INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team geometrica

Geometric computing

Sophia Antipolis - Méditerranée, Saclay - Île-de-France

Theme : Algorithms, Certification, and Cryptography

Activity Report

2010

Table of contents

1. Team

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2. Overall Objectives

2.1. Overall Objectives

Research carried out by the Geometrica project team is dedicated to Computational Geometry and Topology and follows three major directions: (a). mesh generation and geometry processing; (b). topological and geometric inference; (c). data structures and robust geometric computation. The overall objective of the project-team is to give effective computational geometry and topology solid mathematical and algorithmic foundations, to provide solutions to key problems as well as to validate theoretical advances through extensive experimental research and the development of software packages that may serve as steps toward a standard for reliable and effective geometric computing. Most notably, Geometrica, together with several partners in Europe, plays a prominent role in the development of CGAL, a large library of computational geometry algorithms.

2.2. Highlights

The European ERC starting grant proposal "IRON" submitted by P. Alliez has been accepted and will start January 1st, 2011. IRON aims at streamlining the three-dimensional geometry processing pipeline by developing guaranteed techniques for treating defect-laden data.

3. Scientific Foundations

3.1. Mesh Generation and Geometry Processing

Meshes are becoming commonplace in a number of applications ranging from engineering to multimedia through biomedecine and geology. For rendering, the quality of a mesh refers to its approximation properties. For numerical simulation, a mesh is not only required to faithfully approximate the domain of simulation, but also to satisfy size as well as shape constraints. The elaboration of algorithms for automatic mesh generation is a notoriously difficult task as it involves numerous geometric components: Complex data structures and algorithms, surface approximation, robustness as well as scalability issues. The recent trend to reconstruct domain boundaries from measurements adds even further hurdles. Armed with our experience on triangulations and algorithms, and with components from the CGAL library, we aim at devising robust algorithms for 2D, surface, 3D mesh generation as well as anisotropic meshes. Our research in mesh generation primarily focuses on the generation of simplicial meshes, i.e. triangular and tetrahedral meshes. We investigate both greedy approaches based upon Delaunay refinement and filtering, and variational approaches based upon energy functionals and associated minimizers.

The search for new methods and tools to process digital geometry is motivated by the fact that previous attempts to adapt common signal processing methods have led to limited success: Shapes are not just another signal but a new challenge to face due to distinctive properties of complex shapes such as topology, metric, lack of global parameterization, non-uniform sampling and irregular discretization. Our research in geometry processing ranges from surface reconstruction to surface remeshing through curvature estimation, principal component analysis, surface approximationand surface mesh parameterization. Another focus is on the robustness of the algorithms to defect-laden data. This focus stems from the fact that acquired geometric data obtained through measurements or designs are rarely usable directly by downstream applications. This generates bottlenecks, i.e., parts of the processing pipeline which are too labor-intensive or too brittle for practitioners. Beyond reliability and theoretical foundations, our goal is to design methods which are also robust to raw, unprocessed inputs.

3.2. Topological and Geometric Inference

Due to the fast evolution of data acquisition devices and computational power, scientists in many areas are asking for efficient algorithmic tools for analyzing, manipulating and visualizing more and more complex shapes or complex systems from approximating data. Many of the existing algorithmic solutions which come with little theoretical guarantee provide unsatisfactory and/or unpredictable results. Since these algorithms take as input discrete geometric data, it is mandatory to develop concepts that are rich enough to robustly and correctly approximate continuous shapes and their geometric properties by discrete models. Ensuring the correctness of geometric estimations and approximations on discrete data is a sensitive problem in many applications.

Data sets being often represented as point sets in high dimensional spaces, there is a considerable interest in analyzing and processing data in such spaces. Although these point sets usually live in high dimensional spaces, one often expects them to be located around unknown, possibly non linear, low dimensional shapes. These shapes are usually assumed to be smooth submanifolds or more generally compact subsets of the ambient space. It is then desirable to infer topological (dimension, Betti numbers,...) and geometric characteristics (singularities, volume, curvature,...) of these shapes from the data. The hope is that this information will help to better understand the underlying complex systems from which the data are generated. In spite of recent promising results, many problems still remain open and to be addressed, need a tight collaboration between mathematicians and computer scientists. In this context our goal is to contribute to the development of new mathematically well founded and algorithmically efficient geometric tools for data analysis and processing of complex geometric objects. Our main targeted areas of application include machine learning, data mining, statistical analysis, and sensor networks.

3.3. Data Structures and Robust Geometric Computation

GEOMETRICA has a large expertise of algorithms and data structures for geometric problems.We are pursuing efforts to design efficient algorithms from a theoretical point of view, but we also put efforts in the effective implementation of these results.

In the past years, we made significant contributions to algorithms for computing Delaunay triangulations (which are used by meshes in the above paragraph). We are still working on the practical efficiency of existing algorithms to compute or to exploit classical Euclidean triangulations in 2 and 3 dimensions, but the current focus of our research is more aimed towards extending the triangulation efforts in several new directions of research.

One of these directions is the triangulation of non Euclidean spaces such as periodic or projective spaces, with various potential applications ranging from astronomy to granular material simulation.

Another direction is the triangulation of moving points, with potential applications to fluid dynamics where the points represent some particles of some evolving physical material, and to variational methods devised to optimize point placement for meshing a domain with a high quality elements.

Increasing the dimension of space is also a stimulating direction of research, as triangulating points in medium dimension (say 4 to 15) has potential applications and makes new challenges to trade exponential complexity of the problem in the dimension for the possibility to reach effective and practical results in reasonably small dimensions.

On the complexity analysis side, we pursue efforts to obtain complexity analysis in some practical situations involving randomized or stochastic hypotheses. On the algorithm design side, we are looking for new paradigms to exploit parallelism on modern multicore hardware architectures.

Finally, all this work is done while keeping in mind concerns related to effective implementation of our work, practical efficiency and robustness issues which have become a background task of all different works made by GEOMETRICA.

4. Application Domains

4.1. Geometric Modeling and Shape Reconstruction

Modeling 3D shapes is required for all visualization applications where interactivity is a key feature since the observer can change the viewpoint and get an immediate feedback. This interactivity enhances the descriptive power of the medium significantly. For example, visualization of complex molecules helps drug designers to understand their structure. Multimedia applications also involve interactive visualization and include e-commerce (companies can present their products realistically), 3D games, animation and special effects in motion pictures. The uses of geometric modeling also cover the spectrum of engineering, computer-aided design and manufacture applications (CAD/CAM). More and more stages of the industrial development and production pipeline are now performed by simulation, due to the increased performance of numerical simulation packages. Geometric modeling therefore plays an increasingly important role in this area. Another emerging application of geometric modeling with high impact is medical visualization and simulation.

In a broad sense, shape reconstruction consists of creating digital models of real objects from points. Example application areas where such a process is involved are Computer Aided Geometric Design (making a car model from a clay mockup), medical imaging (reconstructing an organ from medical data), geology (modeling underground strata from seismic data), or cultural heritage projects (making models of ancient and or fragile models or places). The availability of accurate and fast scanning devices has also made the reproduction of real objects more effective such that additional fields of applications are coming into reach. The members of GEOMETRICA have a long experience in shape reconstruction and contributed several original methods based upon the Delaunay and Voronoi diagrams.

4.2. Scientific Computing

Meshes are the basic tools for scientific computing using finite element methods. Unstructured meshes are used to discretize domains bounded by complex shapes while allowing local refinements. GEOMETRICA contributes to mesh generation of 2D and 3D possibly curved domains. Most of our methods are based upon Delaunay triangulations, Voronoi diagrams and their variants. Anisotropic meshes are also investigated. We investigate in parallel both greedy and variational mesh generation techniques. The greedy algorithms consist of inserting vertices in an initial coarse mesh using the Delaunay refinement paradigm, while the variational algorithms consists of minimizing an energy related to the shape and size of the elements. Our goal is to show the complementarity of these two paradigms. Quadrangle surface meshes are also of interest for reverse engineering and geometry processing applications. Our goal is to control the final edge alignment, the mesh sizing and the regularity of the quadrangle tiling.

5. Software

5.1. CGAL, the Computational Geometry Algorithms Library

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Manuel Caroli, Olivier Devillers, Michael Hemmer, Pedro Machado Manhães de Castro, Sylvain Pion, Stéphane Tayeb, Monique Teillaud, Mariette Yvinec.

With the collaboration of Hervé Brönnimann, Frédéric Cazals, Frank Da, Christophe Delage, Andreas Fabri, Julia Flötotto, Philippe Guigue, SamuelHornus, Menelaos Karavelas, Sébastien Loriot, Abdelkrim Mebarki, Naceur Meskini, Andreas Meyer, Marc Pouget, François Rebufat, Laurent Rineau, LaurentSaboret, Radu Ursu, and Camille Wormser. *http://www.cgal.org*

CGAL is a C++ library of geometric algorithms and data structures. Its development has been initially funded and further supported by several European projects (CGAL, GALIA, ECG, ACS, AIM@SHAPE) since 1996. The long term partners of the project are research teams from the following institutes: INRIA Sophia Antipolis - Méditerranée, Max-Planck Institut Saarbrücken, ETH Zürich, Tel Aviv University, together with several others. In 2003, CGAL became an Open Source project (under the LGPL and QPL licenses), and it also became commercialized by GEOMETRY FACTORY, a company *Born of INRIA* founded by Andreas Fabri.

The aim of the CGAL project is to create a platform for geometric computing supporting usage in both industry and academia. The main design goals are genericity, numerical robustness, efficiency and ease of use. These goals are enforced by a review of all submissions managed by an editorial board. As the focus is on fundamental geometric algorithms and data structures, the target application domains are numerous: from geological modeling to medical images, from antenna placement to geographic information systems, etc.

The CGAL library consists of a kernel, a list of algorithmic packages, and a support library. The kernel is made of classes that represent elementary geometric objects (points, vectors, lines, segments, planes, simplices, isothetic boxes, circles, spheres, circular arcs...), as well as affine transformations and a number of predicates and geometric constructions over these objects. These classes exist in dimensions 2 and 3 (static dimension) and d (dynamic dimension). Using the template mechanism, each class can be instantiated following several representation modes : one can choose between Cartesian or homogeneous coordinates, use different types to store the coordinates, and use reference counting or not. The kernel also provides some robustness features using some specifically-devised arithmetic (interval arithmetic, multi-precision arithmetic, static filters...).

A number of packages provide geometric data structures as well as algorithms. The data structures are polygons, polyhedra, triangulations, planar maps, arrangements and various search structures (segment trees, d-dimensional trees...). Algorithms are provided to compute convex hulls, Voronoi diagrams, Boolean operations on polygons, solve certain optimization problems (linear, quadratic, generalized of linear type). Through class and function templates, these algorithms can be used either with the kernel objects or with user-defined geometric classes provided they match a documented interface.

Finally, the support library provides random generators, and interfacing code with other libraries, tools, or file formats (ASCII files, QT or LEDA Windows, OpenGL, Open Inventor, Postscript, Geomview...). Partial interfaces with Python, SCILAB and the Ipe drawing editor are now also available.

