INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team reo

Numerical simulation of biological flows

Paris - Rocquencourt

Theme : Observation, Modeling, and Control for Life Sciences

Activity Report

2009

Table of contents

REO is a joint project of the INRIA Research Unit of Rocquencourt and the Jacques-Louis Lions Laboratory (LJLL) of the Pierre et Marie Curie (Paris 6) University.

1. Team

Research Scientist

Jean-Frédéric Gerbeau [Team Leader, Research Director (DR) INRIA, HdR]
Marc Thiriet [Co-team leader, Research Associate (CR) CNRS, HdR]
Miguel Ángel Fernández [Research Associate (CR) INRIA]
Céline Grandmont [Research Associate (CR) INRIA, HdR]
Irène Vignon-Clementel [Research Associate (CR) INRIA]

Faculty Member

Laurent Boudin [Assistant professor Paris 6]
Muriel Boulakia [Assistant professor, on leave from Univ. Paris 6 since september 2008]
Laurent Dumas [Assistant professor Paris 6, HdR]
Vincent Martin [Assistant professor, on leave from Univ. Tech. Compiègne since september 2009]
Sébastien Martin [Assistant professor, on leave from Univ. Paris Sud since september 2009]

Technical Staff

Elsie Phé [part-time MACS project-team, until June 2009]
Michael Grasseau [INRIA, until August 2009]
Julien Castelneau [INRIA]

PhD Student

Matteo Astorino [INRIA, CardioSense3D, Univ. Paris 6]
Cristóbal Bertoglio [INRIA, IP European project "euHeart" since September 2008]
Paul Cazeaux [part time, Univ. Paris 6, since September 2009]
Anne-Claire Egloffe [INRIA, ANR M3RS, since September 2009]
Stéphane Liwarek [INRIA, since November 2009]
Ayman Moussa [part time, ENS Cachan]
Jimmy Mullaert [INRIA and Univ. Paris 6]
Nejib Zemzemi [INRIA and Univ. Paris 11]

Post-Doctoral Fellow

Alfonso Caiazzo [INRIA, ERCIM, since December 2008]
Franz Chouly [INRIA, ANR Pitac]
Driss Yakoubi [INRIA, ANR M3RS]

Visiting Scientist

Leif Rune Hellevik [Professor, on sabbatical leave since september 2009 from University of Science and Technology, Trondheim, Norway]

Administrative Assistant

Maryse Desnous [Secretary, part-time, 20%]

Other

Alexandre Birolleau [Internship, Stanford University]
Keijo Nissen [Internship]
Guillaume Troïanowski [Internship, Stanford University]

2. Overall Objectives

2.1. Introduction

REO is a joint project of the INRIA Research Center of Paris-Rocquencourt and the Jacques-Louis Lions Laboratory (LJLL) of the Pierre and Marie Curie (Paris 6) University. Its research activities are aimed at

- modeling some aspects of the cardiovascular and respiratory systems, both in normal and pathological states;
- developing and analyzing efficient, robust and reliable numerical methods for the simulation of those models;
- developing simulation software to guide medical decision and to design more efficient medical devices.

2.2. Highlights of the year

- Céline Grandmont defended her *Habilitation à Diriger des Recherches* (HDR) entitled *Analyse mathématique et numérique de problèmes d'interaction fluide-structure. Application à la modélisation du système respiratoire*
- Ayman Moussa defended his *Phd thesis* entitled *Mathematical and numerical study of the aerosol transport in the human lung*

3. Scientific Foundations

3.1. Multiphysics modeling

In large vessels and in large bronchi, blood and air flows are generally supposed to be governed by the incompressible Navier-Stokes equations. Indeed in large arteries, blood can be supposed to be Newtonian, and at rest air can be modeled as an incompressible fluid. The cornerstone of the simulations is therefore a Navier-Stokes solver. But other physical features have also to be taken into account in simulations of biological flows, in particular fluid-structure interaction in large vessels and transport of sprays, particles or chemical species.

3.1.1. Fluid-structure interaction

Fluid-structure coupling occurs both in the respiratory and in the circulatory systems. We focus mainly on blood flows since our work is more advanced in this field. But the methods developed for blood flows could be also applied to the respiratory system.

Here "fluid-structure interaction" means a coupling between the 3D Navier-Stokes equations and a 3D (possibly thin) structure in large displacements.

The numerical simulations of the interaction between the artery wall and the blood flows raise many issues: (1) the displacement of the wall cannot be supposed to be infinitesimal, geometrical nonlinearities are therefore present in the structure and the fluid problem have to be solved on a moving domain (2) the densities of the artery walls and the blood being close, the coupling is strong and has to be tackled very carefully to avoid numerical instabilities, (3) "naive" boundary conditions on the artificial boundaries induce spurious reflection phenomena.

Simulation of valves, either at the outflow of the cardiac chambers or in veins, is another example of difficult fluid-structure problems arising in blood flows. In addition, we have to deal with very large displacements and changes of topology (contact problems).

Because of the above mentioned difficulties, the interaction between the blood flow and the artery wall has often been neglected in most of the classical studies. The numerical properties of the fuid-structure coupling in blood flows are rather different from other classical fluid-structure problems. In particular, due to stability reasons it seems impossible to successfully apply the explicit coupling schemes used in aeroelasticity.

As a result, fluid-structure interaction in biological flows raise new challenging issues in scientific computing and numerical analysis : new schemes have to be developed and analyzed.

3.1.2. Aerosol

Complex two-phase fluids can be modeled in many different ways. Eulerian models describe both phases by physical quantities such as the density, velocity or energy of each phase. In the mixed fluid-kinetic models, the diphasic fluid has one dispersed phase, which is constituted by a spray of droplets, with a possibly variable size, and a continuous classical fluid.

This type of model was first introduced by Williams [74] in the frame of combustion. It was later used to develop the Kiva code [55] at the Los Alamos National Laboratory, or the Hesione code [69], for example. It has a wide range of applications, besides the nuclear setting: diesel engines, rocket engines [61], therapeutic sprays, *etc.* One of the interests of such a modeling is that various phenomena on the droplets can be taken into account with an accurate precision: collision, breakups, coagulation, vaporization, chemical reactions, *etc.*, at the level of the droplets.

The model usually consists in coupling a kinetic equation, that describes the spray through a probability density function, and classical fluid equations (typically Navier-Stokes). The numerical solution of this system relies on the coupling of a method for the fluid equations (for instance, a finite volume method) with a method fitted to the spray (particle method, Monte Carlo).

We are mainly interested in modeling therapeutic sprays either for local or general treatments. The study of the underlying kinetic equations should lead us to a global model of the ambient fluid and the droplets, with some mathematical significance. Well-chosen numerical methods can give some tracks on the solutions behavior and help to fit the physical parameters which appear in the models.

3.2. Multiscale modeling

Multiscale modeling is a necessary step for blood and respiratory flows. In this section, we focus on blood flows. Nevertheless, preliminary investigations are currently carried out in our team on respiratory flows.

3.2.1. Arterial tree modeling

Problems arising in the numerical modeling of the human cardiovascular system often require an accurate description of the flow in a specific sensible subregion (carotid bifurcation, stented artery, *etc.*). The description of such local phenomena is better addressed by means of three-dimensional (3D) simulations, based on the numerical approximation of the incompressible Navier-Stokes equations, possibly accounting for compliant (moving) boundaries. These simulations require the specification of boundary data on artificial boundaries that have to be introduced to delimit the vascular district under study. The definition of such boundary conditions is critical and, in fact, influenced by the global systemic dynamics. Whenever the boundary data is not available from accurate measurements, a proper boundary condition requires a mathematical description of the action of the reminder of the circulatory system on the local district. From the computational point of view, it is not affordable to describe the whole circulatory system keeping the same level of detail. Therefore, this mathematical description relies on simpler models, leading to the concept of *geometrical multiscale* modeling of the circulation [70]. The underlying idea consists in coupling different models (3D, 1D or 0D) with a decreasing level of accuracy, which is compensated by their decreasing level of computational complexity.

The research on this topic aims at providing a correct methodology and a mathematical and numerical framework for the simulation of blood flow in the whole cardiovascular system by means of a geometric multiscale approach. In particular, one of the main issues will be the definition of stable coupling strategies between 3D and 1D models that generalize the work reported in [64] to general geometries coming from medical imaging.

When modeling the arterial tree, a standard way consists in imposing a pressure or a flow rate at the inlet of the aorta, *i.e.* at the network entry. This strategy does not allow to describe important features as the overload in the heart caused by backward traveling waves. Indeed imposing a boundary condition at the beginning of the aorta artificially disturbs physiological pressure waves going from the arterial tree to the heart. The only way to catch this physiological behavior is to couple the arteries with a model of heart, or at least a model of left ventricle.

A constitutive law for the myocardium, controlled by an electrical command, is currently developed in the CardioSense3D project [1]. One of our objectives is to couple artery models with this heart model.

A long term goal is to achieve 3D simulations of a system including heart and arteries. One of the difficulties of this very challenging task is to simulate the aortic valve. To this purpose, we plan to mix arbitrary Lagrangian Eulerian and fictitious domain approaches.

3.2.2. Heart perfusion modeling

The heart is the organ that regulates, through its periodical contraction, the distribution of oxygenated blood in human vessels in order to nourish the different parts of the body. The heart needs its own supply of blood to work. The coronary arteries are the vessels that accomplish this task. The phenomenon by which blood reaches myocardial heart tissue starting from the blood vessels is called in medicine perfusion. The analysis of heart perfusion is an interesting and challenging problem. Our aim is to perform a three-dimensional dynamical numerical simulation of perfusion in the beating heart, in order to better understand the phenomena linked to perfusion. In particular the role of the ventricle contraction on the perfusion of the heart is investigated as well as the influence of blood on the solid mechanics of the ventricle. Heart perfusion in fact implies the interaction between heart muscle and blood vessels, in a sponge-like material that contracts at every heartbeat via the myocardium fibers.

