INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Tosca

TO Simulate and CAlibrate stochastic processes

Sophia Antipolis - Méditerranée, Nancy - Grand Est

Theme : Stochastic Methods and Models

Activity Report

2009

Table of contents

TOSCA *is located both at INRIA Nancy – Grand Est (Nancy) and INRIA Sophia Antipolis – Méditerranée.*

1. Team

Research Scientist

Denis Talay [Team Leader, Research Director Inria (Sophia Antipolis – Méditerranée), HdR]
Mireille Bossy [Research Scientist Inria (Sophia Antipolis – Méditerranée)]
Madalina Deaconu [Research Scientist Inria (Nancy – Grand Est), HdR]
Antoine Lejay [Research Scientist Inria (Nancy – Grand Est), HdR]
Etienne Tanré [Research Scientist Inria (Sophia Antipolis – Méditerranée)]
Nicolas Champagnat [Research Scientist Inria (Sophia Antipolis – Méditerranée)]

Faculty Member

Pierre-Emmanuel Jabin [Professor, on leave from Université de Nice – Sophia Antipolis until February 28 and since September 1, HdR]
Samuel Herrmann [Assistant Professor, École des Mines de Nancy, HdR]

External Collaborator

Pierre-Louis Lions [Professor, Collège de France, HdR]
Sylvain Maire [Assistant Professor, Université de Toulon et du Var, HdR]

PhD Student

Julien Claisse [Sophia Antipolis, since October 1]
Olivier Davidau [Sophia Antipolis]
Viet-Dung Doan [Sophia Antipolis, project-team OASIS and TOSCA]
Dalia Ibrahim [Sophia Antipolis, since November 1]
Junbo Huang [Sophia Antipolis/Hong-Kong, until September 31]
Numa Lescot [Paris/Nancy, CIFRE]
Nicolas Perrin [Sophia Antipolis, since October 1]
Julian Tugaut [Nancy]

Post-Doctoral Fellow

Stefania Maroso [Sophia Antipolis, until May 30]

Administrative Assistant

Sophie Drouot [TR Inria (Nancy – Grand Est)]
Christine Faber [TR Inria (Sophia Antipolis – Méditerrannée)]

2. Overall Objectives

2.1. Overall Objectives

The Inria Research team TOSCA is located both at Inria Sophia-Antipolis – Méditerranée and Inria Nancy – Grand Est. The team develops and analyzes stochastic models and probabilistic numerical methods. The present fields of applications are in finance, fluid mechanics, molecular dynamics, chemical kinetics, neurosciences and population dynamics.

The problems where stochastic models arise are numerous, and the critical reasons for which stochastic models are used make analyzes and simulations difficult.

The TOSCA team thus aims to develop calibration and simulation methods for stochastic models in cases where **singularities** in the coefficients or **boundary conditions** make them hard to discretize and estimate. For this, we are willing to tackle theoretical and numerical questions which are motivated by real applications.

We are interested in developing **stochastic numerical methods** and **transverse methodologies** that cover several fields of applications, instead of having chosen a particular field of application (e.g., Biology, or Fluid Mechanics, or Chemistry). We justify this way to proceed as follows:

- From a couple of years now, we have attacked singular problems to answer questions coming from economists, meteorologists, biologists and engineers with whom we collaborate within industrial contracts or research programs such as ACI, ANR, GDR. To solve their problems which are so complex that stochastic processes are involved in the modelling, these colleagues need to combine expertise and knowledge in many fields: deterministic computing, computer science, vision, algorithm analysis, etc. We are incompetent in these fields, and therefore we could not pretend to fully treat any of these problems. A contrario, we are requested to bring our expertise in stochastic modelling and simulation to extremely various domains of applications.
- In spite of this diversity, whatever the application is, one has to simulate stochastic processes solutions to equations of the type

$$\begin{cases} X_t(\omega) &= X_0(\omega) + \left(\int_0^t \int_{\mathbb{R}^d} b(X_s, y)\mu_s(dy)ds\right)(\omega) \\ &\quad + \left(\int_0^t \int_{\mathbb{R}^d} \sigma(X_s, y)\mu_s(dy)dZ_s\right)(\omega), \\ \mu_s &= \text{Law of } X_s \text{ for all } s \geq 0, \end{cases} \tag{1}$$

  in order to compute statistics of the laws of functionals of these solutions. In addition, several fields often produce very similar "pathologies" of the model (1) or of the statistics to compute: for example, Pope's Lagrangian stochastic particles in Fluid Mechanics and models in Molecular Dynamics produce the same degeneracy in (1), namely, one has to substitute 'Conditional Law of components of X_s given the other ones' to 'Law of X_s'; as well, when studying chartist strategies in Finance and stochastic resonance in the electrical working of neurons, we encounter close questions on the density functions of the random passage times of processes (X_t) at given thresholds.
- Theory and numerical experiments show that each 'pathology' of the model (1) requires specific analysis and numerical methods. However, they require common abstract tools (Malliavin calculus, propagation of chaos theory, nonlinear PDE analysis, etc.) and common numerical methodologies (stochastic particle systems, Monte Carlo simulations, time discretization of stochastic differential equations, etc.). Thus each application takes benefit from the modelling and numerical knowledge developed for all the other ones.

The TOSCA team is currently studying models in relation with Geophysics, Neuro-sciences, Fluid Mechanics, Chemical Kinetics, Meteorology, Molecular Dynamics, Population Dynamics, Evolutionary Dynamics and Finance. We also construct and study stochastic particle systems for Fluid Mechanics, coagulation–fragmentation, stationary nonlinear PDEs, variance reduction techniques for Monte-Carlo computations and numerical methods combining deterministic and stochastic steps to solve nonlinear PDEs in Finance.

3. Scientific Foundations

3.1. Scientific Foundations

Most often physicists, economists, biologists, engineers need a stochastic model because they cannot describe the physical, economical, biological, etc., experiment under consideration with deterministic systems, either because of its complexity and/or its dimension or because precise measurements are impossible. Then they renounce to get the description of the state of the system at future times given its initial conditions and try instead to get a statistical description of the evolution of the system. For example, they desire to compute occurrence probabilities for critical events such as overstepping of given thresholds by financial losses or neuronal electrical potentials, or to compute the mean value of the time of occurrence of interesting events such as the fragmentation up to a very low size of a large proportion of a given population of particles. By nature such problems lead to complex modelling issues: one has to choose appropriate stochastic models, which require a thorough knowledge of their qualitative properties, and then one has to calibrate them, which requires specific statistical methods to face the lack of data or the inaccuracy of these data. In addition, having chosen a family of models and computed the desired statistics, one has to evaluate the sensitivity of the results to the unavoidable model specifications. The TOSCA team, in collaboration with specialists of the relevant fields, develops theoretical studies of stochastic models, calibration procedures, and sensitivity analysis methods.

In view of the complexity of the experiments, and thus of the stochastic models, one cannot expect to use closed form solutions of simple equations in order to compute the desired statistics. Often one even has no other representation than the probabilistic definition (e.g., this is the case when one is interested in the quantiles of the probability law of the possible losses of financial portfolios). Consequently the practitioners need Monte Carlo methods combined with simulations of stochastic models. As the models cannot be simulated exactly, they also need approximation methods which can be efficiently used on computers. The TOSCA team develops mathematical studies and numerical experiments in order to determine the global accuracy and the global efficiency of such algorithms.

The simulation of stochastic processes is not motivated by stochastic models only. The stochastic differential calculus allows one to represent solutions of certain deterministic partial differential equations in terms of probability distributions of functionals of appropriate stochastic processes. For example, elliptic and parabolic linear equations are related to classical stochastic differential equations, whereas nonlinear equations such as the Burgers and the Navier–Stokes equations are related to McKean stochastic differential equations describing the asymptotic behavior of stochastic particle systems. In view of such probabilistic representations one can get numerical approximations by using discretization methods of the stochastic differential systems under consideration. These methods may be more efficient than deterministic methods when the space dimension of the PDE is large or when the viscosity is small. The TOSCA team develops new probabilistic representations in order to propose probabilistic numerical methods for equations such as conservation law equations, kinetic equations, nonlinear Fokker–Planck equations.

4. Application Domains

4.1. Application Domains

TOSCA is interested in developing stochastic models and probabilistic numerical methods. Our present motivations come from Finance, Neurosciences and Biology, Fluid Mechanics and Meteorology, Chemical Kinetics, Diffusions in random media, Transverse problems and Softwares and Numerical experiments.