GEOMETRICA is particularly involved in general maintainance, in the arithmetic issues that arise in the treatment of robustness issues, in the kernel, in triangulation packages and their close applications such as alpha shapes, in meshes... Three researchers of GEOMETRICA are members of the CGAL Editorial Board, whose main responsibilities are the control of the quality of CGAL, making decisions about technical matters, coordinating communication and promotion of CGAL.

CGAL is about 700,000 lines of code and supports various platforms: GCC (Linux, Mac OS X, Cygwin...), Visual C++ (Windows), Intel C++... A new version of CGAL is released twice a year, and it is downloaded about 10000 times a year. Moreover, CGAL is directly available as packages for the Debian, Ubuntu and Fedora Linux distributions.

More numbers about CGAL: there are now 13 editors in the editorial board, with approximately 20 additional developers. The user discussion mailing-list has more than 1000 subscribers with a relatively high traffic of 5-10 mails a day. The announcement mailing-list has more than 3000 subscribers.

6. New Results

6.1. Mesh Generation and Geometry Processing

6.1.1. 2D Centroidal Voronoi Tessellations with Constraints

Participants: Pierre Alliez, Olivier Devillers.

In collaboration with Jane Tournois (previously PhD student in our group, and now post-doc at TU Vienna).

We tackle the problem of constructing 2D centroidal Voronoi tessellations with constraints through an efficient and robust construction of bounded Voronoi diagrams, the pseudo-dual of the constrained Delaunay triangulation [23]. We exploit the fact that the cells of the bounded Voronoi diagram can be obtained by clipping the ordinary ones against the constrained Delaunay edges. The clipping itself is efficiently computed by identifying for each constrained edge the (connected) set of triangles whose dual Voronoi vertex is hidden by the constraint. The resulting construction is amenable to Lloyd relaxation so as to obtain a centroidal tessellation with constraints.

6.1.2. Optimizing Voronoi Diagrams for Polygonal Finite Element Computations

Participant: Pierre Alliez.

In collaboration with Daniel Sieger and Mario Botsch from Bielefeld University (Germany).

We present a 2D mesh improvement technique that optimizes Voronoi diagrams for their use in polygonal finite element computations [32]. Starting from a centroidal Voronoi tessellation of the simulation domain we optimize the mesh by minimizing a carefully designed energy functional that effectively removes the major reason for numerical instabilities—short edges in the Voronoi diagram. We evaluate our method on a 2D Poisson problem and demonstrate that our simple but effective optimization achieves a significant improvement of the stiffness matrix condition number. See Figure 1.

6.1.3. Robust Surface Reconstruction from Raw Point Sets

Participants: Pierre Alliez, David Cohen-Steiner.

In collaboration with Fernando de Goes, Patrick Mullen and Mathieu Desbrun from Caltech.

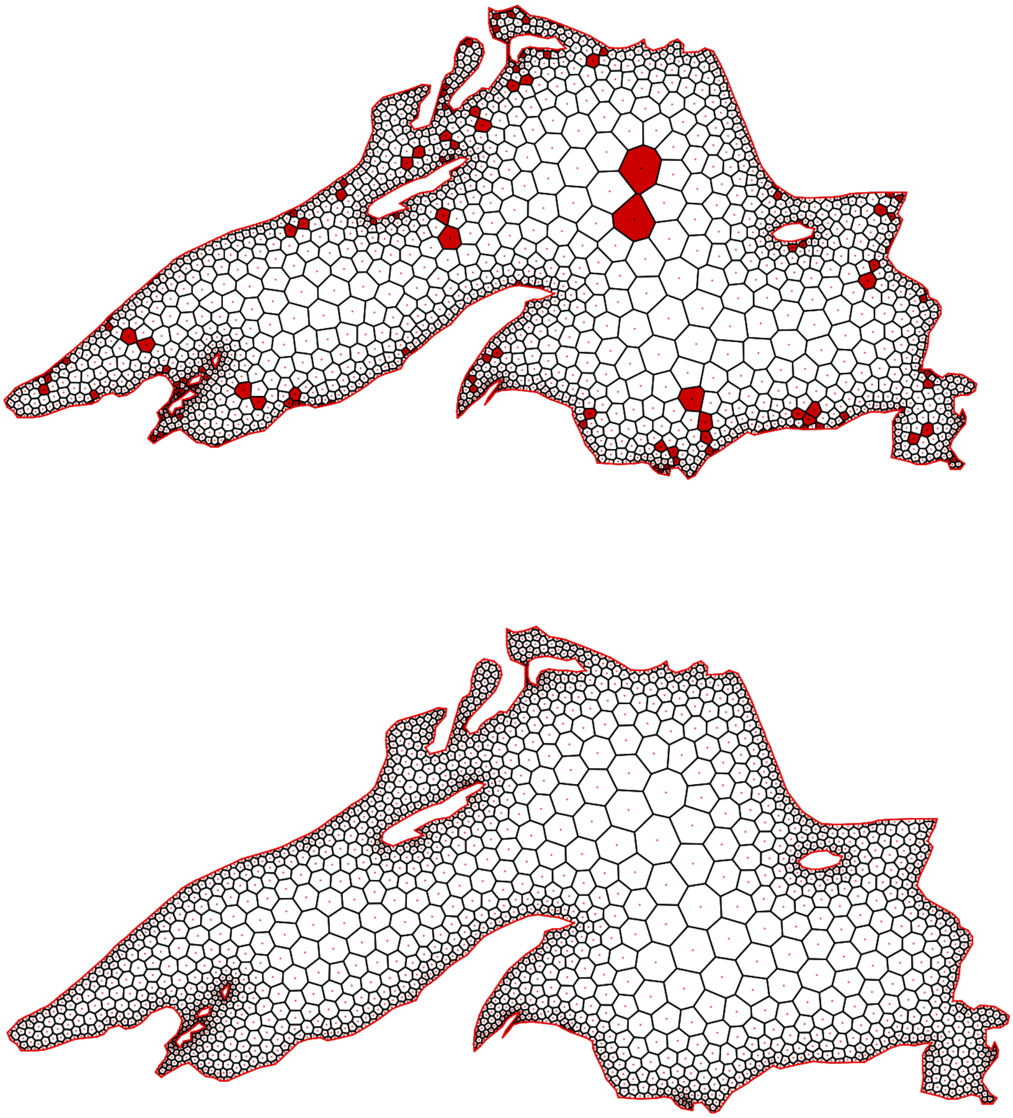


Figure 1. CVT (top) and optimized mesh (bottom) for Lake Superior using a K-Lipschitz sizing function with $K = 0.7$. The underlying Delaunay triangulation contains 4036 triangles. The condition number reduces from 371877 to 190.

We propose a modular framework for robust 3D reconstruction from unorganized, unoriented, noisy, and outlier-ridden geometric data [20]. We gain robustness and scalability over previous methods through an unsigned distance approximation to the input data followed by a global stochastic signing of the function. An isosurface reconstruction is finally deduced via a sparse linear solve. We show with experiments on large, raw, geometric datasets that this approach is scalable while robust to noise, outliers, and holes. The modularity of our approach facilitates customization of the pipeline components to exploit specific idiosyncracies of datasets, while the simplicity of each component leads to a straightforward implementation. See Figure 2.

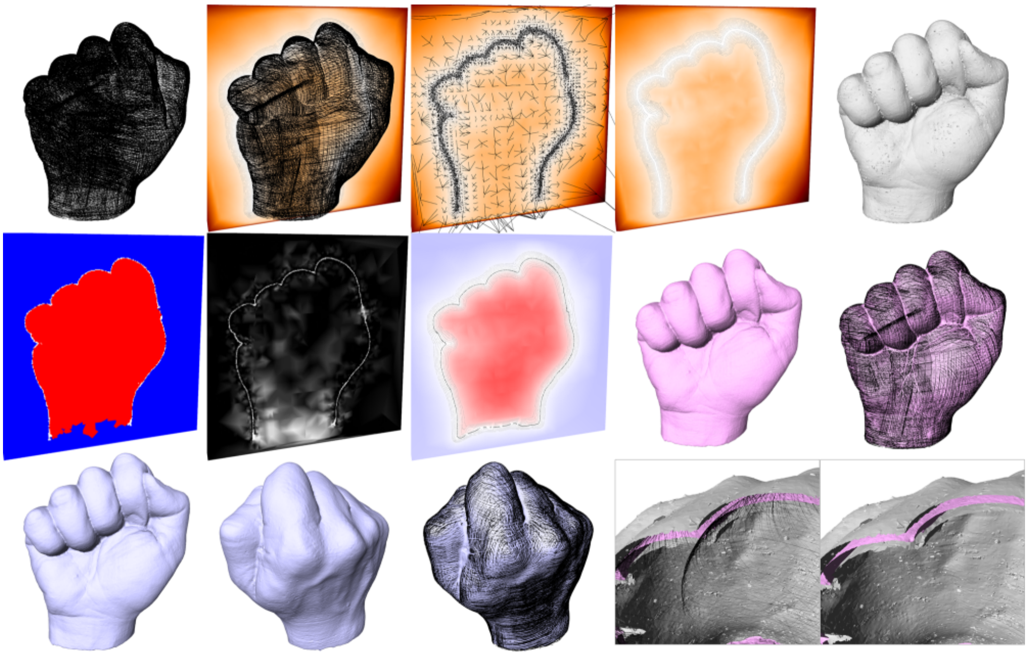


*Figure 2. **Plaster Hand.** Data scanned with a Kreon laser scanner mounted on an articulated arm; the $1.8M$ point sampling is very anisotropic as it was obtained by manual sweeping of a 1D contact sensor. Top: input point set (with a big hole at the bottom and others due to occlusions between the fingers), point set and 2D cut of unsigned function, same 2D cut with nearby edges of the coarse mesh $\mathbb{M}$, same 2D cut alone, and full ϵ-band. Middle: 2D cuts of sign guess (red for inside, blue for outside and white uncertain), confidence (which decreases in the holes), signed function after smoothing, isosurface of the robust unsigned function obtained by marching tetrahedra in the lattice mesh, and same isosurface superimposed with input points. Bottom: views of the reconstructed surface obtained by Delaunay refinement without and with points added, and cut view of the ϵ-band with the reconstructed isosurface of the signed function inside, with and without the input points.*

6.1.4. 3D Periodic Meshes

Participants: Manuel Caroli, Mikhail Bogdanov, Monique Teillaud.

In collaboration with Vissarion Fisikopoulos, Department of Informatics and Telecommunications, University of Athens.

We show how the computation of 3D periodic triangulations can be used in combination of a surface mesh generation method to compute meshes of triply-periodic surfaces (see Figure 3). For smooth surfaces, a sufficiently refined output mesh is guaranteed to be both homeomorphic to the surface and geometrically close to it [50].

