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Project-Team CALVI

Scientific computation and visualization

IN COLLABORATION WITH: Institut Elie Cartan Nancy (IECN), Institut de recherche mathématique avancée (IRMA)

RESEARCH CENTER
Nancy - Grand Est

THEME
Computational models and simulation

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Project-Team CALVI

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2. Overall Objectives

2.1. Overall Objectives

CALVI was created in July 2003.
It is a project associating Institut Elie Cartan (IECN, UMR 7502, CNRS, INRIA and Université Henri Poincaré, Nancy), Institut de Recherche Mathématique Avancée (IRMA, UMR 7501, CNRS and Université de Strasbourg) and Laboratoire des Sciences de l'Image, de l'Informatique et de la Télédétection (LSIIT, UMR 7005, CNRS and Université de Strasbourg) with close collaboration to Laboratoire de Physique des Milieux Ionisés et Applications (LPMIA, UMR 7040, CNRS and Université Henri Poincaré, Nancy).

Our main working topic is modelling, numerical simulation and visualization of phenomena coming from plasma physics and beam physics. Our applications are characterized in particular by their large size, the existence of multiple time and space scales, and their complexity.

Different approaches are used to tackle these problems. On the one hand, we try and implement modern computing techniques like **parallel computing** and **grid computing** looking for appropriate methods and algorithms adapted to large scale problems. On the other hand we are looking for **reduced models** to decrease the size of the problems in some specific situations. Another major aspect of our research is to develop numerical methods enabling us to optimize the needed computing cost thanks to **adaptive mesh refinement** or **model choice**. Work in scientific visualization complement these topics including **visualization of multidimensional data** involving large data sets and **coupling visualization** and **numerical computing**.

3. Scientific Foundations

3.1. Kinetic models for plasma and beam physics

Plasmas and particle beams can be described by a hierarchy of models including N-body interaction, kinetic models and fluid models. Kinetic models in particular are posed in phase-space and involve specific difficulties. We perform a mathematical analysis of such models and try to find and justify approximate models using asymptotic analysis.

3.1.1. Models for plasma and beam physics

The **plasma state** can be considered as the **fourth state of matter**, obtained for example by bringing a gas to a very high temperature ($10^4\,K$ or more). The thermal energy of the molecules and atoms constituting the gas is then sufficient to start ionization when particles collide. A globally neutral gas of neutral and charged particles, called **plasma**, is then obtained. Intense charged particle beams, called nonneutral plasmas by some authors, obey similar physical laws.

The hierarchy of models describing the evolution of charged particles within a plasma or a particle beam includes N-body models where each particle interacts directly with all the others, kinetic models based on a statistical description of the particles and fluid models valid when the particles are at a thermodynamical equilibrium.

In a so-called *kinetic model*, each particle species s in a plasma or a particle beam is described by a distribution function $f_s(\mathbf{x}, \mathbf{v}, t)$ corresponding to the statistical average of the particle distribution in phase-space corresponding to many realisations of the physical system under investigation. The product $f_s \, d\mathbf{x} \, d\mathbf{v}$ is the average number of particles of the considered species, the position and velocity of which are located in a bin of volume $d\mathbf{x} \, d\mathbf{v}$ centered around $(\mathbf{x}, \mathbf{v})$. The distribution function contains a lot more information than what can be obtained from a fluid description, as it also includes information about the velocity distribution of the particles.

A kinetic description is necessary in collective plasmas where the distribution function is very different from the Maxwell-Boltzmann (or Maxwellian) distribution which corresponds to the thermodynamical equilibrium, otherwise a fluid description is generally sufficient. In the limit when collective effects are dominant with respect to binary collisions, the corresponding kinetic equation is the *Vlasov equation*