4.1.1. Finance

For a long time now TOSCA has collaborated with researchers and practitioners in various financial institutions and insurance companies. We are particularly interested in calibration problems, risk analysis (especially model risk analysis), optimal portfolio management, Monte Carlo methods for option pricing and risk analysis, asset and liabilities management. We also work on the partial differential equations related to financial issues, for example the stochastic control Hamilton–Jacobi–Bellman equations. We study existence, uniqueness, qualitative properties and appropriate deterministic or probabilistic numerical methods. At the time being we pay a special attention to the financial consequences induced by modelling errors and calibration errors on hedging strategies and portfolio management strategies.

4.1.2. Neurosciences and Biology

The interest of TOSCA in biology is developing in three main directions: neurosciences, molecular dynamics and population dynamics. In neurosciences, stochastic methods are developed to analyze stochastic resonance effects and to solve inverse problems. For example, we are studying probabilistic interpretations and Monte Carlo methods for divergence form second-order differential operators with discontinuous coefficients, motivated by the 3D MEG inverse problem. Our research in molecular dynamics focus on the development of Monte Carlo methods for the Poisson-Boltzmann equation which also involves a divergence form operator, and of original algorithms to construct improved simulation techniques for protein folding or interactions. Finally, our interest in population dynamics comes from ecology, evolution and genetics. For example, we are studying the emergence of diversity through the phenomenon of evolutionary branching in adaptive dynamics. Some collaborations in biostatistics on cancer problems are also being initiated.

4.1.3. Fluid Mechanics and Meteorology

In Fluid Mechanics we develop probabilistic methods to solve vanishing vorticity problems and to study the behavior of complex flows at the boundary, and their interaction with the boundary. We elaborate and analyze stochastic particle algorithms. Our studies concern the convergence analysis of these methods on theoretical test cases and the design of original schemes for applicative cases. A first example concerns the micro-macro model of polymeric fluids (the FENE model). A second example concerns Pope's Lagrangian modelling of turbulent flows, motivated by the problem of modelling and computing characteristic properties of the local wind activity in areas where windmills are built. Our goal is to estimate local resources in energy which are subject to meteorological randomness by combining large scale wind models and small scale Monte Carlo techniques, and to simulate management strategies of wind resources.

4.1.4. Chemical Kinetics

The TOSCA team is studying coagulation and fragmentation models, that have numerous areas of applications (polymerization, aerosols, cement industry, copper industry, population dynamics...). Our current motivation comes from the industrial copper crushers in Chile. We aim to model and calibrate the process of fragmentation of brass particles of copper in industrial crushers, in order to improve their efficiency at a low cost.

4.1.5. Diffusions in random media

A *random medium* is a material with a lot of heterogeneity which can be described only statistically. Typical examples are fissured porous media with rocks of different types, turbulent fluids or unknown or deficient materials in which polymers evolve or waves propagate. These last few years, the TOSCA team has been in relation with the Geophysics community on problems related to underground diffusions, especially those which concern waste transport or oil extraction. We are extending our previous results on the simulation of diffusion processes generated by divergence form operators with discontinuous coefficients. Such an operator appears for example from the Darcy law for the behavior of a fluid in a porous media. We are also developing another class of Monte Carlo methods to simulate diffusion phenomena in discontinuous media.

4.1.6. Transverse problems

Several of the topics of interest of TOSCA do not only concern a single area of application. This is the case of particular methods for the long time simulations of nonlinear McKean-Vlasov PDEs, the problem of simulation of multivalued models, variance reduction techniques or stochastic partial differential equations. For example, multivalued processes have applications in random mechanics or neurosciences, and variance reduction techniques have applications in any situation where Monte Carlo methods are applicable.

4.1.7. Software, numerical experiments

TOSCA is interested in designing algorithms of resolution of specific equations in accordance with the needs of practitioners. We benefit from our strong experience on the programming of probabilistic algorithms on various architectures including intensive computation architectures. In particular, our activity will concern the development of grid computing techniques to solve large dimensional problems in Finance. We are also interested in intensively comparing various Monte Carlo methods on PDEs and in the development of open source libraries for our numerical methods in Fluid Mechanics, Meteorology, MEG or Chemical Kinetics.

5. Software

5.1. Exitbm library

Participant: Antoine Lejay [correspondant].

The `exitbm` library is a `C` library which allows one to simulate quantities and random variables related to the first exit time and position of the Brownian motion of some particular domains, and may be used in various simulation techniques such as the random walk on spheres or random walk on rectangles. This software is available at http://exitbm.gforge.inria.fr.

6. New Results

6.1. Probabilistic numerical methods, stochastic modelling and applications

Participants: Mireille Bossy, Nicolas Champagnat, Julien Claisse, Olivier Davidau, Madalina Deaconu, Samuel Herrmann, Pierre-Emmanuel Jabin, Antoine Lejay, Pierre-Louis Lions, Sylvain Maire, Nicolas Perrin, Denis Talay, Etienne Tanré, Julian Tugaut.

6.1.1. Monte Carlo methods for diffusion processes and PDE related problems

A. Lejay and A. Kohatsu-Higa (Ōsaka University) are studying the rate of convergence of solutions of Stochastic Differential Equations with an irregular drift. The main tool is the Itô-Krylov estimate and the Malliavin calculus.

6.1.2. Kinetic approximation of divergence form operators

A. Lejay and S. Maire have developed in [53] a numerical method to simulate diffusion processes associated to divergence form operators with piecewise constant coefficients in any dimension. It relies on the combination of efficient random walks in each subdomain and a kinetic approximation at the interfaces between the subdomains.

6.1.3. Poisson-Boltzmann equation in molecular dynamics

M. Bossy, N. Champagnat, S. Maire and D. Talay have studied in [17] the probabilistic interpretation and associated numerical methods for the Poisson-Boltzmann equation of molecular dynamics that allows one to compute the electrostatic potential around a bio-molecular assembly (for example a protein). The first part of the study concerns the probabilistic interpretation of divergence form operators with piecewise constant coefficients in any dimension. The second part of the study concerns several extensions and improvements of previous numerical methods [62] based on a combination of walk on spheres methods and jump procedures at the interfaces. One of these jump procedures relies on the kinetic approximation developed in [53].

During his internship, J. Schneider has developed C/C++ and FORTRAN programs corresponding to these numerical methods.

6.1.4. Stochastic methods in molecular dynamics

N. Perrin has started his Ph.D. thesis under the supervision of M. Bossy, N. Champagnat and D. Talay on stochastics methods in molecular dynamics. In addition to the Poisson-Boltzman equation of the previous paragraph, he is also studying methods due to P. Malliavin (French Academy of Science) and based on the Fourier analysis of covariance matrices with delay in order to identify the fast and slow components of a molecular dynamics and to construct simplified projected dynamics.

6.1.5. Optimal stochastic control of population dynamics

After his four months internship in TOSCA, J. Claisse has started his Ph.D. under the supervision of N. Champagnat and D. Talay on stochastic control of population dynamics. He already proved that the optimal expected value function of a basic controlled stochastic model satisfies a Hamilton-Jacobi-Bellman equation.

6.1.6. Stochastic spectral formulation for elliptic problems

S. Maire and E. Tanré [40] have developed spectral methods for elliptic PDEs by combining standard deterministic linear approximations and the Feynman-Kac formula. These methods require to invert a matrix having asymptotically condition number 1.

6.1.7. Adaptive numerical approximations on hypercubes

C. De Luigi and S. Maire are working on a numerical adaptive method to compute piecewise sparse polynomial approximations and the integral of a multivariate function in medium dimensions (up to 10). This method relies mainly on the quadrature formulae previously developed by the same authors [61].

6.1.8. Selection dynamics for competitive interactions

In [26], P.-E. Jabin and G. Raoul have studied the large time behaviour of population dynamics models with competition between individuals.

6.1.9. Mathematical modelling of immune competition in cancer dynamics

In [18], I. Brazzoli, E. De Angelis and P.-E. Jabin have introduced a simple model of competition between the immune system, cancer cells and endothelial cells. This model takes into account the aggressiveness of cancer cells. When time is large, wave propagation patterns with oscillations are observed.

6.1.10. On existence and well-posedness for the dynamics of a particle in a BV force field in any dimension

In [20], N. Champagnat and P.-E. Jabin have shown well-posedness for the solutions of ordinary differential equations in the phase space when the force field admits less than one derivative. They also obtained a lower bound on the minimal fractional derivative needed to ensure well-posedness in the specific sense they defined.

6.1.11. Some regularizing methods for transport equations and the regularity of solutions to scalar conservation laws

In this work [36], P.-E. Jabin have studied the connection between three regularization phenomena: stationary phase arguments, averaging lemmas and regularizing effects for scalar conservation laws. When the coefficients are smooth enough, all these regularization phenomena can be deduced from a classical stationary phase argument. However, considerable differences appear in more singular cases.