Despite recent advances on the anatomical description and measurements of the coronary tree and on the corresponding physiological, physical and numerical modeling aspects, the complete modeling and simulation of blood flows inside the large and the many small vessels feeding the heart is still out of reach. Therefore, in order to model blood perfusion in the cardiac tissue, we must limit the description of the detailed flows at a given space scale, and simplify the modeling of the smaller scale flows by aggregating these phenomena into macroscopic quantities, by some kind of "homogenization" procedure. To that purpose, the modeling of the fluid-solid coupling within the framework of porous media appears appropriate.

Poromechanics is a simplified mixture theory where a complex fluid-structure interaction problem is replaced by a superposition of both components, each of them representing a fraction of the complete material at every point. It originally emerged in soils mechanics with the work of Terzaghi [73], and Biot [56] later gave a description of the mechanical behavior of a porous medium using an elastic formulation for the solid matrix, and Darcy's law for the fluid flow through the matrix. Finite strain poroelastic models have already been proposed (see references in [45]), albeit with *ad hoc* formulations for which compatibility with thermodynamics laws and incompressibility conditions is not established.

3.2.3. Tumor and vascularization

The same way the myocardium needs to be perfused for the heart to beat, when it has reached a certain size, tumor tissue needs to be perfused by enough blood to grow. It thus triggers the creation of new blood vessels (angiogenesis) to continue to grow. The interaction of tumor and its micro-environment is an active field of research. One of the challenges is that phenomena (tumor cell proliferation and death, blood vessel adaptation, nutrient transport and diffusion, etc) occur at different scales. A multi-scale approach is thus being developed to tackle this issue. The long term objective is to predict the efficiency of drugs and optimize therapy of cancer.

[1] http://www-sop.inria.fr/CardioSense3D/

3.2.4. Respiratory tract modeling

We aim to develop a multiscale modeling of the respiratory tract. Intraprenchymal airways distal from generation 7 of the tracheabronchial tree (TBT), which cannot be visualized by common medical imaging techniques, are modeled either by a single simple model or by a model set according to their order in TBT. The single model is based on straight pipe fully developed flow (Poiseuille flow in steady regimes) with given alveolar pressure at the end of each compartment. It will provide boundary conditions at the bronchial ends of 3D TBT reconstructed from imaging data. The model set includes three serial models. The generation down to the pulmonary lobule will be modeled by reduced basis elements. The lobular airways will be represented by a fractal homogenization approach. The alveoli, which are the gas exchange loci between blood and inhaled air, inflating during inspiration and deflating during expiration, will be described by multiphysics homogenization.

4. Application Domains

4.1. Blood flows

Cardiovascular diseases like atherosclerosis or aneurysms are a major cause of mortality. It is generally admitted that a better knowledge of local flow patterns could improve the treatment of these pathologies (although many other biophysical phenomena obviously take place in the development of such diseases). In particular, it has been known for years that the association of low wall shear stress and high oscillatory shear index give relevant indications to localize possible zones of atherosclerosis. It is also known that medical devices (graft or stent) perturb blood flows and may create local stresses favorable with atherogenesis. Numerical simulations of blood flows can give access to this local quantities and may therefore help to design new medical devices with less negative impacts. In the case of aneurysms, numerical simulations may help to predict possible zones of rupture and could therefore give a guide for treatment planning.

In clinical routine, many indices are used for diagnosis. For example, the size of a stenosis is estimated by a few measures of flow rate around the stenosis and by application of simple fluid mechanics rules. In some situations, for example in the case a sub-valvular stenosis, it is known that such indices often give false estimations. Numerical simulations may give indications to define new indices, simple enough to be used in clinical exams, but more precise than those currently used.

It is well-known that the arterial circulation and the heart (or more specifically the left ventricle) are strongly coupled. Modifications of arterial walls or blood flows may indeed affect the mechanical properties of the left ventricle. Numerical simulations of the arterial tree coupled to the heart model could shed light on this complex relationship.

One of the goals of the REO team is to provide various models and simulation tools of the cardiovascular system. The scaling of these models will be adapted to the application in mind: low resolution for modeling the global circulation, high resolution for modeling a small portion of vessel.

4.2. Respiratory tracts

Breathing, or "external" respiration ("internal" respiration corresponds to cellular respiration) involves gas transport though the respiratory tract with its visible ends, nose and mouth. Air streams then from the pharynx down to the trachea. Food and drink entry into the trachea is usually prevented by the larynx structure (epiglottis). The trachea extends from the neck into the thorax, where it divides into right and left main bronchi, which enter the corresponding lungs (the left being smaller to accommodate the heart). Inhaled air is then convected in the bronchus tree which ends in alveoli, where gaseous exchange occurs. Surfactant reduces the surface tension on the alveolus wall, allowing them to expand. Gaseous exchange relies on simple diffusion on a large surface area over a short path between the alveolus and the blood capillary under concentration gradients between alveolar air and blood. The lungs are divided into lobes (three on the right, two on the left) supplied by lobar bronchi. Each lobe of the lung is further divided into segments (ten segments of the right lung and eight of the left). Inhaled air contains dust and debris, which must be filtered, if possible, before they reach the alveoli. The tracheobronchial tree is lined by a layer of sticky mucus, secreted by the epithelium. Particles which hit the side wall of the tract are trapped in this mucus. Cilia on the epithelial cells move the mucous continually towards the nose and mouth.

Each lung is enclosed in a space bounded below by the diaphragm and laterally by the chest wall and the mediastinum. The air movement is achieved by alternately increasing and decreasing the chest pressure (and volume). When the airspace transmural pressure rises, air is sucked in. When it decreases, airspaces collapse and air is expelled. Each lung is surrounded by a pleural cavity, except at its hilum where the inner pleura give birth to the outer pleura. The pleural layers slide over each other. The tidal volume is nearly equal to 500 ml.

The lungs may fail to maintain an adequate supply of air. In premature infants surfactant is not yet active. Accidental inhalation of liquid or solid and airway infection may occur. Chronic obstructive lung diseases and lung cancers are frequent pathologies and among the three first death causes in France.

One of the goals of REO team in the ventilation field is to visualize the airways (virtual endoscopy) and simulate flow in image-based 3D models of the upper airways (nose, pharynx, larynx) and the first generations of the tracheobronchial tree (trachea is generation 0), whereas simple models of the small bronchi and alveoli are used (reduced-basis element method, fractal homogenization, multiphysics homogenization, lumped parameter models), in order to provide the flow distribution within the lung segments. This activity has been carried out in the framework of successive research programs: RNTS “R-MOD” until 2005, ACI “le-poumon-vous-dis-je” until 2007 and ANR M3RS until 2013.

4.3. Electrophysiology of the heart

The numerical simulation of the electrical activity of the heart is a new topic in our team. It is motivated by our participation in the CardioSense3D project and by a collaboration initiated with the ELA Medical company (pacemaker manufacturer).

Our purpose is to simulate the propagation of the action potential in the heart. A lot of works has already been devoted to this topic in the literature (see *e.g.* [67], [72], [71] and the references therein), nevertheless there are only very few studies showing realistic electrocardiograms obtained from partial differential equations models. Our goal is to find a compromise between two opposite requirements: on the one hand, we want to use predictive models, and therefore models based on physiology, on the other hand, we want to use models simple enough to be parametrized (in view of patient-specific simulations). Our strategy is to select the level of complexity with respect to the “numerical electrocardiograms” produced by the model. We are also interested in various clinical and industrial issues related to pacemakers.

5. Software

5.1. LiFE-V library

Participants: Julien Castelneau, Miguel Ángel Fernández [correspondant], Jean-Frédéric Gerbeau.

LiFE-V[2] is a finite element library providing implementations of state of the art mathematical and numerical methods. It serves both as a research and production library. It has been used already in medical and industrial context to simulate fluid structure interaction and mass transport. LiFE-V is the joint collaboration between three institutions: Ecole Polytechnique Fédérale de Lausanne (CMCS) in Switzerland, Politecnico di Milano (MOX) in Italy and INRIA (REO) in France. It is a free software under LGPL license.

6. New Results

6.1. Mathematical modeling and numerical methods for Partial Differential Equations

6.1.1. *Mathematical analysis*

Participants: Céline Grandmont, Muriel Boulakia.

[2] http://www.lifev.org/

In the submitted paper [43], the coupling between a compressible fluid and an elastic structure is studied. M. Boulakia and S. Guerrero establish the local in time existence and the uniqueness of regular solutions for this model. Contrary to most of the works on this subject, the equations do not contain extra regularizing terms. The result is proved by first introducing a linearized problem and by proving that it admits a unique regular solution. The regularity is obtained through successive estimates on the unknowns and their derivatives in time and through elliptic estimates. At last, a fixed point theorem allows to prove the existence and uniqueness of a regular solution of the nonlinear problem.

Work in progress

M. Boulakia and S. Guerrero are also working on the controllability of fluid-structure interaction problems for a rigid structure immersed in an incompressible fluid. The control acts on a small subdomain of the fluid domain. This work generalizes the paper [58] and deals with the three-dimensional case without any restriction on the geometry of the solid.

6.1.2. Numerical methods in fluid dynamics

Participants: Matteo Astorino, Franz Chouly, Miguel Ángel Fernández, Céline Grandmont, Jean-Frédéric Gerbeau, Jimmy Mullaert.

This activity on fluid-structure interaction is done in close collaboration with the MACS project-team.

6.1.2.1. Algorithms for fluid-structure interaction problem

Following the results reported in [12], M. Astorino, F. Chouly and M.A. Fernández have proposed a semi-implicit coupling scheme for the numerical simulation of fluid-structure interaction systems involving a viscous incompressible fluid. The scheme is stable irrespectively of the so-called added-mass effect and allows for conservative time-stepping within the structure. The efficiency of the scheme is based on the explicit splitting of the viscous effects and geometrical/convective non-linearities, through the use of the Chorin-Temam projection scheme within the fluid. Stability comes from the implicit pressure-solid coupling and a specific Robin treatment of the explicit viscous-solid coupling, derived from Nitsche's method. These results and some numerical experiments have been reported in [13].