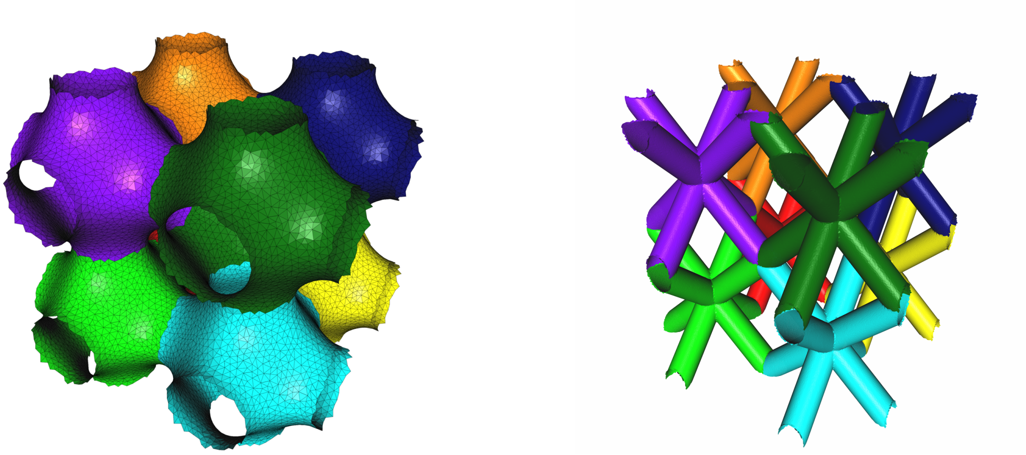


Figure 3. Meshing of triply-periodic surfaces: The Schwarz-p surface (left) and a surface used in bone scaffolding (right, data by courtesy of Maarten Moesen, Department of Metallurgy and Materials Engineering, K.U. Leuven)

We are working on the extension to volume meshing (Figure 4).

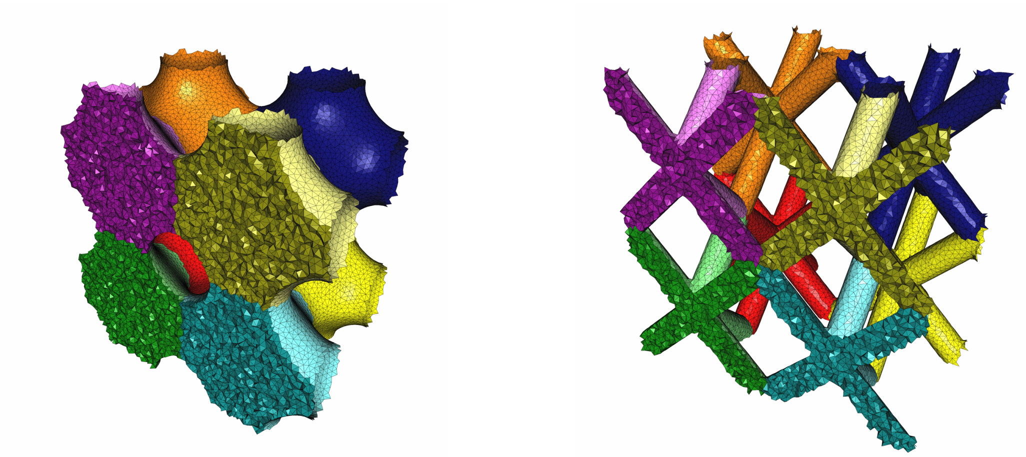


Figure 4. Meshing of triply-periodic volumes.

6.1.5. Feature Preserving Mesh Generation from 3D Point Clouds

Participants: Nader Salman, Mariette Yvinec.

In collaboration with Quentin Mérigot, Stanford University.

We address the problem of generating quality surface triangle meshes from 3D point clouds sampled on piecewise smooth surfaces. Using a feature detection process based on the covariance matrices of Voronoi cells, we first extract from the point cloud a set of sharp features. Our algorithm also runs on the input point cloud a reconstruction process, such as Poisson reconstruction, providing an implicit surface. A feature preserving variant of a Delaunay refinement process is then used to generate a mesh approximating the implicit surface and containing a faithful representation of the extracted sharp edges. See figure 5. Such a mesh provides an enhanced trade-off between accuracy and mesh complexity. The whole process is robust to noise and made versatile through a small set of parameters which govern the mesh sizing, approximation error and shape of the elements. We demonstrate the effectiveness of our method on a variety of models including laser scanned datasets ranging from indoor to outdoor scenes [21].

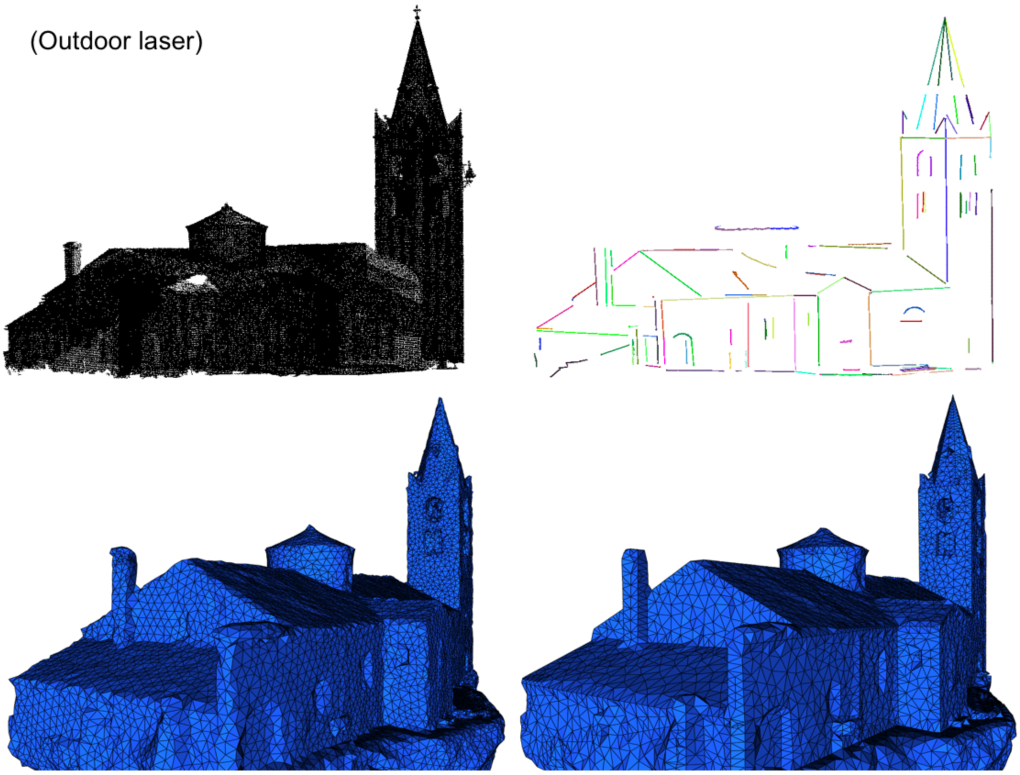


Figure 5. Top left:the point clouds (coutesy of INPG). Top right: the extracted features. Bottom left: a mesh of the implicit surface reconstructed by the Poisson reconstruction. Bottom right: the ouput of our feature preserving mesh generation.

6.1.6. Polygon Mesh Processing

Participant: Pierre Alliez.

In collaboration with Mario Botsch, Leif Kobbelt, Mark Pauly and Bruno Lévy.

Geometry processing, or mesh processing, is a fast-growing area of research that uses concepts from applied mathematics, computer science, and engineering to design efficient algorithms for the acquisition, reconstruction, analysis, manipulation, simulation, and transmission of complex 3D models. Applications of geometry processing algorithms already cover a wide range of areas from multimedia, entertainment, and classical computer-aided design, to biomedical computing, reverse engineering, and scientific computing. Over the last several years, triangle meshes have become increasingly popular, as irregular triangle meshes have developed into a valuable alternative to traditional spline surfaces. This book [39] discusses the whole geometry processing pipeline based on triangle meshes. The pipeline starts with data input, for example, a model acquired by 3D scanning techniques. This data can then go through processes of error removal, mesh creation, smoothing, conversion, morphing, and more. The authors detail techniques for those processes using triangle meshes.

6.2. Topological and Geometric Inference

6.2.1. Triangulating Smooth Submanifolds with Light Scaffolding

Participants: Jean-Daniel Boissonnat, Arijit Ghosh.

We propose an algorithm to sample and mesh a k-submanifold $\mathbb{M}$ of positive reach embedded in $\mathbb{R}^d$. The algorithm first constructs a crude sample of $\mathbb{M}$ using a brute force method. It then refines the sample according to a prescribed parameter ϵ, and builds a mesh that approximates $\mathbb{M}$. Differently from most algorithms that have been developped for meshing surfaces of $\mathbb{R}^3$, the refinement phase does not rely on a subdivision of $\mathbb{R}^d$ (such as a grid or a triangulation of the sample points) since the size of such scaffoldings depends exponentially on the ambient dimension d. Instead, we only compute local stars consisting of k-dimensional simplices around each sample point. By refining the sample, we can insure that all stars become coherent leading to a k-dimensional triangulated manifold $\widehat{\mathbb{M}}$. The algorithm uses only simple numerical operations. We show that the size of the sample is $O(\epsilon^{-k})$ and that $\widehat{\mathbb{M}}$ is a good triangulation of $\mathbb{M}$. More specifically, we show that $\mathbb{M}$ and $\widehat{\mathbb{M}}$ are isotopic, that their Hausdorff distance is $O(\epsilon^2)$ and that the maximum angle between their tangent bundles is $O(\epsilon)$. The asymptotic complexity of the algorithm is $T(\epsilon) = O(\epsilon^{-k^2-k})$ (for fixed $\mathbb{M}$, d and k).

6.2.2. Topological Inference via Meshing

Participant: Steve Oudot.

In collaboration with Benoît Hudson (TTI), Gary Miller and Donald Sheehy (CMU).

We apply ideas from mesh generation to improve the time and space complexities of computing the full persistent homological information associated with a point cloud P in Euclidean space $\mathbb{R}^d$. Classical approaches rely on the Cech, Rips, α-complex, or witness complex filtrations of P, whose complexities scale up very badly with d. For instance, the alpha-complex filtration incurs the $n^{\Omega(d)}$ size of the Delaunay triangulation, where n is the size of P. The common alternative is to truncate the filtrations when the sizes of the complexes become prohibitive, possibly before discovering the most relevant topological features. In this work we propose a new collection of filtrations, based on the Delaunay triangulation of a carefully-chosen superset of P, whose sizes are reduced to $2^{O(d^2)}n$. A nice property of these filtrations is to be interleaved multiplicatively with the family of offsets of P, so that the persistence diagram of P can be approximated in $2^{O(d^2)}n^3$ time in theory, with a near-linear observed running time in practice. Thus, our approach remains tractable in medium dimensions, say 4 to 10 [31].

6.2.3. Persistence-based Segmentation of Deformable Shapes

Participants: Frédéric Chazal, Primoz Skraba.

In collaboration with Maks Ovsjanikov and Leo Guibas (Stanford).