6.1.12. Averaging Lemmas and Dispersion Estimates for kinetic equations

In [25], P.-E. Jabin first reviews some of the main results on averaging lemmas and present their proofs in as self contained a way as possible. The use of kinetic formulations for the well posedness of scalar conservation laws is then explained as an example of application.

6.1.13. On mean discounted numbers of passage times in small balls of Itô processes observed at discrete times

In collaboration with F. Bernardin (CETE de Lyon) and M. Martinez (Université Paris-Est – Marne-la-Vallée), M. Bossy and D. Talay obtained in [15] estimates on mean values of the number of times that Itô processes observed at discrete times visit small balls in $\mathbb{R}^d$. This technique, in the infinite horizon case, is inspired by Krylov's arguments [58]. In the finite horizon case, motivated by an application in stochastic numerics, the number of visits is discounted by a locally exploding coefficient, and the proof involves accurate properties of last passage times at 0 of one dimensional semimartingales.

6.1.14. Approximation of equilibrium distributions of some stochastic systems with McKean-Vlasov interactions

M. Bossy and D. Talay have continued their collaboration with Á. Ganz (Pontificia Universidad Católica de Chile) after her Ph.D. thesis under their supervision. They consider a McKean-Vlasov model admitting an equilibrium measure. This year, they have established a quasi-optimal rate of convergence for the approximation by interacting particles systems of the integral of any test function with respect to the equilibrium measure.

6.1.15. Mathematical analysis of Lagrangian stochastic models and their confinement

In collaboration with J.-F. Jabir (CMM, Santiago de Chile), M. Bossy has studied theoretical problems on Lagrangian stochastic models in simplified cases where, in particular, the pressure term is neglected.
Motivated by the Stochastic Downscaling Model (SDM) in meteorology and based on Lagrangian stochastic models (see Section 7.1), they constructed a Lagrangian system confined within a regular domain $\mathcal{D}$ of $\mathbb{R}^d$ and satisfying the mean no-permeability boundary condition:

$$\mathbb{E}\left((\mathcal{U}_t \cdot n_{\mathcal{D}}(X_t)) \mid X_t = x\right) = 0 \text{ for } x \in \partial\mathcal{D}, \tag{2}$$

where $n_{\mathcal{D}}$ is the outward normal unit vector related to $\mathcal{D}$. This year, the problem of the existence of traces associated to (2) have been studied in the case $\mathcal{D} = \mathbb{R}^{d-1} \times (0, +\infty)$. Under suitable hypotheses, the distribution $\rho_t(x, u)$ of the corresponding confined process $(X_t, \mathcal{U}_t)$ has been shown to admit a strong trace $\gamma(\rho)(t, x, u)$ for $x \in \partial\mathcal{D}$ satisfying the specular boundary condition

$$\gamma(\rho)(t,x,u) = \gamma(\rho)(t,x,u - 2(u \cdot n_{\mathcal{D}}(x))n_{\mathcal{D}}(x)) \text{ for } (t,x,u) \in (0,T) \times \partial\mathcal{D} \times \mathbb{R}^d. \tag{3}$$

6.1.16. Interacting particle systems in Lagrangian modeling and simulation of turbulent flows

M. Bossy continued her collaboration initiated in 2006 with A. Rousseau (MOISE INRIA Grenoble – Rhône-Alpes) and F. Bernardin (CETE Clermont-Ferrand), on the Stochastic Lagrangian simulation methodology and the software development of Stochastic Downscaling Method (SDM, see section 7.1). SDM refines the characteristics of the wind (velocity and variability) inside a mesh given by a coarse Numerical Weather Prediction (NWP) Model. The wind velocity is simulated by fluid particles, living inside a domain $\mathcal{D}$, whose evolution is governed by stochastic differential equations. Such a model raises several original problems which require a strong interaction between mathematics, scientific computation and physics.

This year, improvements in SDM concern mainly the simulation of the meteorological forcing at the boundary of the computational domain. The numerical scheme now approximates the SDM Lagrangian dynamics submitted to non homogeneous Dirichlet boundary conditions and/or periodic conditions on the Eulerian velocities. In [48], we present a numerical validation of the confinement scheme used for SDM on a simple confined one-dimensional Langevin Ornstein Uhlenbeck process. Numerical tests in the meteorological context and comparison with a LES method are in progress and presented in Section 7.1.

6.1.17. Hamilton Jacobi equations with constraints in population dynamics

P.-E. Jabin and N. Champagnat have obtained existence and uniqueness results for Hamilton-Jacobi PDEs with a maximum constraint that are naturally obtained from models of evolutionary dynamics belonging to the biological framework of “adaptive dynamics”. These equations were alreary formally obtained [56], but the well-posedness of the problem appeared to be delicate. The results they obtained give the good conditions to impose on the solution in order to ensure uniqueness in a simplified model without mutation. This work is currently being written.

6.1.18. Large deviation principle for adaptive dynamics diffusion models

In [19], N. Champagnat has obtained large deviation results on diffusions models of adaptive dynamics. He has also obtained results on the problem of exit from a domain, which can be biologically interpreted as the phenomenon of evolution by punctuated equilibria. The main difficulty in this work comes from the degeneracy of the diffusion coefficient of the model and the discontinuity of the drift coefficient.

6.1.19. On Dirichlet eigenvectors for two-dimensional neutral birth and death processes

In collaboration with L. Miclo (CNRS, Univ. Paul Sabatier, Toulouse), N. Champagnat obtained the full spectral decomposition of the transition matrices of neutral two-dimensional birth and death Markov chains. Because of the specific form of the eigenvectors, they were also able to characterize the maximal Dirichlet eigenvalues in a family of subdomains of the original state space of the process. These results have then been applied to determine the limiting quasi-stationary distribution of nearly neutral two-dimensional birth and death processes. This work is currently being written.

6.1.20. Study of particular self-stabilizing diffusions

The well-known Kramers-Eyring law characterizes the time needed by classical diffusions to exit from some bounded domain. This description was extended by S. Herrmann, P. Imkeller and D. Peithmann [57] to particular self-stabilizing diffusions using the large deviation theory. These diffusions represent typically the motion of a particle subject to three sources of forcing. Firstly, it wanders in a landscape whose geometry is determined by a convex potential. Secondly, its trajectories are perturbed by Brownian noise of small amplitude. The third source of forcing can be thought of as self-stabilization: the particle is attracted by its own distribution. The convexity of the potential is essential in this previous study. S. Herrmann and J. Tugaut investigate non-convex situations namely symmetric double-well potentials. They focus their attention on invariant measures for selfstabilizing diffusions living in these potentials. This part is in fact the first step in the study of large deviations phenomenons and exit problems. The authors point out the existence of several stationary measures [49], one symmetric and two others “outlying”. Asymptotic analysis permits to give more informations about the set of all invariant measures as the noise intensity of the global stochastic system becomes small [51]. Finally the uniqueness problem concerning these measures is analysed and the main result concerns the uniqueness of the symmetric invariant measure [50]. All these detailled studies permit finally to emphasize large deviations principles for self-stabilizing diffusions in non convex landscapes.

6.2. Financial Mathematics

Participants: Mireille Bossy, Nicolas Champagnat, Olivier Davidau, Dalia Ibrahim, Junbo Huang, Stefania Maroso, Denis Talay, Etienne Tanré.

6.2.1. Indifference pricing for carbon emission allowances

In collaboration with N. Maïzi (CMA - Mines Paristech) and O. Pourtallier (INRIA -COPRIN Team), M. Bossy, O. Davidau and I. Hammad studied the indifference pricing for carbon emission allowances, as a short term model value of carbon (see also Section 7.2). The indifference pricing methodology describes the way an industrial agent on the emission allowances market chooses his production strategy. An utility function represents the preferences of the producer and its risk aversion. The outputs of its production have stochastic prices on the market, so that the optimal production strategy arises as the solution of a stochastic control problem.

In that case, the resulting stochastic control problem is degenerate, but its well-posedness could be proved: the value function is the unique solution of a Hamilton-Jacobi-Bellman equation. This value function may then be computed through finite difference methods. The *indifference price* of the producer for an amount of allowances could be then computed in a simplified model.

The participants are currently working on the sensitivity analysis of the indifference price function with respect to some allowance market parameters. Forward Backward Stochastic Differential Equations formulations are considered as representation tools for the viscosity solution of the HJB equation. The objective is to establish a representation formula for the value function which allows us to obtain tractable dependencies on the derivatives with respect to the interesting parameters.

