IN PARTNERSHIP WITH:
CNRS

Institut polytechnique de Grenoble

Université Pierre Mendes-France (Grenoble 2)

Université Joseph Fourier (Grenoble 1)

Activity Report 2011

Project-Team BIPOP

Modelling, Simulation, Control and Optimization of Non-Smooth Dynamical Systems

IN COLLABORATION WITH: Laboratoire Jean Kuntzmann (LJK)

RESEARCH CENTER
Grenoble - Rhône-Alpes

THEME
Modeling, Optimization, and Control of Dynamic Systems

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Project-Team BIPOP

Keywords: Modeling, System Analysis And Control, Nonsmooth Analysis, Optimization, Simulation

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2. Overall Objectives

2.1. Overall Objectives

Generally speaking, this project deals with nonregular systems, control, modelling and simulation, with emphasis on

- dynamic systems, mostly mechanical systems with unilateral constraints and Coulomb friction, but also electrical circuits with ideal diodes and transistors Mos[1], etc;
- numerical methods for nonsmooth optimization, and more generally the connection between continuous and combinatorial optimization.

[1] metal-oxyde semiconductor

3. Scientific Foundations

3.1. Dynamic non-regular systems

Dynamical systems (we limit ourselves to finite-dimensional ones) are said to be *non-regular* whenever some nonsmoothness of the state arises. This nonsmoothness may have various roots: for example some outer impulse, entailing so-called *differential equations with measure*. An important class of such systems can be described by the complementarity system

$$\left\{\begin{array}{l} \dot{x} = f(x,u,\lambda)\,, \\ 0 \leq y \perp \lambda \geq 0\,, \\ g(y,\lambda,x,u,t) = 0\,, \\ \text{re-initialization law of the state } x(\cdot)\,, \end{array}\right. \tag{1}$$

where $\perp$ denotes orthogonality; u is a control input. Now (1) can be viewed from different angles.

- Hybrid systems: it is in fact natural to consider that (1) corresponds to different models, depending whether $y_i = 0$ or $y_i > 0$ (y_i being a component of the vector y). In some cases, passing from one mode to the other implies a jump in the state x; then the continuous dynamics in (1) may contain distributions.
- Differential inclusions: $0 \leq y \perp \lambda \geq 0$ is equivalent to $-\lambda \in \mathrm{N}_K(y)$, where K is the nonnegative orthant and $\mathrm{N}_K(y)$ denotes the normal cone to K at y. Then it is not difficult to reformulate (1) as a differential inclusion.
- Dynamic variational inequalities: such a formalism reads as $\langle \dot{x}(t) + F(x(t),t), v - x(t)\rangle \geq 0$ for all $v \in K$ and $x(t) \in K$, where K is a nonempty closed convex set. When K is a polyhedron, then this can also be written as a complementarity system as in (1).

Thus, the 2nd and 3rd lines in (1) define the modes of the hybrid systems, as well as the conditions under which transitions occur from one mode to another. The 4th line defines how transitions are performed by the state x. There are several other formalisms which are quite related to complementarity. A tutorial-survey paper has been published [4], whose aim is to introduce the dynamics of complementarity systems and the main available results in the fields of mathematical analysis, analysis for control (controllability, observability, stability), and feedback control.

3.2. Nonsmooth optimization

Here we are dealing with the minimization of a function f (say over the whole space R^n), whose derivatives are discontinuous. A typical situation is when f comes from dualization, if the primal problem is not strictly convex – for example a large-scale linear program – or even nonconvex – for example a combinatorial optimization problem. Also important is the case of spectral functions, where $f(x) = F(\lambda(A(x)))$, A being a symmetric matrix and λ its spectrum.

For these types of problems, we are mainly interested in developing efficient resolution algorithms. Our basic tool is bundling (Chap. XV of [10]) and we act along two directions:

- To explore application areas where nonsmooth optimization algorithms can be applied, possibly after some tayloring. A rich field of such application is combinatorial optimization, with all forms of relaxation [12], [11].
- To explore the possibility of designing more sophisticated algorithms. This implies an appropriate generalization of second derivatives when the first derivative does not exist, and we use advanced tools of nonsmooth analysis, for example [13].

4. Application Domains

4.1. Introduction

Many systems (either actual or abstract) can be represented by (1). Some typical examples are:

- Mechanical systems with unilateral constraints and dry friction (the biped robot is a typical example), including kinematic chains with slack, phenomena of liquid slosh, etc.
- Electrical circuits with ideal diodes and/or transistors Mos.
- Optimal control with constraints on the state, closed loop of a system controlled by an MPC algorithm[2], etc.

