Activity Report 2011

Project-Team ALGORITHMS

Algorithms

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THEME
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2. Overall Objectives

2.1. Introduction

The primal objective of the project is the field of analysis of algorithms. By this is meant a precise quantification of complexity issues associated to the most fundamental algorithms and data structures of computer science. Departing from traditional approaches that, somewhat artificially, place emphasis on worst-case scenarii, the project focuses on average-case and probabilistic analyses, aiming as often as possible at realistic data models. As such, our research is inspired by the pioneering works of Knuth.

The need to analyze, dimension, and finely optimize algorithms requires an in-depth study of random discrete structures, like words, trees, graphs, and permutations, to name a few. Indeed, a vast majority of the most important algorithms in practice either “make bets” on the likely shape of input data or even base themselves of random choices. In this area we are developing a novel approach based on recent theories of combinatorial analysis together with the view that discrete models connect nicely with complex-analytic and asymptotic methods. The resulting theory has been called “Analytic combinatorics”. Applications of it have been or are currently being worked out in such diverse areas as communication protocols, multidimensional search, data structures for fast retrieval on external storage, data mining applications, the analysis of genomic sequences, and data compression, for instance.

The analytic-combinatorial approach to the basic processes of computer science is very systematic. It appeared early in the history of the project that its development would greatly benefit from the existence of symbolic manipulation systems and computer algebra. This connection has given rise to an original research programme that we are currently carrying out. Some of the directions pursued include automating the manipulation of combinatorial models (counting, generating function equations, random generation), the development of "automatic asymptotics", and the development of a unified view of the theory of special functions. In particular, the project has developed the Maple library *Algolib*, that addresses several of these issues.

2.2. Highlights

Philippe Flajolet, head of the project and of former related projects at Inria, died suddenly on March 22. He is celebrated for opening new lines of research in analysis of algorithms, developing powerful new methods, and solving difficult open problems. A conference with more than 250 participants has been organized by the project in December. It will pay homage to the man as well as the multi-faceted mathematician and computer-scientist.

3. Scientific Foundations

3.1. Analysis of Algorithms

While we know the laws of basic physics and while probabilists have been setting up a coherent theory of stochastic processes for about half a century, the "laws of combinatorics", in the sense of the laws governing random structured configurations of large sizes, are much less understood. Accordingly, our knowledge in the latter area is still very much fragmentary. Some of the difficulties arise from the large variety of models that tend to surface in real-life applications—the world of computer scientists and algorithmic designers is really an artificial world, much more "free" than its physical counterpart. Some of us have then engaged in the long haul project of trying to offer a unified perspective in this area. The approach of analytic combinatorics has evolved from there.

Analytic combinatorics leads to discovering randomness phenomena that are "universal" (a term actually borrowed from statistical physics) across seemingly different applications. For instance, it is found that similar laws govern the behaviour of prime factors in integers, of irreducible factors in polynomials, of cycles in permutations, and of components in mappings of a finite set. Once detected, such phenomena can then be exploited by specific algorithms that factor integers (a problem relevant to public-key cryptography), decompose polynomials (this is needed in computer algebra systems), reorganize tables in place (this is of obvious interest in the manipulation of various data sets), and use collisions to estimate the cardinality of massive data ensembles. The underlying technology bases itself on generating functions, which exactly describe discrete models, as well as an interpretation of these generating functions as analytic transformations of the complex plane. Singularities together with the associated perturbative theory then deliver a number of very precise estimates regarding important characteristics of random discrete structures. The process can be largely made formal and accessible to computer algebra (see below) and it may be adapted to the broad area of analysis of algorithms.

3.2. Computer Algebra

Computer algebra at large aims at making effective large portions of mathematics, paying due attention to complexity issues. For reasons mentioned above, our project specifically investigates the way mathematical objects originating in complex analysis can be dealt with in an algorithmic way by computer algebra systems. Our main contributions in this area concern the automation of asymptotic analysis and the handling of special functions. The mathematical foundations of our algorithms are deeply rooted in differential algebra (Hardy fields for asymptotic expansions and Ore algebras for special functions).

Over the years, in order to automate the average-case analysis of ever larger classes of algorithms, we have developed algorithms and implementations for the following problems: the specification of formally specified combinatorial structures; the corresponding problems of enumeration and random generation; the automatic construction of asymptotic scales which is necessary for extracting the singular behaviour of generating functions; the automatic computation of asymptotic expansions in such scales; the automatic computation of asymptotic expansions satisfied by coefficients of generating series. An *Encyclopedia of Combinatorial Structures*, available on the web, gathers roughly one thousand structures for which generating series, recurrences, and asymptotic behaviour have been determined automatically using our libraries.