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0,$$

which expresses that the distribution function f is conserved along the particle trajectories which are determined by their motion in their mean electromagnetic field. The Vlasov equation which involves a self-consistent electromagnetic field needs to be coupled to the Maxwell equations in order to compute this field

$$\begin{aligned} -\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{B} &= \mu_0 \, \mathbf{J}, \\ \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} &= 0, \\ \div \mathbf{E} &= \frac{\rho}{\varepsilon_0}, \\ \div \mathbf{B} &= 0, \end{aligned}$$

which describes the evolution of the electromagnetic field generated by the charge density

$$\rho(\mathbf{x}, t) = \sum_s q_s \int f_s(\mathbf{x}, \mathbf{v}, t) \, d\mathbf{v},$$

and current density

$$\mathbf{J}(\mathbf{x}, t) = \sum_s q_s \int f_s(\mathbf{x}, \mathbf{v}, t) \mathbf{v} \, d\mathbf{v},$$

associated to the charged particles.

When binary particle-particle interactions are dominant with respect to the mean-field effects then the distribution function f obeys the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} = Q(f, f),$$

where Q is the nonlinear Boltzmann collision operator. In some intermediate cases, a collision operator needs to be added to the Vlasov equation.

The numerical solution of the three-dimensional Vlasov-Maxwell system represents a considerable challenge due to the huge size of the problem. Indeed, the Vlasov-Maxwell system is nonlinear and posed in phase space. It thus depends on seven variables: three configuration space variables, three velocity space variables and time, for each species of particles. This feature makes it essential to use every possible option to find a reduced model wherever possible, in particular when there are geometrical symmetries or small terms which can be neglected.

3.1.2. Mathematical and asymptotic analysis of kinetic models

The mathematical analysis of the Vlasov equation is essential for a thorough understanding of the model as well for physical as for numerical purposes. It has attracted many researchers since the end of the 1970s. Among the most important results which have been obtained, we can cite the existence of strong and weak solutions of the Vlasov-Poisson system by Horst and Hunze [76], see also Bardos and Degond [58]. The existence of a weak solution for the Vlasov-Maxwell system has been proved by Di Perna and Lions [65]. An overview of the theory is presented in a book by Glassey [73].

Many questions concerning for example uniqueness or existence of strong solutions for the three-dimensional Vlasov-Maxwell system are still open. Moreover, their is a realm of approached models that need to be investigated. In particular, the Vlasov-Darwin model for which we could recently prove the existence of global solutions for small initial data [59].

On the other hand, the asymptotic study of the Vlasov equation in different physical situations is important in order to find or justify reduced models. One situation of major importance in tokamaks, used for magnetic fusion as well as in atmospheric plasmas, is the case of a large external magnetic field used for confining the particles. The magnetic field tends to incurve the particle trajectories which eventually, when the magnetic field is large, are confined along the magnetic field lines. Moreover, when an electric field is present, the particles drift in a direction perpendicular to the magnetic and to the electric field. The new time scale linked to the cyclotron frequency, which is the frequency of rotation around the magnetic field lines, comes in addition to the other time scales present in the system like the plasma frequencies of the different particle species. Thus, many different time scales as well as length scales linked in particular to the different Debye length are present in the system. Depending on the effects that need to be studied, asymptotic techniques allow to find reduced models. In this spirit, in the case of large magnetic fields, recent results have been obtained by Golse and Saint-Raymond [74], [79] as well as by Brenier [63]. Our group has also contributed to this problem using homogenization techniques to justify the guiding center model and the finite Larmor radius model which are used by physicist in this setting [70], [68], [69].

Another important asymptotic problem yielding reduced models for the Vlasov-Maxwell system is the fluid limit of collisionless plasmas. In some specific physical situations, the infinite system of velocity moments of the Vlasov equations can be closed after a few of those, thus yielding fluid models.

3.2. Development of simulation tools

The development of efficient numerical methods is essential for the simulation of plasmas and beams. Indeed, kinetic models are posed in phase space and thus the number of dimensions is doubled. Our main effort lies in developing methods using a phase-space grid as opposed to particle methods. In order to make such methods efficient, it is essential to consider means for optimizing the number of mesh points. This is done through different adaptive strategies. In order to understand the methods, it is also important to perform their mathematical analysis. Since a few years we are interested also with solvers that uses Particle In Cell method. This new issue allows us to enrich some parts of our research activities previously centered on the Semi-Lagrangian approach.