6.2.2. Mean field games

M. Bossy, O. Davidau and I. Hammad have formulated a *mean field* type model associated with an optimization problem of power consumption. They model an agent who wants to use electricity in a world price of electricity which depends on the probability distribution of individual consumptions, modelled by a controlled diffusion.

This model is the limit model of a mean field game in the sense of Lasry and Lions [59], [60]. The agents interact only through the functional costs. This simple model served as benchmark tests for the development of numerical methods for the mean field game equations.

J. Huang and D. Talay have studied a mean-field game which models the time evolution of an asset price whose mean instantaneous return is controlled. Two different numerical methods have been developed and tested successfully.

6.2.3. Artificial boundary conditions for nonlinear PDEs in finance

M. Bossy and D. Talay continued their collaboration with M. Cissé (ENSAE-Sénégal) after his Ph.D. thesis under their supervision. They studied the problem of artificial boundary conditions for nonlinear PDEs such as variational inequalities characterizing prices of American options.

They established a representation formula for the space derivative of the viscosity solution of the variational inequalities with Neumann boundary condition. The formula relies on generalized RBSDEs coupled with a reflected forward stochastic differential equations.

Motivated by applications in Finance, where the space derivative of the solution of such PDEs allows one to construct hedging strategies of American options, they estimated the localization error on this derivative inside the domain in terms of the misspecification of the Neumann boundary condition.

6.2.4. Mathematical modelling for technical analysis techniques

After her summer internship in the team, D. Ibrahim started her Ph.D. under supervision of E. Tanré and D. Talay on mathematical modelling for technical analysis techniques in finance. She is studying rigorously the “Bollinger Bandes” indicator for an investor using a risky asset whose instantaneous rate of volatility changes at an unknown random time.

6.2.5. Super-replication of barrier options

This work is part of the contract with NATIXIS (see Section 7.10).

N. Champagnat, S. Maroso and E. Tanré have studied the problem of super-replication of up-and-out barrier options under Gamma constraints. Extending the results of Cheridito, Soner and Touzi [55], they identified the Hamilton-Jacobi-Bellman equation solved by the minimal super-replication cost.

6.2.6. Impulse control with delay

This work is part of the contract with NATIXIS (see Section 7.10).

N. Champagnat, S. Maroso, D. Talay, and E. Tanré have continued their theoretical and numerical study of the problem of hedging of barrier options with strategies having a minimal delay between successive portfolio reallocations. They have obtained the Hamilton-Jacobi-Bellman equation solved by the minimal quadratic risk and described several discretization algorithms (explicit and implicit-explicit). They studied numerically their performances by comparing the value function and the optimal strategy with those obtained under two sets of strategies: first, the set of strategies with fixed, discrete portfolio reallocation times (which is the strategy commonly used by practitioners), and second, the set of strategies without constraints (which corresponds to the Black and Scholes hedging of the option).

6.2.7. *Modelling of financial techniques: resistance and support levels*

One of the concepts that appears frequently in technical analysis is resistance. We will refer to resistance level as a given price level when market pressures seem to force a repeated maximum. Although the study of some historical data shows the eventual appearance of resistance levels, most of the mathematical models used for stock prices, including the well known Black and Scholes model, do not exhibit this kind of phenomenon.

B. Bérard-Bergery (Univ. of Nancy), C. Garcia, B. Guan (Interns at INRIA, Erasmus Mundus program), C. Profeta (Univ. of Nancy) and E. Tanré have developed a mathematical model derived from the Black and Scholes model to include resistance phenomena and a trading strategy for optimizing the logarithm of the returns.

They also developed numerical tests based on Monte Carlo estimations of the wealths obtained with different strategies: the classical Black and Scholes strategy, a simple technical analysis strategy and the optimal strategy for the resistance model. These tests show that the optimal strategy with resistance is robust according to variations of the parameters of the model and produces greater wealth than the other strategies.

6.2.8. *Rate of convergence in the Robbins-Monro algorithm*

Under the supervision of D. Talay, J. Huang continued the study on the convergence of the Robbins-Monro (RM) algorithm. He defended his thesis [12] in July.

By using the smoothing inequality for characteristic functions, they provided a Berry-Esseen type rate of convergence in the central limit theorem for martingales. This result is applied to precise the convergence rate of the RM algorithm, thus establishing a Berry-Esseen bound for the RM algorithm

$$\theta_n = \theta_{n-1} + \gamma_n h(\theta_{n-1}) + \gamma_n \eta_n,$$

where h is assumed to be a smooth function with bounded derivatives and η_n is a martingale difference.

7. Contracts and Grants with Industry

7.1. Collaboration with ADEME: local modeling for the wind velocity

Participant: Mireille Bossy.

Started in 2005, our joint collaboration with the Laboratoire de Météorologie Dynamique (Université Paris 6, École Polytechnique, École Normale Supérieure) is funded by the French Environment and Energy Management Agency (ADEME) and concerns the modeling and the simulation of local wind energy resources. We collaborate with É. Peirano (ADEME), P. Drobinski and T. Salameh (LMD). The second phase of this collaboration has begun in October 2007 and includes as partners A. Rousseau (MOISE Inria Grenoble – Rhône-Alpes) and F. Bernardin (CETE Clermont-Ferrand). We investigate a new method for the numerical simulation of the wind at small scales (see Section 6.1.16). Thanks to boundary data provided at large scales by the weather forecasting code *MM5*, we propose a Langevin model that rules the behavior of stochastic particles. This model called *SDM* (Stochastic Downscaling Method) is adapted from previous works introduced by S.B. Pope [63].

A first meteorological validation, based on simulations of a simple meteorological framework using observations from measurement campaigns, was successful and published in [14]. It underlined the high potential of our Lagrangian Stochastic Model to simulate the wind variability.

This year, major improvements to the SDM model concerned its ability to take into account the forcing conditions at the boundary of the computational domain. We also studied the sensitivity of the (numerical) incompressibility constraints with respect to the forcing values.

This work is preparatory to numerical tests that aim to validate and improve the SDM model for a near wall flow. These tests are based on a comparison with LES method.

7.2. Collaboration with ADEME: carbon value and carbon tax in the context of renewable energies deployment

Participants: Mireille Bossy, Olivier Davidau.

Started in January 2009, this collaboration funded by the French Environment and Energy Management Agency (ADEME), involves the Center for Applied Mathematics (CMA), at Mines ParisTech, COPRIN and TOSCA teams at INRIA. It focuses on a short term carbon value derived from the so-called financial *carbon market*, the European Union Emission Trading Scheme (EU ETS), which is a framework for GHG emissions reduction in European industry.

The objective of this project is to study the compatibility and complementarity of a carbon tax and a target for renewable energy deployment.

As a first step, we are developing a method for assessing the EU ETS value. We consider the constraints related to emission allowances distributed through national plans of allocation (NAP) and the mechanisms of taxes that are taking place. The work will focus on electricity producers, key players in the market in its first phase (NAP-I, 2005-2007). The impact of the *Renewable Energies* park of the electricity producers on their own carbon value will be particularly studied.

We selected the financial concept of indifference price as a relevant methodology. Modelling strategies of production and emission of market quotas therefore rely on stochastic optimal control problems and associated Hamilton-Jacobi-Bellman equations.

7.3. ANR MAEV: Modélisation Aléatoire et Évolution du Vivant (stochastic modelling of the evolution of living systems)

Participant: Nicolas Champagnat.

The general goal of this research project is the analysis of new stochastic models of evolution which take into account the interactions and the diversity of scales in evolution. The partners (probabilists and evolutionary biologists mainly in Paris, Marseille and Grenoble) are exploring four research directions:

- Evolution at the molecular scale: new models of the evolution of genes taking into account the interactions between sites and the main factors of global changes in the genome (genes duplication, transfer...).
- Adaptive evolution: macroscopic models of adaptive evolution that are deduced from the microscopic, individual scale and from genes to the organism.
- Shape of random trees: random tree models for molecular evolution or for species evolution, and the mathematical tools to compare them in order to analyze the evolutionary relations between populations or species.
- Coalescence: coalescent processes coding for the evolution of a group of genotypes inside a large population, allowing one to study the polymorphism when dependence between individuals and various scales are taken into account.

The works of N. Champagnat on adaptive dynamics and population dynamics (see Sections 6.1.17 and 6.1.19 and references [19], [21]) are part of this project.

7.4. ANR ECRU: Exploration des Chemins RUgueux

Participant: Antoine Lejay.

The goal of this ANR project that started in October is to explore new directions in the field of rough paths theory. Part of these explorations are the development of new numerical methods for rough paths, and to provide alternative views on the theory, as well as the use of the rough paths machinery for solving new classes of problems (Stochastic Partial Differential Equations...).

