INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Apics

Analysis and Problems of Inverse type in Control and Signal processing

Sophia Antipolis - Méditerranée

Theme : Modeling, Optimization, and Control of Dynamic Systems

Activity Report

2010

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1. Team

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2. Overall Objectives

2.1. Overall Objectives

The Team aims at designing and developing constructive methods in modeling, identification and control of dynamical, resonant and diffusive systems.

2.1.1. Research Themes

- Function theory and approximation theory in the complex domain, with applications to frequency identification of linear systems and inverse boundary problems for the Laplace and Beltrami operators:
  - System and circuit theory with applications to the modeling of analog microwave devices. Development of dedicated software for the synthesis of such devices.
  - Inverse potential problems in 2-D and 3-D and harmonic analysis with applications to non-destructive control (from magneto/electro-encephalography in medical engineering or plasma confinement in tokamaks for nuclear fusion).
- Control and structure analysis of non-linear systems with applications to orbit transfer of satellites.

2.1.2. International and industrial partners

- Collaboration under contract with Thales Alenia Space (Toulouse, Cannes, and Paris), CNES (Toulouse), XLim (Limoges), CEA-IRFM (Cadarache).
- Exchanges with UST (Villeneuve d'Asq), University Bordeaux-I (Talence), University of Orléans (MAPMO), University of Pau (EPI Inria Magique-3D), University Marseille-I (CMI), CWI (the Netherlands), SISSA (Italy), the Universities of Illinois (Urbana-Champaign USA), California at San Diego and Santa Barbara (USA), Michigan at East-Lansing (USA), Vanderbilt University (Nashville USA), Texas A&M (College Station USA), ISIB (CNR Padova, Italy), Beer Sheva (Israel), RMC (Kingston, Canada), University of Erlangen (Germany), Leeds (UK), Maastricht University (The Netherlands), Cork University (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien (Austria), TFH-Berlin (Germany), CINVESTAV (Mexico), ENIT (Tunis), KTH (Stockholm).
- The project is involved in the ANR projects AHPI (Math., coordinator) and Filipix (Telecom.), in a EMS21-RTG NSF program (with Vanderbilt University, Nashville, USA), in an NSF Grant with Vanderbilt University and the MIT, in an EPSRC Grant with Leeds University (UK), in a Inria-Tunisian Universities program (STIC, with LAMSIN-ENIT, Tunis).

3. Scientific Foundations

3.1. Identification and approximation

Identification typically consists in approximating experimental data by the prediction of a model belonging to some model class. It consists therefore of two steps, namely the choice of a suitable model class and the determination of a model in the class that fits best with the data. The ability to solve this approximation problem, often non-trivial and ill-posed, impinges on the effectiveness of a method.

Particular attention is payed within the team to the class of stable linear time-invariant systems, in particular resonant ones, and in isotropically diffusive systems, with techniques that dwell on functional and harmonic analysis. In fact one often restricts to a smaller class—*e.g.* rational models of suitable degree (resonant systems, see section 4.2) or other structural constraints—and this leads us to split the identification problem in two consecutive steps:

1. Seek a stable but infinite (numerically: high) dimensional model to fit the data. Mathematically speaking, this step consists in reconstructing a function analytic in the right half-plane or in the unit disk (the transfer function), from its values on an interval of the imaginary axis or of the unit circle (the band-width). We will embed this classical ill-posed issue (*i.e.* the inverse Cauchy problem for the Laplace equation) into a family of well-posed extremal problems, that may be viewed as a regularization scheme of Tikhonov-type. These problems are infinite-dimensional but convex (see section 3.1.1).
2. Approximate the above model by a lower order one reflecting further known properties of the physical system. This step aims at reducing the complexity while bringing physical significance to the design parameters. It typically consists of a rational or meromorphic approximation procedure with prescribed number of poles in certain classes of analytic functions. Rational approximation in the complex domain is a classical but difficult non-convex problem, for which few effective methods exist. In relation to system theory, two specific difficulties superimpose on the classical situation, namely one must control the region where the poles of the approximants lie in order to ensure the stability of the model, and one has to handle matrix-valued functions when the system has several inputs and outputs, in which case the number of poles must be replaced by the McMillan degree (see section 3.1.2).

When identifying elliptic (Laplace, Beltrami) partial differential equations from boundary data, point 1. above can be recast as an inverse boundary-value problem with (overdetermined Dirichlet-Neumann) data on part of the boundary of a plane domain (recover a function, analytic in a domain, from incomplete boundary data). As such, it arises naturally in higher dimensions when analytic functions get replaced by gradients of harmonic functions (see section 4.1). Motivated by free boundary problems in plasma control and questions of source recovery arising in magneto/electro-encephalography, we aim at generalizing this approach to the real Beltrami equation in dimension 2 (section 6.3.3) and to the Laplace equation in dimension 3 (section 6.3.1).

Step 2. above, i.e., meromorphic approximation with prescribed number of poles—is used to approach other inverse problems beyond harmonic identification. In fact, the way the singularities of the approximant (*i.e.* its poles) relate to the singularities of the approximated function is an all-pervasive theme in approximation theory: for appropriate classes of functions, the location of the poles of the approximant can be used as an estimator of the singularities of the approximated function (see section 6.3.2).

We provide further details on the two steps mentioned above in the sub-paragraphs to come.

3.1.1. Analytic approximation of incomplete boundary data

Participants: Laurent Baratchart, Slah Chaabi, Yannick Fischer, Juliette Leblond, Jean-Paul Marmorat, Jonathan Partington, Stéphane Rigat [Univ. Aix-Marseille I], Emmanuel Russ [Univ. Aix-Marseille III], Fabien Seyfert.

Given a planar domain D, the problem is to recover an analytic function from its values on a subset of the boundary of D. It is convenient to normalize D and apply in each particular case a conformal transformation to meet a "normalized" domain. In the simply connected case, which is that of the half-plane, we fix D to be the unit disk, so that its boundary is the unit circle T. We denote by H^p the Hardy space of exponent p which is the closure of polynomials in the L^p-norm on the circle if $1 \leq p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which make it possible to speak of (traces of) analytic functions on the boundary.

A standard extremal problem on the disk is [67]:

> (P_0) Let $1 \leq p \leq \infty$ and $f \in L^p(T)$; find a function $g \in H^p$ such that $g - f$ is of minimal norm in $L^p(T)$.

When seeking an analytic function in D which approximately matches some measured values f on a sub-arc K of T, the following generalization of (P_0) naturally arises:

> (P) Let $1 \leq p \leq \infty$, K a sub-arc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behavior capsulizing the expected behavior of the model off K, while M is the admissible error with respect to this expectation. The value of p reflects the type of stability which is sought and how much one wants to smoothen the data.

To fix terminology we generically refer to (P) as a *bounded extremal problem*. The solution to this convex infinite-dimensional optimization problem can be obtained upon iteratively solving spectral equations for appropriate Hankel and Toeplitz operators, that involve a Lagrange parameter, and whose right hand-side is given by the solution to (P_0) for some weighted concatenation of f and ψ. Constructive aspects are described in [49], [51], [86], for $p = 2$, $p = \infty$, and $1 < p < \infty$, while the situation $p = 1$ is essentially open.

Various modifications of (P) have been studied in order to meet specific needs. For instance when dealing with loss-less transfer functions (see section 4.2), one may want to express the constraint on $T \setminus K$ in a pointwise manner: $|g - \psi| \leq M$ a.e. on $T \setminus K$, see [53], [52] for $p = 2$ and $\psi = 0$.

The above-mentioned problems can be stated on an annular geometry rather than a disk. For $p = 2$ the solution proceeds much along the same lines [75]. When K is the outer boundary, (P) regularizes a classical inverse problem occurring in nondestructive control, namely to recover a harmonic function on the inner boundary from overdetermined Dirichlet-Neumann data on the outer boundary (see sections 4.1 and 6.3). Interestingly perhaps, it becomes a tool to approach Bernoulli type problems for the Laplacian, where overdetermined observations are made on the outer boundary and we *seek the inner boundary* knowing it is a level curve of the flux (see section 6.3.3). Here, the Lagrange parameter indicates which deformation should be applied on the inner contour in order to improve the fit to the data.

Continuing effort is currently payed by the team to carry over bounded extremal problems and their solution to more general settings.

Such generalizations are twofold: on the one hand Apics considers 2-D diffusion equations with variable conductivity, on the other hand it investigates the ordinary Laplacian in $\mathbf{R}^n$. The targeted applications are the determination of free boundaries in plasma control and source detection in electro/magneto-encephalography (EEG/MEG), as well as discretization issues of the gravitational potential in geophysics (see section 6.3.2).

An isotropic diffusion equation in dimension 2 can be recast as a so-called real Beltrami equation [74]. This way analytic functions get replaced by "generalized" ones in problems (P_0) and (P). Hardy spaces of solutions, which are more general than Sobolev ones and allow one to handle L^p boundary conditions, have been introduced when $1 < p < \infty$ [17]. The expansions of solutions needed to constructively handle such problems have been preliminary studied in [22], [23]. The goal is to solve the analog of (P) in this context to approach Bernoulli-type problems (see section 6.3.1).

At present, bounded extremal problems for the n-D Laplacian are considered on half-spaces or balls. Following [87], Hardy spaces are defined as gradients of harmonic functions satisfying L^p growth conditions on inner hyperplanes or spheres. From the constructive viewpoint, when $p = 2$, spherical harmonics offer a reasonable substitute to Fourier expansions [15]. Only very recently were we able to define operators of Hankel type whose singular values connect to the solution of (P_0) in BMO norms. The L^p problem also makes contact with some nonlinear PDE's, namely to the p-Laplacian. The goal is here to solve the analog of (P) on spherical shells to approach inverse diffusion problems across a conductor layer.

