INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team mathfi

Financial mathematics

Paris - Rocquencourt

Theme : Stochastic Methods and Models

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2. Overall Objectives

2.1. Overall Objectives

MathFi is a joint INRIA project-team with ENPC (CERMICS) and the University Paris-est Marne la Vallée, located in Rocquencourt and Marne la Vallée.

The development of increasingly complex financial products requires the use of advanced stochastic and numerical analysis techniques. The scientific skills of the MathFi research team are focused on probabilistic and deterministic numerical methods and their implementation, stochastic analysis, stochastic control. Main applications concern evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, calibration of financial models, risk management. Special attention is paid to models with jumps, stochastic volatility models, asymmetry of information. The MathFi project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. Mathfi web site: http://www-rocq.inria.fr/mathfi/. Premia web site: http://www.premia.fr.

3. Scientific Foundations

3.1. Simulation of Stochastic Differential Equations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Mohamed Sbai.

Most financial models are described by SDEs. Except in very special cases, no closed-form solution is available for such equations and one has to approximate the solution via time-discretization schemes in order to compute options prices and hedges by Monte Carlo simulations. Usually this is done by using the standard explicit Euler scheme since schemes with higher order of strong convergence involve multiple stochastic integrals which are difficult to simulate. In addition, the weak order of convergence of the explicit Euler scheme can be improved by using Romberg-Richardson's extrapolations. Nevertheless, some schemes with weak order of convergence two or more have been designed recently. The idea is either to replace the multiple Brownian integrals by discrete random variables which share their moments up to a given order or to integrate Ordinary Differential Equations associated with the vector fields giving the coefficients of the Stochastic Differential Equation up to well-chosen random time-horizons. Another interesting new direction of investigation is the design of exact simulation schemes.

Three directions of research have been investigated in the Mathfi project. First, fine properties of the Euler scheme have been studied [74], [69], [77]. Secondly, concerning SDEs for which the Euler scheme is not feasible, A. Alfonsi [63] have proposed and analysed new schemes respectively for Cox-Ingersoll-Ross processes and for equations with locally but not globally Lipschitz continuous coefficients. Last, the team has contributed to the new directions of research described above. For CIR processes, A. Alfonsi has designed a scheme with weak order two even for large values of the volatility parameter. Adapting exact simulation ideas, B. Jourdain and M. Sbai [76] have proposed an unbiased Monte Carlo estimator for the price of arithmetic average Asian options in the Black-Scholes model.

3.2. Numerical methods for option pricing and hedging and model calibration

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Mohamed Sbai, Vlad Bally, Bernard Lapeyre, Agnès Sulem, Ahmed Kebaier, Céline Labart, Jérôme Lelong, David Pommier, Lokmane Abbas-Turki, Abdelkoddousse Ahida, Antonino Zanette, El Hadj Aly Dia.

Efficient computations of prices and hedges for derivative products are major issues for financial institutions (see [80]) .

This is done by using either Monte-Carlo methods or partial differential equation techniques. Monte-Carlo simulations are widely used because of their implementation simplicity and because closed formulas are usually not available. Speeding up the algorithms is a constant preoccupation in the development of Monte-Carlo simulations. The team is mainly concerned with *adaptive versions* which improve the Monte-Carlo estimator by relying only on stochastic simulations.

The team has also been active on numerical methods in models with jumps and large dimensional problems.

This activity in the MathFi team is strongly related to the development of the Premia software.

Model calibration: The modeling of the so called *implied volatility smile* which indicates that the Black-Scholes model with constant volatility does not provide a satisfactory explanation of the prices observed in the market has led to the appearance of a large variety of extensions of this model as the local volatility models (where the stock price volatility is a deterministic function of the price level and time), stochastic volatility models, models with jump, and so on. An essential step in using any such approach is the *model calibration*, that is, the reconstruction of model parameters from the prices of traded options. This is an inverse problem to that of option pricing and as such, typically ill-posed.

The calibration problem is yet more complex in the interest rate markets since in this case the empirical data that can be used includes a wider variety of financial products from standard obligations to swaptions (options on swaps). The underlying model may belong to the class of short rate models like Hull-White [75], [67], CIR [70], Vasicek [90] etc. or to the popular class of LIBOR (London Interbank Offered Rates) market models like BGM [68].

The choice of a particular model depends on the financial products available for calibration as well as on the problems in which the result of the calibration will be used.

The calibration problem is of particular interest for MathFi project because due to its high numerical complexity, it is one of the domains of mathematical finance where efficient computational algorithms are most needed.

3.3. Malliavin calculus and applications in finance

Participants: Vlad Bally, Arturo Kohatsu-Higa, Agnès Sulem, Antonino Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [81]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [83], D. Bell [66] D. Ocone [86], B. Øksendal [93]) give expositions of the subject. See also V. Bally [64] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [85].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [82], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods.

We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be "smooth" in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequences of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M} \delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M} E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [73] and [72] and the papers by Bally et al., Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [79].

L. Caramellino, A. Zanette and V. Bally have been concerned with the computation of conditional expectations using Integration by Parts formulas and applications to the numerical computation of the price and the Greeks (sensitivities) of American or Bermudean options. The aim of this research was to extend a paper of Reigner and Lions who treated the problem in dimension one to higher dimension - which represent the real challenge in this field. Significant results have been obtained up to dimension 5 [65] and the corresponding algorithms have been implemented in the Premia software.

Moreover, there is an increasing interest in considering jump components in the financial models, especially motivated by calibration reasons. Algorithms based on the integration by parts formulas have been developed in order to compute Greeks for options with discontinuous payoff (e.g. digital options). Several papers and two theses (M. Messaoud and M. Bavouzet defended in 2006) have been published on this topic and the corresponding algorithms have been implemented in Premia. Malliavin Calculus for jump type diffusions - and more general for random variables with localy smooth law - represents a large field of research, also for applications to credit risk problems.

More recently the Malliavin calculus has been used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P. Imkeller in [79]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [79]).

A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. These SDEs can be interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals, as introduced by Russo and Valois [88]. This methodology has been applied for utility maximization with insiders.

