INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team RealOpt

Reformulation based algorithms for Combinatorial Optimization

Bordeaux - Sud-Ouest

Theme : Optimization, Learning and Statistical Methods

Activity Report

2009

Table of contents

REALOPT is an INRIA Team joint with University of Bordeaux (UB1) and CNRS (IMB, UMR 5251 and LaBRI, UMR 5800)

1. Team

Research Scientist

Ruslan Sadykov [INRIA, Research Associate (CR)]

Faculty Member

François Vanderbeck [University Bordeaux 1, Team Leader, Professor (Pr)]
Andrew Miller [University Bordeaux 1, Professor (Pr)]
Arnaud Pêcher [INRIA, Assistant Professor (MdC), until Aug. 2009. He got a Professor position at the University of Toulouse and is now an external collaborator of the team, HdR]
Gautier Stauffer [University Bordeaux 1, Assistant Professor on an INRIA Chair of Excellence (MdC), since Sep. 2009]
Philippe Meurdesoif [University Bordeaux 1, Assistant Professor (MdC). Since Aug. 2009 he is an external collaborator]
Pierre Pesneau [University Bordeaux 1, Assistant Professor (MdC)]

External Collaborator

Sophie Michel [University of Le Havre, Assistant Professor (MdC)]
André Raspaud [University Bordeaux 1, Professor (Pr)]
Eric Sopena [University Bordeaux 1, Professor (Pr)]
Annegret Wagler [University of Madgdeburg, Germany, Research Assistant]
Xuding Zhu [National Sun Yat-sen University, Taiwan, Professor (Pr)]

Technical Staff

Céline Saubatte [INRIA, Associate Engineer until Aug 2009]

PhD Student

Yuri Faenza [Univertisy of Rome Tor Vergata]
Laurent Gely [SNCF, CIFRE]
Cédric Joncour [University Bordeaux 1, MENRT grant]
Guillaume Massonnet [G-SCOP, Grenoble]
Mustafa Kılınc [University of Wisconsin]
Mahdi Namazifar [University of Wisconsin]
Benoit Vignac [University Bordeaux 1, University of Montréal scholarship]
Dimitri Sverdlov [University Bordeaux 1, since Sep. 2009]

Administrative Assistant

Nathalie Dupuy [INRIA, until Oct. 2009]
Marie-Christine Echegoyen [INRIA, since Dec. 2009]

2. Overall Objectives

2.1. Overall Objectives

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: investment budgeting, long term planning, the management of scarce resources, or the planning of day-to-day operations. Many optimization problems that arise in decision support applications involve discrete decision variables. Such problems can be modeled as linear or non-linear programs with integer variables. The solution is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space. Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain

problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Central to our field is the characterization of polyhedron defining or approximating the solution set, and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an infeasible point. One must also avoid the drawback of enumerating what are essentially symmetric solutions.
The team's overall goals are:

1. To design tight formulations for specific problems and generic models, relying on delayed cut and column generation, extended formulations, and inputs from non-linear programming (in particular quadratic programming) and graph theory.
2. To contribute to theoretical developments in exact optimization and combinatorial optimization, while extending the scope of their application.
3. To demonstrate the strength of cooperation between complementary exact mathematical programming techniques, constraint programming and combinatorial algorithms.
4. To develop algorithms for large-scale real-life applications that provide provably good approximate solutions by hybridation of heuristic and exact methods.
5. To provide generic software tools that build on our research developments, writing proof-of-concept code, while making our research findings available to external users.

2.2. Highlights

This year, the team got the opportunity to recruit an assistant professor on a Chair of Excellence granted by the INRIA. Gautier Stauffer has been granted this position. After a PhD in Operations Research at the Swiss Institute of Technology in Lausanne, he spent one year at the Massachusetts Institute of Technology (MIT). Before joining us, he worked as a researcher at the IBM research center of Zurich.
The field of his research concerns combinatorial optimization, integer programming and polyhedral combinatorics. His main result was to prove the Ben Rebea conjecture, that was open for 25 years, yielding a better understanding on the stable set polytope, a subject that get a intensive attention the 20 last years.

We can also emphasize the promotion of Arnaud Pecher who took a Professorship at the University of Toulouse.

F. Vanderbeck and L. Wolsey have contributed to the book "50 Years of Integer Programming 1958-2008: From the Early Years to the State-of-the-Art" [37]. This book is a uniquely useful celebration of the past, present and future of this important and active field. The chapter to which we contributed, presents the state-of-the-art in reformulation and decomposition approaches in integer programming. This was a great opportunity to place RealOpt as one of the excellence center in these thematics that are in the core of our project.

3. Scientific Foundations

3.1. Introduction

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are discrete in nature. Binary variables model on/off decisions to buy, invest, hire, send a vehicle, or enforce a precedence. Integer variables model indivisible quantities. Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are among the most widely used modeling tools. They allow a fair description of reality; they are versatile; they can handle many non-linearities (and even non-convexities) in the cost function and in the constraints; they are also well-suited for global optimization. However useful they may be, these models are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm* : (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a factional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. State-of-the-art MIP solvers – such as the commercial solvers CPLEX of Ilog or Dash-Optimization's Xpress-mp – are remarkably effective. But many real-life applications remain beyond their scope, and the scientific community is actively seeking to extend the capabilities of MIP solvers. Developments made in the context of specific applications often become generic tools over time and see their way into commercial software.

The most effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions and hence provide better LP bounds, Lagrangian decomposition methods to produce alternative powerful relaxations, constraint programming to actively reduce the solution domain through logical implications, heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidate solutions, and specialized branch-and-bound or dynamic programming enumeration schemes to find a global optimum. The real challenge is to integrate the most efficient methods into one global system. Another key to further progress is the development of stronger problem formulations whose relaxations provide approximations that enable enhanced truncation of enumerative solution schemes. Tighter formulations are also much more likely to yield good quality approximate solutions through rounding techniques. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation bounds.

Our project brings together researchers with expertise mathematical programming (polyhedral approaches, Dantzig-Wolfe decomposition, quadratic programming, non-linear integer programing, stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems.

3.2. Polyhedral Approaches

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attemps to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [137]. Introduced by Edmonds in 1965 [74], the polyhedral approach has turned out to be one of the main sources of progress in solving NP-hard combinatorial optimization problems in the last two decades. A benchmark problem, in this regard, is the traveling salesman problem [112]. In the early 80's, the best algorithm was able to solve instances with around 300 cities. A recent paper [50] reports that branch-and-cut algorithms are able to solve instances with nearly 25,000 cities. Similar significant improvements have been observed for instance for network design problems arising in telecommunication (see Kerivin and Mahjoub, 2005 [109]) or vehicle routing problems in logistic (see Letchford and Salazar-Gonzalez, 2006 [114]).

The goal of these approaches is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions, $conv(X)$, where X is the discrete set of solutions to the combinatorial problem on hand. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem and solving the separation problem over the associated polyhedron: if $\widetilde{x} \notin conv(X)$, find a linear inequality $\pi\, x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\widetilde{x}$ [92]. Hence, for NP-hard problems, one cannot hope to define either a closed-form description of $conv(X)$ or a polynomial time exact separation routine. Nevertheless, one does not need to know such a description to take advantage of the polyhedral approach. Only a subset of the inequalities can already yield a good approximation of the ideal polytope. Moreover, non-exact separation, using heuristic procedures, turns out to be quite efficient for practical purposes.

Polyhedral theory provides ways to derive automatically new inequalities from an initial polyhedral description P of the problem. For instance, it is known [134] that any valid inequalities for an IP can be obtained by iteratively taking linear combinations of existing constraints and rounding their coefficients. Such *general purposes cuts* have only recently made their way as practical tools: for instance, Gomory fractional cuts are now generated by default into commercial MIP solvers. Recent work [62], [80] has consisted in numerically testing the strength of the formulation obtained by application of a single round of such general purpose cuts (called the first-closure): the separation problem being set as an MIP problem which is solved with a commercial MIP solver. Cornuéjols (2006) [65] provides a comparative review of general purpose cuts such as lift-and-project cuts, Gomory mixed integer cuts, mixed integer rounding cuts, split cuts, and intersection cuts, as well as their practical contributions to dual bound improvements. However, the most promising results have often been obtained with so-called *template cuts*, i.e. family of valid inequalities derived in an application specific context: the close form expression of these additional inequalities is a template from which specific cuts are generated dynamically. To prove validity, one can show that such inequalities can be obtained as a special case of general purpose procedures. If it can be shown that the inequalities define so-called *facets* of $conv(X)$, these inequalities are needed for its description. In practice, one needs to develop efficient procedures (exact or heuristic) to separate these inequalities. Then, numerical evaluation can show the impact of the additional inequalities not only on the strength of the resulting dual bound but also in yielding solutions more likely to satisfy integrality restrictions (which is good for primal heuristics).

The connections between polyhedral structure and graph theory are deep. Many facets of various polyhedra are directly related to special classes of graphs. The literature is rich with such examples of facet-defining systems described by exhibiting a bijection to a collection of subgraphs of the studied graph: forest polytopes (Edmonds, 1961), the matching polytope (Edmonds, 1965, [74]), and many others. There are even results showing that the structure of a specific polyhedron itself is closely related to the structure of a related graph. For instance, Chvátal has shown that the property of adjacency in the stable set polytope of a graph G (i.e., the fact that two solutions satisfy the same facet defining constraint at equality) is characterized as a connectivity property of G itself. A number of researchers have recently described analogous interpretations of other facets of the stable set polytope. For example, Lipták and Lovász (1999) [116] exhibited such a relationship; in [141] we describe another and use it to generate a new set of facets for the stable set polytope of webs.

3.3. Dantzig-Wolfe decomposition and branch-and-price algorithms

An alternative to using LP bounds to prune the branch-and-bound tree is to use bounds from *Lagrangian relaxation*. Lagrangian relaxation consists in transforming some of the hard constraints of a problem into soft constraints that can be violated with a given penalty. The idea is to keep as hard constraints those that define a well-structured combinatorial problem that is much easier to solve than the original. In particular, one may want to relax linking constraints without which the problem decomposes into much smaller and simpler problems. The best bounds can then be obtained by properly adjusting the penalty cost. Choosing the best set of penalties is itself an optimization problem: the so-called *Lagrangian dual*. It can be reformulated as a linear program with an exponential number of variables (that are associated with the generators of the subproblem solution sets): this is the so-called *Dantzig-Wolfe reformulation*. This large LP can be solved using the revised simplex method while generating its variables and associated columns in the course of the optimization: this is the so-called *column generation* procedure. If branch-and-bound enumeration is based on the Dantzig-Wolfe reformulation of the problem, one must use column generation at each node of the branch-and-bound tree giving rise to the so-called *branch-and-price* algorithm.

Branch-and-price algorithms are more recent than branch-and-cut algorithms. Although column generation appeared in the 60's as a technique to handle linear programs with a huge number of variables [49], [86], its combination with branch-and-bound to solve integer programs was only developed in the 90's [54] (we did pioneering work on this subject [161]). Branch-and-price proved to be very useful in solving many practical problems that were intractable by other means: crew and fleet assignment problems faced by airline companies

[70], [152], vehicle routing problems in public and in fret transport systems [71], cutting stock problems experienced in the paper, textile, or steel industry [139], [154], [156], [160], production planning problems [52], [159], and network design problems [148], [40] are such examples. Branch-and-price has now become the reference method for problems well suited to decomposition and it is making its way in industry: for instance, decision aid software developed by consultant firms like Eurodecision (Paris) or Adopt (Montréal) rely on this approach. The use of this method in the practical context of challenging applications has revealed its limitation. Further developments are required to overcome these difficulties.

Indeed, using column generation in the context of integer programming is not straightforward. The primary challenges revolve around the convergence of the dual bound computations, the enforcement of integrality restrictions and the combination with dynamic cutting plane generation. A first step toward targeting these difficulties was to clarify the underlying Dantzig-Wolfe decomposition principle. While the standard view was to see Dantzig-Wolfe decomposition as the linear programming formulation of the Lagrangian dual [134], we presented it as a reformulation technique that gives rise to an integer master program ([155] does this for the integer case, while [153] extends this view to the mixed integer case). Then, the integrality restrictions of the original formulation translate into integrality restrictions in the reformulated problem (this is the *discretization* approach). Our framework based on the concept of generating sets facilitates the handling of branching decisions (to enforce integrality) and adding cutting planes to the formulation.

Natural applications for the Dantzig-Wolfe approach are problems whose constraint matrix has a block diagonal structure augmented with linking constraints, i.e. a constraint matrix A of the form

$$A = \begin{pmatrix} L & L & ... & L \\ B^1 & 0 & ... & 0 \\ 0 & B^2 & ... & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & ... & B^K \end{pmatrix}, \tag{1}$$

with K blocks B^k, $k = 1, ..., K$. Then, dualizing constraints L decomposes the problem into K sub-problems of smaller size. Moreover, if the K blocks are identical, the column generation reformulation eliminates the symmetry in k and there is only one pricing problem that needs to be solved during the column generation algorithm (the same column being optimal for all K subsystems). Beyond this standard application framework, open questions concern the viability of applying the decomposition principle recursively or to multiple independent subsystems. We develop a first application of a nested decomposition in [156]. A decomposition based on multiple independent subsystems (not necessarily disjoint) should provide tighter bounds according to the theory of Lagrangian decomposition [97]. But, there is typically a large number of dualized constraints linking the different systems and hence many dual prices to adjust (therefore one expects slow convergence). To our knowledge the literature does not report of any column generation approach based on multiple subsystems capturing different combinatorial structures. A recent study however comes close to such multiple decomposition: [69] combines variable redefinition applied to one subsystem and column generation applied to another subsystem. The dual bounds that they obtained are shown to be tighter than with alternative approaches but the computational times are much larger.

