

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team opale

Optimization and control, numerical algorithms and integration of complex multidiscipline systems governed by PDE

Sophia Antipolis - Méditerranée, Grenoble - Rhône-Alpes



Theme : Computational models and simulation

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2. Overall Objectives

2.1. Research fields

Optimizing a complex system arising from physics or engineering covers a vast spectrum in basic and applied sciences. Although we target a certain transversality from analysis to implementation, the particular fields in which we are trying to excell can be defined more precisely.

From the *physical analysis* point of view, our expertise relies mostly on Fluid and Structural Mechanics and Electromagnetics. In the former project Sinus, some of us had contributed to the basic understanding of fluid mechanical phenomena (Combustion, Hypersonic Non-Equilibrium Flow, Turbulence). More emphasis is now given to the coupling of engineering disciplines and to the validation of corresponding numerical methodologies.

From the *mathematical analysis* point of view, we are concerned with functional analysis related to partialdifferential equations, and the functional/algebraic analysis of numerical algorithms. Identifying the Sobolev space in which the direct or the inverse problem makes sound sense, tailoring the numerical method to it, identifying a functional gradient in a continuous or discrete setting, analyzing iterative convergence, improving it, measuring multi-disciplinary coupling strength and identifying critical numerical parameters, etc constitute a non-exhaustive list of mathematical problems we are concerned with.

Regarding more specifically the *numerical aspects* (for the simulation of PDEs), considerable developments have been achieved by the scientific community at large, in recent years. The areas with the closest links with our research are:

- 1. *approximation schemes*, particularly by the introduction of specialized Riemann solvers for complex hyperbolic systems in Finite-Volume/Finite-Element formulations, and highly-accurate approximations (e.g. ENO schemes),
- 2. *solution algorithms*, particularly by the multigrid, or multilevel and multi-domain algorithms bestequiped to overcome numerical stiffness,
- 3. parallel implementation and software platforms.

After contributing to some of these progresses in the former project Sinus, we are trying to extend the numerical approach to a more global one, including an optimization loop, and thus contribute, in the long-term, to modern scientific computing and engineering design. We are currently dealing mostly with *geometrical optimization*.

Software platforms are perceived as a necessary component to actually achieve the computational costefficiency and versatility necessary to master multi-disciplinary couplings required today by size engineering simulations.

2.2. Objectives

The project has several objectives : to analyze mathematically coupled PDE systems involving one or more disciplines in the perspective of geometrical optimization or control; to construct, analyze and experiment numerical algorithms for the efficient solution of PDEs (coupling algorithms, model reduction), or multicriterion optimization of discretized PDEs (gradient-based methods, evolutionary algorithms, hybrid methods, artificial neural networks, game strategies); to develop software platforms for code-coupling and for parallel and distributed computing.

Major applications include : the multi-disciplinary optimization of aerodynamic configurations (wings in particular) in partnership with the French or European aeronautical industry and research (Airbus, Dassault Aviation, ONERA, etc) ; the geometrical optimization of antennas in partnership with France Télécom and Thalès Air Défense (see Opratel Virtual Lab.); the development of *Virtual Computing Environments* in collaboration with CNES and Chinese partners (ACTRI).

2.3. Highlights

After last year's successful habilitation thesis (HDR) defense, Paola Goatin was recruited by INRIA and joined our project-team in October. She was also successful in receiving the ERC junior grant for her project, TRAM3, on macroscopic trafic models. This new activity enhances our involvement in the functional and numerical analysis of hyperbolic equations, and includes perspectives in optimization and control as well.

Jean-Paul Zolesio, co-founder of the project-team, has now expressed his intention to focus on more theoretical aspects, in particular in theoretical physics, more closely related to the research conducted at the CNRS INLN Lab (Nice Sophia Antipolis Nonlinear Institute). His collaboration will continue to be highly valued.

The collaboration with ONERA (The French Aerospace Laboratory) has been reinforced, J.-A. Désidéri being now a consultant in the two departments, DSNA and DAAP (Numerical Simulation in Aerodynamics and Aeroacoustics Department, Chatillon; Applied Aerodynamics Department, Meudon). This collaboration develops along the following major areas of interest in aerodynamic optimization : adjoint equations for aerodynamics and coupled aero-structural models, reduced models for shape optimization and control of numerical uncertainties, global multilevel optimization, mesh-adaption methods, game strategies for multidisciplinary optimization (e.g. for aero-structural, or aero-acoustic models).

3. Scientific Foundations

3.1. Functional and numerical analysis of PDE systems

Our common scientific background is the functional and numerical analysis of PDE systems, in particular with respect to nonlinear hyperbolic equations such as conservation laws of gasdynamics.

Whereas the structure of weak solutions of the Euler equations has been thoroughly discussed in both the mathematical and fluid mechanics literature, in similar hyperbolic models, focus of new interest, such as those related to trafic, the situation is not so well established, except in one space dimension, and scalar equations. Thus, the study of such equations is one theme of emphasis of our research.

The well-developed domain of numerical methods for PDE systems, in particular finite volumes, constitute the sound background for PDE-constrained optimization.

3.2. Numerical optimization of PDE systems

Optimization problems involving systems governed by PDEs, such as optimum shape design in aerodynamics or electromagnetics, are more and more complex in the industrial setting.

In certain situations, the major difficulty resides in the costly evaluation of a functional by means of a simulation, and the numerical method to be used must exploit at best the problem characteristics (regularity or smoothness, local convexity).

In many other cases, several criteria are to be optimized and some are non differentiable and/or non convex. A large set of parameters, sometimes of different types (boolean, integer, real or functional), are to be taken into account, as well as constraints of various types (physical and geometrical, in particular). Additionally, today's most interesting optimization pre-industrial projects are multi-disciplinary, and this complicates the mathematical, physical and numerical settings. Developing *robust optimizers* is therefore an essential objective to make progress in this area of scientific computing.

In the area of numerical optimization algorithms, the project aims at adapting classical optimization methods (simplex, gradient, quasi-Newton) when applicable to relevant engineering applications, as well as developing and testing less conventional approaches such as Evolutionary Strategies (ES), including Genetic or Particle-Swarm Algorithms, or hybrid schemes, in contexts where robustness is a very severe constraint.

In a different perspective, the heritage from the former project Sinus in Finite-Volumes (or -Elements) for nonlinear hyperbolic problems, leads us to examine cost-efficiency issues of large shape-optimization applications with an emphasis on the PDE approximation; of particular interest to us:

- best approximation and shape-parameterization,
- convergence acceleration (in particular by multi-level methods),
- model reduction (e.g. by Proper Orthogonal Decomposition),
- parallel and grid computing; etc.

3.3. Geometrical optimization

Jean-Paul Zolesio and Michel Delfour have developed, in particular in their book [2], a theoretical framework for for geometrical optimization and shape control in Sobolev spaces.

In preparation to the construction of sound numerical techniques, their contribution remains a fundamental building block for the functional analysis of shape optimization formulations.