Together with L. Coutin (Université Toulouse 3), A. Lejay worked on the perturbation of linear rough differential equations. A. Lejay also gives a simple account of the theory in the “regular case” [52] and is studying the extension of rough differential equations in the case of non-bounded coefficients.

7.5. ANR MODECOL: Using mathematical MODeling to improve ECOLogical services of prairial ecosystems

Participant: Nicolas Champagnat.

This research project is part of the SYSCOMM ANR program (Complex Systems and Mathematical Modeling).

The general goal of this project is to develop computational ecological modeling of terrestrial plant community via the simulation of a prairie (e.g. interactive terrestrial plant populations) in relation with environmental data. This project focuses on developing an original tool-box that takes advantage of complementary mathematical disciplines to assess ecological problems. Our project proposes the coupling of Partial Differential Equation to approximate environmental conditions and Individual-Based stochastic Models to integrate the population dynamics of prairies. Simulations will be extensively processed thanks to the use of new tools in distributed computing and webcomputing.

An application is proposed as part of this project that stands at the boundary between agronomical and ecological sciences. It concerns the recent European Common Agricultural Policy which requires setting up herbal strips around intensive cereal fields for purificating water from extra nitrate and pesticides. This application will be developed directly with the end-users. We aim to provide softwares in prairies engineering that could be used as a support for decision making concerning this new policy.

7.6. ANR MANDy: Mathematical Analysis of Neuronal Dynamics

Participants: Samuel Herrmann, Denis Talay, Etienne Tanré.

Our project, which gathers mathematicians and neuroscientists, aims at developing mathematically rigorous approaches to neuroscience considering single neurons as well as interconnected neuronal populations. Our target is to conduct the mathematical analysis of existing models where there is still much work to be done and to enrich the modelling by proposing new models.

This project started in September 2009.

7.7. Stochastic Analysis and Mathematical Physics Research Network

Participants: Mireille Bossy, Antoine Lejay, Denis Talay, Etienne Tanré.

Denis Talay is the international coordinator of the MathAmsud program 08MATH05 - Stochastic Analysis and Mathematical Physics Research Network starting in 2009. M. Bossy, D. Talay and E. Tanré have visited Pontificad Universidad of Chile. (mettre une référence avec les travaux avec Angela Ganz). E. Tanré work with R. Cofré (PUC, Chile), R. Fraiman (Univ. de San Andrés, Argentina and Univ. de la República, Uruguay) and R. Rebolledo (PUC, Chile) on some neuronal models: they propose and study a new model for the ions channels.

7.8. Contract with Calyon

Participants: Denis Talay, Etienne Tanré.

This new contract with Calyon started on January 2009 and concerns the study of the liquidity risk on the bond markets. During his internship, F. Bravo has examined the liquidity problems related to the standard interest rate derivatives. An analysis of the swap hedging strategy shows the importance of the liquidity hedging instruments. We now investigate the impact of liquidity in the pricing of standard interest rate options.

C. Michel (Calyon), V. Reutenauer (Calyon), D. Talay and E. Tanré are collaborating on this subject.

7.9. Contract with Natixis: adaptive Monte Carlo methods

Participants: Madalina Deaconu, Antoine Lejay, Numa Lescot, Denis Talay.

The Ph.D. thesis of Numa Lescot started in September 2006 with a CIFRE grant from Natixis. This thesis concerns adaptive Monte Carlo methods where the importance sampling technique is coupled with stochastic algorithms to perform variance reduction. The aim is to combine this technique and Malliavin calculus in order to reduce the variance of Monte Carlo approximations of the Greeks.

7.10. Contract with Natixis: portfolio optimization in incomplete markets

Participants: Nicolas Champagnat, Stefania Maroso, Denis Talay, Etienne Tanré.

This work concerns various aspects of portfolio management in incomplete markets. It is developed in four directions: calibration and estimation of historical probabilities, dynamical portfolio optimization and numerical validation for optimal control. The hedging problem under Gamma constraints and the discrete hedging problem with minimal delay between transactions are also studied in the case of barrier options (see Sections 6.2.5 and 6.2.6).

8. Other Grants and Activities

8.1. NCCR FINRISK

TOSCA participates to the NCCR FINRISK (Financial Risk) forum launched by the Swiss National Science Foundation and managed by the University of Zürich (see Section 6.2.4).

8.2. Natixis Fundation

Participant: Denis Talay.

D. Talay is the Vice-President of the Fondation d'Entreprise Natixis which aims to contribute to develop research in quantitative finance.

He also serves as a member of the Scientific Committee of the Foundation, jointly with M. Crouhy (President, Natixis), N. El Karoui (École Polytechnique), P-L. Lions (Collège de France), J-P. Laurent (Université Claude Bernard, Lyon).

9. Dissemination

9.1. Animation of the scientific community

M. Bossy serves as a member of the Scientific Committee of the *École Doctorale "Sciences Fondamentales et Appliquées"* of the Université de Nice – Sophia Antipolis.

M. Bossy is responsible for the *Suivi Doctoral* Committee of INRIA Sophia Antipolis – Méditerranée. She is also responsible for the *Formation par la recherche* at INRIA Sophia Antipolis – Méditerranée.

M. Bossy serves as a member of the *Cours et Colloques* Committee of INRIA Sophia Antipolis – Méditerranée and serves as a member of the *Commission Consultative Paritaire Scientifique* of INRIA.

N. Champagnat is elected member of the *Comité de Centre* and represents researchers at the *Comité des Projets* of INRIA Sophia Antipolis – Méditerranée. He also served as a member of the *Comité de Sélection* of the Chaire-CNRS position at Univ. Rennes 1.

M. Deaconu is a member of the *Comité des projets* and of the *Commission des Développements Technologiques du Centre de Recherche* of INRIA Nancy Grand Est and of the *Conseil de Laboratoire* of the Institut Élie Cartan (Univ. Nancy 1).

M. Deaconu has been a member of the Committee for junior permanent research positions of INRIA Nancy Grand Est.

M. Deaconu and A. Lejay are permanent reviewers for the *Mathematical Reviews*.

P.-E. Jabin is member of the hiring committee at the Université de Nice – Sophia Antipolis. He is responsible for the third year of *Licence* in mathematics at the Université de Nice – Sophia Antipolis. He is responsible for the colloquium at the maths department of the Université de Nice – Sophia Antipolis. He is the coordinator for the University of Nice of the Erasmus Mundus Master program Mathmods. He is finally in charge of the internship for the Master program IMEA.

A. Lejay has been nominated as editor of the *Séminaire de Probabilités* published yearly.

A. Lejay serves as a member of the *Commission des Personnels* and the *Commission des Utilisateurs des Moyens informatiques pour la Recherche* of INRIA Nancy – Grand Est.

A. Lejay was a member of a *Comité de recrutement* of Université Louis Pasteur in Strasbourg.

D. Talay serves as an Associate Editor of: *Stochastic Processes and their Applications*, *Annals of Applied Probability*, *ESAIM Probability and Statistics*, *Stochastics and Dynamics*, *SIAM Journal on Numerical Analysis*, *Mathematics of Computation*, *Journal of Scientific Computing*, *Monte Carlo Methods and Applications*, *Oxford IMA Journal of Numerical Analysis*, *Stochastic Environmental Research and Risk Assessment*.

D. Talay was the President of the French Applied Mathematics Society SMAI, up to July.

D. Talay served as a member of the Committees for junior permanent research positions at INRIA and for a Professor position at the University of Geneva.

E. Tanré is president of the CUMIR (Comité des Utilisateurs des Moyens Informatiques, Recherche) at INRIA Sophia-Antipolis Méditerranée.

E. Tanré has been a member of *Comité de Sélection* for a position “chaire Université de Nice – INRIA”.

9.2. Animation of workshops

M. Deaconu, A. Lejay and D. Talay have organized in December 2009 a workshop on uncertainties in Nancy, gathering both people from academia and industry in view of developping interactions.

Within the framework of the GDR Mascot-Num, M. Deaconu has organized in collaboration with C. Prieur (Université Grenoble 1) a workshop *Incertitudes et équations d’évolutions* at Institut Henri Poincaré in May 2009.

P.-E. Jabin organized in January the third edition of the *Cours Poupaud* in Nice, with B. Maury as speaker, and in October an *Etat de la recherche de la SMF* session in Paris on “Applications des Mathématiques aux Sciences du Vivant”.