3.4. Quadratic approaches

The communication between the non-linear programming and the combinatorial optimization communities is limited although the latter has much to learn from the former. Several of the latest developments in discrete optimization are imports from convex optimization [59], [88], [66]. Quadratic programming (QP), in particular, offers very powerful modeling tools. A *quadratic program* is a formulation in continuous variables whose cost and/or constraint functions are polynomials of degree 2. Several classical constraint types for combinatorial problems are more efficiently modeled with (QP): binary constraints ($x_i \in \{0, 1\}$ can be equivalently set as $x_i^2 = x_i$); sequencing constraints: for instance in scheduling problems, if job j follows job i, denoted by $x_{ij} = 1$, then their completion times must satisfy $C_j \geq C_i + p_j$ where p_j is the processing time of j; this can be modeled as $x_{ij}\,(C_j - C_i - p_j) \geq 0$; transitivity constraints represent requests of the type "if two variables x_i and x_j are equal to 1, a third one x_k should be 1" (their quadratic formulation $x_k \geq x_i x_j$ is stronger than the linear $x_k \geq x_i + x_j - 1$ when one relaxes the integrality restrictions).

Although Quadratic Programming (QP) is NP-Hard, some special cases or relaxations are polynomially solvable. Minimizing a convex quadratic cost over a feasible region described by linear constraints is easy. This method is implemented in commercial MIP solvers. Convex QP with linear constraints and 0-1 variables can then be solved using branch-and-bound and solving the continuous QP relaxation at each node. Also, the *Semi-Definite Programming* (SDP) relaxation of a QP is polynomially solvable. An SDP is an extension of an LP where variables are the components of a matrix that is constrained to be semi-definite. When the objective is not convex, it can be convexified using an augmented relaxation approach [60], [59]: the Hessian is made convex by adding to the objectives a weighted sum of the quadratic binary constraints and the squared norm of equality constraints (optimized weights are obtained by solving an SDP relaxation). When applied to an already convex objective, this approach is still useful to improve the continuous relaxation bound. Solvers are available for SDP [55], but, in their current implementation, they are very sensitive to the conditioning of the matrices.

Even though the numerical solution of QP and SDP remains problematic, recent applications of these techniques to some combinatorial problems have led to major improvements [117], [107], [89], [59]. Generally, an SDP relaxation can be found for any general Integer Program [140]. Starting from the QP reformulation of an IP, a Lagrangian relaxation procedure is applied to yield an SDP: after Lagrangian dualization, the (unconstrained) QP has a solution (in the primal variables) iff some matrix of coefficients is positive semidefinite. The associated SDP bound is always better than classical LP relaxation [113], although sometimes the size of the SDP formulation is problematic. In former studies, we discovered other SDP formulations for Lovász's bound on vertex coloring [119]: a direct quadratization that appears to be intermediate between the ones of [107] and [117]. The SDP formulation obtained by application of the general scheme of [140] is of huge dimension and, because of symmetry, does not bring more than our (compact) SDP formulation. Yet, it can be fruitfully used to compute bounds on generalizations of Vertex Coloring where symmetry does not hold (List Coloring, some problems of Frequency Assignment) [119].

3.5. Non-linear integer programming

Many engineering, management, and scientific applications involve not only discrete decisions, but also nonlinear relationships that significantly affect the feasibility and optimality of solutions. Mixed-integer nonlinear programming (MINLP) problems combine the difficulties of MIP with the challenges of handling nonlinear functions. MINLP is one of the most flexible modeling paradigms available. Hence, an expanding body of researchers and practitioners, including engineers, operations managers, economists, statisticians, computer scientists, and mathematical programmers are now interested in solving large-scale MINLPs (for recent examples of studies based on MINLP models, see, e.g., [56], [61], [111], [91], [128], [143]).

The wealth of applications that can be accurately modeled by using MINLP is not yet matched by the capability of available optimization solvers. Both MIP and nonlinear programming (NLP) have witnessed tremendous progress over the past 15 years. Some of the factors that have gone into the development of effective MIP algorithms and powerful academic, open source, and commercial MIP solvers are described above; similarly, new paradigms and a better theoretical understanding have created faster and more reliable NLP solvers that work well even under adverse conditions such as failures of constraint qualifications (more details on recent improvements in NLP solvers, as well as on challenges that remain, can be found in [72], [85], [150], [55], [164], among others.)

The time is right to synthesize these advances and inspire new ideas in order to transform MINLP into an area in which researchers and practitioners can access robust tools and methods capable of solving a wide range of important, commonly occurring decision support problems. While there remains enormous room for progress, initial efforts towards the development of such algorithmic tools are already under way. Our team is involved in the enhancement of existing ideas, and the development of new ones, towards these ends (see, e.g., [110],[43]).

3.6. Stochastic programming

Numerous common application areas involve factors that are inherently random or uncertain (such as varying demands, machine failures, surgery duration times, and cost overruns). When such models also have discrete or binary decision variables, for any of the reasons already discussed, the combination of uncertainty and combinatorial nature makes these problems more difficult than if one were trying to address either the uncertain or the discrete aspects of these problems by themselves. Stochastic MIP models have been proposed for applications in resource acquisition planning [57], internet server capacity expansion [47], electric power management [149], and inventory management [58].

Despite their ubiquity, not much is yet known about how to solve practical MIPs in which uncertainty plays a major role, and in particular problems of the size often encountered in the real world. Our team is involved in research that is advancing our knowledge of how to solve large MIPs in which the data is random or uncertain, in application areas ranging from production planning [96], [95] to health care logistics [17].

3.7. Mathematical Programming and Graph Theory

The relationship between graph theory and mathematical programming has led to several famous research advances. Let us cite just a few landmarks. The matching problem (selecting disjoint edges in a graph) is historically the first integer programming problem that could not be solved by linear programming (no compact ideal formulation being known) but for which an efficient (polynomial time) combinatorial algorithm was known [74] (ideal formulations were only known for network flow problems at the time). The combinatorial algorithm lead to a polyhedral description of the matching polytope with exponentially many constraints separable in polynomial time [74].

The stable set problem in claw-free graphs is a fundamental generalization of the matching problem but while the later has been intensively studied since the seminal work of Edmonds [74] in the 60's and is now well understood, the stable set problem in claw-free graphs has still many "facets" to reveal. Our long term goal is to develop this theory further to the point were the stable set problem in claw-free graphs becomes as fundamental as the matching in polyhedral combinatorics.

The algorithmic aspect has attracted the attention of many researchers over the last three decades and is quite well understood. Minty [127] provides the first polynomial time algorithm to solve the problem, then Sbihi [146], Lovasz and Plummer [118], Nakamura and Tamura [131], Schrijver [147], Nobili and Sassano [135] have proposed alternative methods and often improved the complexity. We have recently proposed an alternative algorithm that outperforms all previous ones [136].

In term of polyhedral structure, most of the results about the stable set polytope of claw-free graphs were negative and discouraged researchers to work on the topic : as an example, Giles and Trotter [84] showed that the facets of the polytope could have arbitrary high values and many coefficients. Nevertheless recent decomposition results for claw-free graphs from Chudnovsky and Seymour [64] attracted new attention to the problem. In particular, building upon their results, we could prove that the so-called Ben Rebea conjecture was true for the stable set polytope of quasi-line graphs (a very interesting transition class between linegraphs - i.e. matching - and claw-free) [75].

Thus, the graph theory tools allow to derive better models/formulations for combinatorial optimization problems (f.i. graph theory characterization of forbidden subgraphs can sometimes be directly expressed as constraints in a mathematical program). Moreover, combinatorial procedure from graph theory can also serve as subroutines in mathematical programming approach, f.i., for cut separation or column generation. In particular, on the issue of symmetries, we believe that progress can come from the complementarity between graph theory and mathematical programming. This is illustrated by the work of Fekete and Schepers (2004) [77] on the 2-dimensional placement problem. In our project, we also play the reverse complementarity, i.e. to use mathematical programming techniques to make progress in graph theory.

In this area, there is potential for collaboration between our team and the INRIA team MASCOTTE, among others.

3.8. Integration of Constraint Programming tools

Constraint Programming (CP) [51] focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagate on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms.

A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems (minimizing the maximum of several variable values). Mixed Integer Programming (MIP), on the other hand, is effective for loosely constrained problems and for problems with an objective function defined as the weigthed sum of variables. Many problems belong to the intersection of these two classes. For example, some scheduling and timetabling problems are tightly constrained and have a sum-type objective. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

The integration of MIP and CP methods is currently an important research direction in combinatorial optimization [100], [123]. A wide variety of applications demonstrate the potential of such collaboration. Methods which combine MIP with CP have been successfully applied to scheduling [8], [105], [101], transportation [76], [81], network design [102], [151] and other problems.

3.9. Mathematical programming based heuristics

A heuristic is an algorithm that attempts to build a "good" primal feasible solution to a combinatorial optimization problem with no a priori guarantee on its maximum deviation from optimality. Exact optimization approaches can also serve to build good approximate solutions for complex combinatorial problems, either (i) by truncating an exact algorithm, or (ii) by constructing solution from the relaxation on which the exact approach relies, or (iii) by using exact solvers as subroutines in building heuristic solutions. Point (i) is common practice, even in commercial MIP solvers: one sets a time limit (or another bound on the number of algorithmic steps) or a threshold deviation from optimality. However, the implementation strategies are typically different if the aim is to get quickly good primal solution rather than solving the problem exactly. Point (ii) is key to our project. The chances are that starting from the solution to a stronger relaxation of a problem (i.e. a better formulation), one gets better primal solution in the end. The techniques to build the primal solutions range from greedy constructive procedures, to rounding techniques, using the relaxed solution as a target, or simply exploiting dual information to price choices. Point (iii) is the idea of "MIPPING" questions [78], i.e. to set intermediate questions, such as finding the best solution in the neighborhood of the current solution, as a mathematical program that can be solved with an MIP solver or another combinatorial algorithm. Even though the initial problem might be much too hard for exact methods, the subquestions that arise during its heuristic solution might be within the scope of exact solvers. There too it is important to have a good formulation of the "mipped" question. This points to potential collaboration with other INRIA research teams who develop meta-heuristic approaches (DOLPHIN and TAO).

Heuristics based on exact methods have found a new breath in the recent literature, in part due to the progress of exact commercial solvers. The latest developments are the *Large Scale Neighborhood Search* (an exponential size neighborhood can be explored in a local search procedure, provided that an efficient/polynomial algorithm exists to search it – Ahuja et al. 2002 [48]), the *Relaxation Induced Neighborhood Search* (the components of the LP solution that are close to the best known integer solution are rounded and the residual problem is then solved as an MIP of smaller size – Danna et al. (2005) [68]) and the *feasibility pump algorithm* (the rounded LP solutions defines a target that might be infeasible; the LP is re-optimized with the objective of minimizing the distance to that target; the process iterates in hope of finding good integer solutions – Fischetti et al., 2003, 2005 [79], [78]).

4. Application Domains

4.1. Introduction

Our group has tackled applications in logistics, transportation and routing [130], [129], [121], [122], in production planning [73], [159] and inventory control [121], [122], in network design and traffic routing [82], [103], [120], [40], [67], [104], [108], [142], [148], in cutting and placement problems [138], [139], [154], [156], [157], [160], and in scheduling [8], [145]. Building on this experience, we plan to find our motivation for algorithmic developments in the study of complex combinatorial problems of industrial relevance. In particular, we are currently involved in two industrial partnerships. With Exeo Solutions (a consultant that has worked for Eco-emballages, Suez, and other main stream group), we study planning and routing problems that arise in waste management [122], [121]. With SNCF, we are studying train timetabling problems and their re-optimization after a perturbation in the network [83].

4.2. Transport and logistics

Managerial problems raised by the planning of operations in transport networks, the distribution of goods, and the associated management of inventories have always been central in Operations Research. The tools of mathematical programming can bring substantial savings given the part of operation and transportation costs in the global cost of logistic. One could think that these major issues should be well resolved by now. This is simply not so. The combinatorial difficulties inherent to these problems require new techniques to increase the size of instances that can be treated. Moreover, new managerial practices and the trend of going from a hierarchical to an integrated optimization yield new problems.

Our experience in this domain includes several industrial studies:

- we are pursuing a project that simultaneously optimizes transportation routes and customer inventories (a problem known as *inventory routing*) with our industrial partner Exeo Solutions [121], [122];
- for Routing International, Brussels, we have combined vehicle routing and planning over a fixed time horizon [129], [130];
- we are also working on variants of the traveling salesman problem (TSP): the cumulative TSP arises when the cost of an arc is inversely proportional to its position in the circuit (consider for instance a delivery man that is paid more at the beginning of the journey because of the weight he has to carry).