M.A. Fernández has shown that the stabilized explicit coupling scheme reported in [20] can be cast into a Robin-Robin coupling framework. In particular, this allows the introduction of a general class (i.e. not necessarily within the Nitsche framework) of stabilized explicit coupling schemes based on genuine Robin-Robin transmission conditions. Interestingly, if a Chorin-Temam scheme is used in the fluid, this Robin-Robin interface treatment provides natural stabilization of explicit coupling. The convergence behavior needs, however, further investigations. Some of these results and numerical simulations illustrating the properties of the proposed algorithms have been presented by M.A. Fernández at the 15th International Conference on Finite Elements in Flow Problems (FEF09), April 1–3, 2009, Tokyo, Japan.

In collaboration with C. Farhat, A. Rallu and K. Wang (Stanford), J.-F. Gerbeau studied fluid-structure interaction problems associated with underwater implosions [49]. The structures are supposed to be immersed within the fluid and their two respective grids are completely independent. A numerical method has been proposed to easily switch from a fluid-structure to a fluid-fluid configuration in order to handle the apparition of cracks in the structure.

J.-F. Gerbeau, in collaboration with C. Farhat and A. Rallu (Stanford), proposed an original method to efficiently solve exact Riemann problems of Gas dynamics for arbitrary equations of state (in particular JWL). The algorithm relies on the construction of a metamodel based on sparsegrids [48].

6.1.2.2. Parareal time-stepping

F. Chouly and M.A. Fernández have investigated a parallel time-marching scheme for coupled parabolic-hyperbolic problems, as a prototype of fluid-structure interaction problems involving a linear structure and a viscous fluid. No linearity assumption is made on the parabolic side. The standard Parareal scheme is applied to the parabolic part, while the modified algorithm proposed in [62] is applied to the hyperbolic part. This hybrid Parareal treatment relies on the partitioned formulation of the coupled propagator. Numerical evidence shows that the resulting scheme is stable for a wide range of physical and discretization parameters. This work has been reported in [27].

6.1.2.3. Stabilized finite element

E. Burman (University of Sussex) and M.A. Fernández have proposed a new analysis for the stabilized PSPG (Pressure Stabilized Petrov Galerkin) method applied to the transient Stokes problem. Stability and convergence are obtained under different conditions on the discretization parameters depending on the approximation used in space. For the pressure they prove optimal stability and convergence only in the case of piecewise affine approximation under the standard condition on the time-step. These results have been reported in [39].

Motivated by the results reported in [19], E. Burman, A. Ern (ENPC) and M.A. Fernández have investigated the fully explicit treatment of the stabilization and the advection terms using Runge-Kutta based methods. They have analyzed (second- and third-order) explicit Runge-Kutta schemes in time combined with stabilized finite elements in space to approximate evolution problems with a first-order linear differential operator in space of Friedrichs-type. They establish L^2-norm error estimates with (quasi-)optimal convergence rates for smooth solutions in space and time. These results hold under the usual CFL condition for third-order Runge-Kutta schemes and any polynomial degree in space and for second-order Runge-Kutta schemes and first-order polynomials in space. For second-order Runge-Kutta schemes and higher polynomial degrees in space, a tightened $4/3$-CFL condition is required. These theoretical results and some numerical experiments have been reported in [38].

6.1.3. Kinetics models

Participant: Laurent Boudin.

Kinetics models are typically used in our team to model aerosol in the respiratory tracts. In recent works, L. Boudin has also used them in the context of Sociophysics.

Sociophysics is a research field first introduced in the pioneering paper [65] in the early eighties. The basic idea is the fact that methods and concepts from physics can be used to describe political or social behaviors. A lot of works has been recently devoted to sociophysics in the sociological, physical and mathematical communities. L. Boudin and F. Salvarani (Univ. Pavia, Italy) started from the assessment that the tools of statistical physics fitted the sociophysical framework. More precisely, individuals are considered as particles whose *collective* behavior can be described through kinetic models.

In [16], [17], they studied a kinetic model of opinion formation where “binary interactions” between individuals (collisions) and “self-thinking” (diffusion) are taken into account. They obtained mathematical results (a priori estimates, existence, long-time behaviour) and performed numerical simulations on relevant situations, recovering results from other non kinetic models and obtaining original behaviors in new situations. In [36], with R. Monaco (Politec. Torino, Italy), they proposed a multidimensional opinion formation model with the presence of media. They supervised a Master 2 internship on this kind of model with the addition of contradictory individuals in the population, and the related paper is currently being written. Eventually, they wrote a review chapter on kinetic models for opinion formation, in a book from the “Modeling and Simulation in Science, Engineering and Technology” Series, Birkhauser.

6.2. Respiration tree modeling

6.2.1. Airway flow and related environmental issues

Participants: Laurent Boudin, Julien Castelneau, Céline Grandmont, Michaël Grasseau, Driss Yakoubi.

L. Boudin, D. Götz (Berlin & Darmstadt) and B. Grec (Paris 5) have studied a model of air (and aerosol) diffusion in the lower part of the lung, the so-called Maxwell-Stefan law for multicomponent mixture, which is to be compared with the standard Fick model classically used for this study. They point out that in some situations, Fick’s model does not hold anymore. This work is about to be submitted, and can be seen as a preliminary step on the mathematical and numerical study of the Maxwell-Stefan law. In 2010, it should also involve Prof. Francesco Salvarani (Univ. Pavia, Italy), who is an expert on diffusive models.

What happens outside the human lung also really matters for the respiratory system. In the framework of the ANR "PiTAC", A. Blouza (Rouen), L. Boudin and S.M. Kaber (UPMC) [35] designed suitable parallel in time algorithms coupled with reduction methods for the stiff differential systems integration arising in chemical kinetics. They consider linear as well as nonlinear systems. Numerical efficiency of our approach is illustrated by a realistic ozone production model, which is a key problem in the atmosphere.

Different kind of fluid numerical schemes have been implemented by D. Yakoubi to improve the numerical method introduced in [28] to discretize the multiscale system proposed in [42] and describing the air flow in the proximal part of the bronchial tree.

S. Martin and D. Yakoubi in collaboration with A. Devys and B. Maury studied the diffusion of oxygen in the bronchial tree and the gas exchange capacity of the human lungs [46].

6.2.2. Double-layered airway surface fluid

Participant: Marc Thiriet.

Human conducting airways are mostly lined with a pseudostratified, secretory, and ciliated epithelium that comprises 3 major cell types – ciliated, secretory, and basal cells – with submucosal glands and eventual cartilaginous elements. Mucus is a viscoelastic fluid (thickness 2–5 μm) secreted by the respiratory epithelium that protects tracheobronchial tree mucosa from dehydration and traps inhaled particles (allergens, carcinogens, dust, micro-organisms, and inflammatory debris) that come into contact with it to clear them from airways. Mucus cleans airways, as it flows from either the tracheobronchial tree or upper airways toward the pharynx, where it is swallowed (or expectorated). Ciliary motions propulse mucus. Mucus is more or less continuously secreted, shed, and recycled, discarded, or degraded.

A preliminary study has been carried out this year to investigate the flow of air and mucus in a small 2D exploration domain that incorporates the periciliary fluid and mucus layers as well as the boundary layer of flowing air using a bi-fluid Navier-Stokes equation and the vorticity-velocity formulation. The cilium was assumed to have during the propulsion phase a sinusoidal movement only (at rest during rest and recovery phases) with a frequency that ranges between 5 and 20 Hz. After splitting, a semi-implicite Runge-Kutta 2 scheme and an explicite and implicit Euler scheme were used for the advection step, gravitational component, and diffusion stage, respectively.

The simulations show that when cilia are at rest, the mucus velocity is null even when the air velocity is maximal. But when the mucus velocity is finite, momentum is slightly transfered from mucus to air close to the mucus layer. Consequently, the bidirectional 3D air flow do not strongly interfere with the slow unidirectional mucus motion that cleans airways.

6.2.3. Modeling of aerosol and spray

Participants: Laurent Boudin, Céline Grandmont, Michaël Grasseau, Ayman Moussa, Marc Thiriet, Driss Yakoubi.

The model mathematically studied in [15] fits the behavior of an aerosol in the airflow inside the lung. This Vlasov-Navier-Stokes system belongs to the class of the so-called fluid-kinetic models, which are currently the object of numerous studies. Along with B. Boutin (CEA Saclay & UPMC), B. Fornet (ONERA Toulouse), T. Goudon (INRIA SIMPAF), P. Lafitte (Lille 1 & INRIA SIMPAF), F. Lagoutière (Paris 7 & Paris 11) and B. Merlet (Paris 13), during Cemracs '08, L. Boudin studied an asymptotic analysis of a fluid-particles coupled model, in the bubbling regime. From a theoretical point of view, they extended the analysis done in [59] for the case of an isentropic gas to the case of an ideal gas, thus adding the internal energy, or temperature, which is unknown. they formally derived the bubbling limit system in the same way as in [59] and propose a numerical scheme to solve this limit system. The numerical resolution of the non-limit system, and the numerical analysis of the asymptotic properties of the scheme (e.g. the asymptotic preserving property), as performed in [59], is under study.

L. Boudin, C. Grandmont and A. Moussa are tackling a problem close to the one of [15], but in a moving domain (ALE), i.e. a situation more realistic for the airways. They also try to enhance the numerical scheme in the ALE case, because some difficult points still remains to be solved, like the deposition criterion itself.

L. Boudin, C. Grandmont, B. Grec (Paris 5) and D. Yakoubi extended the FreeFEM++ code developed in [57] to study the influence of the aerosol on the airflow (the retroaction term). In addition to the fact that big particles (radius$>30\mu$m) have a significant effect on the fluid, they find out that they also have an effect on the deposition phenomenon. This work is about to be submitted.