3.2.1. Introduction

The numerical integration of the Vlasov equation is one of the key challenges of computational plasma physics. Since the early days of this discipline, an intensive work on this subject has produced many different numerical schemes. One of those, namely the Particle-In-Cell (PIC) technique, has been by far the most widely used.

Indeed it belongs to the class of Monte Carlo particle methods which are independent of dimension and thus become very efficient when dimension increases which is the case of the Vlasov equation posed in phase space. However these methods converge slowly when the number of particles increases, hence if the complexity of grid based methods can be decreased, they can be the better choice in some situations. This is the reason why one of the main challenges we address is the development and analysis of adaptive grid methods.

3.2.2. Convergence analysis of numerical schemes

Exploring grid based methods for the Vlasov equation, it becomes obvious that they have different stability and accuracy properties. In order to fully understand what are the important features of a given scheme and how to derive schemes with the desired properties, it is essential to perform a thorough mathematical analysis of this scheme, investigating in particular its stability and convergence towards the exact solution.

3.2.3. The semi-Lagrangian method

The semi-Lagrangian method consists in computing a numerical approximation of the solution of the Vlasov equation on a phase space grid by using the property of the equation that the distribution function f is conserved along characteristics. More precisely, for any times s and t, we have

$$f(\mathbf{x},\mathbf{v},t) = f(\mathbf{X}(s;\mathbf{x},\mathbf{v},t),\mathbf{V}(s;\mathbf{x},\mathbf{v},t),s),$$

where $(\mathbf{X}(s;\mathbf{x},\mathbf{v},t),\mathbf{V}(s;\mathbf{x},\mathbf{v},t))$ are the characteristics of the Vlasov equation which are solution of the system of ordinary differential equations

$$\begin{aligned} \frac{d\mathbf{X}}{ds} &= \mathbf{V}, \\ \frac{d\mathbf{V}}{ds} &= \mathbf{E}(\mathbf{X}(s),s) + \mathbf{V}(s) \times \mathbf{B}(\mathbf{X}(s),s), \end{aligned} \tag{1}$$

with initial conditions $\mathbf{X}(t) = \mathbf{x}$, $\mathbf{V}(t) = \mathbf{v}$.

From this property, f^n being known one can induce a numerical method for computing the distribution function f^{n+1} at the grid points $(\mathbf{x}_i,\mathbf{v}_j)$ consisting in the following two steps:

1. For all i,j, compute the origin of the characteristic ending at $\mathbf{x}_i,\mathbf{v}_j$, i.e. an approximation of $\mathbf{X}(t_n;\mathbf{x}_i,\mathbf{v}_j,t_{n+1})$, $\mathbf{V}(t_n;\mathbf{x}_i,\mathbf{v}_j,t_{n+1})$.
2. As

$$f^{n+1}(\mathbf{x}_i,\mathbf{v}_j) = f^n(\mathbf{X}(t_n;\mathbf{x}_i,\mathbf{v}_j,t_{n+1}),\mathbf{V}(t_n;\mathbf{x}_i,\mathbf{v}_j,t_{n+1})),$$

   f^{n+1} can be computed by interpolating f^n which is known at the grid points at the points $\mathbf{X}(t_n;\mathbf{x}_i,\mathbf{v}_j,t_{n+1}),\mathbf{V}(t_n;\mathbf{x}_i,\mathbf{v}_j,t_{n+1})$.

This method can be simplified by performing a time-splitting separating the advection phases in physical space and velocity space, as in this case the characteristics can be solved explicitly.