4.3. Inventory control

In addition of inventory control combined with routing problems, as mentioned above, we are also interested in multi-echelon inventory control. We focus in particular on the simpler case of a network consisting of one warehouse supplying many retailers. In this setting, we want to understand, using game theory, what is the benefit or impact of coordination in such a network. We start with the simpler case where the retailers are facing continuous demands with constant rates. In this case, Roundy [144] could give a very good approximation to the global optimal (within 2% of optimality) and we are currently exploiting this result to show that coordination has a pretty high saving potential. Chen, Federgruen and Zhang [63] have proposed a sophisticated coordination mechanism that ensure global optimality in this context. Nevertheless, this coordination procedure seems to be unpractical and thus we are also studying simple coordination mechanisms that would yield better overall solutions. We also study the new dual-balancing technique proposed by Levi et al. [115] that has proven to be a powerful technique for approximation results in inventory control. We are implementing this technique for the simple case of one warehouse facing correlated and non stationary stochastic demand to compare theoretical guarantee and practical ones. We want to adapt those ideas to provide the first approximation algorithm for the one-warehouse multi-retailer problem under periodic review and holding and penalty costs. This work is a collaboration with IBM Research Zurich Lab (Dr. Richard Boedi, Dr. Eleni Pratsini) and University of Grenoble (Prof. Jean-Philippe Gayon, Prof. Christophe Rapine).

4.4. Telecommunication network design

Network design is a wide research domain arising in railway, highway and telecommunication. Applications in telecoms are very much studied at the moment, a revival due to the arrival of new transfer technologies such as optical fibers. The aim is to conceive cheap and reliable networks with specific requirements on the topology (which links will be created) or the capacity of the links (which amount of information can be in transit on a link at the same time). Sufficient capacities have to be installed to avoid congestion, several paths may have to link given pairs of nodes to ensure transmission in case of breakdown, all this at the cheapest possible cost.

We are actively working on problems arising in network topology design, implementing a survivability condition of the form "at least two paths link each pair of terminals". We have extended polyhedral approaches to problem variants with specific requirements. In [82], we deal with the design of so-called SDH/SONET networks, where the links must form cycles of bounded length; bounded length requirements can also come from re-routing restrictions [104], [67], [103]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes. In previous work, we optimized traffic assignment to minimize such installation cost [148]. We now consider the problem that arises when using new wavelength division multiplexing (WDM) technologies that allow to pack more traffic on optical networks. Several streams can be multiplexed, each of them supported by a different wavelength, in an optical signal [40] (Figure 1 illustrates the design of a european multipexed network). We are also working on the problem of measuring traffic in a network through the placement of markers at minimal cost [120].

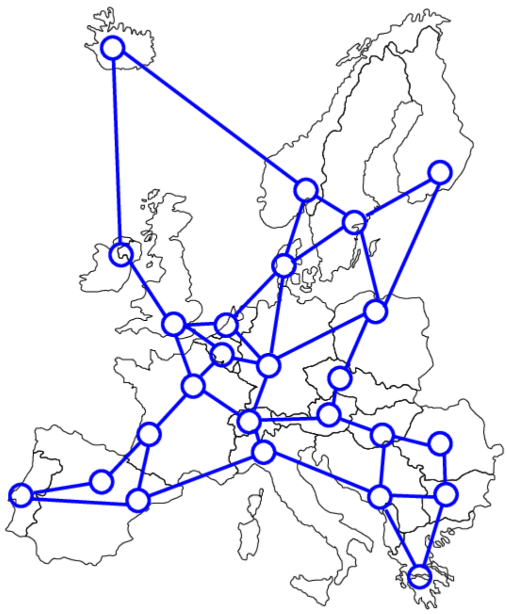

Figure 1. Design of a SDH/SONET european network where demands are multiplexed.

4.5. Production planning and scheduling

These problems present themselves throughout the supply chain in a host of industrial applications. Production planning problems can be defined on a network of facilities on an aggregate scale, or at a more detailed level within a single plant. These problems are concerned with the timing, the location, and the quantities of production. Often multiple product categories compete for scarce resources (raw materials, machine capacities, limited labor), and there are many other factors that must also be taken into account. To list just a few example, there is often external demand for finished products that must be considered; a bill-of-materials that specifies which products require others for their production or assembly must be taken into account; and the restrictions imposed by setup times and/or precedence restrictions must be rigorously accounted for.

One important class of production planning and scheduling problems is *lot-sizing problems*, in which the most important decision typically relate to *how much* to produce of each product category, as well as when (which time period) and where (which machine, line, or facility) to produce it in. In one stream of research, we have been engaging in developing rigorous computational methods to strengthen automatically the formulations of these problems in order to enable optimization software to identify high quality production plans and find strong performance guarantees (lower bounds) for such problems [124], [125], [126]. In another stream, we have been analyzing fundamental MIP models that incorporate uncertainty into lot-sizing problems, deriving results concerning optimality conditions for and the polyhedral structure of these models [93], [94].

Another important category of problems within this larger family are *machine scheduling problems*. In contrast to lot-sizing problems, the emphasis is on determining the *sequence* and *location* of a set of jobs whose specific quantities of production have already been determined. In machine scheduling problems, a set of jobs of different durations must be processed on a set of machines over time. The objective is to minimize a function of the jobs' starting times. Often, jobs can be processed only within availability intervals, or precedence relations between jobs must be followed. Typical applications of machine scheduling can be found in manufacturing and operational planning.

For some machine scheduling problems whose objective is to minimize a weighted sum of the number of late jobs, we have developed branch-and-price and branch-and-cut algorithms [8], [145]. Recently, we have developed heuristic [99] and exact [53], [13] MIP-based algorithms for the problem of scheduling an airborne radar, which can be modeled as a machine scheduling problem.

4.6. Cutting and packing problems

A family of problems related to the applications just discussed are cutting and packing problems. In cutting stock problems, one has a supply of large pieces of raw material in stock and a set of demands for small "order" pieces. One must satisfy these demands by cutting the required small pieces out of the large pieces from the stock. The objective is primarily to minimize the waste that is counted as the unused part of used pieces of stock material. A solution is given by a set of feasible cutting patterns, i.e. assortments of order pieces that can be cut out of a given large piece of stock material, such that their accumulated production covers the demands. There are many variants of the cutting stock problem. The main ones concerns the number of significant dimensions of the forms (1D, 2D, 3D, or even 1.5D), specific restrictions on the cutting process, the geometrical arrangements of pieces, and the number of cutting stages. There might be secondary objectives related to the balancing of the workload between different cutting machines, the minimization of the number of different cutting patterns used, or the respect of due dates for instances.

Packing, placement or loading problems can be stated in similar terms. There, one has a set of resources (vehicles, railway cars, machines) that must be packed with items. The objective is to maximize the value of the resulting packing while respecting the capacity of the resources. A solution is given by a set of feasible individual resource packings which together do not pack the same object more than once. Again, many variants exist depending mainly on the number of dimensions in which the capacity of the resources are measured and the specific restrictions on the loading process. Packing problems arise in particular as subproblems in cutting problems since the question of building "good" cutting patterns typically boils down to packing a resource piece with maximum value items. There are many applications of these cutting and packing problems: for instances, the cutting of paper, steel bars, glass, wood, textile, and plastic film; optimization of newspaper layout; scheduling of parallel machines; line balancing; and more generally scheduling problems with limited resources.

In previous work, we developed specialized algorithms for some variants of the knapsack problem that arise as a subproblem in solving cutting stock problems: problems with class bounds [157] or with setup costs [18]. We also set benchmark results for the 1D cutting stock problem using an exact optimization approach based on branch-and-price [160]. We were first to introduce exact algorithm for the 1D problem with setup minimization (a much harder variant) [154]. We also applied a nested decomposition approach to a 2D multi-cutting-stage variant [156] and considered ways of incorporating industrial side-constraints in an exact approach for the 1D problem [138], [139]. We currently work on 2D orthognal placement problems, combining graph therory and mathematical programmaing approaches.

5. Software

5.1. Application specific solvers

The output of our studies on specific applications typically consists in algorithmic developments and codes that implement the application specific solvers or, in some cases, routines that allow to derive theoretical results (f.i. in graph theory). The algorithms produced this year are:

- GRWA-BCP: A branch-cut-and-price code for routing traffic in a optical telecommunication network [40], [40].
  contact: Benoit Vignac
- BAP-RAD: branch-and-price code for scheduling an airborne radar problem [13].
  contact: Ruslan Sadykov
- 2D-KNAP: a branch-and-bound code for the 2D orthogonal packing problme where feasibility checks are based on graph theory tools [27].
  contact: Cédric Joncour

5.2. BapCod : a generic Branch-and-Price Code

contact: François Vanderbeck

Beyond the above application specific algorithms, we are involved in the development of a generic platform for the implementation of the branch-and-price method. Contrary to the cutting plane approach, column generation has not yet made its way into commercial solvers. Despite its successes, the use of a Dantzig-Wolfe decomposition approach is currently restricted to experts. The difficulties inherent to implementing the method and to combining it with other techniques were only overcome in an application specific context. As a consequence, algorithmic ideas have been developed and tested for specific applications only, without convincing arguments for showing their impact across different applications. The challenge is to show that column generation is also a technique that can be automated and make its way into commercial solvers (as cutting plane approach did recently).

We develop the prototype of a generic branch-and-price code, named *BaPCod*, for solving mixed integer programs by column generation. Previous attempts have been limited to offering "*tool-boxes*" to ease the implementation of algorithms combining branch-and-price-and-cut. With these, the user must implement three basic features for its application: the reformulation, the setting-up of the column generation procedure and the branching scheme. Other available codes that offer more by default were developed for a specific class of applications (such as the vehicle routing problem and its variants). Our prototype is a "*black-box*" implementation that does not require user input and is not application specific. The features are
(i) the automation of the Dantzig-Wolfe reformulation process [158],
(ii) a default column generation procedure with standard initialization and stabilization [1],
(iii) a default branching scheme – recent progress has been made on the issue of generic branching scheme in [22],

(iv) a column generation based primal heuristic. The column generation literature reports many application specific studies where primal heuristics are a key to success. After a review of generic classes of column generation based primal heuristics, we integrated a so-called "diving" method in which we introduce diversification based on Limited Discrepancy Search. While being a general purpose approach, the implementation of our heuristic illustrates the technicalities specific to column generation. The method is numerically tested on variants of the cutting stock and vehicle routing problems [24].
The ongoing work consists in developping preprocessing techniques.

5.3. PMaP: A Parallel Macro Partitioning Framework for Solving Mixed Integer Programs

contact: Andrew Miller

For many applications, it would be interesting to be able to use parallel resources to solve realistic size MIPs that take too long to solve on a single workstation. However, using parallel computing resources to solve MIP is difficult, as parallelizing the standard branch-and-bound framework presents an array of challenges in terms of ramp-up, interprocessor communication, and load balancing. In [133] we propose a new framework (the Parallel Macro Partitioning (PMaP) framework) for MIPs that partitions the feasible domain of an MIP by using concepts derived from recently developed primal heuristics. Initial computational resources suggest they enable PMaP to use many processors effectively to solve difficult problems.

5.4. Primal heuristics for general mixed integer programs

Beside the MIP heuristic developped in a decomposition framework (see 5.2), our team has been working on is the a new family of randomized rounding heurisitics for MIP problems with general integer variables (i.e., not necessarily binary) [132]. We have extensively tested these heuristics within the COIN-OR suite of optimization software [46], and it is our intent to incorporate them within this suite as part of the module CBC [45] in the near future.

6. New Results

6.1. Network Design and Routing

Participants: Philippe Meurdesoif, Pierre Pesneau, François Vanderbeck, Benoit Vignac.

6.1.1. Decomposition approaches for telecom network routing problems

In collaboration with B. Jaumard from Concordia University in Quebec, B. Vignac and F. Vanderbeck are studying routing and associated design decisions in backbone optical networks.

To accomodate the increase of traffic in telecommunication networks, today's optical networks have huge capacity thanks to grooming and wavelength division multiplexing technologies. The wavelength bandwidth utilization is increased by packing several requests on the same wavelength. Moreover, several streams can be multiplexed on an optical signal, each of them supported by a different wavelength. However, packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of dissagregation of trafic at an origin, a destination or a bifurcation node. These conversions requires the installation of expensive ports. Hence, traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronic system installation cost. This optimization problem is known as the *grooming, routing and wavelength assignment* (GRWA) problem.

Given a *physical network*, each link can carry a uniform number of wavelengths. Each wavelength has the same transport capacity. Trafic demand over the network take the form of bandwidth requests defined by their origin and destination and their capacity requirement that is selected from a discrete set of standard granularities, $\{1, 3, 12, 48\}$ that are dividers of the wavelength capacity. A request must be single path routed. Its route is defined by a sequence of *optical hops*, each of which is defined by a path in the physical network along which the signal remains into the optical domain with no electrical conversion at intermediate nodes. Traffic routing consists in defining an *optical path* defined by a sequence of optical hops.

We deal with backbone optical network with relatively few nodes (around 20) but thousands of requests. Such instances cannot be dealt with a traditional multi-commodity network flow approach. Instead, we develop and compare several decomposition approaches [41], [40], [39] (hierarchical versus nested decomposition): column generation is used to solve the LP relaxation of our models and a rounding procedure provides primal solutions; the LP dual bounds are improved using a cutting plane procedure. In ongoing work, we develop a direct branch-and-cut approach on a pseudo-polynomial formulation, for comparison.

We also studied the impact of imposing a restriction on the number of optical hops in any request route. Indeed, electrical convertions may cause important end-to-end delays. To limit such delay we evaluate different bounds on the number of hops in an optical path. Our study [39] shows that limiting path to 1-hop is very restrictive, while restricting the number of hops to 2, has a very limited impact on the design cost; if we use 3 or more hops, the cost decrease is marginal.

6.1.2. Time-dependent formulations for the vehicle routing problem

In collaboration with Teresa Godinho, Luis Gouveia and José Pires of the University of Lisbon and Thomas L. Magnanti of the School of Engineering of the MIT, Pierre Pesneau has studied several time dependent formulations for the unit demand vehicle routing problem [87]. They gave new bounding flow inequalities for a single commodity flow formulation of the problem. They described their impact by projecting on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables.