3.4. Integration platforms

Developing grid computing for complex applications is one of the priorities of the IST chapter in the 6th Framework Program of the European Community. One of the challenges of the 21st century in the computer science area lies in the integration of various expertise in complex application areas such as simulation and optimisation in aeronautics, automotive and nuclear simulation. Indeed, the design of the reentry vehicle of a space shuttle calls for aerothermal, aerostructure and aerodynamics disciplines which all interact in hypersonic regime, together with electromagnetics. Further, efficient, reliable, and safe design of aircraft involve thermal flows analysis, consumption optimisation, noise reduction for environmental safety, using for example aeroacoustics expertise.

The integration of such various disciplines requires powerful computing infrastructures and particular software coupling techniques. Simultaneously, advances in computer technology militate in favour of the use of massively parallel PC-clusters including thousands of processors connected by high-speed gigabits/sec wide-area networks. This conjunction makes it possible for an unprecedented cross-fertilisation of computational methods and computer science. New approaches including evolutionary algorithms, parameterization, multi-hierarchical decomposition lend themselves seamlessly to parallel implementations in such computing infrastructures. This opportunity is being dealt with by the OPALE project since its very beginning. A software integration platform has been designed by the OPALE project for the definition, configuration and deployment of multidisciplinary applications on a distributed heterogeneous infrastructure [56]. Experiments conducted within European projects and industrial cooperations using CAST have led to significant performance results in complex aerodynamics optimisation test-cases involving multi-elements airfoils and evolutionary algorithms, i.e. coupling genetic and hierarchical algorithms involving game strategies [57].

The main difficulty still remains however in the deployment and control of complex distributed applications on grids by the end-users. Indeed, the deployement of the computing grid infrastructures and of the applications in such environments still requires specific expertise by computer science specialists. However, the users, which are experts in their particular application fields, e.g. aerodynamics, are not necessarily experts in distributed and grid computing. Being accustomed to Internet browsers, they want similar interfaces to interact with grid computing and problem-solving environments. A first approach to solve this problem is to define componentbased infrastructures, e.g. the Corba Component Model, where the applications are considered as connection networks including various application codes. The advantage is here to implement a uniform approach for both the underlying infrastructure and the application modules. However, it still requires specific expertise not directly related to the application domains of each particular user. A second approach is to make use of grid services, defined as application and support procedures to standardise access and invocation to remote support and application codes. This is usually considered as an extension of Web services to grid infrastructures. A new approach, which is currently being explored by the OPALE project, is the design of a virtual computing environment able to hide the underlying grid-computing infrastructures to the users. It is currently deployed within the Collaborative Working Environments Unit of the DG INFSO F4 of the European Commission. It is planned to include Chinese partners from the aeronautics sector in 2009 to set up a project for FP7.

4. Application Domains

4.1. Aeronautics and space

The demand of the aeronautical industry remains very strong in aerodynamics, as much for conventional aircraft, whose performance must be enhanced to meet new societal requirements in terms of economy, noise (particularly during landing), vortex production near runways, etc., as for high-capacity or supersonic aircraft of the future. Our implication concerns shape optimization of wings or simplified configurations.

Our current involvement with Space applications relates to software platforms for code coupling.

4.2. Mechanical industry

A new application domain related to the parameter and shape optimization of mechanical structures is under active development. The mechanical models range from linear elasticity of 2D or 3D structures, or thin shells, to nonlinear elastoplasticity and structural dynamics. The criteria under consideration are multiple: formability, stiffness, rupture, fatigue, crash, and so on. The design variables are the thickness and shape, and possibly the topology, of the structures. The applications are performed in collaboration with world-leading industrials, and involve the optimization of the stamping process (Blank Force, Die and Tools shapes) of High Performance steel structures as well as the optimal design of structures used for packaging purposes (cans and sprays under high pressure). Our main contribution relies on providing original and efficient algorithms to capture Pareto fronts, using smart meta-modelling, and to apply game theory approaches and algorithms to propose stable compromise solutions (e.g. Nash equilibria).

4.3. Electromagnetics

In the context of shape optimization of antennas, we can split the existing results in two parts: the twodimensional modeling concerning only the specific transverse mode TE or TM, and treatments of the real physical 3-D propagation accounting for no particular symmetry, whose objective is to optimize and identify real objects such as antennas.

Most of the numerical literature in shape optimization in electromagnetics belongs to the first part and makes intensive use of the 2-D solvers based on the specific 2-D Green kernels. The 2-D approach for the optimization of *directivity* led recently to serious errors due to the modeling defect. There is definitely little hope for extending the 2-D algorithms to real situations. Our approach relies on a full analysis in unbouded domains of shape sensitivity analysis for the Maxwell equations (in the time-dependent or harmonic formulation), in particular, by using the integral formulation and the variations of the Colton and Kreiss isomorphism. The use of the France Telecom software SR3D enables us to directly implement our shape sensitivity analysis in the harmonic approach. This technique makes it possible, with an adequate interpolation, to retrieve the shape derivatives from the physical vector fields in the time evolution processes involving initial impulses, such as radar or tomography devices, etc. Our approach is complementary to the "automatic differentiation codes" which are also very powerful in many areas of computational sciences. In Electromagnetics, the analysis of hyperbolic equations requires a sound treatment and a clear understanding of the influence of space approximation.

4.4. Biology and medecine

A particular effort is made to apply our expertise in solid and fluid mechanics, shape and topology design, multidisciplinary optimization by game strategies to biology and medecine. Two selected applications are priviledged : solid tumours and wound healing.

Opale's objective is to push further the investigation of these applications, from a mathematical-theoretical viewpoint and from a computational and software development viewpoint as well. These studies are led in collaboration with biologists, as well as image processing specialists.

4.5. Traffic flow

The modeling and analysis of traffic phenomena can be performed at a macroscopic scale by using partial differential equations derived from fluid dynamics. Such models give a description of collective dynamics in terms of the spatial density $\rho(t, x)$ and average velocity v(t, x). Continuum models have shown to be in good agreement with empirical data. Moreover, they are suitable for analytical investigations and very efficient from the numerical point of view. Finally, they contain only few variables and parameters and they can be very versatile in order to describe different situations encountered in practice.

Opale's research focuses on the study of macroscopic models of vehicular and pedestrian traffic, and how optimal control approaches can be used in traffic management. The project opens new perspectives of interdisciplinary collaborations on urban planning and crowd dynamics analysis.

4.6. Multidisciplinary couplings

Our expertise in theoretical and numerical modeling, in particular in relation to approximation schemes, and multilevel, multiscale computational algorithms, allows us to envisage to contribute to integrated projects focused on disciplines other than, or coupled with fluid dynamics, such as structural mechanics, electromagnetics, biology and virtual reality, image processing, etc in collaboration with specialists of these fields. Part of this research is conducted in collaboration with ONERA.

