INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Vegas

Effective Geometric Algorithms for Surfaces and Visibility

Nancy - Grand Est

Theme : Algorithms, Certification, and Cryptography

Activity Report

2010

Table of contents

1. Team

Research Scientists

Sylvain Lazard [Team Leader, Research scientist (DR) INRIA, HdR]
Sylvain Petitjean [Vice Team Leader, Research scientist (DR) INRIA, HdR]
Xavier Goaoc [Research scientist (CR) INRIA]
Marc Pouget [Research scientist (CR) INRIA]
Guillaume Moroz [Research scientist (CR) INRIA, started in Oct. 2010]

Faculty Members

Laurent Dupont [Assistant professor, Université Nancy 2]
Hazel Everett [Professor, Université Nancy 2, until Feb 2010, HdR]

PhD Students

Guillaume Batog [MENESR, started in Sept. 2008]
Luis Peñaranda [ATER Université Nancy 1, defended in Dec. 2010]
Maria Pentcheva [defended in Sept. 2010]
Yacine Bouzidi [INRIA, started in Oct. 2010]

Administrative Assistant

Helene Zganic [Research technician (TR) INRIA]

2. Overall Objectives

2.1. Overall Objectives

VEGAS is a research project of LORIA (Lorraine Research Laboratory in Computer Science and Applications), a laboratory shared by INRIA (National Institute for Research in Computer Science and Control), CNRS (National Center for Scientific Research), Université Henri Poincaré Nancy 1, Université Nancy 2, and INPL (National Engineering Institute of Lorraine).

The main scientific objective of the VEGAS research team is to *contribute to the development of an effective geometric computing* dedicated to *non-trivial geometric objects*. Included among its main tasks are the study and development of new algorithms for the manipulation of geometric objects, the experimentation of algorithms, the production of high-quality software, and the application of such algorithms and implementations to research domains that deal with a large amount of geometric data, notably solid modeling and computer graphics.

Computational geometry has traditionally treated linear objects like line segments and polygons in the plane, and point sets and polytopes in three-dimensional space, occasionally (and more recently) venturing into the world of non-linear curves such as circles and ellipses. The methodological experience and the know-how accumulated over the last thirty years have been enormous.

For many applications, particularly in the fields of computer graphics and solid modeling, it is necessary to manipulate more general objects such as curves and surfaces given in either implicit or parametric form. Typically such objects are handled by approximating them by simple objects such as triangles. This approach is extremely important and it has been used in almost all of the usable software existing in industry today. It does, however, have some disadvantages. Using a tessellated form in place of its exact geometry may introduce spurious numerical errors (the famous gap between the wing and the body of the aircraft), not to mention that thousands if not hundreds of thousands of triangles could be needed to adequately represent the object. Moreover, the curved objects that we consider are not necessarily everyday three-dimensional objects, but also abstract mathematical objects that are not linear, that may live in high-dimensional space, and whose geometry we do not control. For example, the set of lines in 3D (at the core of visibility issues) that are tangent to three polyhedra span a piecewise ruled quadratic surface and the lines tangent to a sphere correspond, in projective five-dimensional space, to the intersection of two quadratic hypersurfaces.

Effectiveness is a key word of our research project. By requiring our algorithms to be effective, we imply that the algorithms should be *robust, efficient,* and *versatile*. By robust we mean algorithms that do not crash on degenerate inputs and always output topologically consistent data. By efficient we mean algorithms that run reasonably quickly on realistic data where performance is ascertained both experimentally and theoretically. Finally, by versatile we mean algorithms that work for classes of objects that are general enough to cover realistic situations and that account for the *exact geometry* of the objects, in particular when they are curved.

3. Scientific Foundations

3.1. Theory and applications of three-dimensional visibility

The notion of 3D visibility plays a fundamental role in computer graphics. In this field, the determination of objects visible from a given point, the extraction of shadows or of penumbra boundaries are examples of visibility computations. In global illumination methods, (e.g. radiosity algorithms), it is necessary to determine, in a very repetitive manner, if two points of a scene are mutually visible. The computations can be excessively expensive. For instance, in radiosity, it is not unusual that 50 to 70% of the simulation time is spent answering visibility queries.

Objects that are far apart may have very complicated and unintuitive visual interactions, and because of this, visibility queries are intrinsically global. This partially explains that, until now, researchers have primarily used ad hoc structures, of limited scope, to answer specific queries on-the-fly. Unfortunately, experience has shown that these structures do not scale up. The lack of a well-defined mathematical foundation and the non-exploitation of the intrinsic properties of 3D visibility result in structures that are not usable on models consisting of many hundreds of thousands of primitives, both from the viewpoint of complexity and robustness (geometric degeneracies, aligned surfaces, etc.).

We have chosen a different approach which consists of computing ahead of time (that is, off-line) a 3D global visibility structure for which queries can be answered very efficiently on-the-fly (on line). The 3D visibility complex – essentially a partition of ray space according to visibility – is such a structure, recently introduced in computational geometry and graphics [42], [49]. We approach 3D global visibility problems from two directions: we study, on the one hand, the theoretical foundations and, on the other hand, we work on the practical aspects related to the development of efficient and robust visibility algorithms.

From a theoretical point of view, we study, for example, the problem of computing lines tangent to four among k polytopes. We have shown much better bounds on the number of these tangents than were previously known [2]. These results give a measure of the complexity of the vertices (cells of dimension 0) of the visibility complex of faceted objects, in particular, for triangulated scenes.

From a practical point of view, we have, for example, studied the problem of the complexity for these 3D global visibility structures, considered by many to be prohibitive. The size of these structures in the worst case is $O(n^4)$, where n is the number of objects in the scene. But we have, in fact, shown that when the objects are uniformly distributed, the complexity is linear in the size of the input [6]. This probabilistic result does not prejudice the complexity observed in real scenes where the objects are not uniformly distributed. However, initial empirical studies show that, even for real scenes, the observed complexity is largely inferior to the theoretical worst-case complexity, as our probabilistic result appears to indicate.

We are currently working on translating these positive signs into efficient algorithms. We are studying new algorithms for the construction of the visibility complex, putting the accent on the complexity and the robustness.