An important principle of computer algebra is that it is often easier to operate with equations defining a mathematical object implicitly rather than trying to obtain a "closed-form" expression of it. The class of linear differential and difference equations is particularly important in view of the large variety of functions and sequences they capture. In this area, we have developed the highly successful gfun package (jointly with P. Zimmermann, from the Caramel project) dealing with the univariate case. In the multivariate case, we have developed the underlying theory based on Gröbner bases in Ore algebra, and an implementation in the Mgfun package. The algorithmic advances of the past few years have made it possible to start the implementation of a *Dynamic Dictionary of Special Functions*, providing various information concerning classical functions (of wide use throughout sciences), including Bessel functions, Airy functions, ... The corresponding information is all automatically generated.

4. Software

4.1. Algolib

The Algolib library is a set of Maple routines that have been developed in the project for more than 15 years. Several parts of it have been incorporated into the standard library of Maple, but the most up-to-date version is always available for free from our web pages http://algo.inria.fr/libraries/. This library provides: tools for combinatorial structures (the combstruct package), including enumeration, random or exhaustive generation, generating functions for a large class of attribute grammars; tools for linear difference and differential equations (the gfun package), which have received a very positive review in Computing Reviews and have been incorporated in N. Sloane's superseeker at Bell Labs; tools for systems of multivariate linear operators, definite sums and integrals (the Mgfun package), including Gröbner bases in Ore algebras, that also treat commutative polynomials and have been the standard way to solve polynomial systems in Maple for a long period (although the user would not notice it); Mgfun has also been chosen at Risc (Linz) as the basis for their package Desing, tools for expansions in general asymptotic scales, which make it possible to handle in a transparent and automatic way the problems of finding the proper scale for an expansion and of dealing with the indefinite cancellation problem (the MultiSeries package).

4.2. Mathematics on the Web

We also provide access to our work to scientists who are not using Maple or any other computer algebra system in the form of automatically generated encyclopedias available on the web. The Encyclopedia of Combinatorial Structures at http://algo.inria.fr/ecs/ thus contains more than 1000 combinatorial structures for which generating functions, enumeration sequences, recurrences, and asymptotic approximations have been computed automatically. It gets more than 16,000 hits per month. The Dynamic Dictionary of Mathematical Functions (DDMF) at http://ddmf.msr-inria.inria.fr/ gathers several dozens of special functions for which identities, guaranteed high-precision numerical evaluations, power-series and asymptotic expansions, graphs, ...are generated automatically and on the user's request, starting from a linear differential equation and its initial conditions. The underlying symbolic algorithms and implementations are those of gfun and Mgfun. All the production process being automated, the difficult and expensive step of checking each formula individually is suppressed. A nice specificity of this encyclopedia is its interactivity: the approximations values (numbers, series) are not bound to a statically set precision, rather, the user can fill in the precision he wants in a form,

before clicking to ask for a refined identity to be generated, then displayed. This interactivity is based on a tool DynaMoW at http://ddmf.msr-inria.inria.fr/DynaMoW/ (for Dynamic Mathematics on the Web) that we develop as well. This is an Ocaml library that simultaneously controls external symbolic calculations and web-page generation at the same time. Being available on the web, the DDMF also plays the role of a showcase for part of the packages developed in our project. It is a successor of our former Encyclopedia of Special Functions at http://algo.inria.fr/esf/.

5. New Results

5.1. Analysis of Algorithms

The following articles, conference communications and reports summarize new results in analysis of algorithms over the period: [20], [18], [17], [19], [16], [13], [4], [10], [11], [3], [5].

5.2. Computer Algebra

The following articles, conference communications and reports summarize new results in computer algebra over the period: [9], [21], [2], [8], [22], [7], [15], [1], [12], [6], [14].

6. Contracts and Grants with Industry

6.1. Contracts with Industry

The Algorithms Project and Waterloo Maple Inc. (WMI) have collaborated for many years based on reciprocal interests. Thanks to this collaboration, the company WMI considers Inria as a special partner and grants it a free license for all of its research units.

Our work on automating the derivation of formulæ for special functions is hosted and funded for 6 years (2007–2012) by the Microsoft Research - INRIA Joint Centre as one of its projects, called “Dynamic Dictionary of Mathematical Functions”.

7. Partnerships and Cooperations

7.1. Regional Initiatives

Nicolas Broutin has obtained two years funding from Egide to support a collaboration on geometric data structures with Ralph Neininger from the University of Frankfurt.

