



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team disco

*Dynamical Interconnected Systems in
COMplex Environments*

Saclay - Île-de-France

Theme : Modeling, Optimization, and Control of Dynamic Systems

Activity
R *eport*

2010

Table of contents

1. Team	1
2. Overall Objectives	1
3. Scientific Foundations	2
3.1. Modeling of complex environment	2
3.2. Analysis of interconnected systems	2
3.3. Stabilization of interconnected systems	2
3.4. Synthesis of reduced complexity controllers	3
4. Application Domains	4
4.1. Modeling and analysis of Acute Myeoloid Leukemia	4
4.2. Control of continuous bioreactors	4
4.3. Control of a model of human heart	4
5. Software	4
5.1. YALTA	4
5.2. OreModules	4
5.3. Stafford	5
5.4. QuillenSuslin	5
5.5. OreMorphisms	5
5.6. PurityFiltration	6
5.7. AbelianSystems	6
6. New Results	6
6.1. Analysis of infinite-dimensional and multidimensional systems	6
6.1.1. H_∞ - stability of (possibly fractional) neutral delay systems	6
6.1.2. Numerical methods for (possibly fractional) time-delay systems	6
6.1.3. Differential flatness of analytic linear ordinary differential systems	7
6.1.4. Serre's reduction of multidimensional systems	7
6.1.5. Purity filtration of multidimensional systems	7
6.1.6. Extendability of multidimensional linear systems	8
6.1.7. Symmetries and parametrizations of multidimensional systems	8
6.1.8. Factorization and decomposition problems for 2D Stokes and Oseen equations	8
6.1.9. New domain decomposition methods for linear PD systems	8
6.2. New techniques of observation and control	8
6.2.1. Interval observers	8
6.2.2. Finding positive solutions of systems with delay	9
6.2.3. Lyapunov-based results	9
6.2.4. Predictive control	10
6.2.5. Comparison systems	10
6.2.6. Discrete Markov jump systems	10
6.2.7. Stochastic optimization in energy production systems	10
6.2.8. Use of metaheuristic optimization methods for automatic control	11
6.3. Biological systems	11
6.3.1. Control of continuous bioreactors	11
6.3.2. Study of a model of anaerobic digestion process	12
6.3.3. Control of a model of human heart	12
6.3.4. Modelling of hematopoiesis with application to acute myeloid leukemia	12
7. Other Grants and Activities	12
7.1. Regional Initiatives	12
7.2. National Initiatives	12
7.3. European Initiatives	13
7.4. International Initiatives	13

8. Dissemination	13
8.1. Animation of the scientific community	13
8.1.1. Committee participations	13
8.1.2. Editorial committees	14
8.1.3. Organizations of conferences and school	14
8.1.4. Ph.D. thesis committees	14
8.1.5. Participation at conferences and invitations	15
8.2. Teaching	15
8.2.1. Teaching at Universities	15
8.2.2. Other teaching	15
8.2.3. Ph.D. theses in progress	15
8.2.4. Defended Ph.D theses	15
8.2.5. Internships	16
9. Bibliography	16

DISCO is a joint team with Laboratoire des Signaux Systèmes (L2S) U.M.R. C.N.R.S. 8506, and Supélec, which has been created since January 2010.

1. Team

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2. Overall Objectives

2.1. Objectives

The goal of the project is to better understand and well formalize the effects of complex environments on the dynamics of the interconnections, as well as to develop new methods and techniques for the analysis and control of such systems.

It is well-known that the interconnection of dynamic systems has for consequence an increased complexity of the behavior of the “total” system both in the presence and absence of feedback control loops.

In a simplified way, as the concept of dynamics is well-understood, the interconnections can be seen as associations (by connections of materials or information flows) of distinct systems to ensure a pooling of the resources with the aim of obtaining a better operation with the constraint of continuity of the service in the event of a fault. In this context, the environment can be seen as a collection of elements, structures or systems, natural or artificial constituting the neighborhood of a given system. The development of interactive games through communication networks, control from distance (e.g. remote surgical operations) or in hostile environment (e.g. robots, drones), as well as the current trend of large scale integration of distribution (and/or transport and/or decision) and open information systems with systems of production, lead to new modeling schemes in problems where the dynamics of the environment have to be taken into account.

In order to tackle the control problems arising in the above examples, the team investigates new theoretical methods, develop new algorithms and implementations dedicated to these techniques.

3. Scientific Foundations

3.1. Modeling of complex environment

We want to model phenomena such as a temporary loss of connection (e.g. synchronisation of the movements through haptic interfaces), a nonhomogeneous environment (e.g. case of cryogenic systems) or the presence of the human factor in the control loop (e.g. grid systems) but also problems involved with technological constraints (e.g. range of the sensors). The mathematical models concerned include integro-differential, partial differential equations, algebraic inequalities with the presence of several time scales, whose variables and/or parameters must satisfy certain constraints (for instance, positivity).

3.2. Analysis of interconnected systems

- Algebraic analysis of linear systems

Study of the structural properties of linear differential time-delay systems and linear infinite-dimensional systems (e.g. invariants, controllability, observability, flatness, reductions, decomposition, decoupling, equivalences) by means of constructive algebra, module theory, homological algebra, algebraic analysis and symbolic computation [8], [9], [14], [72], [79], [82].

- Robust stability of linear systems

Within an interconnection context, lots of phenomena are modelled directly or after an approximation by delay systems. These systems might have fixed delays, time-varying delays, distributed delays...

For various infinite-dimensional systems, particularly delay and fractional systems, input-output and time-domain methods are jointly developed in the team to characterize stability. This research is developed at four levels: analytic approaches (H_∞ -stability, BIBO-stability, robust stability, robustness metrics) [1], [2], [5], [6], symbolic computation approaches (SOS methods are used for determining easy-to-check conditions which guarantee that the poles of a given linear system are not in the closed right half-plane, certified CAD techniques), numerical approaches (root-loci, continuation methods) and by means of softwares developed in the team [5], [6].

- Robustness/fragility of biological systems

Deterministic biological models describing, for instance, species interactions, are frequently composed of equations with important disturbances and poorly known parameters. To evaluate the impact of the uncertainties, we use the techniques of designing of global strict Lyapunov functions or functional developed in the team.

However, for other biological systems, the notion of robustness may be different and this question is still in its infancy (see, e.g. [90]). Unlike engineering problems where a major issue is to maintain stability in the presence of disturbances, a main issue here is to maintain the system response in the presence of disturbances. For instance, a biological network is required to keep its functioning in case of a failure of one of the nodes in the network. The team, which has a strong expertise in robustness for engineering problems, aims at contributing at the development of new robustness metrics in this biological context.

3.3. Stabilization of interconnected systems

- Linear systems: Analytic and algebraic approaches are considered for infinite-dimensional linear systems studied within the input-output framework.

In the recent years, the Youla-Kučera parametrization (which gives the set of all stabilizing controllers of a system in terms of its coprime factorizations) has been the cornerstone of the success of the H_∞ -control since this parametrization allows one to rewrite the problem of finding the optimal stabilizing controllers for a certain norm such as H_∞ or H_2 as affine, and thus, convex problem.

A central issue studied in the team is the computation of such factorizations for a given infinite-dimensional linear system as well as establishing the links between stabilizability of a system for a certain norm and the existence of coprime factorizations for this system. These questions are fundamental for robust stabilization problems [1], [2], [8], [9].

We also consider simultaneous stabilization since it plays an important role in the study of reliable stabilization, i.e. in the design of controllers which stabilize a finite family of plants describing a system during normal operating conditions and various failed modes (e.g. loss of sensors or actuators, changes in operating points) [9]. Moreover, we investigate strongly stabilizable systems [9], namely systems which can be stabilized by stable controllers, since they have a good ability to track reference inputs and, in practice, engineers are reluctant to use unstable controllers especially when the system is stable.

- Nonlinear systems

The project aims at developing robust stabilization theory and methods for important classes of nonlinear systems that ensure good controller performance under uncertainty and time delays. The main techniques include techniques called backstepping and forwarding, constructions of strict Lyapunov functions through so-called "strictification" approaches [3] and construction of Lyapunov-Krasovskii functionals [4], [5], [6].

- Predictive control

For highly complex systems described in the time-domain and which are submitted to constraints, predictive control seems to be well-adapted. This model based control method (MPC: Model Predictive Control) is founded on the determination of an optimal control sequence over a receding horizon. Due to its formulation in the time-domain, it is an effective tool for handling constraints and uncertainties which can be explicitly taken into account in the synthesis procedure [7]. The team considers how multiparametric optimization can help to reduce the computational load of this method, allowing its effective use on real world constrained problems.

The team also investigates stochastic optimization methods such as genetic algorithm, particle swarm optimization or ant colony [10] as they can be used to optimize any criterion and constraint whatever their mathematical structure is. The developed methodologies can be used by non specialists.

3.4. Synthesis of reduced complexity controllers

- PID controllers

Even though the synthesis of control laws of a given complexity is not a new problem, it is still open, even for finite-dimensional linear systems. Our purpose is to search for good families of "simple" (e.g. low order) controllers for infinite-dimensional dynamical systems. Within our approach, PID candidates are first considered in the team [2], [99].