3.2.4. Adaptive semi-Lagrangian methods

Uniform meshes are most of the time not efficient to solve a problem in plasma physics or beam physics as the distribution of particles is evolving a lot as well in space as in time during the simulation. In order to get optimal complexity, it is essential to use meshes that are fitted to the actual distribution of particles. If the global distribution is not uniform in space but remains locally mostly the same in time, one possible approach could be to use an unstructured mesh of phase space which allows to put the grid points as desired. Another idea, if the distribution evolves a lot in time is to use a different grid at each time step which is easily feasible with a semi-Lagrangian method. And finally, the most complex and powerful method is to use a fully adaptive mesh which evolves locally according to variations of the distribution function in time. The evolution can be based on a posteriori estimates or on multi-resolution techniques.

3.2.5. Particle-In-Cell codes

The Particle-In-Cell method [62] consists in solving the Vlasov equation using a particle method, i.e. advancing numerically the particle trajectories which are the characteristics of the Vlasov equation, using the equations of motion which are the ordinary differential equations defining the characteristics. The self-fields are computed using a standard method on a structured or unstructured grid of physical space. The coupling between the field solve and the particle advance is done on the one hand by depositing the particle data on the grid to get the charge and current densities for Maxwell's equations and, on the other hand, by interpolating the fields at the particle positions. This coupling is one of the difficult issues and needs to be handled carefully.

3.2.6. Maxwell's equations in singular geometry

The solutions to Maxwell's equations are *a priori* defined in a function space such that the curl and the divergence are square integrable and that satisfy the electric and magnetic boundary conditions. Those solutions are in fact smoother (all the derivatives are square integrable) when the boundary of the domain is smooth or convex. This is no longer true when the domain exhibits non-convex *geometrical singularities* (corners, vertices or edges).

Physically, the electromagnetic field tends to infinity in the neighbourhood of the re-entrant singularities, which is a challenge to the usual finite element methods. Nodal elements cannot converge towards the physical solution. Edge elements demand considerable mesh refinement in order to represent those infinities, which is not only time- and memory-consuming, but potentially catastrophic when solving time dependent equations: the CFL condition then imposes a very small time step. Moreover, the fields computed by edge elements are discontinuous, which can create considerable numerical noise when the Maxwell solver is embedded in a plasma (e.g. PIC) code.

In order to overcome this dilemma, a method consists in splitting the solution as the sum of a *regular* part, computed by nodal elements, and a *singular* part which we relate to singular solutions of the Laplace operator, thus allowing to calculate a local analytic representation. This makes it possible to compute the solution precisely without having to refine the mesh.

This *Singular Complement Method* (SCM) had been developed [57] and implemented [56] in plane geometry.

An especially interesting case is axisymmetric geometry. This is still a 2D geometry, but more realistic than the plane case; despite its practical interest, it had been subject to much fewer theoretical studies [60]. The non-density result for regular fields was proven [64], the singularities of the electromagnetic field were related to that of modified Laplacians [53], and expressions of the singular fields were calculated [54]. Thus the SCM was extended to this geometry. It was then implemented by F. Assous (now at Bar-Ilan University, Israel) and S. Labrunie in a PIC–finite element Vlasov–Maxwell code [55].

As a byproduct, space-time regularity results were obtained for the solution to time-dependent Maxwell's equation in presence of geometrical singularities in the plane and axisymmetric cases [72], [54].

3.3. Large size problems

3.3.1. Introduction

The applications we consider lead to very large size computational problems for which we need to apply modern computing techniques enabling to use efficiently many computers including traditional high performance parallel computers and computational grids.

The full Vlasov-Maxwell system yields a very large computational problem mostly because the Vlasov equation is posed in six-dimensional phase-space. In order to tackle the most realistic possible physical problems, it is important to use all the modern computing power and techniques, in particular parallelism and grid computing.

3.3.2. Parallelization of numerical methods

An important issue for the practical use of the methods we develop is their parallelization. We address the problem of tuning these methods to homogeneous or heterogeneous architectures with the aim of meeting increasing computing resources requirements.

Most of the considered numerical methods apply a series of operations identically to all elements of a geometric data structure: the mesh of phase space. Therefore these methods intrinsically can be viewed as a data-parallel algorithm. A major advantage of this data-parallel approach derives from its scalability. Because operations may be applied identically to many data items in parallel, the amount of parallelism is dictated by the problem size.