Following up on this work, they proved that some new inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

6.1.3. Asymmetric Traveling Salesman Problem

Along with Laurent Alfandari, Sylvie Borne and Lucas Létocart from the University Paris 13, Pierre Pesneau is currently starting a project granted by the working group on operations research (GDR RO) of the CNRS on the study of integer quadratic or integer linear programming formulation for some variants of the Asymmetric Traveling Salesman Problem.

In particular we are studying the case where the distance (in number of links) between the cities of some subset has to be lower bounded. Such case appears, for instance, in the design the shortest circuit for a traveling salesman who has to visit a minimum number of clients before having a break (or a night). Another close application can be found in [90] where the authors consider lower bound on the capacity for a vehicule routing problem.

We have already proposed several formulations of different types for the considered problem. The first two are compact formulations (polynomial number of variables and constraints). Two more formulation will contains a exponential number of constraints leading to the future development of branch-and-cut algorithms. One of these formulation contains an additional set of variables to model the bounding condition in a compact way. We have shown that this formulation is at least as god as the other one. Finally, several formulations where given with a exponential set of variables and for some them, an exponential set of constraints, leading to the conception of branch-and-price(-and cut) algorithms. We plan to compare theoreticaly and practicaly these formulations.

6.2. Packing, Planning and Scheduling

Participants: Sophie Michel, Andrew Miller, Ruslan Sadykov, Gautier Stauffer, François Vanderbeck.

6.2.1. Bin-Packing and Knapsack variants

The bin-packing problem raise the qestion of the minimum number of bin of fixed size we need to pack a set of items of different sizes. We studied a generalization of this problem where items can be in conflicts and thus cannot be put together in the same bin [32]. We show that the instances of the literature with 120 to 1000 items can be solved to optimality with a generic Branch-and-Price algorithm, such as our prototype BaPCod, within competitive computing time (we close 8 of the 10 open instances so far). The approach involves generic primal heuristics, generic branching, but a specific pricing procedure.

This specific precedure consist in a variant of a knapsack problem. This latter consist in choosing the most usefull items among a set while the total weight does not exceed a certain bound. Note that knapsack problems find their application in many concrete industrial and financial problems. Moreover, they also arise as subproblems in a Dantzig-Wolfe decomposition approach to more complex combinatorial optimization problems, where they need to be solved repeatedly and therefore efficiently.

The knapsack variant encountered in our bin packing problem resolution considers conflicts between items. This problem is quite difficult to solve compared to the usual knapsack problem. The latter is already NP-hard, but can be usualy efficiently solved by dynamic programming. We have shown that when the conflict graph (the graph defining the conflicts between the items) is an interval graph, this generalization of the knapsack can also be solved quite efficiently by dynamic programming with the same complexity than the one to solve the common knapsack problem.

We also encountered a variant of the knapsack problem in an inventory routing problem submitted by our industrial partner Exeo Solutions [122]. This problem must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. As a subproblem of a Dantzig-Wolfe decomposition approach applied to this inventory routing problem, we faced a multiple-class integer knapsack problem with setups [18]. Items are partitioned into classes whose use implies a setup cost and associated capacity consumption. Item weights are assumed to be a multiple of their class weight. The total weight of selected items and setups is bounded. The objective is to maximize the difference between the profits of selected items and the fixed costs incurred for setting-up classes. A special case is the bounded integer knapsack problem with setups where each class holds a single item and its continuous version where a fraction of an item can be selected while incurring a full setup. The paper shows the extent to which classical results for the knapsack problem can be generalized to these variants with setups. In particular, an extension of the branch-and-bound algorithm of Horowitz and Sahni is developed for problems with positive setup costs. Our direct approach is compared experimentally with the approach proposed in the literature consisting in converting the problem into a multiple choice knapsack with pseudo-polynomial size.

6.2.2. Workforce Scheduling

In reply to a challenge proposed by France Telecom (the french telecommunication operator), G. Stauffer and S. Pokutta worked on a problem where the purpose is to form technician teams and to plan different tasks in order to minimize the makespan. The tasks are linked by precedence constraints and can be treated only by technician teams having the good expierience. The proposed method is described in [19] along with computationnal experiments coming from real instances.

6.2.3. Machine scheduling

Ruslan Sadykov in collaboration with Prof. Philippe Baptiste from the Ecole Polytechnique (Paris) has worked on the problem to schedule an airborne radar. This research has been done in the framework of a joint project between the Ecole Polytechnique and the DGA. Airborne radars are widely used to perform a large variety of tasks in a fighter aircraft. These tasks include, but are not limited to, searching, tracking and identifying targets. Such tasks play a crucial role for the aircraft and they are repeated in a “more or less” cyclic fashion. This defines a complex scheduling problem that impacts a lot on the quality of the radars output and on the overall safety of the aircraft.

For this problem, three different Mixed Integer Programming formulations have been proposed [13]. Two of the formulations are compact and can be solved directly by an MIP solver. The third formulation relies on a Branch-and-Price algorithm. Theoretical and experimental comparisons of the formulations were reported. A dedicated solver has been implemented for this problem, and real-life instances of a moderate size have been solved to optimality. This work will allow us to estimate the quality of heuristic algorithms for the problem. Only fast heuristic algorithms can be used in practice due to tight resolution time restrictions. The research in this direction has been already started [99].

Another study realised with Prof. Ph. Baptiste concerns the scheduling situation in which a set of jobs subjected to release dates and deadlines are to be performed on a single machine [12]. The objective is to minimize a piecewise linear objective function $\sum_j F_j$ where $F_j(C_j)$ corresponds to the cost of the completion of job j at time C_j. This class of function is very large and thus interesting both from a theoretical and practical point of view: It can be used to model total (weighted) completion time, total (weighted) tardiness, earliness and tardiness, etc. We introduce a new Mixed Integer Program (MIP) based on time interval decomposition. Our MIP is closely related to the well-known time-indexed MIP formulation but uses much less variables and constraints. Experiments on academic benchmarks as well as on real-life industrial problems show that our generic MIP formulation is efficient.

Another research in progress concerns scheduling parallel jobs i.e. which can be executed on more than one processor at the same time. With the emergence of new production, communication and parallel computing system, the usual scheduling requirement that a job is executed only on one processor has become, in many cases, obsolete and unfounded. Therefore, parallel jobs scheduling is becoming more and more widespread. In this work, we consider the NP-hard problem of scheduling a class of parallel jobs to minimize the total weighted completion time or mean weighted flow time criteria. For this problem, we have introduced an important dominance rule which can be used to reduce the search space while searching for an optimal solution [26].

We have also studied in a scheduling problem that takes place at cross docking terminals. In such places, products from incoming trucks are sorted according to there destinations and transferred to outgoing trucks using a temporary storage. Such terminals allow companies to reduce storage and transportation costs in supply chain. This paper focuses on the operational activities at cross docking terminals. In [38], we consider the trucks scheduling problem with the objective to minimise the storage usage during the product transfer. We show that a simplification of this NP-hard problem in which the arrival sequences of incoming and outgoing trucks are fixed is polynomially solvable by proposing a dynamic programming algorithm for it.

6.2.4. Multi-item production planning problems

In research recently accepted for publication ([11]), we have developed rigorous computational methods to find high quality production plans for big bucket lot-sizing problems of realistic size. By *big bucket* we mean problems in which multiple product categories compete for the same capacities (of machines, labor, etc.) Our methods are simple and direct enough to be implemented in a high-level algebraic modeling language (rather than a low level programming language such as C or Fortran). Moreover, they adapt easily to different situations that may arise depending on particularities of the bill-of-materials, specific facility characteristics, etc. There is therefore hope that these methods may prove practical for simple industrial use.

In [42], we have compared various methods for finding performance guarantees (lower bounds) for realistically sized instances of such problems. These methods include both those previously proposed in the literature and those we have developed ourselves. Our methods of comparison were both theoretical and computational; one of the primary contributions of this research is to identify and highlight those aspects of these problems that prevent us from solving them more effectively. This identification could be crucial in improving our ability to solve such models.

The following theoretical research was directly inspired by production planning problems. In [14], the authors discuss a polyhedral study of a generalization of the mixing set where two different, divisible coefficients are allowed for the integral variables. Our results generalize earlier work on mixed integer rounding, mixing, and extensions. As mention before, these results directly apply to applications such as production planning problems involving lower bounds or start-ups on production, when these are modeled as mixed-integer linear programs. We define a new class of valid inequalities and give two proofs that they suffice to describe the convex hull of this mixed-integer set. We give a characterization of each of the maximal faces of the convex hull, as well as a closed form description of its extreme points and rays, and show how to separate over this set in $O(nlogn)$. Finally, we give several extended formulations of polynomial size, and study conditions under which adding certain simple constraints on the integer variables preserves our main result.

6.2.5. Energy production planning

We are currently working on a project aiming to plan the energy production and the maintenance breaks for a set of power plants generating electricity. This problem has two different levels of decisions. The first one consist in determining, for a certain time horizon, when the different power plants will have to stop in order to perform a refueling and to decide the amount of this reflueling. Given a set of scenarios defining variable levels of energy consumption, the second decision level aims to decide the quantity of power each plant will have to produce.

As the number of periods composing the time horizon, and the number of scenario are quite large, the size of any MIP formulation for such problem will forbide a exact resolution of the problem in an acceptable time. However, our objective is to show that exact methods can be used on simpler problems and combined to design heuristics for the solution of large scale problem.

6.2.6. Stochastic optimization of allocation problems

The allocation of surgeries to operating rooms (ORs) is a challenging combinatorial optimization problem. There is moreover significant uncertainty in the duration of surgical procedures, which further complicates assignment decisions. In [17], we present stochastic optimization models for the assignment of surgeries to ORs on a given day of surgery. The objective includes a fixed cost of opening ORs and a variable cost of overtime relative to a fixed length-of-day. We describe two types of models. The first is a two-stage stochastic linear program with binary decisions in the first-stage and simple recourse in the second stage. The second is its robust counterpart, in which the objective is to minimize the maximum cost associated with an uncertainty set for surgery durations. We describe the mathematical models, bounds on the optimal solution, and solution methodologies, including an easy-to-implement heuristic. Numerical experiments based on real data from a large health care provider are used to contrast the results for the two models, and illustrate the potential for impact in practice. Based on our numerical experimentation we find that a fast and easy-to-implement heuristic works fairly well on average across many instances. We also find that the robust method performs approximately as well as the heuristic, is much faster than solving the stochastic recourse model, and has the benefit of limiting the worst-case outcome of the recourse problem.

6.3. Integrating graph theory and mathematical programming approaches

Participants: Cédric Joncour, Philippe Meurdesoif, Arnaud Pêcher, Gautier Stauffer, François Vanderbeck, Annegret Wagler.

6.3.1. Circular chromatic number

A major result of graph theory is that the chromatic number of a perfect graph is computable in polynomial time (Grötschel, Lovász and Schrijver 1981). The circular chromatic number is a well-studied refinement of the chromatic number of a graph. Xuding Zhu noticed in 2000 that circular cliques are the relevant circular counterpart of cliques, with respect to the circular chromatic number, thereby introducing circular-perfect graphs, a super-class of perfect graphs. It is unknown whether the circular-chromatic number of a circular-perfect graph is computable in polynomial time in general. This is an important issue. Indeed, since the chromatic number of a graph is the integer ceiling of its circular-chromatic number, if the circular chromatic number of a circular-perfect graph is computable in polynomial time then it would imply Grötschel, Lovász and Schrijver's result.

In [21], we design a polynomial time algorithm that computes this circular chromatic number when the circular-perfect graphs is claw-free. In 2005, Coulonges, Pêcher and Wagler, introduced the intermediate class of strongly circular-perfect graphs, as those circular-perfect graphs whose complements are also circular-perfect. For the triangle free cases, we managed to fully characterize these graphs, and gave a polynomial time algorithm to recognize them [15]. Coulonges, Pêcher and Wagler also introduced a-perfect graphs, another super-class of perfect graphs. We bounded their imperfection ratio by 1.5 [16].

We have also introduced the circular-clique polytope, and used it to prove that the weighted circular-clique number, and thus the circular chromatic number, of a strongly circular-perfect graph which is also a-perfect is computable in polynomial time. We also proved that the circular chromatic number of a strongly circular-perfect graph is computable in polynomial time. Unexpectedly, we also used the circular-cliques polytope to prove that the circular stability number, and thus the stability number, of a fuzzy circular-interval graph is computable in polynomial time [23].

6.3.2. Stable set problem and polytope in claw-free graphs

Our focus is now two-fold : first we are exploiting our proof of the Ben Rebea conjecture further to give a clearer picture of the polyhedral nature for quasi-line graphs, in particular we want to describe precisely the non rank facets of quasi-line graphs and along the way, to prove a conjecture of Pecher and Wagler [142] for webs (a subclass). In a more general scope, A. Pêcher and A. Wagler describe in [20] new facets for the stable set polytope of claw-free graphs. Second, our current algorithmic developments suggest that we can provide an extended formulation of the problem and therefore an extended characterization of the stable set polytope for claw-free graphs, this would partially answer a 30 years old question. Moreover, we envision that the technique we are developing has a broader spectrum of application than the stable set problem in claw-free graphs. We are currently setting the foundations for a more general setting. This work is a collaboration with University of Tor Vergata, Roma, Italy (Prof. Gianpaolo Oriolo, Yuri Fuenza). A preliminary result has been submitted to the IPCO 2010 conference.

