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Activity Report

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2. Overall Objectives

2.1. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable solution of geometric and algebraic problems, which are encountered in fields such as CAGD (Computer Aided Geometric Design), robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, differential geometry and computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology analysis, and computation with algebraic curves and surfaces.

These geometric investigations lead to algebraic questions, and particularly to the resolution of polynomial equations. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in practical problems.

Approximate numerical computation, usually opposed to symbolic computation, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is defined according to three broad topics: modeling, computing and analysis, in connection with effective algebraic geometry.

3.2. Algebraic Geometric Modeling

We are investigating geometric modeling approaches, based on non-discrete models, mainly of semi-algebraic type. Such non-linear models are able to capture efficiently complexes shapes, using few data. However, they required specific methods to handle and solve the underlying non-linear problems.

Effective algebraic geometry is a naturally framework for handling such representations, in which we are developing new methods to solve these non-linear problems. The framework not only provides tools for modeling but also, it makes it possible to exploit the geometric properties of these algebraic varieties, in order to improve this modeling work. To handle and control geometric objects such as parameterised curves and surfaces or their implicit representations, we consider in particular projections techniques. We focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions we want to compute. Among these formulations, we study in particular *residual* and *toric* resultant theory. The latter approach relates the generic properties of the solutions of polynomial equations, to the geometry of the Newton polytope associated with the polynomials. These tools allow to change geometric representations by computing an implicit model from a parameterised one. We are interested in dedicated methods for solving these type of problems.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations. The use of local algebraic representations also raises problems of approximation and reconstruction, on which we are working on.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. In addition to the development of symbolic geometric computations that exploit these invariant properties, we are also interested in developing compact representations of shapes, based on algebraic/symbolic descriptions. Our aim is to improve geometric computing performances, by using smaller input data, with better properties of approximation and certified computation.

3.3. Algebraic Geometric Computing

The underlying representation behind the geometric model we consider are often of algebraic type. Computing with such models raises algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases. We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

We are also interested in subdivision methods, which are able to localise efficiently the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue in analysing these methods is how to obtain good complexity bounds by exploiting the structure of the problem. Many algebraic problems can be reformulated in terms of linear algebra questions. Thus, it is not surprising to see that complexity analysis of our methods leads to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and their implications in solving polynomial equations.

When solving a system of polynomials equations, a first treatment is to transform it into several simpler subsystems when possible. The problem of decomposition and factorisation is thus also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decomposing a variety into irreducible components.

3.4. Algebraic Geometric Analysis

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many context, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

4. Application Domains

4.1. Shape modeling

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, etc. Optimising these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts that we are developing in Computer Aided Geometric Design.

4.2. Shape processing

Many problems encounter in the application of computer sciences started from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where *Distance geometry* plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tends to be more and more easily used yield large set of data points from which one has to recover compact geometric model. We are working in collaboration with groups in agronomy on the problems of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems. Geometry is also highly involved in the numerical simulation of physical problems such as heat conduction, ship hull design, blades and turbines analysis, mechanical stress analysis. We apply our algebraic-geometric techniques in the isogeometric approach which use the same (bspline) formalism to represent both the geometry and the solutions of partial differential equations on this geometry.

5. Software

5.1. Mathemagix, a free computer algebra environment

Participants: Bernard Mourrain, Daouda N'Diatta, Elias Tsigaridas, Angelos Mantzaflaris.

http://www.mathemagix.org/

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments are based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on the embeding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

The project aims at building a bridge between symbolic computation and numerical analysis. It is structured by collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

- REALROOT: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms; exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterised, implicit with different type of coefficients; algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler AXEL.

Collaborators: Grégoire Lecerf, Olivier Ruatta, Joris van der Hoeven and Philippe Trébuchet.

5.2. Axel, a geometric modeler for algebraic objects

Participants: Angelos Mantzaflaris, Bernard Mourrain, Meriadeg Perrinel, Gang Xu, Julien Wintz [Dream].

http://axel.inria.fr.

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The sofwtare contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

The developments related to isogeometric analysis in Exciting have been integrated as dedicated plugins. Optimisation techniques and solvers for partial differential equations developed by R. Duvigneau (OPALE) have been connected.

A new version of the algebraic-geometric modelers is developed by Meriadeg Perinnel to connect it to the platform Dtk in order to provide a better modularity and a better interface to existing computation facilities and geometric rendering interface.