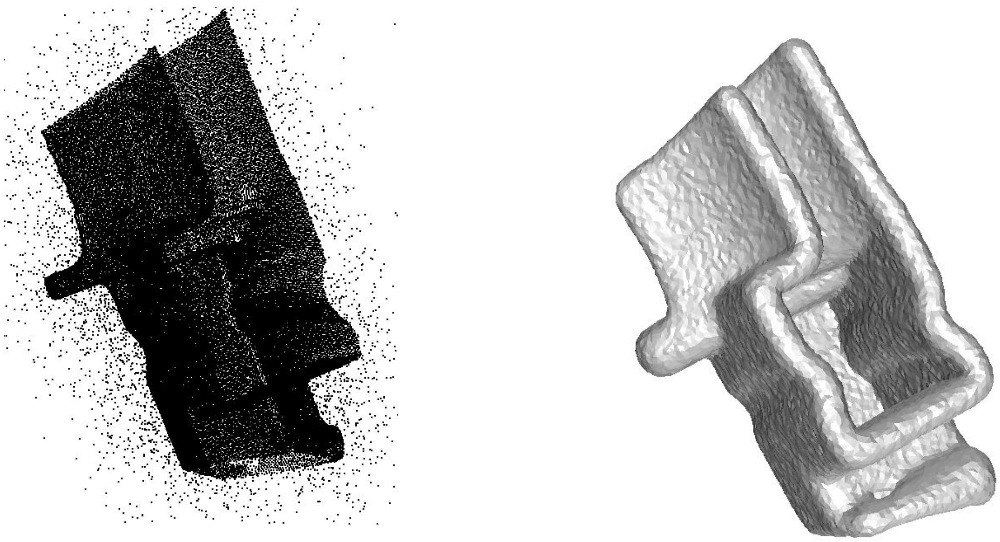
We combine two ideas: persistence-based clustering and the Heat Kernel Signature (HKS) function to obtain a multi-scale isometry invariant mesh segmentation algorithm. The key advantages of this approach is that it is tunable through a few intuitive parameters and is stable under near-isometric deformations. Indeed the method comes with feedback on the stability of the number of segments in the form of a persistence diagram. There are also spatial guarantees on part of the segments. Finally, we present an extension to the method which first detects regions which are inherently unstable and segments them separately. Both approaches are reasonably scalable and come with strong guarantees. We show numerous examples and a comparison with the segmentation benchmark and the curvature function [33].

6.2.4. Geometric Inference for Measures based on Distance Functions.

Participants: Frédéric Chazal, David Cohen-Steiner.

In collaboration with Quentin Mérigot (Stanford).

Data often comes in the form of a point cloud sampled from an unknown compact subset of Euclidean space. The general goal of geometric inference is then to recover geometric and topological features (Betti numbers, curvatures,...) of this subset from the approximating point cloud data. In recent years, it appeared that the study of distance functions allows to address many of these questions successfully. However, one of the main limitations of this framework is that it does not cope well with outliers nor with background noise. In this paper [44], we show how to extend the framework of distance functions to overcome this problem. Replacing compact subsets by measures, we introduce a notion of distance function to a probability distribution in $\mathbb{R}^n$. These functions share many properties with classical distance functions, which makes them suitable for inference purposes. In particular, by considering appropriate level sets of these distance functions, it is possible to associate in a robust way topological and geometric features to a probability measure (see Figure 6). We also discuss connections between our approach and non parametric density estimation as well as mean-shift clustering.



(a) (b)

Figure 6. On the left, a point cloud sampled on a mechanical part to which 10% of outliers (uniformly sampled in a box enclosing the model) have been added. On the right, the reconstruction of an isosurface of the distance function to the uniform probability measure on this point cloud.

6.2.5. Zigzag Persistent Homology in Matrix Multiplication Time

Participant: Primoz Skraba.

This work has been done in collaboration with Nikola Milosavljevic (MPI Saarbrücken) and Dmitriy Morozov (Stanford Univ.).

We present a new algorithm for computing zigzag persistent homology, an algebraic structure which encodes changes to homology groups of a simplicial complex over a sequence of simplex additions and deletions [47]. Provided that there is an algorithm that multiplies two $n \times n$ matrices in $M(n)$ time, our algorithm runs in $O(M(n)logn)$ time if $M(n) = O(n^2)$, and $O(M(n))$ time otherwise, for a sequence of n additions and deletions. In particular, the running time is $O(n^2.376)$, by result of Coppersmith and Winograd. The fastest previously known algorithm for this problem takes $O(n^3)$ time in the worst case.

6.3. Data Structures and Robust Geometric Computation

6.3.1. The Size of some Trees Constructed on Planar Point Sets

Participants: Pedro Machado Manhães de Castro, Olivier Devillers.

The Euclidean minimal k-insertion tree ($EMIT_k$) is obtained for a set of n points obtained by linking the last point to the closest amongst the k last inserted point. $EMIT_1$ is just the chain of points in insertion order and $EMIT_n$ is the minimum spanning tree. If the weight w of an edge e is its Euclidean length to the power of α, we show that $\sum_{e \in EMIT_k} w(e)$ is $O(n \cdot k^{-\alpha/d})$ in the worst case, where d is the dimension, for $d \geq 2$ and $0 < \alpha < d$. We also analyze the expected size of $EMIT_k$ and some stars, when points are evenly distributed inside the unit ball, for any $\alpha > 0$ [16], [48]. These results are used in the next section.

6.3.2. Simple and Efficient Distribution-Sensitive Point Location in Triangulations

Participants: Pedro Machado Manhães de Castro, Olivier Devillers.

Point location in spatial subdivision is one of the most studied problems in computational geometry. In the case of triangulations of $\mathbb{R}^d$, we revisit the problem to exploit a possible coherence between the query-points.

For a single query, walking in the triangulation is a classical strategy with good practical behavior and expected complexity $O(n^{1/d})$ if the points are evenly distributed. Based upon this strategy, we analyze, implement, and evaluate a distribution-sensitive point location algorithm based on the classical Jump & Walk, called Keep, Jump, & Walk. For a batch of query-points, the main idea is to use previous queries to improve the current one. In practice, Keep, Jump, & Walk is actually a very competitive method to locate points in a triangulation.

Regarding point location in a Delaunay triangulation, we show how the Delaunay hierarchy can be used to answer, under some hypotheses, a query q with a $O(\log \#(pq))$ randomized expected complexity, where p is a previously located query and $\#(s)$ indicates the number of simplices crossed by the line segment s.

The Delaunay hierarchy has $O(n \log n)$ time complexity and $O(n)$ memory complexity in the plane, and under certain realistic hypotheses these complexities generalize to any finite dimension.

Finally, we combine the good distribution-sensitive behavior of Keep, Jump, & Walk, and the good complexity of the Delaunay hierarchy, into a novel point location algorithm called Keep, Jump, & Climb. To the best of our knowledge, Keep, Jump, & Climb is the first practical distribution-sensitive algorithm that works both in theory and in practice for Delaunay triangulation—it is actually faster than the Delaunay hierarchy regardless of the spatial coherence of queries, and significantly faster when queries have strong spatial coherence [16], [49].

6.3.3. Delaunay Triangulation of Imprecise Points, Preprocess and Actually Get a Fast Query Time

Participant: Olivier Devillers.

Given a set of disks, we can preprocess them so that given a point in each disk, we can compute the Delaunay triangulation of these points in linear time if the disk are disjoint unit disks [45]. The proposed method is much simpler than previous similar method and is in practice actually faster than computing the Delaunay triangulation from scratch (without the knowledge of the disks).

6.3.4. Oja Medians and Centers of Gravity

Participant: Olivier Devillers.

This work has been done in collaboration with Dan Chen and Pat Morin (Carleton Univ.), John Iacono (Polytechnic, NY), and Stefan Langerman (Univ. Bruxelles).

Given a point set S, various notion of depth can be defined. The Oja depth of a query is the sum of the volume of all simplices formed by the query and points from S, and an Oja center is a point that minimize the Oja depth. In this work, relations between the center of gravity and Oja center are explored [28].

6.3.5. Delaunay Triangulations of Point Sets in Closed Euclidean d-Manifolds

Participants: Manuel Caroli, Monique Teillaud.

We give a definition of the Delaunay triangulation of a point set in a closed Euclidean d-manifold, i.e. a compact quotient space of the Euclidean space for a discrete group of isometries (a so-called Bieberbach group or crystallographic group). We describe a geometric criterion to check whether a partition of the manifold actually forms a triangulation (which subsumes that it is a simplicial complex). We provide an algorithm to compute the Delaunay triangulation of the manifold for a given set of input points, if it exists. Otherwise, the algorithm returns the Delaunay triangulation of a finitely sheeted covering space of the manifold. The algorithm has optimal randomized worst-case time and space complexity.

Whereas there was prior work for the special case of the flat torus, as far as we know this is the first result for general closed Euclidean d-manifolds. This research is motivated by application fields, like computational biology for instance, showing a need to perform simulations in quotient spaces of the Euclidean space by more general groups of isometries than the groups generated by d independent translations [43], [26].

6.3.6. Parallel Geometric Algorithms for Multi-Core Computers

Participant: Sylvain Pion.

In collaboration with Vicente Batista (former INRIA intern), David Millman from University of North Carolina at Chapel Hill, Johannes Singler from Universität Karlsruhe, and Marc Jeanmoungin (INRIA intern from ENS Paris).

Computers with multiple processor cores using shared memory are now ubiquitous. We present several parallel geometric algorithms that specifically target this environment, with the goal of exploiting the additional computing power. The d-dimensional algorithms we describe are (a) spatial sorting of points, as is typically used for preprocessing before using incremental algorithms, (b) kd-tree construction, (c) axis-aligned box intersection computation, and finally (d) bulk insertion of points in Delaunay triangulations for mesh generation algorithms or simply computing Delaunay triangulations. We show experimental results for these algorithms in 3D, using our implementations based on CGAL. This work is a step towards what we hope will become a parallel mode for CGAL, where algorithms automatically use the available parallel resources without requiring significant user intervention [17].

We also started work on parallel mesh generation, built on top of our work just described.

6.3.7. The Design of Core 2: A Library for Exact Numeric Computation in Geometry and Algebra

Participant: Sylvain Pion.

In collaboration with Jihun Yu (New York University), Chee Yap (New York University), Zilin Du (New York University) and Hervé Brönnimann (Polytechnic University Brooklyn).

There is a growing interest in numeric-algebraic techniques in the computer algebra community as such techniques can speed up many applications. This paper is concerned with one such approach called *Exact Numeric Computation* (ENC). The ENC approach to algebraic number computation is based on iterative verified approximations, combined with constructive zero bounds. This paper describes Core 2, the latest version of the Core Library, a package designed for applications such as non-linear computational geometry. The adaptive complexity of ENC combined with filters makes such libraries practical.

Core 2 smoothly integrates our algebraic ENC subsystem with transcendental functions with ϵ-accurate comparisons. This paper describes how the design of Core 2 addresses key software issues such as modularity, extensibility, efficiency in a setting that combines algebraic and transcendental elements. Our redesign preserves the original goals of the Core Library, namely, to provide a simple and natural interface for ENC computation to support rapid prototyping and exploration. We present examples, experimental results, and timings for our new system, released as Core Library 2.0 [34].

6.3.8. On the Complexity of Sets of Free Lines and Line Segments Among Balls in Three Dimensions

Participant: Marc Glisse.

This work has been done in collaboration with Sylvain Lazard from EPI VEGAS.