3.1.2. Meromorphic and rational approximation

Participants: Laurent Baratchart, José Grimm, Martine Olivi, Edward Saff, Herbert Stahl [TFH Berlin].

Let as before D designate the unit disk, T the unit circle. We further put R_N for the set of rational functions with at most N poles in D, which allows us to define the meromorphic functions in $L^p(T)$ as the traces of functions in $H^p + R_N$.

A natural generalization of problem (P_0) is

> (P_N) Let $1 \leq p \leq \infty$, $N \geq 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Problem (P_N) aims, on the one hand, at solving inverse potential problems from overdetermined Dirichlet-Neumann data, namely to recover approximate solutions of the inhomogeneous Laplace equation $\Delta u = \mu$, with μ some (unknown) distribution, which will be discretized by the process as a linear combination of N Dirac masses. On the other hand, it is used to perform the second step of the identification scheme described in section 3.1, namely rational approximation with a prescribed number of poles to a function analytic in the right half-plane, when one maps the latter conformally to the complement of D and solve (P_N) for the transformed function on T.

Only for $p = \infty$ and continuous f it is known how to solve (P_N) in closed form. The unique solution is given by the AAK theory, that allows one to express g_N in terms of the singular vectors of the Hankel operator with symbol f. The continuity of g_N as a function of f only holds for stronger norms than uniform [82].

The case $p = 2$ is of special importance. In particular when $f \in \overline{H}^2$, the Hardy space of exponent 2 of the *complement* of D in the complex plane (by definition, $h(z)$ belongs to $\overline{H}^p$ if, and only if $h(1/z)$ belongs to H^p), then (P_N) reduces to rational approximation. Moreover, it turns out that the associated solution $g_N \in R_N$ has no pole outside D, hence it is a *stable* rational approximant to f. However, in contrast with the situation when $p = \infty$, this approximant may *not* be unique.

The former Miaou project (predecessor of Apics) has designed an adapted steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; it seems today the only procedure meeting this property. Roughly speaking, it is a gradient algorithm that proceeds recursively with respect to the order N of the approximant, in a compact region of the parameter space [47]. Although it has proved rather effective in all applications carried out so far (see sections 4.1, 4.2), it is not known whether the absolute *minimum* can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as done by the Endymion software section 5.5 and RARL2 software, section 5.2).

In order to establish convergence results of the algorithm to the global minimum, Apics has undergone a long-haul study of the number and nature of critical points, in which tools from differential topology and operator theory team up with classical approximation theory. The main discovery is that the nature of the critical points (*e.g. local minima*, saddles...) depends on the decrease of the interpolation error to f as N increases [54]. Based on this, sufficient conditions have been developed for a local *minimum* to be unique. This technique requires strong error estimates that are often difficult to obtain, and most of the time only hold for N large. Examples where uniqueness or asymptotic uniqueness has been proved this way include transfer functions of relaxation systems (i.e., Markov functions) [56], the exponential function, and meromorphic functions [8]. The case where f is the Cauchy integral on a hyperbolic geodesic arc of a Dini-continuous function which does not vanish "too much" has been recently answered in the positive. An analog to AAK theory has been carried out for $2 \leq p < \infty$ [9]. Although not computationally as powerful, it has better continuity properties and stresses a continuous link between rational approximation in H^2 and meromorphic approximation in the uniform norm, allowing one to use, in either context, techniques available from the other[1].

A common feature to all these problems is that critical point equations express non-Hermitian orthogonality relations for the denominator of the approximant. This is used in an essential manner to assess the behavior of the poles of the approximants to functions with branched singularities, which is of particular interest for inverse source problems (*cf.* section 6.3.2).

In higher dimensions, the analog of problem (P_N) is the approximation of a vector field with gradients of potentials generated by N point masses instead of meromorphic functions. The issue is by no means understood at present, and is a major endeavor of future research problems.

Certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus. Such questions have become over years an increasingly significant part of the team's activity (see section 4.2). When translated over to the circle, a prototypical formulation consists in approximating the modulus of a given function by the modulus of a rational function of degree n. When $p = 2$ this problem can be reduced to a series of standard rational approximation problems, but usually one needs to solve it for $p = \infty$. The case where $|f|$ is a piecewise constant function with values 0 and 1 can also be approached via classical Zolotarev problems [85], that can be solved more or less explicitly when the pass-band consists of a single arc. A constructive solution in the case where $|f|$ is a piecewise constant function with values 0 and 1 on several arcs (multiband filters) is one recent achievement of the team. Though the modulus of the response is the first concern in filter design, the variation of the phase must nevertheless remain under control to avoid unacceptable distortion of the signal. This is an important issue, currently under investigation within the team under contract with the CNES. From the point of view of design, rational approximants are indeed useful only if they can be translated into physical parameter values for the device to be built. This is where system theory enters the scene, as the correspondence between the frequency response (i.e., the transfer-function) and the linear differential equations that generate this response (i.e., the state-space representation), which is the object of the so-called *realization* process. Since filters have to be considered as dual-mode cavities, the realization issue must indeed be tackled in a 2×2 matrix-valued context that adds to the complexity. A fair share of the team's research in this direction is concerned with finding realizations meeting certain constraints (imposed by the technology in use) for a transfer-function that was obtained with the above-described techniques (see section 6.7).

[1] When $1 \leq p < 2$, problem (P_N) is still fairly open.

3.1.3. Behavior of poles of meromorphic approximants and inverse problems for the Laplacian

Participants: Laurent Baratchart, Herbert Stahl [TFH Berlin], Maxim Yattselev.

We refer here to the behavior of the poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. If one normalizes the contour to be the unit circle T, we are back to the framework of section 3.1.2 and to problem (P_N); the invariance of the problem under conformal mapping was established in [6]. The research so far has focused on functions whose singular set inside the contour is zero or one-dimensional.

Generally speaking, the behavior of poles is particularly important in meromorphic approximation to obtain error rates as the degree goes large and also to tackle constructive issues like uniqueness. However, the original motivation of Apics is to consider this issue in connection with the approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities. The general theme is thus *how do the singularities of the approximant reflect those of the approximated function?* The approach to inverse problem for the 2-D Laplacian that we outline here is attractive when the singularities are zero- or one-dimensional (see section 4.1). It can be used as a computationally cheap preliminary step to obtain the initial guess of a more precise but heavier numerical optimization.

For sufficiently smooth cracks, or pointwise sources recovery, the approach in question is in fact equivalent to the meromorphic approximation of a function with branch points, and we were able to prove ([4], [6]) that the poles of the approximants accumulate in a neighborhood of the geodesic hyperbolic arc that links the endpoints of the crack, or the sources [50]. Moreover the asymptotic density of the poles turns out to be the equilibrium distribution on the geodesic arc of the Green potential and it charges the end points, that are thus well localized if one is able to compute sufficiently many zeros (this is where the method could fail). The case of more general cracks, as well as situations with three or more sources, requires the analysis of the situation where the number of branch points is finite but arbitrary. This are outstanding open questions for applications to inverse problems (see section 6.3), as also the problem of a general singularity, that may be two dimensional.

Results of this type open new perspectives in non-destructive control, in that they link issues of current interest in approximation theory (the behavior of zeroes of non-Hermitian orthogonal polynomials) to some classical inverse problems for which a dual approach is thereby proposed: to approximate the boundary conditions by true solutions of the equations, rather than the equation itself (by discretization).

Let us point out that the problem of approximating, by a rational or meromorphic function, in the L^p sense on the boundary of a domain, the Cauchy transform of a real measure, localized inside the domain, can be viewed as an optimal discretization problem for a logarithmic potential according to a criterion involving a Sobolev norm. This formulation can be generalized to higher dimensions, even if the computational power of complex analysis is then no longer available, and this makes for a long-term research project with a wide range of applications. It is interesting to mention that the case of sources in dimension three in a spherical or ellipsoidal geometry can be attacked with the above 2-D techniques, as applied to planar sections (see section 6.3).

3.1.4. Matrix-valued rational approximation

Participants: Laurent Baratchart, Martine Olivi, José Grimm, Jean-Paul Marmorat, Bernard Hanzon, Ralf Peeters [Univ. Maastricht].

Matrix-valued approximation is necessary for handling systems with several inputs and outputs, and it generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically. In the matrix case, the McMillan degree (i.e., the degree of a minimal realization in the System-Theoretic sense) generalizes the degree.

The problem we want to consider reads: *Let* $\mathcal{F} \in (H^2)^{m \times l}$ *and* n *an integer; find a rational matrix of size* $m \times l$ *without poles in the unit disk and of McMillan degree at most* n *which is nearest possible to* $\mathcal{F}$ *in* $(H^2)^{m \times l}$. Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The approximation algorithm designed in the scalar case generalizes to the matrix-valued situation [66]. The first difficulty consists here in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (i.e., matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n enter the picture in an essential manner: they play the role of the denominator in a fractional representation of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization).

The set of inner matrices of given degree has the structure of a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (parametrization valid in a neighborhood of a point), and we must handle changes of charts in the course of the algorithm. Such parametrization can be obtained from interpolation theory and Schur type algorithms, the parameters being interpolation vectors or matrices ([42], [10], [11]). Some of these parametrizations have a particular interest for computation of realizations ([10], [11]), involved in the estimation of physical quantities for the synthesis of resonant filters. Two rational approximation codes (see sections 5.2 and 5.5) have been developed in the team.

Problems relative to multiple local minima naturally arise in the matrix-valued case as well, but deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The already investigated case of rational functions of the sought degree (the consistency problem) was solved using rather heavy machinery [7], and that of matrix-valued Markov functions, that are the first example beyond rational functions, has made progress only recently [46].

