurate scheme which can be written for any value of m. We tried to see our numerical scheme as a generalization of the classical θ-schemes for linear PDE, and we introduced a class of “preserving schemes” that fulfill a certain condition leading to a discrete energy preservation. We showed that explicit schemes do not fit this class except for linear models. We also showed that a “partially decoupled scheme”, based on a more intuitive gradient discretization that consists in approximating $\partial_k U$ at the time t^n with its linearization (directional finite difference of U along its k^{th} variable between times $n+1$ and $n-1$ and explicit along all other variables) does neither fit this class except for trivial models. These remarks point out our numerical scheme as a natural extension of θ-schemes for the nonlinear systems’ case. This work has lead to international communications in 2009 (SANUM’09, WAVES’09). A technical INRIA report and an article submission are in progress.

We also extended the numerical scheme to more general energy preserving nonlinear systems : if the function $U \equiv U(\mathbf{p}, \mathbf{q})$ maps now $\mathbb{R}^m \times \mathbb{R}^m$ to $\mathbb{R}^m$, we call $\nabla_1 U$ its gradient along the first variable $\mathbf{p}$ and $\nabla_2 U$ its gradient along the second variable $\mathbf{q}$. The new systems are written under the form $\partial_t^2\mathbf{u} - \partial_x \nabla_1 U(\partial_x \mathbf{u}, \mathbf{u}) + \nabla_2 U(\partial_x \mathbf{u}, \mathbf{u}) = 0$. An efficient computation has been lead in C++, using a modified Newton’s technique for each time step resolution as well as advanced algorithmic optimizations.

6.1.3. Hybrid Meshes for High-Order Discontinuous Galerkin Methods

Participants: Morgane Bergot, Gary Cohen, Marc Duruflé.

In the frame of her thesis directed by G. Cohen and M. Duruflé, M. Bergot studied a way to mix high-order hexahedral elements with tetrahedra, wedges and pyramids for discontinuous Galerkin methods. Hexahedra and tetrahedra have been well studied , and wedge elements are classically obtained by the tensorial product of a triangular element and a 1D element. The main effort is then devoted to the study of pyramidal elements, considering the following elements in the same mesh:

- classical hexahedral elements with Gauss-Lobatto points;
- Hesthaven tetrahedra, with Gauss-Lobatto points on the edges;
- wedges resulting from the tensorial product of a triangle with Hesthaven points, and an edge with Gauss-Lobatto points;
- pyramids joining triangular and quadrangular faces in a continuous way.

A new family of high-order nodal pyramidal finite element which can be used in hybrid meshes which include hexahedra, tetrahedra, wedges and pyramids has been proposed at any order, along with optimal approximate integration rules for the evaluation of the finite element matrices, so that the order of convergence of the method is conserved.Numerical results have been made to demonstrate the efficiency of hybrid meshes compared to pure tetrahedral meshes or hexahedral meshes obtained by splitting tetrahedra into hexahedra.

6.1.4. Coupling Retarded Potentials and Discontinuous Galerkin Methods for time dependent wave propagation

Participant: Patrick Joly.

This topic is developed in collaboration with J. Rodriguez (Santiago de Compostela), T. Abboud (IMACS) and I. Terrasse (EADS) in the framework of the contract ADNUMO with AIRBUS. Let us recall that our objective was to use time-domain integral equations (developed in particular at IMACS and EADS) as a tool for contructing transparent boundary conditions for wave problems in unbounded media. More precisely, we wished couple the use of (possibly high order) discontinuous Galerkin methods for the numerical approximation of the interior equations with a space-time Galerkin approximation of the integral equations that represent the transparent boundary conditions, using possibly a local time stepping for the interior equations (see the activity report of 2008 for the motivation).

This year was the last year of the contract ADNUMO. Two methods have been developed during the last two years for the case of the standard 3D wave equation: a conservative method (CM) (that preserves a discrete energy by construction - see the activity report of 2008) And a so-called post-processed method (PPM) that computes directly a solution that results from a post-processing of the solution given by (CM) (this second method was obtained by adapting the ideas developed in the past for volumic time stepping methods) . These two methods have been implemented and tested numerically. The emphasis has been put on designing an optimal implementation (two algorithms have been considered and compared). The numerical results we have obtained are quite good and have been compiled in a contract report. In short, these results confirm that (PPM) provides a better accuracy than (CM) for an equivalent computational cost. In particular, in the case of local time stepping, it reduces spurious phenomena induced by the aliasing process.

The theoretical aspects of our work have also be completed. This work has been widely presented in various conferences and a esearch article is being written.

The subject is now entering a more operational phase that should achieved in the framework of a new contract ADNUMO2. One additional objective is to extend the work to linearized Euler equations for the applications in aeroacoustics.

6.1.5. Numerical MicroLocal Analysis (NMLA)

Participants: Jean-David Benamou, Anne-Sophie Bonnet-Ben Dhia, Francis Collino, Patrick Joly, Simon Marmorat.

Part I: Application to seismic data

Speed analysis consists in the approximation of the smooth part of the underground wave speed using surface data called seismograms. It is one of the components of seismic imaging and and inverse problem : one tries to recover the seismogram using a numerical model that takes the sought speed as input. Asymptotic models (geometrical optics) are a popular choice because of their cheap cost but also as they correspond top the propagation regime for the class of speed considered. In this context one may wish to perform an assymptotic analysis of the seismograms. We try to apply the NMLA technique develloped at INRIA to this problem.

Part II: Application to Non-destructive control

The numerical simulation of acoustic wave propagation yields a scalar field at all points of the grid discretizing the domain. Focusing on the neighborhood of a fixed chosen point, we want to indentify a small number of significative rays (direction, amplitude, arrival time).

This is a difficult ill-posed non linear optimisation problem with many interesting applications. For instance in the coupling of geometric optics and Finite Difference or Finite Elements codes in Electromagnetics. Seismic imaging also need to extract these data for different processing purposes ...

We have obtained promising results on simple synthetic cases (point sources generated fields in slowly varying smooth media) by extending the numerical microlocal analysis technique originally working in the frequency domain to time domain wave propagation.

6.1.6. *Efficient numerical solution of the Vlasov-Maxwell system of equations*

Participant: Patrick Ciarlet.

A collaboration with Simon Labrunie (Nancy Univ.)

This is a follow-up to a theoretical work initiated with Eric Sonnendrücker and Régine Barthelmé (M3AS, 2007). The aim was to revisit existing approaches (see the works of Heintzé *et al* during the 90's) to compute the motion of charged particles, using an $H(curl, div)$ conforming discretization method to compute the electromagnetic fields. The improvements over the existing methods are twofold: with the help of the Weighted Regularization Method, one can handle 3D singular geometries, that can generate intense fields; also, one can solve systems in which the charge conservation equation is not satisfied rigorously. The numerical analysis has been carried out for explicit and implicit discrete time schemes. Finally, the so-called correction methods, very popular among engineers and applied physicists, are covered by this study. A paper on this topic has been accepted for publication in M3AS.

Similar problems are currently investigated in $2\frac{1}{2}$D axisymmetric geometries, using the Fourier Singular Complement Method, previously developed for scalar problems.

6.1.7. *Numerical Methods for Vlasov-Maxwell Equations*

Participants: Gary Cohen, Marc Duruflé, Alexandre Sinding.

There exist a large number of methods for approximating the motion of charged particles. They rely on a suitable discretization of Maxwell's equations, and a suitable discretization of Vlasov's equation.

A. Sinding's thesis, directed by G. Cohen, is devoted to the coupling of higher order hexaedral finite elements for Maxwell's equations with a Particle in Cell (PIC) method for Vlasov's equations. It is part of a joint work between ONERA Toulouse, CEG and INRIA rocquencourt.

Since continuous approximations seem more fitted to Vlasov'equations, an original implementation of continuous spectral finite elements for solving Maxwell's equations has been constructed. It is based on a mixed formulation of the system. In order to avoid the occurence of spurious modes, a dissipation term corresponding to the tangential jump of the discontinuous fields is added to the formulation. This penalization term is a good way to get rid of the parasitic modes, by sending them into the complex plane with a negative imaginary part (the parasitic waves becoming evanescent).

This solver (high order, continuous fields, numerical dissipation) has given good results in 2D and 3D . An important advantage of this penalization is that it eliminates spurious modes without introducing a divergence term in the formulation so that there will be no problem with non-convex geometries (see Costabel and Dauge). In particular we don't need to use correction techniques such as the Weighted Regularization (Costabel and Dauge) or the Singular Complement Method (Ciarlet et al.) which require the knowledge of the geometry of the computational domain.

So far this technique has been coupled with a Particle In Cell method using a charge conservative coupling such as described by Eric Sonnendrücker and Sebastien Jund for axisymmetric domains and comparisons remains to be done with alternative methods concerning charge and current conservation.

6.1.8. *Modelisation of coupled non linear piano strings*

Participants: Juliette Chabassier, Patrick Joly.

This work is developed in collaboration with Antoine Chaigne (UME, ENSTA).

The linear wave equation does not describe the complexity of the piano strings vibration well enough for physics based sound synthesis, and especially the modelization of a grand piano via numerical simulations. The nonlinear coupling between transversal and longitudinal vibrations has to be taken into account, as does the "geometrically exact" model described by Morse and Ingard. The string's stiffness can moreover be modeled by introducing an angle ϕ that represents the deviation of the string's section from the normal vector. This angle is coupled to the transversal vibration, leading finally to a nonlinear hamiltonian system of wave equations entering the second general class of systems of § 6.1.2.