In April, D. Talay chaired the scientific committee of the *Third Conference on Numerical Methods in Finance* (Marne la Vallée). Jointly with F. Delarue (University of Nice Sophia Antipolis) and N. Touzi (Ecole Polytechnique) he organized a workshop on *Stochastic Analysis for Partial Differential Equations* in June at INRIA Sophia Antipolis.

9.3. Teaching

M. Bossy gave a 15h course on *Continuous Probabilistic Models with Applications in Finance* in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis), and a 9h course on *Risk management on energetic financial markets* in the Master *Ingénierie et Gestion de l'Energie* (École des Mines de Paris) at Sophia-Antipolis. M. Bossy also gave a 12h course on *Particle Methods* at the Master M2 *Probabilité et Applications* at Université Paris 6.

N. Champagnat gave a 15h course on *Continuous Probabilistic Models with Applications in Finance* in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis).

O. Davidau gave 16h of exercice classes on *Continuous Probabilistic Models with Applications in Finance* in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis) and 28h of exercice classes on *Numerical Analysis* in the Licence 3 MAM (*Mathématiques Appliquées & Modélisation*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis).

M. Deaconu gave a course on *Stochastic Calculus* (30h) in the Master 2 Recherche of Applied Mathematics at Université Nancy 1 for 2009–2010.

P.-E. Jabin, in addition to his regular teaching activity as a half-time professor in Nice University, gave a Master course on *Averaging lemmas and kinetic equations* in Granada.

A. Lejay gave a course on *Stochastic Differential Equations* (30h) and *Monte Carlo Methods* (15h) in the Master 2 Recherche of Applied Mathematics at Université Nancy 1 for 2009–2010.

D. Talay has a part time position of Professor at École Polytechnique. He also teaches *probabilistic numerical methods* at Université Paris 6 (Master degree in Probability).

E. Tanré gave a 30 hours course on numerical stochastic methods in the Master 2 MathMods - Erasmus Mundus and a 15 hours course in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis).

9.4. Ph.D. theses and habilitations

Samuel Herrmann defended his habilitation entitled *Calcul asymptotique lié à l'étude de certains processus stochastiques* at Université Henri Poincaré in November.

Junbo Huang defended his Ph.D. thesis entitled *Théorème de Berry-Esseen pour martingales normalisées et algorithmes stochastiques. Application en contrôle stochastique* at Université Pierre et Marie Curie in July.

9.5. Participation to congresses, conferences, invitations...

M. Bossy gave an invited talk at the *Workshop on Rheology of complex fluids: modeling and numerics* at ENPC in January 2009, seminar talks at the *Groupe de Travail "Méthodes Stochastiques et Finance" de l'université de Marne la Vallée et ENPC* in April 2009, at the *Seminaire MFEE* of EDF-R&D in Chatou in May 2009, at the *Seminar del Centro de Modelamiento Matematico* Santiago, Chili, in November 2009, and at the *Seminario de analisis estocastico y fisica matematica*, Pontificia Universidad Católica de Chile, in November 2009.

N. Champagnat gave talks at the *journées dynamiques adaptatives* of the ANR MAEV at Univ. Paris 5 in February, at the *workshop of the ANR MODECOL* at Univ. Bordeaux 1 in February, at the *Journée thématique MEG sur "les modèles mathématiques, déterministes et stochastiques, de populations en interactions"* at Luminy, Univ. Marseille 1, in March, at the Conference on *Probabilistic Models of Evolutionary Biology* at CIRM, Marseille, in May, at the *27th European Meeting of Statisticians (EMS 2009)* at Univ. Toulouse 3 in July, at the *57th Sessions of the International Statistical Institute (ISI 2009)* at Durban, South Africa, in August, at the *Session état de la recherche de la SMF "Applications des Mathématiques en Sciences du vivant"* at IHP, Paris, in October, at the *Workshop on Mathematical Biology* at the Hausdorff Institute of Mathematics, Bonn, in October, at the *11th CLAPEM* at Caracas, Venezuela in November, at the *Inauguration* *de la Chaire "Modélisation Mathématique et Biodiversité"* at the Museum National d'Histoire Naturelle, Paris in November, and at the *4th International Conference on Bio-Inspired Models of Network, Information, and Computing Systems (BIONETICS 2009)* at Avignon in December.

O. Davidau gave a talk at the *23rd European Conference on Operational Research* in July at Bonn.

P.-E. Jabin gave talks at *Journées EDP-Probas*, IHP Paris in January, at the conference on *Mathematical Biology: Modeling and Differential Equations*, Barcelona in February and at the Summer school *Systèmes Dynamiques pour la Biologie des Systèmes*, Les Houches in September.

S. Herrmann gave a talk at the conference *EDP, Analyse stochastique et simulation de processus* at Sophia-Antipolis in June 2009 and a talk at Université Paris 6 for the ANR MANDy in October.

A. Lejay gave in mini-course on rough path theory at the *Conference on Long-Range dependance*, Kiev, in September 2009. A. Lejay gave talks at the *MAMERN Conference*, Pau, in June 2009, at the *Stochastic Processes and their Applications* conference, Berlin, in July 2009, and gave three seminars at Ōsaka University and at the Kyushu University in January, February and March 2009.

S. Maire gave talks at the *LSEET (Laboratoire de Sondages Electromagnétiques de l'Environnement Terrestre) seminar* at Univ. Toulon in December 2008, at the *LATP (Laboratoire d'analyse, topologie et probabilités) seminar* at Univ. de Provence in Marseille in April and at the *Seventh IMACS Seminar on Monte Carlo Methods (MCM2009)* at Bruxelles in September.

D. Talay gave lectures at the *Congrès Français de Mécanique 2009*, at the Pontificia Universidad Catòlica de Chile, at Princeton University, and at seminars of the *'Hybrid' ARC cooperative program on Molecular Dynamics simulations* and of the *'Mandy' ANR cooperative program on Neurosciences*.

E. Tanré gave a seminar at Nice (*Journée Maths du Vivant*), and a talk during a meeting of the ANR-MANDY. He also gave a seminar at the Pontificia Universidad Catòlica de Chile in October.

9.5.1. Invitations

M. Bossy was invited two weeks, in November at the Pontificia Universidad Católica de Chile, Santiago, and one week, in November at the Universidad de Chile, Santiago.

T. Brustle (Univ. de Sherbrook, Canada) is visiting the TOSCA project-team in Sophia Antipolis as part of his sabbatical program since September.

N. Champagnat was invited two weeks in Bonn, Germany, at the HIM (Hausdorff Institute for Mathematics) in October.

M. Cissé (ENSAE-Sénégal) has been visiting the TOSCA project-team in Sophia Antipolis for two weeks in September.

S. Herrmann was invited one week at Université Rennes 1.

P.-E. Jabin spent one month at CRM (Centre de Reserca Matemàtica) in Barcelona, and one month at Univ. of Maryland.

J.-M. Lasry (Univ. Paris Dauphine and Calyon) has been visiting the TOSCA project-team in Sophia Antipolis for two weeks in July.

A. Lejay has been visiting Osaka University from October 2008 till April 2009 for 6 months withing the "sabbatical program" at INRIA.

P. Patie (Univ. of Bern and Univ. Libre de Bruxelles) has been visiting the TOSCA project-team in Sophia-Antipolis for one week in September.

V. Reutenauer (Calyon) has been visiting the TOSCA project-team in Sophia Antipolis four times for one week in the year.

B. de Saporta (Univ. Bordeaux IV) has been visiting the TOSCA project-team in Sophia Antipolis for one week in June.

M. Tahmasebi (Univ. Teheran) has been visiting the TOSCA project-team in Sophia Antipolis for 5 weeks between May and June and for one month in December.

This fall, D. Talay visited the Pontificia Universidad Catòlica de Chile and Princeton University.

E. Tanré spent two weeks in Chile at the Pontificia Universidad Catòlica de Chile in October (MathAmsud Program).

The TOSCA *seminar* organized by N. Champagnat has received the following speakers: François Delarue (Université Paris 7), Eulalia Nualart (Université Publique de Pamploune, Espagne), Hasnaa Zidani (ENSTA and INRIA Saclay), Sylvain Maire (Université du Sud Toulon-Var) and Pierre Patie (University of Bern and Université Libre de Bruxelles)

The seminar *Probabilités* organized at Nancy by S. Tindel has received the following speakers: Frédéric Guilloux (Université de Paris Ouest - Nanterre), Vincent Bansaye (Université de Paris VI), Lluís Quer (Université Autonome de Barcelone), Charles Bordenave (Université de Toulouse 3), Dominique Bontemps (Université de Paris XI), Nicolas Fournier (Université de Paris XII), Marie Albenque (Université de Paris VII,LIAFA), Lorenzo Zambotti (Université de Paris VI), Damien Regnault (LIP, ENS Lyon), Mathieu Hoyrup (LORIA), Nicolas Pouyanne (Université de Versailles-St Quentin), Alessio Figalli (Ecole Polytechnique), Hermine Biermé (MAP5, Université de Paris V), Jean-Michel Loubes (Université de Toulouse), Alexandra Chronopoulou (Université de Purdue), Clément Dombry (Université de Poitiers), Kouji Yano (Kobe University), Joseph Ngatchou-Wandji (école de médecine de Nancy), Zhan Shi (Université Paris 6), Christian Léonard (Université de Nanterre), Guillaume Voisin (Université d'Orléans), Maria Jolis (Universitat Autonoma de Barcelona), Francis Hirsch (Université d'Evry), Loïc Chaumont (Université d'Angers), Laurent Decreusefond (Telecom Paris), and Marie Théret (ENS Ulm).