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.2. Structure and control of non-linear systems

3.2.1. Feedback control and optimal control

Participants: Jean-Baptiste Pomet, Ahed Hindawi, Jana Nemcova, Ludovic Rifford.

Using the terminology of the beginning of section 3.1, the class of models considered here is the one of finite dimensional nonlinear control systems; we focus on control. In many cases, a linear control based on the linear approximation around a nominal point or trajectory is sufficient. However, there are important instances where it is not, either because the magnitude of the control is limited or because the linear approximation is not controllable, or else in control problems like path planning, that are not local in nature.

State feedback stabilization consists in designing a control law which is a function of the state and makes a given point (or trajectory) asymptotically stable for the closed-loop system. That function of the state must bear some regularity, at least enough to allow the closed-loop system to make sense; continuous or smooth feedback would be ideal, but one may also be content with discontinuous feedback if robustness properties are not defeated. One can consider this as a weak version of the optimal control problem which is to find a control that minimizes a given criterion (for instance the time to reach a prescribed state). Optimal control generally leads to a rather irregular dependence on the initial state; in contrast, stabilization is a *qualitative* objective (i.e., to reach a given state asymptotically) which is more flexible and allows one to impose much more regularity.

Lyapunov functions are a well-known tool to study the stability of non-controlled dynamic systems. For a control system, a *Control Lyapunov Function* is a Lyapunov function for the closed-loop system where the feedback is chosen appropriately. It can be expressed by a differential inequality called the “Artstein (in)equation” [44], reminiscent of the Hamilton-Jacobi-Bellmann equation but largely under-determined. One can easily deduce a continuous stabilizing feedback control from the knowledge of a control Lyapunov function; also, even when such a control is known beforehand, obtaining a control Lyapunov function can still be very useful to deal with robustness issues.

Moreover, if one has to deal with a problem where it is important to optimize a criterion, and if the optimal solution is hard to compute, one can look for a control Lyapunov function which comes “close” (in the sense of the criterion) to the solution of the optimization problem but leads to a control which is easier to work with.

A class of systems of interest to us has been the one of systems with a conservative drift and a small control (whose effect is small in magnitude compared to the drift). A prototype is the control of a satellite with low thrust propellers: the conservative drift is the classical Kepler problem and the control is small compared to earth attraction. We developed, starting with Alex Bombrun's PhD [59], original averaging methods, that differ from classical methods in that the average is a *control system*, i.e. the averaging process does not depend on the control strategy. A reference paper is still under preparation [60].

These constructions were exploited in a joint collaborative research conducted with Thales Alenia Space (Cannes), where minimizing a certain cost is very important (fuel consumption / transfer time) while at the same time a feedback law is preferred because of robustness and ease of implementation (see section 4.3).

3.2.2. Optimal transport

Participants: Ahed Hindawi, Jean-Baptiste Pomet, Ludovic Rifford.

Optimal transport is the problem of finding the cheapest transformation that moves a given initial measure to a given final one, where the cost of the transformation is obtained by integrating against the measure a point-to-point cost that may be a squared Euclidean distance or a Riemannian distance on a manifold or more exotic ones where some directions are privileged that naturally lean towards optimal control.

The problem has a long history which goes back to the pioneering works ([79], [76]), and was more recently revised and revitalized by [62] and [78]. At the same time, applications to many domains ranging from image processing to shape reconstruction or urban planning were developed, see a survey in [80].

We are interested in transportation problems with a cost coming from optimal control, i.e. from minimizing an integral quadratic cost, among trajectories that are subject to differential constraints coming from a control system. The optimal transport problem in this setting borrows methods from control and at the same time helps understanding optimal control because it is a more regular problem. The case of controllable affine control systems without drift (in which case the cost is the sub-Riemannian distance) is studied in [43], [41] and [20]. This is a new topic in the team, starting with the arrival of L. Rifford and the PhD of A. Hindawi, whose goal is to tackle the problem of systems with drift. See new results in section 6.10.

3.2.3. Transformations and equivalences of non-linear systems and models

Participants: Laurent Baratchart, Jean-Baptiste Pomet.

The motivations for a detailed study of equivalence classes and invariance of models of control systems under various classes of transformations are two-fold:

- From the point of view of control, a command satisfying specific objectives on the transformed system can be used to control the original system including the transformation in the controller.
- From the point of view of identification and modeling, the interest is either to derive qualitative invariants to support the choice of a non-linear model given the observations, or to contribute to a classification of non-linear models which is missing sorely today. This is a prerequisite for a general theory of non-linear identification; indeed, the success of the linear model in control and identification is due to the deep understanding one has of it.

The interested reader can find a richer overview (in french) in the first chapter of [84].

A *static feedback* transformation is a (non-singular) re-parametrization of the control depending on the state, together with a change of coordinates in the state space. Static equivalence has motivated a very wide literature; in the differentiable case, classification is performed in relatively low dimensions; it gives insight on models and also points out that this equivalence is “too fine”, i.e. very few systems are equivalent and normal forms are far from stable. This motivates the search for a rougher equivalence that would account for more qualitative phenomena. The Hartman-Grobman theorem states that every ordinary differential equation (i.e. dynamical system without control) is locally equivalent, in a neighborhood of a non-degenerate equilibrium, to a linear system via a transformation that is solely bi-continuous, whereas smoothness requires many more invariants. This was a motivation to study *topological* (non necessarily smooth) equivalence. A “Hartman Grobman Theorem for control systems” is stated in [48] under weak regularity conditions; it is too abstract to be relevant to the above considerations on qualitative phenomena: linearization is performed by functional non-causal transformations rather than feedback transformations *stricto sensu*; it however acquires a concrete meaning when the inputs are themselves generated by finite-dimensional dynamics. A stronger Hartman Grobman Theorem for control systems (where transformations are homeomorphisms in the state-control space) in fact cannot hold [55]: almost all topologically linearizable control systems are differentiably (in the same class of regularity as the system itself) linearizable. In general (equivalence between nonlinear systems), topological invariants are still a subject of interest to us.

A *dynamic feedback* transformation consists of a dynamic extension (adding new states, and assigning them new dynamics) followed by a state feedback on the augmented system; dynamic equivalence is another attempt to enlarge classes of equivalence. It is indeed strictly more general than static equivalence: it is known that many systems are dynamic equivalent but not static equivalent to a linear controllable system. The classes containing a linear controllable system are the ones of *differentially flat systems*; it turns out (see [65]) that many practical systems are in this class and that being "flat" also means that all the solutions to the systems are given by a *(Monge) parametrization* that describes the solutions without any integration.

An important question remains open: how can one algorithmically decide whether a given system has this property or not, i.e., is dynamic linearizable or not? The mathematical difficulty is that no a priori bound is known on the order of the differential operator giving the parametrization. Within the team, results on low dimensional systems have been obtained [1]; the above mentioned difficulty is not solved for these systems but results are given with *a priori* prescribed bounds on this order.

For general dynamic equivalence as well as flatness, very few invariants are known. In particular, the fact that the size of the extra dynamics contained in the dynamic transformation (or the order of the above mentioned differential operator, for flatness) is not a priori bounded makes it very difficult to prove that two systems are *not* dynamic feedback equivalent, or that a system is *not* flat. Many simple systems pointed out in [1] are conjectured not to be flat but no proof is available. The only known general necessary condition for flatness is the so-called ruled surface criterion; it was generalised by the team to dynamic equivalence between arbitrary nonlinear systems in [83] and [35].

Another attempt towards conditions for flatness used the differential algebraic point of view: the module of differentials of a controllable system is, generically, free and finitely generated over the ring of differential polynomials in d/dt with coefficients in the ring of functions on the system's trajectories; flatness amounts to existence of a basis consisting of closed differential forms. Expressed in this way, it looks like an extension of the classical Frobenius integrability theorem to the case where coefficients are differential operators. Some non classical conditions have to be added to the classical stability by exterior differentiation, and the problem is open. In [45], a partial answer was given, but in a framework where infinitely many variables are allowed and a finiteness criterion is still missing.

4. Application Domains

4.1. Geometric inverse problems for elliptic partial differential equations

Participants: Laurent Baratchart, Yannick Fischer, José Grimm, Juliette Leblond, Ana-Maria Nicu, Jonathan Partington, Stéphane Rigat [Univ. Aix-Marseille I], Emmanuel Russ [Univ. Aix-Marseille III], Edward Saff, Meriem Zghal [Until October].

This domain is mostly connected to the techniques described in section 3.1.

We are mainly concerned with classical inverse problems like the one of localizing defaults (as cracks, pointwise sources or occlusions) in a two or three dimensional domain from boundary data (which may correspond to thermal, electrical, or magnetic measurements), of a solution to Laplace or to some conductivity equation in the domain. These defaults can be expressed as a lack of analyticity of the solution of the associated Dirichlet-Neumann problem that may be approached, in balls, using techniques of best rational or meromorphic approximation on the boundary of the object (see section 3.1).

Indeed, it turns out that traces of the boundary data on 2-D cross sections (disks) coincide with analytic functions in the slicing plane, that has branched singularities inside the disk [5]. These singularities are related to the actual location of the sources (namely, they reach in turn a maximum in modulus when the plane contains one of the sources). Hence, we are back to the 2-D framework where approximately recovering these singularities can be performed using best rational approximation.

In this connection, the realistic case where data are available on part of the boundary only offers a typical opportunity to apply the analytic extension techniques (see section 3.1.1) to Cauchy type issues, a somewhat different kind of inverse problems in which the team is strongly interested.

The approach proposed here consists in recovering, from measured data on a subset K of the boundary ∂D of a domain D of R^2 or R^3, say the values F_K on K of some function F, the subset $\gamma \subset D$ of its singularities (typically, a crack or a discrete set of pointwise sources), provided that F is an analytic function in $D \setminus \gamma$.