- Predictive control

The synthesis of predictive control laws is concerned with the solution of multiparametric optimization problems. Reduced order controller constraints can be viewed as non convex constraints in the synthesis procedure. Such constraints can be taken into account with stochastic algorithms.

Finally, the development of algorithms based on both symbolic computation and numerical methods, and their implementations in dedicated Scilab/Matlab/Maple toolboxes are important issues in the project.

4. Application Domains

4.1. Modeling and analysis of Acute Myeloid Leukemia

In collaboration with the BANG project-team at INRIA Paris-Rocquencourt, the DRACULA team at INRIA Grenoble - Rhône-Alpes, INSERM, Cordeliers Research Center and St Antoine Hospital, Paris, we consider the modelling of Acute Myeloid Leukemia (AML).

The main goal of this project is the theoretical optimization of drug treatments used in AML, with experimental validation in cell cultures, aiming at proposing efficient therapeutic strategies in clinic.

We work on an discrete maturity-structured model of hematopoiesis introduced in [76]. In this model, several generations of cells are considered and, for the first time, the cell cycle duration is assumed to be distributed. At each level, the population of immature cells are divided into two subpopulations: proliferating and non proliferating cells. Physiological phenomena of re-introduction from the non proliferative into the proliferative subpopulation is modelled in the team as a nonlinear dynamical interconnection between the two sub-populations, and input-output tools allow a complete stability analysis of the model.

4.2. Control of continuous bioreactors

We study problems of coexistence or regulation of species in chemostats with one or several limiting substrate, which are important bioreactor models in bioengineering. We consider some distinct contexts (Monod or Haldane functions as growth functions, presence of pointwise delays, presence of two or an arbitrary number of species, dilution rate and/or input nutrient concentration as controls).

4.3. Control of a model of human heart

We are also working on the control of human heart rate during exercise, which is a problem that has implications for the development of protocols for athletics, assessing physical fitness, weight management, and the prevention of heart failure. We provide new stabilization techniques for a recently-proposed nonlinear model for human heart rate response that describes the central and peripheral local responses during and after treadmill exercise. We plan to consider more realistic systems incorporating delays in the model.

5. Software

5.1. YALTA

Participants: André Fioravanti [correspondent], Catherine Bonnet.

The YALTA package is dedicated to the study of classical and fractional systems with delay in the frequency-domain. Its objective is to provide basic but important information such as, for instance, the position of the neutral chains of poles and unstable poles, as well as the root locus with respect to the delay of the system. The corresponding algorithms are based on recent theoretical results (see, for instance, [12] and [46]) and on classical continuation methods exploiting the particularities of the problem [47], [87]. Although the YALTA package is still in development, it will be freely available in the first semester of 2011.

5.2. OreModules

Participants: Alban Quadrat [correspondent], Daniel Robertz [Univ. Aachen], Frédéric Chyzak [INRIA Rocquencourt, Algorithms Project].

The OREMODULES package [80], based on the commercial Maple package Ore_algebra [81], is dedicated to the study of linear multidimensional systems defined over certain Ore algebras of functional operators (e.g., ordinary or partial differential systems, time-delay systems, discrete systems) and their applications in mathematical systems theory, control theory and mathematical physics. The main novelty of OREMODULES is to combine the recent developments of the Gröbner bases over some noncommutative polynomial rings [89], [94] with new algorithms of algebraic analysis in order to effectively check classical properties of module theory (e.g., existence of a non-trivial torsion submodule, torsion-freeness, reflexiveness, projectiveness, stably freeness, freeness), give their system-theoretical interpretations (existence of autonomous elements or successive parametrizations, existence of minimal/injective parametrizations or Bézout equations) [103], [102], [79] and compute important tools of homological algebra (e.g., (minimal) free resolutions, split exact sequences, extension functors, projective or Krull dimensions, Hilbert power series) [72]. The abstract language of homological algebra used in the algebraic analysis approach carries over to the implementations in OREMODULES: up to the choice of the domain of functional operators which occurs in a given system, all algorithms are stated and implemented in sufficient generality such that linear systems defined over the Ore algebras developed in the Ore_algebra package are covered at the same time. Applications of the OREMODULES package to mathematical systems theory, control theory and mathematical physics are illustrated in a large library of examples. The binary of the package is freely available at <http://wwwb.math.rwth-aachen.de/OreModules/>.

5.3. Stafford

Participants: Alban Quadrat [correspondent], Daniel Robertz [Univ. Aachen].

The STAFFORD package of OREMODULES [80] contains an implementation of two constructive versions of Stafford's famous but difficult theorem [107] stating that every ideal over the Weyl algebra $A_n(k)$ (resp., $B_n(k)$) of partial differential operators with polynomial (resp., rational) coefficients over a field k of characteristic 0 (e.g., $k = \mathbb{Q}, \mathbb{R}$) can be generated by two generators. Based on this implementation and algorithmic results developed in [104] by the authors of the package, two algorithms have been implemented which compute bases of free modules over the Weyl algebras $A_n(\mathbb{Q})$ and $B_n(\mathbb{Q})$. The development of the STAFFORD package was motivated by the problem of computing injective parametrizations of underdetermined linear systems of partial differential equations with polynomial or rational coefficients (the so-called *Monge problem*), differential flatness, the reduction and decomposition problems and Serre's reduction problem. To our knowledge, the STAFFORD package is the only implementation of Stafford's theorems nowadays available. The binary of the package is freely available at <http://wwwb.math.rwth-aachen.de/OreModules/>.

5.4. QuillenSuslin

Participants: Alban Quadrat [correspondent], Anna Fabiańska [Univ. Aachen].

The QUILLEN-SUSLIN package [86] contains an implementation of the famous Quillen-Suslin theorem [106], [108]. In particular, this implementation allows us to compute bases of free modules over a commutative polynomial rings with coefficients in a field (mainly \mathbb{Q}) and in a principal ideal domain (mainly \mathbb{Z}). The development of the QUILLEN-SUSLIN package was motivated by different constructive applications of the Quillen-Suslin theorem in multidimensional systems theory [86] (e.g., the Lin-Bose conjectures, the computation of (weakly) left/right/doubly coprime factorizations of rational transfer matrices, the computation of injective parametrizations of flat linear multidimensional systems with constant coefficients, the reduction and decomposition problems, Serre's reduction problem). To our knowledge, the QUILLEN-SUSLIN package is the only implementation of the Quillen-Suslin theorem nowadays available. For more details, see <http://wwwb.math.rwth-aachen.de/QuillenSuslin>.

5.5. OreMorphisms

Participants: Alban Quadrat [correspondent], Thomas Cluzeau [ENSIL, Univ. Limoges].

The OREMORPHISMS package [84] of OREMODULES [79] is dedicated to the implementation of homological algebraic tools such as the computations of homomorphisms between two finitely presented modules over certain noncommutative polynomial algebras (Ore algebras), of kernel, coimage, image and cokernel of homomorphisms, Galois transformations of linear multidimensional systems and idempotents of endomorphism rings. Using the packages STAFFORD and QUILLEN-SUSLIN, the factorization, reduction and decomposition problems can be constructively studied for different classes of linear multidimensional systems. Many linear systems studied in engineering sciences, mathematical physics and control theory have been factorized, reduced and decomposed by means of the OREMORPHISMS package. The binary of the package is freely available at <http://www-sop.inria.fr/members/Alban.Quadrat/OreMorphisms/index.html>.

5.6. PurityFiltration

Participant: Alban Quadrat [correspondent].

The PURITYFILTRATION package, built upon the OREMODULES package, is an implementation of a new effective algorithm obtained in [63], [72] which computes the purity filtration [77], [78] of linear multidimensional systems (e.g., partial differential systems, differential time-delay systems, difference systems) and their equivalent block-triangular matrices. This package is used to solve in closed form solutions over/underdetermined linear partial differential systems which cannot be directly integrated by the standard computer algebra systems such as Maple and Mathematica. The binary of the package is freely available at <http://www-sop.inria.fr/members/Alban.Quadrat/PurityFiltration.html>.

5.7. AbelianSystems

Participants: Alban Quadrat [correspondent], Mohamed Barakat [Univ. Kaiserslautern].

The ABELIANSYSTEMS package is an implementation of the algorithm developed in [63], [72] for the computation of the purity filtration [77], [78] in the powerful `homa1g` package of GAP4 dedicated to constructive homological algebra methods, developed by Barakat (University of Kaiserslautern) and his collaborators (<http://homa1g.math.rwth-aachen.de/>). This package supersedes the existing `homa1g` procedure which computes purity filtrations based on the computation of spectral sequences and the PURITYFILTRATION package which is based on the rather slow Maple Gröbner basis computation (see Section 5.6). Using the design of the `homa1g` package, where the different layers such as the computational engine (which can be the computer algebra systems Singular, Macaulay2, OREMODULES, MAGMA, SAGE), the module-theoretic results, and the homological ones are separated, the ABELIANSYSTEMS package can be used for the computation of purity filtration of different objects in abelian categories (e.g., purity filtration of sheaf cohomology of projective varieties). This package will be soon available on the `homa1g` website and at <http://www-sop.inria.fr/members/Alban.Quadrat/AbelianSystems.html>.