Measurements have been made on on a stringed soundboard, in order to compare velocities at a certain point of the string with numerical results in similar conditions. Experimental studies on the agraffe have confirmed the assumption that this end of the strings does not significantly move, accordingly with the Dirichlet conditions used in our numerical simulations. For piano strings, the boundary condition at the other end of the strings are not as simple as Dirichlet conditions, because of the bridge vibration. Moreover, most of the notes of a piano are performed with several (one, two or three) strings, between which the bridge is the coupling point. An experimental and modelization work is in progress concerning the bridge, which has been for now considered as a damped linear oscillator but seems to behave as an elastic beam.

Our numerical method is based on a variational formulation of the coupled problem, a high order finite element approximation for the space discretization and energy dissipating schemes, extending the ideas of § 6.1.2, for the time discretization. The energy method allowed us to easily take into account the coupling with a nonlinear hysteretic felt hammer, as well as the coupling between strings and bridge.

6.1.9. Modeling Piezoelectric Sensors

Participants: Gary Cohen, Sébastien Impériale, Patrick Joly.

Non destructive sensors are built up with piezoelectric materials. These materials provide a small displacement when we apply a electric current and vice versa. To simulate their behaviour it is necessary to take into account the coupling between linear elastodynamics equations and Maxwells' equations. A first step has consisted in developing a simplified model. Because of the very large contrast between elastic waves velocity and electromagnetic waves velocity, at the time scale we are interseted in, the appropriate model for the electric part is an electrostatic model. Moreover we are able to restrict the computation of the electric potential on a relatively small subdomain. These approximate models have been justified theoritically, which enable to control the modelling error. For the numerical computation we couple an explicit scheme for the mechanical part of the model with an implicit scheme for the electric part. The different behaviours of the sensors (emission or reception) induce different boundary conditions so appropriate time discretizations have been studied and implemented. Theses discretizations are energy preserving, which ensure the stability of the resolution. Finally, we use a high order spectral finite element method to solve the elastodynamics equations, whereas the particular device's geometry enable us to use a modal resolution to solve the electrostatic problem.

6.1.10. High order time domain tetrahedral finite elements and GPU implementation

Participants: Alexandre Impériale, Sébastien Impériale.

Graphic processing unit offers a cheap way to do intensive computations. The trade off is that the on-chip memory space is small, this constraint motivates the developpment of particular algorithm to exploit GPU technologie for wave propagation problems. We have developped new high order tetrahedral elements. Whereas they do not provide a fully explicit scheme (no mass lumping) the stability properties of theses elements are optimized and nearly optimal. These elements are well suited for GPU since every computations can be made using small local matrices that fit into the GPU memory. Implementation of order 2 and 3 tetrahedral elements on relatively big problems provide encouraging results.

6.1.11. Evolution problems in locally perturbed infinite periodic media

Participants: Julien Coatléven, Patrick Joly.

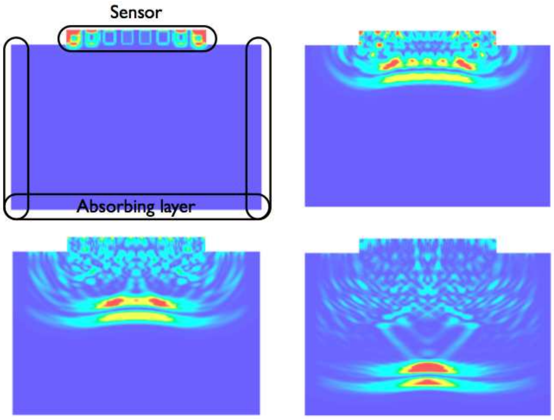

Figure 1. Simulation of a multi-element sensor. Each element of the sensor is excited with a constant voltage for a short period. This voltage is applied with a delay between elements. This delay enable to make the wave focus on a small region.

This work is part of the PhD of J. Coatléven. The principles and theoretical basis of a numerical method has been developped rigorously for the treatement of linear evolution problems in locally perturbed periodic media, for the moment in 1D and in wave-guide geometries. The idea is to take advantage of the periodicity to reduce the effective computations to small neighbourhood of the perturbation, by constructing transparent boundary conditions. The method is based on a semi-discretization in time of the problem in the whole infinite media, and the transparent boundary conditions constructed are showed to provide an interior solution that is exactly the restriction to the computational domain of the solution of the semi-dicretized problem, thus providing automatically stability of the method. The transparent boundary conditions are constructed through the resolution of semi-discretized unperturbed half-guide problems. We show that the solutions of these problems can be computed using a family of recursively defined functions, for which we use the periodicity to restrict our computations to local-cell problems. So far, we have tested numerically the method on several equations : the heat equation, the linear Schrödinger equation and of course the wave equation. The case of a locally perturbed infinite periodic media in $\mathbb{R}^2$ has been theoretically solved, and should be numericaly tested soon.

6.1.12. Ultrasonic Waves Propagation in a Bended Stratified Medium with Local Fold's Undulation Defect

Participants: Gary Cohen, Édouard Demaldent, Sébastien Impériale.

We were dedicated to the study and development of a finite element code that simulates the propagation of ultrasonic waves in a bended stratified elastic medium. This medium is embedded in fluid and can present a local fold's undulation defect.

The originality of this study lies in the behaviour of the elasticity tensor $\mathcal{C}$ which is not expressed in terms of the canonical basis vectors, but in terms of the tangents to the surface layers (including a rotation between each layer) and the normal to their average surface. When the medium is free from defect, the model is based on the shell model of Kirchhof-Love which restricts the set of admissible geometries. First, the function that maps a reference (flat) medium into a bended one has to be a diffeomorphism and, secondly, its cotangent basis has to be orthogonal (this implies a constant thickness in each layer). We define a defect as a transgression of the second law and introduce it by a local variation of the thickness. Figure 2 shows us some computations with and without defect.

A code has been fully developed (written in Fortran 90, almost 30.000 lines): it is operational in the two dimensional space and its extension to the three dimensional space should be confined to pre and post processing tasks. In this code, the mixed spectral finite element method (MSFEM) has been implemented within an inhomogeneous order of approximation technique. This trick is motivated by the low thickness of the layers (that restrains the increase of the order within the classical homogeneous MSFEM), and avoids us to suffer from the gap of celerity between fluid and solid domains (the fluid-solid coupling is nonconformal).

A suitable absorbing layer method had to be introduce to bound the domain of simulation. Classical (cartesian) perfectly matched layer (PML) methods can leads to instabilities since the wavefront depends on the bending, so we have investigated a new procedure to handle this problem. It consists in choosing absorbing directions that coincide with the directions associated to $\mathcal{C}$ (cotangent directions). Therefore, the change of variables that leads to the PML system is made with respect to the parametric coordinates. The procedure we use can treat the same range of anisotropic materials than classical PML and stands perfectly matched when the mapping function is locally linear. Another procedure that stands perfectly matched with any mapping function has also been tested but it increases computational time whereas results are not improved.

From a more general point of view, an alternative (not perfectly matched) absorbing method that can treat all elastic materials without corrupting too much the overall accuracy has been studied, and an original 'velocity-strain' formulation of linear elastodynamic equations that simplifies the PML equations system and reduces computational costs has been introduced.

6.2. Time-harmonic diffraction problems

6.2.1. Robust computation of eigenmodes in electromagnetism

Participant: Patrick Ciarlet.

This is a twofold work.

First, a collaboration with Annalisa Buffa (CNR, Pavia, Italy), Grace Hechme (former PostDoc) and Erell Jamelot (former PhD student).

To overcome the traditional difficulty linked to the apparition of spurious modes of the Maxwell's operator, we have proposed, analyzed and implemented a method based on a saddle-point formulation of the eigenvalue problem. Convincing numerical experiments have been carried out. Two papers on this topic have been published: one in CMAME (Dec. 2008), and one in Numer. Math. (2009).

Second, a collaboration with François Lefèvre and Stephanie Lohrengel (Reims Univ.) and Serge Nicaise (Valenciennes Univ.).

We have extended this method to composite materials, with the help of a generalized Weighted Regularization Method. Numerical results have been obtained. A paper on this topic has been accepted for publication in M2AN.