L. Boudin, C. Grandmont, M. Grasseau, A. Moussa, M. Thiriet and D. Yakoubi are currently studying in collaboration with P. Diot and L. Vecellio (Inserm Tours U618) the behavior of an aerosol inside an experimental device U618 developed. They investigate the aerosol deposition inside the device with respect to the size distribution and average velocities of the aerosol. Preliminary results were presented during ISAM '09 (International Conference on Aerosol in Medicine) by A. Moussa.

L. Boudin, S.M. Kaber (UPMC), C. Majoral (Air Liquide) and L. Vecellio (Inserm Tours) are also working on the way to accurately and rapidly determine the radius distribution of a given aerosol nebulizer using some specific properties of pseudo-invertible matrices.

6.2.4. *Lung tissue modeling*

Participants: Paul Cazeaux, Céline Grandmont.

Lung parenchyma is a foam-like material consisting of millions of alveoli. Sound transmission through the lung plays an important role in the non-invasive diagnosis of many lung diseases. The firts step is to derive new 3D homogenized viscoelastic models taking into account the damping effect of the bronchial tree. It should be noted that, to our knowledge, in most of the studies, the mechanical behaviour of the parenchyma is described by an elastic law or as a porous media. Our choice is here to derive, through homogenization techniques, macroscopic models for a composite material made of an elastic body filled with gaseous bubbles connected through a diadic tree. Once this 3D homogenized viscoelastic model is obtained the questions are: how the obtained PDE can be discretized? How can we couple it with the rest of the respiratory tract? How sound is transmitted is such a media? This is the topic of the just starting PhD thesis of P. Cazeaux, supervised by C. Grandmont and Y. Maday. The thesis and in particular the study of sound propagation will be done in collaboration with Brown university.

Work in progress

P. Cazeaux and C. Grandmont have derived new 3D homogenized viscoelastic models taking into account the non local damping effect of the bronchial tree. They are currently studying the behavior of the solution (for instance its long time behavior) though a theoretical study as well as a numerical study based on Freefem++.

6.2.5. *Inverse problem for air flow modeling*

Participants: Muriel Boulakia, Anne-Claire Egloffe, Céline Grandmont.

One interest of the multiscale model proposed in [42] (based on the Navier-Stokes equations coupled with an ODE representing the motion of the diaphragm muscle) is that there are only a few parameters to fit. Our goal is to parametrize the ODE model. In particular one question of interest is: are we able to recover the stiffness of the lung and the resistance of the small airways via partial easily accessible measurements of volume and flux at the mouth? From a theoretical point of view we would like to investigate the stability of parameters with respect to measurements and from a numerical point of view we would like to try to recover these coefficients by optimization procedures.

Work in progress

A.-C. Egloffe started to work on the topic during her master internship and is now doing her PhD thesis supervised by M. Boulakia and C. Grandmont. She has obtained theoretical stability results for the heat equation with non-standard boundary conditions and encouraging $2D$ numerical results to recover parameters of the multiscale model.

6.3. Blood flows

6.3.1. *Resistive Immersed Surfaces for stent modeling*

Participants: Alfonso Caiazzo, Miguel Ángel Fernández, Jean-Frédéric Gerbeau, Vincent Martin.

A. Caiazzo, M.A. Fernández, V. Martin and J.-F. Gerbeau have proposed and analyzed different finite element formulations to simulate incompressible flow through a particular type of stent, modeled as a resistive immersed surface [63]. They first analyze a variant of the monolithic approach proposed in [63], which does not require extra pressure stabilization. Then they consider a fractional-step formulation based on the Chorin-Temam projection scheme. This requires a proper reformulation of the interface coupling conditions at each substep. They show that an appropriate Nitsche interface treatment (see [66]) of the pressure interface conditions allows to derive uniform stability, within the whole range of resistivity $r \in [0, +\infty]$. The theoretical stability and convergence results are illustrated via numerical experiments. A paper is in preparation [44].

Applications to abdominal aneurisms have been presented in [26].

6.3.2. Inverse problems for blood flow simulation

Participants: Cristóbal Bertoglio, Laurent Dumas, Miguel Ángel Fernández, Jean-Frédéric Gerbeau.

6.3.2.1. Data assimilation for 3D fluid-structure problems

C. Bertoglio, D. Chapelle (MACS), M.A. Ferández, J.-F. Gerbeau and P. Moireau (MACS) have addressed some inverse problems in fluid-structure interaction. For the joint state-parameter estimation they considered a sequential approach, inspired by filtering strategies recently proposed in [68].

The performance of the proposed approach has been investigated theoretically, using simplified models, and illustrated numerically through some inverse problems inspired from vascular mechanics [25].

A model to account for the tissues and organs surrounding the vessels has been proposed, in collaboration with A. Figueroa, C. Taylor and N. Xian (Stanford) [52] and successfully apply to model the effect of the spine on the aorta.

6.3.2.2. Inverse problems for 1D blood flow models

In order to reduce the cost of complex 3D fluid-structure computations of blood flow in arteries, one dimensional model based on the averaging of the general three dimensional equations are commonly used. Under a certain number of hypotheses for the artery flow and geometry, it computes the section area $A(t, z)$ and the volumic flux $Q(t, z)$ at any longitudinal position z and time t.

Such model is interesting for three main reasons: first, it drastically reduces the computational time of the 3D model, secondly, its two unknowns are quantities that can be experimentally obtained by non invasive techniques, like echotracking, and finally, it allows to recover all the other hemodynamic variables like blood pressure that are more difficult to measure.

In a preliminary work, [47], the inverse problem corresponding to the identification of the rigidity function $z \mapsto \beta(z)$, supposed to be piecewise constant, has been successfully solved by Laurent Dumas in the case of a straight artery. He has shown in particular that for an artery with a loss of compliance in some portion, the knowledge of only one area section profile downstream is enough to locate the exact position of this disease portion and also its associated compliance.

The next step in this problem, currently under study with a medical team of Hôpital Georges Pompidou (co-headed by Pr. Boutouyrie), is to construct the simplified numerical network of a given patient by using its own flux and section measurements obtained by echotracking.

6.3.3. Cardiac valves simulation

Participants: Matteo Astorino, Jean-Frédéric Gerbeau, Irène Vignon-Clémentel.

Important progress has been achieved in recent years in simulating the fluid-structure interaction around cardiac valves. An important step in making these computational tools useful to clinical practice is the development of postprocessing techniques to extract clinically-relevant information from these simulations. In collaboration with Shawn Shadden (Stanford University), M. Astorino, I. Vignon-Clementel and J.-F. Gerbeau showed how the concept of Lagrangian Coherent Structures (LCS) could improve insight into the transport mechanics of the flow downstream of the aortic valve [24]. In [53], LCS were further investigated to extract from numerical simulation the Effective Orifice Area (EOA) which is a commonly used clinical index.

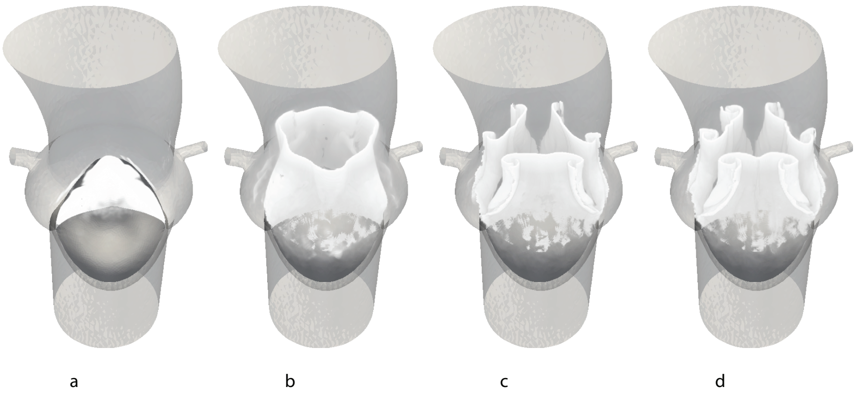

Figure 1. Lagrangian Coherent Structure (LCS) for aortic valve simulation [24], [53]

6.3.4. Perfusion of the myocardium

Participants: Jean-Frédéric Gerbeau, Irène Vignon-Clementel.

This activity on perfusion is done in close collaboration with the MACS project-team, in particular with D. Chapelle and J. Sainte-Marie, in the framework of the CardioSense3D INRIA project.

This work is motivated by the modeling of blood flows through the beating myocardium, namely cardiac perfusion. Perfusion is modeled here as a flow through a poroelastic medium. The main contribution of this study is the derivation of a general poroelastic model valid for a nearly incompressible medium which experiences finite deformations, illustrated by several numerical examples [45].

A general poroelastic formulation valid for finite strains and compatible with incompressibility was introduced, as these two features are deemed to be important in the modeling of living tissues. The strategy – presented in [60] in a linear framework – of deriving the formulation from an appropriate free energy functional, which is crucial to guarantee that fundamental thermodynamics principles are satisfied, was followed.
A numerical procedure to solve the resulting system of equations was then proposed: a fixed point algorithm iteratively couples the "solid" and the " fluid" parts of this system.

To illustrate the behavior of this poroelastic model several numerical examples were run. The first test cases consist of typical poroelastic configurations: swelling (see Figure 2) and complete drainage.

Finally, a simulation of cardiac perfusion was performed in an idealized left ventricle embedded with active fibers. Results showed the complex temporal and spatial interactions of the muscle and blood, reproducing several key phenomena observed in cardiac perfusion.

6.3.5. Interaction between vascularization and tumor development

Participant: Irène Vignon-Clementel.

This is a collaborative project with D. Drasdo (BANG, INRIA) and his coworkers.