6. New Results

6.1. Analysis of infinite-dimensional and multidimensional systems

6.1.1. H_∞ - stability of (possibly fractional) neutral delay systems

Participants: Catherine Bonnet, André Fioravanti, Hitay Özbay, Jonathan R. Partington.

Both in the time-domain and frequency-domain approaches to stability of delay systems, neutral type systems are the most difficult systems to analyze since they may have chains of poles asymptotic to the imaginary axis in the complex plane. In [12], we provide the location of all asymptotic poles for classical systems, and for the fractional case in [46]. In both cases, necessary and sufficient conditions for H_∞ -stability are derived. Preliminary results about the robustness relative to a change in the delay or in the parameters are given.

6.1.2. Numerical methods for (possibly fractional) time-delay systems

Participants: Catherine Bonnet, André Fioravanti, Silviu-Iulian Niculescu, Hitay Özbay.

Many important properties of time-delay systems cannot be obtained in a pure algebraic form. In those cases, numerical methods have proved to be extremely efficient and precise. But most of the time, only classical retarded systems are considered. We studied how those algorithms can be adapted to the cases of neutral and fractional systems, and if not, how to obtain new algorithms for this class of systems [47], [87]. Those results will be implemented in the YALTA toolbox described in Section 5.1.

6.1.3. Differential flatness of analytic linear ordinary differential systems

Participants: Alban Quadrat, Daniel Robertz.

Based on an extension of Stafford's classical theorem in noncommutative algebra [107] developed in [85], we prove in [64] that a controllable linear ordinary differential system with convergent power series coefficients (i.e., germs of real analytic functions) and at least two inputs is differentially flat [88]. This result extends a result obtained in [104] for linear ordinary differential systems with polynomial coefficients. The algorithm developed in [104] for the computation of injective parametrizations and bases of free differential modules with polynomial coefficients can be used to compute injective parametrizations and flat outputs of these classes of differentially flat systems. This algorithm allows us to remove artificial singularities which naturally appear in the computation of injective parametrizations and flat outputs obtained by means of Jacobson normal form computations.

6.1.4. Serre's reduction of multidimensional systems

Participants: Alban Quadrat, Thomas Cluzeau, Mohamed S. Boudellioua.

Given a linear multidimensional system (e.g., ordinary/partial differential systems, differential time-delay systems, difference systems), Serre's reduction aims at finding an equivalent linear multidimensional system which contains fewer equations and fewer unknowns. Finding Serre's reduction of a linear multidimensional system can generally simplify the study of structural properties and of different numerical analysis issues, and it can sometimes help solving the linear multidimensional system in closed form. In [14], [74], [42], a constructive approach to Serre's reduction is developed for determined and underdetermined linear systems. In particular, an algorithm is given for the class of controllable 2D systems, which is used to find explicit Serre's reduction of many classical control differential time-delay systems. Serre's reduction of these systems generally simplifies their analysis and their synthesis.

In [44] and [83], Serre's reduction problem is studied for underdetermined linear systems of partial differential equations with either polynomial, formal power series or analytic coefficients and with holonomic adjoints in the sense of algebraic analysis [77], [78]. These linear partial differential systems are proved to be equivalent to a linear partial differential equation. In particular, an analytic linear ordinary differential system with at least one input is equivalent to a single ordinary differential equation. In the case of polynomial coefficients, we give an algorithm which computes the corresponding linear partial differential equation.

The algorithms obtained in [14], [74], [42], [44] and [83] are implemented in the SERRE package in development.

6.1.5. Purity filtration of multidimensional systems

Participant: Alban Quadrat.

In [63], [72], it is shown that every linear system of partial differential equations in n independent variables is equivalent to a linear system of partial differential defined by an upper block-triangular matrix of partial differential operators: each diagonal block is respectively formed by the elements of the system satisfying an i -dimensional (resp., $(n + i)$ -dimensional) dynamics if the coefficients of the system are either constant or rational functions (resp., the coefficients are either polynomial, formal power series or convergent power series). Hence, the system can be integrated in cascade by successively solving (inhomogeneous) linear j -dimensional systems of partial differential equations to get a Monge parametrization of its solution space [105]. The results are based on an explicit construction of the purity filtration of the differential module associated with the linear systems of partial differential equations, which does not use spectral sequences [77], [78]. These results can be extended to other classes of linear multidimensional systems such as differential time-delay systems or difference systems.

6.1.6. *Extendability of multidimensional linear systems*

Participant: Alban Quadrat.

Within the algebraic analysis approach to multidimensional linear systems defined by linear systems of partial differential equations with constant coefficients [72], [79], [102], we show in [62] how to use different mathematical results developed in the literature of algebraic analysis [91], [95] to obtain new characterizations of the concepts of controllability in the sense of Willems [101] and Pillai-Shankar [100], observability, flatness and autonomous systems in terms of the possibility to extend (smooth or distribution) solutions of the multidimensional system and of its formal adjoint. Each characterization is equivalent to a module-theoretic property that can be constructively checked by means of the OREMODULES and QUILLENUSLIN packages.

6.1.7. *Symmetries and parametrizations of multidimensional systems*

Participants: Thomas Cluzeau, Alban Quadrat.

Within the algebraic analysis approach to linear systems theory [72], [79], [102], [45] shows how the behaviour homomorphisms (namely, transformations which send the solutions of a linear multidimensional system to the solutions of another linear system [82]) induce natural transformations on the autonomous elements of these systems (e.g., on the obstructions to controllability) and on the potentials (e.g., flat outputs) of their parametrizable parts (e.g., controllable parts). Extension of these results are then considered for linear systems inducing a chain of successive parametrizations (e.g., the divergence operator, the first group of Maxwell equations, the stress tensor in linear elasticity).

6.1.8. *Factorization and decomposition problems for 2D Stokes and Oseen equations*

Participants: Thomas Cluzeau, Alban Quadrat.

In [43], within the constructive algebraic analysis approach to linear systems [72], [79], [102], we study classical linear systems of partial differential equations in two independent variables with constant coefficients appearing in mathematical physics and engineering sciences such as the Stokes and Oseen equations studied in hydrodynamics and the Navier-Cauchy equations in linear elasticity. In particular, a precise description of the endomorphism ring of the differential module associated with the Stokes and Oseen, Navier-Cauchy equations is given. Using the fact that the endomorphism ring of the Stokes and Oseen equations in \mathbb{R}^2 defines a cyclic differential module, we decide about the existence of factorizations of the matrices of differential operators defined by these systems and the decomposition of their solution spaces in direct sums.

6.1.9. *New domain decomposition methods for linear PD systems*

Participants: Thomas Cluzeau, Victorita Dolean, Frédéric Nataf, Alban Quadrat.

Within the framework of the PEPS Maths-ST2I SADDLES (Symbolic Algebra, Domain Decomposition, Linear Equations and Systems), the purpose of this work is to use algebraic and symbolic computation techniques such as Smith normal forms and Gröbner basis techniques to develop new Schwarz algorithms for domain decompositions and for preconditionners of linear systems of partial differential equations, especially for the Stokes and Oseen equations studied in hydrodynamics and the Navier-Cauchy equations in linear elasticity. New algorithms are developed to reduce the interface conditions and to solve the completion problem built on physical Smith variables. They are implemented in the SCHWARZ package (to appear) built upon the OREMODULES package. For more details, see <http://www-math.unice.fr/~dolean/saddles/>.

6.2. *New techniques of observation and control*

6.2.1. *Interval observers*

Participants: Frédéric Mazenc, Silviu-Iulian Niculescu, Olivier Bernard.

The *interval observer* method is a recent state estimation technique. It is used in particular in biological contexts, where taking into account the presence of uncertainties is essential. We have completed the theory of the linear interval observers in several works.

1. The contribution of the work [26] is twofold. A first part of our work is devoted to the problem of exhibiting necessary and sufficient conditions which guarantee that, for a time-invariant linear system of dimension 2, a time-invariant linear and exponentially stable interval observer can be constructed. In the second part of the work, when these conditions are violated, we have shown that one can still construct exponentially stable linear interval observers but these interval observers have the remarkable feature of being *time-varying*. Thus, we managed to give a complete picture of the difficulties and solutions which arise for systems of dimension two. To illustrate the power of our approach, we have applied it to a chaotic system which is known to be highly sensitive to uncertainties in the initial conditions.

2. The work [27] (see also [54]) presents a solution to the problem of constructing exponentially stable interval observers for any time-invariant exponentially stable system. This result, which is constructible, relies on two crucial steps. The first step consists in transforming, through a time-invariant change of coordinates, the system under consideration into a system of the Jordan form. The second consists in determining a time-varying change of coordinates which transforms the system in Jordan form into a cooperative time-invariant system (recall that a linear system is cooperative if it is associated to a matrix whose off-diagonal entries are nonnegative).

3. In [97], we investigated the problem of constructing interval observers for exponentially stable linear systems with point-wise delays. First, we proved that classical interval observers for systems without delays are *not robust* with respect to the presence of delays that appear in a specific structure location, no matter how small the delay is. Next, we have shown that, in general, for linear systems classical interval observers endowed with a point-wise delay are not satisfactory because they are exponentially unstable. Finally, we have designed interval observers of a new type. Our construction relies on framers that incorporate *distributed delay* terms. These framers are interval observers when the delay is smaller than an upper bound that we have estimated.