3.4. Optimal stopping

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton.

The theory of American option pricing has been an incite for a number of research articles about optimal stopping. Our recent contributions in this field concern optimal stopping for one dimensional diffusions and American options in exponential Lévy models.

In the context of general one-dimensional diffusions, we have studied optimal stopping problems with bounded measurable payoff functions. We have obtained results on the continuity of the value function and its characterization as the unique solution of a variational inequality in the sense of distributions, both in finite and infinite horizon problems (collaboration between D. Lamberton and Michail Zervos, London School of Economics).

We have explained how to calibrate a continuous and time-homogeneous local volatility function from the prices of perpetual American Call and Put options (A. Alfonsi and B. Jourdain).

The use of jump diffusions in financial models goes back to Merton (1976). More recently, there has been a growing interest for more sophisticated models, involving Lévy processes with no diffusion part and infinite activity (see, in particular, papers by Carr, Geman, Madan and Yor). A number of results on the exercise boundary and on the so called smooth fit property have been established.

3.5. Stochastic Control and Backward Stochastic Differential equations (BSDEs)

Participants: Vlad Bally, Jean-Philippe Chancelier, Marie-Claire Kammerer-Quenez, Agnès Sulem.

B. Øksendal (Oslo University) and Agnès Sulem have written a book on Stochastic control of Jump diffusions [10]). The types of control problems covered include classical stochastic control, optimal stopping, impulse control and singular control. Both the dynamic programming method and the maximum principle method are discussed, as well as the relation between them. Corresponding verification theorems involving the Hamilton-Jacobi Bellman equation and/or (quasi-)variational inequalities are formulated. There are also chapters on the viscosity solution formulation and numerical methods. In the second edition (2007), a chapter on optimal control of stochastic partial differential equations driven by Lévy processes and a section on optimal stopping with delayed information have been added. Applications to portfolio optimization problems and insurance problems have been studied.

In the context of risk measures, M.C. Kammerer-Quenez (Prof Paris VII) has shown how some dynamic measures of risk can be induced by Backward Stochastic Differential Equations and A. Sulem and B. Øksendal in [92] have studied risk-indifference pricing in incomplete markets with jumps using stochastic control theory and PDE methods.

4. Application Domains

4.1. Application domains

- Option pricing and hedging
- Calibration of financial models
- Portfolio optimization
- Risk management
- Market microstructure
- Insurance-reinsurance optimization policy
- Insider modeling, asymmetry of information

5. Software

5.1. PREMIA

Participants: Aurélien Alfonsi, Vlad Bally, Jean-Philippe Chancelier, Benjamin Jourdain, Ahmed Kebaier, Oleg Kudryavtsev, Ismail Laachir, Céline Labart, Anton Kolotaev, Bernard Lapeyre, Jérôme Lelong, Agnès Sulem [correspondant], Xiao Wei, Antonino Zanette, Vadim Zherder.

Figure 1.

Premia is a software designed for option pricing, hedging and financial model calibration. It is provided with it's C/C++ source code and an extensive scientific documentation. Major features of Premia : Pricing Interest Rate Derivatives Pricing Credit Risk Derivatives Pricing Equity in Black-Scholes and Heston models Pricing and Hedging Equity in Jump models Calibration in Jump models
See also the web page http://www-rocq.inria.fr/mathfi/Premia/index.html .

- AMS: 91B28, 65Cxx, 65Fxx, 65Lxx, 65Pxx
- License: Licence Propriétaire (genuin license for the Consortium Premia)
- Type of human computer interaction: Console, interface in Nsp
- OS/Middelware: Linux, Mac OS X, Windows
- Programming language: * C/C++ (librairie Gtk)
- Documentation: the PNL library is interfaced via doxygen
- Size of the software: 237 Mbyte, 36 Mbyte of C/C++ routines; Number of lines of code: 936 339
- Publications: [1] [71] [78] [87] [89], [62]

This project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is:

1. to be a powerful testing platform for comparing different numerical methods between each other;
2. to build a link between professional financial teams and academic researchers;
3. to provide a useful teaching support for Master and PhD students in mathematical finance.

The development of Premia started in 1999 and 12 are released up to now and registered at the APP agency.

5.1.1. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. Equity derivatives:

   The following models are considered:

   Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

   For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods and Malliavin calculus based methods.

   Dynamic Hedging for Black-Scholes and jump models is available.

   Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. Interest rate derivatives

   The following models are considered:

   HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

   Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. Credit derivatives: CDS, CDO

   Reduced form models and copula models are considered.

   Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. Hybrid products:

   PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. Energy derivatives: swing options

   Mean reverting and jump models ar considered.

   Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

5.1.2. Latest features

Premia 12 was delivered to the consortium members in February 2010. It contains new algorithms for pricing equity derivatives in stochastic volatility models (alternative Sabr models, Wishart model), models with jumps (path dependent options) and regime-switching models (using a Wiener-Hopf approach). It also provides algorithms for hedging default risks of CDOs and for calibration and pricing of volatility products using consistent variance curve models.

New algorithms for the release 13 of Premia to be delivered in March 2011 to the Consortium:

- **Interest Rate Derivatives**
  - Pricing and hedging callable Libor exotics in forward Libor models, V.Piterbarg Journal of Computational Finance Volume 8 Number 2, Winter 2004/05 (by I. Laachir)

  - A stochastic volatility forward Libor model with a term structure of volatility smiles. A.Piterbarg, V. (2004), SSRN Working Paper.
  - Iterating cancelable snowballs and related exotics in a many-factor Libor model, John Schoenmakers RISK Sept. 2006, Asia RISK Oct. 2006 (by I. Laachir)
  - A new approach to Libor modeling, M. Keller-Ressel, A.Papapantaleon J.Teichmann: preprint 2009 (by I. Laachir)
  - Jump-adapted discretization schemes for Levy-driven SDEs A.Kohatsu Higa P.Tankov, to appear Stochastic Processes and their Applications

- **Credit Risk Derivatives**
  - R.Cont A.Minca Recovering Portfolio Default Intensities Implied by CDO Quotes, preprint 2008
  - Interacting particle systems for the computation of rare credit portfolio losses, Rene Carmona, Jean-Pierre Fouque and Douglas Vestal, Finance Stochastics Volume 13, Number 4 / September, 2009
  - A Multilevel Approach to Control Variates. Adam Speight. The Journal of Computational Finance, to appear.