6.3.3. Using graph theory for solving orthogonal knapsack problems

With C. Joncour (PhD student), we investigate the orthogonal knapsack problem, with the help of graph theory [106]. Fekete and Schepers managed a recent breakthrough in solving multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* (whose complement admits a transitive orientation: each such orientation of the edges corresponds to a specific placement of the forms) for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to repesent interval graphs.

In [27], we propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch and bound algorithm already used to solve the one-dimension knapsack problem. The first results are encouraging as we are able to solve more problems than the algorithm proposed by Fekete & Schepers. The ongoing work is the development of a branch-and-price algorithm for this problem using this way to represent interval graphs.

6.4. Generic solvers

Participants: Sophie Michel, Andrew Miller, Céline Saubatte, François Vanderbeck, Benoit Vignac.

6.4.1. Branching in branch-and-price: a generic scheme for the non-binary case

We have pursued our work on developing a branching scheme that is compatible with the column generation procedure and that implies no strutural modifications to the pricing problem. We have carry on a comprehensive experimental study that demonstrates the efficiency of what could have been seen otherwise as a theoretic scheme. Moreover, although the scheme was initially presented for a binary integer program, we have shown (in theory and in our computational experiments) how it extends to a general mixed integer program [22]

6.4.2. Primal heuristics based on column generation

In the past decade, significant progress has been achieved in developing generic primal heuristics that made their way into commercial mixed integer programming (MIP) solver. Extensions to the context of a column generation solution approach are not straightforward. The Dantzig-Wolfe decomposition principle can indeed be exploited in greedy, local search, rounding or truncated exact methods. The price coordination mechanism can bring a global view that may be lacking in some "myopic" approaches based on a compact formulation. However, the dynamic generation of variables requires specific adaptation of heuristic paradigms.

Based on our application specific experience with these tehcniques [122], [130], [162], [163], and on a review of generic classes of column generation based primal heuristics, in [24] we focus on a so-called "diving" method in which we introduce diversification based on Limited Discrepancy Search. While being a general purpose approach, the implementation of our heuristic illustrates the technicalities specific to column generation. The method is numerically tested on variants of the cutting stock and vehicle routing problems.

6.4.3. PMaP: A Parallel Macro Partitioning framework for solving mixed integer programs

In [133] we propose a new framework (the Parallel Macro Partitioning (PMaP) framework) that partitions the feasible domain of an MIP by using concepts derived from recently developed primal heuristics. The ideas from these heuristics, which include LP-and-fix, RINS, and local branching, among others, enable us both to define quickly many subproblems whose feasibility region is much smaller than that of the original problem. They also allow us to generate complementary cuts in order to ensure that the solver does not search the same region on many different processors. The result is that PMaP is able to quickly partition the feasible region at a high level into a large set of subproblem MIPs that can be solved simultaneously in parallel on separate processors. Initial computational resources suggest that PMaP has significant promise as a framework capable of bringing many processors to bear effectively on difficult problems. In particular, they suggest that the overall approach of high-level partitioning is more promising as a means of enumeration than classical branch-and-bound in terms of being able to use effectively hundreds or even thousands of processors in order to solve difficult problems. Given the increasing prevalence that parallel computing architectures are likely to plan in the future, it seems at least possible that our approach, when it reaches maturity, may come to be seen as a breakthrough achievement.

Partially as a result of the development of this software, Mahdi Namazifar spent a summer internship at IBM's T.J. Watson Research Center in 2008 working on these and other ideas. We expect to integrate PMaP into COIN-OR (maintained by an IBM team at Watson) in the near future.

6.4.4. Other mixed integer programming heuristics

Most MIP heuristics that have been developed perform best on problems with binary variables. In [132] we propose methods for problems with *general* (i.e., not necessary binary) integer variables. Called *randomized rounding* heuristics, these methods do much more than simply rounding a single LP fractional solution. They attempt to find feasible solutions by randomly walking within a specially-constructed polyhedron, and then performing rounding operations from the points traversed. The polyhedron in question is expressed as the convex hull of some of the "interesting" extreme points of the LP relaxation of the original problem, where the extreme points chosen to be of interest have a high probability of being in the region of the LP relaxation pointed to by the objective function. Preliminary computational results for this heuristic approach suggest that it may be the most effective primal heuristic known for certain classes of general MIP problems.

6.4.5. Mixed integer non-linear programming

While our research on MINLP problems is just beginning, there is already evidence of potential for significant success. For example, we have been considering how to exploit the information generated by solvers as they make branching decisions (in particular, which variable to branch on) during the branch-and-bound process. This has allowed us to define a structured family of disjunctive cuts whose generation requires no additional effort beyond that necessary to perform classical strong branching. Even for MIPs, these ideas seem capable of singificantly reducing the size of branch-and-bound trees; moreover, the ideas themselves are directly applicable to MINLPs, and we are currently investigating how best to apply them [110].

Another factor in defining effective branch-and-solvers for MINLPs is defining solvers that are capable of *re*-optimizing NLP problems efficiently. Classical interior point methods have well-known difficulties in performing warm re-starts; for this and other methods, *active set* methods seem to have more potential for efficient re-optimization. In [43], we have defined a new active set method for linear programming and generalized it for quadratic programming problems. This algorithm has a number of desirable properties for these problems (including re-optimization capabilities), and we are hopeful to generalize this method further to other families of NLP problems.

Multilinear functions appear in many global optimization problems, including blending and electricity transmission, among many others. In [30], the authors study convex envelopes for a product of variables that have lower and upper bounds, and which is itself bounded. Since global branch-and-bound solvers for such problems use polyhedral relaxations of such sets to compute bounds, having tight relaxations can improve performance. For two variables, the well-known McCormick inequalities define the convex hull for an unbounded product. For a bounded product, we define valid linear inequalities that support as many points of the convex hull as possible. Though uncountably infinite in number, these inequalities can be separated for exactly in polynomial time. We have extended extensions of such results to products of more than two variables, considering both convex hull descriptions and separation.

For global optimization problems with sums of multiple product terms, a common technique for creating relaxations for these problems is to decompose each product term into bilinear terms (i.e., products of two variables) and then use the McCormick envelope for each term separately. The authors of [31] are investigating an approach which generates a (stronger) relaxation directly from the multilinear terms. Computational results demonstrate that significant advantages can result from using this approach.

7. Contracts and Grants with Industry

7.1. SNCF

Participants: Laurent Gely, Philippe Meurdesoif, Arnaud Pêcher, Pierre Pesneau, François Vanderbeck.

Our contract with SNCF, "Innovation et Recherche" ended officialy in March. In previous work, L. Gély produced timetables in the aim of maximizing the throughput (number of trains) that can be handled by a given network [98]. In this project, we considered the problem of managing perturbations [83]. Network managers must re-optimize train schedules in the event of a significant unforeseen event that translates into new constraints on the availability of resources. The control parameters are the speed of the trains, their routing and sequencing. The aim is to re-schedule trains to return as quickly as possible to the theoretic timetable and to restrict the consequences of the perturbation to a limited area. The question of formulation is again central to the approach that shall be developed here. The models of the literature are not satisfactory. Continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing). Other standard models based on arc flow in time-space graph blow-up in size. Formulations in time-space graphs have therefore been limited to tackling single line timetabling problems. We have develop a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiuous time model having fixed the precedence based on a solution to the discrete time model.

8. Other Grants and Activities

8.1. International Grants and Activities

Participants: Andrew Miller, Arnaud Pêcher, Pierre Pesneau.

8.1.1. Next Generation Mixed Integer Nonlinear Programming Solvers: Structure, Search and Implementation

With Jeffrey T. Linderoth and James Luedtke of the University of Wisconsin-Madison, and Sven Leyffer and Todd R. Munson of Argonne National Laboratory (a research unit of the United States Department of Energy), Andrew Miller was awarded two grants in 2008 from United States government sources for the project “Next Generation Mixed Integer Nonlinear Programming Solvers: Structure, Search and Implementation".

The first grant (Department of Energy grant number DE-PS02-08ER08-13) began on August 15, 2008 and lasts through August 14, 2011. The second grant (grant number CCF 0830153 of the National Science Foundation) started on January 1, 2009, and continue through December 31, 2011.

8.1.2. ANR Gratel

André Raspaud launched in 2005 a fruitful cooperation with the Department of Applied Mathematics of the Sun Yat-Sen University of Kaohsiung, Taiwan. This project takes its roots from this three years cooperation (including one PhD student in double doctoral degree scheme). The priority scientific theme “Telecommunications” is a well-known key application area of graph theory. The scientific aim of this proposal is to tackle telecommunications problems, especially wireless communications problems, with the help of graph colorings and polyhedral graph theory.

This project will be funded for three years starting in January 2010.

8.2. National Grants

Participant: Pierre Pesneau.

A project with Laurent Alfandari, Sylvie Borne and Lucas Létocart of University Paris 13, on the Hop-constrained Asymmetric traveling salesman problem has been granted in 2009 by the operations research group of the CNRS (GDR RO). The main objectives of this project is to propose and compare several formulations for the treated problem and develop exact methods for its solution.

9. Dissemination

9.1. Organization of scientific meetings and activities in scientific life

Andrew Miller organized on March the 19th and 20th in Bordeaux a Spring Workshop on Computational Issues in Mixed Integer Nonlinear Programming. In the continuity of this succesfull workshop, Andrew Miller is organizing in April 2010 a European Workshop on Mixed Integer Nonlinear Programming in Marseille.

Pierre Pesneau is an active member of the organizing committee of the working group on Polyhedra and Combinatorial Optimization affiliated to the french operation research society (ROADEF) and the operation research group of the CNRS. The purpose of this working group is to promote the field of polyhedra in the research domain of combinatorial optimization. To this aim, the group organizes every year the Polyhedra and Combinatorial Optimization Days. These days gather young and less young researchers who want to discuss or discover the polyhedral optimisation. These days are preceded by a doctoral school.

This year, the edition of this workshop took place at the IMB (Institut de Mathématiques de Bordeaux) in June 2009 and was organized by the RealOpt team. It was preceded by a doctoral school on decomposition methods in combinatorial optimization.

To celebrate the 75th birthday of Jack Edmonds, the POC working group organized a 3-days workshop on Pretty Structure, Existential Polytime and Polyhedral Combinatorics.

We also regularly organize day-long workshops on more specific individual subjects. Such workshops are scheduled around a tutorial on the subject and several talks; some slots are also dedicated to open questions. For example, in November 2009, Pierre Pesneau organized such a workshop on the subject of multi-cuts and graph partitionning.

A. Raspaud and A. Pêcher were the chairs of the international conference Eurocomb 2009, held in September at the University of Bordeaux 1. This conference covered the full range of Combinatorics and Graph Theory including applications in other areas of Mathematics, Computer Science and Engineering. There were 200 participants and the European Prize in Combinatorics was attributed to two laureates: Peter Keevash and Balazs Szegedy.

Finaly, F. Vanderbeck has been elected for a 2-years mandate as the vice-president in charge of industrial and international relations in the french operations research society board.

9.2. Conferences

- M. Namazifar, P. Belotti and A.J. Miller — "Linear Envelopes of Bounded Multilinear Functions" — BR-OPT. May 28, 2009, Pittsburgh, USA.
- M. Namazifar, J.T. Linderoth, J. Luedtke and A.J. Miller — "Strong Relxations and Computations for Global Optimization Problems with Multilinear Terms" — ISMP 2009. August 23–28, 2009, Chicago, USA.
- C. Joncour and A. Pêcher — "Consecutive ones matrices for orthogonal packing problems" — ISMP 2009. August 23–28, 2009, Chicago, USA.
- C. Joncour and A. Pêcher — "Consecutive ones matrices for orthogonal packing problems" — JPOC6. June 10–12, 2009, Bordeaux, France.
- G. Stauffer — "An extended formulation for the stable set polytope of claw-free graphs" — AIRO2009, The fortieth Annual Conference of the Italian Operational Research Society. September 8–11, 2009, Siena, Italy.
- A. Pêcher and A. Wagler — "On the polynomial time computability of the circular chromatic number for some superclasses of perfect graph" — LAGOS'09. November 3–7, 2009, Grammado, Brazil.
- R. Sadykov and F. Vanderbeck — "Branch-and-price for bin packing with conflicts" — ISMP 2009. August 23–28, 2009, Chicago, USA.
- F. Vanderbeck and L.A. Wolsey — "Reformulation and Decomposition of Integer Programs" — JFRO09. March 6, 2009, Paris, France.
- F. Vanderbeck and L.A. Wolsey — "Reformulation and Decomposition of Integer Programs" — JPOC6. June 10–12, 2009, Bordeaux, France.
- S. Michel and F. Vanderbeck — "Review and Classification of Branching Schemes for Branch-and-Price" — ISMP 2009. August 23–28, 2009, Chicago, USA.
- B. Vignac, B. Jaumard, and F. Vanderbeck — "A Hierarchical Optimization Approach to Optical Network Design where Traffic Grooming and Routing is Solved by Column Generation" — INOC 2009. April 26–29, 2009, Pisa, Italy.

9.3. Invitations / Workshop

- In collaboration with L. Wolsey, F. Vanderbeck has written a paper that made the subject of main presentations of two workshops, namely JFRO and JPOC6.
- A. Pêcher spent two weeks in the Department of Applied Mathematics, Sun Yat-sen University, Kaohsiung, Taiwan in January.This invitation take place in the context of its collaboration with Prof. Xuding Zhu.
- A. Pêcher was also invited to give a seminar to the Laboratoire d'Informatique Fondamentale of Marseille, France in April 2.
- He was also invited to give a plenary talk in the 6th edition of the Polyhedral and Combinatorial Optimization Days organized in Bordeaux.