6.2.2. Finding positive solutions of systems with delay

Participants: Frédéric Mazenc, Silviu-Iulian Niculescu.

In [34] (see also [59]), a new technique of design of feedbacks for systems with pointwise delays is developed. It relies on the introduction of an operator which has remote connections with the ones used in reduction model approaches. Using this operator, one succeeds, in many cases, to rewrite the closed-loop system we consider into the interconnection of an ordinary differential equation with an integral equation. An important advantage of this representation, is that it allows to derive simple conditions, in terms of initial conditions, ensuring that the resulting solutions of the closed-loop systems are positive. It is worth mentioning that our wish to determine positive solutions for systems with delay had several strong motivations: when this objective is reached, one can easily solve more general problems: in particular, we have shown that our new result can be used to generate solutions which respect to more general constraints than the constraint of sign and solutions which can be compared between each other, which is useful when is only available an approximate knowledge of the initial condition and an estimation of the state variables at each instant is desirable.

6.2.3. Lyapunov-based results

Participants: Frédéric Mazenc, Claudio De Persis, Michael Malisoff, Marcio De Queiroz, Olivier Bernard.

We did some works in which strict Lyapunov functions play a central role.

1. Systems with quantized feedback.

Quantized control systems are systems in which the control law is a piece-wise constant function of time taking values in a finite set. The design of quantized control systems is based on a partition of the state space. The aim of [18] was to design control laws for general families of quantized time-delay control systems. Our approach relies on the construction of Lyapunov-Krasowskii functionals and provided with quantized feedbacks which are parametrized with respect to the quantization density. Our approach leads to a set of conditions to design quantized control systems which are robust with respect to delays.

2. Strict Lyapunov functions under LaSalle conditions.

In [33], we provided new techniques for building explicit global strict Lyapunov functions for broad classes of periodic time-varying nonlinear systems satisfying LaSalle conditions. Our new constructions are simpler than the designs available in the literature. We illustrated our work using the Lotka-Volterra model, which plays a fundamental role in bioengineering. We used our strict Lyapunov constructions to prove robustness of the Lotka-Volterra tracking dynamics to uncertainty in the death rates.

3. Adaptive control.

In [30] and [56], we studied adaptive tracking problems for nonlinear systems with unknown control gains. We constructed controllers that yield uniform global asymptotic stability for the error dynamics, hence tracking and parameter estimation for the original systems. Our result is based on a new explicit, global, strict Lyapunov function construction. We illustrated our work using a brushless DC motor turning a mechanical load. We quantified the effects of time-varying uncertainties on the motor electric parameters.

6.2.4. Predictive control

Participant: Sorin Olaru.

1. In [39], new results have been obtained toward predictive control design for nonlinear systems upon optimization-in-the-loop techniques for an air ventilation problem. From a methodological point of view, the use of local embeddings of the nonlinearity led to polytopic differential inclusions [19]. This were further used for the nonlinear predictive control synthesis. A generic procedure which deals with the state-space partitioning for a multi-model description and subsequently obtain the control law by means of LMIs have been presented in [19]. An interesting aspect of this result is the explicit formulation of the control law in terms of patchy feedback gains.

2. On NCS related topics, several results can be mentioned with respect to the construction of polytopic embeddings for linear systems affected by variable time delays. In [21], the Cayley-Hamilton approach was investigated while in [48] a comparison is made with respect to the alternative methods (Taylor series approximations, truncations, Jordan normal forms). For the same class of systems, several results have been reported for the adaptation of predictive control techniques for their constraints handling mechanisms (see [52] and [53]). For the stability point of view, an interesting aspect is the characterization of terminal invariant sets [53], a research subject which receives currently a renewed attention.

3. The fragility of proportional-derivative controllers has been studied in relation with robotics/tele-operation application in the two recent publications [50], [51].

6.2.5. Comparison systems

Participants: André Fioravanti, José Geromel [Unicamp], Rubens Korogui [Unicamp].

A new idea to the study of H_∞ -stability and filter [92] and control [93] designs can be obtained from the Rekasius transformation of the delay term in frequential term. After the relations of stability and norms between the comparison and real systems are obtained, classical techniques involving Riccati equation from the H_∞ theory of LTI systems are used to derive infinite-dimensional filters and controllers for time-delay systems. All implementation issues are discussed in order to provide a new and easy to implement technique.

6.2.6. Discrete Markov jump systems

Participants: André Fioravanti, José Géromel [Unicamp], Alim Gonçalves [Unesp].

In the last years, the interest on networked control has enormously increased. Actually, communication networks inherently introduce packet dropouts, quantisation errors, time-delays and limited bandwidths. Up to now, the most successful way to model packet dropouts are stochastic markovian systems, since they are the basic mathematical tool for the models of many different networks. In [22] and [23], the H_2 and H_∞ -filtering, and in [67], the state feedback design problems of those systems are addressed.

6.2.7. Stochastic optimization in energy production systems

Participants: Henri Borsenberger, Philippe Dessante, Jorge Luis Reyespesantez, Guillaume Sandou.

Continuing the collaboration with the Energy Department of Supélec, the use of robust optimization methods has allowed the computation of energy production control laws taking into account various uncertainties on the plant. Main uncertainties, which have been taken into account, are the consumer load prediction, the maximum unit production level and the production costs. Results exhibit a more robust technical behavior together with a decrease of global operation costs [13], [41], [66].

6.2.8. Use of metaheuristic optimization methods for automatic control

Participants: Gilles Duc, Bianca Minodora Heiman, Saïd Ighbriouen, Gabriela Raduinea, Guillaume Sandou, Mohamed Yagoubi.

The development of generic methodologies for automatic control based on metaheuristic methods has led to several promising results. Among them are the automatic computation of weighing filters and the design of static H_∞ output feedbacks using Particle Swarm Optimization, and the use of ant colony optimization for the identification of nonlinear systems. Some promising results have also been obtained using multiobjective Particle Swarm Optimization [49], [73]

6.3. Biological systems

We solved several problems of control and stability analysis for different types of biological models.

6.3.1. Control of continuous bioreactors

Participants: Frédéric Mazenc, Jiang Zhong Ping, Michael Malisoff.

The paper [29] is devoted to the persistence issue for chemostat models with an arbitrary number of species competing for a single limiting substrate. In a first part, we have shown that there exist fundamental limitations for the existence of nonlinear feedback control which ensure persistence of several species in some chemostats. More precisely, we have exhibited models for which there are some admissible bounded periodic trajectories for which there is no feedback control law guaranteeing their local asymptotic stability. In a second part, for models with an arbitrary number of species associated to growth rates of Monod type, we have shown that a dilution rate and input substrate time-varying nonlinear controllers can be designed so that a positive trajectory of the chemostat model becomes globally asymptotically stable. In this case, the designed control laws ensure persistence of all the species.

The paper [28] extends the results of [29] and [96] by designing a dilution rate and input substrate feedback controllers when only the substrate concentration is measured. More precisely, we achieved the coexistence by designing a novel output-feedback controller that globally asymptotically stabilizes a periodic reference state trajectory of the system. It is worth mentioning that, in practice, measuring the values of the concentration of each species is not feasible but measuring the substrate concentration is. Therefore, considering the substrate concentration as the output is a reasonable choice which is frequently made in the literature. The dynamic output feedback we proposed relies on an observer.

The stabilization of equilibria in chemostats with measurement delays is a complex and challenging problem, and is of significant ongoing interest in bioengineering and population dynamics. In [32] (see also [58]), we solved an output feedback stabilization problem for chemostat models having two species, one limiting substrate, and either Haldane or Monod growth functions. Our feedback stabilizers depend on a given linear combination of the species concentrations, which are both measured with a constant time delay. Our work is based on a Lyapunov-Krasovskii argument.

In [31] (see also [57]), we studied feedback stabilization problems for chemostats with two species and one limiting substrate, which play an important role in systems biology and population dynamics. We constructed new dilution rate output feedbacks that stabilize a componentwise positive equilibrium, and only depend on the sum of the species levels. We proved that the feedbacks are robust to model uncertainty. The novelty and importance of our new contribution is in our dropping the usual assumption on the relative sizes of the growth yield constants, and our allowing uncertain uptake functions that are not necessarily concave. The proofs are based on classical results of ordinary differential equations (as for instance the Poincaré-Bendixson theorem and Dulac' criterion).

6.3.2. Study of a model of anaerobic digestion process

Participants: Frédéric Mazenc, Miled El Hajji, Jérôme Harmand.

In [24], a mathematical model involving the syntrophic relationship of two major populations of bacteria (acetogens and methanogens), each responsible for a stage of the methane fermentation process is proposed. We carried out a detailed qualitative analysis. We performed the local and global stability analyses of the equilibria. Under general assumptions of monotonicity, we demonstrated relevant from an applied point of view, the global asymptotic stability of a positive equilibrium point which corresponds to the coexistence of acetogenic and methanogenic bacteria. The proofs are based on classical results of ordinary differential equations.

6.3.3. Control of a model of human heart

Participants: Frédéric Mazenc, Michael Malisoff, Marcio De Queiroz.