7.2. National Initiatives

Aléa is a national working group dedicated to the analysis of algorithms and random combinatorial structures. It is a meeting place for mathematicians and computer scientists working in the area of discrete models. It is currently supported by CNRS (GDR IM) and was globally animated by Philippe Flajolet. In March 2011 the yearly meeting has gathered in Luminy over 80 participants from about 20 different research laboratories throughout France.

In September 2009, the Algorithms project has started a new participation in the programme funded by the National Research Agency (ANR) entitled BOOLE for “Quantifying Boolean Frameworks”. Four teams are involved: Algorithms from Inria Paris–Rocquencourt, the Universities of Caen, Versailles (coordinator), and Provence Aix–Marseille 1; the project is for 4 years until August 2013. The Inria Team also includes researchers at the École Normale Supérieure (ENS Ulm): Guilhem Semerjian and Jean Vuillemin.

7.3. International Initiatives

7.3.1. Visits of International Scientists

James Davenport, from the University of Bath (UK) has been invited for one month in April 2011. At this occasion, [15] was completed.

Other visitors for shorter periods were: Manuel Kauers, RISC, Linz (Austria), Ziming Li (Key Laboratory of Mathematics Mechanization of the Chinese Academy of Mathematics and System Sciences, Beijing),

8. Dissemination

8.1. Animation of the scientific community

The Algorithms project runs a biweekly seminar devoted to the analysis of algorithms and related topics. A. Bostan and F. Chyzak organize this seminar. Several partner teams in the grand Paris area attend on a regular basis, and also take part in two regular workshops, Alea and the "Journées Nationales de Calcul Formel", both gathering around 80 people in Luminy.

Alin Bostan has joined the Advisory Board of the MEGA conference (Effective Methods in Algebraic Geometry). He has served as a member of the hiring committees in computer science at the University Paris 13, and at the University Lille 1. He has been a member of the PhD thesis committee for Carole El Bacha, who defended on Nov. 25 her thesis "Méthodes algébriques pour la résolution d'équations différentielles matricielles d'ordre arbitraire" at the University of Limoges.

Nicolas Broutin is co-organizer with Luc Devroye (McGill, Montreal) of the 23rd International Meeting on Probabilistic, Combinatorial and Asymptotic Methods for the Analysis of Algorithms (AofA 2012), to be held in Montreal next June. With Luc Devroye and Gabor Lugosi (Pompeu Fabra, Barcelona), they have received the funds to organize a one-week workshop at the Banff International Research Station (Canada), on "Sparse random graphs and network algorithms", to be held in February 2012.

Bruno Salvy is a member of the editorial board of the Journal of Symbolic Computation and of the Journal of Algebra (section Computational Algebra). He is organizing the working group Computer Algebra of the CNRS GDR IM (Mathematical Computer Science). This year, he is a member of the program committee of AofA 2012, Montreal. He is a co-organizer of the conference "Philippe Flajolet and Analytic Combinatorics" that pays homage to Philippe Flajolet in December. He has been a member of the PhD committees of Marc Mezzarobba and Jérémy Berthomieu (Polytechnique) and of the committee for the habilitation of Damien Stehlé (ENS Lyon). He has also been a member of the committees hiring researchers at Inria (CR, DR) and Professors in combinatorics at LIPN (Paris 13).

8.2. Participation in conferences, seminars, invitations

Alin Bostan gave talks on "Symbolic analysis for lattice path combinatorics" at the Institute of Systems Science, Beijing, China (June 3), at the FOCM'11 conference, Budapest, Hungary (July 13), at the workshop "Rencontres autour des séries formelles arithmétiques", Institut Fourier, Grenoble (Sept. 8), and at the Algo Seminar of the GREYC, Univ. Caen (Oct. 25). He presented a poster on "Fast computation of common left multiples of linear ordinary differential operators" at the ISSAC'11 conference, San Jose, USA (June 9).

Nicolas Broutin has given lectures at the workshop on random graphs (Lille), the meeting on "random trees, information and algorithms" in Oberwolfach, the "Colloque SMF Etats de la Recherche" on statistical learning (IHP), the seminar organized by the members of the A3 ANR project (ENS), the colloquium on Analysis of Algorithms (Poznan), the meeting of the Applied Probability Society (Stockholm), the Paris-Bath meeting on branching Structures (IHP), the "Séminaire de Combinatoire Philippe Flajolet" (IHP), and the joint Mainz-Frankfurt Seminar on probability. He also presented his experience of research at Inria to the students of Ecole Polytechnique during the round table on careers in research organized at the occasion of the "Forum métiers".