- **Electricity Derivatives**
  - Variance optimal hedging for processes with independent increments and applications. Applications to electricity market. F. Russo S. Goutte N. Oudjane. Preprint

- **Equity Derivatives**
  - J. Infante Acevedo, T. Lelievre and D.Pommier: Greedy algorithms for high-dimensional problems. Application to option pricing.
  - On The Heston Model with Stochastic Interest Rates Lech, A. Grzelak, Cornelis W. Oosterlee
  - A Novel Option Pricing Method based on Fourier-Cosine Series Expansions. F. Fang, C.W. Oosterlee, SIAM J. Sci. Comput., to appear.
  - Pricing early-exercise and discrete barrier options by Fourier-cosine series expansions. C.W. Oosterlee, F. Fang
  - Kudryavtsev O.: Wiener-Hopf techniques for Bates and Heston model
  - Fourier space time-stepping for option pricing with Levy models by Kenneth R. Jackson, Sebastian Jaimungal and Vladimir Surkov The Journal of Computational Finance Volume 12 / Number 2, Winter 2008
  - Monte Carlo for pricing Asian options in jump models. E. Dia, D. Lamberton
  - An Improved Convolution Algorithm for Discretely Sampled Asian Options, Cerny, A. and I. Kyriakou, to appear in Quantitative Finance
  - Estimating Greeks in Simulating Levy-Driven Models. Paul Glasserman and Zongjian Liu. The Journal of Computational Finance, to appear.
  - Computing exponential moments of the discrete maximum of a Levy process and lookback options, L. Feng and V. Linetsky The Journal of Computational Finance Volume 13, Number 4 / September, 2009
  - Smart expansion and fast calibration for jump diffusions E. Benhamou, E. Gobet and M. Miri Finance Stochastics Volume 13, Number 4 / September, 2009
  - Saddlepoint methods for option pricing Peter Carr and Dilip Madan The Journal of Computational Finance to appear

- Saddlepoint Approximations for Affine Jump-Diffusion Models P. Glasserman and K. Kim, Journal of Economic Dynamics and Control, vol 33, 37-52, 2009.
- Multi-level Monte Carlo path simulation. *M.B. Giles, Operations Research, 56(3):607-617, 2008*.
- Pricing options under stochastic volatility: a power series approach Antonelli, F. and Scarlatti, S. Finance Stochastics Volume XIII (2009), issue 1
- Gamma Expansion of the Heston Stochastic Volatility Model P. Glasserman and K. Kim, Finance and Stochastics, to appear.
- Pricing and hedging American-style options: a simple simulation-based approach : Yang Wang and Russel Caflisch The Journal of Computational Finance to appear (by I. Laachir)
- Empirical martingale simulation of asset prices Duan, J.-C. and Simonato, J.-G. Management Science(1998) 44-9, 1218-1233
- Doubly Reflected BSDEs with Call Protection and their Approximation, J.F. Chassagneux S. Crépey. Preprint (by C. Labart)
- Pricing Convertible Bonds with Call Protection, S.Crépey and A.Rahal. Preprint (by C. Labart)

- **Developmentof the PNL library by J. Lelong**
  - Complete rewriting of tridiagonal and band matrices related code to be Lapack compliant.
  - New Makefile system based on Automake for using PNL in an external project.
  - All functions related to solving linear systems rely on Blas & Lapack.
  - New implementation of *PnlBasis* to enable multivariate polynomial approximation without any limitation on the number of variates. An internal mechanism enables to compute the first and second derivatives of such a polynomial chaos decomposition. Polynomial bases can also be sparse : the construction of the basis is based on the use of an hyperbolic set of indices which only retain low order interactions. To improve the numerical behavior of the approximation, the polynomial bases can be centered and renormalized to match a given domain.
  - Numerical integration now relies on routines from *QuadPack*: qng, qage, qagi, qags.
  - A top level object has been added to emulate some inheritance between the different objects : this enabled to factorize a lot of code.
  - Multidimensional root finding is now based on *Minpack*.
  - A doubly linked list object has been added.
  - MPI bindings for manipulating *PnlObjects* natively
  - A great effort has been made to make random number generators and related routines thread-safe. For now, all random generators except Sobol have been rewritten to become thread-safe. This has also let to the implementation of a new interface to random number generators in which they are manipulated as *PnlObjects*. The “Dynamically created Mersenne Twister” generator has been added to the library for its very good properties in parallel computing.
  - Save/Load mechanism for all Pnl objects based on MPI Pack/Unpack facilities.

6. New Results

6.1. Simulation and numerical methods

Participants: Benjamin Jourdain, Bernard Lapeyre, P. Sabino, Aurélien Alfonsi, A. Ahdida, Ahmed Kebaier.

6.1.1. Variance reduction by stratification.

B. Jourdain, B. Lapeyre and P. Sabino have proposed a new algorithm to stratify normal random vectors along several non-orthogonal directions. They have discussed the choice of such non-orthogonal directions and checked the efficiency of their algorithm by an extensive numerical comparison with other variance reduction techniques.

6.1.2. Simulation of Wishart processes.

A. Alfonsi and A. Ahdida have completed a paper on the simulation of Wishart processes. It has been submitted last June. Now, they are focusing on the modeling of correlation matrices and will investigate some calibration issue in finance.

6.2. Option Pricing and calibration

Participants: Damien Lamberton, El Hadj Aly DIA, Sidi Mohamed Ould Aly, Oleg Kudryavtsev, Antonino Zanette.

6.2.1. Monte-Carlo methods for exotic options in models with jumps.

(El Hadj Aly DIA's thesis, defended on July 1st, 2010). Some results concerning lookback and barrier options in jump-diffusion models have been obtained, in particular, estimates for expectations involving discrete vs continuous maxima of the sample paths of a Lévy process. We also have results concerning the effect of truncating small jumps and replacing them by Brownian motion.