3.2. Reliable geometric computations on curves and surfaces

Simple algebraic surfaces cover a variety of forms sufficient for representing the majority of objects encountered in the fields of design, architecture and industrial manufacturing. For instance, it has been estimated that 95% of all mechanical pieces can be well modeled by quadric patches (degree 2 surfaces, including planes, spheres, cylinders and cones) and torii [50]. It is important, then, to be able to process these surfaces in a robust and efficient manner.

In comparison with polygonal representations, modeling and manipulating scenes made of curved objects pose a large variety of new issues and require entirely different tools. It is for instance no longer realistic to assume that simple operations like intersecting two primitives take constant time. The usual notion of complexity has to be revised and needs to incorporate the arithmetic complexity of operations.

Geometric computing with curved objects is plagued with robustness issues. The numerical instability of geometric algorithms is intimately linked to the double nature of geometric objects. Indeed, a geometric object is two things: a combinatorial structure which encodes the incidence properties between the elements constituting the object and numerical quantities (coordinates, equations) describing the embedding of the object in space. Manipulating geometric data, without breaking the consistency constraints that govern the relation between combinatorial and numerical quantities, is usually hard and has led to the unfolding of the *exact geometric computing* paradigm.

The dependence of combinatorial decisions on numerical computations is encapsulated in the notion of *geometric predicates*. When working with algebraic objects, evaluating a geometric predicate often means determining the sign of a polynomial expression in the coefficients of the input. This sign encodes the answer to simple geometric queries like “are three given points aligned?” or “is a given line tangent to a given surface?”. The paradigm of exact geometric computing requires the predicates to be evaluated exactly, ensuring that the branching of the algorithm are correct, that the software will not crash, loop indefinitely or output a wrong answer, and thus that the topological structure of the output is correct.

In the context of exact geometric computing, we work on key problems involving curved objects, mainly two-dimensional curves, and low-degree three-dimensional surfaces such as quadrics. For instance, we study intersections of quadrics both from an algorithmic and an algebraic-geometric point of view. On the algorithmic side, we work on finding simple and usable parameterizations of the intersection of two arbitrary quadrics. On the algebraic side, we deal with finding simple (and ideally optimal) geometric predicates for classifying the intersection pattern and the positional relationship of two quadrics.

We also work on computing arrangements of curved objects, i.e. the partitioning of space induced by the objects, such as arrangements of curves on a surface, or arrangements of quadrics in 3D space. Note that intersections of 2 and 3 quadrics are building blocks for the constructions of quadric arrangements. We work on constructing simpler sub-arrangements, like the BRep (Boundary Representation) of a solid model (CSG). Exact CSG-to-BRep conversion is a key and long-standing problem in CAGD, where many conventional modelers work with volumes and rendering software based on the global illumination approach need surface patches.

Finally, we deal with geometric problems where low-degree surfaces appear indirectly, not in the input but as intermediate structures. A major problem in this category is the computation of the Voronoi diagram, or medial axis, of polyhedra in 3D. In particular, we work on the simpler instance where only lines and line segments in 3D are considered, the bisectors of pairs of lines being quadric surfaces.

4. Application Domains

4.1. Computer graphics

Our main application domain is photorealistic rendering in computer graphics. We are especially interested in the application of our work to virtual prototyping, which refers to the many steps required for the creation of a realistic virtual representation from a CAD/CAM model.

When designing an automobile, detailed physical mockups of the interior are built to study the design and evaluate human factors and ergonomic issues. These hand-made prototypes are costly, time consuming, and difficult to modify. To shorten the design cycle and improve interactivity and reliability, realistic rendering and immersive virtual reality provide an effective alternative. A virtual prototype can replace a physical mockup for the analysis of such design aspects as visibility of instruments and mirrors, reachability and accessibility, and aesthetics and appeal.

Virtual prototyping encompasses most of our work on effective geometric computing. In particular, our work on 3D visibility should have fruitful applications in this domain. As already explained, meshing objects of the scene along the main discontinuities of the visibility function can have a dramatic impact on the realism of the simulations.

4.2. Solid modeling

Solid modeling, i.e., the computer representation and manipulation of 3D shapes, has historically developed somewhat in parallel to computational geometry. Both communities are concerned with geometric algorithms and deal with many of the same issues. But while the computational geometry community has been mathematically inclined and essentially concerned with linear objects, solid modeling has traditionally had closer ties to industry and has been more concerned with curved surfaces.

Clearly, there is considerable potential for interaction between the two fields. Standing somewhere in the middle, our project has a lot to offer. Among the geometric questions related to solid modeling that are of interest to us, let us mention: the description of geometric shapes, the representation of solids, the conversion between different representations, data structures for graphical rendering of models and robustness of geometric computations.

5. Software

5.1. QI: Quadrics Intersection

QI stands for “Quadrics Intersection”. QI is the first exact, robust, efficient and usable implementation of an algorithm for parameterizing the intersection of two arbitrary quadrics, given in implicit form, with integer coefficients. This implementation is based on the parameterization method described in [10], [39], [40], [41] and represents the first complete and robust solution to what is perhaps the most basic problem of solid modeling by implicit curved surfaces.

QI is written in C++ and builds upon the LiDIA computational number theory library [33] bundled with the GMP multi-precision integer arithmetic [32]. QI can routinely compute parameterizations of quadrics having coefficients with up to 50 digits in less than 100 milliseconds on an average PC; see [10] for detailed benchmarks.

Our implementation consists of roughly 18,000 lines of source code. QI has being registered at the Agence pour la Protection des Programmes (APP). It is distributed under the free for non-commercial use INRIA license and will be distributed under the QPL license in the next release. The implementation can also be queried via a web interface [34].

Since its official first release in June 2004, QI has been downloaded six times a month on average and it has been included in the geometric library EXACUS developed at the Max-Planck-Institut für Informatik (Saarbrücken, Germany). QI is also used in a broad range of applications; for instance, it is used in photochemistry for studying the interactions between potential energy surfaces, in computer vision for computing the image of conics seen by a catadioptric camera with a paraboloidal mirror, and in mathematics for computing flows of hypersurfaces of revolution based on constant-volume average curvature.