10. Bibliography

Major publications by the team in recent years

[1] C. BLANCHET-SCALLIET, A. DIOP, R. GIBSON, D. TALAY, E. TANRÉ. *Technical analysis compared to mathematical models based methods under parameters mis-specification*, in "Journal of Banking and Finance", vol. 31, n^{o} 5, 2007, p. 1351–1373.

[2] M. BOSSY, E. GOBET, D. TALAY. *A symmetrized Euler scheme for an efficient approximation of reflected diffusions*, in "J. Appl. Probab.", vol. 41, n^{o} 3, 2004, p. 877–889.

[3] M. BOSSY, B. JOURDAIN. *Rate of convergence of a particle method for the solution of a 1D viscous scalar conservation law in a bounded interval*, in "Ann. Probab.", vol. 30, n^{o} 4, 2002, p. 1797–1832.

[4] N. CHAMPAGNAT. *A microscopic interpretation for adaptive dynamics trait substitution sequence models*, in "Stochastic Process. Appl.", vol. 116, n^{o} 8, 2006, p. 1127–1160.

[5] M. DEACONU, N. FOURNIER, E. TANRÉ. *A pure jump Markov process associated with Smoluchowski's coagulation equation*, in "Ann. Probab.", vol. 30, n^{o} 4, 2002, p. 1763–1796.

[6] S. HERRMANN, P. IMKELLER, D. PEITHMANN. *Transition times and stochastic resonance for multidimensional diffusions with time periodic drift: a large deviations approach*, in "Ann. Appl. Probab.", vol. 16, n^{o} 4, 2006, p. 1851–1892.

[7] A. LEJAY. *An introduction to rough paths*, in "Séminaire de Probabilités XXXVII, Berlin", Lecture Notes in Math., vol. 1832, Springer, 2003, p. 1–59.

[8] A. LEJAY, M. MARTINEZ. *A scheme for simulating one-dimensional diffusion processes with discontinuous coefficients*, in "Ann. Appl. Probab.", vol. 16, n^o 1, 2006, p. 107–139.

[9] B. ROYNETTE, P. VALLOIS, M. YOR. *Pénalisations et quelques extensions du théorème de Pitman, relatives au mouvement Brownien et à son maximum unilatère*, in "In memoriam Paul-André Meyer: Séminaire de Probabilités XXXIX, Berlin", Lecture Notes in Math., vol. 1874, Springer, 2006, p. 305–336.

[10] D. TALAY, Z. ZHENG. *Approximation of quantiles of components of diffusion processes*, in "Stochastic Process. Appl.", vol. 109, n^o 1, 2004, p. 23–46.

Year Publications

Doctoral Dissertations and Habilitation Theses

[11] S. HERRMANN. *Calcul asymptotique lié à l'étude de certains processus stochastiques*, Université Henri Poincaré, Nancy I, November 2009, Habilitation à Diriger les Recherches, Ph. D. Thesis.

[12] J. HUANG. *Théorème de Berry-Esseen pour martingales normalisées et algorithmes stochastiques. Application en contrôle stochastique*, Université Pierre et Marie Curie, Paris 6, July 2009, Ph. D. Thesis.

Articles in International Peer-Reviewed Journal

[13] X. BARDINA, I. NOURDIN, C. ROVIRA, S. TINDEL. *Weak approximation of a fractional SDE*, in "Stochastic Proces. Appl.", vol. 120, n^o 1, 2010, p. 39–65, http://dx.doi.org/10.1016/j.spa.2009.10.008ES.

[14] F. BERNARDIN, M. BOSSY, C. CHAUVIN, P. DROBINSKI, A. ROUSSEAU, T. SALAMEH. *Stochastic Downscaling Methods : Application to Wind Refinement*, in "Stoch. Environ. Res Risk. Assess", vol. 23, n^o 6, 2009.

[15] F. BERNARDIN, M. BOSSY, M. MARTINEZ, D. TALAY. *On mean numbers of passage times in small balls of discretized Itô processes*, in "Electron. Commun. Probab.", vol. 14, 2009, p. 302–316.

[16] O. BOKANOWSKI, B. BRUDER, S. MAROSO, H. ZIDANI. *Numerical approximation for a superreplication problem under gamma constraints*, in "SIAM J. Numer. Anal.", vol. 47, n^o 3, 2009, p. 2289–2320, http://hal.archives-ouvertes.fr/hal-00278077/en/.

[17] M. BOSSY, N. CHAMPAGNAT, S. MAIRE, D. TALAY. *Probabilistic interpretation and random walk on spheres algorithms for the Poisson-Boltzmann equation in molecular dynamics*, in "M2AN Math. Model. Numer. Anal.", 2009, to appear.

[18] I. BRAZZOLI, E. DE ANGELIS, P.-E. JABIN. *A Mathematical Model of Immune Competition Related to Cancer Dynamics*, in "M2AN Math. Model. Numer. Anal.", 2009, to appear IT .

[19] N. CHAMPAGNAT. *Large deviations for singular and degenerate diffusion models in adaptive evolution*, in "Markov Process. Related Fields", vol. 15, n^o 3, 2009, p. 289–342, Inhomogeneous random systems (IHP, 2008).

[20] N. CHAMPAGNAT, P.-E. JABIN. *The dynamic of one particle in any dimension with non BV force tems*, in "Comm. Partial Diff. Eq.", 2009, to appear.

[21] N. CHAMPAGNAT, S. MÉLÉARD. *Polymorphic evolution sequence and evolutionary branching*, in "Probab. Theory and Relat. Fields", 2009, http://hal.inria.fr/inria-00345399/en/, to appear.

[22] M. DEACONU, A. LEJAY. *Simulation of a diffusion process using the importance sampling paradigm*, in "Ann. Appl. Probab.", 2009, http://hal.archives-ouvertes.fr/inria-00126339/en/, to appear.

[23] P. DEL MORAL, F. PATRAS, S. RUBENTHALER. *Tree based functional expansions for Feynman-Kac particle models*, in "Ann. Appl. Probab.", vol. 19, n^o 2, 2009, p. 778–825.

[24] M. GUBINELLI, S. TINDEL. *Rough evolution equations*, in "Ann. Probab.", 2009, to appear.

[25] P.-E. JABIN. *Averaging Lemmas and Dispersion Estimates for kinetic equations*, in "Riv. Mat. Univ. Parma", 2009, to appear.

[26] P.-E. JABIN, G. RAOUL. *On Selection dynamics for competitive interactions*, in "J. Math. Biol.", 2009, to appear.

[27] A. LEJAY. *On rough differential equations*, in "Electronic Journal of Probability", vol. 14, 2009, p. 341-364, http://hal.inria.fr/inria-00278246/en/.

[28] M. MARTINEZ, S. RUBENTHALER, E. TANRÉ. *Approximations of a continuous time filter. Application to optimal allocation problems in finance*, in "Stoch. Anal. Appl.", vol. 27, n^o 2, 2009, p. 270–296, http://dx.doi.org/10.1080/07362990802678846.

[29] A. NEUENKIRCH, I. NOURDIN, A. RÖSSLER, S. TINDEL. *Trees and asymptotic expansions for fractional stochastic differential equations*, in "Ann. Inst. Henri Poincaré Probab. Stat.", vol. 45, n^o 1, 2009, p. 157–174 DE .

[30] B. ROYNETTE, P. VALLOIS, M. YOR. *Penalisations of multidimensional Brownian motion. VI*, in "ESAIM Probab. Stat.", vol. 13, 2009, p. 152–180.

[31] B. ROYNETTE, P. VALLOIS, M. YOR. *Brownian penalisations related to excursion lengths*, in "Ann. Inst. Henri Poincaré Probab. Stat.", vol. 45, n^o 2, 2009, p. 421–452.