Parallelism, for such data-parallel PDE solvers, is achieved by partitioning the mesh and mapping the submeshes onto the processors of a parallel architecture. A good partition balances the workload while minimizing the communications overhead. Many interesting heuristics have been proposed to compute near-optimal partitions of a (regular or irregular) mesh. For instance, the heuristics based on space-filing curves [75] give very good results for a very low cost.

Adaptive methods include a mesh refinement step and can highly reduce memory usage and computation volume. As a result, they induce a load imbalance and require to dynamically distribute the adaptive mesh. A problem is then to combine distribution and resolution components of the adaptive methods with the aim of minimizing communications. Data locality expression is of major importance for solving such problems. We use our experience of data-parallelism and the underlying concepts for expressing data locality [80], optimizing the considered methods and specifying new data-parallel algorithms.

As a general rule, the complexity of adaptive methods requires to define software abstractions allowing to separate/integrate the various components of the considered numerical methods (see [78] as an example of such modular software infrastructure).

Another key point is the joint use of heterogeneous architectures and adaptive meshes. It requires to develop new algorithms which include new load balancing techniques. In that case, it may be interesting to combine several parallel programming paradigms, i.e. data-parallelism with other lower-level ones.

Moreover, exploiting heterogeneous architectures requires the use of a run time support associated with a programming interface that enables some low-level hardware characteristics to be unified. Such run time support is the basis for heterogeneous algorithmics. Candidates for such a run time support may be specific implementations of MPI such as MPICH-G2 (a grid-enabled MPI implementation on top of the GLOBUS tool kit for grid computing [67]).

Our general approach for designing efficient parallel algorithms is to define code transformations at any level. These transformations can be used to incrementally tune codes to a target architecture and they warrant code reusability.

4. Application Domains

4.1. Thermonuclear fusion

Controlled fusion is one of the major prospects for a long term source of energy. Two main research directions are studied: magnetic fusion where the plasma is confined in tokamaks using a large external magnetic field and inertial fusion where the plasma is confined thanks to intense laser or particle beams. The simulation tools we develop can be applied for both approaches.

Controlled fusion is one of the major challenges of the 21st century that can answer the need for a long term source of energy that does not accumulate wastes and is safe. The nuclear fusion reaction is based on the fusion of atoms like Deuterium and Tritium. These can be obtained from the water of the oceans that is widely available and the reaction does not produce long-term radioactive wastes, unlike today's nuclear power plants which are based on nuclear fission.

Two major research approaches are followed towards the objective of fusion based nuclear plants: magnetic fusion and inertial fusion. In order to achieve a sustained fusion reaction, it is necessary to confine sufficiently the plasma for a long enough time. If the confinement density is higher, the confinement time can be shorter but the product needs to be greater than some threshold value.

The idea behind magnetic fusion is to use large toroidal devices called tokamaks in which the plasma can be confined thanks to large applied magnetic field. The international project ITER[1] is based on this idea and aims to build a new tokamak which could demonstrate the feasibility of the concept.

The inertial fusion concept consists in using intense laser beams or particle beams to confine a small target containing the Deuterium and Tritium atoms. The Laser Mégajoule which is being built at CEA in Bordeaux will be used for experiments using this approach.