5.2. Isotop: Topology and Geometry of Planar Algebraic Curves

ISOTOP is a Maple software for computing the topology of an algebraic plane curve, that is, for computing an arrangement of polylines isotopic to the input curve. This problem is a necessary key step for computing arrangements of algebraic curves and has also applications for curve plotting. This software has been developed since 2007 in collaboration with F. Rouillier from INRIA Paris - Rocquencourt (SALSA). It is based on the method described in [16] which incorporates several improvements over previous methods. In particular, our approach does not require generic position (nor shearing) and avoids the computations of sub-resultant sequences. Our preliminary implementation is competitive with other implementations (such as ALCIX and INSULATE developed at MPII Saarbrücken, Germany and TOP developed at Santander Univ., Spain). It performs similarly for small-degree curves and performs significantly better for higher degrees, in particular when the curves are not in generic position.

5.3. CGAL: Computational Geometry Algorithms Library

Born as a European project, CGAL (http://www.cgal.org) has become the standard library for computational geometry. It offers easy access to efficient and reliable geometric algorithms in the form of a C++ library. CGAL is used in various areas needing geometric computation, such as: computer graphics, scientific visualization, computer aided design and modeling, geographic information systems, molecular biology, medical imaging, robotics and motion planning, mesh generation, numerical methods...

In computational geometry, many problems lead to standard, though difficult, algebraic questions such as computing the real roots of a system of equations, computing the sign of a polynomial at the roots of a system, or determining the dimension of a set of solutions. we want to make state-of-the-art algebraic software more accessible to the computational geometry community, in particular, through the computational geometric library CGAL. On this line, S. Lazard and L. Peñaranda proposed an extension to the already existing Number Types package. They contributed two new number types Gmpfr and Gmpfi to the CGAL library (see Sections 5.4, 5.7, and 5.9 of [46]); these new types interface the multiple-precision floating-point arithmetic library MPFR, and corresponding interval arithmetic library MPFI. They also contributed a model of the *Univariate Algebraic Kernel* concept for algebraic computations [30] (see Sections 8.2.2 and 8.4). This CGAL package improves, for instance, the efficiency of the computation of arrangements of polynomial functions in CGAL [48]. This implementation uses the RS library developed by F. Rouillier at INRIA Paris - Rocquencourt (SALSA) for isolating real roots of polynomials. All these packages have been reviewed and accepted accepted by the editorial board of CGAL and have been released this year.

6. New Results

6.1. 3D visibility, theory and applications

In recent years, our activity in the area of 3D visibility focused on three main directions: (i) the computation and complexity analysis of the *3D visibility complex* or *skeleton*, (ii) the computation and complexity analysis of the boundary of shadows cast by area light sources, and (iii) the study of some fundamental questions in geometric transversal theory.

6.1.1. Complexity of sets of free lines and line segments among balls in three dimensions

We presented in [26] two new fundamental lower bounds on the worst-case combinatorial complexity of sets of free lines and sets of maximal free line segments in the presence of balls in three dimensions.

We first proved that the visibility complex of n disjoint *unit* balls, or equivalently the set of maximal non-occluded line segments among n disjoint unit balls, has complexity $\Omega(n^4)$, which matches the trivial $O(n^4)$ upper bound. This improves the trivial $\Omega(n^2)$ bound and also the $\Omega(n^3)$ lower bound for the restricted setting of arbitrary-size balls, proved by Devillers and Ramos in 2001. This result settles, negatively, the natural conjecture that this set of line segments, or, equivalently, the visibility complex, has smaller worst-case complexity for disjoint fat objects than for skinny triangles.

We also proved an $\Omega(n^3)$ lower bound on the complexity of the set of non-occluded lines among n balls of arbitrary radii, improving on the trivial $\Omega(n^2)$ bound. This new bound almost matches the recent $O(n^{3+\epsilon})$ upper bound [51].

We submitted the final version of this paper to the journal *Discrete and Computational Geometry* in a special issue dedicated to the best papers from the 2010 Symposium on Computational Geometry.

6.1.2. Succinct 3D visibility skeleton

The 3D visibility skeleton is a data structure that encodes the global visibility information of a set of 3D objects. While it is useful in answering global visibility queries, its large size often limits its practical use. We addressed this issue in [24], [27] by proposing a subset of the visibility skeleton, which is about 25% to 50% of the whole set. We showed that the rest of the data structure can be recovered from the subset as needed, partially or completely. Our recovery method is efficient in the sense that it is output-dependent. We also proved that this subset is minimal for the complexity of our recovery method.

6.1.3. Shadow boundary computation

Computing a *geometric* description (as opposed to, say, a bitmap rendering) of the boundaries of the shadow regions in a 3D scene with extended light-sources is a difficult problem. Typical solutions are based on *visual event surfaces*, certain patches of ruled surfaces that partition the 3D spaces in regions from which the view of the scene is invariant. The number of visual event surfaces can be prohibitive, but only a few of them contribute to defining the shadow boundaries. In his PhD thesis in 2008, Julien Demouth, a former student of the group, identified a small superset of visual event surfaces that are relevant for this problem and developped a prototype code that compute the boundaries of shadows cast by extended convex polyhedra. This year, from January to July, JeongHwan Jang, a master student from KAIST worked on improving the prototype of Julien Demouth. Specifically, the prototype could only trace the shadows on the "ground" of the 3D world due to certain simplifying assumptions; JeongHwan lifted these assumptions by, in particular, making a detailed analysis of the predicates for the intersection of a plane and a visual event suface. The prototype can now trace the shadow boundaries on the objects themselves. This work is the basis for JeongHwan's master thesis, to be defended in December in KAIST.

6.1.4. Lower bounds to Helly numbers of line transversals to disjoint congruent balls

A line ℓ is a transversal to a family F of convex objects in $\mathbb{R}^d$ if it intersects every member of F. We showed that for every integer $d > 2$ there exists a family of $2d-1$ pairwise disjoint unit balls in $\mathbb{R}^d$ with the property that every subfamily of size $2d-2$ admits a transversal, yet any line misses at least one member of the family. This answers a question of Danzer from 1957. This work was accepted for publication in the *Israel Journal of Mathematics* [18].

6.1.5. Lines pinning lines

A line g is a transversal to a family F of convex polytopes in 3-dimensional space if it intersects every member of F. If, in addition, g is an isolated point of the space of line transversals to F, we say that F is a pinning of g. We showed that any minimal pinning of a line by convex polytopes such that no face of a polytope is coplanar with the line has size at most eight. If, in addition, the polytopes are disjoint, then it has size at most six. We completely characterize configurations of disjoint polytopes that form minimal pinnings of a line. This work was accepted for publication in a special issue of the journal *Discrete & Computational Geometry* [14] devoted to the *workshop on Transversal and Helly-type Theorems in Geometry, Combinatorics and Topology*, Banff, 2009.