In [32], Drasdo, Vignon-Clementel and co-authors, have developed a multiscale model. Agent-based and continuum models are coupled to study the dynamic interplay of the tumor mass and its environment, including blood vessels and their remodeling, nutrients and other factors such as angiogenic growth factors. In their models the tumor cells are represented by individual agents. The models consider the competition between contact inhibition-limited and nutrient/oxygen limited growth. For these questions it turns out to be sufficient to model individual cells within a cellular automaton model where the dynamics is rule-based. The vessels are modeled explicitly as discrete objects with a simplified lumped model relating flow and pressure inside them while the diffusion of oxygen, nutrients and growth factors is represented by continuum equations. Drasdo et

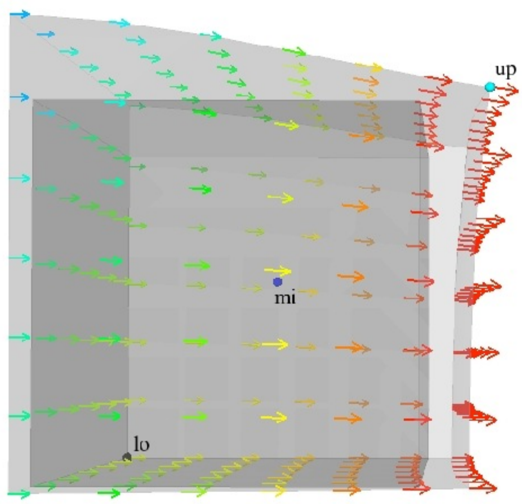

Figure 2. Swelling test of a cube. No external force is applied on the skeleton but a fluid pressure gradient is imposed between two opposite faces whereas a null flux condition is applied on the four other faces. Dark grey represents the initial cube, and light grey the deformed cube. The arrows are the velocity vectors, colored by their magnitude [45]

al. compared the growth kinetics of cell populations in 3D from a single precursor cell up to a cell population of several thousands of cells for (1) oxygen-and nutrient unlimited growth, (2) growth in a *static* vascular environment and (3) growth if the vascular environment is remodeled by angiogenesis. The model is able to explain the growth characteristics found in approximately spherically growing tumors such as multi-cellular spheroids, an experimental system used to mimic tumors in their avascular phase, and Xenografts of NIH3T3 mouse fibroblast cells which also have a spherical shape.
Work in progress includes the extension of this model to specific in-vitro and in-vivo situations, in collaboration with IGR (Institut Gustave Roussy, UPRES 4040) and the *LungSys* consortium (http://www.lungsys.de/, Germany).

6.4. Electrophysiology

Participants: Muriel Boulakia, Miguel Ángel Fernández, Jean-Frédéric Gerbeau, Céline Grandmont, Nejib Zemzemi.

6.4.1. Decoupled time-marching schemes

M.A. Fernández and N. Zemzemi have investigated the approximation of the cardiac bidomain equations, either isolated or coupled with the torso, via first order semi-implicit time-marching schemes involving a fully decoupled computation of the unknown fields (ionic state, transmembrane potential, extracellular and torso potentials). For the isolated bidomain system, M.A. Fernández and N. Zemzemi show that the Gauss-Seidel and Jacobi like splittings do not compromise energy stability; they simply alter the energy norm. Time-step constraints are only due to the semi-implicit treatment of the non-linear reaction terms. Within the framework of the numerical simulation of electrocardiograms (ECG), these bidomain splittings are combined with an explicit Robin-Robin treatment of the heart-torso coupling conditions. They show that the resulting schemes allow a fully decoupled (energy) stable computation of the heart and torso fields, under an additional mild CFL like condition. Numerical simulations, based on anatomical heart and torso geometries, illustrate the stability and accuracy of the proposed schemes. These results have been reported in [40].

6.4.2. Inverse problems

M. Boulakia and C. Grandmont have been working with A. Osses (University of Chili, Santiago) on the parameter identification problem for the Allen-Cahn or bistable equation which can be viewed as a simplified model in cardiac electrophysiology. In [18], through suitable Carleman estimates, they recover parameters of the ionic model from volume or surface measurements and they obtain Lipschitz stability results.

Work in progress

M. Boulakia, M.A. Fernández, J.-F. Gerbeau and N. Zemzemi are working on the numerical identification of parameters of the cardiac model from measurements and in particular from electrocardiograms. Since the computation of the direct problem is quite costly, we first consider reduced order models obtained using for instance the POD method.

7. Contracts and Grants with Industry

7.1. Cardio3 BioSciences

Participants: Jean-Frédéric Gerbeau, Keijo Nissen, Irène Vignon-Clémentel.

Industrial contract. Period: January 2009 - December 2009
Cardio3 BioSciences is a company focused on regenerative therapies for the treatment of heart failure. The purpose of this contract is develop a mathematical model for the retention of cells injected in the myocardium and guide catheter design. This project mixes understanding of the biomedical context, modeling and numerical simulation techniques. This lead in particular to a common presentation at the 21rst Annual Transcatheter Cardiovascular Therapeutics Conference [30].

8. Other Grants and Activities

8.1. National research program

8.1.1. ANR Project "M3RS"

Participants: Laurent Boudin, Muriel Boulakia, Paul Cazeaux, Anne-Claire Egloffe, Céline Grandmont [Principal Investigator].

Period: 2008-2013.
This project, coordinated by C. Grandmont, aims at studying mathematical and numerical issues raised by the modeling of the lungs.

8.1.2. ANR Project "Endocom"

Participants: Miguel Ángel Fernández, Jean-Frédéric Gerbeau [correspondant].

Period: 2008-2012.
This project [3] is funded by the TECSAN call (health technology) of the ANR. It aims at developing a pressure sensor embedded on an endoprosthesis.

8.1.3. ANR Project "PITAC"

Participants: Laurent Boudin, Franz Chouly, Miguel Ángel Fernández [correspondant].

Period: 2007-2011.
This project [4] is funded by the CIS call (High-Performance Computing and Simulation) of the ANR. It aims at developing and studying parallel-in-time numerical methods.

[3] http://www.endocom.upmc.fr
[4] http://www.ann.jussieu.fr/PITAC/http://www.ann.jussieu.fr/PITAC/

8.1.4. CARDIOSENSE3D (INRIA Large Initiative Action)

Participants: Matteo Astorino, Miguel Ángel Fernández [correspondant], Jean-Frédéric Gerbeau, Irène Vignon-Clementel, Nejib Zemzemi.

Period: 2005-2009.

The REO project is a member of the "CardioSense3D project", an INRIA "Large Initiative Action" aimed at developing an electro-mechanical model of the heart[5].

8.1.5. Other grants

- The post-doc of Alfonso Caiazzo (Dec. 2008- Nov. 2009) is funded by an ERCIM grant.
- Muriel Boulakia is a member of MATH-AmSud project on "Controllability and inverse problems in PDEs"
- REO is a member of the following CNRS working groups:
  - GDR *Math-Bio* coordinated by Emmanuel Grenier and Didier Bresch
  - GDR *Fluid-structure interaction in blood flows* coordinated by V. Deplano
  - GDR *Fluid-structure interaction* coordinated by Mhamed Souli
  - GDRE "Control of Partial Differential Equations" coordinated by F. Alabau, O. Glass, P. Cannarsa and F. Ancona

8.2. European research program

8.2.1. European Integrated Project "euHeart"

Participants: Matteo Astorino, Cristóbal Bertoglio, Miguel Ángel Fernández, Jean-Frédéric Gerbeau [correspondant].

Period: 2008-2012
REO is a member of the Integrated Project "euHeart"[6] whose goal is the development of individualized, computer-based, human heart models. The project euHeart consists of seventeen industrial, clinical and academic partners. REO is specifically involved in the modeling and simulation of cardiac valves.

8.3. Bilateral international relations

8.3.1. Foreign Associated Team "Cardio" (INRIA/Stanford)

Participants: Matteo Astorino, Jean-Frédéric Gerbeau, I. Vignon-Clementel [correspondant], Guillaume Troianowski, Alexandre Birolleau.

Period: 2008-2011.
The aim of this project is to foster the collaboration between the Cardiovascular Biomechanics Research Laboratory (CVBRL) of C.A. Taylor (Stanford University, USA) and the project-team REO, through research on cardiovascular related topics (boundary conditions for complex flow [23], [54], [51], patient-specific modeling of congenital heart disease, image-based fluid solid interaction, postprocessing of numerical simulations).

8.3.2. Foreign Associated Team "CFT" (INRIA/Centre de Recherche Mathématiques, Canada and SCCS, National Taiwan University)

Participant: Marc Thiriet.

The aim of this project is to perform numerical simulations and experimental measures of various biomechanical systems (aortic valve, carotid, liver,...).

[5] http://www-sop.inria.fr/CardioSense3D/
[6] http://www.euheart.eu/

8.4. Visiting professors and invited researchers

- J.-F. Gerbeau was a visiting professor at Stanford University from September 2008 to August 2009 (sabbatical leave).
- Leif Rune Hellevik has been a visiting professor in REO project-team since September 2009.