6.2.2. Exotic options and stochastic volatility models.

(Sidi Mohamed OULD ALY's thesis, now in his third year). Sidi-Mohamed has results on the effective computation of option prices in a stochastic volatility model, in the context of variance swap modeling. He has worked out a new model, in the spirit of Bergomi's approach. This model has remarkable features in terms of tractability and calibration. Sidi Mohamed has just submitted a paper on this topic.

6.2.3. Swing options under Lévy models.

OlegKudryavtsev and AntoninoZanette have developed efficient pricing methods for swing options under Lévy models

6.3. American options

6.3.1. American Put option with discrete dividends

Participants: Benjamin Jourdain, M. Jeunesse, M. Vellekoop.

B. Jourdain and M. Vellekoop are interested in the regularity of the optimal exercise boundary for the American Put option when the underlying asset pays discrete dividends at known times during the lifetime of the option. The dividend amounts are deterministic functions of the asset prices just before the dividend dates. In case these functions are linear, they have proved that the exercise boundary is right-continuous and that the high-contact principle holds for the pricing function. They have also checked that, in a left-hand neigbourhood of each dividend date, the exercise boundary is a non-increasing and continuous function of time which tends to 0 as time approaches the dividend date. M. Jeunesse and B. Jourdain are currently investigating the extension of these results to general dividend functions.

6.3.2. American options in exponential Lévy models

Participants: Damien Lamberton, Ayech Bouselmi.

(Mohammed Mikou's thesis, defended on December 2nd, 2009). We have results on the regularity of the American put price in the case of general exponential Lévy models. We also derive the asymptotic behavior of the early exercise boundary near maturity in non-classical cases. In particular, when the underlying Lévy process has finite variation, the behavior is linear. D. Lamberton's PhD student A. Bouselmi is working on American options within multi-dimensional exponential Lévy models.

6.4. Stochastic control, Stochastic Maximum principles and BSDEs

Participants: John-Joseph-Absalom Hosking, Agnès Sulem.

J. Hosking is studying stochastic maximum principles (SMPs) for certain forms of stochastic differential games (SDG); he has extended to the SDG setting some recent research by B. Øksendal and X. Y. Zhou on SMP using spike variations and Malliavin calculus. He was able to construct a SMP that provided necessary conditions for the existence of Nash equilibria in a certain form of 2-agent SDG, and began to apply this result to risk-indifference pricing problem.

On further analysis of this Malliavin calculus approach, he was able to indicate, using known results from the theory of backward stochastic differential equations (BSDEs), a close relationship between this approach and an earlier approach of S. Peng where the relevant adjoint equations are represented in terms of the solutions to certain BSDEs. In order: to state this relationship exactly, to better understand the possible generality of this form of connection and any possible comparative advantages between the two approaches, the Malliavin calculus based approach and BSDE based approach; The mean-fiel type problem was also studied and SMPs have been obtained for this new problem by extending each of the two forms of approach.

In June 2010, John Hosking began to work on a type of backward stochastic equation approach for a general form of SDG of a certain mean-field type. He has constructed a SMP for the considered SDG in which the adjoint processes are given by a mean-field BSDE and a standard BSDE. The proof relies on a third pair of adjoint processes (but which do not appear in the final SMP equations) that are given by a conditional mean-field BSDE. We are planning to collaborate with B. Djehiche (KTH Stockholm) who recently visited the Mathfi, on the dynamic version of this type of problem for Nash-equilibria in a game setting.

Two other research projects are in progress:

- A joint project with M. Davis (Imperial College London), concerning the representation of solutions to a jump-diffusion martingale problem.
- A joint project with G. Di Nunno and S. Sjursen (University of Oslo), aiming to construct, via Malliavin calculus methods, a stochastic maximum principle for a system where one of the drivers is a doubly stochastic random Poisson measure, and to apply the result to the problem of risk-indifference pricing in a market containing a defaultable asset.

Agnès Sulem has worked on stochastic control and stochastic differential games with jumps, delays, model uncertainty, partial observation in collaboration with B. Øksendal, M.C. Quenz, A. Cretarola and J. Hosking. She has studied in particular stochastic maximum principles for singular control problems, optimal control problems with delays, robust control problems and connections with BSDEs. Applications deal with portfolio optimization, pricing in incomplete markets and risk management with uncertainty([37], [30], [84],[49], [48], [91], [92]).

6.5. Risk

Participants: Aurélien Alfonsi, Céline Labart, Jérôme Lelong, J. Acevedo, Agnès Sulem, Andreea Minca, Jean-Philippe Chancelier.

6.5.1. Credit risk

A. Alfonsi, and J. Lelong have studied closed form extension of the Black-Cox model (paper submitted). They are studying with C. Labart loss models that are used in practice.

6.5.2. Liquidity risk - Limit Order books

(with A. Schied, A. Slynko and J. Acevedo): A. Alfonsi is pursuing his collaboration with A. Schied (paper with A. Schied and A. Slynko has been submitted last December). the collaboration with A. Schied will go on, since he will visit the CERMICS twice two weeks in 2011. On the other hand, A. Alfonsi has started to study with his phD student J Acevedo impact models for which the limit order book shape evolves along the time.

6.5.3. Systemic risk

Participants: Andreea Minca, Agnès Sulem, Jean-Philippe Chancelier.

A. Minca is pursuing her research activity in the second year of the PhD Thesis "Contagion in Financial Markets" under the supervision of Rama CONT (UPMC), funded by the Natixis Foundation of Quantitative Research. With J-Ph. Chancelier and A. Sulem, she is extending the model by adding a control to model contagion with intervention.

6.6. Malliavin calculus

Participant: Vlad Bally.

6.6.1. Malliavin Calculus for jump type processes.

The classical Malliavin Calculus for jump type processes as it was developed in the book of Bichteler Gravereu and Jacod 1985 concerns Poisson point measure in which the jump amplitudes represent independent random variables. Our aim is to consider equation driven by Poisson random measures in which the law of the jumps depends on the position of the solution of the stochastic equation. This corresponds to physical models as the Bolzmann equation for example. The two preprints in collaboration with E. Clément and N. Fournier concern this topic. We mention that this is also a continuation of the work which we have done with M.P. Bavouzet and Marouen Messaoud (former PhD students) concerning the sensitivity computations in jump type models in finance.