6.1.6. Pinning a line by balls or ovaloids in $\mathbb{R}^3$

We show that if a line ℓ is an isolated line transversal to a finite family F of (possibly intersecting) balls in $\mathbb{R}^3$ and no two balls are externally tangent on ℓ , then there is a subfamily $G \subseteq F$ of size at most 12 such that ℓ is an isolated line transversal to G. We generalize this result to families of semialgebraic ovaloids. This work was accepted for publication in a special issue of the journal *Discrete & Computational Geometry* [20] devoted to the *workshop on Transversal and Helly-type Theorems in Geometry, Combinatorics and Topology*, Banff, 2009.

6.2. Certified geometric computing for curves and surfaces

6.2.1. Topology of real algebraic plane curves

We revisited in the problem of computing the topology and geometry of a real algebraic plane curve. The topology is of prime interest but geometric information, such as the position of singular and critical points, is also relevant. A challenge is to compute efficiently this information for the given coordinate system even if the curve is not in generic position.

Previous methods based on the cylindrical algebraic decomposition use sub-resultant sequences and computations with polynomials with algebraic coefficients. A novelty of our approach is to replace these tools by Gröbner basis computations and isolation with rational univariate representations. This has the advantage of avoiding computations with polynomials with algebraic coefficients, even in non-generic positions. Our algorithm isolates critical points in boxes and computes a decomposition of the plane by rectangular boxes. This decomposition also induces a new approach for computing an arrangement of polylines isotopic to the input curve. We also present an analysis of the complexity of our algorithm. An implementation of our algorithm demonstrates its efficiency, in particular on high-degree non-generic curves; see Section Software. These results were presented in Luis Peñaranda's Ph.D. thesis [13] and published in [16].

6.2.2. Algebraic tools for geometric computing

In computational geometry, many problems lead to standard, though difficult, algebraic questions such as computing the real roots of a system of equations, computing the sign of a polynomial at the roots of a system, or determining the dimension of a set of solutions. We want to make state-of-the-art algebraic software more accessible to the computational geometry community, in particular, through the computational geometric library CGAL. We have contributed to the CGAL library a model of the *Univariate Algebraic Kernel* concept for algebraic computations [30] (see Sections 8.2.2 and 8.4). This CGAL package improves, for instance, the efficiency of the computation of arrangements of polynomial functions in CGAL [48]. This implementation uses the RS library developed by F. Rouillier at INRIA Paris - Rocquencourt (SALSA) for isolating real roots of polynomials. We have also contributed an extension to the already existing CGAL Number Types package. We proposed two new number types Gmpfr and Gmpfi to the CGAL library (see Sections 5.4, 5.7, and 5.9 in [46]); these new types interface the multiple-precision floating-point arithmetic library MPFR, and corresponding interval arithmetic library MPFI. All these packages have been reviewed and accepted by the editorial board of CGAL have been released this year. This work was also presented in Luis Peñaranda's Ph.D. thesis [13].

6.2.3. Boundary evaluation of quadric-based solids

Few approaches have been proposed for computing exactly and efficiently a representation of the boundary (BRep) of a solid defined as unions and intersections of elementary solids delimited by quadrics. All of them assume, in particular, that the objects considered are in “generic position”, therefore bypassing the all-important issue of degeneracies. Using our algorithm for parameterizing the intersections of two projective quadrics [7] as a building block, we have presented an algorithm for exactly and robustly extracting the surface patches appearing on the BRep and giving an explicit representation of their borders. This algorithm was presented this year in M. Pentcheva's Ph.D. thesis [12].

It should be stressed that this project, which consists in computing a part of an arrangement of quadrics, has many applications other than the boundary evaluation of quadric-based solids. For instance, problems such as computing the convex hull of a set of quadric patches or computing one cell of the Voronoi diagram of polyhedra can be solved by computing the boundary of a quadric-based solid.

6.2.4. Computing the edge-adjacency graph of an arrangement of quadrics

In [22], we presented a complete, exact and efficient implementation to compute the edge-adjacency graph of an arrangement of quadrics, i.e. surfaces of algebraic degree 2. This is a major step towards the computation of the full 3D arrangement. We enhanced an implementation for an exact parameterization of the intersection curves of two quadrics [7], such that we can compute the exact parameter value for intersection points and from that the edge-adjacency graph of the arrangement. Our implementation is complete in the sense that it can handle all kinds of inputs including all degenerate ones, i.e. singularities or tangential intersection points. It is exact in that it always computes the mathematically correct result. It is efficient measured in running times, i.e. it compares favorably to the only previous implementation.

6.2.5. Characterizing the intersection pattern of two projective conics

We showed how to efficiently detect the type of the intersection of two arbitrary plane projective conics. The characterization uses geometric predicates of bidegree $(6, 6)$ in the two input conics, which is optimal. While we already proved similar results recently (cf. [11]), the approach spelled out here attaches geometric meaning to the predicates and overall brings substantial geometric insight to the characterization problem, which was previously treated purely algebraically. The idea is as follows: we first consider a special conic pencil to which can be attached in a simple way a quartic binary form. We show that the root pattern of the quartic is in one-to-one correspondence with the intersection pattern of the two conics. Then, following old ideas of Lindemann, we relate invariants/covariants of any two conics in the special pencil to invariants/covariants of the quartic, using the symbolic approach to classical invariant theory. We compute the Bezoutian of the quartic and show that its inertia (as a quadratic form) characterizes the intersection pattern of the two conics. We finally express it in terms of covariants of the two conics and show that its expression and the properties attached to it extend to the case of a general conic pencil.

6.2.6. Geometric predicates as arrangements of hypersurfaces

Geometric predicates can be formulated as an arrangement of hypersurfaces (usually algebraic varieties) in a high-dimensional space, where each cell of the arrangement corresponds to an outcome of the predicate, and an evaluation of the predicate maps to point-location queries in this arrangement. To do this successfully, the arrangement has to be decomposed by the aid of subsidiary hypersurfaces, the degree of which plays a fundamental role in the algebraic complexity of the predicate, with respect to the input coefficients. We show in [29] that the widely used predicate of root comparison of quadratic polynomials can be mapped to an arrangement of lines and a parabola. For cubics, it becomes an arrangement of planes and a quartic surface, when a monic polynomial of degree d is represented as a point in $\mathbb{R}^d$. Minimizing the degree of the subsidiary equations is an outstanding open problem.