Nonlinear wave-wave interactions are primary mechanisms by which nonlinear fields evolve in time. Understanding the detailed interactions between nonlinear waves is an area of fundamental physics research in classical field theory, hydrodynamics and statistical physics. A large amplitude coherent wave will tend to couple to the natural modes of the medium it is in and transfer energy to the internal degrees of freedom of that system. This is particularly so in the case of high power lasers which are monochromatic, coherent sources of high intensity radiation. Just as in the other states of matter, a high laser beam in a plasma can give rise to stimulated Raman and Brillouin scattering (respectively SRS and SBS). These are three wave parametric instabilities where two small amplitude daughter waves grow exponentially at the expense of the pump wave, once phase matching conditions between the waves are satisfied and threshold power levels are exceeded. The illumination of the target must be uniform enough to allow symmetric implosion. In addition, parametric instabilities in the underdense coronal plasma must not reflect away or scatter a significant fraction of the incident light (via SRS or SBS), nor should they produce significant levels of hot electrons (via SRS), which can preheat the fuel and make its isentropic compression far less efficient. Understanding how these deleterious parametric processes function, what non uniformities and imperfections can degrade their strength, how they saturate and interdepend, all can benefit the design of new laser and target configuration which would minimize their undesirable features in inertial confinement fusion. Clearly, the physics of parametric instabilities must be well understood in order to rationally avoid their perils in the varied plasma and illumination conditions which will be employed in the National Ignition Facility or LMJ lasers. Despite the thirty-year history of the field, much remains to be investigated.

Our work in modelling and numerical simulation of plasmas and particle beams can be applied to problems like laser-matter interaction, the study of parametric instabilities (Raman, Brillouin), the fast ignitor concept in the laser fusion research as well as for the transport of particle beams in accelerators. Another application is devoted to the development of Vlasov gyrokinetic codes in the framework of the magnetic fusion programme in collaboration with the Department of Research on Controlled Fusion at CEA Cadarache. Finally, we work in collaboration with the American Heavy Ion Fusion Virtual National Laboratory, regrouping teams from laboratories in Berkeley, Livermore and Princeton on the development of simulation tools for the evolution of particle beams in accelerators.

4.2. Nanophysics

[1] http://www.iter.org

Kinetic models like the Vlasov equation can also be applied for the study of large nano-particles as approximate models when ab initio approaches are too costly.

In order to model and interpret experimental results obtained with large nano-particles, ab initio methods cannot be employed as they involve prohibitive computational times. A possible alternative resorts to the use of kinetic methods originally developed both in nuclear and plasma physics, for which the valence electrons are assimilated to an inhomogeneous electron plasma. The LPMIA (Nancy) possesses a long experience on the theoretical and computational methods currently used for the solution of kinetic equation of the Vlasov and Wigner type, particularly in the field of plasma physics.

Using a Vlasov Eulerian code, we have investigated in detail the microscopic electron dynamics in the relevant phase space. Thanks to a numerical scheme recently developed by Filbet et al. [66], the fermionic character of the electron distribution can be preserved at all times. This is a crucial feature that allowed us to obtain numerical results over long times, so that the electron thermalization in confined nano-structures could be studied.

The nano-particle was excited by imparting a small velocity shift to the electron distribution. In the small perturbation regime, we recover the results of linear theory, namely oscillations at the Mie frequency and Landau damping. For larger perturbations nonlinear effects were observed to modify the shape of the electron distribution.

For longer time, electron thermalization is observed: as the oscillations are damped, the center of mass energy is entirely converted into thermal energy (kinetic energy around the Fermi surface). Note that this thermalization process takes place even in the absence of electron-electron collisions, as only the electric mean-field is present.

5. Software

5.1. SeLaLib

SeLaLib (the Semi-Lagrangian Library) is a library providing numerical methods for the kinetic models of plasma physics, in particular different types of Vlasov equations including the gyrokinetic model coupled to field solvers based on the Poisson, Maxwell or gyrokinetic quasi-neutraly equations. The PDEs are solved on structured mapped meshes or a collection of patches of such meshes, where the meshes are defined by a mapping for a cartesian logical grid. It is developed with an ADT and is strongly related to the INRIA large scale initiative Fusion.

One of its aims is to provide numerical building blocks for the GYSELA code developed at CEA Cadarache in collaboration with the Calvi project-team. GYSELA is used by physicists for simulating the development of turbulence in magnetic fusion plasmas in particular in view of the ITER project.

6. New Results

6.1. Mathematical analysis of kinetic models

Participants: Aurore Back, Nicolas Besse, Emmanuel Frénod, Mathieu Lutz.