The control of human heart rate during exercise is an important problem that has implications for the development of protocols for athletics, assessing physical fitness, weight management, and the prevention of heart failure. In [55], we provided a new stabilization technique for a recently-proposed nonlinear model for human heart rate response that describes the central and peripheral local responses during and after treadmill exercise. The control input is the treadmill speed, and the control objective is to make the heart rate track a prescribed reference trajectory. We used a strict Lyapunov function analysis to design new state and output feedback tracking controllers that render the error dynamics globally exponentially stable. This allowed us to prove robustness stability properties for our feedback stabilized systems relative to actuator errors.

6.3.4. Modelling of hematopoiesis with application to acute myeloid leukemia

Participants: José Luis Avila Alonso, Houda Benjelloun, Catherine Bonnet, Jean Clairambault [BANG Project-Team, INRIA Paris-Rocquencourt], Jean-Pierre Marie [INSERM Paris (team 18 of UMR 872), Cordeliers Research Center ans St Antoine Hospital, Paris], Faten Merhi [BANG Project-Team, INRIA Paris-Rocquencourt], Hitay Özbay, Ruopping Tang [INSERM Paris (team 18 of UMR 872), Cordeliers Research Center ans St Antoine Hospital, Paris].

We have worked on a nonlinear PDE model of hematopoeisis designed by Adimy and Crauste [76], more precisely on its approximation by a nonlinear system with multiple distributed delays.

A complete stability analysis with therapeutic implications has been performed in [61], [60] and [98]. Moreover, a simulation program of this model is already available.

Through the DIGITEO project ALMA, parameters of this model will be identified through experiments and the model will be changed in order to take more precisely into account some biological phenomena in hematopoiesis.

7. Other Grants and Activities

7.1. Regional Initiatives

- C. Bonnet is the coordinator of the DIGITEO project ALMA (December 2010 - December 2013), which involves the BANG project team at INRIA Paris-Rocquencourt, L2S, INSERM Paris (team 18 of UMR 872), Cordeliers research center and the COMMANDS team at INRIA Saclay-Île-de-France.

7.2. National Initiatives

- C. Bonnet is a member of the ANR Program *Bimod* (December 2010 - December 2014) coordinated by V. Volpert and involving 3 partners: CNRS (Institut Camille Jordan), University of Bordeaux II and INRIA (Paris-Rocquencourt and Saclay-Île-de-France).

- F. Mazenc is a member of the ARC Vitelbio (January 2009 - January 2011). Participants: EPI MERE (Montpellier), EPI IPSO (Rennes), Society ITK (Montpellier), UMR Geosciences (Rennes), INRA UREP (Clermont-Ferrand) UMR Eco et Sols (Montpellier).
- S.-I. Niculescu is a member of a PEPS INST2I with the Centre d'Enseignement et de Recherche sur l'Environnement et la Société (CERES).
- A. Quadrat is a member of the PEPS Maths-ST2I: SADDLES (Symbolic Algebra, Decomposition Domains, Linear Equations and Systems) in collaboration with V. Dolean (University of Nice), F. Nataf (CNRS, Paris 6) and T. Cluzeau (University of Limoges). A. Quadrat has a long term collaboration with T. Cluzeau, M. Barkatou and J.-A. Weil (University of Limoges, XLIM).

7.3. European Initiatives

- C. Bonnet has a long term collaboration with J. R. Partington, School of mathematics, Leeds, United Kingdom.
- S. Olaru is the project manager for the bi-lateral PHC projects Brâncusi with the University of Craiova, Romania, and van Gogh with the University of Eindhoven, The Netherlands.
- S.-I. Niculescu is a member of the MAE with the Institute of Physics, Belgrade, Serbia, a member of the MAE with the University of Bilkent, Ankara, a member of a PHC Brâncusi with the University of Craiova, Romania, and a member of a PHC van Gogh with the University of Eindhoven, The Netherlands.
- A. Quadrat has developed a strong collaboration with the Lehrstuhl B für Mathematik of Wilhelm Plesken, RWTH Aachen University, and particularly with Daniel Robertz and Mohamed Barakat. He is a member of an accepted PHC Procope developed in collaboration with the University of Limoges (XLIM) and the Lehrstuhl B für Mathematik, RWTH Aachen University (2011-2012).

7.4. International Initiatives

- F. Mazenc has a strong collaboration with Michael Malisoff and Marcio De Queiroz of, respectively the Department of Mathematics and the Department of Mechanical Engineering of the Louisiana State University.
- S.-I. Niculescu is the head of a CNRS exchange convention with the Laboratory for Advanced Robotics at the Korea University of Seoul, South Korea.

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Committee participations

- C. Bonnet is a member of the IFAC Robust Control Technical Committee, of the CNU 61 (National Council of Universities), of the scientific council of Île-de-France region (CCRRESTI) until April 2010, in charge with B. Eurin of the working group on Student Life. She is also in the board of the association *Femmes et Mathématiques* and a member of the National INRIA working group on Scientific Mediation.
- F. Mazenc is member of the COST (Scientific and technological Orientation Council) in the team GTAI (Groupe de travail Actions Incitatives). The main mission of the GTAI: organization, selection and supervision of INRIA's incentive initiatives, such as the Cooperative research Initiatives (ARC) of the Scientific Management and the Software Development Operations (ODL) and Standardization operations of the Department of Development and Industrial Relations.

- S.-I. Niculescu is the head of the Laboratory of Signals and Systems (L2S), UMR 8506. Moreover, he is a member of the “commission de spécialistes” CCSU 60-61-62 of the University of Paris-Sud, a member of the scientific and administrative board of the Romanian National University Research Council, a member of the Programme Committee of DIGITEO, Saclay, Île-de-France, and a member of council Ecole Doctorale STITS of the University Paris-Sud.
- S. Oлару is the co-organizer of the meetings of the French Research Group on Nonlinear Predictive Control within GDR MACS.
- A. Quadrat was a member of a Selection Committee for an assistant professorship position at the University Limoges (sections 25-26) and at the University Toulouse III (section 61).

8.1.2. Editorial committees

- C. Bonnet was a member of the scientific committee of the 10th forum des Jeunes Mathématiciennes, CIRM, Marseille, France.
- F. Mazenc was an Associate Editor, Conference Editorial Board, for the American Control Conference (ACC) 2010.
- S.-I. Niculescu is an Associate Editor of Journal of Control Science and Engineering. Moreover, he was International Program Committee (IPC) of the Conférence Internationale Francophone d’Automatique (CIFA) 2010, Nancy, France, of the 18th Mediterranean Conference on Control and Automation, Marrakech, Morocco, of the 2nd IFAC Workshop on Distributed Estimation and Control in Networked Systems (NecSys’10), Annecy, France, of the 14th International Conference on Systems Theory and Control, Sinaia, Romania, of the IFAC World Congress, Milano, Italy, of the 9th IFAC Workshop on Time-Delay Systems, Prague, Czech Republic, of the International Conference on Control, Automation and Systems (ICCS), Gyeonggi-do, South Korea, of the IFAC Symposium on System Structure and Control (SSSC’10), Ancona, Italy, and of the 2011 IEEE International Conference on Communications, Computing and Control Applications (CCCA’11), Hammamet, Tunisia. He was also an associated editor of the special issue “Time-delay systems” of IMA Journal on Mathematical Control and Information (2011). Finally, he is member of IFAC Technical Committee on Linear Systems (since 2002) and is the responsible of the IFAC Research Group on “Time-delay systems” since its creation in October 2007.
- S. Oлару was member of the program committee of the 14th International Conference on System Theory and Control (SINTES 2010), Sinaia, Romania, and has been appointed in the program committee for 2011 JD MACS, Marseille, France.
- A. Quadrat is an Associate Editor of the international journal Multidimensional Systems and Signal Processing (Springer).

8.1.3. Organizations of conferences and school

- F. Mazenc co-organized with M. Malisoff and H. Ito the invited session *New directions in stability and stabilization* at the American Control Conference, Baltimore, USA.
- S. Oлару organized the invited session *Predictive control* at CIFA 2010 (with A. Chemori, LIRMM as chairman), Nancy, France.
- A. Quadrat organized 3 invited sessions *New mathematical methods in multidimensional systems theory* at the 19th International symposium on Mathematical Theory of Networks and Systems (MTNS2010), Budapest, Hungary.

8.1.4. Ph.D. thesis committees

- F. Mazenc was a referee of Ixbalank Torres’ Ph.D. thesis, *Simulation and control of denitrification biofilters described by PDEs*, University of Toulouse III - Paul Sabatier (May 2010). Moreover, he was a referee of Pierre Masci’s Ph.D. defense committee, *Control and optimization of ecosystems in bioreactors for bioenergy production*, University of Nice Sophia-Antipolis (November 2010).

- S. Olaru was appointed member of the Ph.D. defense committee of Khaoula Nagoudi-Layerle, University of Rouen.