This class of models is not too large (to allow thorough studies), yet rich enough to include many applications. This goes in contrast to a study of general hybrid systems. Note for example that (1) is a “continuous” hybrid system, in that the continuous variables x and u prevail in the evolution (there is no discrete control to commute from a mode to the other: only the input u can be used). Let us cite some specific applications.

4.2. Nonsmooth mechanical systems

This constitutes a major topic of the team. Walking robots, flexible beams that model hair dynamics, circuit breakers, granular materials, structures like 2D or 3D blocks on moving grounds, are important examples of mechanical systems that involve unilateral contact with friction. Their analysis, control, simulation, modelling, require specific tools from convex or nonsmooth analysis, complementarity theory, variational inequalities theory, and impact mechanics.

4.3. Computational neuroscience

Modeling in neuroscience makes extensive use of nonlinear dynamical systems with a huge number of interconnected elements. Our current theoretical understanding of the properties of neural systems is mainly based on numerical simulations, from single cell models to neural networks. To handle correctly the discontinuous nature of integrate-and-fire networks, specific numerical schemes have to be developed. Our current works focus on event-driven, time-stepping and voltage-stepping strategies, to simulate accurately and efficiently neuronal networks. Our activity also includes a mathematical analysis of the dynamical properties of neural systems. One of our aims is to understand neural computation and to develop it as a new type of information science.

4.4. Electronic circuits

Whether they are integrated on a single substrate or as a set of components on a board, electronic circuits are very often a complex assembly of many basic components with non linear characteristics. The IC technologies now allow the integration of hundreds of millions of transistors switching at GHz frequencies on a die of 1cm^2. It is out of question to simulate a whole such IC with standard tools such as the SPICE simulator. We currently work on a dedicated plug-in able to simulate a whole circuit comprising various components, some modelled in a nonsmooth way.

4.5. Walking robots

As compared to rolling robots, the walking ones – for example hexapods – possess definite advantages whenever the ground is not plane or free: clearing obstacles is easier, holding on the ground is lighter, adaptivity is improved. However, if the working environment of the system is adapted to man, the biped technology must be preferred, to preserve good displacement abilities without modifying the environment. This explains the interest displayed by the international community in robotics toward humanoid systems, whose aim is to back man in some of his activities, professional or others. For example, a certain form of help at home to disabled persons could be done by biped robots, as they are able to move without any special adaptation of the environment.

[2] model predictive control

4.6. Optimization

Optimization exists in virtually all economic sectors. Simulation tools can be used to optimize the system they simulate. Another domain is parameter *identification* (Idopt or Estime teams), where the deviation between measurements and theoretical predictions must be minimized. Accordingly, giving an exhaustive list of applications is impossible. Some domains where Inria has been implied in the past, possibly through the former Promath and Numopt teams are: production management, geophysics, finance, molecular modeling, robotics, networks, astrophysics, crystallography, ...Our current applicative activity includes: the management of electrical production (deterministic or stochastic), the design and operation of telecommunication networks.

4.7. Computer graphics Animation

A new application in Bipop is the simulation of complex scenes involving many interacting objects. Whereas the problem of collision detection has become a mature field those recent years, simulating the collision response (in particular frictious contacts) in a realistic, robust and efficient way, still remains an important challenge. Another related issue we began to study is the simulation of heterogeneous objects such as granular or fibrous materials, which requires the design of new high-scales models for dynamics and contacts; indeed, for such large systems, simulating each interacting particle/fiber individually would be too much time-consuming for typical graphics applications. We also pursue some study on the design of high-order models for slender structures such as rods, plates or shells. Finally, our current activity includes the static inversion of mechanical objects, which is of great importance in the field of artistic design, for the making of movies and video games for example. Such problems typically involve geometric fitting and parameters identification issues, both resolved with the help of constrained optimization.

5. Software

5.1. Nonsmooth dynamics: Siconos

Participants: Vincent Acary, Maurice Bremond, Olivier Bonnefon.

In the framework of the European project Siconos, Bipop was the leader of the Work Package 2 (WP2), dedicated to the numerical methods and the software design for nonsmooth dynamical systems. The aim of this work is to provide a common platform for the simulation, modeling, analysis and control of abstract nonsmooth dynamical systems. Besides usual quality attributes for scientific computing software, we want to provide a common framework for various scientific fields, to be able to rely on the existing developments (numerical algorithms, description and modeling software), to support exchanges and comparisons of methods, to disseminate the know-how to other fields of research and industry, and to take into account the diversity of users (end-users, algorithm developers, framework builders) in building expert interfaces in Python and end-user front-end through Scilab.