We present two new fundamental lower bounds on the worst-case combinatorial complexity of sets of free lines and sets of maximal free line segments in the presence of balls in three dimensions. We first prove that the set of maximal non-occluded line segments among n disjoint *unit* balls has complexity $\Omega(n^4)$, which matches the trivial $O(n^4)$ upper bound. This improves the trivial $\Omega(n^2)$ bound and also a previously known $\Omega(n^3)$ lower bound for the restricted setting of arbitrary-size balls. This result settles, negatively, the natural conjecture that this set of line segments, or, equivalently, the visibility complex, has smaller worst-case complexity for disjoint fat objects than for skinny triangles. We also prove an $\Omega(n^3)$ lower bound on the complexity of the set of non-occluded lines among n balls of arbitrary radii, improving on the trivial $\Omega(n^2)$ bound. This new bound almost matches the $O(n^{3+\epsilon})$ upper bound obtained recently by Rubin [29].

6.3.9. Reverse Nearest Neighbors Search in High Dimensions using Locality-Sensitive Hashing

Participant: Steve Oudot.

In collaboration with David Arthur (Stanford then Google).

We investigate the problem of finding reverse nearest neighbors efficiently. Although provably good solutions exist for this problem in low or fixed dimensions, to this date the methods proposed in high dimensions are mostly heuristic. We introduce a method that is both provably correct and efficient in all dimensions, based on a reduction of the problem to one instance of ϵ-nearest neighbor search plus a controlled number of instances of *exhaustive* r-PLEB, a variant of *Point Location among Equal Balls* where all the r-balls centered at the data points that contain the query point are sought for, not just one. The former problem has been extensively studied and elegantly solved in high dimensions using Locality-Sensitive Hashing (LSH) techniques. By contrast, the latter problem has a complexity that is still not fully understood. We revisit the analysis of the LSH scheme for exhaustive r-PLEB using a somewhat refined notion of locality-sensitive family of hash function, which brings out a meaningful output-sensitive term in the complexity of the problem. Our analysis, combined with a non-isometric lifting of the data, enables us to answer exhaustive r-PLEB queries (and down the road reverse nearest neighbors queries) efficiently. Along the way, we obtain a simple algorithm for answering exact nearest neighbor queries, whose complexity is parametrized by some *condition number* measuring the inherent difficulty of a given instance of the problem [41].

6.3.10. Certified Complex Root Isolation via Adaptive Root Separation Bounds

Participant: Michael Hemmer.

In collaboration with Michael Sagraloff from MPII and Michael Kerber from IST.

We address the problem of *root isolation* for polynomial systems: for an affine, zero-dimensional polynomial system of N equations in N variables, we describe an algorithm to encapsulate all complex solutions into disjoint regions, each containing precisely one solution (called *isolating regions*). Our approach also computes the multiplicity of each solution. The main novelty is a new approach to certify that a set of computed regions is indeed isolating. It is based on an adaptive root separation bound obtained from combining information about the approximate location of roots and resultant calculus. Here we use simple subdivision method to determine the number of roots within certain regions. The resultant calculus only takes place over prime fields to avoid the disadvantageous coefficient growth in symbolic methods, without sacrificing the exactness of the output. The presented approach is complete for uni- and bivariate systems, and in general applies in higher dimensions as well, possibly after a coordinate change.

6.3.11. A Complete, Exact and Efficient Implementation for Computing the Edge-Adjacency Graph of an Arrangement of Quadrics

Participant: Michael Hemmer.

In collaboration with Sylvain Petitjean and Laurent Dupont from EPI VEGAS and Elmar Schömer from the University of Mainz.

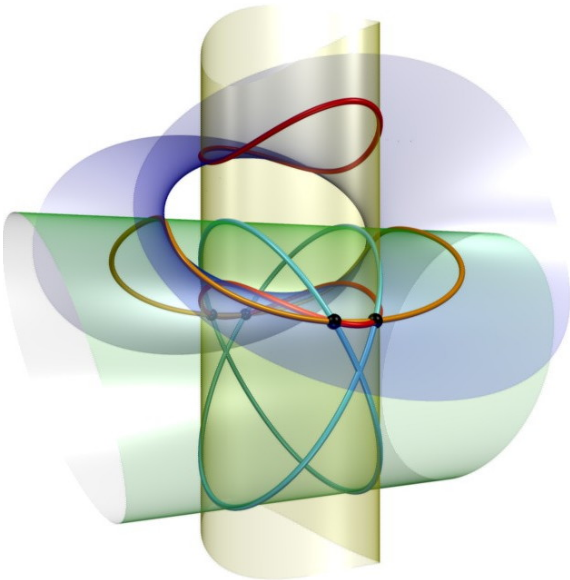


Figure 7. Arrangement of quadrics

We present a complete, exact and efficient implementation to compute the edge-adjacency graph of an arrangement of quadrics, i.e. surfaces of algebraic degree 2 (Figure 7). This is a major step towards the computation of the full 3D arrangement. We enhanced an implementation for an exact parameterization of the intersection curves of two quadrics, such that we can compute the exact parameter value for intersection points and from that the edge-adjacency graph of the arrangement. Our implementation is *complete* in the sense that it can handle all kinds of inputs including all degenerate ones, i.e. singularities or tangential intersection points. It is *exact* in that it always computes the mathematically correct result. It is *efficient* in terms of running times, i.e. it compares favorably to the only previous implementation [19].

6.3.12. Constructing the Exact Voronoi Diagram of Arbitrary Lines in Space, with Fast Point-Location

Participant: Michael Hemmer.

In collaboration with Ophir Setter and Dan Halperin from the University of Tel Aviv.

Supplementary material and in particular the prototypical code of our implementation can be found in the website: http://acg.cs.tau.ac.il/projects/internal-projects/3d-lines-vor/project-page

We introduce a new, efficient, and complete algorithm, and its exact implementation, to compute the Voronoi diagram of lines in space (Figure 8). This is a major milestone towards the robust construction of the Voronoi diagram of polyhedra. As we follow the exact geometric-computation paradigm, it is guaranteed that we always compute the mathematically correct result. The algorithm is complete in the sense that it can handle all configurations, in particular all degenerate ones. The algorithm requires $O(n^{3+\varepsilon})$ time and space, where n is the number of lines. The Voronoi diagram is represented by a data structure that permits answering point-location queries in $O(\log^2 n)$ expected time. The implementation employs the CGAL packages for constructing arrangements and lower envelopes together with advanced algebraic tools [30], [46].

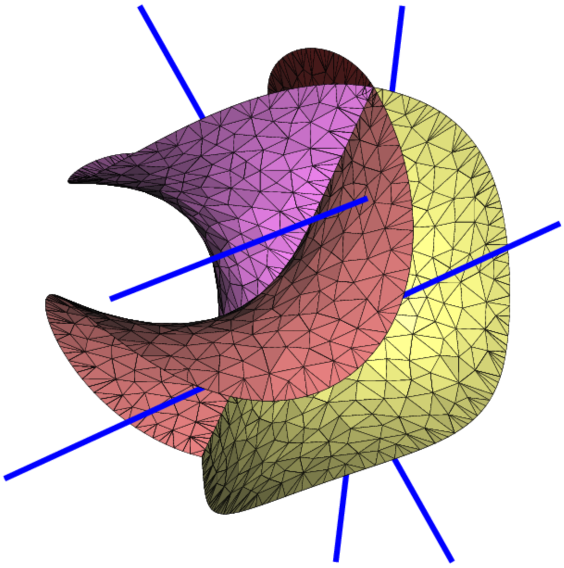


Figure 8. Voronoi diagram of lines

6.3.13. A Generic Algebraic Kernel for Non-linear Geometric Applications

Participant: Michael Hemmer.

In collaboration with Eric Berberich from MPII and Michael Kerber from IST.

We report on a generic (uni- and bivariate) algebraic kernel that becomes available to the public with CGAL 3.7. It comprises complete, correct, though efficient state-of-the-art implementations on polynomials, roots of polynomial systems, and the support to analyze algebraic curves defined by bivariate polynomials. The kernel is accompanied with a ready-to-use interface to enable arrangements induced by algebraic curves, that have already been used as basis for various geometric applications, as arrangements on Dupin cyclides or the triangulation of algebraic surfaces. We present two novel applications: arrangements of rotated algebraic curves and Boolean set operations on polygons bounded by segments of algebraic curves. We also provide exhaustive experiments showing that our implementation is competitive and often outperforms existing implementation on non-linear curves available in CGAL, which demonstrates the general usefulness of the presented software [42].

6.4. Software

6.4.1. CGAL

Two major new releases of CGAL, versions 3.6 and 3.7, have been been made available in 2010. These releases contain the following new features, involving GEOMETRICA researchers:
— *Algebraic Kernel* [36]. This package, introduced in CGAL 3.6, is targeted to provide black-box implementations of state-of-the-art algorithms to determine, compare and approximate real roots of univariate polynomials and bivariate polynomial systems. So far the package only provides models for the univariate kernel. Nevertheless, it already defines concepts for the bivariate kernel, since this settles the interface for upcoming implementations.
— *3D Periodic Alpha-Shapes*. The packages 3D alpha-shapes [38] and 3D periodic triangulations [37] have been interfaced in CGAL 3.6, which allows to compute 3D periodic alpha-shapes.

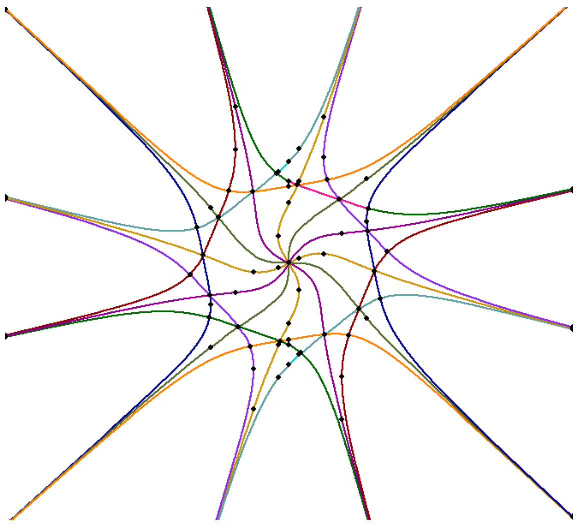


Figure 9. Arrangements of rotated algebraic curves

—*3D Mesh Generation*. The mesh generation package was introduced in CGAL 3.5. From release CGAL 3.6, the package proposes, after Delaunay refinement phase, an optimization phase to improve the quality of the mesh, in particular to get rid of slivers (see figure 10). The release CGAL 3.7 includes a demo of the mesh generation package and the code has been optimized for efficiency [35].

The new release also contains new packages implemented by our CGAL partners and improvements to some existing packages: a detailed list can be found on the CGAL web site.

A one-week CGAL developers meeting has been organized in June at INRIA by Monique Teillaud. There were 18 participants.

7. Contracts and Grants with Industry

7.1. Geometry Factory

The initial development phase of the CGAL library has been made by a European consortium. In order to achieve the transfer and diffusion of CGAL in the industry, a company called GEOMETRY FACTORY has been founded in January 2003 by Andreas Fabri (http://www.geometryfactory.com).

The goal of this company is to pursue the development of the library and to offer services in connection with CGAL (maintenance, support, teaching, advice). GEOMETRY FACTORY is a link between the researchers from the computational geometry community and the industrial users.

It offers licenses to interested companies, and provides support. There are contracts in various domains such as CAD/CAM, medical applications, GIS, computer vision...