6.2.2. Time harmonic aeroacoustics

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

We are still working on the finite element approximation of the time harmonic Galbrun's equation for simulating the acoustic scattering and radiation in presence of a mean flow. This is now the object of an ANR project, AEROSON, in collaboration with Florence Millot and Sébastien Pernet at CERFACS, Nolwenn Balin at EADS and Vincent Pagneux at the Laboratoire d'Acoustique de l'Université du Maine. Remember that we use an augmented variational formulation to overcome the lack of coerciveness of the original model. The regularizing term requires the evaluation of the vorticity ψ (curl of the main unknown which is the displacement $\mathbf{u}$) which solves a hydrodynamic equation. Finally the complete problem couples the time harmonic augmented Galbrun's equation with a second order time harmonic advection equation. Our recent results concern mainly the theoretical and numerical study of the following time harmonic advection equation:

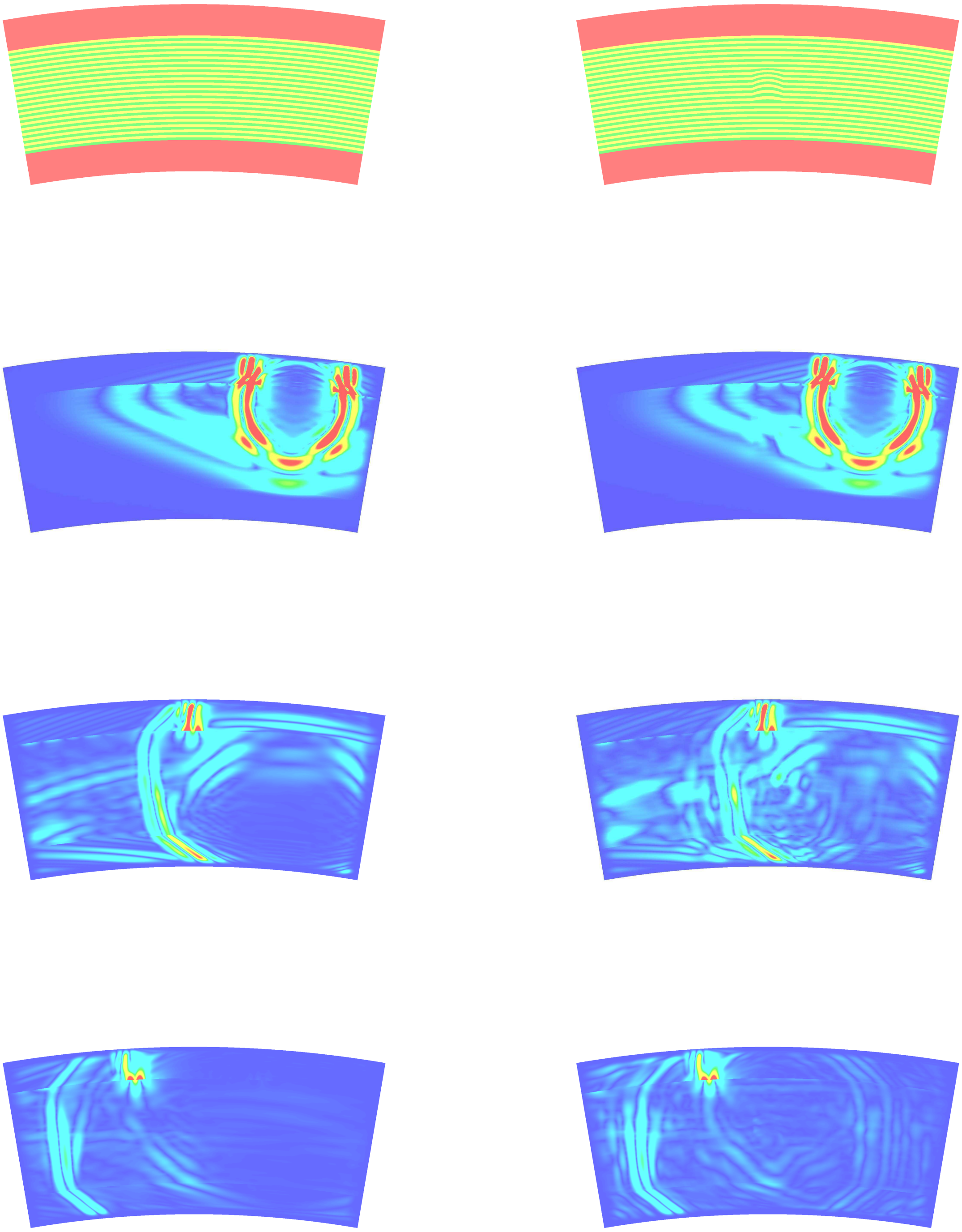

Figure 2. Velocity field for a stratified medium (two kinds of material) embedded in fluid (red part): free from defect (left) and with defect (right).

$$-i\omega\psi + \mathbf{V}\cdot\psi = f \quad \text{in } \Omega \quad \text{and} \quad \psi = 0 \quad \text{on } \Gamma^-$$

where the flow $\mathbf{V}$ is incompressible and Γ^- is the inflow boundary of Ω. Available results in the literature concern the case $\omega = ia$ with $a \in \mathbb{R}$ and the extension to the case $\omega \in \mathbb{R}$ is not straightforward. We proved that:

- The problem is well-posed if the flow $\mathbf{V}$ is Ω filling, in the sense that the streamlines coming from Γ^- cover all the domain Ω. In particular, recirculation streamlines are forbidden.
- The solution can be approximated by a Discontinuous Galerkin scheme, the convergence being established in some appropriate discrete norm.

Using these results, we have proved that the complete problem, restricted to a bounded computational domain by using Perfectly Matched Layers, is coercive + compact, if the flow varies slowly enough. The numerical method which couples continuous nodal finite element for $\mathbf{u}$ with the DG scheme for ψ has been implemented at CERFACS and the validation is in progress.

6.2.3. Harmonic wave propagation in locally perturbed infinite periodic media

Participants: Julien Coatléven, Sonia Fliss, Patrick Joly.

Two main domains of applications are concerned :

- the propagation of electromagnetic waves in photonic crystals, a subject that we study in the framework of the ANR Project SimNanoPhot, in collaboration with the Institut d'Electronique Fondamentale (IEF, Orsay University). This project ends at December 2008.
- The propagation of ultrasonic waves in composite materials, in view of their nondestructive testing, a subject developped within a collaboration with EADS.

There is a need for efficient numerical methods for computing the propagation of waves inside media that are not necessarily periodic but differ from periodic media only in bounded regions (small with respect to the total size of the propagation domain) containing scatterers (obstacles, cracks, inhomogeneities). For these problems, a major difficulty is the reduction of the pure numerical computations to bounded domains surrounded the perturbations. The key point is to take advantage of the periodic structure of the problem outside to construct artificial 9but exact) boundary conditions. That is why we have investigated then the generalization of the DtN approach to periodic media.

This was the subject of the PhD Thesis of Sonia Fliss who has defended her thesis in May 2009. In this work, we proposed a solution for construction DtN operator for the 2D Helmholtz equation in the case of a single perturbation. We show in particular that we can factorize the DtN operator as a product of two operators. The first operator corresponds to a halfspace DtN operator and is constructed with a method which combines analytical tools such as the Floquet Bloch Transform with the numerical solution of local cell problems. The second operator requires the numerical solution of non stantdard integral equations in the (x, k) plane, where k denotes the dual variable in the Floquet Bloch Transform. The method is well established, rigorously justified and successfully tested in the case of absorbing media.

The treatment of non absorbing media raises complicated and new questions and requires the definition of an appropriate numerical procedure that should correspond to the continuous limiting absorption principle. The difficulties concern the discretization of the (x, k)-compact set and the resolution of the non standard integral equation.

Moreover, for theoretical and numerical issues, we have extended the method to the construction of Robin-to-Robin operator : instead of relating the Dirichlet and the Neumann traces of the solution, we want to relate two different Robin traces of the solution. From a theoretical point of view, this approach makes sense for a non absorbing media. Indeed, the construction of the DtN operator needs the resolution of family of local cell problems with Dirichlet conditions. For non absorbing media, this introduces a set of frequencies corresponding to the eigenvalues of the laplace operator with periodic coefficients on periodicity cell. This set of forbidden frequencies, due to the construction of DtN operator, is artificial and disappears with the construction of RtR operator. From a numerical point of view, one of the interest of RtR operators is that, contrary to DtN operators for instance, they are bounded operators with bounded inverse and their discretization leads to well-conditionned matrices. This extension is the subject of a chapter that we have written for the E-Book "Wave Propagation in Periodic Media Analysis, Numerical Techniques and practical Applications".

The first part of the PhD of J. Coatléven has been devoted to devicing a unified vision for multi-scattering problems, showing that for any situation where one could identify a reference medium and several compact perturbations, the DtN map on the boundary of the union of the scatterers can be expressed directly from the resolution of songle-scattering problems, thus allowing to treat multiple-scattering in a periodic medium (see the activity report of 2008). This work has been concretized by a talk during the 9th International Conference on Mathematical and Numerical Aspects of Waves (WAVES'09) in Pau, another during the Progress In Electromagnetics Research Symposium (PIERS 2009) in Moscow, and an article has been submitted. Recent work has also showed that the theoretical framework developped readily extends to the situation of a perturbed interface problem (seen as a multiple scattering problem), and thus will allow quasi-realistic non-destructive testing simulations in near future.

Moreover, we intend to treat more general pertubations and in particular line defects (i.e. the perturbation is infinite in one dimension). In optics, such defects are created to construct an (open) waveguide to concentrate light. The existence and the computation of the eigenmodes is a crucial issue. This is related to a seladjoint eigenvalue problem associated to a PDE in an unbounded domain (namely in the directions orthogonal to the line defect), which makes both the analysis and the computation hard. We believe that, by adapting the DtN approach developed for scattering problems, we shall offer a rigorously justified alternative to existing methods and an improvement of the computational time since we can reduce the numerical computation to a small neighborhood of the defect. However, there is a price to be paid : the reduction of the problem leads to a nonlinear eigenvalue problem, of a fixed point nature.

This research offers a number of interesting perspectives and developpments from theoretical, numerical and practical point of view. Applied to photonic crystals modelization, this work enters in the framework of the collaboration with the laboratory of Fundamental Electronics of Orsay University. For now, our collaboration has focused on the modelling of the light refraction at the surface of a photonic crystal and has been concretized by a co-signed article submitted to Physical Review B. Some perspectives of the collaboration with the electronic and optic communities is to generalize this study to a 3D problem, to analyse the extension of this method to electromagnetism and to study more general defects. These perspectives encourage us to ask for an ARC project with the BACCHUS team project of Inria Bordeaux for their high precision numerical expertise and with the laboratory of Fundamental Electronics for the modelling and the comparison with realistic experimentations.