9. Dissemination

9.1. Scientific community animation

9.1.1. Various academic responsibilities

- L. Boudin
  - Member of three “comités de sélection” (Paris 6, Compiègne, Orsay-Sup’Elec)
  - Scientific head of the cooperation between Paris 6 and Univ. St-Joseph, Beirut, Lebanon.
- L. Dumas
  - Jury member of *Agrégation de Mathématiques*.
  - External member of the *Comité de sélection* of Evry university.
  - Elected member of UFR929 (Mathematics) at UPMC.
- M.A. Fernández
  - Co-organizer of a CEA-EDF-INRIA winter school on electrophysiology (with M. Clerc and J.-F. Gerbeau)Badly nested begin-end blocks
  - PhD thesis committees: N. Zemzemi (Paris 11)
- J.-F. Gerbeau
  - Editorial activity : *ESAIM Proceedings* (editor-in-chief), *Mathematical modeling and Numerical Analysis* M2AN (associate editor)
  - Member of the evaluation committee of INRIA
  - Vice-president of the project-teams committee at INRIA Paris-Rocquencourt.
  - Member of the board of the Department of Mathematics of Paris 6 University (*conseil de l’UFR 929*).
  - PhD thesis committees: Florian Blanc (Toulouse, reviewer), Mauro Perego (Politecnico di Milano), Sara Minisini, (Politecnico di Milano), Andrea Mola (Politecnico di Milano), Tiziano Passerini (Politecnico di Milano), Mariarita de Luca (Politecnico di Milano), Nejib Zemzemi (Paris 11)
  - Habilitation committee: Céline Grandmont (HDR, Paris 6)
  - Co-organizer of a CEA-EDF-INRIA winter school on electrophysiology (with M. Clerc and M. Fernández)
- C. Grandmont
  - PhD thesis committees: C. Vannier (Orsay), A. Moussa (ENS Cachan)
- I. Vignon-Clémentel
  - Organizing the monthly seminar at INRIA Paris-Rocquencourt on “modeling and scientific computing” since January 2008.

- Co-organizing the weekly internal seminar at INRIA Paris-Rocquencourt "openBang" to foster cross-knowledge between the researchers at INRIA that work on bio-related topics since October 2007.
- Member of the "Conseil d'orientation scientifique et technologique" (scientific and technologic orientation council) of l'INRIA, in the subgroup "GT Actions Incitatives" (incentive action working group) since December 2007.
- Mediator between PhD students and their supervisors for INRIA Paris-Rocquencourt since October 2007.
- Talk and discussion with high school students at the information and orientation center of Saint Germain en Laye (March, 11th 2009)
- Presentation and experience sharing of associated teams with foreign countries (Internal communication, INRIA - May, 15th 2009)
- Coordinator of the associated team CARDIO between REO and C.A. Taylor's lab at Stanford University, USA since 2008

9.2. Teaching

- Laurent Boudin
  - Coordinator for the applied mathematics of the bidisciplinary applied maths / computer science licence cursus created on Sept. 2009: this cursus is a selective one and offers an enhanced teaching to its students (oral interrogations, summer school in Brown Univ...)
  - Series and Integrals (24h), Licence, Univ. Paris 6
  - Multiple variables functions (72h), Licence, Univ. Paris 6
  - Hilbertian analysis (18h), Licence, Univ. Paris 6
  - Basics of Applied Mathematics with Scilab (75h), Master, Univ. Paris 6
- Laurent Dumas
  - Master course "Introduction to Fluent and Gambit" (20h), Univ. Paris 6.
  - Master course "Numerical Optimization and its applications" (30h), Ecole Centrale Paris.
- Céline Grandmont
  - "Fluid-structure interaction. Application to physiological flows", (with Y. Maday), Master of numerical analysis (12 h), Univ. Paris 6
- Miguel Á. Fernández
  - "Numerical methods in bio-fluids", Master of Mathematical Methods and Numerical Simulation in Engineering and Applied Sciences, University of Vigo (6h), Spain
  - "Inverse problems" (35h) Ecole Supérieure d'Ingénieurs Léonard de Vinci
  - "Scientific computing" (30h) École Nationale des Ponts et Chaussées
- Jean-Frédéric Gerbeau
  - Functional analysis (33 h), Ecole Nationale des Ponts et Chaussées.
  - Finite Element for incompressible fluid mechanics (36 h), Stanford University.
- Driss Yakoubi
  - Numerical Analysis, École Nationale Supérieure d'Informatique pour l'Industrie et l'Entreprise (ENSIIE) (45h), Evry, France.

  - Colle, Mathematics (MPSI), lycée Jeanne D'Albret, Saint Germain en Lay, France (2h/Hebd)

- Marc Thiriet
  - Biomechanics & Biomathematics (36h) TIMS, NTU
  - Cours de remise en forme. Introduction à la modélisation des processus biologiques et aux écoulements physiologiques, Univ. Paris 6.

- Nejib Zemzemi
  - *Moniteur* in Department of Mathematics at Palais de la Découverte (64 h)

9.3. Participation in conferences, workshops and seminars

- Matteo Astorino
  - Seminar at Cours Frédéric Poupaud 2009, January 28th, 2009, Nice, France.
  - Contributed talk of a Mini-symposium at FEF09, April 1st–3rd, 2009, Tokyo, Japan.
  - Contributed talk at GDR 2760, June 11th–12th, 2009, Marseille, France.
  - Contributed talk of a Mini-symposium at CMBE09 1st International Conference on Computational & Mathematical Biomedical Engineering, Swansea - United Kingdom, June 29th – July 1st, 2009.
  - Seminar at Laboratoire de Mathématiques Appliquées UTC, September 29th, 2009, Compiègne, France.

- Laurent Boudin
  - Seminar in Caen
  - Talk at the meeting between J.-L. Lions Lab. and "Institut Jean Le Rond d'Alembert"

- Cristoba Bertoglio
  - Seminar, Department of Structural Engineering, Pontificia Universidad Católica, October 13th, 2009, Santiago, Chile.
  - Contributed talk at Current & New Trends in Scientific Computing, October 5th–9th, 2009, Santiago, Chile.
  - Seminar, CEMRACS summer school, August 14th, Marseille, France.
  - Contributed talk of a Mini-symposium at 1st International Conference on Mathematical & Computational Biomedical Engineering, June 29th – July 1st, 2009, Swansea, UK.

- Muriel Boulakia
  - Seminar, University Evry, December, 2009
  - Talk at Laboratoire J.-L. Lions-Institut d'Alembert Meeting, UPMC Paris 6, January, 2009
  - Invited talk at "Mathematical Modeling and Computing in Electrocardiology", Nantes, June 8-9, 2009
  - Talk at Fraunhofer Kaiserslautern Meeting, UPMC Paris 6, July 2009
  - Talk at 7th ISAAC Congress, Imperial College, London, July 13-18, 2009
  - Invited talk at Mathematical Physics and PDE Congress, Levico Terme, Italy, September 6-11, 2009
  - Talk at International Workshop on Biomathematics and Biomechanics, Tozeur, Tunisia, November 20-23, 2009

- Alfonso Caiazzo
  - Contributed talk at workshop in Computational Multiscale Methods, June 16th – June 20th, 2009, Oberwolfach, Germany.
  - Contributed talk at CMBE, June 29th – July 1st, 2009, Swansea, UK.
  - Seminar, Arbeitsgruppe Numerik, October 18th, 2009, Konstanz, Germany.
- Laurent Dumas
  - Invited talk at CIMMNF workshop, April 28th, 2009, Errachidia, Maroc.
  - Contributed talk at MAMERN conference, June, 10th, 2009, Pau, France.
  - Contributed talk at CNTSC conference, October, 8th, 2009, Santiago, Chile.
  - Invited talk at an half day workshop on atherosclerosis, October 23th, 2009, Paris 5 University.
  - Invited talk at four seminars (Evry, Orsay, Paris 13, Besançon).
- Miguel Ángel Fernández
  - Invited lecturer at the Biomat School on Mathematics and Life Sciences, June 1–5, University of Granada, Spain.
  - Seminar at the CEA-EDF-INRIA School on Cardiac and brain electrophysiology: modeling and simulation, November 16–19, 2009, Rocquencourt, France.
  - Talk at a Mini-symposium of the 15th International Conference on Finite Elements in Flow Problems (FEF09), April 1–3, 2009, Tokyo, Japan.
- Jean-Frédéric Gerbeau
  - Talk at a minisymposium, SIAM Conference on Analysis of Partial Differential Equation, December 7-10, 2009
  - Colloquium of Applied Mathematics, Université Paris 5, France, November 6, 2009
  - Invited talk at the Bioengineering Conference, October 2009, Oxford, UK
  - Invited talk at a satellite symposium, MICCAI conference, October 2009, London, UK
  - Organization of a minisymposium "Cardiovascular Mathematics", SIAM Conference on Computational Science and Engineering, March 2-6, 2009, Miami, USA
- Céline Grandmont
  - Seminar INSA Toulouse (01/09).
  - Talk at the meeting between J.-L. Lions Lab. and "Institut Jean Le Rond d'Alembert"
- Leif Rune Hellevik
  - Talk and poster in 3rd International Meeting of the French Society of Hypertension, December 17-18, 2009
- Ayman Moussa
  - Seminars in Orsay, Paris 6, Caen, Strasbourg
  - Posters at ISAM '09 (with abstract published in Journal of Aerosol Medicine and Pulmonary Drug Delivery 22(2), 2009) and SMAI 2009
- Irène Vignon-Clementel
  - Invited talk, 15th International Conference on Finite Elements in Flow Problems, April 1rst-3rd, Tokyo, Japan
  - Invited seminar, Oshima's group, April 6th, U. of Tokyo, Japan

  - Poster at the System Biology for Cancer Conference, June 7th-11th, Rostock, Germany
  - Invited talk, NIH-INRIA seminar, June 3rd-4th, INRIA
  - Poster at the 21rst Annual Transcatheter Cardiovascular Therapeutics Conference, San Francisco, USA
- Nejib Zemzemi
  - Talk at International Workshop on Biomathematics and Biomechanics, Tozeur, Tunisia, November 20-23, 2009
  - Poster at *1er colloque annuel de l'IMTO Technologies pour la santé*, Paris, France. October 23th 2009.
  - Seminar, CEMRACS summer school, August 02nd–21st, Marseille, France.
  - Mathematical Modeling and Computing in Electrocardiology, Nantes, France. June 8-9, 2009.
  - Talk at International Conference Functional Imaging and Modeling of the Heart, Nice, France. Jun 02nd 2009.
  - Chaste Users & Developers Workshop. Oxford, UK, 2009. March 22nd– 26 th 2009
- Driss Yakoubi
  - Seminar, Groupe de travail Moélisation Numérique et Image, Université de Paris–Descartes, Avril 10th 2009, Paris, France.