5. Software

5.1. Design optimization platform FAMOSA

Opale team is developing the software platform FAMOSA (C++), that is devoted to multidisciplinary design optimization in engineering. It integrates the following components:

- an optimization library composed of various algorithms : several descent methods from steepestdescent method to quasi-Newton BFGS method (deterministic, smooth), the Multi-directional Search Algorithm (deterministic, noisy), the Covariance Matrix Adaption Evolution Strategy (semistochastic, multi-modal) and the Efficient Global Optimization method (deterministic, multi-modal). It also contains the Pareto Archived Evolution Strategy to solve multiobjective optimization problems;
- an evaluation library managing the performance estimation process (communication with external simulation tools);
- a metamodel library that contains tools to build a database and kriging models that are used to approximate the objective function for different purposes;
- a scenario library that allows to use the previous components to achieve various tasks:
 - Construct a design of experiments ;
 - Construct a metamodel ;
 - Find the design that minimizes a cost functional ;
 - Find the Pareto front for two cost functionals
 - Play a Nash game to find the equilibrium between two criteria;
 - Apply a multiple gradient descent strategy to improve simultaneously two criteria.

The FAMOSA platform is employed by Opale Project-Team to test its methodological developments in multidisciplinary design optimization (MDO). The platform is also used by the Fluid Mechnics Laboratory at Ecole Centrale de Nantes and by the K-Epsilon company (http://www.k-epsilon.com) for hydrodynamic design applications. Moreover, it is presently tested by Peugeot Automotive industry for aerodynamic design purpose.

5.2. Multidisciplinary simulation platform NUM3SIS

NUM3SIS is a modular platform devoted to scientific computing and numerical simulation. It is not restricted to a particular application field, but is designed to host complex multidisciplinary simulations. Main application fields are currently Computational Fluid Dynamics (CFD), Computational Structural Mechanics (CSM), Computational Electro-Magnetics (CEM, in collaboration with Nachos Project-Team). Nevertheless, NUM3SIS can be a framework for the development of other numerical simulation tools for new application areas, such as pedestrian traffic modeling for instance. The platform is especially designed for large scale parallel computations.

The most important concept in NUM3SIS is the concept of node. It is a visual wrapper around derivatives of fundamental concepts such as data, algorithm or viewer. Atomic nodes are provided for convenience in order to manipulate computational data (such as grids or fields), apply computational methods (such as the building of a finite-element matrix or the construction of a finite-volume flux) and vizualize computational results (such as vector or tensor fields, on a screen or in an immersive space). For a given abstract node, different implementations can be found, each of them being embedded in a plugin system that is managed by a factory.

The second important concept in NUM3SIS is the concept of composition. It consists of the algorithmic pipeline used to link the nodes together. The use of these two concepts, composition and nodes, provides a highly flexible, re-usable and efficient approach to develop new computational scenarii and take benefit from already existing tools. This is a great advantage with respect to classical monolithic softwares commonly used in these fields.

This work is being carried out with the support of two engineers in the framework of an ADT (Action de Développement Technologique) program.



Figure 1. Illustration of the graphic user interface of the NUM3SIS platform: at the top the composition space, at the bottom the visualization space.

5.3. Isogeometric analysis and design plugins for AXEL

Opale team is developing plugins in the framework of the algebraic modeler Axel, whose development is carried out by Galaad team. These plugins aim at implementing a prototype of isogeometric solver and optimizer, in interaction with the CAD library contained in Axel platform.

Two simulation tools for heat conduction and compressible flows have been implemented, in conjunction with some deterministic and semi-stochastic optimization algorithms for optimum-shape design.

5.4. Integration platform for multidiscipline optimization applications

A prototype software integration platform is developed and tested for multidiscipline optimization applications. It is based on a workflow management system called YAWL (http://www.yawlfoundation.org). The goal is to design, develop and assess high-performance distributed scientific workflows featuring resilience, faulttolerance and exception-handling capabilities. The platform is used to experiment new resilience algorithms, including monitoring and management of application-level errors. The platform is tested against usecases provided by the industry partners in the OMD2 project supported by the French Agence Nationale de la Recherche. This work is part of Laurentiu Trifan's PhD thesis.

6. New Results

6.1. Computational methods, numerical analysis and validation

6.1.1. Isogeometric analysis and design

Participants: Louis Blanchard, Jean-Antoine Désidéri, Régis Duvigneau, Bernard Mourrain [Galaad Project-Team], Mohammed Ziani.

Design optimization stands at the crossroad of different scientific fields (and related software): Computer-Aided Design (CAD), Computational Fluid Dynamics (CFD) or Computational Structural Dynamics (CSM), parametric optimization. However, these different fields are usually not based on the same geometrical representations. CAD software relies on Splines or NURBS representations, CFD and CSM software uses grid-based geometric descriptions (structured or unstructured), optimization algorithms handle specific shape parameters. Therefore, in conventional approaches, several information transfers occur during the design phase, yielding approximations and non-linear transformations that can significantly deteriorate the overall efficiency of the design optimization procedure.

The isogeometric approach proposes to definitely overcome this difficulty by using CAD standards as a unique representation for all disciplines. The isogeometric analysis consist in developing methods that use NURBS representations for all design tasks:

- the geometry is defined by NURBS surfaces;
- the computation domain is defined by NURBS volumes instead of meshes;
- the solution fields are obtained by using a finite-element approach that uses NURBS basis functions instead of classical Lagrange polynomials;
- the optimizer controls directly NURBS control points.

Using such a unique data structure allows to compute the solution on the exact geometry (not a discretized geometry), obtain a more accurate solution (high-order approximation), reduce spurious numerical sources of noise that deteriorate convergence, avoid data transfers between the software. Moreover, NURBS representations are naturally hierarchical and allows to define multi-level algorithms for solvers as well as optimizers.

A prototype of isogeometric solver and design optimizer has been developed in collaboration with Galaad team[37], [51]. The heat conduction equation (elliptic) has been first considered to assess the convergence properties (high-order approximation schemes) and test the possible refinement preocedures (h- and p-refinement). Then, a compressible flow solver based on 2D Euler equations (hyperbolic) has been developed, including a stabilization procedure similar to the SUPG (Streamwise Upwind Petrov Galerkin) approach. The accuracy of the solver has been demonstrated on some testcases. However, some stabilization issues have been found, that should still be addressed.

A study of hierarchical optimization strategies has been achieved. In particular, two algorithms (hierarchical optimization, hierarchical optimization and modeling) have been developed and tested for inverse problems in heat conduction. It has been found that such strategies permit to avoid local minima and speed-up the convergence process^[44].

A collaboration with the Technical University of Munich has been set-up for applications in structural design[38].

6.1.2. Computational models for aero-acoustics

Participants: Gérald Carrier [IR ONERA/DAAP], Jean-Antoine Désidéri, Régis Duvigneau, Andrea Minelli, Itham Salah El Din [IR ONERA/DAAP].