6.6.2. Lower bounds for densities.

The last two contributions in this area are papers in collaboration with A. Meda and B. Fernandez from the University of Mexico (tubes estimates) and another one in collaboration with A. Kohatsu-Higa (lower bounds for the density of Asian options models).

6.6.3. Regularity of the law using the Riesz transform.

Following an idea of Malliavin and Thalmaier, L. Caramellino and V. Bally use the Riesz transform in order to give regularity criterions for the law of a random variable under week hypothesis.

7. Contracts and Grants with Industry

7.1. Consortium Premia

- **Consortium Premia**

  The consortium Premia is centered on the development of the pricer software Premia. It is presently composed of the following financial institutions: CALYON, Société Générale, Natixis, Bank Austria, Raiffeisen Zentralbank Österreich AG.
- **Fondation Natixis** grant (PhD Thesis of Andreea Minca): (2009-2012).

8. Other Grants and Activities

8.1. Regional Initiatives

Pôle compétitivité : "Pôle Finance Innovation"

Project "Credinext" on credit risk derivatives (2009-2011).

Partners: Euronext Paris, Lunalogic, Pricing Partners, CMAP (Ecole Polytechnique), CERMICS/ENPC, Université Paris-Est Marne la Vallée (Laboratoire de Mathématiques Appliqués), INRIA (projet Mathfi).

8.2. National Initiatives

- **ANR-08-BLAN Program: Big'MC** (Issues in large scale Monte Carlo). (2009-2012)

  Partners ENST, ENPC, University Paris-Dauphine.

  ENPC coordinator: B. Jourdain
- **Chair "Risques financiers, Fondation du Risque :** 2007-2012

  Partners: Ecole des Ponts ParisTech, Ecole Polytechnique, Société Générale.

8.3. European Initiatives

Part of the European network "Advanced Mathematical Methods for Finance" (AMaMef). This network is supported by the European Science Foundation (ESF).

9. Dissemination

9.1. Seminars and Conferences Organisation

- A. Alfonsi

  Co-organizer of the working group seminar of MATHFI "Méthodes stochastiques et finance".
- V. Bally, E. Clément and D. Lamberton :

  Conference on Malliavin Calculus for Jump Processes, University od Paris-Est, November 18-20 2010, http://congres-math.univ-mlv.fr/malliavin_calculus_for_jump_processes_2010
- F. Russo
  1. Session "Stochastic processes" of the 10th French-Rumenian Conference in Applied Mathematics, August 26-31 2010
  2. *Séminaire Probabilités-Statistiques-Contrôle* ENSTA-ParisTech (October-December 2010)
- A. Sulem

  Organisation of a session on "Stochastic Control and Simulation in Finance" , International Research Forum " , 15-17 December 2010, the Hong Kong Polytechnic University , Hong Kong, http://www.fb.polyu.edu.hk/Forum2010/.

9.2. Participation to workshops, conferences and invitations

- A. Alfonsi
  - "Exact and High order discretization schemes for Wishart processes and their affine extensions", IMS conference, Gothenburg (11th of August).
  - "Optimal execution and price manipulations in limit order book models", Colloque Paris-Dauphine "Markets with Frictions: Transaction Costs and Liquidity Risk" (16th of September). item "Exact and High order discretization schemes for Wishart processes and their affine extensions" (in June at the Seminar of the Chaire 'X-Ponts-Société Générale, in November at the ETH Zürich).
  - "Optimal execution and price manipulations in limit order book models" (in April at Marne-la-vallée).
  - invited at ETH Zürich, by Joseph Teichmann (2nd to the 5th of November)
- V. Bally

  - 23-24 March: s invited by Marta Sanz to give a talk in the University of Barcellona. Title: "Integration by parts formula and applications to equations with jumps".
  - 19-30 April: visit of the University Tor Vergata for a scientific collaboration with Lucia Caramellino.
  - 26-31 August 2010. : invited to give a talk in the "Dixième Colloque Franco-Roumain de Mathématiques Appliquées" in Poitiers, France
  - 8-10 September: invited to give a talk in the conference on "Kolmogorov equations in Physics and Finance" in Modena, Italy.
  - October 25 - 28: invited to give a talk in the "New advances in Backward SDEs for financial engineering applications" in Tamerza (Tunisia).

- J. Hosking
  - Groupe de travail Méthodes Stochastiques et Finance, Université Paris-Est Marne-la-Vallée, Champs sur Marne, France. Title of talk: A weak integration-by-parts formula for a Malliavin calculus of pure jump Lévy functionals. Date of talk: 19 February 2010.
  - The Fifth General Conference on Advanced Mathematical Methods in Finance (AMaMeF 2010), Bled, Slovenia, 4-8 May 2010. Title of talk: A stochastic maximum principle for stochastic differential games via Malliavin calculus and spike variation. Date of talk: 7 May 2010.
  - Workshop on Malliavin Calculus for jump processes, Université Paris-Est, Champs sur Marne, France, 18-20 November 2010. Title of talk: On Sobolev spaces of pure jump Lévy functionals. Date of talk: 18 November 2010.
  - Stochastic Analysis Seminar, Centre of Mathematics for Applications, University of Oslo, Oslo, Norway. Title of talk: A stochastic maximum principle for mean-field stochastic differential games. Date of talk: 14 September 2010.
  - Guest of the Centre of Mathematics for Applications, University of Oslo, Oslo, Norway, 23 August to 18 September 2010.

- B. Jourdain
  - seminar of the credit chair at Evry university, 4 march, Regularity of the Exercise Boundary for American Put Options on Assets with Discrete Dividends.
  - Bachelier seminar, 16 april, Regularity of the Exercise Boundary for American Put Options on Assets with Discrete Dividends.
  - workshop on numerical methods in finance at Bordeaux university, 1-2 june, Regularity of the Exercise Boundary for American Put Options on Assets with Discrete Dividends.