After the requirement elicitation phase, the Siconos Software project has been divided into 5 work packages which are identified to software products:

1. SICONOS/NUMERICS This library contains a set of numerical algorithms, already well identified, to solve non smooth dynamical systems. This library is written in low-level languages (C,F77) in order to ensure numerical efficiency and the use of standard libraries (Blas, Lapack, ...)
2. SICONOS/KERNEL This module is an object-oriented structure (C++) for the modeling and the simulation of abstract dynamical systems. It provides the users with a set of classes to describe their nonsmooth dynamical system (dynamical systems, intercations, nonsmooth laws, ...) and to perform a numerical time integration and solving.
3. SICONOS/FRONT-END. This module is mainly an auto-generated wrapper in Python which provides a user-friendly interface to the Siconos libraries. A scilab interface is also provided in the Front-End module.

4. SICONOS/CONTROL This part is devoted to the implementation of control strategies of non smooth dynamical systems.
5. SICONOS/MULTIBODY. This part is dedicated to the modeling and the simulation of multi-body systems with 3D contacts, impacts and Coulomb's friction. It uses the Siconos/Kernel as simulation engine but relies on a industrial CAD library (OpenCascade and pythonOCC) to deal with complex body geometries and to compute the contact locations and distances.

Further informations may be found at http://siconos.gforge.inria.fr/

5.2. Humanoid motion analysis and simulation

Participant: Pierre-Brice Wieber.

The HuMAnS toolbox offers tools for the modelling, control and analysis of humanoid motion, be it of a robot or a human. It is a C/C++/Scilab/Maple-based set of integrated tools for the generation of dynamical models of articulated bodies with unilateral contact and friction, their simulation with an event-driven integration scheme, their 3D visualization, the computation of stability measures, optimal positions and trajectories, the generation of control laws and observers, the reconstruction of movements from different sensing systems.

5.3. AMELIF

Participants: Pierre-Brice Wieber, François Keith.

The AMELIF framework is an integrative framework that proposes an API for the representation and simulation of virtual scenes including articulated bodies. AMELIF was devised to realize interactive scenario studies with haptic feedback while providing an interface enabling fast and general prototyping of humanoids (avatars or robots). It is entirely developed in C++ and is cross-platform. The framework is articulated around a core library, upon which several modules have been developed for collision detection, dynamic simulation (contact handling in a time stepping scheme), 3D rendering, haptic interaction, posture generation. This framework is developed mostly at the CNRS/AIST UMI JRL, but we started using it in the Bipop team and therefore started contributing actively to its development.

5.4. Optimization

Participant: Claude Lemaréchal.

Essentially two possibilities exist to distribute our optimization software: library programs (say Modulopt codes), communicated either freely or not, depending on what they are used for, and on the other hand specific software, developed for a given application.

The following optimization codes have been developed in the framework of the former Promath project. They are generally available at http://www-rocq.inria.fr/~gilbert/modulopt/; M1QN3 is also distributed under GPL.

5.4.1. Code M1QN3

Optimization without constraints for problems with many variables ($n \geq 10^3$, has been used for $n = 10^6$). Technically, uses a limited-memory BFGS algorithm with Wolfe's line-search (see Chap. 4 of [3] for the terminology).

5.4.2. Code M2QN1

Optimization with simple bound-constraints for (small) problems: D is a parallelotope in R^n. Uses BFGS with Wolfe's line-search and active-set strategy.

5.4.3. Code N1CV2

Minimization without constraints of a convex nonsmooth function by a proximal bundle method (Chap. XV of [10], Chap. 9 of [3]).

5.4.4. Modulopt

In addition to codes such as above, the Modulopt library contains application problems, synthetic or from the real world. It is a field for experimentation, functioning both ways: to assess a new algorithm on a set of test-problems, or to select among several codes one best suited to a given problem.

5.5. Simulation of fibrous materials

Participants: Florence Bertails-Descoubes, Gilles Daviet.