One aspect of the design of a supersonic business jet is the compliance with environmental regulations concerning noise propagation. In the perspective of aerodynamic shape optimization, the *sonic boom* phenomenon must be accurately computed, ad this requires the development of at least two computational elements :

- the coupling of a purely aerodynamic model by the Euler of RANS equations with an appropriate acoustic wave propagation model;
- the development of mesh adaption techniques to resolve the fine flow structures.

This topic has been initiated by A. Minelli and will be continued at ONERA within the course of his doctoral thesis.

A multiobjective optimization has been developed for the design of the fuselage of a supersonic business jet. The attention has been focused on the coupling between the CFD solver (Num3sis) developed at INRIA and the acoustic propagation code TRAPS in use at ONERA. The accuracy of the propagation prediction over the entire domain is fundamental for obtaining a valuable solution. In order to achieve this result an iterative procedure for unstructured mesh adaptation has been developed. Mesh adaptation has the potential to reduce the computational time increasing the density of the mesh only where required. In this preliminary work, the optimization has been carried out in a hybrid process in which an evolutionary strategy (ES) was used for exploration, and a gradient-based method in exploitation. The Pareto front has been approximated using the Pareto Archive Evolutionary Strategy as well as the classical Adapted Weighted Sum method. This design methodology has the ability to generate non intuitive configurations. [47]

This study is now being enhanced to serve as a building block of the multidiciplinary optimization process.

6.1.3. Analysis and numerical approximation of macroscopic models of vehicular and pedestrian traffic

Participants: Paola Goatin, Rinaldo M. Colombo [Brescia University, Italy], Mauro Garavello [Piedmont University, Italy], Benedetto Piccoli [Rutgers University - Camden, USA], Massimiliano D. Rosini [Warsaw University, Poland].

Paola Goatin was part-time on secondment from University of Toulon until August 2010. She is now permanent INRIA researcher since October 2010.

The research activity during 2010 focused on the mathematical analysis of traffic flow models on road networks or subject to unilateral constraints. In particular, [26] is devoted to a hyperbolic 2 phase model for traffic flow on a network. The model is rigorously described and the existence of solutions is proved, without any restriction on the network geometry. Another traffic flow model with phase-transition is proposed and described in [54], [29].

In [27], [32], [39], we deal with the mathematical description of toll gates along roads, or of the escape dynamics for crowds. This problem requires the introduction of unilateral constraints on the observable flow. We present a rigorous approach to these constraints, and numerical integrations of the resulting models are included to show their practical usability.

The project "TRAM3 - Traffic management by macroscopic models", has received fundings (809 Keuros) from the ERC Starting Grant 2010 for 60 months starting from October 2010. The project intends to investigate traffic phenomena from the macroscopic point of view, using models derived from fluid-dynamics consisting in hyperbolic conservation laws.

Our scope is to develop a rigorous analytical framework and fast and efficient numerical tools for solving optimization and control problems, such as queues lengths control or buildings exits design. This will allow to elaborate reliable predictions and to optimize traffic fluxes. To achieve this goal, we will move from the detailed structure of the solutions in order to construct ad hoc methods to tackle the analytical and numerical difficulties arising in this study. The foreseen applications target the sustainability and safety issues of modern society.

The major mathematical difficulties related to this study follow from the mandatory use of weak (possibly discontinuous) solutions in distributional sense. Indeed, due to the presence of shock waves and interactions among them, standard techniques are generally useless for solving optimal control problems, and the available results are scarce and restricted to particular and unrealistic cases. This strongly limits their applicability.

6.2. Numerical algorithms for optimization and optimum-shape design

6.2.1. Hierarchical (multilevel) and adaptive parameterization

6.2.1.1. Multi-level algorithms based on an algebraic approach

Participants: Jean-Antoine Désidéri, Régis Duvigneau, Adrien Zerbinati.

The hierarchical approach developed during the last years was based on the degree-elevation property of Bézier curves. It has been extended to other parameterization types in order to be able to solve general parametric optimization problems. The proposed approach rely on the construction of a hierarchical basis of the design space, originating from the eigenmodes of the Hessian matrix of the cost functional.

This approach has been applied to the multidisciplinary design of a supersonic business jet (aerodynamics, structure, propulsion, flight mechanics), proposed by Dassault-Aviation as multi-disciplinary optimization benchmark for "OMD" project [40]. The algebraic method is now being extended by A. Zerbinati to optimize the flow in an automobile engine in the framework of the OMD ANR Project (Phase 2).

6.2.1.2. Multi-level algorithms for local / global shape parameterization

Participants: Praveen Chandrashekarappa [Tata Institute, Bangalore], Régis Duvigneau.

In several engineering problems, the system to optimize is characterized by some parameters that define global shape properties, while remaining parameters define local shape modifications. Of course, these two sets of parameters do not play the same role and have not the same impact on the cost functional value. Therefore, we are studying how to construct an optimization strategy that can take benefit of this global / local splitting of parameters. Typically, we try to develop and compare algorithms based on successive optimization, hierarchical optimization or virtual game strategies. First tests have been carried out in aerodynamics, by considering a wing shape defined by global parameters (planform, incidence and twist angles), as well as local airfoil sections[31].

6.2.2. Metamodels

6.2.2.1. Metamodel-based optimization

Participants: Praveen Chandrashekarappa [Tata Institute, Bangalore], Régis Duvigneau.

Design optimization in Computational Fluid Dynamics or Computational Structural Mechanics is particularly time consuming, since several hundreds of expensive simulations are required in practice. Therefore, we are currently developing approaches that rely on *metamodels*, i.e. models of models, in order to accelerate the optimization procedure by using different modelling levels. Metamodels are inexpensive functional value predictions that use data computed previously and stored in a database. Different techniques of metamodelling (polynomial fitting, Radial Basis Functions, Kriging) have been developed and validated on various engineering problems. Our developments have been particularly focused on the construction of algorithms that use both metamodels and models based on PDE's solving to drive a semi-stochastic optimization, with various couplings[41].

6.2.2.2. Surrogate models based on function and derivative values for aerodynamic global optimizaton Participants: Manuel Bompard [doctoral student, ONERA/DSNA], Jean-Antoine Désidéri, Jacques Peter [IR ONERA/DSNA].

The application of global algorithms in aerodynamic shape optimization remains today very limited due to the required CPU time. To avoid this limitation, one method is to replace the objective function by a surrogate model. One important achievement of the recent years is the development of accurate and robust surrogate models, minimiz- ing the quantity of information (computed via costly CFD simulations) required for their construction.

In this study, we have proposed to use derivatives in addition to function values to determine the meta model. With adjoint methods, these derivatives may be obtained at a reduced cost, independent of the number of design variables. Three distinct meta model constructions including derivatives are presented and adapted. Then some aerodynamic design applica- tions are presented. The benefit of including derivatives in the surrogate models is clearly evident. [30]

6.2.2.3. Hermitian interpolation in presence of uncertainties

Participants: Manuel Bompard [doctoral student, ONERA/DSNA], Jean-Antoine Désidéri, Jacques Peter [IR ONERA/DSNA].