- The analytic approximation techniques (section 3.1.1) first allow us to extend F from the given data F_K to all of ∂D, if $K \neq \partial D$, which is a Cauchy type issue for which our algorithms provide robust solutions, in plane domains (see [15] for 3D spherical situations, also discussed in section 6.3).
- From these extended data on the whole boundary, one can obtain information on the presence and the location of γ, using rational or meromorphic approximation on the boundary (section 3.1). This may be viewed as a discretization of γ by the poles of the approximants [4].

  This is the case in dimension 2, using classical links between analyticity and harmonicity [2], but also in dimension 3, at least in spherical or ellipsoidal domains, working on 2-D plane sections, [5], [77].

The two above steps were shown to provide a robust way of locating sources from incomplete boundary data in a 2-D situation with several annular layers. Numerical experiments have already yielded excellent results in 3-D situations and we are now on the way to process real experimental magneto-encephalographic data, see also sections 5.7, 6.3.2. The PhD theses of A.-M. Nicu and M. Zghal [13] are concerned with these applications, in collaboration with the Athena team of Inria Sophia Antipolis, and with neuroscience teams in partner-hospitals (hosp. Timone, Marseille).

Such methods are currently being generalized to problems with variable conductivity governed by a 2-D Beltrami equation, see [17], [22], [23]. The application we have in mind is to plasma confinement for thermonuclear fusion in a Tokamak, more precisely with the extrapolation of magnetic data on the boundary of the chamber from the outer boundary of the plasma, which is a level curve for the poloidal flux solving the original div-grad equation. Solving this inverse problem of Bernoulli type is of importance to determine the appropriate boundary conditions to be applied to the chamber in order to shape the plasma [58]. These issues are the topics of the PhD theses of S. Chaabi and Y. Fischer, and of a joint collaboration with the CEA-IRFM (Cadarache), the Laboratoire J.-A. Dieudonné at the Univ. of Nice-SA, and the CMI-LATP at the Univ. of Marseille I (see section 6.3.3).

Inverse potential problems are also naturally encountered in magnetization issues that arise in nondestructive control. A particular application, which is the object of a joint NSF-supported project with Vanderbilt University and MIT, is to geophysics where the remanent magnetization a rock is to be analyzed using a squid-magnetometer in order to analyze the history of the object; specifically, the analysis of Martian rocks is conducted at MIT, for instance to understand if inversions of the magnetic field took place there. Mathematically speaking, the problem is to recover the (3-D valued) magnetization m from measurements of the vector potential:

$$\int_\Omega \frac{\operatorname{div} m(x')\, dx'}{|x-x'|},$$

outside the volume Ω of the object.

In turns out that discretization issues in geophysics can also be undertaken by these approximation techniques. Indeed, in geodesy or for GPS computations, one may need to get a best discrete approximation of the gravitational potential on the Earth's surface, from partial data there. This is also the topic of the PhD theses of A.-M. Nicu, and of a beginning collaboration with a physicist colleague (IGN, LAREG, geodesy). Related geometrical issues (finding out the geoid, level surface of the gravitational potential) should be considered either.

4.2. Identification and design of resonant systems: hyperfrequency filter identification

Participants: Laurent Baratchart, Stéphane Bila [XLim, Limoges], José Grimm, Jean-Paul Marmorat, Martine Olivi, Fabien Seyfert.

This domain is mostly connected to the techniques described in section 3.1.

One of the best training grounds for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption works well in the considered range of frequency, and whose specifications are made in the frequency domain. Resonant systems, either acoustic or electromagnetic based, are prototypical devices of common use in telecommunications.

In the domain of space telecommunications (satellite transmissions), constraints specific to onboard technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study (of the Helmholtz equation) states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Figure 1. Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that there are 16 quantities that should be optimized. Quantities like the diameter and length of the cavities, or the width of the 11 slits are fixed in the design phase.

Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all the cavities have the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 Ghz.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modeled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plugged on a resistor at one end and fed with some potential at the other end. We are then interested in the power which is transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the design, particularly the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (i.e., the underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80Mhz in the example).
- solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, cf. section 3.1.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.
- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this example) via the Endymion or RARL2 software (cf. section 3.1.4).
- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 2). Non-physical couplings are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, useful for the synthesis of repeating devices.

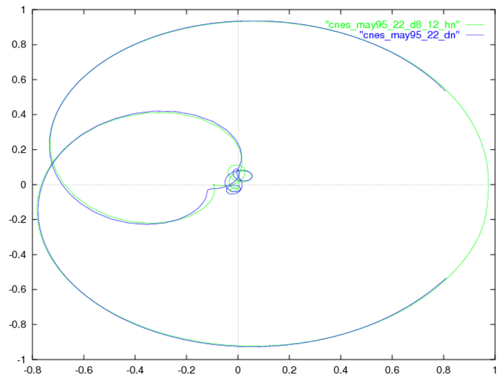

Figure 2. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}

The team investigates today the design of output multiplexors (OMUX) where several filters of the previous type are coupled on a common guide. In fact, it has undergone a rather general analysis of the question "How does an OMUX work?" With the help of numerical simulations and Schur analysis, general principles are being worked out to take into account:

- the coupling between each channel and the "Tee" that connects it to the manifold,
- the coupling between two consecutive channels.

The model is obtained upon chaining the corresponding scattering matrices, and mixes up rational elements and complex exponentials (because of the delays) hence constitutes an extension of the previous framework. Its study is being conducted under contract with Thales Alenia Space (Toulouse) (see sections 7.1).

4.3. Spatial mechanics

Participants: Ludovic Rifford, Jana Nemcova, Jean-Baptiste Pomet.

This domain is mostly connected to the techniques described in section 3.2.

Generally speaking, aerospace engineering requires sophisticated control techniques for which optimization is often crucial, due to the extreme functioning conditions. The use of satellites in telecommunication networks motivates a lot of research in the area of signal and image processing; see for instance section 4.2 for an illustration. Of course, this requires that satellites be adequately controlled, both in position and orientation (attitude). This problem and similar ones continue to motivate research in control. The team has been working for six years on control problems in orbital transfer with low thrust engines, including four years under contract with Thales Alenia Space (formerly Alcatel Space) in Cannes.

Technically, the reason for using these (ionic) low thrust engines, rather than chemical engines that deliver a much higher thrust, is that they require much less "fuel"; this is decisive because the total mass is limited by the capacity of the launchers: less fuel means more payload, while fuel represents today an impressive part of the total mass.

From the control point of view, the low thrust makes the transfer problem delicate. In principle of course, the control law leading to the right orbit in minimum time exists, but it is quite heavy to obtain numerically and the computation is non-robust against many unmodelled phenomena. Considerable progress on the approximation of such a law by a feedback has been carried out using *ad hoc* Lyapunov functions. These approximate surprisingly well time-optimal trajectories. The easy implementation of such control laws makes them attractive as compared to genuine optimal control. Here the $n-1$ first integrals are an easy means to build control Lyapunov functions since any function of these first integrals can be made monotone decreasing by a suitable control. See [59] and the references therein.

5. Software

5.1. Tralics

Participant: José Grimm [correspondant].

The development of the LaTeX to XML translator, named Tralics, was continued (see section 6.1). Binary versions are available for Linux, Windows and MacOS X. Its web page is http://www-sop.inria.fr/apics/tralics. It is now licensed under the CeCILL license version two, see http://www.cecill.info. Latest release is version 2.13.7, dated 24-11-2010.

5.2. RARL2

Participants: Jean-Paul Marmorat, Martine Olivi [corresponding participant].

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see section 3.1.4) http://www-sop.inria.fr/apics/RARL2/rarl2-eng.html.

This software takes as input a stable transfer function of a discrete time system represented by

- either its internal realization,
- or its first N Fourier coefficients,
- or discretized values on the circle.

It computes a local best approximant which is *stable, of prescribed McMillan degree*, in the L^2 norm.

It is akin to the arl2 function of Endymion (see section 5.5) from which it differs mainly in the way systems are represented: a polynomial representation is used in Endymion, while RARL2 uses realizations, this being very interesting in certain cases. It is implemented in Matlab. This software handles *multi-variable* systems (with several inputs and several outputs), and uses a parametrization that has the following advantages

- it incorporates the stability requirement in a built-in manner,
- it allows the use of differential tools,
- it is well-conditioned, and computationally cheap.

An iterative research strategy on the degree of the local minima, similar in principle to that of arl2, increases the chance of obtaining the absolute minimum by generating, in a structured manner, several initial conditions.

RARL2 performs the rational approximation step in our applications to filter identification (section 4.2) as well as sources or cracks recovery (section 4.1). It was released to the universities of Delft, Maastricht, Cork and Brussels. The parametrization embodied in RARL2 was recently used for a multi-objective control synthesis problem provided by ESTEC-ESA, The Netherlands. An extension of the software to the case of triple poles approximants is now available. It gives nice results in the source recovery problem. It is used by FindSources3D (see 5.7).

5.3. RGC

Participants: Fabien Seyfert [corresponding participant], Jean-Paul Marmorat.

The identification of filters modeled by an electrical circuit that was developed by the team (see section 4.2) led us to compute the electrical parameters of the underlying filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called "the arrow form" [57] which is of particular interest since it is unique for a given transfer function and also easily computed. The computation of this realization is the first step of RGC. Subsequently, if the target realization is not in arrow form, one can nevertheless show that it can be deduced from the arrow-form by a complex- orthogonal change of basis. In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has a lot of local and global minima. In fact, there is not always uniqueness of the realization of the filter in the given geometry. Moreover, it is often interesting to know all the solutions of the problem, because the designer cannot be sure, in many cases, which one is being handled, and also because the assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software gives no guarantee to obtain a single realization that satisfies the prescribed constraints. The software Dedale-HF (see 5.6), which is the successor of RGC, solves in a guaranteed manner this constraint realization problem.