The package is distributed as binary packages for Linux as well as for MacOSX. It is hosted at the INRIA's gforge (http://gforge.inria.fr) and referenced by many leading software websites such as http://apple.com. The first version of the software has been downloaded more than 15000 times, since it is available.

5.3. Multires, a Maple package for multivariate resolution problems

Participants: Laurent Busé, Bernard Mourrain.

http://www-sop.inria.fr/galaad/software/multires/.

The Maple package MULTIRES contains a set of routines related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, but not their effectiveness, which is achieved in a more adapted environment such as MATHEMAGIX. It provides methods for building matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay, Jouanolou for projective resultants, Bezout and (sparse) resultants on a toric variety, residual resultants of complete intersections, functions for computing the degree of residual resultants, algorithms for the geometric decomposition of algebraic varieties. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residues for complete intersections of dimension 0.

Collaborators: Ioannis Emiris, Olivier Ruatta and Philippe Trébuchet.

5.4. diffalg, a Maple package for differential algebra

Participant: Evelyne Hubert.

The Maple package *diffalg* is a collection of routines to handle systems of polynomial differential equations and inequations. The functionalities include differential elimination, expansion of the solutions into formal power series and analysis of singular solutions. The underlying theory and terminology belongs to differential algebra.

Collaborators: François Boulier and François Lemaire from University of Lille.

5.5. AIDA, a Maple package for algebraic invariants and their differential algebra

Participant: Evelyne Hubert.

http://www-sop.inria.fr/members/Evelyne.Hubert/aida/.

The Maple AIDA package is a collection of routines to explore algebra of differential invariants: computation of generating sets of invariants, rewritings, syzygies, and their differential analogues. The package builds on the Maple libraries Groebner, Vessiot and diffalg.

6. New Results

6.1. Geometric modeling with semi-algebraic sets

6.1.1. The Hilbert scheme of points and its link with border basis

Participants: Mariemi Alonso, JÃ©rÃ´me Brachat, Bernard Mourrain.

In this work, we give new explicit representations of the Hilbert scheme of μ points in $\mathbb{P}^r$ as a projective subvariety of a Grassmanniann variety. This new explicit description of the Hilbert scheme is simpler than the previous ones and global. It involves equations of degree 2. We show how these equations are deduced from the commutation relations characterizing border bases. Next, we consider infinitesimal perturbations of an input system of equations on this Hilbert scheme and describe its tangent space. We propose an effective criterion to test if it is a flat deformation, that is if the perturbed system remains on the Hilbert scheme of the initial equations. This criterion involves in particular formal reduction with respect to border bases. A paper has been submitted for publication [25].

6.1.2. Matrix-based implicit representations of rational algebraic curves and applications

Participants: Laurent Busé, Thang Luu Ba.

Given a parameterization of an algebraic rational curve in a projective space of arbitrary dimension, we introduce and study a new implicit representation of this curve which consists in the locus where the rank of a single matrix drops. Then, we illustrate the advantages of this representation by addressing several important problems of Computer Aided Geometric Design: The point-on-curve and inversion problems, the computation of singularities and the intersection problem between two rational curves. This work has been published in [16].

6.1.3. Convolution surfaces based on polygons for infinite and Compact Support Kernels

Participant: Evelyne Hubert.

We provided mathematical formulae to create 3D smooth shapes fleshing out a skeleton made of line segments and polygons. The boundary of the shape created is a level set of a convolution function. This latter is obtained through the integration of a kernel function along the skeleton. The technique has been proposed and developed for geometric modeling in computer graphics. Given the additivity of integration, the convolution function of a complex skeleton is the sum of the convolution functions for its simpler elements. Providing the closed form formulae for the convolution function generated by a polygon is the main contribution of [27]. We apply Green's theorem and improve on previous results in several ways. First we do not require the prior triangulation of the polygon. Then, we obtain formulae for complete families of kernels, either with infinite or compact supports. The families are indexed by an integer that controls sharpness and smoothness. Full generality is achieved through recurrence formulae. Last, but not least, the geometric computations needed, in the case of compact support kernels, are restricted to intersections of spheres with line segments, rather than intersections of spheres with triangles in previous works.

6.1.4. Optimal analysis-aware parameterization of computational domain in isogeometric analysis

Participants: Gang Xu, Régis Duvigneau [OPALE], André Galligo, Bernard Mourrain.