In PDE-constrained optimization, iterative algorithms are commonly efficiently ac- celerated by techniques relying on approximate evaluations of the functional to be minimized by an economical, but lower-fidelity model ("metamodel"). Various types of metamodels exist (interpolation polynomials, neural networks, Kriging models, etc). Such metamodels are con-structed by precalculation of a database of functional values by the costly high-fidelity model. In adjoint-based numerical methods, derivatives of the functional are also available at the same cost, although usually with poorer accuracy. Thus, a question arises : should the derivative information, known to be less accurate, be used to construct the metamodel or ignored ? As a first step to answer this question, we consider the case of the Hermitian interpolation of a function of a single variable, when the function values are known exactly, and the derivatives only approximately, assuming a uniform upper bound, ϵ , on this approximation is known. The classical notion of best approximation has been revisited in this context, and a criterion has been introduced to define the best set of interpolation points. This set is identified by either analytical or numerical means. If n + 1 is the number of interpolation points, it is advantageous to account for the derivative information when $\epsilon \leq \epsilon_0$, where ϵ_0 decreases with n, and this is in favor of piecewise, lowdegree Hermitian interpolants. In all our numerical tests, we have found that the distribution of Chebyshev points is always close to optimal, and provides bounded approximants with close-to-least sensitivity to the uncertainties. [45]

6.2.3. Multidisciplinary optimization

Participants: Gérald Carrier [IR ONERA/DAAP], Jean-Antoine Désidéri, Régis Duvigneau, Imane Ghazlane [doctoral student, ONERA/DAAP], Abderrahmane Habbal, Andrea Minelli [doctoral student, ON-ERA/DAAP], Adrien Zerbinati. In the most competitive engineering fields, such as aeronautics, multicriterion and multidisciplinary design has gained importance in order to cope with new and acute needs of society. In the literature, contributions to single discipline and/or single-point design optimization abound. In recent years, for purely-aerodynamic design, we had proposed to introduce a new approach combining the adjoint method with a formulation derived from game theory for multipoint design problems [24]. Transonic flows around lifting airfoils were analyzed by Euler computations. Airfoil shapes were optimized according to various aerodynamic criteria. The notion of player was introduced. In a competitive Nash game, each player attempts to optimize its own criterion through a symmetric exchange of information with others. A Nash equilibrium is reached when each player constrained by the strategy of the others, cannot improve further its own criterion. Specific real and virtual symmetric Nash games were implemented to set up an optimization strategy for design under conflict.

When devising a numerical shape-optimization method in the context of a practical engineering situation, the practitioner is faced with an additional difficulty related to the participation of several relevant physical criteria in a realistic formulation. For some problems, a solution may be found by treating all but one criteria as additional constraints. In some other problems, mainly when the computational cost is not an issue, Pareto fronts can be identified at the expense of a very large number of functional evaluations. However the difficulty is very acute when optimum-shape design is sought w.r.t. an aerodynamic criterion as well as other criteria for two main reasons. The first is that aerodynamics alone is costly to analyze in terms of functional evaluation. The second is that generally only a small degradation of the performance of the absolute optimum of the aerodynamic criterion alone is acceptable (suboptimality) when introducing the other criteria.

We have proposed a numerical methodology for the treatment of such problems of concurrent engineering [8]. After completion of the parametric, possibly-constrained minimization of a single, primary functional J_A , approximations of the gradient and the Hessian matrix are available or calculated using data extracted from the optimization loop itself. Then, the entire parametric space (a subset of \mathbb{R}^N) is split into two supplementary subspaces on the basis of a criterion related to the second variation. The construction is such that from the initial convergence point of the primary functional, normalized perturbations of the parameters lying in one of the two subspaces, of specified dimension $p \leq N - 1$, cause the least possible degradation to the primary functional. The latter subspace is elected to support the parameterization of a secondary functional, J_B , in a concurrent optimization realized by an algorithm simulating a Nash game between players associated with the two functionals. We prove a second result indicating that the original global optimum point of the full-dimension primary problem is Pareto-optimal for a trivial concurrent problem. This latter result permits us to define a continuum of Nash equilibrium points originating from the initial single-criterion optimum, in which the designer could potentially make a rational election of operating point [43].

In his thesis, B. Abou El Majd, had successfully realized a wing-shape aero-structural optimization despite the strong antagonism of the criteria in conflict in the concurrent reduction of the wing drag in Eulerian flow and a stress integral of the structural element treated as a shell subject to linear elasticity (see Figure 2). B. Abou El Majd's experiments have served to illustrate several publications, including articles in press in *Mécanique et Industrie* and the *Journal of Optimization and Applications*.

The technique of territory splitting is viewed as a systematic way to organize competition. A counterpart is the Multiple-Gradient Descent Algorithm [10] to perform a preliminary phase of cooperative optimization [9]. Combining these two complementary phases constitutes our basic methodology for multidisciplinary optimization. This methodology is currently being applied to the challenging problems, in part at ONERA :

- Aerodynamic and structural wing shape optimization : I. Ghazlane is developing a more complete aero-structural model, using the RANS equations for the flow, and a beam model for the wing, as a structural element. A functional gradient is calculated that accounts for coupled terms. The optimization process to be made subsequently will be realized by a Nash Game, here with the shape gradient information available.
- Aerodynamic and acoustic wing shape optimization of a supersonic business jet : the model developed in 6.1.2 will serve in this optimization.

6.2.4. Shape Optimization in Multidisciplinary Non-Linear Mechanics



a) Initial aerodynamic optimum solution

b) Aerostructural Nash game solution using the orthogonal decomposition



Figure 2. Illustration of a Nash game with an adapted split of territory. On top, the Mach number field corresponding to the aerodynamic global optimum solution. Below, the aerostructural Nash equilibrium solution, demonstrating a minor aerodynamic degradation (slightly-increased shock intensity). As a structural element, the wing is treated as a shell of given thickness under aerodynamic forces and subject to linear elasticity. The game has been organized in an orthogonal basis of parameters and devised to best preserve the aerodynamic criterion, thus suboptimality, while reducing significantly a stress-tensor-based structural criterion.

Participants: Aalae Benki, Jean-Antoine Désidéri, Abderrahmane Habbal.

The aim of our collaboration with the Research Center for Automotive Applications, an ArcelorMittal Division, is to develop algorithms well adapted to the design of metal structures that undergo nonlinear deformations, the criteria to be considered being multiple and antagonistic. A first industrial application is related to the bi-objective shape optimization of Dome Growth (DG) and Dome Reversal Pressure (DRP) of axisymmetric cans and sprays. Large deformation elasto-plastic analyses are performed using the LS-DYNA commercial code.

6.2.5. Optimization of Addendum Surfaces in Stamping

Participants: Fatima Zahra Oujebbour, Jean-Antoine Désidéri, Abderrahmane Habbal.