5.4. PRESTO-HF

Participant: Fabien Seyfert.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for microwave filters http://www-sop.inria.fr/apics/personnel/Fabien.Seyfert/Presto_web_page/presto_pres.html. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single shot:

- determination of delay components, that are caused by the access devices (automatic reference plane adjustment),
- automatic determination of an analytic completion, bounded in modulus for each channel,
- rational approximation of fixed McMillan degree,
- determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies either on hyperion (see 5.5) (Unix or Linux only) or RARL2 (platform independent), two rational approximation engines developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Thales Alenia Space in Toulouse and a license agreement has been recently negotiated with Thales airborne systems. XLim (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements are currently being considered with the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada).

5.5. Endymion

Participant: José Grimm.

The core of the *Endymion* system (a follow-up to hyperion) is formed by a library that handles numbers (short integers, arbitrary size rational numbers, floating point numbers, quadruple and octuple precision floating point numbers, arbitrary precision real numbers, complex numbers), polynomials, matrices, etc. Specific data structures for the rational approximation algorithm *arl2* and the bounded extremal problem *bep* are also available. One can mention for instance splines, Fourier series, Schur matrices, etc. These data structures are manipulated by dedicated algorithms (matrix inversion, roots of polynomials, a gradient-based algorithm for minimizing ψ, Newton method for finding a critical point of ψ, etc), and input-output functions that allow one to save data on disk, restore them, plot them, etc. The software is interactive: there is a symbolic interpreter based upon a Lisp interpreter.

The development of *Endymion*, http://www-sop.inria.fr/apics/endymion/index.html has come to an end. The software is still maintained and sources are available on the ftp server.

5.6. Dedale-HF

Participant: Fabien Seyfert.

Dedale-HF is a software meant to solve exhaustively the coupling matrix synthesis problem in reasonable time for the users of the filtering community. For a given coupling topology, the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of given filter characteristics (see section 6.7). Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements (see Figure 3).

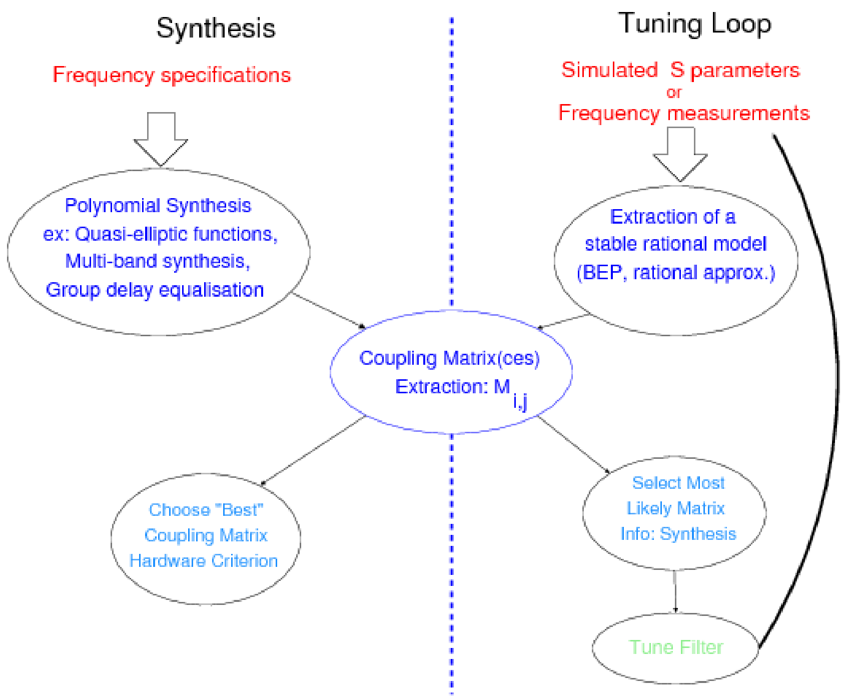

Figure 3. Overall view of the design and tuning process of a microwave filter

Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. problem of the user, i.e., the one corresponding to user-specified filter characteristics. The reference files are computed off line using Groebner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such a continuation technique combined with an efficient implementation of the integrator produces a drastic reduction of the computational time, say, by a factor of 20.

Access to the database and integrator code is done via the web on http://www-sop.inria.fr/apics/Dedale/WebPages. The software is free of charge for academic research purposes: a registration is however needed in order to access full functionality. Up to now 90 users have registered among the world (mainly: Europe, U.S.A, Canada and China) and 4000 reference files have been downloaded.

As mentioned in 6.7 an extension of this software that handles symmetrical networks is under construction.

5.7. FindSources3D

Participants: Maureen Clerc [EPI Athena], Juliette Leblond [corresponding participant], Jean-Paul Marmorat, Théo Papadopoulo [EPI Athena].

FindSources3D is a software dedicated to source recovery for the inverse EEG problem, in 3-layer spherical settings, from pointwise data (see http://www-sop.inria.fr/apics/FindSources3D/). Through the algorithm described in section 4.1, it makes use of RARL2 (section 5.2) for the rational approximation step in plane sections. The data transmission preliminary step ("cortical mapping") is solved using boundary element methods through the software OpenMEEG (its CorticalMapping features) developed by the Athena Team (see http://www-sop.inria.fr/athena/software/OpenMEEG/). A first release of FindSources3D is now available, which will be demonstrated and distributed within the medical teams we are in contact with (see figure 4).

6. New Results

6.1. Tralics: a LaTeX to XML Translator

Participant: José Grimm.

The major use of Tralics remains the production of the RaWeb (Scientific Annex to the Annual Activity Report of Inria) [24]. The software is described in [70], [73], [72], [71]. Other applications of Tralics consist in putting scientific papers on the Web; for instance Cedram (http://www.cedram.org, Centre de diffusion de revues académiques mathématiques), that publishes the *Journal de théorie des nombres de Bordeaux*, uses Tralics for the abstracts and plans to translate full papers; on the other hand the Connexions project of the Rice University is an environment for collaboratively developing, freely sharing, and rapidly publishing scholarly content on the Web, it uses it as LaTeX importer. Tralics has been debianized by Ross J. Reedstrom in 2008 for versions 2.11-2.12. Tralics is also used by Zentralblatt for converting comments, rewiews and abstracts. The Software has been presented at the DML2010 conference [32]; the slides of the presentation are available on the web.

6.2. Proving Bourbaki with Coq

Participant: José Grimm.

This new research theme started in 2009. Our objective is to use the proof assistant Coq in order to formally prove a great number of theorems in Algebra. We started with the first book (Theory of sets [61]) of the series "Elements of Mathematics". The first chapter describes Formal Mathematics, and we have shown that it is possible to interpret it in the Coq language, thanks to a bunch of axioms designed by Carlos Simpson (CNRS, Nice). The second chapter of Bourbaki covers the theory of sets proper. It defines ordered pairs, correspondences, union, intersection and product of a family of sets, as well as equivalence relations. The implementation is described in the technical report [39] and led to a publication in the Journal of Formalized Reasoning [25].

The third chapter of Bourbaki covers the theory of ordered sets, well-ordered sets, equipotent sets, cardinals, natural integers, and infinite sets; its implementation in Coq is described in [40]. The software has been rewritten, using the ssreflect library.

6.3. Inverse problems for elliptic operators

Participants: Laurent Baratchart, Aline Bonami [Univ. Orléans], Slah Chaabi, Maureen Clerc [EPI Odyssée], Yannick Fischer, Sandrine Grellier [Univ. Orléans], Doug Hardin [Vanderbilt Univ.], Juliette Leblond, Jean-Paul Marmorat, Ana-Maria Nicu, Théo Papadopoulo [EPI Odyssée], Jonathan Partington, Stéphane Rigat [Univ. Aix-Marseille I], Emmanuel Russ [Univ. Aix-Marseille III], Edward Saff, Meriem Zghal.

6.3.1. Boundary value problems for Laplace equation in 3-D

Solving overdetermined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to treat incomplete experimental data is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG since the latter involves propagating the initial conditions from the boundary to the center of the domain where the singularities (i.e., the sources) are sought. Here, the domain is typically made of several homogeneous layers of different conductivities.

Such problems offer an opportunity to state and solve extremal problems for harmonic fields for which an analog of the Toeplitz operator approach to bounded extremal problems [49] has been obtained [15]. Still, a best approximation on the subset of a general vector field generated by a harmonic gradient under a L^2 norm constraint on the complementary subset can be computed by an inverse spectral equation for some Toeplitz operator. Constructive and numerical aspects of the procedure (harmonic 3-D projection, Kelvin and Riesz transformation, spherical harmonics) and encouraging results have been obtained on numerically simulated data [15]. Issues of robust interpolation on the sphere from incomplete pointwise data are also under study (splines, spherical harmonics, spherical wavelets, spherical Laplace operator, ...), in order to improve numerical accuracy of our reconstruction schemes.

The analogous problem in L^p, $p \neq 2$, is quite interesting but considerably more difficult. A collaborative work is going on, in the framework of the ANR project AHPI, aiming mainly at the case $p = \infty$. We investigate the connections between the BMO[2] distance of a bounded vector field on the sphere to a BMO harmonic gradient and the spectral properties of a "big" Hankel-like operator, acting on L^2 harmonic gradients and valued in its orthogonal space embedded in L^2 vector fields on the sphere whose tangent component is a gradient. This issue is also considered in L^p, $1 < p < \infty$, where it leads to analyze particular solutions to the the p-Laplacian on the sphere.