10. Bibliography

Major publications by the team in recent years

[1] L. BOUDIN, L. DESVILLETTES, R. MOTTE. *A modeling of compressible droplets in a fluid*, in "Commun. Math. Sci.", vol. 1, n^{o} 4, 2003, p. 657–669.

[2] M. BOULAKIA. *Existence of weak solutions for an interaction problem between an elastic structure and a compressible viscous fluid*, in "J. Math. Pures Appl. (9)", vol. 84, n^{o} 11, 2005, p. 1515–1554.

[3] E. BURMAN, M. A. FERNÁNDEZ. *Stabilization of explicit coupling in fluid-structure interaction involving fluid incompressibility*, in "Comput. Methods Appl. Mech. Engrg.", 2008.

[4] E. BURMAN, M. A. FERNÁNDEZ. *Galerkin Finite Element Methods with Symmetric Pressure Stabilization for the Transient Stokes Equations: Stability and Convergence Analysis*, in "SIAM Journal on Numerical Analysis", vol. 47, n^{o} 1, 2008, p. 409–439.

[5] P. CAUSIN, J.-F. GERBEAU, F. NOBILE. *Added-mass effect in the design of partitioned algorithms for fluid-structure problems*, in "Comp. Meth. Appl. Mech. Engng.", vol. 194, n^{o} 42-44, 2005.

[6] B. DRUEZ, L. DUMAS, N. LECERF. *Adaptive Hybrid Optimization of aircraft engine blades*, in "J. Comp. Appl. Math.", vol. 232, n^{o} 1, 2009, p. 54-60.

[7] M. A. FERNÁNDEZ, J.-F. GERBEAU, C. GRANDMONT. *A projection semi-implicit scheme for the coupling of an elastic structure with an incompressible fluid*, in "Internat. J. Numer. Methods Engrg.", vol. 69, n^{o} 4, 2007, p. 794–821.

[8] C. FETITA, S. MANCINI, D. PERCHET, F. PRÊTEUX, M. THIRIET, L. VIAL. *Computational model of oscillatory flow in the proximal part of tracheobronchial trees*, in "Computer Methods in Biomechanics and Biomedical Engineering", vol. 8, 2005, p. 279-293.

[9] C. GRANDMONT. *Existence of weak solutions for the unsteady interaction of a viscous fluid with an elastic plate*, in "SIAM J. Math. Anal.", vol. 40, n° 2, 2008, p. 716–737.

[10] M. THIRIET. *Biology and Mechanics of Blood Flows, part I: Biology of Blood Flows (652 p.), part II: Mechanics and Medical Aspects of Blood Flows (464 p.)*, CRM Series in Mathematical Physics, Springer, 2008.

[11] I. E. VIGNON-CLEMENTEL, C. A. FIGUEROA, K. E. JANSEN, C. A. TAYLOR. *Outflow Boundary Conditions for Three-dimensional Finite Element Modeling of Blood Flow and Pressure in Arteries*, in "Computer Methods in Applied Mechanics and Engineering", vol. 195, 2006, p. 3776-3796.

Year Publications

Articles in International Peer-Reviewed Journal

[12] M. ASTORINO, F. CHOULY, M. A. FERNÁNDEZ. *An added-mass free semi-implicit coupling scheme for fluid-structure interaction*, in "C. R. Math. Acad. Sci. Paris", vol. 347, n° 1-2, 2009, p. 99–104.

[13] M. ASTORINO, F. CHOULY, M. A. FERNÁNDEZ. *Robin Based Semi-Implicit Coupling in Fluid-Structure Interaction: Stability Analysis and Numerics*, in "SIAM Journal on Scientific Computing", vol. 31, n° 6, 2009, p. 4041–4065.

[14] M. ASTORINO, J.-F. GERBEAU, O. PANTZ, K. TRAORÉ. *Fluid-structure interaction and multi-body contact. Application to aortic valves*, in "Comp. Meth. Appl. Mech. Engng.", vol. 198, 2009, p. 3603-3612.

[15] L. BOUDIN, L. DESVILLETTES, C. GRANDMONT, A. MOUSSA. *Global existence of solutions for the coupled Vlasov and Navier-Stokes equations*, in "Differential Integral Equations", vol. 11–12, 2009, p. 1247–1271.

[16] L. BOUDIN, F. SALVARANI. *A kinetic approach to the study of opinion formation*, in "M2AN Math. Model. Numer. Anal.", vol. 43, n° 3, 2009, p. 507–522 IT .

[17] L. BOUDIN, F. SALVARANI. *The quasi-invariant limit for a kinetic model of sociological collective behavior*, in "Kinet. Relat. Models", vol. 2, n° 3, 2009, p. 433–449 IT .

[18] M. BOULAKIA, C. GRANDMONT, A. OSSES. *Some inverse stability results for the bistable reaction-diffusion equation using Carleman inequalities*, in "C. R. Math. Acad. Sci. Paris", vol. 347, n° 11-12, 2009, p. 619–622, http://dx.doi.org/10.1016/j.crma.2009.03.022Chili.

[19] E. BURMAN, M. A. FERNÁNDEZ. *Finite element methods with symmetric stabilization for the transient convection-diffusion-reaction equation*, in "Comput. Methods Appl. Mech. Engrg.", vol. 198, n° 33-36, 2009, p. 2508–2519.

[20] E. BURMAN, M. A. FERNÁNDEZ. *Stabilization of explicit coupling in fluid-structure interaction involving fluid incompressibility*, in "Comput. Methods Appl. Mech. Engrg.", vol. 198, n° 5-8, 2009, p. 766–784.

[21] A. DECOENE, J.-F. GERBEAU. *Sigma transformation and ALE formulation for three-dimensional free surface flows*, in "Internat. J. Numer. Methods Fluids", vol. 59, n^o 4, 2009, p. 357-386.

[22] J.-F. GERBEAU, T. LELIÈVRE. *Generalized Navier Boundary Condition and Geometric Conservation Law for surface tension*, in "Comput. Meth. Appl. Mech. Engng.", vol. 198, 2009, p. 644-756.

[23] H. J. KIM, I. E. VIGNON-CLEMENTEL, C. A. FIGUEROA, J. F. LADISA, K. E. JANSEN, J. A. FEINSTEIN, C. A. TAYLOR. *On coupling a lumped parameter heart model and a three-dimensional finite element aorta model*, in "Annals of Biomedical Engineering", vol. 37, n^o 11, 2009, p. 2153-2169, http://dx.doi.org/10.1007/s10439-009-9760-8US.

International Peer-Reviewed Conference/Proceedings

[24] M. ASTORINO, J.-F. GERBEAU, S. C. SHADDEN, I. E. VIGNON-CLEMENTEL. *Realistic Blood Flow Through the Aortic Valve: Simulation and Analysis*, in "1st International Conference on Mathematical and Computational Biomedical Engineering - CMBE", P. NITHIARASU, R. LÖHNER (editors), 2009 US .

[25] C. BERTOGLIO, M. A. FERNÁNDEZ, J.-F. GERBEAU, P. MOIREAU. *Filtering-based data assimilation in fluid-structure interaction: towards personalization of vascular models*, in "1st International Conference on Mathematical and Computational Biomedical Engineering - CMBE", P. NITHIARASU, R. LÖHNER (editors), 2009.

[26] A. CAIAZZO, M. A. FERNÁNDEZ, J.-F. GERBEAU, V. MARTIN, I. E. VIGNON-CLEMENTEL. *Flow through stents in Abdominal Aortic Aneurysms*, in "1st International Conference on Mathematical and Computational Biomedical Engineering - CMBE", P. NITHIARASU, R. LÖHNER (editors), 2009.

[27] F. CHOULY, M. A. FERNÁNDEZ. *An Enhanced Parareal Algorithm for Partitioned Parabolic-Hyperbolic Coupling*, in "AIP Conference Proceedings", T. SIMOS, G. PSIHOYIOS, T. CH. (editors), vol. 1168, n^o 1, AIP, 2009, p. 1517–1520.

[28] A. DEVYS, C. GRANDMONT, B. GREC, B. MAURY, D. YAKOUBI. *Numerical method for the 2D simulation of the respiration.*, in "CEMRACS 2008 - Modelling and Numerical Simulation of Complex Fluids", ESAIM: Proceedings, vol. 28, 2009.

[29] L. R. HELLEVIK. *Analytical results for wave propagation in compliant vessels*, in "Proceedings of the Twenty Second Nordic Seminar on Computational Mechanics", P. NITHIARASU, R. LÖHNER (editors), vol. 11, 2009, p. 83-86.

Workshops without Proceedings

[30] J.-P. LATERE, I. E. VIGNON-CLEMENTEL, K. NISSEN, J.-F. GERBEAU, C. HOMSY. *Injection of cells into the heart: a comprehensive model to increase retention*, in "American Journal of Cardiology", vol. 104, n^o 6A, 2009, 78D, 21rst Annual Transcatheter Cardiovascular Therapeutics Conference BE .

[31] I. E. VIGNON-CLEMENTEL, T. VARESCHI, J.-F. GERBEAU. *Kinematics and hemodynamics of arteries on the beating heart*, in "15th International Conference on Finite Elements in Flow Problems, Tokyo, Japan", April 1st-3rd 2009.

Scientific Books (or Scientific Book chapters)

[32] D. DRASDO, N. JAGIELLA, I. RAMIS-CONDE, I. E. VIGNON-CLEMENTEL, W. WEENS. *Modeling Steps from a Begnin Tumor to an Invasive Cancer: Examples of Instrinsically Multiscale Problems*, in "Cell mechanics: from single scale based models to multiscale modeling", A. CHAUVIÈRE, L. PREZIOSI, C. VERDIER (editors), Taylor & Francis, 2009.