The goal of the MECHE ADT, which started in September 2009 and was completed in fall 2011, was to develop a software for simulating the dynamics of assemblies of thin rods (such as hair), subject to contact and friction. This software combines a panel of well-accepted models for rods (ranging from reduced coordinates to maximal coordinates models, and including models recently developed by some members of the group) with classical as well as innovative schemes for solving the problem of frictional contact (incorporating the most recent results of the group, as well as the new contact solver we published in [21]). The aim of this software is twofold: first, we were able to compare and analyze the performance of nonsmooth schemes for the frictional contact problem, in terms of realism (capture of dry friction, typically), robustness, and computational efficiency. This study was conducted onto the different rod models that are available in the software. Second, we believe such a software will help us understand the behavior of a fibrous material (such as hair) through virtual experiments, thanks to which we hope to identify and understand some important emergent phenomena. A careful validation study against experiments started to be conducted in 2011 in collaboration with physicists from L'Oréal. Once this discrete elements model will be fully validated, our ultimate goal would be to build a continuous macroscopic model for the hair medium relying on nonsmooth laws.

An associate engineer, Gilles Daviet, has been hired in September 2009 to work full-time on the MECHE project. His contract was extended until October the 31st, funded by the L'Oréal collaboration project.

6. New Results

6.1. Modeling

6.1.1. Simulation of electrical circuits as nonsmooth dynamical systems

Participants: Vincent Acary, Olivier Bonnefon, Bernard Brogliato.

DC-DC converters are usually difficult to simulate with classical tools like SPICE because of the highly nonlinear behaviour of some components and the frequent occurrence of intrinsically generated switching events.

The simulation of such circuits modelled as nonsmooth systems has been successfully achieved with a clear advantage over several SPICE simulators and a simulator belonging to the hybrid modelling approach [1] [48].

6.1.2. Spiking neuronal networks dynamics

Participant: Arnaud Tonnelier.

Precise spatiotemporal sequences of spikes are observed in many neural systems and are thought to be involved in the neural processing of sensory stimuli. In [58] we examine the capability of spiking neural networks to propagate stably spatiotemporal sequences of spikes. We derive some analytical results for the wave speed and show that the stability of simple waves is determined by the Schur criteria. The transmission of a sequence of several spikes is related to the existence of stable composite waves, i.e. the existence of stable spatiotemporal periodic traveling waves. We show that the stability of composite waves is related to the roots of a system of multivariate polynomials.

A fundamental aspect that shapes the properties of traveling waves in networks is the underlying lattice-structure of the space . Discreteness has a strong effect on propagating activity patterns and, for instance, anisotropy or propagation failure can be observed. Numerical simulations and analytical calculations have been carried out to characterize more precisely these properties [47].

6.1.3. Computational Toxicology

Participant: Arnaud Tonnelier.

It is now well recognized that toxicology has entered a new era. Previously mainly based on animal testing, toxicology is now turning to in vitro and in silico experiments. To assess the risk of chemicals but also to gather and to interpret the massive amounts of experimental data generated by modern toxicology, the development of mathematical and computational tools are essential. An important element in risk assessment of chemicals is the human bioaccumulative potential. We developed a predictive tool for human bioaccumulation assesment using a physiologically based toxicokinetic model [28].

6.1.4. High-order models of mechanical rods

Participants: Florence Bertails-Descoubes, Romain Casati.

Reduced-coordinates models for rods such as the articulated rigid body model or the super-helix model [50] are able to capture the bending and twisting deformations of thin elastic rods while strictly and robustly avoiding stretching deformations. In this work we are exploring new reduced-coordinates models based on a higher-order geometry. Typically, elements are defined by a polynomial curvature function of the arc length, of degree $d \geq 1$. The main difficulty compared to the super-helix model (where $d = 0$) is that the kinematics has no longer a closed form. We have already investigated the clothoidal case ($d = 1$) in the 2d case [51], relying on Romberg numerical integration, and a general approach in 3d based on power series expansion was formulated in the master thesis of R. Casati, for a single element. R. Casati is currently extending the method to a chain of linked elements as well as to an arbitrary degree d of the curvature function.

6.1.5. Inverse modeling of mechanical rods

Participants: Florence Bertails-Descoubes, Alexandre Derouet-Jourdan.

Controlling the input shape of slender structures such as rods is desirable in many design applications (such as hairstyling, reverse engineering, etc.), but solving the corresponding inverse problem is not straightforward. In [29] we started to extend to 3d our 2d method introduced in [8] for automatically converting a smooth sketched curve into a dynamic curve at stable equilibrium under gravity. The main challenge in 3d amounts to converting an input curve into a continuous piecewise helix. Using a least-squares optimization approach is a natural option, however it may suffer from both robustness and computational issues due to the presence of multiple local minima in the objective function. To overcome these issues, we have recently proposed to reformulate the problem as a geometric interpolation problem. In this new method, only tangents are strictly interpolated while points are displaced in an optimal way so as to lie in a feasible configuration, *i.e.*, a configuration that is compatible with the interpolation by a helix. Our method proves to be much more robust and faster compared to the global optimization approach. We plan to publish these results in 2012.