6.3.2. Sources recovery in 3-D domains, application to MEEG and geophysics

The problem of sources recovery can be handled in 3-D balls by using best rational approximation on 2-D cross sections (disks) from traces of the boundary data on the corresponding circles (see section 4.1).

The team started to consider more realistic geometries for the 3-D domain under consideration. A possibility is to parametrize it in such a way that its planar cross-sections are quadrature domains or R-domains. In this framework, best rational approximation can still be performed in order to recover the singularities of solutions to Laplace equations, but complexity issues are delicate. The preliminary case of an ellipsoid, which requires the preliminary computation of an explicit basis of ellipsoidal harmonics, has been studied in [77] and is one of the topics of [13].

In 3-D, functional or clinical active regions in the cortex are often represented by pointwise sources that have to be localized from measurements on the scalp of a potential satisfying a Laplace equation (EEG, electroencephalography). A breakthrough was made which makes it possible now to proceed via best rational approximation on a sequence of 2-D disks along the inner sphere [5].

A dedicated numerical software "FindSources3D" (see section 5.7) has been developed, in collaboration with the team Athena.

[2] bounded mean oscillation

Further, it appears that in the rational approximation step of these schemes, *multiple* poles possess a nice behaviour with respect to the branched singularities (see figure 4). This is due to the very basic physical assumptions on the model (for EEG data, one should consider *triple* poles). Though numerically observed, there is no mathematical justification so far why these multiple poles have such strong accumulation properties, which remains an intriguing observation. This is the topic of [64].

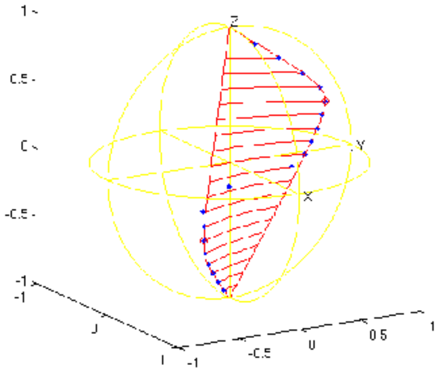

Figure 4. Localization of 2 sources (red circles) by a single triple pole in plane sections (blue points).

Also, magnetic data from MEG (magneto-encephalography) will soon become available, which should enhance sources recovery.

This approach also appears to be interesting for geophysical issues, concerning the discretization of the gravitational potential by means of pointwise masses. This is another topic of A.-M. Nicu's PhD thesis and of our present collaboration with LAMSIN-ENIT, hence the reason why she also made a long working stay there (Univ. El Manar, Tunis, Tunisia, September) and with IGN (Paris, LAREG, geodesy).

Magnetic sources localization from observations of the field away from the support of the magnetization is a topic under investigation in a joint effort with the Math. department of Vanderbilt University and the Earth Sciences department of MIT. The goal is to recover the magnetic properties of rock samples (meteorites) from fine measurements very close to the sample that can nowadays be obtained using SQUIDs (supraconducting coil device).

We completed the analysis of the kernel of the magnetization operator for 3-D samples, which is the Riesz potential of the divergence of the magnetization assumed to be of bounded variation. It can be described in terms of measures whose balayage on the boundary of the sample vanishes, but this is not so effective, computationally.

The case of a thin slab (the magnetization is then modelled as a vector field on a portion of the plane) has proved more amenable. In the uni and bi-directional cases, using Hodge decomposition, one can show that the kernel contains only divergence free tangential vector field that are constant on lines, showing in particular that it reduces to zero for compactly supported magnetizations (which is always the case). A paper is being written on these results. Meanwhile, the severe ill-posedness of the reconstruction challenges discrete Fourier methods, one of the main problems being the truncation of the observations outside the range of the SQUID measurements. The next step will be to develop the extrapolation techniques initiated by the project team, using bounded extremal problems, in an attempt to overcome this issue.

6.3.3. Boundary value problems for 2-D conductivity equations, application to plasma control

In collaboration with the CMI-LATP (University Marseille I) and in the framework of the ANR AHPI, the team considers 2-D diffusion processes with variable conductivity. In particular its complexified version, the so-called *real Beltrami equation*, was investigated. In the case of a smooth domain, and for Lipschitz conductivity, we analyzed the Dirichlet problem for solutions in Sobolev and then in Hardy classes [17].

Their traces merely lie in L^p $(1 < p < \infty)$ of the boundary, a space which is suitable for identification from pointwise measurements. Again these traces turn out to be dense on strict subsets of the boundary. This allows us to state Cauchy problems as bounded extremal issues in L^p classes of generalized analytic functions, in a reminiscent manner of what was done for analytic functions as discussed in section 3.1.1.

This year we generalized the construction to finitely connected Dini-smooth domains and $W^{1,q}$-smooth conductivities, with $q \leq 2$. The case of an annular geometry is the relevant one for the application to plasma shaping mentioned below. An article is currently being written on these topics.

The application that initially motivated this work came from free boundary problems in plasma confinement (in tokamaks) for thermonuclear fusion. This work was initiated in collaboration with the Laboratoire J. Dieudonné (University of Nice) and is now the topic of a collaboration with two teams of physicists from the CEA-IRFM (Cadarache).

In the transversal section of a tokamak (which is a disk if the vessel is idealized into a torus), the so-called poloidal flux is subject to some conductivity outside the plasma volume for some simple explicit smooth conductivity function, while the boundary of the plasma (in the Tore Supra Tokamak) is a level line of this flux [58]. Related magnetic measurements are available on the chamber, which furnish incomplete boundary data from which one wants to recover the inner (plasma) boundary. This free boundary problem (of Bernoulli type) can be handled through the solutions of a family of bounded extremal problems in generalized Hardy classes of solutions to real Beltrami equations, in the annular framework. Such approximation problems also allow us to approach a somewhat dual extrapolation issue, raised by colleagues from the CEA for the purpose of numerical simulation. It consists in recovering magnetic quantities on the outer boundary (the chamber) from an initial guess of what the inner boundary (plasma) is.

In the particular case at hand, the conductivity is $1/x$ and the domain is an annulus embedded in the right half-plane. We obtained a basis of solutions (exponentials times Legendre functions) upon separating variables in toroidal coordinates. This may be viewed as a generalization to the annulus of the Bessel type expansions derived in [22], [23] for simply connected geometries. This provides a computational setting to solve the extremal problems mentioned before, and is the topic of the PhD thesis of Y. Fischer.

On the half-plane, the conductivity $1/x$ is severely unbounded but the analysis of this test case is quite important for the convergence of extrapolation algorithms to recover magnetic quantities on the chamber. Additive decompositions into Hardy solutions inside the outer boundary and outside the inner boundary, with controlled vanishing on the imaginary axis, have been obtained as part of the PhD work of S. Chaabi.

In the most recent tokamaks, like Jet or ITER, an interesting feature of the level curves of the poloidal flux is the occurrence of a cusp (a saddle point of the poloidal flux, called an X point), and it is desirable to shape the plasma according to a level line passing through this X point for physical reasons related to the efficiency of the energy transfer. This issue is still untouched, but should be the topic of future studies, once the present approach will have been validated numerically.

6.4. Interpolation and parametrizations of lossless functions

Participants: Martine Olivi, Bernard Hanzon, Ralf Peeters [Univ. Maastricht].

Lossless systems and their transfer functions play a central role in system theory mainly, but not only, due to the Douglas-Shapiro-Shields factorization. Lossless matrix-valued functions generalize, in some sense, the notion of denominator to the matrix case. As such they are involved in many representations of stable systems. They are also present in many applications, as the scattering matrix of a resonant filter or the polyphase matrix of a filter bank, for example. Their parametrization is an important issue for many purposes going from optimization, model reduction to physical parameters recovery.

In [10], by making appropriate choices in the Schur algorithm, construction of balanced canonical forms is achieved in a recursive way with unitary matrix multiplications. Each step of the recursion involves an interpolation condition at a point located within the analyticity domain. Thereby, we get parametrizations which combine the computational interest of canonical forms and the conceptual interest of Schur analysis. However, for several reasons, it may prove helpful to allow for interpolation points on the boundary of the analyticity domain (the circle in discrete-time and the imaginary axis in continuous-time). In [81], the tridiagonal canonical form of Ober in continuous-time was obtained from a recursion involving interpolation conditions *at infinity*. Vanishing moments, diagonal Markov parameters, can be interpreted in term of boundary interpolation conditions.

This year, we extended the work of [10] on *discrete-time lossless systems* to allow for interpolation conditions *on the unit circle*. We investigate the possibility to parametrize orthogonal wavelets with vanishing moments using these results. A vanishing moment condition can be expressed as a boundary interpolation condition for the lossless polyphase filter. We got explicit parametrizations of 2×2 polyphase matrices of arbitrary order n with (up to) 3 vanishing moments built in, in terms of angular derivative (positive) parameters [34].

6.5. Orthogonal rational functions and non-stationary stochastic processes

Participants: Laurent Baratchart, Stanislas Kupin [Univ. Bordeaux 1].

The theory of orthogonal polynomials on the unit circle is a most classical piece of analysis which is still the object of intensive studies. The asymptotic behaviour of orthogonal polynomials is of special interest for many issues pertaining to approximation theory and the spectral theory of differential operators. Its connection with prediction theory of stationary stochastic processes has long been known [69]. Namely, the n-th orthonormal polynomial with respect to the spectral measure of the process yields the optimal regression coefficients of a linear one-step ahead predictor from the $n-1$-st last values, in the sense of minimum variance of the error. Likewise, the (inverse of) the dominant coefficient of the polynomial gives the prediction error. In particular, asymptotics for the dominant coefficient determine the asymptically optimal prediction error from the past as time goes large.