6.2.4. Modeling of meta-materials in electromagnetism

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Ciarlet, Lucas Chesnel.

Meta-materials can be seen as particular dielectric media whose dielectric and/or magnetic constant are negative, at least for a certain range of frequency. This type of behaviour can be obtained, for instance, with particular periodic structures. Of special interest is the transmission of an electromagnetic wave between two media with opposite sign dielectric and/or magnetic constants. As a matter of fact, applied mathematicians have to address challenging issues, both from the theoretical and the discretization points of view.

The (simplified) scalar model can be solved efficiently by the most "naive" discretization. It turns out that the convergence of the numerical approximation can be proved via a uniform stability estimate. Interestingly, it can also be solved numerically by introducing some dissipation in the model (first topic addressed by L. Chesnel, during his Master intership).

Last year, we considered the study of the transmission problem in a 3D electromagnetic setting from a theoretical point of view: to achieve well-posedness of this problem, we had to proceed in several steps, proving in particular that the space of electric fields is compactly embedded in L^2. For that, we assumed some regularity results on the interface. The second topic now studied by L. Chesnel is how to remove this assumption, and allow for instance to solve the problem around an interface with corners.

Finally, we are currently investigating configurations in which the problem is ill-posed. The aim is to define a suitable functional framework to recover well-posedness.

6.2.5. A Multiscale Finite Element Method for 2D Photonic Crystals

Participant: Kersten Schmidt.

This is a joint project with Christoph Schwab and Holger Brandsmeier (ETH Zürich).

Photonic crystals (PhC) structures are dielectric materials with a periodic fine structure. Light injected into the PhC is diffracted and refracted by the many dielectric scatterers arranged in the periodic arrays. Some properties of photonic crystal structures can predicted by the model of the infinite crystal with the same pattern which leads to the well-known band structure. For each wave-length comparable to the size of the periodicity there exist a set of functions – the Bloch modes – propagating in the infinite crystal. We propose a multiscale finite element method for finite structures of photonic crystals with basis functions of two scales. The small scale is represented by Bloch modes where the large scale functions are polynomials. We derived a fast quadrature formula leading to computational costs independent of the number of crystal periods inside a macroscopic cell.

At a first model we investigated the TE and TM modes in a photonic crystal band with finite extend in one and infinite in the other. We hold the number of Bloch modes and increase the macroscopic polynomial degree. We observed the relative error decaying rapidly when increasing hte polynbomial degree is independent of the number of crystal periods inside a macroscopic cell. The next step are true two-dimensional PhC structures.

6.2.6. Accurate computation of influence coefficients

Participants: Marc Lenoir, Nicolas Salles.

The dramatic increase of the efficiency of the variational integral equation methods for the solution of scattering problems must not hide the difficulties remaining for an accurate numerical computation of some influence coefficients, especially when the panels are close and almost parallel.

A complete set of explicit formulas has been derived for the case of constant basis functions and plane triangular panels, when the singularity of the kernel is homogeneous of order -1. The object of the thesis of Nicolas Salles is to test the accuracy and efficiency of these formulas, to extend their range of use to affine basis functions and other degrees of homogeneity and to implement them in a finite elements computer software.

6.3. Absorbing boundary conditions and absorbing layers

6.3.1. High order Absorbing Boundary Conditions for Anisotropic Models

Participant: Éliane Bécache.

Our collaboration with Dan Givoli (Technion, Israel), Tom Hagstrom (Southern Methodist University) on high order ABC goes on.

The first topic of the collaboration concerns the extension of high-order ABC (the so-called Hagstrom-Warburton or H-W ABC) that were introduced by Givoli et al, and Hagstrom et al for isotropic models, to a very general anisotropic scalar model, which includes in particular the anisotropic scalar wave equation and the convective (dispersive and non-dispersive) wave equation. We have proposed the construction of high order ABC for outgoing waves (in the sense of the sign of the group velocity). We have worked on the analysis of these ABC (well-posedness showed with the Kreiss theory and in some cases with energy estimates, reflection coefficients....). The efficiency of these conditions have been shown with numerical results obtained for the anisotropic wave equation. A paper containing this work has been submitted and accepted for publication.

The second topic has been done in collaboration with Daniel Rabinovich (post-doc at Technion, Israel). It is a numerical work which compares the accuracy of high-order ABC and PMLs for the Helmholtz equation. The comparison between the two methods is done for the same cost of computation, which is determined either by the order of the ABC or by the number of elements in the absorbing layer. A paper on this topic has been submitted for publication.

We are now working (still in collaboration with D. Rabinovich) on the extension of the H-W high-order ABC to the isotropic elastic waves.

6.3.2. Leaky modes and PML techniques for non-uniform waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Benjamin Goursaud, Christophe Hazard.

This topic was initiated in the framework on the ANR SimNanoPHot (with the Institut d'Electronique Fondamentale, Orsay), about the simulation of tapers in integrated optics, or more generally varying cross section open waveguides. Our motivation was to study the possible use of the so-called *leaky modes* in the numerical simulation of such devices. The work presented in the previous report concerned the case of two-dimensional waveguides. The generalization to the three-dimensional case was recently studied. Using an infinite PML surrounding the core of the waveguide, which amounts to a complex stretching of spatial coordinates, the leaky modes appear as the eigenvalues of the transverse component of the stretched Helmholtz operator defined in a two-dimensional section. We have achieved the spectral analysis of this operator in the case of radial PMLs, but not for the more familiar cartesian PMLs for which open questions remain. Using a PML of finite width yields a numerical approximation of the leaky modes. Numerical results were obtained for both radial and cartesian PMLs. As for 2D waveguides, an instability related to the exponentially increasing spatial behaviour of leaky modes was observed. Again, the remedy consists in reducing the intermediate zone between the core and the PML. Two kinds of 3D numerical computations have been made. On one hand, the knowledge of the leaky modes furnishes a very efficient way of computing the Green's function of a 3D uniform waveguide. On the other hand, these leaky modes can be used to express a diagonal form of the Dirichlet-to-Neumann (DtN) operator of a semi-infinite uniform waveguide. We have dealt with the case of the junction of two such waveguides: the problem can be reduced to a bounded region containing the junction using PMLs in the transverse direction and the DtN operators at both ends of the junction.

6.3.3. Perfectly matched layers for one-way wave equations

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard, Patrick Joly, Jérôme Le Rousseau.

Perfectly matched layers are used as absorbing boundary layers to simulate the wave equation as they yield no reflection. When one is interested in the part of the wavefield that propagates in a preferred direction, one-way wave equations can be derived. The wave equation is diagonalized according to this direction. The one-way operators are of pseudo-differential type away from transverse propagation. The issue of designing proper absorbing layers is also important for these one-way wave equations. We investigate how the PMLs introduced by Berenger can be transposed to the one-way setting. Even in constant media, the PML-one-way operator is intricate and its numerical evaluation is rather involved. We are studying ways to obtain efficient numerical methods.

This is a joint project in collaboration with Alison Malcolm of the Massachusetts Institute of Technology (Cambridge, MA, USA), with application to seismic imaging as performed in exploration geophysics.

6.4. Waveguides, resonances, and scattering theory

6.4.1. Elastic waveguides

Participants: Vahan Baronian, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Éric Lunéville.

In partnership with the CEA, we are developing numerical tools to simulate ultrasonic non-destructive testing in elastic waveguides.

During the PhD of Vahan Baronian (which has been defended in November), a finite element method has been developed to compute the scattering by an arbitrary local perturbation of an isotropic 2D or 3D guide. By using modal expansions, specific boundary conditions are written on the artificial boundaries which are perfectly transparent, allowing the FE computation zone to be reduced as small as possible.

During this year, Vahan Baronian has extended the method to the more complicated configuration of a junction of several possibly different waveguides. From a theoretical point of view, he also proved that the method works in an orthotropic waveguide, as soon as one symmetry axis of the material is parallel to the axis of the waveguide.

We are now investigating the case of a 3D fully anisotropic plate, in partnership with CEA and EADS. We first consider the diffraction by a 2D obstacle (a stiffener for instance) at oblique incidence, which leads to a 2D problem.

6.4.2. Acoustic propagation in a lined waveguide

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

We have continued our work in collaboration with Emmanuel Redon of University of Dijon on the acoustic radiation in an infinite 2D lined guide (absorbing boundary condition on a wall) with a uniform mean flow. The aim was, in order to bound the domain, to build transparent boundary conditions, introduced by means of a Dirichlet to Neumann (DtN) operator based on a modal decomposition. Such decomposition is easy to carry out in a hard-walled guide. With absorbing lining and in presence of a mean flow, many difficulties occur. Even without flow, the eigenvalue problem is no longer selfadjoint but a relation for which the modes are orthogonal is found. In the flow case is only found a generalized orthogonality relation asymptotically satisfied by the high-order modes, which allows to define transparent boundary conditions.

We focus now on improving the mathematical framework of this problem and in characterizing the modes : clarifying their asymptotic behavior and proving their completness. In the no flow case, the modes behave asymptotically like the modes of a hard-walled guide. Then, following the approach developed in the literature for water waves, completness of the modes by comparison with the rigid modes and well-posedness of the radiation problem (defining a coercive+compact problem) can be proved. In the flow case, the asymptotic behavior of the modes is unexpected : they correspond to a soft boundary, even for a slow flow. Our objective is now to extend the completness proof of the no flow case.