- A. Kebaier
  - Invited speaker to the AMAMEF Conference 2010, Slovenia

- O. Kudryavtsev
  - 29 January 2010 : “Efficient pricing options under regime switching”, Seminar “Stochastic methods in finance” at University Paris-Est-Marne-la-Vallée, Paris, France (autumn session), Sochi, Russia, October 16-19
  - 3 May 2010 : “Efficient pricing barrier options under Lévy models”, Seminar at Dipartimento di Finanza dell’Impresa e dei Mercati Finanziari, Udine University (Italy)
  - 7 May 2010 : “Efficient pricing barrier options under Lévy models” the Fifth General Conference on Advances Mathematical Methods in Finance, Hotel Golf, Bled, Slovenia, May 3 - 9, 2010.

- C. Labart
  - Seminar at Savoie University, January 2010,
  - Seminar at Evry University, April 2010.
  - Conference JERAA, Lyon, November 2010.
- D. Lamberton
  - Risk & Stochastics and Financial Mathematics Joint Seminars, London School of Economics, February 2010. "Critical price near maturity for the American put in exponential Lévy models".
  - AMaMeF 5th General Conference, Bled, Slovenia, May 2010, "Critical price near maturity for the American put in an exponential Lévy models".
  - Workshop Kolmogorov Equations in Physics and Finance, Modena, Italy, September 2010. "American options and integro-differential equations".
- J. Lelong
  - Journées MAS, Bordeaux, September 2010.
- A. Minca
  - 3 July 2010, Séminaire de Recherche Quantitative, Natixis
  - 18 Janvier 2010, Groupe de travail CMAP, Ecole Polytechnique
  - 18-19 November, San Francisco, SIAM Conference on Financial Mathematics and Engineering (invited talk)
  - 2 September 2010, Paris, Risque de contrepartie, Risque systémique et Chambres de compensation (invited talk)
  - 26-31 August 2010, Poitiers, Colloque Franco-Roumain de Mathématiques Appliquées(contributed talk)
  - 24-28 May 2010, Toronto, Workshop on financial derivatives and risk management (invited talk)
  - 19-21 May 2010, Toronto, Financial Networks and Risk Assessment(contributed talk)
- F. Russo
  - Levico (Italy), January 2010 CIRM-Workshop Stochastic Analysis, SPDEs, Particle Systems, Optimal Transport.
  - Ecole Polytechnique (France), February 2010 Séminaire du CMAP.
  - Marne-la-Vallée (France), March 2010 Groupe de travail (Mathématiques financières).
  - Roscoff (France), March 2010. Stochastic control and finance.
  - Cambridge (United Kingdom), April 2010. Conference “Stochastic PDEs and applications”.
  - Loughborough (United Kingdom), April 2010. East Midlands Stochastic analysis seminar.
  - Angers (France), May 2010. Séminaire triangulaire Le Mans, Angers, Brest.
  - Poitiers (France), August 2010. Colloque franco-roumain de mathématiques appliquées.
  - Dijon (France), October 2010. Conference in memory of Paul Malliavin.
  - Paris 6-7 (France), November 2010. Séminaire de Probabilités.
  - Marrakech (Morocco), December 2010. Workshop “Stochastic Control Problems for FBSDEs and Applications”

  - One month invitation (April 2010) to the "Isaac Newton Institute" in Cambridge. Special semester in SPDEs.

- A. Sulem
  - 4 th Bachelier Colloquium , Metabief, France January 24-31. “Maximum principles for Singular control problems and connections with reflected BSDEs”
  - Worshop on Stochastic Analysis and Applications, May 20-21, Växjö, Sweden
  - AMaMef workshop "Advanced Mathematical Methods in Finance", Berlin, 27-30 September
  - Probability seminar at the University of Evry, June 10th
  - Seminar at the University of Marne la Vallée, October 8th
  - Conference athe “International Research Forum” in Hongkong (15-17 December)
  - Invited for a visit in Shandong University (8-14 december) and 2 invited lechises

- A. Zanette
  - AMASES CONFERENCE 2010 Macerata (Italy), M. Gaudenzi, A. Zanette : Pricing Ratchet equity-indexed annuities with early surrender risk in a CIR++ model.
  - AMAMEF CONFERENCE 2010 BLED(Slovenia), M. Gaudenzi , A. Zanette : Fast binomial procedures for pricing Parisian options.

9.3. Animation of the scientific community

- B. Jourdain
  - Deputy head of the doctoral school ICMS, University Paris-Est

- D. Lamberton
  - In charge of the master programme “Mathématiques et Applications" (Universities of Marne-la-Vallée, Créteil and Evry, and Ecole Nationale des Ponts et Chaussées).
  - Directeur de l’UFR de mathématiques, Université Paris-Est Marne-la-Vallée.
  - Member of the Steering Committee of the ESF European Network "Amamef" (2005-2010, http://www.iac.rm.cnr.it/amamef/).

- A. Sulem
  - Participation to the selection committee for hiring a professor at the University Evry Val d’Essonne

- F. Russo
  - Member of the Professors council of the graduate program of LUISS University (Rome) on “ Mathematical methods for finance, economics and insurance.”
  - Advisor for mathematics at Liceo cantonale Lugano, Switzerland.

9.4. Editorship

- D. Lamberton
  - "Associate Editor" of *Mathematical Finance*, co-editor of *ESAIM P&S*.