Frédéric Chyzak presented his joint work with Alexis Darrasse (postdoc at the MSR-INRIA) on "Using Camlp4 for presenting dynamic mathematics on the web: DynaMoW, an OCaml language extension for the run-time generation of mathematical contents and their presentation on the web (an experience report)" [14] at ICFP'11 (Tokyo).

Christoph Koutschan gave an invited talk at ICASF (International Conference on Asymptotics and Special Functions, Hong Kong, June 2011) on "Software for Special Functions". He presented his work on "Lattice Green's Functions of the Higher-Dimensional Face-Centered Cubic Lattices" at the Joint Mathematical Conference of the Austrian Mathematical Society together with the Catalan, Czech, Slovak, and Slovenian Mathematical Societies (Krems, Austria) and at the Conference on Applied Algebraic Geometry (Raleigh, USA). He also gave a talk on "Advanced Computer Algebra for Evaluating Determinants" at the Journees Nationales de Calcul Formel 2011 (Luminy).

Bruno Salvy has been an invited speaker at the conference "Computational and Analytical Mathematics" in honour of Jonathan's Borwein 60th birthday, Vancouver, May 2011, where he talked on automatic proofs of identities. He also presented variants of this talk at seminars in Waterloo, Ontario, the Fourier Institute (Grenoble), Labri (Bordeaux) and at the "Séminaire Philippe Flajolet" in IHP (Paris). He gave a talk on "Newton Iteration for Combinatorial Systems" at the Discrete Mathematics Seminar in Vancouver, at the Alea days in Luminy and at a seminar in ENS Lyon. Finally, he was invited to give a talk entitled "Computational Variations on Linear Differential Equations" at the workshop "Differential Equations and Galois Theory" organized at IHES for Galois' 200th birthday.

Flavia Stan gave two talks on her work on symbolic summation for Feynman-integral calculus (at a seminar of the LIPN and at the workshop Combinatorial Physics III in Kraków, Poland). She also gave a seminar talk on her work on symbolic summation and Mellin-Barnes integrals at the University of Limoges (XLIM).

8.3. Teaching

Alin Bostan, Frédéric Chyzak, and Bruno Salvy have set up and taught a 48h course in computer algebra together with Marc Giusti (from École polytechnique). This course is part of the Master Parisien de Recherche en Informatique (MPRI).

Frédéric Chyzak gave a mini-course on "Le télescopage créatif pour l'intégration et la sommation paramétrées" at the JNCF 2011 (Journées Nationales de Calcul Formel) at Luminy.

Alin Bostan and Bruno Salvy participated to the training of ENS students preparing the French "agrégation".

Bruno Salvy is also a PCC (professeur chargé de cours) at École polytechnique starting this September, where he participates in the courses "Logic, Models and Computation" and "Algorithms and Programming".

During the month of November, Nicolas Broutin has taught an intensive course on the topic of "random trees and applications" in the joint program in Mathematics and Computer Science at the J.W. Goethe University of Frankfurt (Germany).

In terms of volume, this is summarized as follows:

Master : Efficient Algorithms in Computer Algebra, 54h ETP, M2, MPRI, France.

License : Logic, Models and Computation, 54h ETP, L3, École polytechnique, France.

Master : préparation à l'agrégation, 14h ETP, M1, ENS Cachan, France.

Doctorate : Random trees and applications, 24h ETP, Joint program in Maths and CS, J. W. Goethe University of Frankfurt, Germany.

Doctorate : Le télescopage créatif pour l'intégration et la sommation paramétrées, 4.5h ETP, Journées Nationales de Calcul Formel, France.

And here is the data concerning our PhD students:

PhD : Shaoshi Chen, Quelques applications de l'algébre différentielle et aux différences pour le télescopage créatif, École polytechnique, February 2011, F. Chyzak and Z. Li.

PhD : Marc Mezzarobba, Autour de l'évaluation numérique des fonctions D-finies, École polytechnique, October 2011.

PhD in progress : Alexandre Benoit, Algorithmes pour des familles de fonctions D-finies, started September 2008, B. Salvy.

PhD in progress : Pierre Lairez, Algorithmique efficace pour la création télescopique et ses applications, started September 2011, A. Bostan and B. Salvy.

PhD in progress : Basile Morcrette, Urnes analytiques, started September 2009, Ph. Dumas.

PhD in progress : Minmin Wang, Asymptotique des arbres aléatoires inhomogènes, started September 2010, N. Broutin and T. Duquesne (Paris 6).

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[9] A. BOSTAN, T. COMBOT. *A binomial-like matrix equation*, in "American Mathematical Monthly", 2012.

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