6.4.3. Propagation in non uniform open waveguide

Participants: Anne-Sophie Bonnet-Ben Dhia, Benjamin Goursaud, Christophe Hazard.

This topic follows the work in collaboration with Lahcene Chorfi (University of Annaba, Algeria) and Ghania Dakhia (University of Biskra, Algeria) about the problem of the scattering of a time-harmonic acoustic wave by a defect located in a two-dimensional uniform open waveguide. The theoretical analysis of this problem using a generalized Fourier transform has been presented in the previous activity reports, and has led to an article published in 2009. The natural continuation of this work concerns the propagation in a non uniform open waveguide, more precisely the question of the junction of two different uniform open waveguides. The question we consider concerns the existence and the uniqueness of the solution to the propagation equations together with two *modal radiation conditions* imposed to the scattered field on both sides of the junction. The direction we have followed consists in a combination of the technique used in the above mentioned work and the idea developed a few year ago in the lab by Anne-Sophie Bonnet-Ben Dhia and Axel Tillequin in the case of an abrupt junction (along a line perpendicular to the direction of propagation). We have made a lot of progress this year, but some difficulties concerning the question of uniqueness remain.

6.4.4. Multiple scattering in a duct

Participants: Éric Lunéville, Jean-François Mercier.

We study the multiple scattering of acoustic waves by rigid obstacles randomly placed in a duct. Multiple scattering regime corresponds to the propagation of a field with a wavelength comparable to the scatterers size. The aim is to find the characteristic parameters of a homogeneous medium modelizing this heterogeneous medium. The usual approach to find such equivalent medium is to send a plane wave and to calculate the mean field by averaging on many configurations of scatterers. We have developped a fast numerical method to compute such field. It is based on the coupling of finite element in a neighborhood of the scatterers (to reduce the mesh size) with an integral representation of the scattered field. A difficulty is that this last part requires the computation of the Green's function of the guide, which expresses as an expansion converging slowly. To enhance the reduction of computation time, perturbed periodic arrangements of scatterers are considered : they are placed on a periodic lattice and then randomly locally moved. Then it is possible to parallelize the computations : the scattering area is splited in slices and scattering matrices of each slice is computed. We prove numerically that only the plane wave has a coherent part in the heterogeneous medium : all the amplitudes of the other guided modes vanish after averaging. Moreover we obtain that the transmission through the averaged medium follows the law predicted by analytical models originally developed in the free space (not in a duct) excepting at the band gap frequencies of the periodic lattice, where the transmission vanishes as in a periodic medium.

6.4.5. Modelling of TE and TM Modes in Photonic Crystal Wave-Guides

Participant: Kersten Schmidt.

This is a joint project with Roman Kappeler (ETH Zürich).

In this project we study the band structure in photonic crystal wave-guides of finite width and infinite periodicity in the other direction. The TE and TM modes for a given frequency ω are determined by a quadratic eigenvalue problem in the quasi-momentum k in the unit cell. The unit cell – an infinite strip – is truncated and the decay conditions are replaced by approximative boundary conditions. We perform numerical experiments for realistic wave-guides with the p-version of the finite element method on meshes with curved cells.

6.5. Asymptotic methods and approximate models

6.5.1. Multiscale modelling in electromagnetism

Participants: Bérangère Delourme, Patrick Joly.

This topic is developed in collaboration with the CEA-Grenoble (LETI) and H.Haddar (INRIA-Saclay-DEFI) and is dedicated to the study of asymptotic models associated with the scattering of electromagnetic waves from a complex periodic structure. More precisely, this structure is made of a dielectric ring that contains two layers of wires winding around it (see figure 3). We are interested in situations where the thickness of the ring and the distance between two consecutive wires are very small compared to the wavelength of the incident wave and the diameter of the ring. One easily understands that in those cases, direct numerical computations of the solution would become prohibitive as the small scale (denoted by δ) goes to 0, since the used mesh would need to accurately follow the geometry of the heterogeneities.

In order to overcome this difficulty, we derive approximate models where the periodic ring is replaced by effective transmission conditions. The numerical discretization of approximate problems is expected to be much less expensive than the exact one, since the used mesh has no longer to be constrained by the small scale.

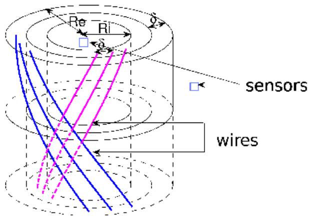

Figure 3. the periodic ring

In a first part, we have studied a simplified 2D case: we have constructed a complete and explicit expansion of the solution with respect to the small parameter δ and we have derived approximate models. These models are theoretically and numerically validated. For one year, we have been interesting in the 3D Maxwell case (which is the interesting model for the application) which presents new difficulties. The first one is due to the finite length of the ring: we need to understand the behavior of the electromagnetic field in the vicinity of the two extremities of the ring. This work has been partially done in collaboration with X.Claeys: we have studied a simplified two dimensional case and proved the relative accuracy of the first order intuitive approximate model. However, the building of approximate models of higher orders seems to be difficult. To avoid this first difficulty, we now consider the 3D Maxwell case with periodic boundary conditions on the extremities of the ring. We have derived an asymptotic expansion of the solution and an approximate model. From both hand computations and functional analysis points of view, the study of Maxwell's equations is more difficult than the study of Helmholtz equation.

6.5.2. *Quasi-singularities and electrowetting*

Participants: Patrick Ciarlet, Thu Huyen Dao.

A collaboration with Claire Scheid (Nice Univ.).

This is a twofold work.

First, the Master internship of Thu Huyen Dao. Following the PhD thesis of Samir Kaddouri (2007), she studied quasi-singularities for the 2D-cartesian electrostatic model around rounded tips, using the electric field as the primary unknown, instead of the potential. To complement this work, we are investigating the computation of accurate maps of the values of the electric field, to model corona discharge phenomena around 2D and $2\frac{1}{2}$D tips.

Handling very small amount of liquid on a solid surface is of great industrial interest. In this field, electrowetting process is now broadly used: one charges a droplet posed on a solid by applying a given voltage between this droplet and a counter-electrode placed beneath the insulator. This allows one to control precisely the wetting of the drop on the solid. For modeling purposes, one has to compute very accurately the shape of the drop near the counter-electrode by solving an electrostatic problem with a piecewise constant electric permittivity. We recently considered 3D configurations, based on the numerical approximation of the electric field, using a generalized Weighted Regularization Method (see § 6.2.1). A paper on this topic has been accepted for publication in M2AN.

6.5.3. *Asymptotic models for junctions of thin slots*

Participants: Katrin Boxberger, Patrick Joly, Adrien Semin.

We have almost finished the work started in 2007 for the acoustic case. We have considered the most general possible case (a finite number of slots and junctions, and the slots may have different width). We have studied the two differents aspects of this problem:

- the theoretical point of view: as for the case of two junctions and one slot, we completly justify the asymptotic expansion. We plan to publish (at least) one INRIA Research Report and one article in Asymptotic Analysis,
- the numerical point of view: with the fellowship of Katrin Boxberger, we developed a C++ oriented-object code named "Net Waves" (this code is available on the INRIA GForge web site at url http://gforge.inria.fr/projects/netwaves). This code is particular in the sense that there's no code at our knowledge which solves acoustic wave equation on a general finite network, even with classical Kirchhoff conditions. This code boards graphical output and is still maintained in the project.

6.5.4. Wave propagation on infinite trees

Participants: Patrick Joly, Adrien Semin.

We have continued the work started in 2007, on two different ways.

- Firstly, we have implemented an approximation of transparent boundary conditions for the Helmoltz equation on a self-similar p-adic tree in the code "Net Waves" mentionned in § 6.5.3. We are currently making some regressions tests to be able to test these conditions. The next step, to be able to do many computations, is to write a GPU version of this code.
- Secondly, we started a collaboration with Serge Nicaise from the University of Valenciennes since July 2009 to look the functionnal framework and the notion of trace at infinity on a general (not necessarily self-similar) p-adic tree.

6.5.5. Approximate models in aeroacoustics

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Joly, Lauris Joubert, Ricardo Weder.

This is the subject of the PhD thesis of Lauris Joubert and the object of a collaboration with M. Duruflé.

Two aspects of the subject have been considered.

First we have completed our work on a simplified model for the propagation of acoustic waves in a duct in the presence of a laminated flow. The theoretical analysis of this model has been completed in two directions:

- The stability analysis of the model in function of the Mach profile has been achieved completely. In the unstable case, an analogy with the known results about Kelvin-Helmholtz instabilities for incompressible fluids (Rayleigh anf Fjorjtoft criteria) has been established. An article has been submitted for publication.
- We have developed a general method for obtaining a quasi-analytic representation of the solution that results into a priori estimates. This method in based on the use of the Fourier-Laplace transform and complex analysis methods. An article has been submitted.

The quasi-analytic representation of the solution has been exploited numerically (see for instance the result of figure in the case of a parabolic Mach profile). The comparison with results obtained by discretizing the full model (Galbrun's equations) is under way.

The second aspect we have first developed is the construction of effective boundary conditions for taking into account boundary layers in aeroacoustics. On the basis of the analysis of the thin duct problem, we have proposed a first effective condition whose stabilty has been proven. This condition has the practical disadvantage to be nonlocal with respect to the normal coordinate inside the boundary layer. One can obtain a local condition after approximating the exact Mach profile by a piecewise linear profile. The study and the implementation of this new condition will be the subject of our next contribution to the problem.