GEOMETRY FACTORY is keeping close contacts with the original consortium members, and in particular with GEOMETRICA.

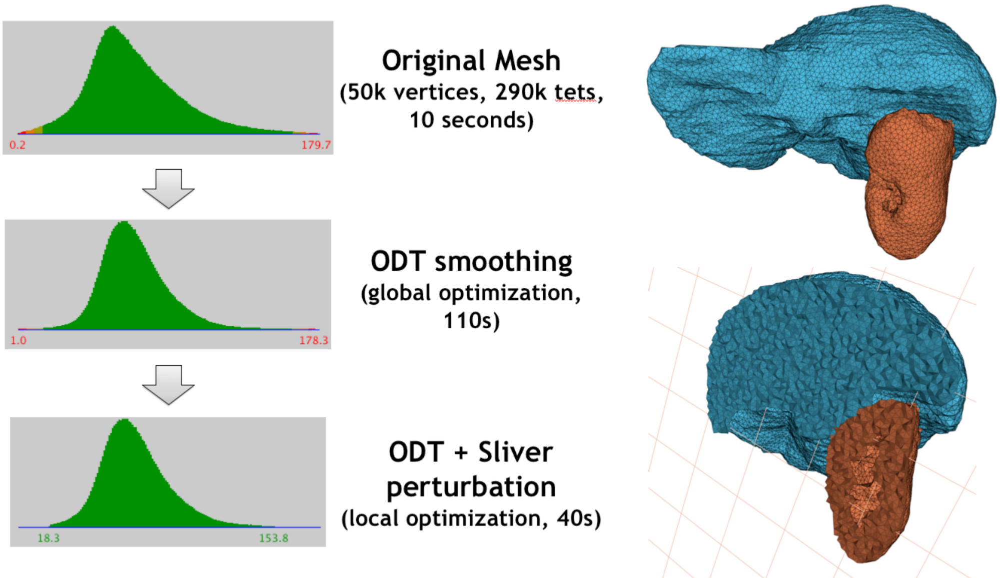


Figure 10. A mesh generated from a 3D segmented medical image of a liver. On the left part, the histograms of dihedral angles in the mesh are shown repectively before optimization (top), after ODT smoothing (middle), after ODT smoothing and vertex perturbation (bottom). The figures correspond in each case to the measure in degrees of the smallest and the biggest dihedral angles in the mesh.

In 2010, GEOMETRY FACTORY had the following new customers for CGAL packages developed by GEOMETRICA: Dr.D Studios (spatial sorting, animation, Australia), Forum8 (2D mesh generation, animation, New Zealand), Esri (AABB tree, GIS, USA), NexGeo (2D mesh generation, GIS, Corea), Polytec (2D mesh generation, Laser Measurement, Germany), Sierra Nevada Corp. (2D triangulations, GIS, USA), Alberta Sustainable Resource Development (2D triangulations, fire fight, Canada).
Additionally, there were two confidential customers, one for surface reconstruction, and one for 3D alpha-shapes.

Moreover, research licenses (in-house research usage for all of CGAL) have been purchased by: Volkswagen (car industry, Germany), Navteq (image processing, USA), InfoTerra (image processing, F), Boeing (air planes, USA), Lawrence Berkeley National Laboratory (USA), KU Leuven (B), Australian National University (Australia), ZIB - Zuse-Institut Berlin (Germany), BRGM (geophysics, F), and by one confidential customer.

7.2. Dassault Systèmes

Participants: Frédéric Chazal, Nicolas Montana.

In collaboration with André Lieutier (Dassault Systèmes)

The goal of this study is to develop and implement robust and efficient 3D Boolean operators and surface regularization tools for industrial use.

The motivation of this work comes from machining simulation where the computation of the part of the space swept by a moving tool involves a huge amount of Boolean operations (unions, intersections, differences). Such computations meet two main difficulties (that are both theoretical and technical): First, 3D Boolean operations face robustness issues and second, the output of large sequences of Boolean operations usually consists of very complicated meshes containing many irrelevant topological and geometric features that need to be removed for further processing. In this study, we intend to develop a software based on an original theoretical approach which overcomes these difficulties.

7.3. Lumiscaphe

Participants: Pierre Alliez, Mariette Yvinec.

In collaboration with Jean-Christophe Leducq (Lumiscaphe) and Alain Tayeb (master intern in our project-team).

Our goal was to devise a new intersection oracle for the mesh generators of CGAL, specialized for parametric NURBS surfaces (common in CAD). The main added value to proceed by Delaunay refinement and access the input surface through an oracle is to be able to mesh the union of NURBS surface patches at once instead of meshing each patch separately. For models comprising hundreds of patches we obtain meshes with lower complexity and with only well-shaped triangles. For a given intersection query with a line segment, our methodology consists of using a Newton iteration in the parameter space of all NURBS patches in order to track the intersection point(s). As future work we wish to elaborate upon a robust version of the intersection oracle, so as to deal with defect-laden NURBS surfaces. The current prototype code will be later consolidated and added to the CGAL library.

8. Other Grants and Activities

8.1. National Actions

8.1.1. ADT CGAL-Mesh

Participants: Pierre Alliez, Mariette Yvinec, Jean-Daniel Boissonnat, Stéphane Tayeb, Dobrina Boltcheva.

CGAL-Mesh is a two-year INRIA technological development action started in March 2009. Building upon components from CGAL, we have started implementing generic and robust mesh generation algorithms for surfaces, 3D domains as well as time-varying 3D domains. We primarily target applications which involve data acquired from the physical world: geology, medicine, 3D cartography and reverse engineering. We wish to establish for the whole duration of the action a close collaboration with industrial and academic partners so as to maximize the impact of the platform for a number of applications and research experiments.

- Starting date: March 2009

- Duration: 2 years

8.1.2. ANR Triangles

Participants: Manuel Caroli, Pedro Machado Manhães de Castro, Olivier Devillers, Sylvain Pion, Monique Teillaud.

Web site: http://www.inria.fr/sophia/geometrica/collaborations/triangles/

We lead the TRIANGLES project funded by the ANR. The project involves:
— the «Laboratoire d'InfoRmatique en Image et Systèmes d'information» (LIRIS), Lyon,
— the «Département d'informatique de l'ENS»
— the GEOMETRICA team.

Triangulations are essential in many applications, in particular for meshing and shape reconstruction. We want to develop and distribute new results for academic and industrial researchers. The goal of the project is the development of robust and effective algorithms for the manipulation of large sets of points, of moving sets of points and points in non Euclidean spaces such as periodic spaces (torus, cylinder), projective, oriented projective or hyperbolic spaces. The results obtained will be implemented in the CGAL library and will be applied to computer vision (visual envelopes, camera calibration), fluid dynamics, astronomy, computer graphics and medical applications.

In the GEOMETRICA team, Triangles is co-funding the scholarship of Pedro de Castro (with «Région PACA») and funding travel expenses and computers. Several meetings have been organized between participants, details can be found on the project's web page. A workshop has been organized with the associated team OrbiCG in december (see Section Workshops below).

- Starting date: November 2007

- Duration: 3 years + 6 months prolongation.

8.1.3. ANR GAIA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, Arijit Ghosh, David Cohen-Steiner.

The aim of this project is to formalize a collaboration between researchers from computational geometry, machine learning and computer vision to study distortions and in particular Bregman divergences, information theory, statistics, Riemannian geometry, and convex analysis.

The other partners of the project are the Université des Antilles et de la Guyane (R. Nock, coordinator), the Ecole Polytechnique (F. Nielsen) and the Lear project-team (C. Schmid).

- Starting date: November 2007

- Duration: 4 years

8.1.4. ANR Galapagos

Participant: Sylvain Pion.

In this project, we wish to apply computerized theorem proving tools to two aspects of geometry. One aspect concerns computational geometry. The second aspect is focused on verifying geometric reasoning steps in usual constructions, such as constructions with rules and compass. Other participants in this contract are the universities of Strasbourg and Poitiers, the ENSIEE in Evry and the Ecole Normale Supérieure in Lyon. The leader of the project is the MARELLE project-team.

- Starting date: November 2007.

- Duration: 3 years.

8.1.5. ANR GIGA

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Mariette Yvinec, Steve Oudot, Marc Glisse, Primoz Skraba.

GIGA stands for Geometric Inference and Geometric Approximation. GIGA aims at designing mathematical models and algorithms for analyzing, representing and manipulating discretized versions of continuous shapes without losing their topological and geometric properties. By shapes, we mean sub-manifolds or compact subsets of, possibly high dimensional, Riemannian manifolds. This research project is divided into tasks which have Geometric Inference and Geometric Approximation as a common thread. Shapes can be represented in three ways: a physical representation (known only through measurements), a mathematical representation (abstract and continuous), and a computerized representation (inherently discrete). The GIGA project aims at studying the transitions from one type to the other, as well as the associated discrete data structures.

Some tasks are motivated by problems coming from data analysis, which can be found when studying data sets in high dimensional spaces. They are dedicated to the development of mathematically well-founded models and tools for the robust estimation of topological and geometric properties of data sets sampled around an unknown compact set in Euclidean spaces or around Riemannian manifolds.

Some tasks are motivated by problems coming from data generation, which can be found when studying data sets in lower dimensional spaces (Euclidean spaces of dimension 2 or 3). The proposed research activities aim at leveraging some concepts from computational geometry and harmonic forms to provide novel algorithms for generating discrete data structures either from mathematical representations (possibly deriving from an inference process) or from raw, unprocessed discrete data. We target both isotropic and anisotropic meshes, and simplicial as well as quadrangle and hexahedron meshes.

This project coordinated by GEOMETRICA also involves researchers from the INRIA team-project ABS, CNRS (Grenoble), and a representative from the industry holding a PAST position (Visiting Professor from Industry) at the university of Grenoble.

- Starting date: October 2009.

- Duration: 4 years.

8.1.6. ANR Gyroviz

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Nader Salman, Mariette Yvinec.

The Gyroviz project was selected by the ANR in the framework of the call Audivisual and Multimedia techniques. The project, which was launched in December 2007 for three years, involves the SME Sofresud (Toulon, coordinator) and IXSEA and research teams from the CEA, INRIA and SupMECA Toulon. The project addresses the challenge of automatic modeling of 3D physical scenes from located frames. The aim of the project is to couple new accurate inertial sensors with an image acquisition device and efficient reconstruction algorithms to obtain an automatic image-based modeling system.

- Starting date: December 2007.

- Duration: 3 years.

8.1.7. DIGITEO project GAS: Geometry Algorithms and Statistics

Participants: Claire Caillerie, Frédéric Chazal, David Cohen-Steiner, Bertrand Michel, Steve Oudot.

The project GAS was selected by the DIGITEO consortium in the framework of the “Domaines d’Intérêt Majeur” call of the Région Île-de-France. The project intends to explore and to develop new research at the crossing of information geometry, computational geometry and statistics. It started in September 2008 for an expected duration of 2 years. The other partners of the project are the Ecole Polytechnique (F. Nielsen) and the SELECT project-team (G. Celeux, P. Massart).

- Starting date: September 2008.

- Duration: 2 years.