8.1.5. Participation at conferences and invitations

- C. Bonnet was an invited speaker at the Conference MACS4 (Modélisation, Analyse et Contrôle des Systèmes), Meknès, Marocco. She also participated at the 19th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2010), Budapest, Hungary.
- André Fioravanti won the Fall 2010 Matlab Programming Contest.
- F. Mazenc participated at the American Control Conference, Baltimore. He also participated at the IFAC Symposium on Nonlinear Control Systems (NOLCOS 2010) and at the IEEE Conference on Decision and Control, Atlanta, USA.
- S.-I. Niculescu was invited speaker at the IFAC Symposium on System, Structure and Control (SSSC'10), Ancona, Italy. He was also invited at the Ecole Polytechnique and Concordia University of Montreal, Canada.
- A. Quadrat was invited speaker at the French National Days on Symbolic Computation (JNCF 2010), CIRM, France. He was an invited speaker at the Sage Days 24, RISC, Austria. He also participated at the the 19th International symposium on Mathematical Theory of Networks and Systems (MTNS 2010), Budapest, Hungary.

8.2. Teaching

8.2.1. Teaching at Universities

- S. Olaru is an associate professor in Supélec.
- G. Sandou is an associate professor in Supélec (250 hours).

8.2.2. Other teaching

- S.-I. Niculescu taught a course on Robust Control at ESIEE (8 hours) and tutorials in Mathematics at Ecole Nationale Supérieure des Mines de Paris (30 hours).
- A. Quadrat gave a lecture at the French National Days on Symbolic Computation (JNCF), CIRM, Luminy, France (3 hours).
- G. Sandou is a free lance teacher at the University of Evry-Val d'Essonne (7 hours), at the Ecole Centrale Paris (20 hours), at the Ecole des Mines de Nantes (22 hours), at the Ecole Nationale Supérieure de Techniques Avancées (ENSTA) (21 hours) and at the Ecole Militaire (15 hours).

8.2.3. Ph.D. theses in progress

- Mounir Bekaik, *Commande et observation de systèmes variant dans le temps à retard.*
- Abdelkarim Chakhar, *Etude des systèmes différentiels quasi-linéaires par l'analyse algébrique et l'algèbre différentielle.*
- André Fioravanti, *H_∞ -analysis and control of some classes of (possibly fractional) time-delay systems by frequential methods.*
- Warody Lombardi, *Synthèse des lois de commande basées sur prédiction pour des systèmes à retard.*
- Bogdan Liacu, *Commande prédictive MPC pour la télé-opération.*

8.2.4. Defended Ph.D theses

- Benjamin Bradu, *Modélisation, simulation et contrôle des installations cryogéniques du CERN, L2S, CERN, Geneva, Switzerland, March 2010.*

8.2.5. Internships

- Houda Benjelloun, *Mathematical analysis of acute myeloid leukemia*, final project, INSA Rouen (6 months). She was also an engineer in the team for three months.
- Saïd Ighobriouen, *Use of metaheuristic optimization methods for the design of controllers*, final project, master RVSI (University of Evry - Val d'Essonne) (5 months).
- Bianca Minodora Heiman, *Ant colony optimization for the identification of non linear systems*, final project (University of Bucarest) (4 months).
- Catalin Raduinea, *Predictive control for a NCS for a positioning benchmark*, final project (University of Bucarest) (4 months).
- Gabriela Raduinea, *Particle Swarm Optimization for the design of LPV controllers*, final project, master ATSI (Supélec, University Paris XI) (5 months).
- Jorge Luis Reyespesantez, *Robust optimization for energy management*, final project, master ATSI (Supélec, University Paris XI), (5 months).
- Andrei Spanu, *Fault detection and isolation for a wind turbine*, final project, master ATSI (Supélec, University Paris XI), (5 months).

9. Bibliography

Major publications by the team in recent years

- [1] C. BONNET, J. PARTINGTON. *Analysis of fractional delay systems of retarded and neutral type*, in "Automatica", 2002, vol. 38, p. 1133–1138.
- [2] C. BONNET, J. PARTINGTON. *Stabilization of some fractional delay systems of neutral type*, in "Automatica", 2007, vol. 43, p. 2047–2053.
- [3] M. MALISOFF, F. MAZENC. *Constructions of Strict Lyapunov Functions*, Communications and Control Engineering Series, Springer-Verlag London Ltd., 2009.
- [4] F. MAZENC, P. BLIMAN. *Backstepping design for time-delay nonlinear systems*, in "IEEE Transactions on Automatic Control", January 2006, vol. 51, n^o 1, p. 149–154.
- [5] W. MICHIELS, S.-I. NICULESCU. *Stability and Stabilization of Time-Delay Systems. An Eigenvalue-Based Approach*, Advances in Design and Control, SIAM: Philadelphia, 2007, vol. 12.
- [6] S.-I. NICULESCU. *Delay Effects on Stability: a Robust Control Approach*, Lecture Notes in Control and Information Sciences, Springer, 2001, vol. 269.
- [7] S. OLARU, D. DUMUR. *Avoiding constraints redundancy in predictive control optimization routines*, in "IEEE Trans. Automat. Control", 2005, vol. 50, n^o 9, p. 1459–1465.
- [8] A. QUADRAT. *The fractional representation approach to synthesis problems: an algebraic analysis viewpoint. Part I: (Weakly) doubly coprime factorizations, Part II: Internal stabilization*, in "SIAM J. Control & Optimization", 2003, vol. 42, n^o 1, p. 266–299, 300–320.
- [9] A. QUADRAT. *On a general structure of the stabilizing controllers based on stable range*, in "SIAM J. Control & Optimization", 2004, vol. 42, n^o 6, p. 2264–2285.

- [10] G. SANDOU. *Particle swarm optimization: an efficient tool for the design of automatic control law*, in "European Control Conference", Budapest, Hungary, August 23rd-26th 2009.

Publications of the year

Doctoral Dissertations and Habilitation Theses

- [11] A. QUADRAT. *Systèmes et Structures : Une approche de la théorie mathématique des systèmes par l'analyse algébrique constructive*, University of Nice, September 2010, Habilitation à diriger des Recherches, Ph. D. Thesis.

Articles in International Peer-Reviewed Journal

- [12] C. BONNET, A. FIORAVANTI, J. PARTINGTON. *Stability of neutral systems with commensurate delays and poles asymptotic to the imaginary axis*, in "SIAM Journal of Control and Optimization", 2010, to appear.
- [13] H. BORSENBERGER, P. DESSANTE, G. SANDOU. *Unit Commitment with production cost uncertainty: a recourse programming method*, in "Journal of Energy and Power Engineering", 2010, to appear.
- [14] M. S. BOUDELLIOUA, A. QUADRAT. *Serre's reduction of linear functional systems*, in "Mathematics in Computer Science", 2010, to appear, DOI 10.1007/s11786-010-0057-y.
- [15] B. BRADU, P. GAYET, S.-I. NICULESCU, E. WITRANT. *Modeling of the very low pressure helium flow in the LHC cryogenic distribution line after a quench*, in "Cryogenics", 2010, vol. 50, p. 71–77.
- [16] J. CHEN, P. FU, S.-I. NICULESCU, Z. GUAN. *An eigenvalue perturbation approach to stability analysis, Part I: Eigenvalue series of matrix operator*, in "SIAM Journal on Optimization and Control", 2010, to appear.
- [17] J. CHEN, P. FU, S.-I. NICULESCU, Z. GUAN. *Eigenvalue perturbation with application to time-delay systems, Part II: When will zeros cross imaginary axis?*, in "SIAM Journal on Optimization and Control", 2010, to appear.
- [18] C. DE PERSIS, F. MAZENC. *Stability of quantized time-delay nonlinear systems: A Lyapunov-Krasovskii-functional approach*, in "Mathematics of Control, Signals, and Systems", 2010, vol. 21, p. 337–370.
- [19] P. FALUGI, S. OLARU, D. DUMUR. *Multi-model predictive control based on LMI. From the adaptation of the state-space model to the analytic description of the control law*, in "International Journal of Control", 2010, vol. 83, n^o 8, p. 1548–1563.
- [20] R. GIELEN, S. OLARU, W. HEEMELS, N. VAN DE WOUW, S.-I. NICULESCU. *On polytopic approximations as a modeling framework for systems with time-varying delays*, in "Automatica", 2010, vol. 46, n^o 3, p. 615–619.
- [21] R. GIELEN, S. OLARU, M. LAZAR, W. HEEMELS, N. VAN DE WOUW, S.-I. NICULESCU. *On polytopic inclusions as a modeling framework for systems with time-varying delays*, in "Automatica", 2010, vol. 46, n^o 3, p. 615 - 619.
- [22] A. GONÇALVES, A. FIORAVANTI, J. GEROMEL. *Filtering of discrete-time Markov jump linear systems with uncertain transition probabilities*, in "International Journal of Robust and Nonlinear Control", 2010, to appear.