In isogeometric analysis (IGA for short) framework, computational domain is exactly described using the same representation as that employed in the CAD process. For a CAD object, we can construct various computational domains with the same shape but with different parameterizations. One basic requirement is that the resulting parameterization should have no self-intersections. In [24], a linear and easy-to-check sufficient condition for injectivity of planar B-spline parameterization is firstly proposed. By an example of 2D thermal conduction problem, we show that different parameterization of computational domain has different impact on the simulation result and efficiency in IGA. For problems with exact solutions, we propose a shape optimization method to obtain optimal parameterization of computational domain. The proposed injective condition is used to check the injectivity of initial parameterization constructed by discrete Coons method. Several examples and comparisons are presented to show the effectiveness of the proposed method. Compared with the initial parameterization during refinement, the optimal parameterization can achieve the same accuracy but with less degrees of freedom. This work is published in the proceedings of a conference on Geometric Modeling and Processing 2010 [24]. The generalization to 3D case is studied in [29].

6.1.5. Parametrization of computational domain in isogeometric analysis: methods and comparison

Participants: Gang Xu, Régis Duvigneau [OPALE], André Galligo, Bernard Mourrain.

Parameterization of computational domain plays an important role in isogeometric analysis as mesh generation in finite element analysis. In [30], we investigate this problem in the 2D case, i.e, how to parametrize the computational domains by planar B-spline surface from the given CAD objects (four boundary planar B-spline curves). Firstly, two kinds of sufficient conditions for injective B-spline parameterization are derived with respect to the control points. Then we show how to find good parameterizations of computational domains by solving a constraint optimization problem, in which the constraint condition is the injectivity sufficient conditions of planar B-spline parametrization, and the optimization term is the minimization of quadratic energy functions related to the first and second derivatives of planar B-spline parameterization. By using this method, the resulted parameterization has no self-intersections, and the isoparametric net has good uniformity and orthogonality. After introducing a posteriori error estimation for isogeometric analysis, we propose r-refinement method to optimize the parameterization by repositioning the inner control points such that the estimated error is minimized. Several examples are tested on isogeometric heat conduction problem to show the effectiveness of the proposed methods and the impact of the parameterization on the quality of the approximation solution. Comparison examples with known exact solutions are also presented. This work is being submitted for publication and is available at http://hal.inria.fr/inria-00530758/en/.

6.1.6. Tree reconstructions from scanned point clouds

Participants: Evelyne Hubert, Bernard Mourrain, Xinghua Song.

We present a new method for extracting skeletal points from point clouds of laser scanned trees. First, a vector flow of the input data is computed, which can be used to determine a cross section plane through each data point. Then, for a given data point $\mathbf{p}_i$ and its associated cross section plane π_i, the minimal bounding circle of the neighbor points of $\mathbf{p}_i$ which are close to the plane π_i is computed. The center of the minimal bounding circle is denoted as a skeletal point.

This work is done in collaboration with Chakkrit Preuksakarn, Fréderic Boudon and Christophe Godin (Virtual plants) and in relation with the Arc PlantScan3D 7.1.1.

6.2. Algebraic Geometric Computing

6.2.1. Multivariate continued fraction solvers for polynomial equations

Participants: Angelos Mantzaflaris, Bernard Mourrain, Elias Tsigaridas.

We develop on a correspondence between the coefficients of a multivariate polynomial represented in the Bernstein basis and in a tensor-monomial basis, which leads to homography representations of polynomial functions, that use only integer arithmetic (in contrast to Bernstein basis) and are feasible over unbounded regions. Then, we study an algorithm to split this representation and we obtain a subdivision scheme for the domain of multivariate polynomial functions. This implies a new algorithm for real root isolation, MCF, that generalizes the Continued Fraction (CF) algorithm of univariate polynomials.

A partial extension of *Vincent's Theorem* for multivariate polynomials is presented, which allows us to prove the termination of the algorithm. Bounding functions, projections and preconditioning are employed to speed up the scheme. The resulting isolation boxes have optimized rational coordinates, corresponding to the first terms of the *continued fraction expansion* of the real roots. Finally, we present new complexity bounds for a simplified version of the algorithm in the bit complexity model, and also bounds in the real RAM model for a family of subdivision algorithms in terms of the real *condition number* of the system. Examples computed with our C++ implementation illustrate the practical aspects of our method.

This work [28] is accepted for publication in the Journal of Theoretical Computer Science.

6.2.2. The DMM bound: multivariate (aggregate) separation bounds

Participants: Ioannis Emiris, Bernard Mourrain, Elias Tsigaridas.