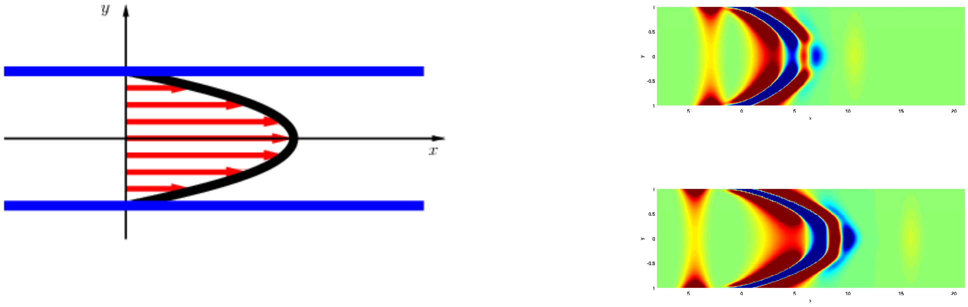

Figure 4. Asymptotic wave propagation in a thin duct. We plotted solution with two differents times.

6.5.6. Impedance boundary conditions for the aero-acoustic wave equations in the presence of viscosity

Participants: Bérangère Delourme, Patrick Joly, Kersten Schmidt.

This is a joint work with Sébastien Tordeux (INSA Toulouse).

In compressible fluids the propagating sound can be described by linearised and perturbed Navier-Stokes equations. This project is dedicated to the case of viscous fluid without mean flow. By multiscale expansion and matched asymptotic expansion we are deriving impedance boundary conditions taking into account the viscosity of the fluid. The first case is a plain wall and the second a wall with periodic perforations where we apply surface homogenisation.

6.6. Imaging and inverse problems

6.6.1. Quasi-reversibility

Participants: Laurent Bourgeois, Jérémi Dardé, Éric Lunéville.

We have continued our works on the method of quasi-reversibility to solve second order ill-posed Cauchy problems for an elliptic equation (as they appear in standard inverse problems). This constitutes part of the subject of the PhD thesis of J. Dardé. In particular, we have developped a new strategy to identify obstacles in a domain from partial Cauchy data on the boundary of such domain. This strategy uses a coupling between the method of quasi-reversibility and a level set method. Too types of level set method have been studied in the case of an obstacle which is characterized by a Dirichlet boundary condition $u = 0$. The first one relies on the solution of an eikonal equation, while the second one relies on the solution of a simple Poisson equation. The second one turns out to be particularly easy to implement and very efficient. Some theoretical justifications have been provided for each method. We have also extended this strategy to the case of non standard boundary conditions such as $|\nabla u| = 1$. Such condition arises in the field of mechanical engineering, precisely in the problem of identification of plastic zones from simultaneous measurements of displacements and forces on the boundary. A promising application is the Non Destructive Evaluation of cracks in elastoplastic media.

6.6.2. Linear sampling Method

Participants: Laurent Bourgeois, Frédérique Le Louër, Éric Lunéville.

We aim at extending some previous work concerning the Linear Sampling Method in an acoustic waveguide to the case of an elastic waveguide. This constitutes the subject of the Post Doc of F. Le Louër and we have begun the first investigations. The main challenge relies on the fact that we can no longer use a family of transverse modes in order to project the displacement field. One possibility consists in using a family of two vector fields formed with particular combinations of displacement and stress components, for which a bi-orthogonality relationship in the transverse section has been proved (Fraser relationship). We hence expect to use propagating modes based on these vector fields as incident waves and the corresponding scattered waves measured at long distance in order to retrieve unknown obstacles, in the framework of the Linear Sampling Method.

6.6.3. Identification of generalized boundary conditions

Participants: Laurent Bourgeois, Nicolas Chaulet, Houssem Haddar.

This work is a collaboration between POEMS and DEFI projects and constitutes the subject of the PhD thesis of N. Chaulet. In the context of acoustics in the harmonic regime, we have first considered the problem of identification of some Generalized Impedance Boundary Conditions (GIBC) at the boundary of an object (which is supposed to be known) from far field measurements associated with a single incident plane wave at a fixed frequency. The GIBCs are approximate models for thin coatings or corrugated surfaces. We have addressed the theoretical questions of uniqueness, stability, as well as the numerical reconstruction of these boundary parameters.

6.7. Other topics

6.7.1. Spectral analysis of elliptic operators and application to control

Participants: Laurent Bourgeois, Jérôme Le Rousseau, Matthieu Léautaud.

The techniques used in control of evolution PDEs depend very often on some spectral properties of the generator if one uses the semigroup point of view. Such spectral properties rely on powerful functional inequalities such as Carleman estimates. The controlled systems are of parabolic type here.

We are interested in several subjects: (i) the case of non smooth coefficients, (ii) systems of coupled equations, and (iii) discretization issues.

(i) Non smooth coefficients

Carleman estimates have been known for a long time in the case of smooth coefficients in the principal part of the operator. In the case of a regularity as low as Hölder there are some counterexamples. Here, we are interested in coefficients that exhibit jumps across interfaces, for elliptic and parabolic operators. Classical techniques to prove such estimates fail to work in this situation. We manage to prove such Carleman estimates by relying on tools from semi-classical microlocal analysis (e.g. calderéron projectors).

We are also interested in models where diffusion phenomena take place in co-dimension one at the interface. This is joint work with Luc Robbiano (Université de Versailles).

(ii) systems of coupled equations

We address here the controllability of several coupled parabolic equations. This type of problem arises in particular in biology, considering for example coupled reaction and diffusion processes. The difficulty is that we want to control two different evolution problems with only one control force. We hence have to prove spectral estimates (by means of Carleman estimates) with only one observation. Moreover, these phenomena can be non-symmetric. In this situation the generator operator is no more diagonalizable. To treat this Problem, we have to introduce a different functional setting, relying on resolvent estimates for perturbed selfadjoint operators.

We are also interested in the stabilization of coupled hyperbolic equations (such as wave equations). This is joint work with Fatiha Alabau-Boussouira (Université de Metz).

(iii) Discretization issues

Discretization and control properties do not “commute” well in general. Exactly controllable equations can become uncontrollable (even for approximate controllability) after discretization. This happens for the wave equation and also for parabolic equations. For the parabolic case, we prove discrete Carleman estimates that yield uniform controllability results for the lower part of the spectrum in the case of a semi-discretization in space. Uniformity is then with respect to the discretization parameters. The analysis does not rely on filtering out high frequencies. In fact we prove that the high-frequency content of the controlled solution tends to zero exponentially as we refine the discretization. We also address the numerical analysis of the fully discretized problem. This is joint work in collaboration with Florence Hubert and Franck Boyer (Aix-Marseille universities), in the framework of the ANR project CoNum led by Jérôme Le Rousseau

6.7.2. Reactive transport in porous media

Participant: Adrien Semin.

This is a joint work with Jean-Baptiste Apoung from University of Paris-Sud, Pascal Havé from IFP, Jean-Gabriel Houot from University of Paris 5 and Michel Kern from team Estime (INRIA Rocquencourt).

We started a work last year about reactive transport in porous media. In this work, we developped a numerical method for coupling transport with chemistry in porous media. Our method is based on a fixed-point algorithm that enables us to coupled different transport and chemistry modules. This work finished at the beginning of this year with a publication in ESAIM Proceedings.

6.7.3. Linear elasticity

Participant: Patrick Ciarlet.

A collaboration with Philippe Ciarlet (City Univ. of Hong Kong).

Following a number of theoretical studies (carried out with the help of G. Geymonat and F. Krasucki (CRAS, 2007), C. Amrouche (Analysis and Applications, to appear)), we are considering the numerical approximation of linearized elasticity problems, via the St-Venant approach. Within this framework, one builds conforming finite element subspaces of the stress tensors. In this sense, this approach is different from the mixed approaches, whose aim is to compute approximations of both the displacement and stress.

This problem has been solved for the planar case. The stress tensors are approximated with the help of discrete, piecewise constant, symmetric tensor fields, defined on triangles, and one solves a discrete optimization problem. A paper on this topic has been published in M3AS (2009).

Higher degree approximations and extensions to quadrilaterals and to 3D configurations are currently investigated with Stefan Sauter (Zürich Univ.) and Blandine Vicard (ENSTA).

7. Contracts and Grants with Industry

7.1. Contract POEMS-CEA-LIST

Participants: Gary Cohen, Édouard Demaldent.

G. Cohen perticipates to Projet CASSIS headed by the LIST laboratory of CEA and funded by the EADS Fundation which started in June 2008. This project aims to simulate elastic waves in thin layered anisotropic media for non-destructive testing. In collaboration with E. Demaldent, who will start a post-doc at Inria in the beginning of 2009, G. Cohen must provide a code based on spectral element methods to model these waves.

7.2. Contract POEMS-ONERA-CE Gramat

Participants: Gary Cohen, Marc Duruflé, Morgane Bergot.

In collaboration with ONERA-DEMR, G. Cohen participates with M. Bergot to the FEMGD project funded by CEG (Centre d’Études de Gramat), which started in 2004. This project is devoted to the construction of a software using spectral discontinuous Galerkin methods for Maxwell’s equations. This project came to an end in December 2008.

7.3. Contract POEMS-CE Gramat

Participants: Gary Cohen, Alexandre Sinding.