- Agnès Sulem
  - Associate editor of SIAM Journal on Financial mathematics *SIFIN*
  - Associate editor of Journal of Applied Mathematics and Stochastic Analysis *JAMSA*

9.5. Teaching

- A. Alfonsi
  - “Probabilités et Statistiques”, first year course at the Ecole des Ponts.
  - “Modéliser, Programmer et Simuler”, second year course at the Ecole des Ponts.
  - “Calibration, Volatilité Locale et Stochastique”, third-year course at ENSTA (Master with Paris I).
  - “Mesures de risque”, Master course of UPEMLV and Paris VI.
- A. Minca
  - 20-24 September 2010, Luminy: Continuous limits of random trees and graphs
- V. Bally
  - Malliavin Calculus and applications in finance (30h) Master II, UMLV-ENPC, Finance.
  - Interest rates. (20h) Master II, UMLV-ENPC, Finance.
  - Risk analysis. Master II, IMIS UMLV.
  - Probability Master I, UMLV.
- B. Jourdain
  - course "Probability theory and statistics", first year ENPC
  - course "Introduction to probability theory", 1st year, Ecole Polytechnique
  - course "Stochastic numerical methods", 3rd year, Ecole Polytechnique
  - projects in finance and numerical methods, 3rd year, Ecole Polytechnique
- B. Jourdain, B. Lapeyre
  - course "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée
- J.-F. Delmas, B. Jourdain
  - course "Jump processes with applications to energy markets", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée
- B. Lapeyre Ecole des Ponts M1 (20h) ; Université Paris Est M2 (20h)
- J. Lelong
  - Lectures on “Parallel programming in financial mathematics” at Ensimag (third year course)
  - Lectures on “Monte-Carlo methods in financial engineering” at Ensimag (third year course)
  - Lectures on “Numerical methods for pricing American options” at Ensimag (third year course)
  - Lectures on “Numerical programming in C++” at Ensimag (second year course)
  - Lectures on “Random models and simulation” at Ensimag (second year course)
  - Supervision of second year students for long term projects on numerical finance.
- D. Lamberton
  - Third year of Licence de mathématiques (differential calculus), Université Paris-Est Marne-la-Vallée.
  - Préparation à l’agrgation interne de mathmatiques, Université Paris-Est Marne-la-Vallée.

  - Master course “Calcul stochastique et applications en finance", Université Paris-Est Marne-la-Vallée.

- A. Kebaier
  - stochastic calculus and Mathematical Finance: Master 2 university of Paris 13,
  - numerical probability in Finance: Master Finance HEC Paris,
  - credit risk: Master 2 university of Paris 13,
  - propagation of uncertainty: Master 2 university of Paris 13.

- C. Labart
  - Lectures on “Numerical methods for finance” at ENSTA (2nd year course).
  - Practicals on “Probability”, 12 hours, Polytech’Paris.
  - Practicals on “Numerical methods for differential equations”, 48 hours at University Pierre et Marie Curie (Licence 3).
  - Lectures and practicals on “Stochastic modelisation for finance and Insurance ” at Savoie University (Master IDESSE) 58 hours.
  - Practicals on “Probability and Statistics ”, at Savoie University (L3 MASS), 35 hours.
  - Practicals on “Numerical Methods ”, at Savoie University (L3 MATH), 18 hours.

- A. Sulem

  Finite difference methods in Finance, Master program MASEF, Université Paris-Dauphine, 21 hours

9.6. PhD advising

- A. Alfonsi
  - Ahdida Abdelkoddousse (from Oct. 2008). Wishart processes and correlation modeling in large dimension.
  - José Infante Acevedo (from Oct. 2009). Half of this thesis would be dedicated to liquidity risk.

- V. Bally
  - Stephano de Marco - has defended his Phd thesis the 23 november in the Scola Normale Superioore de Pisa, Italy.
  - Victor Rabiet. Second year. Supported by a fellowship of ENS Cachant.

- F. Russo
  - Nadia Belaribi, MSc, University Paris 13. Subject: Probabilistic and deterministic numerical methods for a PDE of porous media type with discontinuous coefficient.
  - Cristina Di Girolami, University of Paris 13 and Luiss Rome. Subject: Infinite dimensional stochastic calculus via regularizations with financial perspectives. July 4th 2010.
  - Stéphane Goutte, University of Paris 13 and Luiss Rome. Subject: Pricing and hedging of derivatives in the electricity market. July 4th 2010.
  - Mirko Stefano Mega, University of Paris 13 and Luiss Rome. Subject: Financial instruments for the telecommunications network. July 4th 2010.
  - Ida Kruk, University of Paris 13. Subject: Malliavin-Skorohod calculus for general Gaussian processes. December 9th 2010.

- B. Jourdain.
  - "Study of some numerical methods in finance", Maxence Jeunesse
- A. Kebaier
  - Since October 2010, supervision in collaboration with Eulalia Nualart the PhD thesis of Kaouther Hajji at University of Paris 13. Kaouther Hajji obtained financial resarch support from university paris 13. In this work, we study a new algorithm for variance reduction based on combining a stochastic algorithm and the multilevel Monte Carlo method. We employ the obtained results in order to reduce variance in option pricing problems.
- D. Lamberton
  - El Hadj Aly DIA. Monte-Carlo methods for exotic options in models with jumps. Thesis defense on July 1st, 2010.
  - Sidi Mohamed OULD ALY (4th year). Exotic options and stochastic volatility models. Allocataire de recherche, Université Paris-Est, doctorant-conseil at Natixis.
  - Lokmane ABBAS TURKI (2nd year, started in March 2009). Modelling of correlation in high dimensions. This thesis is funded by Credinext.
  - Ayech BOUSELMI (2nd year, started in October 2009). Allocataire de recherche, Université Paris-Est. Lévy processes and multi-dimensional models in finance.
- A. Sulem

  Andrea Minca: Control of Systemic risk (main adviser: Rama Cont)(2nd year)

9.7. PhD Defenses

- Stephano de Marco defended 23 november in the Scola Normale Superiore de Pisa, Italy. Advisor: V. Bally
- Stephane Goutte, July 5 2010, Universite Luiss, Roma (cotutelle Paris 13) adviser F. Russo. “Pricing and hedging of derivatives in the electricity market”.
- Ida Kruk, University of Paris 13. “Malliavin-Skorohod calculus for general Gaussian processes”. adviser: F. Russo defended December 9th 2010
- Cristina Di Girolami, July 5 2010, Universite Luiss, Roma (cotutelle Paris 13) “Infinite dimensional stochastic calculus via regularizations with financial perspectives”. adviser F. Russo
- Mirko Stefano Mega, July 5 2010, Universite Luiss, Roma (cotutelle Paris 13) “Financial instruments for the telecommunications network”. adviser F. Russo
- El Hadj Aly DIA. “Monte-Carlo methods for exotic options in models with jumps”. Thesis defense took place on July 1st, 2010. Adviser: D. Lamberton

9.8. Internship advising

- B. Jourdain
  - Marie Amory : Implementation into Premia of power series approximation formulas for options in stochastic volatility models given by Fabio Antonelli and Sergio Scarlatti, "Pricing options under stochastic volatility: a power series approach", Finance Stoch. 13 (2009), no. 2, 269303
- A. Kebaier
  - Supervision of the internship of Kaouther Hajji in the mathfi team from November 2009 to May 2010 on a credit risk problem based on an interacting particles algorithm by Carmona, Fouques and Vestal.