In [21], we derive aggregate separation bounds, named after Davenport-Mahler-Mignotte, on the isolated roots of polynomial systems, specifically on the minimum distance between any two such roots. The bounds exploit the structure of the system and the height of the sparse (or toric) resultant by means of mixed volume, as well as recent advances on aggregate root bounds for univariate polynomials, and are applicable to arbitrary positive dimensional systems. We improve upon Canny's gap theorem by a factor of $O(d^{n-1})$, where d bounds the degree of the polynomials, and n is the number of variables. One application is to the bitsize of the eigenvalues and eigenvectors of an integer matrix, which also yields a new proof that the problem is polynomial. We also compare against recent lower bounds on the absolute value of the root coordinates by Brownawell and Yap, obtained under the hypothesis there is a 0-dimensional projection. Our bounds are in general comparable, but exploit sparseness; they are also tighter when bounding the value of a positive polynomial over the simplex. Our analysis provides a precise asymptotic upper bound on the number of steps that subdivision-based algorithms perform in order to isolate all real roots of a polynomial system. This leads to the first complexity bound of Milne's algorithm in 2D.

6.2.3. Semi-algebraic set approximation methods by means of subdivision

Participants: Angelos Mantzaflaris, Bernard Mourrain.

Semi-algebraic sets naturally occur when dealing with implicit models and boolean operations between them. Most notably, the computation of topology of a real curve or surface as well as the arrangement of several implicit shapes are special cases of semi-algebraic set computation. In this work, we study algorithms and techniques to efficiently and in a *certified* way compute the connected components of semi-algebraic sets, which are usually given by intersection or union of conjunctions of bi-variate equalities and inequalities. For any given precision, a flexible representation is a polygonal and isotopic approximation of the underlying exact set. We have derived an algorithm [22] to provide such approximations. The main idea is to localize the *boundary curves* by subdividing the space and then deduce their shape within small enough cells using only boundary information. Then a systematic traversal of the boundary curve graph yields polygonal regions *isotopic* to the connected components of the semi-algebraic-set. Space subdivision is supported by a kd-tree structure and localization is done using Bernstein representation. We have developed a C++ implementation of this method in the 2D case, and we continue to work towards computing in 3D space.

6.2.4. Symmetric tensor decomposition

Participants: Jérôme Brachat, Pierre Comon, Bernard Mourrain, Elias Tsigaridas.

In [14], we present an algorithm for decomposing a symmetric tensor, of dimension n and order d as a sum of rank-1 symmetric tensors, extending the algorithm of Sylvester devised in 1886 for binary forms. We recall the correspondence between the decomposition of a homogeneous polynomial in n variables of total degree d as a sum of powers of linear forms (Waring's problem), incidence properties on secant varieties of the Veronese variety and the representation of linear forms as a linear combination of evaluations at distinct points. Then we reformulate Sylvester's approach from the dual point of view. Exploiting this duality, we propose necessary and sufficient conditions for the existence of such a decomposition of a given rank, using the properties of Hankel (and quasi-Hankel) matrices, derived from multivariate polynomials and normal form computations. This leads to the resolution of polynomial equations of small degree in non-generic cases. We propose a new algorithm for symmetric tensor decomposition, based on this characterization and on linear algebra computations with these Hankel matrices. The impact of this contribution is two-fold. First it permits an efficient computation of the decomposition of any tensor of sub-generic rank, as opposed to widely used iterative algorithms with unproved global convergence (e.g. Alternate Least Squares or gradient descents). Second, it gives tools for understanding uniqueness conditions, and for detecting the rank.

6.2.5. Singular factors of rational plane curves

Participant: Laurent Busé.

In this work, a complete factorization of the invariant factors of resultant matrices built from birational parameterizations of rational plane curves is given in terms of the singular points of the curve and their multiplicity graph. This result allows to prove the validity of some conjectures about these invariants stated by Chen, Wang and Liu. As a byproduct, we also give a complete factorization of the D-resultant, originally introduced by Abhyankar for polynomial parameterizations, for rational functions in terms of the similar data extracted from the multiplicities.

This is a joint work with Carlos D'Andrea, University of Barcelona.

6.3. Randomness, Approximation, Certification

6.3.1. Approximate GCD of several univariate polynomials, small degree perturbations

Participants: Mohamed Elkadi, André Galligo, Thang Luu Ba.

We consider the following computational problem, posed by von sur Gathen and al: Given a family of generic univariate polynomials $f := (f_0, ..., f_s)$, contruct an algorithm to find polynomial pertubations $u := (u_0, ..., u_s)$ with "small" degrees such that the GCD of the perturbed $f + u$ has a "large" degree. In this work, we propose an algorithm which solves this problem in polynomial time under a generic condition generalizing the normal degree sequence used by von sur Gathen and al for the case $s = 1$.