-
- [23] A. GONÇALVES, A. FIORAVANTI, J. GEROMEL. *Markov jump linear systems and filtering through network transmitted measurements*, in "Signal Processing", 2010, vol. 90, n^o 10, p. 2842–2850.
- [24] M. E. HAJJI, F. MAZENC, J. HARMAND. *A mathematical study of a syntrophic relationship of a model of anaerobic digestion process*, in "Mathematical Biosciences and Engineering", 2010, vol. 7, p. 641–656.
- [25] X. LI, A. CELA, S.-I. NICULESCU, A. REAMA. *Some problems in the stability of networked-control systems with periodic scheduling*, in "International Journal of Control", 2010, vol. 83, n^o 5, p. 996–1008.
- [26] F. MAZENC, O. BERNARD. *Asymptotically stable interval observers for planar systems with complex poles*, in "IEEE Transactions on Automatic Control", 2010, vol. 55, p. 523–527.
- [27] F. MAZENC, O. BERNARD. *Interval observers for linear time-invariant systems with disturbances*, in "Automatica", 2010, Brief Paper, in press, DOI:10.1016/j.automatica.2010.10.019.
- [28] F. MAZENC, Z.-P. JIANG. *Global output feedback stabilization of a chemostat with an arbitrary number of species*, in "IEEE Transactions on Automatic Control", 2010, vol. 55, p. 2579–2575.
- [29] F. MAZENC, Z.-P. JIANG. *Persistence for a chemostat with many species.*, in "Dynamics of Continuous, Discrete and Impulsive Systems", 2010, to appear.
- [30] F. MAZENC, M. MALISOFF, M. DE QUEIROZ. *Uniform global asymptotic stability of adaptive cascaded nonlinear systems with unknown high-frequency gains*, in "Nonlinear Analysis", 2010, to appear.
- [31] F. MAZENC, M. MALISOFF. *Remarks on output feedback stabilization of two-species chemostats models*, in "Automatica", 2010, vol. 46, p. 739–724.
- [32] F. MAZENC, M. MALISOFF. *Stabilization of a chemostat model with Haldane growth functions and a delay in the measurements*, in "Automatica", 2010, vol. 46, p. 1428–1436.
- [33] F. MAZENC, M. MALISOFF. *Strict Lyapunov function constructions under LaSalle conditions with an application to Lotka-Volterra systems*, in "IEEE Transactions on Automatic Control", 2010, vol. 55, p. 841–854.
- [34] F. MAZENC, S.-I. NICULESCU. *Generating positive and stable solutions through delayed state feedback*, in "Automatica", 2010, to appear.
- [35] S.-I. NICULESCU, P. KIM, K. GU, P. LEE, D. LEVY. *Stability crossing boundaries of delay systems modeling immune dynamics in leukemia*, in "Discrete and Continuous Dynamical Systems - Series B (DCDS-B)", January 2010, vol. 15, n^o 1, p. 129–156.
- [36] S.-I. NICULESCU, M. PUTINAR. *A toric positivstellensatz with applications to time delay system*, in "Comptes rendus de l'académie des sciences (série Mathématiques)", 2010, to appear.
- [37] R. SIPAHI, S.-I. NICULESCU. *Stability of car following with human memory effects and automatic headway compensation*, in "Philosophical Transactions of the Royal Society A", 2010, to appear.

- [38] R. VILLAFUERTE, S. MONDIÉ, S.-I. NICULESCU. *Stability analysis and estimate of the region of attraction of a human respiratory model*, in "IMA Journal on Mathematical Control and Information", 2010, to appear.
- [39] E. WITRANT, A. D'INNOCENZO, G. SANDOU, F. SANTUCCI, M. DI BENEDETTO, A. ISAKSSON, K. JOHANSSON, S.-I. NICULESCU, S. OLARU, E. SERRA, S. TENNINA, U. TIBERI. *Wireless ventilation control for large-scale systems: The mining industrial case*, in "International Journal of Robust and Nonlinear Control", 2010, vol. 20, n^o 2, p. 226–251.
- [40] E. WITRANT, S.-I. NICULESCU. *Modeling and control of large convective flows with time-delays*, in "Mathematics in Engineering, Science and Aerospace", 2010, vol. 1, n^o 2, p. 191–205.

International Peer-Reviewed Conference/Proceedings

- [41] H. BORSENBARGER, P. DESSANTE, G. SANDOU. *Unit commitment with production cost uncertainty, a recourse programming method*, in "Proceedings of the IFAC Conference on Control Methodologies and Technology for Energy Efficiency", Villamoure (Portugal), March 2010.
- [42] M. S. BOUDELLOUA, A. QUADRAT. *Further results on Serre's reduction of multidimensional linear systems*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.
- [43] T. CLUZEAU, A. QUADRAT. *Module structure of classical multidimensional linear systems appearing in mathematical physics*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.
- [44] T. CLUZEAU, A. QUADRAT. *Serre's reduction of linear partial differential systems based on holonomy*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.
- [45] T. CLUZEAU, A. QUADRAT. *Symmetries, parametrizations and potentials of multidimensional linear systems*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.
- [46] A. FIORAVANTI, C. BONNET, H. OZBAY. *Stability of fractional neutral systems with multiple delays and poles asymptotic to the imaginary axis*, in "IEEE Conference on Decision and Control", Atlanta, USA, 2010, 2010.
- [47] A. FIORAVANTI, C. BONNET, H. OZBAY, S.-I. NICULESCU. *A numerical method to find stability windows and unstable poles for linear neutral time-delay systems*, in "9th IFAC Workshop on Time-Delay Systems", 2010.
- [48] W. HEEMELS, N. VAN DE WOUW, R. GIELEN, M. DONKERS, L. HETEL, S. OLARU, M. LAZAR, J. DAAFOUZ, S.-I. NICULESCU. *Comparison of overapproximation methods for stability analysis of networked control systems*, in "Proceedings of the 13th ACM international conference on Hybrid systems: computation and control", ACM, 2010, p. 181–190.
- [49] S. IGHOUBRIOUEN, G. SANDOU. *Optimisation multi-objectifs par essaim particulaire pour la synthèse de correcteurs*, in "Proceedings of 10th Conférence Internationale Francophone d'Automatique", Nancy (France), June 2010.

-
- [50] B. LIACU, C. MENDEZ-BARRIOS, S.-I. NICULESCU, S. OLARU. *Some remarks on the fragility of PD controllers for SISO systems with I/O delays*, in "4th International Conference on System Theory and Control, Sinaia, Romania", 2010.
- [51] B. LIACU, C. MENDEZ-BARRIOS, S.-I. NICULESCU, S. OLARU. *Some remarks on the fragility of transparency and stability in general 4-channel architecture for bilateral teleoperation with delay*, in "International Conference on Control, Automation and Systems, Korea", 2010.
- [52] W. LOMBARDI, A. LUCA, S. OLARU, S.-I. NICULESCU, J. CHEONG. *Feedback Stabilization and motion synchronization of systems with time-delay in the communication network*, in "9th IFAC Workshop on Time-Delay Systems", 2010.
- [53] W. LOMBARDI, A. LUCA, S. OLARU, S.-I. NICULESCU. *State admissible sets for discrete systems under delay constraints*, in "Proc. IEEE ACC", 2010.
- [54] F. MAZENC, O. BERNARD. *Time-varying interval observers for linear systems with additive disturbances*, in "IFAC Symposium on Nonlinear Control Systems (NOLCOS 2010)", 2010.
- [55] F. MAZENC, M. MALISOFF, M. DE QUEIROZ. *Model-based nonlinear control of the human heart rate during treadmill exercising*, in "IEEE Conference on Decision and Control", Atlanta, USA, 2010, 2010.
- [56] F. MAZENC, M. MALISOFF, M. DE QUEIROZ. *On uniform global asymptotic stability of adaptive systems with unknown control gains*, in "American Control Conference", Baltimore, MD, 2010, 2010.
- [57] F. MAZENC, M. MALISOFF. *Further results on robust output feedback control for the chemostat dynamics*, in "IEEE Conference on Decision and Control", Atlanta, USA, 2010, 2010.
- [58] F. MAZENC, M. MALISOFF. *Stabilization of two-species chemostats with delayed measurements and Haldane growth functions*, in "American Control Conference", Baltimore, MD, 2010, 2010.
- [59] F. MAZENC, S.-I. NICULESCU. *Stabilizing controllers for delays systems subject to positivity constraints*, in "9th IFAC Workshop on Time Delay Systems, TDS 2010", 2010.
- [60] H. OZBAY, H. BENJELLOUN, C. BONNET, J. CLAIRAMBAULT. *Absolute stability of a system with distributed delays modeling cell dynamics in leukemia*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.
- [61] H. OZBAY, H. BENJELLOUN, C. BONNET, J. CLAIRAMBAULT. *Stability conditions for a system modeling cell dynamics in leukemia*, in "9th IFAC Workshop on Time-Delay Systems", MTNS, 2010.
- [62] A. QUADRAT. *Extendability of multidimensional linear systems*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.
- [63] A. QUADRAT. *Purity filtration of 2-dimensional linear systems*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.

- [64] A. QUADRAT, D. ROBERTZ. *Controllability and differential flatness of linear analytic ordinary differential systems*, in "Proceedings of 19th International Symposium on Mathematical Theory of Networks and Systems", Budapest (Hungary), MTNS, 05-09/07 2010.
- [65] G. SANDOU, S. OLARU, E. WITRANT, S.-I. NICULESCU. *Receding horizon control: an effective methodology for energy management in complex systems*, in "Proceedings of the IFAC Conference on Control Methodologies and Technology for Energy Efficiency", Villamoure (Portugal), March 2010.