6.2.7. Bounded-curvature shortest paths

We considered the problem of computing shortest paths having curvature at most one almost everywhere and visiting a sequence of n points in the plane in a given order. This problem arises naturally in path planning for point car-like robots in the presence of polygonal obstacles, and is also a sub-problem of the Dubins Traveling Salesman Problem.

We showed in [31] that a shortest bounded-curvature path through a sequence of points $p_1, ..., p_n$ such that consecutive points are distance at least 4 apart can be computed by minimizing a function from $\mathbb{R}^n$ to $\mathbb{R}$ which is strictly convex over at most 2^k convex polyhedra, and realizes its minimum over these polyhedra; each polyhedron is defined by $4n - 1$ inequalities, and k denotes the number of *sharp turns*, that is, informally, the number of points p_i such that $\angle(p_{i-1}, p_i, p_{i+1})$ is small. The function to be optimized maps $(\theta_1, ..., \theta_n) \in \mathbb{R}^n$ to the length of a shortest curvature-constrained path that visits the points $p_1, ..., p_n$ in order and whose tangent in p_i makes an angle θ_i with the x-axis.

We also reveal a connection between the above problem and the question of finding a shortest path of bounded curvature between two given points in the presence of polygonal obstacles. As a consequence, we obtain that if the sequence of points where a shortest path touches the obstacles is known then "connecting the dots" reduces, under certain conditions, to a family of convex optimization problems.

6.3. Other results in computational geometry

6.3.1. Rigid graph

Given an abstract graph and its edge lengths, we considered the problem of counting its embeddings in the plane. This problem is important in computational geometry [36], [35] and has applications in robot kinematics [52], [38], [44] and structural biology [47], [45], [43]. We focused on graphs with 11 edges and 7 vertices, that is the smallest case for which the maximal number of embeddings was unknown. We proved that they can have at most 56 embeddings and showed an example of such a graph.

The rigidity of these graphs was captured by a polynomial system derived from Cayley-Menger matrices. The upper bound were obtained using the theory of mixed-volume for sparse systems. And for the lower bound, we used stochastic optimisation methods.

Moreover, this result allowed us to slightly improve the lower bound for any n on the number of embeddings of a n-vertices graph, raising the record obtained by Borcea and Streinu from $\Omega(2.289^n)$ to $\Omega(2.300^n)$.

This work has been submitted to IFToMM 2011, one of the major conferences in mechanical design.

6.3.2. Homotopic Fréchet distance between curves

The Fréchet distance between two curves in the plane is the minimum length of a leash that allows a dog and its owner to walk along their respective curves, from one end to the other, without backtracking. We proposed in [15] a natural extension of Fréchet distance to more general metric spaces, which requires the leash itself to move continuously over time. For example, for curves in the punctured plane, the leash cannot pass through or jump over the obstacles ("trees"). We describe a polynomial-time algorithm to compute the *homotopic* Fréchet distance between two given polygonal curves in the plane minus a given set of polygonal obstacles.

6.3.3. Universal sets of n points for one-bend drawings of planar graphs with n vertices

We showed in [19] that any planar graph with n vertices can be point-set embedded with at most one bend per edge on a universal set of n points in the plane. An implication of this result is that any number of planar graphs admit a simultaneous embedding without mapping with at most one bend per edge.

6.3.4. Farthest-polygon Voronoi diagrams

Given a family of k disjoint connected polygonal sites in general position and of total complexity n, we consider the farthest-site Voronoi diagram of these sites, where the distance to a site is the distance to a closest point on it. We showed in [17] that the complexity of this diagram is $O(n)$, and give an $O(n \log^3 n)$ time algorithm to compute it. We also prove a number of structural properties of this diagram. In particular, a Voronoi region may consist of $k-1$ connected components, but if one component is bounded, then it is equal to the entire region.

6.4. International initiatives

6.4.1. Associated Teams and Other International Projects

- KAIST-INRIA associated team. This INRIA program is a joint project between VEGAS and the Theory of Computation Laboratory of the KAIST University of Daejeon, in Korea, more particularly the group of Otfried Cheong. It started in 2008, following a 2-years PHC grant. The research theme is Discrete and Computational Geometry, in general, with a particular emphasis on questions where both continuous and discrete aspects come into play and interact. In 2010, this collaboration continued through mutual visits (see below) and a research workshop that we organized in September 2010 to discuss problems on *interactions between discrete and algebraic geometry*. The projects on which we collaborate include line geometry [14], [18], combinatorial geometry [37] bounded curvature path planning [31] and geometric data structures [17].

  In 2010, this cooperation was supported for 13 kE by INRIA and for 4 kE by our partners.

- Sylvain Petitjean started a collaboration with Pr. Gert Vegter of the University of Groningen on "Certified Geometric Approximation". This collaboration is funded by the Netherlands Organization for Scientific Research (NWO) - 2008–2012.

6.5. Visiting scientists

Visitors:

- JeongHwan Jang, KAIST, January-July, 6 months.

- Otfried Cheong, KAIST, June, 1 week.
- Hyo-Sil Kim, KAIST, June, 2 weeks.
- Mira Lee, KAIST, July, 10 days.
- Martin Tancer, Charles University (Prague), September 1 week.
- Jae-Soon Ha, KAIST, September, 1 week.
- Christian Knauer, Bayreuth University, September, 1 week.

Visits:

- Xavier Goaoc, Univ. Magdeburg, Germany, 2 days; Charles University, Czech Republic, 2x1 week; Frankfurt, Germany, 2 days; KAIST, Korea, 2 weeks.
- Laurent Dupont, KAIST, Korea, 2 weeks.
- Sylvain Lazard, Inria Sophia Antipolis, 2 weeks.

7. Dissemination

7.1. Teaching

All of the teaching activities were carried out in Nancy. The research Masters program is a joint degree with Univ. Nancy 1, Univ. Nancy 2 and the engineering school INPL. These three institutes are jointly known as University of Nancy.