6.1.6. Multiple impacts modelling

Participants: Bernard Brogliato, Hongjian Zhang, Ngoc-Son Nguyen.

The work consists of studying two systems: the rocking block and tapered chains of balls, using the Darboux-Keller model of multiple impacts previously developed. The objectives are threefold: 1) show that the model predicts well the motion by careful comparisons with experimental data found in the literature, 2) study the system's dynamics and extract critical kinetic angles that allow the engineer to predict the system's gross motion, 3) develop numerical code inside the SICONOS platform that incorporates the model of multiple impact. Results are in [42].

6.1.7. Simulating contact with Coulomb friction in fiber assemblies

Participants: Florence Bertails-Descoubes, Gilles Daviet.

We have developed a new frictional contact solver in [21] which is able to robustly and efficiently handle large fiber problems composed of thousands self-contacting rods with exact Coulomb friction. The solver relies on a Gauss-Seidel iterative approach, where each local one-contact solver is based on a hybrid strategy. The solution to the one-contact problem is first searched for using a nonsmooth Newton method based upon a generalized Fischer-Burmeister formulation. This primary solver manages to solve the local problem in 99.9% of the cases. When the solver fails to converge to an acceptable solution, the method switches to a more costly but exact solver, based on the α-formulation introduced in [39]. This hybrid strategy experimentally allows us to always find a solution to the local problem, which greatly contributes to improve the robustness of the global solver. We have compared our new solver against other solvers of the literature (*e.g.*, damped Newton solvers relying on the Alart-Curnier function) and observed a noticeable gain, both in terms of robustness and computational efficiency.

6.2. Optimization

6.2.1. Nonsmooth analysis and optimization on matrix manifolds

Participant: Jérôme Malick.

Optimization on matrix manifolds is an emerging fields of research in optimization, driven by applications in robotics. We have contributed on two different projects.

- **Numerical efficiency of optimization methods.** Newton method on manifolds would require to compute a geodiesic (that is to solve a ODE). It is clear that replacing classical differential-geometric objects with certain approximations, resulting in faster and possibly more robust algorithms. With our colleague P.-A. Absil from the Department of Mathematical Engineering of the Ecole Polytechnique de Louvain (Belgique), we propose in [16] a way to construct "retractions" (a key step when applying optimization algorithms on matrix manifolds) by projecting onto the submanifold. We show that the operation remains a retraction if the projection is generalized to a projection-like procedure that consists of coming back to the submanifold along "admissible" directions. This theory offers a framework in which previously-proposed retractions can be analyzed, as well as a toolbox for constructing new ones. Illustrations are given for projection-like procedures on some specific manifolds for which we have an explicit, easy-to-compute expression.
- **Towards the application of matrix optimization techniques to spectral manifolds.** Spectral sets are sets of matrices that depend only on the constraints on the eigenvalues: S is a spectral set if $S = \lambda^{-1}(C)$ with C a subset of $\mathbb{R}^n$. A spectral set S inherits from properties of the underlying set C, such as convexity. We prove in [46] that the spectral sets associated to smooth manifolds in $\mathbb{R}^n$ (having some local symmetry) are themselves manifolds in the space of matrices. This result looks simple but generalizes several useful particular cases, and was extremely difficult to prove: we brace together tools from nonsmooth analysis, differential geometry, group theory and spectral analysis.

6.2.2. Semidefinite programming and combinatorial optimization

Participants: Nathan Krislock, Jérôme Malick.

We have worked with Frederic Roupin (Prof. at Paris XIII) on the use of semidefinite programming to solve combinatorial optimization problems to optimality. Within exact resolution schemes (branch-and-bound), "good" bounds are those with a "good" balance between tightness and computing times.

We proposed a new family of semidefinite bounds for 0-1 quadratic problems with linear or quadratic constraints [26], [54]. An interesting feature is that the final accuracy level is controlled by real parameter acting like a cursor. This gives ways to trade computing time for a (small) deterioration of the quality of the usual semidefinite bounds, in view of enhancing this efficiency in exact resolution schemes. Extensive numerical comparisons et tests showed the superior quality of our bounds on standard test-problems (unconstrained 0-1 quadratic problems, heaviest k-subgraph problems, and graph bisection problems).

We have embedded the new bounds within branch-and-bound algorithms to solve 2 standard combinatorial optimization problems to optimality.