6.1.1. Asymptotic analysis of gyrokinetic models

Proceeding [52] presents the method that allows us to get the gyrokinetic Approximation of the Dynamical System satisfied by the trajectory of a particle submitted to a Strong Magnetic Field. The goal of the method is to build a change of coordinates in order to make the fast dynamics of two components of the trajectory to disappear. This change of coordinates is based on a Darboux mathematical Algorithm and on a Lie Transform. It is the first work of a forthcoming series of papers which goal is to make the Geometrical gyrokinetic Approximation a mathematically affordable theory. Review paper [51] presents the results of Two-Scale Convergence Theory and an application to Homogenization of linear Singularly Perturbed Hyperbolic Partial Differential Equations. It consists in the theoretical basis of the Two-Scale Numerical Methods.

6.1.2. Two-scale convergence with differential form

In the framework of the thesis of Aurore Back [11], we developed at two-scale convergence theory using the tools of exterior calculus and differential forms. A geometric formulation of the Vlasov-Maxwell equations was introduced and some geometric conservation properties were proved.

6.1.3. Analysis of multi-water-bag models

In the case of toroidal geometry, thanks to the strong anisotropy between parallel (to the magnetic field) and transverse direction, we could perform an asymptotic analysis of the eigenvalue problem for the integro-differential gyrowaterbag operator with two independent dimensions, the third being represented by Fourier toroidal modes. This analysis enabled us to reduce the two-dimensional integro-differential operator to a series of one-dimensional integro-differential operators the solution of which enables to obtain as well the poloidal as the radial envelope of the global eigenmodes as well as their local frequency. Note that the terms of the series can be computed numerically independently from each other as the differential variables decouple, which leads to an embarassingly parallel algorithm. The global dispersion relation for the global eigenfrequency appears as an integral quantification relation involving the local frequency due to the property of conservation of the action. On the other hand the mathematical analysis, in particular the spectral properties, of the obtained operators has been performed. Several mathematical results on the well-posedness of gyrowaterbag models have been obtained as an exact geometric reduction of the Vlasov equation, the notion of solution differing depending on the nature of the problem being considered [14], [15], [16].

6.2. Numerical analysis

Participants: Martin Campos Pinto, Nicolas Crouseilles, Michel Mehrenberger, Eric Sonnendrücker.

6.2.1. Analysis of numerical methods for the Vlasov-Poisson system

In [47], we derive the order conditions for fourth order time splitting schemes in the case of the $1D$ Vlasov-Poisson system. Computations to obtain such conditions are motivated by the specific Poisson structure of the Vlasov-Poisson system : this structure is similar to Runge-Kutta-Nyström systems. The obtained conditions are proved to be the same as RKN conditions derived for ODE up to the fourth order. Numerical tests are performed and show the benefit of using high order splitting schemes in that context.

In [37], we prove enhanced error estimates for high order semi-lagrangian discretizations of the Vlasov-Poisson equation. It provides new insights into optimal numerical strategies for the numerical solution of this problem. The new error estimate $O(\min(\frac{\Delta x}{\Delta t}, 1)\Delta x^p + \Delta t^2)$ is based on advanced error estimates for semi-lagrangian schemes, also equal to shifted Strang schemes, for the discretization of the advection equation.

6.2.2. Analysis of a new particle method with deformable shapes

Particle methods are known to be simple and efficient in most practical cases, however they suffer from weak convergence properties: they only converge in a strong sense when the particles present an extended overlapping (i.e., when the number of overlapping particles tends to infinity as the mesh size h of their initialization grid tends to 0), and additional constraints such as vanishing moments. In practice, extended particle overlapping can be expensive and it involves an additional parameter to be optimized, such as the overlapping exponent $q < 1$ for which the particles radius behaves like h^q. In PIC codes for instance, extended overlapping requires increasing the number of particles per cell together with the number of cells, which determine the radius of the particles. In many practical cases such conditions are not met, which leads to strong oscillations in the solutions. To smooth out the oscillations some methods (like the Denavit redeposition scheme, recently revisited as a Forward semi-Lagrangian scheme) use periodic remappings, but frequent remappings introduce unwanted numerical diffusion which seems to contradict the benefit of using low-diffusion particle schemes. Moreover, the vanishing moment condition prevents high orders to be achieved with positive particles.