Several members of the group, in particular the professors, assistant professors and Ph.D. students, actively teach at Université Nancy 2, Université Henri Poincaré Nancy 1, and INPL. Members of the group also teach in the Master of Computer Science of Nancy; namely, H. Everett contributed to the module "Modelisation of geometric data". Inria researchers also intervene: X. Goaoc teaches computational geometry in the Master's program of the geology school at INPL and an "algorithm in C" course in the "formation continue" program at Nancy University. S. Lazard also teaches the L3 course on "Algorithms and Complexity" at Nancy University.

7.2. Animation of the scientific community

Program and Paper Committee:

- Hazel Everett: Program committee of the ACM Symposium on Computational Geometry 2010 (SoCG'10).
- Sylvain Lazard: Program committee of the European Symposium on Algorithms 2010 (ESA'10)

Editorial responsibilities:

- Hazel Everett: Editor of the *Journal of Computational Geometry*.
- Xavier Goaoc: Editor of the *Journal of Computational Geometry*.
- Sylvain Lazard: Guest editor (with L. Gonzalez-Vega, Santander Univ., Spain) of *Mathematics in Computer Science* (special issue on Computational Geometry and CAGD) [21]. Guest editor of *Computational Geometry: Theory and Applications* (special issue on selected papers from EuroCG'08) [23].
- Sylvain Petitjean: Editor of *Graphical Models*.

Workshop organizations:

- Hazel Everett and Sylvain Lazard co-organized with S. Whitesides (Victoria University) the 9th Workshop on Geometry Problems in Computer Graphics[1] (Bellairs Research Institute of McGill University) in Feb. (1 week workshop on invitation).
- Xavier Goaoc co-organized with Otfried Cheong (KAIST, Korea) and Frank Sottile (TAMU, USA). the Workshop on Interactions between discrete and algebraic geometry[2] (Val d'Ajol) in Sept. (1 week workshop on invitation).

Thesis and habilitation committee:

- Sylvain Lazard: member of the PhD. committee of P. Marin, INPL, Nancy.
- Sylvain Petitjean: member of the PhD. committee of D. Robert, Nancy-Université.

Other responsibilities:

- Hazel Everett: Head of the LORIA laboratory (Jan.-Feb. 2010).
- Sylvain Lazard: Head of the INRIA Nancy-Grand Est *PhD and Post-doc hiring committee* (since 2009). Member of the *Bureau du Département Informatique de Formation Doctorale* of the *École Doctorale IAE+M* (since 2009).
- Laurent Dupont: Director of the departement *Services et réseaux de communication* of IUT Charlemagne, University Nancy 2 (since 2008).
- Xavier Goaoc: Member of the hiring committee for computer science, University Paris 6. *Correspondent Europe* of INRIA Nancy Grand-Est.
- Sylvain Petitjean: *Scientific delegate* of INRIA Nancy Grand-Est and chairman of its *Project committee* (since 2009). Member of the *Executive committee* of INRIA Nancy Grand-Est, member of its *Commission des développements technologiques*. Member of INRIA's *Evaluation committee*.
- Marc Pouget: Member of the CGAL Editorial Board (since 2008).

8. Bibliography

Major publications by the team in recent years

[1] C. Borcea, X. Goaoc, S. Lazard, S. Petitjean. *Common Tangents to Spheres in* $\mathbb{R}^3$, in "Discrete & Computational Geometry", 2006, vol. 35, n° 2, p. 287-300 [*DOI :* 10.1007/s00454-005-1230-y], http://hal.inria.fr/inria-00100261/en.

[2] H. Brönnimann, O. Devillers, V. Dujmovic, H. Everett, M. Glisse, X. Goaoc, S. Lazard, H.-S. Na, S. Whitesides. *Lines and free line segments Tangent to Arbitrary Three-dimensional Convex Polyhedra*, in "SIAM Journal on Computing", 2007, vol. 37, n° 2, p. 522-551 [*DOI :* 10.1137/S0097539705447116], http://hal.inria.fr/inria-00103916/en.

[3] H. Brönnimann, H. Everett, S. Lazard, F. Sottile, S. Whitesides. *Transversals to line segments in three-dimensional space*, in "Discrete & Computational Geometry", 2005, vol. 34, n° 3, p. 381 - 390 [*DOI :* 10.1007/s00454-005-1183-1], http://hal.inria.fr/inria-00000384/en.

[4] J. Cheng, S. Lazard, L. Peñaranda, M. Pouget, F. Rouillier, E. P. Tsigaridas. *On the topology of real algebraic plane curves*, in "Mathematics in Computer Science", 2010, http://hal.inria.fr/inria-00517175/en.

[1] Workshop on Problems in Computational Geometry
[2] KAIST-INRIA associated team

[5] O. CHEONG, X. GOAOC, A. HOLMSEN, S. PETITJEAN. *Hadwiger and Helly-type theorems for disjoint unit spheres*, in "Discrete & Computational Geometry", 2008, vol. 39, n^{o} 1-3, p. 194-212, http://hal.inria.fr/inria-00103856/en.

[6] O. DEVILLERS, V. DUJMOVIC, H. EVERETT, X. GOAOC, S. LAZARD, H.-S. NA, S. PETITJEAN. *The expected number of 3D visibility events is linear*, in "SIAM Journal on Computing", 2003, vol. 32, n^{o} 6, p. 1586-1620 [*DOI :* 10.1137/S0097539702419662], http://hal.inria.fr/inria-00099810/en.

[7] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: I. The Generic Algorithm; II. A Classification of Pencils; III. Parameterizing Singular Intersections*, in "Journal of Symbolic Computation", 2008, vol. 43, p. 168–191, 192–215, 216–232, autres url : http://hal.inria.fr/inria-00186090/en/, http://hal.inria.fr/inria-00186091/en/, http://hal.inria.fr/inria-00186089/en/.

[8] H. EVERETT, D. LAZARD, S. LAZARD, M. SAFEY EL DIN. *The Voronoi diagram of three lines*, in "Journal of Discrete and Computational Geometry", 2009, vol. 42, n^{o} 1, p. 94-130 [*DOI :* 10.1007/s00454-009-9173-3], http://www.springerlink.com/content/f5601q6324664k2p/?p=6d7bb74bf9df40b0b7756b3a5153809f&pi=5, http://hal.inria.fr/inria-00431518/en.