- A. Miller was invited in January 5-6 and in August 10-12 by J. Linderoth and J. Luedtke at the University of Wisconsin for a collaboration on Mixed Integer Nonlinear Programming.
- A. Miller was also invited by Pietro Belotti in August 17-18 at Lehigh University to work on the same field.
- A. Miller also attends a working group meeting on Combinatorial Optimization under Uncertainty (November 29 to December 4) in Banff, Alberta, Canada. The purpose of this 20 people meeting was to create new collaborations in this field.

9.4. Visitors

Several international and national colleagues visited us (short visits for scientific exchanges and seminars presentations):

- Pietro Belotti, Assistant Professor, Lehigh University. December 14-18.
  "Disjunctive cuts for non-convex MINLP".
- L. Alfandari, S. Borne and L. Létocart have visited the laboratory in December, 4 and 5, in the framework of the project granted by the operation research working group (GDR RO) of the CNRS.
- Valery Gordon, Research Director, United Institute of Informatics Problems, Minsk, Belarus. November 15-18.
  "Single Machine Scheduling with Positionally Dependent Processing Times".
- Yuri Faenza, Università di Roma Tor Vergata. October 11 - November 11.
  "Extended formulations for some 0-1 symmetry-breaking polytopes".
- Kerem Akartunali, Postdoctoral Research Fellow, University of Melbourne. September 1-11.
  "Two-period Convex Hull Closures for Big Bucket Lot-sizing Problems".
- Jeffrey T. Linderoth, Associate Professor, University of Wisconsin-Madison. March 16-20 and July 6-17.
  "A Different Perspective on Perspective Cuts".
  "Orbits and Integer Programming. Abstract".
- Frédéric Babonneau, Researcher-consultant, ORDECSYS, Geneva. April 7.
  "Robust capacity expansion solutions for telecommunication networks with uncertain demands".
- Alain Hertz, École Polytechnique de Montréal et GERAD. April 3.
  "Problèmes de tournées de véhicules avec contraintes de gestion de stock".
- Gautier Stauffer, IBM Research Lab, Ruschlikon, Switzerland. March 4.
  "On using the EOQ formula for inventory control in one-warehouse multi-retailer systems".
- Pierre Bonami, Chargé de Recherche CNRS, Laboratoire d'Informatique Fondamentale de Marseilles. February 3.
  "Coupes disjonctives et projection pour la programmation mixte sous contraintes quadratiques".

9.5. PhD Theses

- Yuri Faenza has defended is PhD thesis on extended formulations for several polyhedra in combinatorial optimization and related topics. He would defend his PhD thesis in beginning of 2010. Advisors: G. Oriolo and G. Stauffer.
- Cédric Joncour started in September 2007. His doctoral study is on 2-D packing problems. Advisors: A. Pêcher, P. Pesneau, F. Vanderbeck.
- Mustafa Kılınc (Advisors: J. Linderoth and A. Miller) started his PhD thesis in 2006 on Computational approaches for mixed integer nonlinear programming. His defense is planed for 2010.
- Guillaume Massonnet started his PhD thesis in September 2009 on cost balancing technics for inventory control with uncertainty. Advisors: Jean-Phillippe Gayon, Christophe Rapine and G. Stauffer.
- Mahdi Namazifar began his doctoral work in August 2006. His thesis will cover various theoretical and computational aspects of MIP and MINLP. In 2008 he had one paper published in a competitive conference proceedings ([133]; this paper was one of only 22 short papers accepted, out of 61 submissions), and is currently working on several others. Advisors: A.J. Miller, M.C. Ferris, and J.T. Linderoth.
- Benoit Vignac (Advisors: B. Jaumard, G. Laporte, F. Vanderbeck) has submitted his doctoral research. His defence is scheduled in January.

9.6. Teaching and Administrative Duties

Each member of the team is quite involved in teaching in the thematic specialties of the project, including in the research track of the Masters in applied mathematics or computer science. Moreover, we are largely implied in the organization of the curriculum:

- Andrew Miller will be the head of the operations management specialty in next habilitation of the Master of Applied Mathematics, Statistics and Econometrics.
- Pierre Pesneau is head of the professional curriculum of the operations management specialty.
- François Vanderbeck is head of the Master of Applied Mathematics, Statistics and Econometrics.
- Gautier Stauffer is the project organizer for the operations management specialty of the Master of Applied Mathematics, Statistics and Econometric.

External collaborators of the group are also quite involve locally. In particular, E. Sopena is head of the doctoral school in Mathematics and Computer Science.

9.7. INRIA Internship Program

Participants: Ruslan Sadykov, François Vanderbeck.

The master student Dimitri Fedorov from Moscow Institute of Physics and Technologies made his master training course of 5 months (August-December 2009) under the supervision on Ruslan Sadykov and François Vanderbeck. He worked on algorithms for the knapsack problem with conflicts. He is expected to defend his master thesis in 2010.

10. Bibliography

Major publications by the team in recent years

[1] O. BRIANT, C. LEMARÉCHAL, P. MEURDESOIF, S. MICHEL, N. PERROT, F. VANDERBECK. *Comparison of Bundle and Classical Column Generation*, in "Mathematical Programming", vol. 113, n^o 2, 2008, p. 299–344.

[2] L. GOUVEIA, P. PESNEAU. *On extended formulations for the precedence constrained asymmetric traveling salesman problem*, in "Networks", vol. 48, n° 6, 2006, p. 77–89.

[3] Y. GUAN, A. J. MILLER. *Polynomial-Time Algorithms for Stochastic Uncapacitated Lot-Sizing Problems*, in "Operations Research", vol. 56, n° 2, 2008, p. 1172–1183.

[4] P. MEURDESOIF. *Strengthening the Lovász $\theta(\overline{G})$ bound for graph coloring*, in "Mathematical Programming", vol. 102, n° 3, 2005, p. 577–588.

[5] A. MILLER, L. WOLSEY. *Tight formulations for some simple mixed integer programs and convex objective integer programs*, in "Mathematical Programming", vol. 98, 2003, p. 73-88.

[6] A. PÊCHER, A. WAGLER. *Almost all webs are not rank-perfect*, in "Mathematical Programming", vol. 105, n° 2-3, 2006, p. 311–328.

[7] A. PÊCHER, X. ZHU. *On the Circular Chromatic Number of Circular Partitionable Graphs*, in "Journal of Graph Theory", vol. 52, n° 4, aug 2006, p. 294–306.

[8] R. SADYKOV, L. A. WOLSEY. *Integer Programming and Constraint Programming in Solving a Multimachine Assignment Scheduling Problem with Deadlines and Release Dates*, in "INFORMS Journal on Computing", vol. 18, n° 2, 1 2006, p. 209–217.

[9] F. VANDERBECK. *On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm*, in "Operations Research", vol. 48, n° 1, 2000, p. 111–128.

[10] F. VANDERBECK. *Computational Study of a Column Generation algorithm for Bin Packing and Cutting Stock problems*, in "Mathematical Programming, Ser. A", vol. 86, 1999, p. 565–594.

Year Publications

Articles in International Peer-Reviewed Journal

[11] K. AKARTUNALI, A. MILLER. *A heuristic approach for big bucket multi-level production planning problems*, in "European Journal of Operational Research", 2009, p. 396-411, http://hal.archives-ouvertes.fr/hal-00387052/en/AU.

[12] P. BAPTISTE, R. SADYKOV. *On Scheduling a Single Machine to Minimize a Piecewise Linear Objective Function : A Compact MIP Formulation*, in "Naval Research Logistics / Naval Research Logistics An International Journal", vol. 56, n° 6, 2009, p. 487–502, http://hal.inria.fr/inria-00387012/en/.

[13] P. BAPTISTE, R. SADYKOV. *Time Indexed Formulations for Scheduling Chains on a Single Machine: An Application to Airborne Radars*, in "European Journal of Operational Research", 2009, http://hal.inria.fr/inria-00339639/en/.

[14] M. CONSTANTINO, A. MILLER, M. VAN VYVE. *Mixing MIR Inequalities with Two Divisible Coefficients*, in "Mathematical Programming, Series A", 2009, Published online, http://hal.archives-ouvertes.fr/hal-00387098/en/BEPT.

[15] S. COULONGES, A. PECHER, A. WAGLER. *Triangle-Free Strongly Circular-Perfect Graphs*, in "Discrete Mathematics", vol. 309, 2009, p. 3632-3643, http://hal.archives-ouvertes.fr/hal-00308145/en/DE.

[16] S. COULONGES, A. PÊCHER, A. WAGLER. *Characterizing and bounding the imperfection ratio for some classes of graphs*, in "Mathematical Programming, Series A", vol. 118, n^{o} 1, 2009, p. 37-46, http://hal.archives-ouvertes.fr/hal-00308150/en/DE.

[17] B. DENTON, A. MILLER, H. BALASUBRAMANIAN, T. HUSCHKA. *Optimal Allocation of Surgery Blocks to Operating Rooms Under Uncertainty*, in "Operations Research", 2009, Accepted for publication, http://hal.archives-ouvertes.fr/hal-00386469/en/US.

[18] S. MICHEL, N. PERROT, F. VANDERBECK. *Knapsack Problems with Setups*, in "European Journal of Operational Research", vol. 196, 2009, p. 909-918, http://hal.inria.fr/inria-00232782/en/.

[19] S. POKUTTA, G. STAUFFER. *France Telecom Workforce Scheduling problem : A Challenge*, in "RAIRO Operational Research", 2009, http://hal.inria.fr/inria-00442261/en/DE.

[20] A. PÊCHER, A. WAGLER. *On facets of stable set polytopes of claw-free graphs with stability number three*, in "Discrete Mathematics", vol. 310, 2010, p. 493-498, http://hal.archives-ouvertes.fr/hal-00283138/en/DE.

[21] A. PÊCHER, X. ZHU. *Claw-free circular-perfect graphs*, in "Journal of Graph Theory", 2010, http://hal.archives-ouvertes.fr/hal-00431241/en/, to appearTW.

[22] F. VANDERBECK. *Branching in Branch-and-Price: a Generic Scheme*, in "Mathematical Programming, Series A", 2009, http://hal.inria.fr/inria-00311274/en/, to appear.

Invited Conferences

[23] A. PÊCHER. *Circular-clique polytopes and circular-perfect graphs*, in "2009 Workshop on Graph Theory, Taiwan Kaohsiung", 2009, http://hal.archives-ouvertes.fr/hal-00336143/en/.

International Peer-Reviewed Conference/Proceedings

[24] C. JONCOUR, S. MICHEL, R. SADYKOV, D. SVERDLOV, F. VANDERBECK. *Column Generation based Primal Heuristics*, in "Submitted to ISCO 2010", 2010.

[25] A. PÊCHER, A. WAGLER. *On the polynomial time computability of the circular chromatic number for some superclasses of perfect graph*, in "LAGOS'09, Electronic Notes in Discrete Mathematics, Bresil", vol. 35, 2009, p. 53-58 DE .

[26] R. SADYKOV. *On scheduling malleable jobs to minimise the total weighted completion time*, in "13th IFAC Symposium on Information Control Problems in Manufacturing, Russie Moscow", 2009, http://hal.inria.fr/inria-00339646/en/.

Workshops without Proceedings

[27] C. JONCOUR, A. PÊCHER. *Consecutive ones matrices for orthogonal packing problems*, in "ISMP 2009, Chicago, USA", 2009.

[28] C. JONCOUR, A. PÊCHER. *Consecutive ones matrices for orthogonal packing problems*, in "JPOC6, Bordeaux, France", 2009.

[29] S. MICHEL, F. VANDERBECK. *Review and Classification of Branching Schemes for Branch-and-Price*, in "ISMP 2009, Chicago, USA", 2009.

[30] M. NAMAZIFAR, P. BELOTTI, A. MILLER. *Linear Envelopes of Bounded Multilinear Functions*, in "BR-OPT, Pittsburgh, USA", 2009 US .

[31] M. NAMAZIFAR, J. LINDEROTH, J. LUEDTKE, A. MILLER. *Strong Relxations and Computations for Global Optimization Problems with Multilinear Terms*, in "ISMP 2009, Chicago, USA", 2009 US .

[32] R. SADYKOV, F. VANDERBECK. *Branch-and-price for bin packing with conflicts*, in "ISMP 2009, Chicago, USA", 2009.

[33] G. STAUFFER. *An extended formulation for the stable set polytope of claw-free graphs*, in "AIRO2009, The fortieth Annual Conference of the Italian Operational Research Society, Siena, Italy", 2009.

[34] F. VANDERBECK, L. WOLSEY. *Reformulation and Decomposition of Integer Programs*, in "JFRO09, Paris, France", 2009 BE .

[35] F. VANDERBECK, L. WOLSEY. *Reformulation and Decomposition of Integer Programs*, in "JPOC6, Bordeaux, France", 2009 BE .

[36] B. VIGNAC, B. JAUMARD, F. VANDERBECK. *A Hierarchical Optimization Approach to Optical Network Design where Traffic Grooming and Routing is Solved by Column Generation*, in "INOC 2009, Pisa, Italy", 2009 CA .

Scientific Books (or Scientific Book chapters)

[37] F. VANDERBECK, L. WOLSEY. *Reformulation and Decomposition of Integer Programs*, in "50 Years of Integer Programming 1958-2008: From the Early Years to the State-of-the-Art", Springer, 2010 BE .