[33] M. A. FERNÁNDEZ, L. FORMAGGIA, J.-F. GERBEAU, A. QUARTERONI. *The derivation of the equations for fluids and structures*, in "Cardiovascular mathematics", MS&A. Model. Simul. Appl., vol. 1, Springer Italia, Milan, 2009, p. 77–121 IT .

[34] M. A. FERNÁNDEZ, J.-F. GERBEAU. *Algorithms for fluid-structure interaction problems*, in "Cardiovascular mathematics", MS&A. Model. Simul. Appl., vol. 1, Springer Italia, Milan, 2009, p. 307–346.

Research Reports

[35] A. BLOUZA, L. BOUDIN, S. KABER. *Parallel in time algorithms with reduction methods for solving chemical kinetics*, UPMC-LJLL, 2009, http://www.ann.jussieu.fr/publications/2009/R09040.html, Technical report.

[36] L. BOUDIN, R. MONACO, F. SALVARANI. *A kinetic model for multidimensional opinion formation in presence of media*, Dipartimento di Matematica, Politecnico di Torino, 2009, Technical report IT .

[37] M. BOULAKIA, S. CAZEAU, M. A. FERNÁNDEZ, J.-F. GERBEAU, N. ZEMZEMI. *Mathematical Modeling of Electrocardiograms: A Numerical Study*, INRIA, 2009, http://hal.inria.fr/inria-00400490/fr/, RR-6977, To appear in Annals of Biomedical Engineering, Rapport de recherche.

[38] E. BURMAN, A. ERN, M. A. FERNÁNDEZ. *Explicit Runge-Kutta schemes and finite elements with symmetric stabilization for first-order linear PDE systems*, n^{o} 409, CERMICS, ENPC, 2009, http://hal.archives-ouvertes.fr/hal-00380659, Submitted, Research Report.

[39] E. BURMAN, M. A. FERNÁNDEZ. *Analysis of the PSPG method for the transient Stokes' problem*, n^{o} RR-7074, INRIA, 2009, http://hal.archives-ouvertes.fr/inria-00426777, Submitted, Research ReportUK.

[40] M. A. FERNÁNDEZ, N. ZEMZEMI. *Decoupled time-marching schemes in computational cardiac electrophysiology and ECG numerical simulation*, n^{o} RR-7022, INRIA, 2009, http://hal.archives-ouvertes.fr/inria-00411510, Submitted, Research Report.

Other Publications

[41] M. ASTORINO, C. GRANDMONT. *Convergence analysis of a projection semi-implicit coupling scheme for fluid-structure interaction problems*, 2009, http://hal.inria.fr/inria-00406493/fr/, submitted.

[42] L. BAFFICO, C. GRANDMONT, B. MAURY. *Multiscale modelling of the respiratory tract*, 2009, http://hal.inria.fr/inria-00343629/fr/, to appear in Math. Models Methods Appl. Sci. (M3AS).

[43] M. BOULAKIA, S. GUERRERO. *A regularity result for a solid-fluid system associated to the compressible Navier-Stokes equations*, 2009, submitted.

[44] A. CAIAZZO, M. A. FERNÁNDEZ, J.-F. GERBEAU, V. MARTIN. *Projection schemes for fluid flows through a porous interface*, 2009, in preparation.

[45] D. CHAPELLE, J.-F. GERBEAU, J. SAINTE-MARIE, I. E. VIGNON-CLEMENTEL. *A poroelastic model valid in large strains with applications to perfusion in cardiac modeling*, 2009, http://dx.doi.org/10.1007/s00466-009-0452-x, to appear in Computational Mechanics.

[46] A. DEVYS, S. MARTIN, B. MAURY, D. YAKOUBI. *Oxygen Diffusion During Human Breathing*, 2009, submitted.

[47] L. DUMAS. *An optimal model of blood flow in arteries*, 2009, submitted to International Journal for Numerical Methods in Fluids, special issue of CNTSC09.

[48] C. FARHAT, J.-F. GERBEAU, A. RALLU. *Three dimensional two-phase flows with general equations of states*, 2009, in preparation.

[49] C. FARHAT, J.-F. GERBEAU, K. WANG, A. RALLU. *Algorithms for the Discretization of Fluid-Structure Transmission Conditions on Embedded Meshes*, 2009, in preparation.

[50] M. A. FERNÁNDEZ, J.-F. GERBEAU. *Cardiovascular modelling: two examples of industrial applications*, vol. 46, 2009, http://www.mafy.lut.fi/EcmiNL/issues.php?issueNumber=46.

[51] H. J. KIM, I. E. VIGNON-CLEMENTEL, J. S. COOGAN, C. A. FIGUEROA, K. E. JANSEN, C. A. TAYLOR. *Patient-specific modeling of blood flow and pressure in human coronary arteries*, 2009, submitted US .

[52] P. MOIREAU, N. XIAO, M. ASTORINO, C. A. FIGUEROA, D. CHAPELLE, C. A. TAYLOR, J.-F. GERBEAU. *External tissue support and fluid-structure simulation in blood flows*, 2009, in preparation.

[53] S. C. SHADDEN, M. ASTORINO, J.-F. GERBEAU. *Computational analysis of an aortic valve jet with Lagrangian coherent structures*, 2009, to appear in Chaos.

[54] I. E. VIGNON-CLEMENTEL, C. A. FIGUEROA, K. E. JANSEN, C. A. TAYLOR. *Outflow boundary conditions for three-dimensional simulations of non-periodic blood flow and pressure fields in deformable arteries*, 2009, In press US .

References in notes

[55] A. AMSDEN, P. O'ROURKE, T. BUTLER. *A computer program for chemically reactive flows with sprays*, n^{o} LA-11560-MS, Los Alamos National Laboratory, 1989, Technical report.

[56] M. A. BIOT. *Theory of propagation of elastic waves in a fluid-saturated porous solid. II higher frequency range*, in "J. Acoust. Soc. Am.", vol. 28, 1956, p. 179–191.

[57] L. BOUDIN, L. WEYNANS. *Spray impingement on a wall in the context of the upper airways*, in "Mathematical and numerical modelling of the human lung", ESAIM: Proceedings, vol. 23, 2008, p. 1-9.

[58] M. BOULAKIA, A. OSSES. *Local null controllability of a two-dimensional fluid-structure interaction problem*, in "ESAIM Control Optim. Calc. Var.", vol. 14, n^{o} 1, 2008, p. 1–42.

[59] J.-A. CARRILLO, T. GOUDON, P. LAFITTE. *Simulation of fluid and particles flows: asymptotic preserving schemes for bubbling and flowing regimes*, in "J. Comput. Phys.", vol. 227, n^o 16, 2008, p. 7929–7951, http://dx.doi.org/10.1016/j.jcp.2008.05.002.

[60] O. COUSSY. *Mechanics of porous continua*, Wiley, New-York, 1995.

[61] J. DUPAYS, Y. FABIGNON, P. VILLEDIEU, G. LAVERGNE, G. ESTIVALEZES. *Some aspects of two phase flows in solid propellant rocket motors, in Solid propellant chemistry, combustion and interior ballistics*, in "Progress in Astronautics and Aeronautics", V. YANG, T. BRILL, W. PEN (editors), vol. 185, Academic Press, 2000.

[62] C. FARHAT, J. CORTIAL, C. DASTILLUNG, H. BAVESTRELLO. *Time-parallel implicit integrators for the near-real-time prediction of linear structural dynamic responses*, in "International Journal for Numerical Methods in Engineering", vol. 67, 2006, p. 697-724.

[63] M. A. FERNÁNDEZ, J.-F. GERBEAU, V. MARTIN. *Numerical simulation of blood flows through a porous interface*, in "Math. Model. Num. Anal. (M2AN)", vol. 42, 2008, p. 961-990.

[64] L. FORMAGGIA, J.-F. GERBEAU, F. NOBILE, A. QUARTERONI. *On the coupling of 3D and 1D Navier-Stokes equations for flow problems in compliant vessels*, in "Comput. Methods Appl. Mech. Engrg.", vol. 191, n^o 6-7, 2001, p. 561–582.

[65] S. GALAM, Y. GEFEN, Y. SHAPIR. *Sociophysics: a new approach of sociological collective behaviour. I. Mean-behaviour description of a strike.*, in "J. Math. Sociol.", vol. 9, 1982, p. 1–23.

[66] M. JUNTUNEN, R. STENBERG. *Nitsche's method for general boundary conditions*, in "Math. Comp.", vol. 78, n^o 267, 2009, p. 1353–1374.

[67] G. LINES, P. GROTTUM, A. TVEITO. *Modeling the electrical activity of the heart. A bidomain model of the ventricles embedded in a torso.*, in "Comput. Visual. Sci.", vol. 5, 2003, p. 195-213.

[68] P. MOIREAU, D. CHAPELLE, P. LE TALLEC. *Joint state and parameter estimation for distributed mechanical systems*, in "Comput. Methods Appl. Mech. Engrg.", vol. 197, n^o 6-8, 2008, p. 659–677.

[69] R. MOTTE. *A numerical method for solving particle-fluid equations*, in "Trends in numerical and physical modeling for industrial multiphase flows, Cargèse, France", 2000.

[70] A. QUARTERONI, S. RAGNI, A. VENEZIANI. *Coupling between lumped and distributed models for blood flow problems*, in "Comput. Visual Sci.", vol. 4, 2001, p. 111–124.

[71] F. SACHSE. *Computational Cardiology: Modeling of Anatomy, Electrophysiology, and Mechanics*, Springer-Verlag, 2004.

[72] J. SUNDNES, G. LINES, X. CAI, B. NIELSEN, K.-A. MARDAL, A. TVEITO. *Computing the electrical activity in the heart*, Springer-Verlag, 2006.

[73] K. TERZAGHI. *Theoretical Soil Mechanics*, John Wiley and Sons, New-York, 1943.

[74] F. WILLIAMS. *Combustion theory*, 2nd, Benjamin Cummings, 1985.