- *Heaviest k-subgraph problems*. Our algorithm [26] takes advantage of the new bounds to prune very well in the search tree. Its performances are then comparable with the best method (based on convex quadratic relaxation using CPLEX as an engine). In practice, our method works particularly fine on the most difficult instances (with a large number of vertices, small density and small k).
- *Max-cut.* We are working on extending our algorithm to max-cut problems [53]. It dynamically manages polyedral and semidefinite relaxations to outperform the state-of-the-art solver ([56]) on the large test-problems.

6.2.3. *Marginal prices in electricity production*

Participants: Claude Lemaréchal, Jérôme Malick, Welington Oliveira, Sofia Zaourar.

Two subjects were involved this year in our ongoing collaboration with EdF.

- **Stabilizing prices**. Unit-commitment optimization problems in electricity production are large-scale, nonconvex and heterogeneous, but they are decomposable by Lagrangian duality. Realistic modeling of technical production constraints makes the dual objective function computed inexactly though. An inexact version of the bundle method has been dedicated to tackle this difficulty [52]. However, the computed optimal dual variables show a noisy and unstable behaviour, that could prevent their use as price indicator. We propose a simple and controllable way to stabilize the dual optimal solutions, by penalizing the total variation of the prices [59]. Our illustrations on the daily electricity production optimization of EDF show a strinking stabilization at a negligible cost.
- **Accelerating** the solution phase by the so-called disaggregation technique [49], using the fact that (see Activity Report of 2010) the dual objective function is the sum of two terms: one coming from primal cost, one coming from valorization of constraints (plus possibly a third term when price stabilization is present). The resulting CPU time is drastically improved, sometimes divided by 10.

6.3. Control

6.3.1. *Digital sliding mode control*

Participants: Vincent Acary, Bernard Brogliato.

The problem of digital sliding mode controllers is a long-standing issue not yet satisfactorily solved. We propose in ideas which are inspired from the numerical methods of contact mechanics [2] and which permit a) to suppress the numerical chattering, b) to obtain a smooth stabilization on the sliding surfaces. The work is continued together with Yury Orlov in more general cases where the system is acted upon by disturbances and a disturbance estimation is added [19].

6.3.2. *Discrete-time discontinuous systems*

Participants: Vincent Acary, Bernard Brogliato, Carmina Georgescu, Scott Greenhalg, Thorsten Schindler.

We focus on some classes of discontinuous dynamical systems like relay systems, linear complementarity systems. The objectives are to show that the time-stepping numerical schemes like Moreau's algorithm can be used to successfully simulate such systems (like in the case of biological systems like gene networks), and also to study the properties of these schemes for finite nonzero time steps (like preservation of disssipativity properties). Results are in [23], [35]. Further work deals with timestepping schemes for nonsmooth dynamical systems. So far, these schemes are locally of order one both in smooth and nonsmooth segments. This is inefficient for applications with few events like circuit breakers. To consistently improve the behavior during smooth episodes, the traditional schemes are being embedded in time discontinuous Galerkin methods. After establishing the correct mathematical setting, a Petrov-Galerkin distributional differential inclusion is outlined. The bouncing ball example illustrates its capabilities.

6.4. Locomotion analysis

6.4.1. Synchronous imitation of human motion by a humanoid robot

Participants: Mehdi Benallegue, Pierre-Brice Wieber.

Interactions between humans and robots require that each one is able to understand and interpret each other's actions. From the point of view of the robot, this means: (i) to move in a way that can be naturally interpreted by humans and (ii) to be able to understand the humans' actions. Studies in Neuroscience in the case of interactions between humans indicate that these two abilities might be tightly linked in the human's brain: we understand actions when we map the observed action onto our motor representation of the same action [57]. In this work, we consider that the "motor representation" of a task is the control law, and "mapping an observed action" means finding the corresponding control parameter, in an observer-based approach.

Considering a correspondance between two different control laws can be seen a modeling error. This modeling error can be seen as an unknown arbitrary perturbation on the modeled system, or an unknown input sent to the observed system. We developed an observer that can cancel the effects of unknown inputs on the dynamics of discrete time linear systems with unknown inputs. To do so, the observer has to satisfy a delayed invertibility condition and use delayed outputs. In other words, the observer has to wait for several measurement after a given instant to collect enough data to reconstruct the state at that instant.

6.4.2. Hierarchic QP solver

Participant: Pierre-Brice Wieber.