National Peer-Reviewed Conference/Proceedings

- [66] H. BORSENBERGER, P. DESSANTE, G. SANDOU. *Optimisation de la production dans un réseau d'énergie soumis à des incertitudes sur le coût de production*, in "Proceedings of the 11ième congrès de la société française de recherche opérationnelle et d'aide à la décision, ROADEF 2010", Toulouse (France), February 2010.
- [67] A. GONÇALVES, A. FIORAVANTI, J. GEROMEL. *Filtering for discrete-time Markov jump systems with network transmitted model*, in "IEEE Conference on Decision and Control", Atlanta, USA, 2010, December 2010.
- [68] A. GONÇALVES, A. FIORAVANTI, J. GEROMEL. *Filtragem de medidas transmitidas por rede usando sistemas com saltos Markovianos a tempo discreto*, in "XVIII Congresso Brasileiro de Automática (CBA2010)", September 2010.
- [69] R. KOROGUI, A. FIORAVANTI, J. GEROMEL. *Análise de estabilidade e controle H_∞ de sistemas lineares com atraso*, in "XVIII Congresso Brasileiro de Automática (CBA2010)", September 2010.

Workshops without Proceedings

- [70] C. BONNET. *H_∞ -stability of neutral delay systems and neutral fractional systems*, in "Workshop Analyse et commande des systèmes (MACS4)", 2010.
- [71] H. OZBAY, C. BONNET, H. BENJELLOUN, J. CLAIRAMBAULT. *Global stability analysis of a system modeling cell dynamics in acute myelogenous leukemia*, in "The Third Conference on Computational and Mathematical Population Dynamics (CMPD3)", June 2010, 186.

Scientific Books (or Scientific Book chapters)

- [72] A. QUADRAT. *An introduction to constructive algebraic analysis and its applications*, in "Les cours du CIRM", Journées Nationales de Calcul Formel (2010), CIRM, 2010, vol. 1, n^o 2, p. 281–471.
- [73] G. SANDOU, S. IGHOBRIOEN. *Using monobjective and multiobjective particle swarm optimization for the tuning of process control laws*, in "Particle Swarm Optimization: Theory, Techniques, Applications", A. E. OLSSON (editor), Nova Science Publishers, 2010, p. 1–30.

Research Reports

- [74] M. S. BOUDELLOUA, A. QUADRAT. *Serre's reduction of linear functional systems*, INRIA, 2010, <http://hal.archives-ouvertes.fr/inria-00459722/fr/>.

- [75] A. QUADRAT. *An introduction to constructive algebraic analysis and its applications*, INRIA, 2010, <http://hal.archives-ouvertes.fr/inria-00506104/fr/>.

References in notes

- [76] M. ADIMY, F. CRAUSTE, A. EL. *Discrete maturity-structured model of cell differentiation with application to acute myeloid leukemia*, in "J. Biological systems", 2008, vol. 16, n^o 3, p. 395–424.
- [77] J. E. BJORK. *Rings of Differential Operators*, North Holland, 1979.
- [78] J. E. BJORK. *Analytic \mathcal{D} -modules and Applications*, Academic Press, 1987, vol. 2.
- [79] F. CHYZAK, A. QUADRAT, D. ROBERTZ. *Effective algorithms for parametrizing linear control systems over Ore algebras*, in "Appl. Algebra Engrg. Comm. Comput.", 2005, vol. 16, p. 319–376.
- [80] F. CHYZAK, A. QUADRAT, D. ROBERTZ. OREMODULES: *A symbolic package for the study of multidimensional linear systems*, in "Applications of Time-Delay Systems", J. CHIASSON, J.-J. LOISEAU (editors), Lecture Notes in Control and Information Sciences 352, Springer, 2007, p. 233–264, <http://wwwb.math.rwth-aachen.de/OreModules>.
- [81] F. CHYZAK, B. SALVY. *Non-commutative elimination in Ore algebras proves multivariate identities*, in "J. Symbolic Comput.", 1998, vol. 26, p. 187–227.
- [82] T. CLUZEAU, A. QUADRAT. *Factoring and decomposing a class of linear functional systems*, in "Linear Algebra Appl.", 2008, vol. 428, p. 324–381.
- [83] T. CLUZEAU, A. QUADRAT. *Serre's reduction of linear partial differential systems with holonomic adjoints*, 2010, submitted for publication.
- [84] T. CLUZEAU, A. QUADRAT. ORE MORPHISMS: *A homological algebraic package for factoring, reducing and decomposing linear functional systems*, in "Topics in Time-Delay Systems: Analysis, Algorithms and Control", J.-J. LOISEAU, W. MICHIELS, S.-I. NICULESCU, R. SIPAHI (editors), Lecture Notes in Control and Information Sciences 388, Springer, 2010, p. 179–196, <http://www-sop.inria.fr/members/Alban.Quadrat/OreMorphisms/index.html>.
- [85] S. C. COUTINHO, M. P. HOLLAND. *Module structure of rings of differential operators*, in "Proc. London Math. Soc.", 1988, vol. 57, p. 417–432.
- [86] A. FABIAŃSKA, A. QUADRAT. *Applications of the Quillen-Suslin theorem to multidimensional systems theory*, in "Gröbner Bases in Control Theory and Signal Processing", H. PARK, G. REGENSBURGER (editors), Radon Series on Computation and Applied Mathematics, de Gruyter, 2007, vol. 3, p. 23–106, <http://wwwb.math.rwth-aachen.de/QuillenSuslin>.
- [87] A. FIORAVANTI, C. BONNET, H. OZBAY, S.-I. NICULESCU. *Stability windows and unstable root-loci for linear fractional time-delay systems*, in "The 18th IFAC World Congress", Milan, Italy, September 2011, submitted.

-
- [88] M. FLIESS, J. LÉVINE, P. MARTIN, P. ROUCHON. *Flatness and defect of nonlinear systems: introductory theory and examples*, in "Internat. J. Control", 1995, vol. 61, p. 1327–1361.
- [89] A. KANDRI-RODY, V. WEISPFENNING. *Non-commutative Gröbner Bases in Algebras of Solvable Type*, in "J. Symbolic Computation", 1990, vol. 9, p. 1–26.
- [90] H. KITANO. *Biological robustness*, in "Nature", 2004, vol. 5, 826837.
- [91] H. KOMATSU. *Relative cohomology of sheaves of solutions of differential equations*, in "Hyperfunctions and Pseudo-Differential Equations", H. KOMATSU (editor), Lecture Notes in Mathematics 287, Springer, 1971, p. 192–263.
- [92] R. KOROGUI, A. FIORAVANTI, J. GEROMEL. *On a rational transfer function-based approach to H_∞ filtering design for time-delay linear systems*, in "IEEE Transactions on Signal Processing", 2009, to appear.
- [93] R. KOROGUI, A. FIORAVANTI, J. GEROMEL. *H_∞ output feedback for time-delay linear systems: a rational transfer function based approach*, September 2011, submitted.
- [94] H. KREDEL. *Solvable Polynomial Rings*, Shaker, Aachen, 1993.
- [95] B. MALGRANGE. *Systèmes à coefficients constants*, in "Séminaire Bourbaki 1962/63", N. BOURBAKI (editor), Séminaire Bourbaki, Benjamin, 1963, p. 1–11.
- [96] F. MAZENC, M. MALISOFF, J. HARMAND. *Stabilization in a two-species chemostat with Monod growth functions*, in "IEEE Transactions on Automatic Control", 2009, vol. 54, p. 855–861.
- [97] F. MAZENC, S.-I. NICULESCU, O. BERNARD. *Exponentially stable interval observers for linear systems with delay*, 2010, Submitted.
- [98] H. OZBAY, C. BONNET, H. BENJELLOUN, J. CLAIRAMBAULT. *Stability analysis of a distributed delay system modeling cell dynamics in leukemia*, 2010, in revision.
- [99] H. OZBAY, C. BONNET, A. FIORAVANTI. *PID controller design for fractional-order systems with time delays*, 2008, in revision.
- [100] H. PILLAI, S. SHANKAR. *A behavioural approach to control of distributed systems*, in "SIAM J. Control Optim.", 1998, vol. 37, p. 388–408.
- [101] J. W. POLDERMAN, J. C. WILLEMS. *Introduction to Mathematical Systems Theory. A Behavioral Approach*, Texts in Applied Mathematics 26, Springer, 1998.
- [102] J.-F. POMMARET, A. QUADRAT. *Algebraic analysis of linear multidimensional control systems*, in "IMA J. Math. Control and Inform.", 1999, vol. 16, p. 275–297.
- [103] J.-F. POMMARET, A. QUADRAT. *Localization and parametrization of linear multidimensional control systems*, in "Systems Control Lett.", 1999, vol. 37, p. 247–260.

- [104] A. QUADRAT, D. ROBERTZ. *Computation of bases of free modules over the Weyl algebras*, in "Journal of Symbolic Computation", 2007, vol. 42, p. 1113–1141.
- [105] A. QUADRAT, D. ROBERTZ. *Baer's extension problem for multidimensional linear systems*, in "Proceedings of MTNS 2008", Blacksburg, Virginia (USA), 2008.
- [106] D. QUILLEN. *Projective modules over polynomial rings*, in "Invent. Math.", 1976, vol. 36, p. 167–171.
- [107] S. T. STAFFORD. *Module structure of Weyl algebras*, in "J. London Math. Soc.", 1978, vol. 18, p. 429–442.
- [108] A. A. SUSLIN. *Projective modules over polynomial rings are free*, in "Soviet Math. Dokl.", 1976, vol. 17, p. 1160–1164.