As compared to orthogonal polynomials, orthogonal rational functions have not been much considered up to now. They were apparently introduced by Dzrbasjan but the first systematic exposition seems to be the monograph by Bultheel et al. [63] where the emphasis is more on the algebraic side of the theory. In fact, the asymptotic analysis of orthogonal rational functions is still in its infancy.

Last year, we developed an analog of the Kolmogorov-Krein-Szegö theorem [16] for orthogonal rational functions which is first of its kind in that it allows for the poles of these functions to approach the unit circle, generalizing previously known results for compactly supported singular set. Dwelling on this asymptotic analysis of orthogonal rational functions, we developed this year a prediction theory for certain, possibly *nonstationary* stochastic processes that we call *Blaschke varying* processes[3]. These are characterized by a spectral calculus where time shift corresponds to multiplication by an elementary Blaschke product (that may depend on the time instant considered). This class of processes contains the familiar Gaussian stationary processes, but it contains many more that exhibit a much more varied behaviour. For instance, the process may be asymptotically deterministic along certain subsequences and nondeterministic along others. The optimal predictor is constructed from the spectral measure *via* orthogonal rational functions, and its asymptotic behaviour is characterized by the above-mentioned generalization of the Kolmogorov-Krein-Szegö theorem. The main open issues facing this theory now are the characterization of Blaschke processes from their covariance sequences and the development of suitable identification tools. The latter question is tantamount to construct optimal Schur rational approximants to a given Szeő function.

6.6. Rational and meromorphic approximation

Participants: Laurent Baratchart, Herbert Stahl [TFH Berlin], Maxim Yattselev.

[3] http://arxiv.org/abs/1007.1363

Having demonstrated last year, under mild smoothness assumptions, the possibility of convergent rational interpolation to Cauchy integrals of complex measures on analytic Jordan arcs and their strong asymptotics [18], we started investigating the case of Cauchy integrals on so-called symmetric contours for the logarithmic potential. These correspond to functions with more than two branched singularities, like those arising in the slicing method for source recovery in a sphere when there is more than one source (see section 6.3.2). Recently we obtained weak asymptotics in this case, through an existence result and a characterization of compact of minimum capacity outside of which the function is single-valued, teaming up with results from [68]. A manuscript has just been completed on the subject.

To study strong asymptotics, at present we limit ourselves to a threefold geometry, and to the case of Padé approximants (interpolation at a single point with high order). The result is that uniform convergence can only take place if the weights of the branches of the threefold with respect to the equilibrium distribution are rationally dependent. If they are algebraically dependent, a spurious pole clusters to certain curves within the domain of analyticity, and if they are algebraically independent, exactly one pole exhibits chaotic behaviour in the complex plane. This generalizes results of Suetin on Cauchy integrals on disconnected pieces of a smooth contour. This investigation will continue.

6.7. Circuit realisations of filter responses: determination of canonical forms and exhaustive computations of constrained realisations

Participants: Smain Amari [Royal Military College, Kingston, Canada], Jean Charles Faugère [EPI SALSA, INRIA Rocquencourt], Giuseppe Macchiarella [Politecnico di Milano, Milan, Italy], Uwe Rosenberg [Design and Project Engineering, Osterholz-Scharmbeck, Germany], Matteo Oldoni [Politecnico di Milano, Milan, Italy], Fabien Seyfert.

We continued our work on the circuit realisations of filters' responses with mixed type (inductive or capacitive) coupling elements and constrained topologies. Our first paper describing the algebraic theory governing these equation was published [14]. We focused on some practical application of it in [36] where compact filter using zero generating irises were studied and designed. We now focus on the use of resonating couplings in the design of asymmetric filter's characteristics without the use cross-coupling in order to simplify their practical implementation. In parallel efforts are being spent to improve the synthesis method to higher order filters, having in mind some application to diplexer's filter with high number of symmetrically placed transmission zeros.

In another area of circuit realisation, we also started some work on lossy filter synthesis [33].

6.8. Synthesis of compact multiplexers and the polynomial structure of reciprocal lossless scattering matrices

Participants: David Avanessoff, Martine Olivi, Fabien Seyfert, Stéphane Bila [Xlim], Hussein Ezzedin [Xlim], Giuseppe Macchiarella [Politecnico di Milano, Milan, Italy], Matteo Oldoni [Politecnico di Milano, Milan, Italy].

The objective of our work is the derivation of efficient algorithms for the synthesis of microwave multiplexers. Although the functioning of such devices relies primary on Maxwell equations, classical modal analysis techniques yield equivalent reciprocal electrical circuits that exhibit good approximation properties in a restricted frequency range. These models present an interesting trade off between flexibility and accuracy that justifies their central position among the techniques used for the design of microwave devices. In particular, the design of microwave filters relies heavily on the polynomial structure of 2×2 reciprocal lossless squattering matrices. This structure allows to cast the design, under modulus constraints, of the response of a filter to a quasi-convex optimisation Zolatariov problem.

The polynomial structure of 3×3 reciprocal lossless squattering matrices turn out to be more involved and mainly unknown. An important step was performed this year in that direction using the results obtained in [30] on reciprocal Darlington synthesis. We derived a method [28] to count and compute all possible 3×3 reciprocal lossless extensions of a rational Schur function p/q (a reflection of the scattering matrix). From a practical synthesis point of view, the extension process that starts with the numerators p_1 and p_2 of the transmission elements is more relevant. Divisibility conditions have been derived which can be interpreted as interpolation conditions connecting p/q on one hand and p_1 and p_2 on the other hand. The Shur function p/q can thus be computed from p_1 and p_2 running a Schur type algorithm. The study of particular forms of the polynomial model in connexion with some special circuit topologies used for the implementation of the diplexer are also currently under investigation. Of particular interest is the case where the underlying circuit is composed of two filters connected at a common port. Based on some of the characteristics the rational response posseses in this case, the design and realization of a compact diplexer was performed in the context of the ANR Filipix [31].

Some progress was made on the de-embedding of filter responses when starting from the external measurements of a diplexer. The problem states as followed. Let S be the measured scattering matrix of a diplexer composed of a junction with response T and two filtering devices with response A and B as plotted on figure 5. The de-embedding question is the following: given S and T, is it possible to derive A and B. It was shown that unless some additional hypothesis are made on the responses A and B the problem has no unique solution. More precisely, at every frequency point the solution set is a complex algebraic variety of dimension one which, for example, can be parametrised by the transmission term of one the two filtering devices. Under some losslessness assumption on all system components the dimension of the solution set drops to a single real one and the phase of one transmission term (of A or B) can be taken as a free parameter. Eventually the problem is being studied under some hypothesis about the rationality of the device's response or when junctions with more than one possible state are used (resulting in several measurements of the diplexer). This work is pursued in collaboration with Thales Alenia Space and the Polytecnico di Milano as well as with the support of the ANR Filipix.

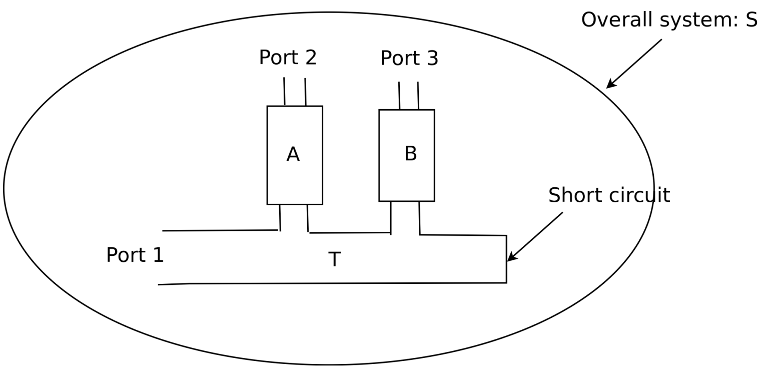

Figure 5. Diplexer made of a junction T and two filtering devices A and B

6.9. Simple nonlinear models for river flows

Participants: Xavier Litrico [CEMAGREF, Montpellier], Jean-Baptiste Pomet.

There has been a recurrent collaboration with X. Litrico on simple nonlinear models obtained from a family of linear systems with delay. This comes from situations where a PDE is linearized in the neighborhood of a steady-state behavior and then approached by a linear system with delay. In the paper [27], we develop this method, aiming at control or forecast of river flow.

6.10. Optimal transport

Participants: Ahed Hindawi, Jean-Baptiste Pomet, Ludovic Rifford.

In [26], we give results on existence and continuity of the optimal transport map between absolutely continuous measures for a point-to-point cost coming from controllable linear-quadratic optimal control. This is the simplest case of a cost coming from a system with drift; it bears some interest because the cost can be explicitly computed. A. Hindawi's PhD also aims at more general systems.

Publications [21] and [19] are related to optimal transport in that they study geometric conditions in Riemannian geometry that are either sufficient or necessary for continuity of transport map. [21] is a deep study of the Ma-Trudinger-Wang Tensor while [19] studies convexity of the injectivity domain; both are restricted to surfaces.

7. Contracts and Grants with Industry

7.1. Contracts CNES-IRCOM-INRIA

Contract (reference Inria: 2470, CNES: 60465/00) involving CNES, XLim and Inria, whose objective is to work out a software package for identification and design of microwave devices. The work at Inria concerns the design of multiband filters with constraints on the group delay. The problem is to control the logarithmic derivative of the modulus of a rational function, while meeting specifications on its modulus.

Contract (reference CNES: RS10/TG-0001-019) involving CNES, University of Bilbao and Inria whose objective is to set up a methodology for testing the stability of amplifying devices. The work at Inria concerns the design of frequency optimization techniques to identify the linearized response and analyze the linear periodic components.

8. Other Grants and Activities

8.1. Regional Initiatives

Apics is linked with the CEA-IRFM (Cadarache), through a grant with the **Région PACA**, for the thesis of Y. Fischer.