6.3.2. Hierarchical spline approximation of the signed distance function

Participants: Bert Juettler [J. Kepler Univ, Linz], Adrien Poteaux [J. Kepler Univ, Linz], Xinghua Song.

In [23], we present a method to approximate the signed distance function of a smooth curve or surface by using polynomial splines over hierarchical T-meshes (PHT splines). In particular, we focus on closed parametric curves in the plane and implicitly defined surfaces in space.

6.3.3. A symbolic-numeric algorithm for computing the alexander polynomial of a plane curve singularity.

Participants: Madalina Hodorog [RICAM, Linz], Bernard Mourrain, Josef Schicho [RICAM, Linz].

We give a symbolic-numeric algorithm for computing the Alexander polynomial of each singularity of a plane complex algebraic curve defined by a polynomial with coefficients of limited accuracy, i.e. the coefficients are both exact and inexact data. We base the algorithm on combinatorial methods from knot theory which we combine with computational geometry algorithms in order to compute efficient and accurate results. Nonetheless the problem we are dealing with is ill-posed, in the sense that tiny perturbations in the coefficients of the defining polynomial cause huge errors in the computed results. This work was presented at 12th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing, Timisoara, Romania [26].

7. Other Grants and Activities

7.1. National Initiatives

7.1.1. PlantScan3D

PlantScan3D is an ARC between coordinated by the EPI Virtual Plants (UMR DAP, INRIA-CIRAD, Montpellier), with the EPI Galaad (INRIA, Méditerranée) and Evasion (INRIA Rhône-Alpes, Grenoble).

A close collaboration between specialists in plant structures modelling, algebraic geometry, and 3D computer graphic is required to address plant structure reconstruction from laser scanned point clouds. Indeed it is required to take into account efficiently knowledge from topology and geometry to allow mapping and reconstruction of data despite noise, occlusions, and thinness of structure. The objective of the project is to provide as output a compact geometrical model that model smoothly branching point of tubular structure and organs (like leaves). At the end, this model should make it possible an interactive visualisation and automatize different measurement operators needed by biological partners.

More information available at http://plantscan3d.gforge.inria.fr/wiki.

7.2. European actions

7.2.1. SAGA

SAGA (ShApe, Geometry and Algebra, 2008-2012) is a Marie-Curie Initial Training Network of the call FP7-PEOPLE-2007-1-1-ITN.

The project aims at promoting the interaction between Geometric Modeling and Real Algebraic Geometry and, in general, at strengthening interdisciplinary and inter-sectorial research and development concerning CAD/CAM. Its objective is also to train a new generation of researchers familiar with both academic and industry viewpoints, while supporting the cooperation among the partners and with other interested collaborators in Europe. The partners are:

- SINTEF, Oslo, Norway (Leader);
- University of Oslo, Norway;
- Johannes Kepler Universitaet Linz, Austria;
- Universidad de Cantabria, Santander, Spain;
- Vilniaus Universitetas, Lithuany;
- National and Kapodistrian University of Athens, Greece;
- INRIA Méditerranée, France;
- GraphiTech, Italy;
- Kongsberg SIM GmbH, Austria;
- Missler Software, France;

More information available at http://saga-network.eu/.

The Ph.D. thesis of Angelos Mantzaflaris on robust algebraic methods for geometric computations is supported by the Marie-Curie ITN SAGA.

7.2.2. Exciting

Exciting – Exact geometry simulation for optimized design of vehicles and vessels – FP7-CP-SST-2007-RTD-1- 218536 (2008–2011).

This project focuses on computational tools for the optimized design of functional free-form surfaces. Specific applications are ship hulls and propellers in naval engineering and car components, frames, and turbochargers in the automotive and railway transportation industries. The objective is to base the corresponding computational tools on the same exact representation of the geometry. This should lead to huge benefits for the entire chain of design, simulation, optimization, and life cycle management, including a new class of computational tools for fluid dynamics and solid mechanics, simulations for vehicles and vessels based. This seamless integration of CAD and FEM will have direct applications in product design, simulation and optimization of core components of vehicles and vessels. The partners are:

- Johannes Kepler University, Linz Autriche (Leader);
- SINTEF, Oslo, Norway;
- Siemens AG, Germany;
- National Technical University of Athens, Greece;
- Hellenic Register of Shipping, Greece;
- University of Technology, Munich Germany;
- INRIA Méditerranée, France;
- VA Tech Hydro, Austria;
- Det Norske Veritas AS, Norway.