Research Reports

[38] R. SADYKOV. *A polynomial algorithm for a simple scheduling problem at cross docking terminals*, INRIA, 2009, http://hal.inria.fr/inria-00412519/en/, RR-7054, Rapport de recherche.

[39] B. VIGNAC, B. JAUMARD, F. VANDERBECK. *Hierarchical Heuristic for the GRWA Problem in WDM Networks with Delay Constraints*, INRIA, 2009, http://hal.inria.fr/inria-00415513/en/, Rapport de rechercheCA.

[40] B. VIGNAC, F. VANDERBECK, B. JAUMARD. *Nested Decomposition Approach to an Optical Network Design Problem*, INRIA, 2009, http://hal.inria.fr/inria-00415500/en/, Rapport de rechercheCA.

[41] B. VIGNAC, F. VANDERBECK, B. JAUMARD. *Reformulation and Decomposition Approaches for Traffic Routing in Optical Networks*, INRIA, 2009, http://hal.inria.fr/inria-00392256/en/, http://www.math.u-bordeaux.fr/ fv/papers/grwaWP.pdf, Rapport de rechercheCA.

Other Publications

[42] K. AKARTUNALI, A. J. MILLER. *A Computational Analysis of Lower Bounds for Big Bucket Production Planning Problems*, 2009, http://hal.archives-ouvertes.fr/hal-00387105/en/AU.

[43] A. J. MILLER. *A Combinatorial Active Set Algorithm for Linear and Quadratic Programming*, 2009, http://hal.archives-ouvertes.fr/hal-00387108/en/.

[44] M. MONTASSIER, A. PÊCHER, A. RASPAUD, X. ZHU. *Edge-partitions of sparse graphs and their applications to game coloring*, 2009, http://hal.archives-ouvertes.fr/hal-00368828/en/TW.

References in notes

[45] *CBC Home Page*, 2008, https://projects.coin-or.org/Cbc.

[46] *COIN-OR: open source for the operations research community*, 2008, http://www.coin-or.org/.

[47] S. AHMED, A. KING, G. PARIJA. *A Multi-Stage Stochastic Integer Programming Approach for Capacity Expansion under Uncertainty*, in "Journal of Global Optimization", vol. 26, 2003, p. 3-24.

[48] R. K. AHUJA, J. B. ORLIN, A. TIWARI. *A survey of very large scale neighborhood search techniques*, in "Discrete Applied Mathematics", vol. 123, 2002, p. 75–102.

[49] L. APPELGREN. *A column generation algorithm for a ship scheduling problem*, in "Transportation Science", 1969, p. 53–68.

[50] D. APPLEGATE, W. COOK, R. BIXBY, V. CHVATAL. *Implementing the Dantzig-Fulkerson-Johnson algorithm for large traveling salesman problems*, in "Mathematical Programming, series B", vol. 97, 2003, p. 91–153.

[51] K. APT. *Principles of Constraint Programming*, Cambridge University Press, New York, NY, USA, 2003.

[52] A. BALAKRISHNAN, F. VANDERBECK. *Setup Optimization and Workload Balancing for Mixed-model Electronics Assembly Operations*, in "Operations Research", vol. 47, n^o 3, 1999, p. 395–409.

[53] P. BAPTISTE, R. SADYKOV. *Formulations PLNE pour l'ordonnancement des chaînes sur une machine*, in "Livre des résumé du 9ème Congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision", February 2008, p. 43–44.

[54] C. BARNHART, E. JOHNSON, G. NEMHAUSER, M. SAVELSBERGH, P. VANCE. *Branch-and-Price: Column Generation for Solving Huge Integer Programs*, in "Operations Research", vol. 46, 1998, p. 316–329.

[55] S. BENSON, Y. YE. *DSDP5 : Software For Semidefinite Programming*, n^o ANL/MCS-P1289-0905, Mathematics and Computer Science Division, Argonne, IL, 2005, Technical report.

[56] D. BIENSTOCK, S. MATTIA. *Using Mixed-Integer Programming to Solve Power Grid Blackout Problems*, in "Discrete Optimization", vol. 4, 2007, p. 115-141.

[57] D. BIENSTOCK, J. F. SHAPIRO. *Optimizing Resource Acquisition Decisions by Stochastic Programming*, in "Management Science", vol. 34, 1988, p. 215–229.

[58] D. BIENSTOCK, N. ÖZBAY. *Computing robust basestock levels*, in "Discrete Optimization", vol. 5, n^o 2, 2008, p. 389 - 414, In Memory of George B. Dantzig.

[59] A. BILLIONNET, S. ELLOUMI. *Using a Mixed Integer Quadratic Programming Solver for the Unconstrained Quadratic 0-1 Problem*, in "Mathematical Programming", vol. 109, n^o 1, 2007, p. 55–68.

[60] A. BILLIONNET, S. ELLOUMI, M.-C. PLATEAU. *Programmation Quadratique en Variables 0-1 avec Contraintes Linéaires*, 2006, ROADEF 06.

[61] R. BOORSTYN, H. FRANK. *Large-Scale Network Topological Optimization*, in "IEEE Transactions on Communications", vol. 25, 1977, p. 29-47.

[62] A. CAPRARA, M. FISCHETTI. $0-1/2$ *Chvàtal-Gomory cuts*, in "Mathematical Programming", vol. 74, 1996, p. 221–235.

[63] F. CHEN, A. FEDERGRUEN, Y.-S. ZHENG. *Coordination Mechanisms for a Distribution System with One Supplier and Multiple Retailers*, in "Management Science", vol. 47, 2001, p. 693–708.

[64] M. CHUDNOVSKY, P. SEYMOUR. *Claw free Graphs IV. Global structure.*, in "Journal of Combinatorial Theory. Ser B", vol. 98, 2008, p. 1373-1410.

[65] G. CORNUÉJOLS. *Valid Inequalities for Mixed Integer Linear Programs*, in "Mathematical Programming series A and B", vol. 112, n^o 1, July 2007, p. 3–44.

[66] G. CORNUÉJOLS, C. LEMARÉCHAL. *A convex-analysis perspective on disjunctive cuts*, in "Mathematical Programming", vol. 106, n^o 3, 2005, p. 567–586.

[67] G. DAHL, D. HUYGENS, A. R. MAHJOUB, P. PESNEAU. *On the k-edge disjoint 2-hop-constrained paths polytope*, in "Operations Research Letters", vol. 34, 2005, p. 577–582.

[68] E. DANNA, E. ROTHBERG, C. L. PAPE. *Exploring relaxation induced neighborhoods to improve MIP solutions*, in "Mathematical Programming", vol. 102, n^o 1, 2005, p. 71–90.

[69] Z. DEGAEVE, R. JANS. *A New Dantzig-Wolfe Reformulation and Branc-and-Price Algorithm for the Capacitated Lot Sizing Problem with Set Up Times*, ERIM, Erasmus Universiteit Rotterdam, 2003, Technical report.

[70] G. DESAULNIERS, J. DESROSIERS, Y. DUMAS, S. MARC, B. RIOUX, M. SOLOMON, F. SOUMIS. *Crew Pairing at Air France*, in "European Journal of Operational Research", vol. 97, n^o 2, 1997, p. 245–259.

[71] J. DESROSIERS, Y. DUMAS, M. SOLOMON, F. SOUMIS. *Time Constrained Routing and Scheduling, Chapter 2*, in "Handbooks in Operations Research and Management Sciences: Networks", M. BALL, T. MAGNANTI, C. MONMA, G. NEMHAUSER (editors), vol. 8, Elsevier, 1995.

[72] A. S. DRUD. *CONOPT – A Large-Scale GRG Code*, in "ORSA Journal on Computing", vol. 6, 1994, p. 207–216.

[73] L. DUBOST, R. GONZALEZ, C. LEMARÉCHAL. *A primal-proximal heuristic applied to the French unit-commitment problem*, in "Mathematical Programming", vol. 104, n° 1, 2005, p. 129–152.

[74] J. EDMONDS. *Maximum matching and a polyhedron with 0, 1 vertices*, in "Journal of Research National Bureau of Standards", vol. 69B, 1965, p. 125–130.

[75] F. EISENBRAND, G. ORIOLO, G. STAUFFER, P. VENTURA. *The stable set polytope of quasi-line graphs*, in "Combinatorica", vol. 28, n° 1, 2008, p. 45-67.

[76] T. FAHLE, U. JUNKER, S. E. KARISCH, N. KOHL, M. SELLMANN, B. VAABEN. *Constraint Programming Based Column Generation for Crew Assignment*, in "Journal of Heuristics", vol. 8, n° 1, 01 2002, p. 59–81.

[77] S. FEKETE, J. SCHEPERS. *A combinatorial characterization of higher-dimensional orthogonal packing*, in "Mathematics of Operations Research", 2004, p. 353–368.

[78] M. FISCHETTI, F. GLOVER, A. LODI. *The Feasibility Pump*, in "Mathematical Programming A", vol. 104, 2005, p. 91–104.

[79] M. FISCHETTI, A. LODI. *Local Branching*, in "Mathematical Programming B", vol. 98, 2003, p. 23–47.

[80] M. FISCHETTI, A. LODI. *Optimizing over the first Chvàtal closure*, in "Mathematical Programming B", vol. 110, n° 1, 2007, p. 3–20.

[81] F. FOCACCI, A. LODI, M. MILANO. *A Hybrid Exact Algorithm for the TSPTW*, in "INFORMS Journal on Computing", vol. 14, n° 4, 1 2002, p. 403–417.

[82] B. FORTZ, A. R. MAHJOUB, S. T. MCCORMICK, P. PESNEAU. *Two-edge connected subgraphs with bounded rings: polyhedral results and branch-and-cut*, in "Mathematical Programming", vol. 105, n° 1, 2006, p. 85–111.

[83] L. GELY. *Real-time train scheduling at SNCF*, in "1st Workshop on Robust Planning and Rescheduling in Railways", ARRIVAL meeting on Robust planning and Rescheduling in Railways, Utrecht, April 2007.

[84] R. GILES, L. E. JUN. TROTTER. *On stable set polyhedra for $K_{(1,3)}$-free graphs*, in "Journal on Combinatorial Theory", vol. 31, 1981, p. 313–326.

[85] P. E. GILL, W. MURRAY, M. A. SAUNDERS. *SNOPT: An SQP algorithm for large–scale constrained optimization*, in "SIAM Journal on Optimization", vol. 12, n° 4, 2002, p. 979-1006.

[86] P. GILMORE, R. GOMORY. *A linear programming approach to the cutting stock problem: part II*, in "Operations Research", vol. 11, 1963, p. 863–888.

[87] M. T. GODINHO, L. GOUVEIA, T. L. MAGNANTI, P. PESNEAU, J. PIRES. *On a Time-Dependent Model for the Unit Demand Vehicle Routing Problem*, n° 11-2007, Centro de Investigacao Operacional da Universidade de Lisboa, 2007, Technical report.

[88] M. GOEMANS. *Semidefinite programming in combinatorial optimization*, in "Mathematical Programming", vol. 79, 1997, p. 143–161.

[89] M. GOEMANS, D. WILLIAMSON. *Improved approximation algorithms for maximum cut and satisfiability problems using semidefinite programming*, in "Journal of Association for Computing Machinery", vol. 42, n° 6, 1995, p. 1115–1145.

[90] L. GOUVEIA, J. RIERA, J.-J. SALAZAR. *On the Vehicule Routing Problem with lower bound capacities*, in "INOC 2009 International Network Optimization Conference", 2009.

[91] I. E. GROSSMANN, R. W. H. SARGENT. *Optimal Design of Multipurpose Batch Plants*, in "Industrial & Engineering Chemistry Process Design and Development", vol. 18, 1979, p. 343-348.

[92] M. GRÖTSCHEL, L. LOVÁSZ, A. SCHRIJVER. *The ellipsoid method and its consequences in combinatorial optimization*, in "Combinatorica", vol. 1, n° 2, 1981, p. 169–197.

[93] Y. GUAN, S. AHMED, A. MILLER, G. NEMHAUSER. *On formulations of the stochastic uncapacitated lot-sizing problem*, in "Operations Research Letters", vol. 34, 2006, p. 241-250.

[94] Y. GUAN, S. AHMED, G. NEMHAUSER, A. MILLER. *A branch-and-cut algorithm for the stochastic uncapacitated lot-sizing problem*, in "Mathematical Programming", vol. 105, 2006, p. 55-84.

[95] Y. GUAN, A. J. MILLER. *A Polynomial Time Algorithm for the Stochastic Uncapacitated Lot-Sizing Problem with Backlogging*, in "IPCO 2008, LNCS 5035, Bertinoro, Italy", 2008, p. 450-462.

[96] Y. GUAN, A. J. MILLER. *Polynomial-Time Algorithms for Stochastic Uncapacitated Lot-Sizing Problems*, in "Operations Research", vol. 56, n° 2, 2008, p. 1172–1183.

[97] M. GUIGNARD, S. KIM. *Lagrangean decomposition: A model yielding strong Lagrangean bounds*, in "Matematical Progamming", vol. 39, 1987, p. 215–228.

[98] L. GÉLY. *Master Thesis*, Université Bordeaux 1, 2006, In partnership with SNCF, Masters thesis.

[99] Y. HENDEL, R. SADYKOV. *Timing problem for scheduling an airborne radar*, in "Proceedings of the 11th International Workshop on Project Management and Scheduling, Istanbul, Turkey", April 2008, p. 132-135.

[100] J. N. HOOKER. *Logic, Optimization, and Constraint Programming*, in "INFORMS Journal on Computing", vol. 14, n° 4, 1 2002, p. 295–321.