- Agnès Sulem
  - Mlle Xuyang Ma, CMAP (6 months) "Singular control of jump processes with delay"

9.9. PhD Committee

A. Sulem: Participation of the PhD committee of:

- Stephane Goutte, July 5 2010, Universite Luiss, Roma (cotutelle Paris 13) (Head of defense committee)
- Cristina Di Girolami, July 5 2010, Universite Luiss, Roma (cotutelle Paris 13) (Head of defense committee)
- Mirko Mega, July 5 2010, Universite Luiss, Roma (cotutelle Paris 13) (Head of defense committee)
- Gilles-Edouard Espinosa, June 9th, CMAP, Ecole Polytechnique
- Brahim El Asri, April 30th, Le Mans
- Thomas Lim, July 7th, Universite Paris 7
- Armand Ngoupeyou, July 7th, Universite d'Evry (Referee)
- Francisco Silva, November 29th, CMAP, Ecole Polytechnique

10. Bibliography

Major publications by the team in recent years

[1] *Numerical Methods implemented in the Premia Software*, 2009, Bankers, Markets, Investors, Introduction by A. Sulem and A. Zanette.

[2] A. ALFONSI, A. FRUTH, A. SCHIED. *Optimal execution strategies in limit order books with general shape functions*, in "Quantitative Finance", 2009, vol. 10, n^{o} 2, p. 143-157, DOI:10.1080/14697680802595700.

[3] A. ALFONSI, B. JOURDAIN. *Exact volatility calibration based on a Dupire-type Call-Put duality for perpetual American options*, in "Nonlinear Differential Equations and Applications", 2009, vol. 16, n^{o} 4, p. 523-554.

[4] V. BALLY, M.-. BAVOUZET, M. MESSAOUD. *Computations of Greeks using Malliavin Calculus in jump type market models*, in "Annals of Applied Probability", 2007, vol. 17, p. 33-66.

[5] B. JOURDAIN. *Probabilités et statistique*, Ellipses, 2009.

[6] B. JOURDAIN, J. LELONG. *Robust Adaptive Importance Sampling for Normal Random Vectors*, in "Annals of Applied Probability", 2009, vol. 19, n^{o} 5, p. 1687-1718, http://arxiv.org/pdf/0811.1496v1+.

[7] A. KOHATSU-HIGA, A. SULEM. *Utility maximization in an insider influenced market*, in "Mathematical Finance", 2006, vol. 16, n^{o} 1, p. 153–179.

[8] D. LAMBERTON. *Optimal stopping with irregular reward functions*, in "Stochastic Processes and their Applications", 2009, vol. 119, p. 3253-3284.

[9] D. LAMBERTON, M. MIKOU. *The critical exercise price for the American put in an exponential Lévy model*, in "Finance & Stochastics", 2008, vol. 12, p. 561-581.

[10] B. ØKSENDAL, A. SULEM. *Applied Stochastic Control of Jump Diffusions*, Universitext, Second Edition, Springer, Berlin, Heidelberg, New York, 257 pages 2007.

[11] B. ØKSENDAL, A. SULEM. *Maximum principles for optimal control of forward-backward stochastic differential equations with jumps*, in "SIAM J. Control Optimization", 2009, vol. 48, n^{o} 5, p. 2845–2976.

Publications of the year

Articles in International Peer-Reviewed Journal

[12] A. ALFONSI. *High-order discretization scheme for the CIR process: application to the Heston model*, in "Mathematics of Computation", 2010, vol. 79, n^{o} 269, p. 209-237.

[13] A. ALFONSI, A. FRUTH, A. SCHIED. *Optimal Trade Execution and Absence of Price Manipulations in Limit Order Book Models*, in "SIAM J. Finan. Math.", 2010, vol. 1, p. 490-522, hal-00397652.

[14] V. BALLY, E. CLÉMENT. *Integration by parts formula and applications to equations with jumps*, in "Probabilty Theory and related fields", 2010, http://www.springerlink.com/content/0q1130511145p6n4/.

[15] V. BALLY, B. FERNANDEZ, A. MEDA. *Estiamates of the Prbability Itô Processes remain around a curve and applications in finance*, in "Stoch. Proc. Appl.", 2010, to appear.

[16] V. BALLY, N. FOURNIER. *Regularization properties of the 2D homogeneous Boltzmann equation without cutoff*, in "Probability Theory and Related Fields", 2010, to appear.

[17] V. BALLY, A. KOHATSU-HIGA. *Lower bounds for densities of Asian type stochastic differential equations*, in "J. Funct. Anal.", 2010, vol. 258, n^{o} 9, p. 3134–3164.

[18] V. BARBU, M. RÖCKNER, F. RUSSO. *Probabilistic representation for solutions of an irregular porous media type equation: the degenerate case*, in "Probability and related fields", 2010, (50 pages), to appear, http://hal.inria.fr/inria-00410248/fr/.

[19] P. BLANCHARD, M. RÖCKNER, F. RUSSO. *Probabilistic representation for solutions of an irregular porous media type equation*, in "Ann. Probab.", 2010, vol. 38, n^{o} 5, p. 1870–1900.

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[21] P. ETORÉ, B. JOURDAIN. *Adaptive optimal allocation in stratified sampling methods*, in "Methodology and Computing in Applied Probability", 2010, n^{o} 12(3), p. 335-360.

[22] M. GAUDENZI, M. LEPELLERE, A. ZANETTE. *The Singular Points method for Pricing American Path-Dependent Options*, in "Journal of Computational Finance", 2010, vol. 14, n^{o} 1.

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