More information available at http://exciting-project.eu/.

7.2.3. DECONSTRUCT

Participants: Alessandra Bernardi, Jérôme Brachat, Pierre Comon, Bernard Mourrain.

The project DECONSTRUCT (Decomposition of Structured Tensors, Algorithms and Characterization) is an Intra-European Fellowships (FP7-PEOPLE-2009-IEF) for Alessandra Bernardi. The activity is connected to several domains: In algebra, it concerns the big Waring problem (J. Alexander and A. Hirschowitz, 1995); the canonical forms for homogeneous polynomials, the interpolation of multiple points in the projective space (B. Harbourne, A. Gimigliano, A. Hirschowitz conjecture). In algebraic geometry, it concerns the dimensions and equations of secant varieties of varieties parameterizing fixed rank tensors and/or homogeneous polynomials; the application domains are electrical engineering, computational statistics, data analysis on algorithms for the computation of tensor rank and potentially many other domains where tensor decomposition is involved.

7.3. Bilateral actions

7.3.1. PAI STAR South Corea collaboration

Participants: Laurent Busé, André Galligo, Gang Xu, Evelyne Hubert, Angelos Mantzaflaris, Bernard Mourrain.

The objective of this collaboration is to conduct research in algebraic techniques for solving geometric modeling problems. More specially, we are interested in developing efficient and robust methods to solve non-linear constraints which appear in geometric computation. These methods will be used in applications such as shape design and reconstruction for solving interpolation or approximation problems. A typical area in which we will apply our methods is ship design. Experimentation and validation will lead to open source software implementation.

Collaborators from Seoul National University: Park Sung Ha, Tae-Wan Kim, Sharma Rajiv, Hur Seok.

B. Mourrain visited Seoul National University (June 27 - July 3); A. Mantzaflaris and B. Mourrain visited Seoul National University (December 11-18). Tae-way Kim visited Nice (8-14 March) and participated to the winter SAGA School in Auron (15-19 March).

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Seminar organization

We organize a seminar called “Formes & Formules”. The list of talks is archived at http://www-sop.inria.fr/teams/galaad/.

8.1.2. Comittee participations

- Evelyne Hubert was part of the program committee for the international conference DART 4 (*Differential Algebra and Related Topics*) that was held this time in Beijing and sponsored by several Chinese societies and foundations.
- Bernard Mourrain was co-chair of the conference Geometric Modeling and Processing (with S. Schaefer and Xu Guo Liang). He was also member of the program committee of ADG (Automated Deduction in Geometry’10) and SPM (Solid and Physical Modeling’10).

8.1.3. Editorial committees

- Evelyne Hubert and Bernard Mourrain are members of the editorial board of the Journal of Symbolic Computation.
- B. Mourrain (with I. Kostireas and V.Y. Pan) are guest editors of the special issue of Theoretical Computer Science related to SNC’09.
- B. Mourrain (with S. Schaefer and Xu Guo Liang) are guest editors of special issues of CAD and CAGD for the conference GMP’10.

8.1.4. Organisation of conferences and schools

- Laurent Busé, Angelos Mantzaflaris and Bernard Mourrain coorganized the Winter School of SAGA, March 15-19, Auron, France.

8.1.5. Ph.D. thesis committees

- Laurent BusÃ© was a member of the Ph.D. committee of Nicolas Botbol, University of Pierre et Marie Curie, September 29th 2010.
- Bernard Mourrain was a member of the Ph.D. committee of C. Bertone (Univ. Turin, March 25), of A. Urbanska-Marzalek (Univ. Grenoble, April 27); of D. Eklund (Univ. Stockholm, November 30) and, as a referee, of S. Belhaj (Univ. de Franche-ComtÃ©, May 7).
- AndrÃ© Galligo was a member of the Ph.D. committee of C. Bertone (Univ. Turin, March 25). He was also a member of the "Habilitation Ã Diriger des Recherches" of Alban Quadrat, University of Nice, September 13th 2010 and Mohamed Elkadi, University of Nice, November 23th 2010.

8.1.6. Other committees

- L. Busé is an elected member of the administrative council of the SMF (the French Mathematical Society).
- A. Galligo is one of the 3 members of the steering committee of ISSAC.
- Evelyne Hubert served as grant referee for NSERC (Canada) and joined the ACM-SIGSAM advisory board.
- Bernard Mourrain served as referee for the journal of Symbolic Computation, AAECC (Applied Algebra and Error Correcting Codes), Computer Aided Design, Computer Aided Geometric Design, Mathemactis of Computation. He is also chair of the local INRIA Committee for Courses and Conferences.