The energy consumption of cars is critically proportional to their overall weight, and so is the safe mechanical behavior of their components. Thus, designing a light structure that meets safety requirements is a complex task, particularly because of the complexity of the manufacturing process. The consortium OASIS gathers partners who are leaders in each of the process milestones from mathematics to packaging industry. Within the OASIS consortium, INRIA OPALE Team-Project is the leader for the optimization algorithms workpackage. The aim is to design, study and implement original and efficient optimization algorithms in a multidisciplinary shape and parameter design context. A particular attention is paid for the use of metamodels to compute approximate Pareto Fronts, and game theory concepts to select efficient non dominated solutions.

6.2.6. Uncertainty estimation and robust design

Participants: Régis Duvigneau, Massimiliano Martinelli.

A major issue in design optimization is the capability to take uncertainties into account during the design phase. Indeed, most phenomena are subject to uncertainties, arising from random variations of physical parameters, that can yield off-design performance losses.

To overcome this difficulty, a methodology for *robust design* is currently developed and tested, that includes uncertainty effects in the design procedure, by maximizing the expectation of the performance while minimizing its variance.

Two strategies to propagate the uncertainty are currently under study[28], [42] :

- the use of metamodels to predict the uncertainties of the objective function from the uncertainties of the input parameters of the simulation tool. During the optimization procedure, a few simulations are performed for each design variables set, for different values of the uncertain parameters in order to build a database used for metamodels training. Then, metamodels are used to estimate some statistical quantities (expectation and variance) of the objective function and constraints, using a Monte-Carlo method.
- the use of the automatic differentiation tool Tapenade (developped by Tropics Project-Team) to compute first and second order derivatives of the performance with respect to uncertain parameters. The first order derivatives are computed by solving the adjoint system, that is built by using Tapenade in reverse mode. For the computation of the second derivatives, two strategies can be employed: the use of two successive tangent mode differentiations or the use of the tangent mode on the result of the reverse mode differentiation. The efficiency of these strategies depends on the number of the parameters considered. Once these derivatives have been computed, one can easily derive statistic estimations by integrating the Taylor series expansion of the performance multiplied by the probability density function. This work is carried out in collaboration with Tropics Project-Team.

6.2.7. Numerical shape optimization of axisymmetric radiating structures

Participants: Benoît Chaigne, Claude Dedeban [France Télécom R & D], Jean-Antoine Désidéri.

This activity aims at devising efficient numerical methods for shape optimization of three-dimensional axisymmetric radiating structures incorporating and adapting various general numerical advances [58] (multi-level parameterization, multi-model methods, etc) within the framework of the time-harmonic Maxwell equations.

The optimization problem consists in finding the shape of the structure that minimizes a criterion related to the radiated energy. In a first formulation, one aims at finding the structure whose far field radiation fits a target radiation pattern. The target pattern can be expressed in terms of radiated power (norm of the field) or directivity (normalized power). In a second formulation, we assume that the structure is fed by a special device named the waveguide. In such a configuration, one wishes to reduce the so-called reflexion coefficient in the waveguide. Both formulations make sense when the feeding is monochrome (single frequency feeding). For multiple frequency optimization, several classical criteria have been used and various multipoint formulations considered (min-max, aggregated criterion, etc.).

After the successful thesis of B. Chaigne [55], and in order to provide a theoretical basis to this multilevel method, a shape reconstruction problem has been considered. The convergence of an ideal two-level algorithm has been studied. In a first step, the matrix of the linear iteration equivalent to the two-grid cycle is computed. Then, by means of similar transformations and with the help of Maple, the eigenvalues problem has been solved. Hence, the spectral radius of the ideal cycle is deduced. Provided that an adequate prolongation operator is used, we were able to show that the convergence rate is independent of the dimension of the search space. An article on this analysis is in press in the journal *Inverse Problems in Science and Engineering*.

6.3. Numerical algorithms for interactive and immersive simulation

Participant: Régis Duvigneau.

Simulation and visualization are two tasks in scientific computing that are usually carried out successively. These tasks are possibly performed on different computers: the simulation task is computationally expensive and hence uses high-performance computing facilities. On the contrary, visualization is often considered as a secondary task that can be performed using a simple workstation.

However, there is a growing demand for interactive processes on one side, and for the use of more sophisticated visualization devices on the other side. Indeed, the more and more intensive use of simulation methods in engineering design requires the development of tools that allow the user to modify interactively simulation parameters and visualize changes in the physical solution at real time. Then, engineers can rapidly have an answer to the question: what if ... ? Moreover, it is more and more obvious that simple screens are not able to represent adequately 3D solution fields for complex simulations, such as multiphase or turbulent flow simulations. The use of 3D immersive facilities could be a new and effective way to visualize and apprehend a physical phenomenon.

In the current study, we have experimented a new methodology to adress both issues: a method for fast estimation of nearby solutions has been employed to compute at real time solution fields, as the user interactively modifies some parameters. The proposed approach relies on the continuous sensitivity equation method to apply a first-order Taylor series expansion to a reference solution field. Moreover, this approach has been linked to a virtual reality software developed by SimplySim company (http://www.simplysim.net) to visualize solution fields as 3D scenes in the immersive space Gouraud-Phong at INRIA Sophia-Antipolis[46].

6.4. Application of shape and topology design to biology and medecine

6.4.1. Mathematical modeling of dorsal closure DC

Participants: Abderrahmane Habbal, L. Almeida [Univ. Nice], P. Bagnerini [Univ. Genova], F. Deman [Univ. Nice], S. Noselli [Univ. Nice], G. Edwards [Duke Univ.].





Figure 3. These figures illustrate the computation and 3D immersive visualization of a solution field at real time. At left, a room including a window and a heater, that can be moved interactively by the user. At right, the 3D temperature field constructed at real time.

We studied a simple mathematical model for DC that involves a reduced number of parameters directly linked to the intensity of the forces in the presence and which is applicable to a wide range of geometries of the leading edge (LE). This model is a natural generalization of the very interesting model proposed in Hutson et al. (Science, 2003). Being based on an ordinary differential equation (ODE) approach, the previous model had the advantage of being even simpler, but this restricted significantly the variety of geometries that could be considered and thus the number of modified dorsal closures that could be studied. A partial differential equation (PDE) approach, as the one developed with our partners, allows considering much more general situations that show up in genetically or physically perturbed embryos and whose study will be essential for a proper understanding of the different components of the DC process. Even for native embryos, our model has the advantage of being applicable since an early stages of DC when there is no antero-posterior symmetry (approximately verified only in the late phases of DC). We validated our model in a native setting and also tested it further in embryos where the zipping force is perturbed through the expression of spastin (a microtubule severing protein). We obtained variations of the force coefficients that are consistent with what was previously described for this setting.

Modeling and validation related to the mechanical behavior of the leading edge is going on with Luis Almeida (now in CNRS Biopark, Grenoble) and Glenn Edwards (Duke University).