8.1.8. DIGITEO Chair C3TTA: Cell Complexes in Computational Topology: Theory and Applications

Participants: Claire Caillerie, Frédéric Chazal, David Cohen-Steiner, Steve Oudot, Primoz Skraba, Amit Patel.

The primary purpose of this project is to bring about a close collaboration between the chair holder Dr Vin de Silva and Digiteo teams working on the development of topological and geometric methods in Computer Science. The research program is motivated by problems coming from the increasing need of studying and analyzing the (often huge) data sets that are now available in many scientific and economic domains. Indeed, due to the improvements of measurement devices and data storage tools, the available data about complex shapes or complex systems are growing very fast. These data being often represented as point clouds in high dimensional (or even infinite dimensional) spaces there is a considerable interest in analyzing and processing data in such spaces. Despite the high dimensionality of the ambiant space, one often expects them to be located around an unknown, possibly non linear, low dimensional shape. It is then appealing to infer and analyse topological and geometric characteristics of that shape from the data. The hope is that this information will help to process more efficiently the data and to better understand the underlying complex systems from which the data are generated. In the last few years, topological and geometric approaches to obtain such information have encountered an increasing interest. The goal of this project is to bring together the complementary expertises in computational topology and geometry of the involved Digiteo teams and in applied geometry and algebraic topology of V. de Silva to develop new topological approaches to the previous mentioned domain. The project intends to develop both the theoretical and practical sides of this subject. The other partners of the project are the Ecole Polytechnique (L. Castelli-Aleardi and F. Nielsen) and the CEA (E. Goubault).

- Starting date: January 2009.

- Duration: 3 years.

8.2. Actions Funded by the EC

8.2.1. Coordination action FOCUS K3D

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Mariette Yvinec.

Web page: http://www.focusk3d.eu/.

FOCUS K3D (ICT-2007-214993) is a Coordination Action of the European Union's 7th Framework Programme. The other consortium members are:
– Istituto di Matematica Applicata e Tecnologie Informatiche - Unità Organizzativa di Genova - Consiglio Nazionale delle Ricerche (CNR-IMATI-GE), Italy.
– Center for Research and Technology - Thessaly - Laboratory for Information Technology Systems and Services (CERETETH), Greece.
– École Polytechnique Federale de Lausanne - VRlab (EPFL), Switzerland.
– Fraunhofer-Institut für Graphische Datenverarbeitung, Germany.
– Université de Genève - MIRALab, Switzerland.
– SINTEF, Norway.
– Utrecht University, The Netherlands.
The aim of FOCUS K3D was to foster the comprehension, adoption and use of knowledge intensive technologies for coding and sharing 3D media content in application communities by: (i) exploiting the scientific and technological advances in the representation of the semantics of 3D media to increase awareness of the new technologies for intelligent 3D content creation and management; (ii) building user-driven scenarios to evaluate and adapt the technologies so far developed to the requirements of application environments; and (iii) fostering a shift of role of 3D content users, from passive consumers of technologies to active creators.

- Dates: March 2008 - March 2010.

- Duration: 2 years.

8.2.2. CG Learning

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Olivier Devillers, Marc Glisse, Steve Oudot, Mariette Yvinec.

Web page: http://cglearning.eu/.

Computational Geometric Learning (ICT-2007-255827) is FET Open project of the European Union's 7th Framework Programme. The consortium members are:
– Friedrich-Schiller Universität Jena
– National and Kapodestrian University of Athens
– Technische Universität Dortmund
– Institut National de Recherche en Informatique
– Tel Aviv University
– Eidgenössische Technische Hochschule Zürich
– Rijksuniversität Groningen
– Freie Universität Berlin
High dimensional geometric data are ubiquitous in science and engineering, and thus processing and analyzing them is a core task in these disciplines. The Computational Geometric Learning project (CG Learning) aims at extending the success story of geometric algorithms with guarantees, as achieved in the CGAL library and the related EU funded research projects, to spaces of high dimensions. This is not a straightforward task. For many problems, no efficient algorithms exist that compute the exact solution in high dimensions. This behavior is commonly called the curse of dimensionality. We plan to address the curse of dimensionality by focusing on inherent structure in the data like sparsity or low intrinsic dimension, and by resorting to fast approximation algorithms. The following two kinds of approximation guarantee are particularly desirable: first, the solution approximates an objective better if more time and memory resources are employed (algorithmic guarantee), and second, the approximation gets better when the data become more dense and/or more accurate (learning theoretic guarantee). To lay the foundation of a new field—computational geometric learning—we will follow an approach integrating both theoretical and practical developments, the latter in the form of the construction of a high quality software library and application software.

- Dates : November 2010, November 2013.

- Duration: 3 years.

8.3. International initiatives

8.3.1. Associate team TGDA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Quentin Mérigot, Steve Oudot.

We are involved in an INRIA associated team with the group of Prof. Leonidas Guibas at Stanford University since January 2008. Our collaboration focuses on Topological and Geometric Data Analysis. More precisely, our aim is to develop new topological and geometric frameworks and algorithms for the analysis of data sets represented by point clouds in possibly high-dimensional or non-Euclidean spaces. Several visits took place in 2010 leading to several joint publications. Among the scientific outcomes of this collaboration are a new stability theory for topological persistence, a new analysis method for scalar fields defined over sampled Riemannian manifolds, and a clustering algorithm based on persistence.

8.3.2. Associate team DDGM

Participants: Pierre Alliez, David Cohen-Steiner.

We are involved in an INRIA associate team with Prof. Desbrun's group at Caltech since January 2009. Our goal is to collaborate on topics commonly referred to as Geometry Processing. This year we have exchanged on robust surface reconstruction. In addition to Prof. Desbrun three students from Caltech were involved in the collaboration. We applied for renewal of the associate team for 2011.

8.3.3. Associate team OrbiCG

Participants: Mikhail Bogdanov, Manuel Caroli, Monique Teillaud.

The associate team OrbiCG started in 2009. It is a joint project with two institutes of the University of Groningen: the Institute of Mathematics and Computing Science led by Gert Vegter, and Rien van de Weijgaert from the Kapteyn Astronomical Institute. This research was originally motivated by the needs of astronomers in Groningen who study the evolution of the large scale mass distribution in our universe by running dynamical simulations on periodic 3D data. Our goal is to extend the traditional focus of computational geometry on the Euclidean space $\mathbb{R}^d$ ("urbi") to encompass various spaces ("orbi"), in particular orbit spaces of the Euclidean space, of the hyperbolic space, and of the sphere.

8.4. Exterior research visitors

— Gert Vegter, *Institute of Mathematics and Computing Science, University of Groningen, NL*, two weeks in February, two weeks in october, one week in December.
— Jonathan Shewchuk, *University of Berkeley*, 3 months in February-April.
— Johan Hidding, *Institute of Mathematics and Computing Science, University of Groningen, NL*, one week in December.
— Pratyush Pranav, *Kapteyn Astronomical Institute, University of Groningen, NL*, three weeks in June.
— Rien van de Weijgaert, *Kapteyn Astronomical Institute, University of Groningen, NL*, one week in December.
— Mathieu Desbrun, *Caltech*, one week in July.
— Fernando de Goes, *Caltech*, one month in June-July.
— Amir Vaxman, *Technion*, one week in September.
— Leonidas Guibas *Stanford*, 2 weeks in Sophia Antipolis in August and 3 weeks in Saclay in October-September.

9. Dissemination

9.1. Animation of the scientific community

9.1.1. Editorial boards of scientific journals

— P. Alliez is an associate editor *ACM Transactions on Graphics* and *Graphical Models*. He was an associate editor of *The Visual Computer* until October 2010.
— J-D. Boissonnat is a member of the editorial board of the *Journal of the ACM*, *Discrete and Computational Geometry*, *Algorithmica*, the *International Journal of Computational Geometry and Applications* and the electronic *Journal of Computational Geometry*. He is also a member of the editorial advisory board of the Springer Verlag book series *Geometry and Computing*.
— F. Chazal is an associate editor of *Graphical Models* and *SIAM journal on Imaging Science*.
— M. Teillaud is a member of the editorial board of CGTA, *Computational Geometry: Theory and Applications* and of IJCGA, the *International Journal of Computational Geometry and Applications*.
— M. Yvinec is a member of the editorial board of *Journal of Discrete Algorithms*.
— P. Alliez, M. Hemmer, S. Pion (chair), M. Teillaud (review manager), and M. Yvinec are members of the CGAL editorial board.

9.1.2. Conference program committees

— P. Alliez was paper co-chair of Pacific Graphics 2010. He was a programme committee member of Eurographics Symposium on Geometry Processing, ACM Symposium on Solid and Physical Modeling, Shape Modeling International and Advances in Architectural Geometry. He also organized a mini-symposium on Open Source Software for Curves and Surfaces for the seventh international conference on Curves and Surfaces.
— J-D. Boissonnat was a programme committee member of Eurographics Symposium on Geometry Processing and a member of the organization committee of the seventh international conference on Curves and Surfaces.
— F. Chazal was a programme committee member of ATMCS 2010, Shape Modeling International.
— D. Cohen-Steiner was a programme committee member of Eurographics Symposium on Geometry Processing 2010.
— S. Pion was a programme committee member of ESA 2010, European Symposium on Algorithms.
— Monique Teillaud was a member of the *Third International Congress on Mathematical Software*, held in Kobe University, Kobe, Japan, September 13-17, 2010.

9.1.3. Steering committees

— Monique Teillaud is a member of the Computational Geometry Steering Committee.

9.1.4. Ph.D. thesis and HDR committees

— P. Alliez was a member of the PhD committee of Mathieu Bredif (Telecom ParisTech and IGN, France), and thesis reviewer for Thierry Stein (INRIA Rhône-Alpes) and Julie Digne (ENS Cachan).
— J-D. Boissonnat was a member of the PhD committee of Pooran Memari (Université de Nice - Sophia Antipolis), Nicolas Montana (Université Paris-Sud), Jean-Marie Mirebeau (Paris 6), Nader Salman (Université de Nice - Sophia Antipolis).
— F. Chazal was a member of the PhD committee of Nicolas Montana (Université Paris-Sud), Julie Digne (ENS Cachan) and Maks Ovsjanikov (Stanford).
— O. Devillers was a member of the PhD committee of P. M. M. de Castro (University of Nice-Sophia Antipolis).
— M. Teillaud was a reviewer and a member of the HDR committee of Guillaume Damiand (Université Claude Bernard Lyon 1), a reviewer and a member of the PhD committee of Maria Pentcheva (Université Nancy 2), and a member of the PhD committee of Luis Peñaranda (Université Nancy 2) and of Manuel Caroli (Université de Nice - Sophia Antipolis).
— Mariette Yvinec was a member of the PhD committee of Nader Salman (Université de Nice - Sophia Antipolis).

9.1.5. INRIA committees

— P. Alliez is member of the COST GTAI (conseil d'orientation scientifique et technologique, groupe de travail actions incitatives), of the commission d'animation scientifique (CAS) and of the comité de suivi doctoral (CSD).
— J-D. Boissonnat was a member of the CR2/CR1 recruitment committee of INRIA Rhône-Alpes.
— F. Chazal is a member (chair since November) of the "Commission scientifique" at INRIA Saclay - Île de France.
— M. Teillaud is a member of the INRIA Evaluation Board, the INRIA Sophia Antipolis - Méditerranée CDT (Committee for Technologic Development), and the national INRIA CDT.