In collaboration with ONERA-DEMR, G. Cohen participates with A. Sinding to the NADEGE project funded by CEG (Centre d'Études de Gramat), which started in September 2008. This project is devoted to the construction of a software based on FEMGD for solving Vlasov-Maxwell's equations by a PIC method.

7.4. Contract POEMS-DGA

Participants: Laurent Bourgeois, Éric Lunéville.

This contract concerns the identification of buried objects using the linear sampling method.

7.5. Contract POEMS-Airbus

Participants: Patrick Joly, Jerónimo Rodríguez.

This contract (Project ADNUMO) is about the hybridation of time domain numerical techniques in aeroacoustics (Linearized Euler equations).

8. Other Grants and Activities

8.1. National Cooperations

- GDR Ultrasons: this GDR, which regroups more than regroup 15 academic and industrial research laboratories in Acoustics and Applied Mathematics working on nondestructive testing. It has been renoveled this year with the participation of Great Britain.
- ANR (RNTL) project *MOHYCAN: MOdélisation HYbride et Couplage semi-ANalytique pour la simulation du CND.*

  Topic: *On the coupling of the finite element code ATHENA with the semi-analitic code CIVA. Nondestructif testing.* Collaborators: CEA-LIST (main contact), EDF and CEDRAT.
- ANR project *AEROSON: Simulation numérique du rayonnement sonore dans des géométries complexes en présence d'écoulements réalistes*

8.2. International Cooperations

- The research project, “Improved Imaging in Non-Smooth Media”, is a starting collaboration with Prof. Alison Malcolm (Massachusetts Institute of Technology, Dept. of Earth, Atmospheric, and Planetary Sciences). It has obtained the support of the MIT-France fund (MIT-France Seed Fund for Collaborative Research) for the years 2008–2009.

8.3. Visiting researchers

- Peter Monk, Professor at University of Delaware, USA.
- Ricardo Weder, Professor at University of Mexico, Mexico.
- Alison Malcolm, Professor at MIT, USA.
- Elie Turkel, Professor at Tel-Aviv, Israël.

9. Dissemination

9.1. Various academic responsibilities

- A. S. Bonnet-Ben Dhia is the Head of the Electromagnetism Group at CERFACS (Toulouse)
- A. S. Bonnet-Ben Dhia is in charge of the relations between l'ENSTA and the Master "Dynamique des Structures et des Systèmes Couplés (Responsible : Etienne Balmes)".
- P. Ciarlet is an editor of DEA (Differential Equations and Applications) since July 2008
- P. Joly is a member of the scientific committee of CEA-DAM.
- P. Joly is a member of the Commission de Spécialistes of the University Paris VII.
- P. Joly is a member of the Hiring Committee of Ecole Polytechnique in Applied Mathematics.
- P. Joly is a member of the Post Docs Commission of INRIA Rocquencourt.
- P. Joly is a member of the Scientific Committee of the Seminar in Applied Mathematics of College de France (P. L. Lions).
- P. Joly is an editor of the journal Mathematical Modeling and Numerical Analysis.
- P. Joly is a member of the Book Series Scientific Computing of Springer Verlag.
- P. Joly is an expert for the MRIS (Mission pour l'Innovation et la Recherche Scientifique) of DGA (Direction Générale de l'Armement)
- P. Joly is a scientific expert for the "Fondation de Recherche pour l'Aéronautique et l'Espace" in the thematic "Mathématiques Appliquées au domaine de l'Aéronautique et Espace".
- M. Lenoir is a member of the Commission de Spécialistes of CNAM.
- M. Lenoir is in charge of Master of Modelling and Simulation at INSTN.
- J. Le Rousseau is coordinator of the CoNum project, "Numerical control, application to biology", Projet ANR JeuneS chercheurs.
- J. R. Li is one of the guest editors of the special issue of the Journal of Computational mathematics that will follow the WAVES2007 Conference.
- E. Lunéville is the Head of UMA (Unité de Mathématiques Appliquées) at ENSTA.
- The Project organizes the monthly Seminar Poems (Coordinators: J. Chabassier, J. Dardé)

9.2. Teaching

- Éliane Bécache
  - *Introduction à la théorie et l'approximation de l'équation des ondes*, 3ème année ENSTA-Paris et Master 2 UVSQ
  - *Introduction à la discrétisation des équations aux dérivées partielles*, 1ère année ENSTA, Paris
  - *Compléments sur la méthode des éléments finis*, 2ème année ENSTA, Paris
- Laurent Bourgeois
  - *Outils élémentaires pour les EDP*, ENSTA, Paris
  - *Contrôle optimal des EDP*, ENSTA, Paris
- Anne-Sophie Bonnet-Ben Dhia
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles. MA102*, Cours de Tronc Commun de 1ère année à l'ENSTA
  - *Propagation d'ondes*, Cours commun au Master de Dynamique des Structures et des Systèmes Couplés et à l'Option de Mécanique (filière VO) de l'Ecole Centrale de Paris
  - *Propagation dans les guides d'ondes. C7-3*, Cours de 3ème année à l'ENSTA. En collaboration avec Éric Lunéville.

  - *Théorie spectrale des opérateurs autoadjoints et application aux guides optiques. MAE21*, Cours de 2ème année à l'ENSTA. En collaboration avec Christophe Hazard et Jean-François Mercier.
- Juliette Chabassier
  - Math315 : *Calcul scientifique, Initiation à MATLAB*. (License 3e année)
  - Math315 : *Calcul scientifique, travaux pratiques sur machine (MATLAB)*. (License 3e année)
  - Math208 : *Simulations numériques, travaux pratiques sur machine (MATLAB)*. (License 2e année)
  - Math266 : *Algèbre III et géométrie, travaux dirigés en petite classe*. (License 2e année)
- Patrick Ciarlet
  - *The finite element method*, ENSTA (2nd year)
  - *Distributed computing: a theoretical viewpoint*, ENSTA (3rd year), and Master "Modeling and Simulation" (2nd year)
  - *Maxwell's equations and their discretization*, ENSTA (3rd year), and Master "Modeling and Simulation" (2nd year)
- Julien Coatléven
  - *Méthode des éléments finis*, ENSTA, Paris (2nd year)
  - *Systèmes dynamiques : Stabilité et commande*, ENSTA, Paris (1st year)
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris (1st year)
- Bérangère Delourme
  - *Travaux dirigés d'Analyse*, Licence 1ère année, Université Paris Dauphine
- Sonia Fliss
  - *Méthode des éléments finis*, ENSTA, Paris (2nd year)
  - *Simulation Numérique*, ENSTA, Paris (2nd year)
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris (1st year)
- Benjamin Goursaud
  - *Analyse numérique et optimisation*, ENSTA, Paris
  - *Fonction d'une variable complexe*, ENSTA, Paris
  - *Introduction à matlab*, ENSTA, Paris
  - *Introduction au calcul scientifique pour les admis sur titres*, ENSTA, Paris
- Christophe Hazard
  - *Outils élémentaires d'analyse pour les EDP*, 1ère année, ENSTA Université Paris XI
  - *Théorie spectrale des opérateurs auto-adjoints et applications aux guides optiques*, 3ème année, ENSTA
- Sebastien Impériale
  - *Introduction à la discrétisation des EDP*, ENSTA, Paris
- Patrick Joly

  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
  - *Outils élémentaires d'analyse pour les EDP*, ENSTA, Paris
  - *Problèmes directs et inverses en théorie de la diffraction*, Master 2 Paris 6, Parcours Analyse Numérique et EDP
  - *Méthodes volumiques et couches PML pour les problèmes de propagation d'ondes en régime transitoire*, Collège Polytechnique.

- Lauris Joubert
  - *Chaîne de Markov*, 2ème année ENSTA
  - *Equations différentielles et introduction à l'automatique*, 1ère année ENSTA
  - *Introduction au calcul des probabilités et à la statistique*, 1ère année ENSTA
- Matthieu Léautaud
  - *Calcul Matriciel*, Licence 1, UPMC Paris 6
  - *Fonctions de plusieurs variables*, Licence 2, UPMC Paris 6
  - *Suites, séries et intégrales*, Licence 2, UPMC Paris 6
- Éric Lunéville
  - *Introduction au calcul scientifique*, Cours de 2ème année à l'ENSTA, Paris
  - *Programmation scientifique et simulation numérique*, Cours de 2ème année à l'ENSTA, Paris
  - *Propagation dans les guides d'ondes*, Cours de 3ème année à l'ENSTA, Paris
- Jean-François Mercier
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, Travaux dirigés de 1ère année à l'ENSTA
  - *Variable complexe*, Travaux dirigés de 2ème année à l'ENSTA
  - *Ondes dans les milieux continus*, Cours de 3ème année à l'ENSTA
  - *Théorie spectrale des opérateurs autoadjoints et application aux guides optiques. MAE21*, Cours de 2ème année à l'ENSTA. En collaboration avec Anne-Sophie Bonnet-Ben Dhia et Christophe Hazard.
- Adrien Semin
  - *Algorithmique et programmation*, Licence 3ème année, Université Paris XI

9.3. Participation in Conferences, Workshops and Seminars

- Éliane Bécache
  - *Questions de stabilité des couches PML pour des modèles anisotropes.*, Séminaire Sciences Numériques pour la Mécanique, Laboratoire Mécanique des Sols, Structures et Matériaux, ECP, April 28.
  - *The Ficitious domain method and applications in wave propagation. Part B: The acoustic case*, Minisymposium "Computational Methods for Waves" at the 2nd International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2009), June 22-24, Rhodes, Greece.
- Morgane Bergot
  - *Higher-Order Finite Elements for Hybrid Meshes*, MAFELAP 09, June 2009;