6.5. Image Processing

6.5.1. Nash game approach to image processing

Participants: Abderrahmane Habbal, R. Aboulaich [Univ. Rabat], M. Moakher [Univ. Tunis], M. Kallel [Univ. Tunis].

We propose a game theory approach to simultaneously restore and segment noisy images. We define two players: one is restoration, with the image intensity as strategy, and the other is segmentation with contours as strategy. Cost functions are the classical relevant ones for restoration and segmentation, respectively. The two players play a static game with complete information, and we consider as solution to the game the so-called Nash Equilibrium. For the computation of this equilibrium we present an iterative method with relaxation. The results of numerical experiments performed on some real images show the relevance and striking efficiency of the proposed algorithm.

6.6. Resilient workflows for distributed multidiscipline optimization

Participants: Toan Nguyen, Laurenciu Triffan.

A resilience algorithm is being designed for large-scale distributed simulation applications. This work uses a novel approach by which the usual checkpoint-restart procedures are improved to answer the petascale and future exascale computing requirements. Indeed, it is widely recognized that the traditional approach suffer from inherent drawbacks in terms of CPU, memory and bandwidth requirements for the upcoming petascale and exascale applications which are multidiscipline, distributed and high-performance. Therefore, new approaches are required that significantly improve the checkpoint sizes, the data and jobs migrations, as well as the restart latency. A new scheme called assymetric checkpoint is developed and deployed on a distributed workflow platform based on the open-source YAWL workflow management system (http:// www.yawlfoundation.org). The platform testbed uses a virtualized infrastructure relying on the Oracle VM Virtualbox software (formerly SUN Virtualbox). Application tescases are implemented using the industry contribution of the OMD2 partners, involving 2D and 3D optimization of vehicle parts. This project is supported by the French Agence Nationale de la Recherche (grant ANR-08-COSI-007). This work is Laurentiu Trifan's PhD subject.

7. Contracts and Grants with Industry

7.1. Industrial contracts

7.1.1. Optimized steel solutions

A partnership with industry was launched with the R & D *Automotive Applications Centre* of Arcelor Mittal in Montataire, France. This partnership is related to the optimization of steel (automobile) elements with respect to mechanical criteria (crash, fatigue). I supports the PhD thesis of Aalae Benki.

The project team was solicited to audit the GEAR2 optimization team in Montataire.

7.2. National Initiatives

7.2.1. Project "OASIS"

The OASIS project, Optimization of Addendum Surfaces In Stamping, is an R&D consortium (CS, Arcelor-Mittal, ErDF, INRIA, UTC, EURODECISION, ESILV, NECS, DeltaCAD, SCILAB-DIGITEO) of the Pole Systemtic Paris-Region dedicated to develop an optimal design framework (methods-software platformsapplications) for stamping processes. The EPI OPALE/INRIA is the leader within the consortium for the Optimization workpackage (one of six WP). The OASIS project yields 2.4 Meuro total financial support (one Ph.D thesis, two post-doctoral positions and 12 months internship for OPALE).

7.2.2. Project "OMD2", Optimisation Multi-Disciplinaire Distribuée(Distributed Multidisciplinary Optimization)

This project funded by ANR deals with the development of a software platform devoted to Multidisciplinary Design Optimization (MDO) in the context of distributed computing.

The notion of optimization platform based on distributed and parallel codes is undertaken with a distributed workflow management system running on a grid infrastructure using the ProActive middleware from INRIA, in collaboration with the OASIS project at INRIA Sophia-Antipolis.

Renault is the coordinator of this project, which involves also EMSE, ENS Cachan, EC Nantes, Université de Technologie de Compiègne, CD-Adapco, Sirehna, Activeon, and INRIA project TAO, OASIS and OPALE. This contract provides the grant supporting two PhD theses (A. Zerbinati and L. Trifan)

7.3. European Initiatives

7.3.1. ERC Starting Grant TRAM3

P. Goatin is the Principal Investigator of the project "Traffic management by macroscopic models", that has received fundings (809 Keuros) from the European Research Council for 60 months starting from October 2010. Other project members are J. A. Desideri (INRIA, Opale Project), J. P. Zolesio (CNRS), R. M. Colombo (Brescia University) and M. Garavello (Piedmont University).

The project intends to investigate traffic phenomena from the macroscopic point of view, using models derived from fluid-dynamics consisting in hyperbolic conservation laws. The scope is to develop a rigorous analytical framework and fast and efficient numerical tools for solving optimization and control problems, such as queues lengths control or buildings exits design.

7.3.2. European project EXCITING

Opale and Galaad Project-Teams participate in the European project EXCITING (EXaCt-geometry sImulTIoN for optimized desiGn of vehicules and vessels). The objective is to develop simulation and design methods and software based on the isogeometric concepts, that unify Computer-Aided Design (CAD) and Finite-Elements (FE) representation bases. Applications concern hull shape, turbine and car structure design. This project supports the Post-doct grants of M. Ziani and L. Blanchard.

7.3.3. European project GRAIN

The main objectives of GRAIN are to identify and assess the future development of large scale simulation methods and tools needed for greener technologies reaching the Vision 2020 environmental goals. GRAIN will prepare the R&T development and exploitation with new large scale simulation tools used on distributed parallel environments to deeper understand and minimize the effects of aircraft/engine design on climate and noise impact. The participating institutions will focus on future collaborative applied research concerning modeling, experiments, simulation, control and optimization for greener aircraft and engines technologies including: emissions, drag and noise reduction and green materials with an emphasis on multidiscipline approaches for environmentally friendly aircraft.

7.3.4. European project MARS

Opale Project-Team participates in the European project MARS (MAnipulation of Reynolds Stress), whose objective is to study flow control devices for aeronautical applications. This project gathers twelve European partners (Airbus and Dassault-Aviation as industrial partners) and twelve Chinese partners for a common work that includes both experimental and numerical studies. Opale Project-Team is in charge of developing numerical algorithms to optimize flow control devices (vortex generators, synthetic jets). This project supports the PhD thesis of Jérémy Labroquière.

7.4. International Initiatives

7.4.1. Euromed 3+3 program SCOMU

A. Habbal is the French responsible for the project "Scientific Computing and Multidisciplinary Optimization" (SCOMU), a Euro-Mediterranean Euromed 3+3 program. The project SCOMU involves institutions from France (INRIA, Opale Project, Nice Sophia Antipolis University), Italy (University of Genova), Spain (University of Corogna), Tunisia (ENIT, Tunis) and Morocco (Ecole Mohammedia, University Mohamed V, Rabat). The project is a three-year financed action, 2009-2011. The principle aim of this project is to allow researchers from the Maghreb and Euro-Mediterranean regions to exchange their modeling and analysis skills in the fields of optimization and game theory as well as to make use of their acquired algorithms. The originality of this project lies in the fact that the partner teams intend to develop and implement applications of game theory in new areas which have strategic interests such as imaging, mathematical finance and mathematics for live sciences.