8.1.7. WWW server

- http://www-sop.inria.fr/teams/galaad.
- http://www-sop.inria.fr/teams/galaad/wiki. We launched a MediaWiki platform where the members of the project collect content related to their research using this fast and collaborative interface.

8.2. Participation at conferences and invitations

- Laurent Busé participated to *SAGA Winter School*, Auron, France, March 14-19; to the *Journées Nationales de Calcul Formel*, May 3-7, Luminy, France; to the Fall School *Shapes, Geometry, and Algebra*, October 4-8, Kolympari, Greece; to the conference *Commutative algebra and its interactions with algebraic geometry*, November 2-6, Luniny, France. He also gave an invited talk at the Seminar of algebraic geometry of the university of Barcelona, June 2-4, and attended meeting of the SMF on January 16, June 1 and June 26.
- André Galligo gave a talk at the University of Torino (Italy) on March 25th 2010 and then at the University of Pisa (Italy) on May 7th 2010 ; was an invited speaker at the conference *Real Geometry, Computer Algebra and Math Education*, May 17 - 21, Castro Urdiales, Spain; was an invited speaker at the conference *Rencontres ArithmÃ©tiques* Caen (France), June 23-26 2010; was a participant of ISSAC'10 in Munich (Germany), July 25-28 2010; gave a talk at Seminaire *Algorithmes* INRIA Rocquencourt, October 25th 2010; was an invited speaker at the conference *Analyse et Probability*, Nice, November 15-17 2010.

- The highlight for Evelyne Hubert was to deliver a keynote lecture at ISSAC this year in Munich (Germany). She was also a plenary lecturer at EACA, the spanish national meeting on computational algebra, that was held in Santiago de Compostela. She was invited to give a talk at the worshop on *Curve Flows and Moving Frame* of the AMS (American Mathematical Society) meeting in St Paul (Minnesota, USA) and at the conference *Harmony Gröbner Bases and the Modern Industrial Society*, sponsored by the JST (Japan Science and Technology), in Osaka (Japan). She attended the *SAGA winter school* in Auron (France) and the conference on *Differential Algebra and Related Topics* at the Chinese Academy of Sciences in Beijing. In November, she gave a colloquium talk at the department of mathematics of La Trobe university in Melbourne (Australia). That happened during her one month stay as a guest of P. van der Kamp. She was also invited for a week at Utah State University in Logan (USA) by I. Anderson.
- Angelos Mantzaflaris attended SAGA Winter School & Workshop, Auron, France and presented a tutorial on Geometric Modeling and Axel software. On April 15 he delivered an talk titled *An Introduction to Geometric Modeling using Axel* at Algorithms' General Seminar of the Department of Informatics & Telecoms, University of Athens. He participated to the conference *Geometric Modeling & Processing (GMP) 2010*, which took place in June, 16â18 at Castro Urdiales, Spain and presented a paper. He participated in the 2010 edition of EuroScience Open Forum (ESOF) at Torino, Italy (July 1–7) and held a poster entitiled *Algebraic Geometry in CAGD and Formula One* in the Marie Curie Satellite Event of ESOF. He was awarded a travel grant to participate in the *Journées Informatique et Géométrie (JIG)* held at GIPSA-Lab, Grenoble, France (Sept. 27–28) and gave a talk titled *Robust semi-algebraic set computation in the plane*. Finally, he participated in Fall School 'Shapes, Geometry, and Algebra', at Kolympari, Greece (October 4–8).
- Bernard Mourrain participated to SAGA Winter School, Auron, France, March 14-19; to the meeting of Exciting project March 22-24 at SINTEF, Oslo, Norway; gave a talk at the *Journées Nationales de Calcul Formel*, May 3-7, Luminy, France; was an invited speaker at the conference *Real Geometry, Computer Algebra and Math Education*, May 17-21, Castro Urdiales, Spain; participated to the conference *Geometric Modeling & Processing (GMP) 2010*, June 16â18 Castro Urdiales, Spain; was an invited speaker at the conference CIAM on Applied Mathematics, July 6-9, Bandung, Indonesia; gave an introductory course on effective algebraic geometry at the Univ. UGM, July 13th, Yogjakarta, Indonesia; particpated to the meeting of the ARC PlantScan3D, September 1-3, Grenoble, France; was an invited speaker of the conference *Convex Optimization and Algebraic Geometry*, September 28 -October 1, IPAM, UCLA, US; participated to the Fall School *Shapes, Geometry, and Algebra*, October 4-8, Kolympari, Greece; participated to the meeting of the project Exciting, October 11-13, Strobl, Austria.
- Xinghua Song participated to SAGA Winter School, Auron, France, March 14-19; participated to the conference *Shape Modeling Internatiola Conference, SMI'10*, June 21-23, Aix-en-provence, France; participated to the meeting of the ARC PlantScan3D, September 1-3, Grenoble, France.
- Gang Xu attended SAGA Autumn School & Workshop, Auron, France (15-19 March) and presented a tutorial on Isogeometric Analysis and Axel software. He attended the 4th meeting of the EU project Exciting at SINTEF, Oslo, Norway, March 22-24. He participated to the conference *Geometric Modeling & Processing (GMP) 2010*, which took place in June, 16-18 at Castro Urdiales, Spain and gave a presentation about parameterization of computational domains. He attended the *Workshop on Non-Standard Numerical Methods for PDE's, Pavia, Italy*, (29 June-2 July). Finally, he participated to the 5th meeting of the EU project Exciting at Strobl, Linz, Austria, October 11-13, and gave a talk there about the isogeometric toolbox in Axel software.
- Jérôme Brachat participated to *SAGA Winter School*, Auron, France (15-19 March) and talked there, participated to the *Journées Nationales de Calcul Formel*, May 3-7, Luminy, France, and gave a talk there on Hilbert schemes; participated to the *DTA Workshop*, Sept. 13- 17, Monopoly, Italy and gave a talk there on tensor decomposition.