We are working in collaboration with the LAAS-CNRS and the CEA-LIST on solving multi-objective Quadratic Programs with Lexicographic ordering: Hierarchic QPs [25]. The focus this year has been on the regularization of the problem when the Quadratic Program approaches singularities. There is indeed a problem of discontinuity of the solution when reaching such singularities, what's not a rare event in robotic applications. This discontinuity has been related to the fact that Lexicograpically ordered QPs correspond to the limit of weighed multi-objective QPs when weights go singular, and that the regularization is itself a weighting of objectives which goes to a limit when approaching singularities, and those two limit processes interfere. The solution we found so far is to cancel the first limiting process and move back from strict Hierarchic QPs to weighted QPs staying at a small distance from singularity [33]. But this solution is not really satisfying and we have to find a better one.

6.4.3. Numerical modeling of muscle contraction under FES

Participant: Pierre-Brice Wieber.

We have been working in collaboration with the EPI DEMAR in Montpellier on modeling muscle contraction under Functional Electric Stimulation (FES). With respect to the literature in the domain, our contributions are mostly linked to the model of the contractile element, through the introduction of the recruitment at the fibre scale, formalizing the link between FES parameters, recruitment and Calcium signal paths. The resulting controlled model is able to reproduce both short term (twitch) and long term (tetanus) responses. It also matches some of the main properties of the dynamic behaviour of muscles, such as the Hill force-velocity relationship or the instantaneous stiffness of the Mirsky-Parmley model. The specific contribution of the BIPOP team has been on the numerical implementation of the contraction model as a Linear Complementarity Problem (LCP) allowing fast and precise numerical simulations [22].

6.4.4. Modeling of human balance in public transports

Participants: Pierre-Brice Wieber, Zohaib Aftab.

In our ongoing collaboration with the IFSTTAR (previously INRETS) on modeling human balance in public transports, we have aggregated biomechanical studies and numerical models proposed in robotics, and compared how they match or mismatch in situations of strong perturbations requiring a step to recover balance. We began developing a specific Model Predictive Control scheme for the prediction of recovery step locations with adaptive step timings, reproducing various balance recovery strategies as observed in humans. Initial results for stepping predictions have been validated against a balance recovery scenario found in the literature [45].

6.4.5. Model Predictive Control for Biped Walking

Participants: Pierre-Brice Wieber, Andrei Herdt, Jory Lafaye, François Keith.

We improved our Linear MPC-based walking motion generator by incorporating explicitly the robot's kinematic constraints: polyhedral constraints on the position of the CoM ensure the kinematic feasibility of the generated walking motions for arbitrary vertical motions of the CoM. This more precise kinematic model within the LMPC allowed considering toe rotations in a safer way, considerably improving energy efficiency, naturalness of the motion, and maximal speed.

We proposed a formulation of dynamic constraints for 3D motion through simple bounds on the variables, leading to faster resolution of the corresponding optimization problem. This allowed generating three-dimensional walking on non-planar ground in real-time. Thanks to specifically enforcing leg compliance, this scheme managed additionally to reproduce the natural profiles of the CoM and the contact forces observed in human walking.

We finally refined our numerical scheme for solving Linear MPC problems in walking motion generation. We switched the underlying QP solver to enable reductions of the number of iterations through warmstart, non-empty initial active sets, and obtaining feasible solutions at each iterations, what considerably reduced the computation time, allowing 1 ms feedback loops [30], [32].

6.5. Software development

6.5.1. MECHE toolbox

Participants: Florence Bertails-Descoubes, Gilles Daviet.

The main tool developed in 2011 in the MECHE software was the hybrid iterative solver for Coulomb friction, published in [21]. In 2011, the MECHE software was extensively used to validate this new solver on large data consisting of thousands interacting fibers (subject to tens of thousands frictional contacts). Code parallelization and optimization were performed so as to speed up computations.

6.5.2. Platform development: Siconos

Participants: Vincent Acary, Olivier Bonnefon, Maurice Brémond, Franck Pérignon.

The main achievements for the SICONOS platform are

1. Automatic serialization of the whole set of classes in SICONOS
2. Improvements and development of a full auto-generated Python wrapper in the SICONOS/FRONT-END
3. Development of the Siconos/Multi-body library and validation on industrial examples (C60 circuit breaker of Schneider Electric)
4. New algorithms for the resolution of the discrete frictional contact problem
5. Development of (θ/γ)-schemes for first order dynamical systems
6. Development of routines for sliding mode control

6.5.3. AMELIF framework

Participants: Pierre-Brice Wieber, François Keith, Jory Lafaye.