[9] M. GLISSE, S. LAZARD. *An Upper Bound on the Average Size of Silhouettes*, in "Discrete & Computational Geometry", 2008, vol. 40, n^{o} 2, p. 241-257 [*DOI :* 10.1007/s00454-008-9089-3], http://hal.inria.fr/inria-00336571/en.

[10] S. LAZARD, L. PEÑARANDA, S. PETITJEAN. *Intersecting Quadrics: An Efficient and Exact Implementation*, in "Computational Geometry", 2006, vol. 35, n^{o} 1-2, p. 74–99, http://hal.inria.fr/inria-00000380/en.

[11] S. PETITJEAN. *Invariant-based characterization of the relative position of two projective conics*, in "Non-Linear Computational Geometry", I. Z. EMIRIS, F. SOTTILE, T. THEOBALD (editors), Springer, 2008, http://hal.inria.fr/inria-00335968/en.

Publications of the year

Doctoral Dissertations and Habilitation Theses

[12] M. PENTCHEVA. *Conversion CSG-BRep de scènes définies par des quadriques*, Université Nancy II, Sep 2010, http://hal.inria.fr/tel-00540015/en.

[13] L. PEÑARANDA. *Geometrie algorithmique non lineaire et courbes algebriques planaires*, Université Nancy II, December 2010, http://hal.inria.fr/tel-00547829/en.

Articles in International Peer-Reviewed Journal

[14] B. ARONOV, O. CHEONG, X. GOAOC, R. GÜNTER. *Lines Pinning Lines*, in "Discrete & Computational Geometry", 2010, http://hal.inria.fr/inria-00518028/en.

[15] E. W. CHAMBERS, E. COLIN DE VERDIÈRE, J. ERICKSON, S. LAZARD, F. LAZARUS, S. THITE. *Homotopic Fréchet Distance Between Curves or, Walking Your Dog in the Woods in Polynomial Time*, in "Computational Geometry", 2010, vol. 43, n^{o} 3, p. 295-311 [*DOI :* 10.1016/J.COMGEO.2009.02.008], http://www.sciencedirect.com/science?_ob=MImg&_imagekey=B6TYS-4W6XVWR-1-1&_cdi=5626&_user=6068174&_orig=search&_cov zSkWz&md5=77170fe13ab0fd311adc0c72b956d4ad&ie=/sdarticle.pdf, http://hal.inria.fr/inria-00438463/en.

[16] J. CHENG, S. LAZARD, L. PEÑARANDA, M. POUGET, F. ROUILLIER, E. P. TSIGARIDAS. *On the topology of real algebraic plane curves*, in "Mathematics in Computer Science", 2010, http://hal.inria.fr/inria-00517175/en.

[17] O. CHEONG, H. EVERETT, M. GLISSE, J. GUDMUNDSSON, S. HORNUS, S. LAZARD, M. LEE, H.-S. NA. *Farthest-Polygon Voronoi Diagrams*, in "Computational Geometry", 2010, http://hal.inria.fr/inria-00442816/en.

[18] O. CHEONG, X. GOAOC, A. HOLMSEN. *Lower Bounds to Helly Numbers of Line Transversals to Disjoint Congruent Balls*, in "Israël Journal of Mathematics", 2010, http://hal.inria.fr/inria-00518035/en.

[19] H. EVERETT, S. LAZARD, G. LIOTTA, S. WISMATH. *Universal Sets of n Points for One-bend Drawings of Planar Graphs with n Vertices*, in "Journal of Discrete and Computational Geometry", 2010, vol. 43, n^o 2, p. 272-288 [*DOI :* 10.1007/s00454-009-9149-3], http://www.springerlink.com/content/1851pv541v2714v5/, http://hal.inria.fr/inria-00431769/en.

[20] X. GOAOC, S. KOENIG, S. PETITJEAN. *Pinning a Line by Balls or Ovaloids in R^3*, in "Discrete & Computational Geometry", 2010 [*DOI :* 10.1007/s00454-010-9297-5], http://hal.inria.fr/inria-00518033/en.

[21] L. GONZALEZ-VEGA, S. LAZARD. *Foreword: Special Issue on Computational Geometry and Computer-Aided Geometric Design*, in "Mathematics in Computer Sciences", 2010, vol. 4, n^o 1, p. 1-2 [*DOI :* 10.1007/s11786-010-0059-9], http://www.springerlink.com/content/axg1846xv51w277n/, http://hal.inria.fr/inria-00537757/en.

[22] M. HEMMER, L. DUPONT, S. PETITJEAN, E. SCHOMER. *A Complete, Exact and Efficient Implementation for Computing the Edge-Adjacency Graph of an Arrangement of Quadrics*, in "Journal of Symbolic Computation", Nov 2010, http://hal.inria.fr/inria-00537592/en.

[23] S. LAZARD. *Editorial of the 24th European Workshop on Computational Geometry, EuroCG'08*, in "Computational Geometry", 2010, vol. 43, n^o 2, 67, http://dx.doi.org/10.1016/j.comgeo.2009.06.006, http://hal.inria.fr/inria-00431610/en.

[24] S. LAZARD, C. WEIBEL, S. WHITESIDES, L. ZHANG. *A Succinct 3D Visibility Skeleton*, in "Discrete Mathematics, Algorithms and Applications (DMMA)", 2010, http://hal.inria.fr/inria-00511233/en.

International Peer-Reviewed Conference/Proceedings

[25] G. BATOG, X. GOAOC, J. PONCE. *Admissible Linear Map Models of Linear Cameras*, in "23rd IEEE Conference on Computer Vision and Pattern Recognition - CVPR 2010", États-Unis San Francisco, IEEE, Jun 2010, p. 1578 - 1585 [*DOI :* 10.1109/CVPR.2010.5539784], http://hal.inria.fr/inria-00517899/en.

[26] M. GLISSE, S. LAZARD. *On the Complexity of Sets of Free Lines and Line Segments Among Balls in Three Dimensions*, in "26th annual symposium on Computational geometry - SCG 2010", États-Unis Snowbird, Utah, ACM Press, 2010, http://hal.inria.fr/inria-00442751/en.

[27] S. LAZARD, C. WEIBEL, S. WHITESIDES, L. ZHANG. *On the Computation of the 3D Visibility Skeleton*, in "16th Annual International Computing and Combinatorics Conference - COCOON 2010", Viet Nam Nha Trang, Lecture Notes in Computer Science, Springer, 2010, http://hal.inria.fr/inria-00472839/en.