9.1.6. Other committees

— J.-D. Boissonnat chaired the visiting committee of LIAMA (Pékin), november 2010.
— J.-D. Boissonnat is a member of the working groups GP1 (Modèles et calcul) and GP2 (Logiciels et systèmes informatiques) de l'Alliance des sciences et technologies du numérique (Allistène).
— S. Pion is a member of the experts group of AFNOR for the standardization of the C++ language within the ISO/WG21 working group.
— S. Pion is a member of the IEEE-1788 working group for standardization of interval arithmetic.

9.1.7. Conference organization

— J-D. Boissonnat chairs the scientific committee of the Jacques Morgenstern colloquium.
— O. Devillers, C. French, and M. Teillaud co-organized the Triangles-OrbiCG workshop on computational geometry. http://www.inria.fr/sophia/geometrica/collaborations/triangles/Workshop/
— M. Yvinec and P. Alliez co-organized the workshop on Semantic 3D Media and Content within the framework of the EU coordinated action Focus K3D (http://195.251.17.14/conference/).

9.1.8. Web site

M. Teillaud is maintaining the Computational Geometry Web Pages http://www.computational-geometry.org/, hosted by INRIA. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual Symposium on Computational Geometry.

9.2. Teaching

9.2.1. Teaching responsibilities

— Monique Teillaud is a member of the jury of the Agrégation de Mathématiques.

9.2.2. Teaching at universities

We give here the details of graduate courses. Web pages of these courses can be found on the web site : http://www.inria.fr/sophia/geometrica/

2009-2010 courses (taught in 2010)
— Master IFI 1ère année (Nice), *Computational Geometry*, O. Devillers (12h).
2010-2011 courses (taught in 2010)
— International Chair of Tsinghua University (Beijing), *Shape reconstruction*, J-D. Boissonnat (2 days).
— Master IFI (Sophia Antipolis), *Geometric algorithms, theory and practice*, P. Alliez, O. Devillers and M. Teillaud (28h).
— Ecole des Ponts ParisTech (Paris), *Meshes and Applications*, P. Alliez with collaboration from G. Peyré (21h).
— Master at EFREI (Paris), *Geometric algorithms*, P. Alliez and B. Pellenard (45h).
— Winter school, ENS Lyon, *Algorithms for geometric approximation* J.-D. Boissonnat, F. Chazal and M. Yvinec (24).
— Master MPRI (Paris) *Computational Geometric Learning* J.-D. Boissonnat, F. Chazal and M. Yvinec (24h).

9.2.3. Internships

Internship proposals can be found on the web at http://www.inria.fr/sophia/geometrica/
— Mikhail Bogdanov, 3D periodic volume meshes, Moscow Institute of Physics and Technology.
— Thu-Hien Nguyen Thi, Tangential complex, MPRI.
— Fei (Sophie) Che, 3D triangulation demo, in the framework of the Google Summer of Code, University of Delaware (PhD student)
— Amir Vaxman, Oracle for subdivision surfaces, in the framework of the Google Summer of Code, Technion (PhD student).
— Alain Tayeb, Meshing NURBS surfaces, in collaboration with Lumiscaphe (Master student).
— Boris Dalstein, Quadrangle surface tiling, (student from ENS Lyon).
— Marc Jeanmougin, Parallel Mesh Generation, (student from ENS Paris).
— Kacper Rzepecki, Improving and simplifying the CGAL Triangulation API, in the framework of the Google Summer of Code, Warsaw, Poland.
— Maxime Brénon, Experimental study of nearest and reverse nearest neighbor searches using Locality-Sensitive Hashing (Master student).

9.2.4. Ongoing Ph.D. theses

— Mikhail Bogdanov, *Triangulations in non-Euclidean spaces*, Université de Nice-Sophia Antipolis.

— Alexandre Bos, *Topological methods for geometric data classification*, Université Paris XI.
— Claire Caillerie, *Sélection de modèles pour l'inférence géométrique*, Université Paris XI.
— Arijit Ghosh, *Computational Information Geometry*, Université de Nice-Sophia Antipolis.
— Bertrand Pellenard, *Surface and Domain Tiling*, Université de Nice-Sophia Antipolis.

9.2.5. Ph.D. and HDR defenses

— Manuel Caroli, *Triangulating Point Sets in Orbit Spaces*, Université de Nice-Sophia Antipolis, December 10th.
— Pedro Machado Manhães de Castro, *Practical Ways to Accelerate Delaunay Triangulations*, Université de Nice-Sophia Antipolis, October 25th.
— Nicolas Montana, *Calcul robuste d'enveloppes de solides en mouvement. Application à la simulation de l'enlèvement de matière en usinage*, Université Paris-Sud, June 2010.
— Pooran Memari, *Geometric tomography with topological guarantees*, Université de Nice-Sophia Antipolis, March 26th.
— Nader Salman, *3D point clouds to feature preserving meshes*, Université de Nice-Sophia Antipolis, December 16th.

9.3. Participation to conferences, seminars, invitations

9.3.1. Invited Talks

Invited talks at conferences :
— P. Alliez, "Digital Geometry Processing", Le modèle et l'algorithme, INRIA Rocquencourt, January 2010.
— J-D. Boissonnat, "Maillage et reconstruction de variétés", LAAS, March 2010.
— J-D. Boissonnat, "Star stitching", Journée conjointe MSPC/AFA Génération de maillages : théorie et applications", November 2010.
— J-D. Boissonnat, "Histoire naturelle de la géométrie algorithmique", Lycée Alphonse Daudet, Nîmes, Septembre 2010.
— P. Alliez, "Traitement numerique de la geometrie", Lycée Alphonse Daudet, Nîmes, Septembre 2010.
— F. Chazal, "Geometric inference for probability measures: extracting robust geometric information from noisy data", Journées STAR, Rennes II, October 2010.
— F. Chazal, "Geometric inference for probability measures: extracting robust geometric information from noisy data", Algebra and Topology: Methods, Computation and Science, June 2010.
— F. Chazal, "Geometric Inference", Curves and Surfaces 2010, Avignon, June 2010.
— S. Oudot, "Analysis of Scalar Fields over Point Cloud Data", Algebra and Topology: Methods, Computation and Science, June 2010.

9.3.2. Conferences and Seminars

Members of the project have presented their published articles at conferences. The reader can refer to the bibliography to obtain the corresponding list. We list below all other talks given in seminars, summer schools and other workshops.
— J.-D. Boissonnat, "Manifold Reconstruction using Tangential Delaunay Complexes", Stanford university, January 2010.
— F. Chazal, "Geometric inference for probability measures: extracting robust geometric information from noisy data", séminaire parisien de statistique, IHP, January 2010.
— F. Chazal, "Geometric Inference", Stanford, Feb. 2010.
— J.-D. Boissonnat, "Manifold Reconstruction using Tangential Delaunay Complexes", Journées de Géométrie Algorithmique, Luminy, mars 2010.
— C. Caillerie, "Sélection de modèle pour l'approximation simpliciale", Journées de Géométrie Algorithmique, Luminy, mars 2010.
— M. Caroli, "Computing 3D Periodic Triangulations", Journées de Géométrie Algorithmique, Luminy, mars 2010.

— O. Devillers, “Influence du bruit sur le nombre de points extrêmes”, Journées de Géométrie Algorithmique, Luminy, mars 2010.
— P. Skraba, “Soft Clustering and Persistent Homology”, Journées de Géométrie Algorithmique, Luminy, mars 2010.
— P. M. M. de Castro, “Self-Adapting Point Location”, Journées de Géométrie Algorithmique, Luminy, mars 2010.
— F. Chazal, “Calcul géométrique : de la modélisation des formes 3D à la recherche de structures géométriques dans les masses de données”, séminaire Maths et Société, Neuchatel, avril 2010.
— F. Chazal, “Geometric inference for probability measures: extracting robust geometric information from noisy data”, séminaire de mathématiques, Neuchâtel, avril 2010.
— P. Alliez, “Robust Surface Reconstruction from Raw Point Sets”, ENS Cachan, Juin 2010.
— O. Devillers, “Triangulation de Delaunay de points dans des disques. Concilier complexité et efficacité”, Journées Informatique et Géométrie, Grenoble, septembre 2010.
— M. Glisse, “Persistance topologique et stabilité”, Journées Informatique et Géométrie, Grenoble, septembre 2010.
— M. Yvinec, “CGAL-mesh”, seminar at CEMRACS 2010, SMAI, Marseille Luminy, september 2010.
— M. Yvinec, “CGAL-mesh”, Tetrahedron workshop, Swansea U.K., septembre 2010.
— F. Chazal, “Geometric inference for probability measures: extracting robust geometric information from noisy data”, séminaire de mathématiques, Mulhouse, novembre 2010.

9.3.3. The Geometrica seminar

http://www.inria.fr/sophia/geometrica/
The GEOMETRICA seminar featured presentations from the following visiting scientists:
— I. Bloch and T. Boubekeur (Telecom Paris Tech) : Modélisation réaliste du corps humain: intérprétation d’images et informatique graphique.
— A. Vaxman (Technion) : A multi-resolution approach to heat kernels on discrete surfaces.
— A. Vaxman (Technion) : On line reconstruction of 3D objects from arbitrary cross-section data.
— L. Guibas (Stanford) : The information is in the maps.
— T. Lewiner (PUC) : Fast generation of pointerless octree duals.
— G. Tavares (PUC) : Geometry processing and application to 3D oil reservoir reconstruction.
— F. de Goes (Caltech) : Exoskeleton: shape abstraction driven by perceptual parts.
— L. de Luca (CNRS/MCC Marseille) : Methods, formalisms and tools for digital surveying and representation of architectural heritage.
— Y. Wei (LIAMA) : Real-time simulation of blood flow interactions in vascular procedures.
— M. Fisher (Stanford) : Context-aware scene modeling.
— J.M. Favreau (CNR-IMATI) : Outils pour le pavage de surfaces.
— G. Vegter (Groeningen) : Complexity of curve approximation.
— N. Wolpert (Stuttgart) : Maintaining exactly the convex hull of points moving along circles.

9.3.4. Scientific visits

— Pierre Alliez, Caltech, Nov 19-Dec 22.
— Jean-Daniel Boissonnat, Stanford, Jan. 20- Feb, 3, 2010.
— Manuel Caroli, University of Groningen, May 3-28.
— Manuel Caroli, Stanford University, October 26-27.
— Frédéric Chazal, Stanford, Feb 22 - March 5.
— Frédéric Chazal, Stanford, Nov 26-Dec 4.
— Monique Teillaud, University of Groningen, May 19-28.

9.3.5. Distinctions

— Michael Hemmer received the Dan David Prize Fellowship, Dan David Foundation, Tel Aviv, Israel, May 10.

— F. Chazal and P. Skraba received with M. Ovsjanikov and L. Guibas from Stanford the best paper award [33] at the NORDIA workshop 2010 (CVPR 2010).

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[14] N. MONTANA. *Calcul robuste d'enveloppes de solides en mouvement. Application à la simulation de l'enlèvement de matière en usinage*, Université Paris XI, France, 2010.

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