[101] J. N. HOOKER. *Planning and Scheduling by Logic-Based Benders Decomposition*, in "Operations Research", vol. 55, n° 3, 5 2007, p. 588–602.

[102] J. N. HOOKER, H. YAN, I. E. GROSSMANN, R. RAMAN. *Logic cuts for processing networks with fixed charges*, in "Comput. Oper. Res.", vol. 21, n° 3, 1994, p. 265–279.

[103] D. HUYGENS, M. LABBÉ, A. R. MAHJOUB, P. PESNEAU. *The two-edge connected hop-constrained network design problem: Valid inequalities and branch-and-cut*, in "Networks", vol. 49, n° 1, 2007, p. 116-133.

[104] D. HUYGENS, A. R. MAHJOUB, P. PESNEAU. *Two edge-disjoint hop-constrained paths and polyhedra*, in "SIAM Journal of Discrete Mathematics", vol. 18, n^{o} 2, 2004, p. 287–312.

[105] V. JAIN, I. E. GROSSMANN. *Algorithms for Hybrid MILP/CP Models for a Class of Optimization Problems*, in "INFORMS Journal on Computing", vol. 13, n^{o} 4, 1 2001, p. 258–276.

[106] C. JONCOUR, A. PÊCHER, P. PESNEAU, F. VANDERBECK. *Mathematical programming formulations for the orthogonal 2d knapsack problem*, in "Livre des résumé du 9ème Congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision", February 2008, p. 255–256, http://hal.archives-ouvertes.fr/hal-00307152/en/.

[107] D. KARGER, R. MOTWANI, M. SUDAN. *Approximate graph coloring by semidefinite programming*, in "Journal of the ACM", vol. 45, n^{o} 2, March 1998, p. 246–265.

[108] H. KERIVIN, A. MAHJOUB. *On Survivable Network Polyhedra*, in "Discrete Mathematics", vol. 290, 2005, p. 183–210.

[109] H. KERIVIN, A. R. MAHJOUB. *Design of Survivable Networks: A Survey*, in "Networks", vol. 46, n^{o} 1, 2005, p. 1–21.

[110] M. R. KILINÇ, J. T. LINDEROTH, A. J. MILLER. *Strong Branching Disjunctive Cuts for MIP*, in "INFORMS National Meeting, Washington, D.C., USA", 2008.

[111] G. R. KOCIS, I. E. GROSSMANN. *Global Optimization of Nonconvex Mixed–Integer Nonlinear Programming (MINLP) problems in Process Synthesis*, in "Industrial Engineering Chemistry Research", vol. 27, 1988, p. 1407-1421.

[112] E. L. LAWLER, J. K. LENSTRA, A. H. G. RINNOOY KAN, D. B. SHMOYS. *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization*, Wiley and Sons, New-York, 1985.

[113] C. LEMARÉCHAL, F. OUSTRY. *SDP relaxations in combinatorial optimization from a Lagrangian point of view.*, in "Advances in Convex Analysis and Global Optimization", N. HADJISAVVAS, P. M. PARDALOS (editors), Kluwer, 2001, p. 119–134.

[114] A. N. LETCHFORD, J. J. SALAZAR-GONZALEZ. *Projection results for vehicle routing*, in "Mathematical Programming", vol. 105, 2006, p. 251–274.

[115] R. LEVI, M. PAL, R. ROUNDY, D. SCHMOYS. *Approximation Algorithms for Stochastic Inventory Control Models*, in "Mathematics of Operations Research", vol. 32, 2007, p. 284–302.

[116] L. LIPTAK, L. LOVÁSZ. *Critical Facets of the Stable Set Polytope*, in "Combinatorica", vol. 21, n^{o} 1, 2001, p. 61–88.

[117] L. LOVÁSZ. *On the Shannon capacity of a graph*, in "IEEE Transactions on Information Theory", vol. 25, n^{o} 1, 1979, p. 1–7.

[118] L. LOVÁSZ, M. PLUMMER. *Matching Theory*, North Holland, Amsterdam, 1986.

[119] P. MEURDESOIF. *Approches convexes pour la coloration de graphes avec application à l'allocation de fréquences*, University of Grenoble 1, 2002, Ph. D. Thesis.

[120] P. MEURDESOIF, P. PESNEAU, F. VANDERBECK. *Meter installation for monitoring network traffic*, in "International Conference on Network Optimization, INOC", International Network Optimization Conference (INOC), Spa, Belgium, 2007.

[121] S. MICHEL. *Optimisation des tournées de véhicules combinées à la gestion de stock*, Université Bordeaux 1, Dec 2006, Ph. D. Thesis.

[122] S. MICHEL, F. VANDERBECK. *A Column Generation based Tactical Planning Method for Inventory Routing*, INRIA, 2008, http://hal.inria.fr/inria-00169311/en/, Research Report.

[123] M. MILANO. *Constraint and Integer Programming: Toward a Unified Methodology (Operations Research/Computer Science Interfaces", 27)*, Kluwer Academic Publishers, Norwell, MA, USA, 2003.

[124] A. MILLER, G. NEMHAUSER, M. SAVELSBERGH. *Solving Multi–Item Capacitated Lot–Sizing Problems with Setup Times by Branch–and–Cut*, August 2000, CORE DP 2000/39, Université Catholique de Louvain, Louvain-la-Neuve, Belgium.

[125] A. MILLER, G. NEMHAUSER, M. SAVELSBERGH. *On the Polyhedral Structure of a Multi–Item Production Planning Model with Setup Times*, in "Mathematical Programming", vol. 94, 2003, p. 375–405.

[126] A. MILLER, L. WOLSEY. *Tight MIP Formulations for Multi-Item Discrete Lot-Sizing Problems*, in "Operations Research", vol. 51, 2003, p. 557-605.

[127] G. J. MINTY. *On maximal independent sets of vertices in claw-free graphs*, in "Journal on Combinatorial Theory", vol. 28, 1980, p. 284–304.

[128] J. MOMOH, R. KOESSLER, M. BOND, B. STOTT, D. SUN, A. PAPALEXOPOULOS, P. RISTANOVIC. *Challenges to Optimal Power Flow*, in "IEEE Transaction on Power Systems", vol. 12, 1997, p. 444–455.

[129] M. MOURGAYA, F. VANDERBECK. *The Periodic Vehicle Routing Problem: classification and heuristic*, in "RAIRO-OR", vol. 40, n^o 2, April-June 2006, p. 169–194.

[130] M. MOURGAYA, F. VANDERBECK. *Column generation based heuristic for tactical planning in multi period vehicle routing*, in "European Journal of Operational Research", vol. 183, n^o 3, 2007, p. 1028-1041.

[131] D. NAKAMURA, A. TAMURA. *A revision of Minty's algorithm for finding a maximum weighted stable set of a claw-free graph*, in "Journal of the Operations Research Society of Japan", vol. 44, n^o 2, 2001, p. 194–204.

[132] M. NAMAZIFAR, R. LOUGEE-HEIMER, A. J. MILLER. *Randomized Rounding; A New Primal Heuristic for General Mixed Integer Programming*, in "INFORMS Computing Society Conference, Charleston, USA", January 2008.

[133] M. NAMAZIFAR, A. J. MILLER. *A Parallel Macro Partitioning Framework for Solving Mixed Integer Programs*, in "CPAIOR 2008, LNCS 5015, Paris, France", 2008, p. 343-348.

[134] G. L. NEMHAUSER, L. A. WOLSEY. *Integer and Combinatorial Optimization*, John Wiley & Sons, 1988.

[135] P. NOBILI, A. SASSANO. *Personal Communication*, 2009.

[136] G. ORIOLO, U. PIETROPAOLI, G. STAUFFER. *A new algorithm for the maximum weighted stable set problem in claw-free graphs*, in "Proceedings Thirteenth IPCO Conference", A. LODI, A. PANCONESI, G. RINALDI (editors), 2008, p. 77–96.

[137] M. PADBERG, G. RINALDI. *A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems*, in "SIAM Review", vol. 33, n^o 1, 1991, p. 60–100.

[138] N. PERROT. *Advanced IP column generation strategies for the cutting stock stock problem and its variants*, University Bordeaux 1, 2005, Ph. D. Thesis.

[139] N. PERROT, F. VANDERBECK. *Industrial Cutting Stock Problem*, in "Collection Sciences, Technologie Informatique", PRESSES UNIVERSITAIRES DE VALENCIENNES (editor), ROADEF 2006, 2006.

[140] S. POLJAK, F. RENDL, H. WOLKOWICZ. *A recipe for semidefinite relaxation for 0-1 quadratic programming*, in "Journal of Global Optimization", vol. 7, 1995, p. 51–73.

[141] A. PÊCHER, A. WAGLER. *A construction for non-rank facets of stable set polytopes of webs*, in "European Journal of Combinatorics", vol. 27, n^o 7, oct 2006, p. 1172–1185.

[142] A. PÊCHER, A. K. WAGLER. *Almost all webs are not rank-perfect*, in "Mathematical Programming", vol. 105, 2006, p. 311–328.

[143] A. J. QUIST, R. VAN GEMEERT, J. E. HOOGENBOOM, T. ÍLLES, C. ROOS, T. TERLAKY. *Application of nonlinear optimization to reactor core fuel reloading*, in "Annals of Nuclear Energy", vol. 26, 1998, p. 423-448.

[144] R. ROUNDY. *98%-effective integer-ratio lot-sizing for one-warehouse, retailer systems*, in "Management Science", vol. 31, 1985, p. 1416–1430.

[145] R. SADYKOV. *A branch-and-check algorithm for minimizing the sum of the weights of the late jobs on a single machine with release dates*, in "European Journal of Operations Research", vol. 189, n^o 3, 2008, p. 1284–1304.

[146] N. SBIHI. *Algorithme de recherche d'un stable de cardinalité maximum dans un graphe sans étoile*, in "Discrete Mathematics", vol. 29, 1980, p. 53–76.

[147] A. SCHRIJVER. *Combinatorial Optimization : Polyhedra and Efficiency*, CVI, 2003.

[148] A. SUTTER, F. VANDERBECK, L. WOLSEY. *Optimal Placement of Add/Drop Multiplexers: Heuristic and Exact Algorithms*, in "Operations Research", vol. 46, n^o 5, 1998, p. 719–728.

[149] S. TAKRITI, J. BIRGE, E. LONG. *A Stochastic Model for the Unit Commitment Problem*, in "IEEE Transactions on Power Systems", vol. 11, n^o 3, 1996, p. 1497–1508.

[150] M. TAWARMALANI, N. V. SAHINIDIS. *Global Optimization of Mixed integer Nonlinear Programs: A Theoretical and Computational Study*, in "Mathematical Programming", vol. 99, 2004, p. 563-591.

[151] M. TÜRKAY, I. GROSSMANN. *Logic-based MINLP algorithms for the optimal synthesis of process networks*, in "Computers and Chemical Engineering", n° 20, 1996, p. 959–978.

[152] P. H. VANCE, C. BARNHART, E. L. JOHNSON, G. L. NEMHAUSER. *Airline Crew Scheduling: A New Formulation and Decomposition Algorith*, in "Operations Research", vol. 45, 1997, p. 188–200.

[153] F. VANDERBECK, M. SAVELSBERGH. *A generic view of Dantzig-Wolfe decomposition in mixed integer programming*, in "Operations Research Letters", vol. 34, n° 3, 2006, p. 296–306.

[154] F. VANDERBECK. *Exact algorithm for minimising the number of setups in the one-dimensional cutting stock problem*, in "Operations Research", vol. 48, n° 5, 2000, p. 915–926.

[155] F. VANDERBECK. *On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm*, in "Operations Research", vol. 48, n° 1, 2000, p. 111–128.

[156] F. VANDERBECK. *A nested decomposition approach to a 3-stage 2-dimensional cutting stock problem*, in "Management Science", vol. 47, n° 2, 2001, p. 864–879.

[157] F. VANDERBECK. *Extending Dantzig's Bound to the Bounded Multiple-class Knapsack Problem*, in "Mathematical Programming, Ser. A", vol. 94, n° 1, 2002, p. 125–136.

[158] F. VANDERBECK. *Towards a generic branch-and-price solver: progress report*, in "CORS/Optimization Days, May 2008, Quebec", 2008, plenary talk.

[159] F. VANDERBECK. *Lot-sizing with start-up times*, in "Management Science", vol. 44, n° 10, 1998, p. 1409–1425.

[160] F. VANDERBECK. *Computational study of a column generation algorithm for bin packing and cutting stock problems*, in "Mathematical Programming, Ser. A", vol. 86, 1999, p. 565–594.

[161] F. VANDERBECK, L. WOLSEY. *An Exact Algorithm for IP Column Generation*, in "Operations Research Letters", vol. 19, n° 4, 1996, p. 151–159.

[162] B. VIGNAC, B. JAUMARD, F. VANDERBECK. *An Hierarchical Optimization approach to Optical Network Design where Traffic Grooming and Routing is Solved by Column Generation*, 2008, submitted to INOC 2009.

[163] B. VIGNAC, B. JAUMARD, F. VANDERBECK. *Hierarchical Optimization Procedure for Traffic Grooming in WDM Optical Networks*, 2008, submitted to IEEE conference ONDM2009.

[164] R. WALTZ. *Current Challenges in Nonlinear Optimization*, 2007, Presentation at San Diego Supercomputer Center: CIEG Spring Orientation Workshop, available at www.sdsc.edu/us/training/workshops/2007sac_studentworkshop/docs/SD