The main improvements to the AMELIF framework developed this year are:

- A new package specific to torque control has been developed, that contains the algorithms required to realize a given motion with a humanoid robot: estimation of contact forces and torque computation (feedforward), feedback methods ensuring the contact force convergence. These algorithms have been tested for two humanoid platforms: the robot Romeo and the robot HRP-2.
- The dynamics algorithm has been improved, based on the expertise coming from the HuMAnS toolbox. Besides, inverse dynamic algorithms and Runge Kutta integration methods have been added.
- Finally, the bridge with the stack-of-tasks framework [55], that computes the inverse kinematics and the inverse dynamics of humanoid systems, has been enhanced to handle the binding with Python. With this framework, it is possible to use the Model Predictive Control algorithm aforementioned and to simulate the behaviour of a humanoid in a dynamic simulation realized by AMELIF. Tests are still in progress.

7. Contracts and Grants with Industry

7.1. Contracts

Participants: Pierre-Brice Wieber, Florence Descoubes, Vincent Acary, Gilles Daviet, Olivier Bonnefon, Jerome Malick, Claude Lemarechal, Bernard Brogliato.

- ANR Saladyn: Numerical tools for simulating dynamics systems in mechanics; Partners: INRIA Bipop, LMGC Montpellier, EdF, Schneider Electric.
- FUI Romeo: Partners: Aldebaran Robotics, Acapela, As An Angel, INRIA, Institut de la Vision, LAAS, LIMSI, LIST, LISV, LPPA, Spirops, Telecom Paris Tech, Voxler. http://www.projetromeo.com/index_en.html
- ANR Multiple Impact: INRIA Bipop, Peking's university PKU (State Key Laboratory for Turbulence and Complex Systems).
- ANR ChaSlim: Chattering Free Sliding Mode Control; Partners: INRIA Bipop, INRIA Non-A, IRCCYN Nantes. http://chaslim.gforge.inria.fr/
- EdF: Documentation of the `noisedf` software
- L'OREAL: Contrat de recherche et de transfert with L'Oréal, performed from February to November 2011 for validating our model of frictional contact within large fiber assemblies.

8. Partnerships and Cooperations

8.1. European Initiatives

8.1.1. FP7 Projet

8.1.1.1. FlexibleRobotBehav

Title: FlexibleRobotBehavior

Type: FP7-PEOPLE-2007-4-1-IOF

Instrument: Marie Curie International Outgoing Fellowships for Career Development (IOF)

Duration: June 2008 - May 2011

Coordinator: INRIA (France)

Others partners: Japanese Advanced Institute for Science and Technology (AIST)

Abstract: The main objective of this research and training project is to enhance the algorithms and control laws of existing humanoid robots in order to obtain a walking behavior versatile and safe enough to be integrated into higher level tasks such as manipulation, vision, tele-operation, interaction with humans, which all require a strong capacity to face unforeseen events in an efficient way. And the objective is to solve this problem with a solution general enough to be applied also to the case of manipulator robots.

8.2. International Initiatives

8.2.1. INRIA Associate Teams

- 2009 -2011: SHARE associated team between INRIA (BiPop and EVASION) and the University of British Columbia (Canada).

9. Dissemination

9.1. Animation of the scientific community

- B. Brogliato member of International Program Committee of IFAC-ADHS 2012 (Eindhoven), CIFA 2012 (Grenoble); Guest Editor for the special issue Discontinuous Dynamical Systems of Mathematics and Computers in Simulation. reviewer for IEEE Transactions on Automatic Control, SIAM J. Control Optimization, Multibody System Dynamics, Asian J. of Control, European J. of Mechanics A/Solids, Automatica, Proc. Royal Society A.
- V. Acary is co-animator (with R. Leine ETH Zurich) of the European network for nonsooth dynamics. Member of the ENOC (EUROMECH Nonlinear Oscillations Conference) comittee. Reviewer in 2011 for IEEE Transactions on Automatic Control, Math reviews, ASME Journal of Applied Mechanics, Applied Numerical Mathemetics, American Control Conference
- F. Bertails-Descoubes has been a reviewer in 2011 for ACM SIGGRAPH, ACM SIGGRAPH Asia, Eurographics, ACM/EG Symposium on Computer Animation, Pacific Graphics and IEEE Transactions on Vizualization and Computer Graphics.

9.2. Teaching

Licence : F. Descoubes (Optimisation numérique, 22.5 equiv. TD), J. Malick (Optimisation numérique, 50 equiv. TD)

Master : B. Brogliato (Nonsmooth Dynamical Systems, Master 2 Acsyon, Université de Limoges, 13.5 equiv. TD)

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