Apics is part of the regional working group SBPI (Signal, Noise, Inverse Problems), with teams from Observatoire de la Côte d'Azur and Géoazur (CNRS) http://www-sop.inria.fr/apics/sbpi.

8.2. National Initiatives

8.2.1. ANR project "AHPI"

AHPI (Analyse Harmonique et Problèmes Inverses), is a "Projet blanc" in Mathematics involving Inria-Sophia (L. Baratchart coordinator), the Université de Provence (LATP, Aix-Marseille), the Université Bordeaux I (LATN), the Université d'Orléans (MAPMO), Inria-Bordeaux and the Université de Pau (Magique 3D). It aims at developing Harmonic Analysis techniques to approach inverse problems in seismology, electro-encephalography, tomography and nondestructive control.

8.2.2. ANR project "Filipix"

Filipix (FILtering for Innovative Payload with Improved fleXibility) is a "Projet Thématique en Télécommunications", involving Inria-Sophia (Apics), XLim, Thales Alenia Space (Centre de Toulouse, coordinator).

8.3. European Initiatives

APICS is part of the European Research Network on System Identification (ERNSI)

8.4. International Initiatives

NSF EMS21 RTG is a students exchange program with Vanderbilt University (Nashville, USA).

NSF CMG collaborative research grant DMS/0934630, "Imaging magnetization distributions in geological samples", with Vanderbilt University and the MIT (USA).

Cyprus NF grant "Orthogonal polynomials in the complex plane: distribution of zeros, strong asymptotics and shape reconstruction."

A program **Inria-Tunisian Universities** (STIC) links Apics to the LAMSIN-ENIT (Tunis).

8.5. Exterior research visitors

- Elodie Pozzi (Univ. Lyon I).
- Yannick Privat (CNRS, ENS Cachan, antenne Bretagne).
- Doug Hardin (Vanderbilt University).
- Vladimir Peller (Michigan State University).
- Nikos Stylianopoulos (University of Cyprus).
- Maxim Yattselev (University of Oregon at Eugene).
- Andrea Gombani (University of Padova).
- Smain Amari (Royal Military College of Canada)

8.6. Community service

L. Baratchart is Inria's representative at the « conseil scientifique » of the Univ. Provence (Aix-Marseille). He was a member of the "Comité de sélection" of the Univ. of Bordeaux I (section 25).

J. Grimm is a representative at the « comité de centre » (Research Center INRIA-Sophia).

J. Leblond is a member of the « Commission d'Évaluation » (CE) of INRIA[4]. She is a member of the « Commission d'Animation Scientifique » (CAS) and is associated to the « pôle REI » of the Research Center.

M. Olivi is a member of the CSD (Comité de Suivi Doctoral) of the Research Center.

J.-B. Pomet is a representative at the « comité technique paritaire » (CTP) of INRIA.

9. Dissemination

9.1. Conferences and workshops

L. Baratchart was a semi-plenary speaker at the Conference "Mathematical theory of Networks and Systems", Budapest (Hungary). He was an invited speaker at the conference "Finite and infinite-dimensional complex analysis and applications", Macau (China). He gave a communication at the Workshop "Boundary value problems and related questions" Beijing (China). He was a "colloquium speaker" at the Morningside Institute of the Chinese Academy of Sciences (Beijing). He was an invited participant to the Workshop "New perspectives on univariate and multivariate orthogonal polynomials", BIRS, Banff (Canada). He gave a seminar at the Technische Universität (Berlin).

Y. Fischer gave a presentation at the meeting of the ANR project AHPI, Bordeaux (Jan.), and at the seminar of the team APICS (Oct.). He presented a poster at the conference PICOF 2010, "Inverse Problems, Control and Shape Optimization", Carthagène (Spain, Apr.).

[4]Thus, of several recruitment or hiring committees of Inria researchers, or evaluation seminars of Inria teams, and participates to working groups.

J. Leblond was an invited speaker at the Workshop in “Operator Theory and Complex Analysis”, Lyon (Nov.), She was invited to give a talk at the working group of the team Défi, INRIA-Saclay and CMAP-Poytechnique (June) and at the seminar of the team Analyse et Géométrie, LATP-CMI, Univ Aix-Marseille I (Dec.), and gave a communication at the meeting of the ANR project AHPI, INRIA-Sophia (Nov.). She was a member of the “Conseil Scientifique” of the X^{th} Forum for Young Mathematicians, CIRM, Luminy (Nov.).

J. Grimm gave a talk at the DML 2010 conference. He presented [32].

A.-M. Nicu gave a communication at the meeting of the ANR project AHPI, INRIA-Sophia (Nov.) and at the X^{th} Forum for Young Mathematicians, CIRM, Luminy (Nov.). She presented a poster at the conference PICOF 2010, Carthagène (Spain, Apr.).

M. Olivi co-organized with A. Gombani two sessions on "Interpolation and approximation" and gave a communication at the Conference “Mathematical theory of Networks and Systems”, Budapest (Hungary). She gave a talk at the 2010 ERNSI Meeting in Cambridge (UK).

F. Seyfert co-organized with S.Bila (Xlim) a workshop on “Advanced topics in design and realization of microwave filters" at the European Microwave Conference in Paris (Sept.).

A meeting of the ANR AHPI was organized at INRIA (17-19 Nov.).

9.2. Animation of the scientific community

L. Baratchart is a member of the editorial board of *Computational Methods and Function Theory* and *Complex Analysis and Operator Theory*.

9.3. The Apics Seminar

The following scientists gave a talk at the team’s seminar:

- *Jana Nemcova*, Rational Systems in Control and System Theory.
- *Pierre Vacher* (ONERA, Toulouse), Recherche de paramétrisations adaptées à l’identification de systèmes linéaires MIMO.
- *Nicolas Brisebarre* (CNRS, LIP, Arénaire, ENS Lyon) and *Guillaume Hanrot* (ENS Lyon, LIP, Arénaire), Approximants polynomiaux efficaces en machine.
- *Sylvain Chevillard* (INRIA Nancy - Équipe-projet Caramel), Outils pour l’évaluation efficace de fonctions numériques.
- *Nataliya Shcherbakova* (ENSEEIHT, Toulouse), Optimal control of 2-level dissipative quantum control systems.
- *Laurent Baratchart*, Problèmes inverses de magnétisation et décomposition de Hodge pour les champs à variation bornée.
- *Lydiya Yuschenko* (Centre de Physique Théorique, Luminy), Approximants de Padé à N points complexes pour les fonctions de Stieltjes.
- *Bernard Bonnard* (Institut de Mathématiques de Bourgogne), Applications du contrôle des spins 1/2 dissipatifs en imagerie médicale.
- *Elodie Pozzi* (Institut Camille Jordan, Lyon), Universalité du shift et des opérateurs de composition.
- *Andrea Gombani* (ISIB-Information, Padova, Italy), On the partial realization problem.
- *Guillaume Charpiat* (EPI PULSAR), Estimation de métriques convenant à une variété empirique de formes.
- *Maxym Derevyagin* (Technische Universität Berlin), On the convergence of Pade approximants to rational perturbations of Markov functions.
- *Vladimir Peller* (Michigan State University), Perturbations d’opérateurs normaux.
- *Yannick Privat* (ENS Cachan), Comment déterminer le domaine de contrôle optimal d’une membrane en vue de sa stabilisation ?
- *Yannick Fischer*, Problèmes extrémaux bornés pour une équation elliptique et applications à la résolution d’un problème inverse pour le tokamak Tore Supra.

9.4. Teaching

9.4.1. Courses

- Martine. Olivi (with Maureen. Clerc), Mathématiques pour l'ingénieur (Fourier analysis and integration), section Mathématiques Appliquées et Modélisation, 3rd year, École Polytechnique Univ. Nice-Sophia Antipolis (EPU).

9.4.2. Ph.D. Students

- Slah Chaabi, « Problèmes extrémaux pour l'équation de Beltrami réelle 2-dimensionnelle et application à la détermination de frontières libres », co-advised, Univ. Aix-Marseille I.
- Yannick Fischer, « Problèmes inverses pour l'équation de Beltrami et extrapolation de quantités magnétiques dans un Tokamak », Univ. Nice-Sophia Antipolis.
- Ahed Hindawi « Transport optimal en contrôle », Univ. Nice-Sophia Antipolis.
- Ana-Maria Nicu, « Inverse potential problems in MEEG and in geophysics », Univ. Nice-Sophia Antipolis.
- Meriem Zghal, [13], co-advised, Univ. Tunis El Manar (Tunisia), until October.

9.4.3. HDR

- Martine Olivi, «Parametrization of rational lossless matrices with application to linear system theory», defended in october [12].

9.4.4. Committees

- Jean-Baptiste Pomet was a member of the PhD defense committees of Li Shunjie (Université de Rouen), Gauthier Picot and Gabriel Janin (Université de Bourgogne, Dijon).

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Major publications by the team in recent years

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Publications of the year

Doctoral Dissertations and Habilitation Theses

[12] M. OLIVI. *Parametrization of rational lossless matrices with application to linear system theory*, Université de Nice - Sophia Antipolis, October 2010, Habilitation à Diriger des Recherches (HDR).

[13] M. ZGHAL. *Problèmes inverses pour l'équation de Laplace en dimension 3. Application à l'éléctroencéphalographie*, Univ. Tunis El Manar (Tunisie), October 2010.

Articles in International Peer-Reviewed Journal

[14] S. AMARI, F. SEYFERT, M. BEKHEIT. *Theory of Coupled Resonator Microwave Bandpass Filters of Arbitrary Bandwidth*, in "Microwave Theory and Techniques, IEEE Transactions on", August 2010, vol. 58, n° 8, p. 2188 -2203.

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