- Thang Luu Ba participated to *SAGA Workshop, Auron, France* (15-19 March) and talked there, participated to *SAGA Fall School, Kolympari, Crete, Greece* (October 4-8) and gave a talk at this conference. He attended the two week at *The Fifth RISC/SCIENCE Training School in Symbolic Computation, Johannes Kepler University of Linz,Hagenberg, Austria* (June 28-July 9).

8.3. Formation

8.3.1. Teaching at Universities

- Laurent Busé gave a course "Curves and Surfaces" of 12 hours (with 48h of TDs) in MAM4 at EPU.
- Mohamed Elkadi gave courses of Algebra in Master2 of mathematics, of linear algebra in Licence3 of computer sciences, and of computer algebra for preparation at the Agregation. All together, these courses sum up to 192 hours.
- André Galligo gave courses in Licence of mathematics, 192 hours.
- Evelyne Hubert participated to the jury *Modeling - Computer Algebra* for the *Agregation de Mathématiques* (July).
- Bernard Mourrain gave a course *Computational Algebra for Real Geometry* (M2, 30h) at the Master of Mathematics, Lab. J.A. DieudonnÃ©.

8.3.2. Ph.D. thesis in progress

- Elimane Ba, *Résultants, calculs et applications*, UNSA.
- Jérôme Brachat, *Dualité effective pour la résolution d'équations polynomiales*, bourse AMX, UNSA.
- Angelos Mantzaflaris, *Robust algebraic methods for geometric computations*, bourse ITN SAGA, UNSA.
- Thang Luu Ba, *Using matrix-based representations for CAGD*, bourse du gouvernement vietnamien and INRIA, UNSA.

8.3.3. Defended Ph.D. thesis and Habilitation thesis

- Cristina Bertone, *Polynomial factorization and curve decomposition algorithms*, Ph.D. thesis of the University of Nice and the University of Torino, defended march 26 2010.
- Mohamed Elkadi, *Contributions à la théorie de l'élimination et Applications*, Habilitation à diriger des recherches of the University of Nice, defended November 23 2010.

8.3.4. Internships

See the web page of our internships.

- Andjela Davidovic, *Modeling locks with convolution surfaces*, August 7th to September 17th, MathMod Master Mundus.
- Reza Sadoughian, *Symmetric tensor decomposition*, June 1st - September 15, M2 of Maths at Univ. of Versailles.
- Nelly Villamizar, *Spline spaces over triangulated domains*, November 8 - December 8, Ph.D. candidate, collaboration in the context of the network SAGA.

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Doctoral Dissertations and Habilitation Theses

[11] C. BERTONE. *Polynomial factorization and curve decomposition algorithms*, University of Nice and University of Torino, 2010.

[12] M. ELKADI. *Contributions à la théorie de l'élimination et Applications*, University of Nice Sophia-Antipolis, 2010, Habilitation à Diriger des Recherches, Ph. D. Thesis.

Articles in International Peer-Reviewed Journal

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