  - *Higher-Order Finite Elements for Hybrid Meshes*, Waves 09, June 2009.
- Anne-Sophie Bonnet-Ben Dhia
  - *Simulation numérique du rayonnement acoustique dans un écoulement*, Séminaire au Laboratoire de Mathématiques Appliquées de Compiègne, February 2009.
  - *Topographic elastic waveguides.* Workshop on the fundamental aspects of edge resonances in elasticity, April 2009, Paris.
  - Invited talh at the Workshop Polaritons 2009, Aussois, May 2009. *Numerical simulation of the interface between a metamaterial and a dielectric.*
  - WITH J.-F. MERCIER, F. MILLOT AND S. PERNET *Time-harmonic acoustic scattering in a complex flow : a full coupling between acoustics and hydrodynamics.* Waves'09, Pau, June 2009.
  - *Simulation de l'interface entre un métamatériau et un diélectrique.* Séminaire à l'Université de Reims, October 2009.
- Laurent Bourgeois
  - *Séminaire du laboratoire* LMAM, Université de Metz, France, March 6th
  - *Séminaire du projet DEFI*, Ecole Polytechnique, France, April 9th
  - *Applied Inverse Problems 2009*, University of Vienna, Austria, July 20th-24th
  - *Séminaire du laboratoire Dieudonné*, Université de Nice, France, November 12th
- Juliette Chabassier
  - *Energy preserving scheme for a nonlinear system of piano strings*, SANUM 09, Stellenbosch, South Africa, April 2009.
  - *Energy preserving schemes for nonlinear systems of wave equations. Application to piano strings*, WAVES 09, Pau, France, June 2009
- Patrick Ciarlet
  - MAFELAP 2009, Jun. 2009
- Julien Coatléven
  - *Operator Factorization for Multiple Scattering Problems and an Application to Periodic Media*, 9th International Conference on Mathematical and Numerical Aspects of Waves (WAVES'09), jun, 2009, Pau
  - *Conditions aux limites parfaitement transparentes pour la résolution numérique de problèmes d'évolution en milieux périodiques présentant un défaut localisé*, Seminaire POEMS, Oct, 2009.
- Bérangère Delourme
  - *Approximate models for wave propagation across thin periodic interfaces*, Waves'09, June 18th, Pau
- Édouard Demaldent
  - *Fast High-Order Finite Elements for the Integral Equations of Time-Harmonic Maxwell Problems*, Waves'09, June 18th, Pau
- Sonia Fliss
  - *Exact boundary conditions for wave propagation problems in periodic media including a local perturbation*, Applied Math seminar, Stanford University, Feb. 2009.

  - *Transparent boundary conditions for periodic media : numerical issues*, 9th International Conference on Mathematical and Numerical Aspects of Waves (WAVES'09), jun. 2009, Pau
  - *Numerical and mathematical analysis of wave propagation in locally perturbed infinite periodic media*, Séminaire Analyse numérique et EDP, Laboratoire P. Painlevé, Université Lille 1, Oct. 2009.
  - *Transparent boundary conditions in periodic media*, Institut für Numerische und Angewandte Mathematik (NAM), Georg-August-Universität Göttingen, Nov. 2009.
  - *Conditions aux limites transparentes pour les milieux périodiques*, Séminaire Sciences Numériques pour la Mécanique, Ecole Centrale Paris, Dec. 2009.

- Benjamin Goursaud
  - *Leaky modes in an open waveguide*, Waves'09, Pau, June 2009.
- Christophe Hazard
  - *A multimodal method for non-uniform open waveguides*, in collaboration with A.-S. Bonnet-Bendhia, B. Goursaud and A. Prieto, International Congress on Ultrasonics, Santiago, Chile, January 11-17, 2009.
  - *A modal radiation condition in open waveguides* in collaboration with A.-S. Bonnet-Bendhia, L. Chorfi and G. Dakhia, Waves'09, June, Pau.
- Sebastien Impériale
  - *Mixed Spectral Elements for the Cauchy-Poisson problem in unbounded doamins*, WAVES 2009, Pau, France.
- Patrick Joly
  - *Coupling Discontinuous Galerkin Methods and Retarded Potentials for Time Dependent Wave Propagation Problems.* 25th Annual GAMM-Seminar Leipzig on FEM and BEM for time-dependent wave problems, Leipzig, January 2008.
  - *Coupling Discontinuous Galerkin Methods and Retarded Potentials for Time Dependent Wave Propagation Problems.* Seminar at Göttigen University, January 2008.
  - *Asymptotic models in aeroacoustics.* South African Symposium on Numerical and Applied Mathematics , Stellenbosch, South Africa, April 2009
  - *Stability Analysis of Acoustic Wave Propagation in a Thin Duct in the Presence of a Shear Flow*, Conference WAVES2009, Pau, June 2009.
  - *Numerical modelling of multiple-scattering problems in periodic media*, Conference PIERS 2009, Moscow, August 2009.
  - *Asymptotic models in aeroacoustics.* Workshop MACADAM (Multiscale Asymptotics and Computational Approximation for surface Defects and Applications in Mechanics), ENS Cachan Antenne de Bretagne, Rennes, September 2009.
  - *Asymptotic models in aeroacoustics.* Seminar at BCAM (Basque Center for Applied Mathematics), Bilbao, September 2009.
  - *Asymptotic models in aeroacoustics.* Seminar at the University of Santiago de Compostela, Santiago de Compostela, September 2009.
- Lauris Joubert
  - *A quasi-1D model for aeroacoustics in thin ducts*, WAVES09, Pau, France, juin 2009
- Matthieu Léautaud

  - *A spectral approach of the null controllability of coupled non-selfadjoint parabolic systems*, Control of Distributed Parameter Systems 2009, Toulouse, France, juillet 2009
  - *Inégalités spectrales pour des opérateurs elliptiques non autoadjoints et application la contrôlabilité des systèmes paraboliques*, Journées Metz-Nancy-Besançon-Strasbourg de contrôle des EDP, Besançon, octobre 2009
- Éric Lunéville
  - *Mise en oeuvre des méthodes de couplage EF/Spectral Atelier Mélina*, Dinard, May 2009
- Jean-François Mercier
  - *Dirichlet-to-Neumann operators for an acoustic waveguide with a uniform flow and an impedance boundary condition*. Waves'09, Pau, June 2009.
  - *Time-harmonic acoustic scattering in a complex flow : a full coupling between acoustics and hydrodynamics*. Waves'09, Pau, June 2009.
- Kersten Schmidt
  - *Modelling of TE and TM Modes in Photonic Crystal Wave-Guides*, Waves'09, 15th June 2009, Pau, France.
  - *Anisotropic hp-adaptive H(curl)-conforming FE space in 2D on meshes with hanging nodes*, ICOSAHOM'09, 24th June 2009, Trondheim, Norway.
  - *hp-adaptive Finite Elements in the C++ library Concepts*, Zurich NanoOptics Workshop 2009, 8th July 2009, Zürich, Switzerland.
  - *High order transmission conditions for conductive thin sheets - asymptotic expansions versus thin sheet bases*, Macadam Workshop, 1st Sept. 2009, Rennes, France.
  - *Transmission conditions by asymptotic expansion versus thin sheet bases*, Swiss Numerics Colloqium, 24th April 2009, Basel, Switzerland.
- Adrien Semin
  - *Étude de la propagation des ondes dans des jonctions de fentes minces*, University of Paris-Sud, January 2009
  - *Reactive Transport in porous media*, OpenBang Seminar, February 2009
  - *Résolution de l'équation des ondes dans un réseau infini*, Chevaleret, April 6th
  - *Construction and Analysis of Improved Kirchhoff Conditions for Acoustic Wave Propagation in a Junction of Thin Slots*, Waves'09, June 18th, Pau
- Alexandre Sinding
  - *Éléments finis mixtes spectraux continus pour le calcul de modes propres électromagnétiques*, IRMAR Seminar, Rennes, February 2009
  - *Continuous Mixed Spectral Finite Elements for the Maxwell Eigenproblem*, Waves'09, Pau, June 2009

9.4. Miscellanous

The textbook entitled "The Finite Element Method. From Theory to Practice. I. General Concepts" (in French) by Patrick Ciarlet and Eric Lunéville, has been published (2009) in the Series Les Cours, Coll. ENSTA.

10. Bibliography

Year Publications

Doctoral Dissertations and Habilitation Theses

[1] V. Baronian. *Couplage des méthodes modale et éléments finis pour la diffraction des ondes élastiques guidées. Application au Contrôle Non Destructif*, École Polytechnique, 11 2009, Ph. D. Thesis.

[2] É. Demaldent. *Etude de schémas de discrétisation d'ordre élevé pour les équations de Maxwell en régime harmonique*, Université Paris Dauphine, 1 2009, Ph. D. Thesis.

[3] S. Fliss. *Etude mathématique et numérique de la propagation des ondes dans des milieux périodiques localement perturbés*, École Polytechnique, 5 2009, Ph. D. Thesis.

Articles in International Peer-Reviewed Journal

[4] J.-B. Apoung, P. Havé, J.-G. Houot, M. Kern, A. Semin. *Reactive Transport in porous media*, in "ESAIM Proceeding", vol. 28, 11 2009, p. 227–245, http://dx.doi.org/10.1051/proc/2009049.

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