Workshops without Proceedings

[28] G. BATOG. *From invariants to predicates: example of line transversals to lines*, in "26th European Workshop on Computational Geometry - EuroCG 2010", Allemagne Dortmund, Mar 2010, http://hal.inria.fr/inria-00517885/en.

[29] G. TZOUMAS. *Geometric predicates as arrangements of hypersurfaces: Application to comparison of algebraic numbers*, in "Fall School Shapes, Geometry, and Algebra", Grèce Kolympari, Oct 2010, Poster, http://hal.inria.fr/inria-00538839/en.

Scientific Books (or Scientific Book chapters)

[30] E. BERBERICH, M. HEMMER, S. LAZARD, L. PEÑARANDA, M. TEILLAUD. *Algebraic kernel*, in "CGAL User and Reference Manual, 3.6 edition", CGAL EDITORIAL BOARD (editor), CGAL Editorial board, 2010, http://hal.inria.fr/inria-00537545/en.

Research Reports

[31] X. GOAOC, H.-S. KIM, S. LAZARD. *Bounded-Curvature Shortest Paths through a Sequence of Points*, INRIA, 2010, n^o RR-7465, http://hal.inria.fr/inria-00539957/en.

References in notes

[32] *GMP: the GNU MP Bignum Library*, The Free Software Foundation, http://gmplib.org/.

[33] *LiDIA: a C++ Library for Computational Number Theory*, Darmstadt University of Technology, http://www.informatik.tu-darmstadt.de/TI/LiDIA.

[34] *QI: a C++ package for parameterizing intersections of quadrics*, 2005, LORIA, INRIA Lorraine, VEGAS project, http://www.loria.fr/equipes/vegas/qi.

[35] M. THORPE, P. DUXBURY (editors). *Rigidity Theory and Applications*, Fundamental Materials Research, Kluwer Academic/ Plenum Publishers, New York, 1999.

[36] C. BORCEA, I. STREINU. *The Number of Embeddings of Minimally Rigid Graphs*, in "Discrete and Computational Geometry", 2004, vol. 31, p. 287-303, 10.1007/s00454-003-2902-0, http://dx.doi.org/10.1007/s00454-003-2902-0.

[37] O. CHEONG, X. GOAOC, C. NICAUD. *Set Systems and Families of Permutations with Small Traces*, 2009, Manuscript.

[38] C. L. COLLINS. *Forward kinematics of planar parallel manipulators in the Clifford algebra of P2*, in "Mechanism and Machine Theory", 2002, vol. 37, n^o 8, p. 799 - 813 [*DOI : * DOI: 10.1016/S0094-114X(02)00023-X], http://www.sciencedirect.com/science/article/B6V46-45RFM59-1/2/66c85ba1c16daae578e0f61edf13d87d.

[39] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: I. The Generic Algorithm*, in "Journal of Symbolic Computation", 2008, vol. 43, p. 168–191, http://hal.inria.fr/inria-00186089/en/.

[40] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: II. A Classification of Pencils*, in "Journal of Symbolic Computation", 2008, vol. 43, p. 192–215, http://hal.inria.fr/inria-00186090/en/.

[41] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: III. Parameterizing Singular Intersections*, in "Journal of Symbolic Computation", 2008, vol. 43, p. 216–232, http://hal.inria.fr/inria-00186091/en/.

[42] F. DURAND. *Visibilité tridimensionnelle : étude analytique et applications*, Université Joseph Fourier - Grenoble I, 1999.

[43] I. Z. EMIRIS, B. MOURRAIN. *Computer Algebra Methods for Studying and Computing Molecular Conformations*, in "Algorithmica", 1999, vol. 25, p. 372-402, 10.1007/PL00008283, http://dx.doi.org/10.1007/PL00008283.

[44] J. C. FAUGÈRE, D. LAZARD. *Combinatorial classes of parallel manipulators*, in "Mechanism and Machine Theory", 1995, vol. 30, n^{o} 6, p. 765 - 776 [*DOI :* DOI: 10.1016/0094-114X(94)00069-W], http://www.sciencedirect.com/science/article/B6V46-3YMFRHR-10/2/56698ff2034a8f5242f91c8dcf059d16.

[45] T. F. HAVEL. *Distance Geometry: Theory, Algorithms, and Chemical Applications*, in "Encyclopedia of Computational Chemistry", John Wiley & Sons, Inc., 2002.

[46] M. HEMMER, S. HERT, L. KETTNER, S. PION, S. SCHIRRA. *Number Types*, in "CGAL User and Reference Manual", CGAL Editorial Board, 2010, http://www.cgal.org/Manual/3.7/doc_html/cgal_manual/packages.html#Pkg:NumberTypes.

[47] D. J. JACOBS, A. RADER, L. A. KUHN, M. THORPE. *Protein flexibility predictions using graph theory*, in "Proteins: Structure, Function, and Bioinformatics", 2001, p. 150-165.

[48] S. LAZARD, L. PEÑARANDA, E. P. TSIGARIDAS. *Univariate Algebraic Kernel and Application to Arrangements*, in "International Symposium on Experimental Algorithms – SEA Experimental Algorithms, 8th International Symposium, SEA 2009", Allemagne Dortmund, Springer, 2009, vol. 5526/2009, p. 209-220, http://hal.inria.fr/inria-00431559/en/.

[49] M. POCCHIOLA, G. VEGTER. *The visibility complex*, in "International Journal of Computational Geometry and Applications", 1996, vol. 6, n^{o} 3, p. 1-30.

[50] A. REQUICHA, H. VOELCKER. *Solid modeling: a historical summary and contemporary assessment*, in "IEEE Computer Graphics and Applications", 1982, vol. 2, n^{o} 1, p. 9-24.

[51] N. RUBIN. *Lines avoiding balls in three dimensions revisited*, in "26th annual symposium on Computational geometry - SCG 2010", États-Unis Snowbird, Utah, ACM Press, 2010, p. 58–67.

[52] D. WALTER, M. HUSTY. *On a nine-bar linkage, its possible configurations and conditions for paradoxical mobility*, in "Porceedings of Twelfth World Congress In Mechanism And Machine Science, IFToMM 2007, Besaniçon, France", J.-P. MERLET, M. DAHAN (editors), 2007.