[32] B. ROYNETTE, M. YOR. *Local limit theorems for Brownian additive functionals and penalisation of Brownian paths, IX*, in "ESAIM PS", 2009, http://hal.archives-ouvertes.fr/hal-00257593/en/.

[33] P. VALLOIS, C. S. TAPIERO. *A claims persistence process and insurance*, in "Insurance Math. Econom.", vol. 44, n^o 3, 2009, p. 367–373.

Articles in Non Peer-Reviewed Journal

[34] M. BOSSY, O. DAVIDAU, N. MAÏZI, O. POURTALLIER. *Valuing CO2 emission allowances with stochastic control*, in "ERCIM News", vol. 78, n^o 4, 2009, p. 24 – 25.

International Peer-Reviewed Conference/Proceedings

[35] A. LEJAY. *Monte Carlo methods for discontinuous media*, in "3rd International Conference on Approximation Methods and numerical Modeling in Environment and Natural Resources MAMERN 2009, France Pau", B.

AMAZIANE, D. BARRERA, M. FORTES, M. IBÁNEZ, M. ODUNLAMI, A. PALOMARES, M. PASADAS, M. RODRÍGUEZ, D. SBIBIH (editors), vol. 2, Universidad de Granada, 2009, p. 591-596, http://hal.inria.fr/inria-00393738/en/.

National Peer-Reviewed Conference/Proceedings

[36] P.-E. JABIN. *Some regularizing methods for transport equations and the regularity of solutions to scalar conservation laws*, in "Séminaire X-EDP", 2009, to appear.

Workshops without Proceedings

[37] A. GAIKWAD, V. D. DOAN, M. BOSSY, F. ABERGEL, F. BAUDE. *Superquant financial-benchmark suite for performance analysis of grid middlewares*, in "4th Int. Conference on High Performance Scientific Computing, Modeling, Simulation and Optimization of Complex Processes", Institute of Mathematics, Vietnamese Academy of Science and Technology, March 2009, 70, Full version submitted for post-proceeding publication by Springer.

Scientific Books (or Scientific Book chapters)

[38] C. BLANCHET-SCALLIET, R. GIBSON BRANDON, B. DE SAPORTA, D. TALAY, E. TANRÉ. *Viscosity solutions to optimal portfolio allocation problems in models with random time changes and transaction costs*, in "Advance Financial Modelling", H. ALBRECHER, W. RUNGGALDIER, W. SCHACHERMAYER (editors), Radon Series on Computational and Applied Mathematics, vol. 8, Walter de Gruyter GmbH & Co. KG, Berlin, 2009, p. 53–90 CH .

[39] A. LEJAY. *Yet another introduction to rough paths*, in "Séminaire de Probabilités XLII", Lecture Notes in Mathematics, vol. 1979, Springer-Verlag, 2009, p. 1–101.

[40] S. MAIRE, E. TANRÉ. *Stochastic spectral formulations for elliptic problems*, in "Monte Carlo and quasi-Monte Carlo methods 2008, Berlin", Springer, 2009, http://hal.inria.fr/inria-00340708/en/, to appear.

[41] J. NAJNUDEL, B. ROYNETTE, M. YOR. *A global view of Brownian penalisations*, MSJ Memoirs, vol. 19, Mathematical Society of Japan, Tokyo, 2009.

[42] D. TALAY. *Around model risk in finance*, in "Modèles Alétaoires en Finance Mathématique, Courses of the 2007 Cimpa-Unesco-Marocco School on Stochastic Models in Mathematical Finance", M. EDDAHBI, S. HAMADÈNE, Y. OUKNINE (editors), Travaux en Cours, vol. 77, Hermann, 2009, p. 161–229.

Research Reports

[43] A. BERGAOUI, M. DEACONU, M. GHAZAI, I. HENCHIRI, S. HERRMANN, A. LEJAY, V. REUTENAUER, D. TALAY, E. TANRÉ, Y. WANG. *Méthodes de réduction de variance originales et de simulation exacte de prix et de grecques en finance*, INRIA / Calyon, 2009, rapport final de collaboration Calyon-Inria, Technical report.

[44] C. BLANCHET-SCALLIET, R. GIBSON BRANDON, B. DE SAPORTA, D. TALAY, E. TANRÉ. *Mathematical Model for Resistance and Optimal Strategy*, n^{o} 552, NCCR-FINRISK, 2009, Working Paper Series CH .

[45] M. BOSSY, M. CISSÉ, D. TALAY. *Stochastic representations of derivatives of solutions of one dimensional parabolic variational inequalities with Neumann boundary conditions*, INRIA, 2009, http://hal.inria.fr/inria-00381854/en/, RR-6921, Rapport de rechercheSN.

[46] B. BÉRARD-BERGERY, C. PROFETA, E. TANRÉ. *Mathematical Model for Resistance and Optimal Strategy*, n° 524, NCCR-FINRISK, 2009, Working Paper Series.

[47] V. D. DOAN, A. GAIKWAD, M. BOSSY, F. BAUDE, F. ABERGEL. *A financial engineering benchmark for performance analysis of grid middlewares*, INRIA, 2009, http://hal.inria.fr/inria-00387324/en/, RT-365, Technical report.

Other Publications

[48] F. BERNARDIN, M. BOSSY, C. CHAUVIN, J.-F. JABIR, A. ROUSSEAU. *Stochastic Lagrangian Method for Downscaling Problems in Meteorology*, 2009, http://hal.inria.fr/docs/00/41/09/32/PDF/BBCJR-09.pdf, preprintCL.

[49] S. HERRMANN, J. TUGAULT. *Non uniqueness of stationary measures for self-stabilizing diffusions*, 2009, http://hal.archives-ouvertes.fr/hal-00368177, preprint.

[50] S. HERRMANN, J. TUGAULT. *Self-stabilizing processes: uniqueness problem for stationary measures and convergence rate in the small noise limit*, 2009, http://hal.archives-ouvertes.fr/hal-00434254, preprint.

[51] S. HERRMANN, J. TUGAULT. *Stationary measures for self-stabilizing processes: asymptotic analysis in the small noise limit*, 2009, http://hal.archives-ouvertes.fr/hal-00397470, preprint.

[52] A. LEJAY. *Controlled differential equations as Young integrals: a simple approach*, 2009, http://hal.inria.fr/inria-00402397/en/, preprint.

[53] A. LEJAY, S. MAIRE. *Simulating diffusions with piecewise constant coefficients using a kinetic approximation*, 2009, http://hal.inria.fr/inria-00358003/en/, preprint.

[54] A. LEJAY, V. REUTENAUER. *A variance reduction technique using a quantized Brownian motion as a control variate*, 2009, http://hal.inria.fr/inria-00393749/en/, preprint.

References in notes

[55] P. CHERIDITO, H. M. SONER, N. TOUZI. *The multi-dimensional super-replication problem under gamma constraints*, in "Ann. Inst. H. Poincaré Anal. Non Linéaire", vol. 22, n° 5, 2005, p. 633–666, http://dx.doi.org/10.1016/j.anihpc.2004.10.012.

[56] O. DIEKMANN, P.-E. JABIN, S. MISCHLER, B. PERTHAME. *The dynamics of adaptation: An illuminating example and a Hamilton-Jacobi approach*, in "Theor. Pop. Biol.", vol. 67, 2005, p. 257–271.

[57] S. HERRMANN, P. IMKELLER, D. PEITHMANN. *Large deviations and a Kramers' type law for self-stabilizing diffusions*, in "Ann. Appl. Probab.", vol. 18, n° 4, 2008, p. 1379–1423.

[58] N. V. KRYLOV. *Controlled diffusion processes*, Springer-Verlag, New-Yor,Heidelberg, Berlin, 1980.

[59] J.-M. LASRY, P.-L. LIONS. *Jeux à champ moyen. I : Le cas stationnaire*, in "C. R. Acad. Sci. Paris", 2006.

[60] J.-M. LASRY, P.-L. LIONS. *Mean field games. II : Finite horizon and optimal control*, in "C. R. Acad. Sci. Paris", 2006.

[61] S. MAIRE, C. DE LUIGI. *Quasi-Monte Carlo quadratures for multivariate smooth functions*, in "Appl. Numer. Math.", vol. 56, n^{o} 2, 2006, p. 146–162, http://dx.doi.org/10.1016/j.apnum.2005.02.014.

[62] M. MASCAGNI, N. A. SIMONOV. *Monte Carlo methods for calculating some physical properties of large molecules*, in "SIAM J. Sci. Comput.", vol. 26, n^{o} 1, 2004, p. 339–357.

[63] S. POPE. *Turbulent flows*, Cambridge Univ. Press, 2003.