8. Dissemination

8.1. Teaching

The members of the OPALE project participate in the Educational effort within different areas at the University of Nice Sophia-Antipolis.

8.1.1. Ecole Polytechnique Universitaire (EPU), Nice Sophia-Antipolis

A. Habbal teaches the following courses ("Information Systems"):

- Numerical Engineering methods (third year, 75 hrs)
- Programming mathematics (third year, 16 hrs)
- Numerical Methods in Finance (fifth year, 18 hrs)
- Introduction to biomathematics (fourth year, 14 hrs)

J.-A. Désidéri, R. Duvigneau and A. Habbal jointly teach the following course ("Applied Mathematics and Modelling"):

• Shape optimization (fifth year, 54 hrs)

J.-A. Désidéri teaches the following course ("Applied Mathematics and Modelling"):

• Numerical Methods I (third year, 39 hrs)

R. Duvigneau teaches the following course ("Applied Mathematics and Modelling"):

- Project on Partial Differential Equations (third year, 20 hrs)
- P. Goatin teaches the following course ("Applied Mathematics and Modelling"):
 - Conservation laws and traffic flow models (fifth year, 24 hrs)
- A. Zerbinati teaches training classes ("Applied Mathematics and Modelling"):
 - Numerical Analysis (third year, 52 hrs)

8.1.2. Collège de Polytechnique

J.-A. Désidéri is coresponsible with J. Peter (ONERA) of the program of the Shortcourse "Code verification in mechanics of continuous media" (Collège de Polytechnique). At this occasion, he delivers the lecture "Error analysis in classical PDE systems" (2.5 hrs) also delivered at ONERA and UNSA.

8.2. Participation in international courses

- J.-A. Désidéri delivered the course "Hierarchical Shape Optimization: Cooperation and Competition in Multi-Disciplinary Approaches" at the French-German Summer-School "Modeling, Numerical Simulation and Optimization in Fluid Mechnics", Bad Herrenalb, Germany, September 2010.
- R. Duvigneau delivered the course "Numerical Algorithms for Optimization and Control of PDE Systems" at the French-German Summer-School "Modeling, Numerical Simulation and Optimization in Fluid Mechnics", Bad Herrenalb, Germany, September 2010.

8.3. Theses and educational trainings

- Aalae Benki, doctoral student, University of Nice Sophia Antipolis; topic: concurrent optimization of thin shells in elasto-plastic ans crash regimes.
- Manuel Bompard, doctoral student, University of Nice Sophia Antipolis, at ONERA-DSNA (Châtillon s/ Bâgneux); topic: Gradient computations for local optimization and multilevel strategies.

- Imane Ghazlane, doctoral student, University of Nice Sophia Antipolis, at ONERA-DAAP (Meudon); topic: Aerodynamic and structural optimization of a transport aircraft wing shape.
- Mohammed Kaicer, doctoral student, University of Mohammed V, Rabat; topic: Analysis of asymmetric information using game theory. Analysis design and implementation of a Simulator Adapted to MicroFinance Market.
- Jérémy Labroquère, doctoral student, University of Nice Sophia Antipolis; topic: Optimization of flow control devices.
- Andrea Minelli, doctoral student, University of Nice Sophia Antipolis, at ONERA-DAAP (Meudon); topic: Aerodynamic and acoustic shape optimization of a supersonic business jet.
- Maxime Nguyen Dinh, doctoral student, University of Nice Sophia Antipolis, at ONERA-DSNA (Châtillon s/ Bâgneux); topic: Qualification of numerical simulations by anisotropic mesh adaption.
- Samira El Moumen, doctoral student, University of Mohammed V, Rabat; topic: Portfolio management in finance.
- Nouredine Moussaid, doctoral student, "École Mohammedia" Engineering School of Rabat, Marroco; topic: Nash games in topological optimization. Thesis defended on September 2010.
- Fatima Zahra Oujebbour, doctoral student, University of Nice Sophia Antipolis; topic: Game strategies in multidisciplinary design optimization for distributed systems.
- Laurentiu Trifan, doctoral student, University of Grenoble I (Joseph Fourier); topic: Collaborative platforms for high performance multidiscipline optimization.
- Adrien Zerbinati, doctoral student, University of Nice Sophia Antipolis; topic: Multidisciplinary optimization using collaborative and competitive strategies.
- Jeannifer Vides Higueros and Sergio Minera Rebulla, MathMods- Mathematical Modelling in Engineering (M1), University of Nice Sophia Antipolis; topic: A reactive dynamic user equilibrium model for pedestrian flows: a continuum modeling approach.
- Maria Laura Delle Monache and Chandrasekhar Karnam, MathMods- Mathematical Modelling in Engineering (M1), University of Nice Sophia Antipolis; topic: Vehicular traffic flow modelling

8.4. Participation in scientific committees

- J.-A. Désidéri is being appointed part-time by ONERA (Department of Numerical Simulation and Aeroacoustics, DSNA-Chatillon, and Department of Applied Aerodynamics, DAAP-Meudon). This appointment includes the direction of the following theses: M. Bompard, I. Ghazlane, A. Minelli, M. Nguyen Dinh.
- R. Duvigneau is member of the CFD (Computational Fluid Dynamics) committee of ECCOMAS (European Community for Computational Methods in Applied Science).
- A. Habbal is head of the Modeling and Applied Mathematics Department of "Ecole Polytech Nice".
- A. Habbal is member of the executive board of "Ecole Polytech Nice".
- T. Nguyen is member of the Program Committee for the 7th Intl. Conf. on Collaborative Design, Visualization and Engineering, CDVE2010, Majorque (Spain), September 2010.
- T. Nguyen is member of the Program Committee for the 1st Intl. Conf. on Cloud Computing, Grids and Virtualization, Cloud Computing 2010, Lisbonne (Portugal), November 2010.
- T. Nguyen is member of the Database Testcase Committee for the Open Database Workshop for Multiphysics Software Validation. University of Jyvaskyla (Finland). 10-12 March 2010.
- T. Nguyen is member of the Program Committee of the Fourth International Conference on Advanced Engineering Computing and Applications in Sciences, ADVCOMP 2010, Florence, october 2010.

8.5. Invited or keynote lectures

- "Isogeometric analysis: a new paradigm for engineering design", International Workshop on Industrial Geometry: Variational PDEs and level set methods in image processing and shape optimization, Obergurgl, Austria, April 7th-9th, 2010 (R. Duvigneau)
- "Vehicular traffic management by conservation laws", Workshop "Hyperbolic systems and control in networks", Institut Henri Poincaré, Paris, October 18th October 20th, 2010 (P. Goatin).
- "Large Scale Simulations with HPC on Parallel Environments", in "Open Software Horizon Forum", University of Jyvaskyla, Finland, March 10-12, 2010. (T. Nguyen)

Additionally, the paper "A Distributed Workflow Platform for Simulation" [33] has been selected as a 'Best Paper' ("based on the reviews of the original submission, the camera-ready version, and the presentation during the conference").

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