In [44] we present a new class of particle methods with deformable shapes for transport problems that converge in the supremum norm without requiring remappings, extended overlapping or vanishing moments for the particles. UIndeed, unlike the classical error analysis based on a smoothing kernel argument, our estimates hold for any particle collection with Lipschitz smoothness and compact supports that have the same scale than their initialization grid. Our results are threefold. On the theoretical side we first show that for arbitrarily smooth characteristic flow, high order convergence rates are obtained by deforming the particles with local polynomial mappings. On the practical side we provide an explicit implementation of the first order case: the resulting linearly-transformed particle (LTP) scheme consists of transporting the particle centers along the numerical flow, together with finite difference approximations of the local Jacobian matrices of the flow. For the fully discrete scheme we establish rigorous a priori error estimates and demonstrate the uniform boundedness of the particle overlapping. Finally, we describe an adaptive multilevel version of the LTP scheme that includes a local correction filter for positivity-preserving approximations.

In [45] we apply the LTP method to the 1+1d Vlasov-Poisson problem with a simple deposition scheme and show that deforming the particles helps removing the noise traditionally observed with standard PIC schemes.

6.2.3. Two-Scale Asymptotic-Preserving issues

In the submitted paper [48], we build a Two-Scale Macro-Micro decomposition of the Vlasov equation with a strong magnetic field. This consists in writing the solution of this equation as a sum of two oscillating functions with circonscribed oscillations. The first of these functions has a shape which is close to the shape of the Two-Scale limit of the solution and the second one is a correction built to offset this imposed shape. The aim of such a decomposition is to be the starting point for the construction of Two-Scale Asymptotic-Preserving Schemes.

The aim of using Two-Scale Asymptotic-Preserving Schemes is first, to deal efficiently with long time scales with solutions having high frequency oscillations and second, to manage the transition between different regimes, in a unified framework.

The aim of a new starting project is to test on a simplified model the Two-Scale Asymptotic-Preserving Schemes. The model, a two dimensional in phase space Vlasov-Poisson equation with small parameter, is used for a long time simulation of a beam in a focusing channel. This work was already done in [71] in the case where the solution is approximated by the two scale limit. The goals are first to improve this approximation, by going further, to the first order one, and secondly, to replace this approximation by an exact decomposition, using the macro-micro framework. This last approach will permit to treat the case of a not necessary small parameter.

In order to accomplish the first task we started to write a PIC code which is to be integrated in SeLaLib.

6.3. Development of numerical methods

Participants: Aurore Back, Nicolas Besse, Jean-Philippe Braeunig, Anaïs Crestetto, Emmanuel Frénod, Philippe Helluy, Sever Hirstoaga, Ahmed Ratnani, Eric Sonnendrücker.

6.3.1. Application of isogeometric analysis to plasma physics simulations

Mainly around the PhD thesis of Ahmed Ratnani [13] which has been defended in October 2011, we have been using the concept of isogeometric analysis introduced by Hughes and co-workers [77] which consists in representing the computational domain as well as the numerical solution of the equations with NURBS (Non Uniform Rational B-Splines).

In [26] we introduced a time-domain conforming Finite Element solver using arbitrary order B-Splines as basis functions. The discrete function spaces used in the Finite Element formulation form a De Rham sequence, which has proved to be an important property for numerical Maxwell solvers. In particular, they allow to have a simple relation between spline coefficients of the magnetic and electric field that is independent on order and geometry for one of Ampère's or Faraday's law. The other then necessarily involves a discrete Hodge operator which depends on order and geometry. High-order energy conserving leap-frog schemes have been validated with this solver.

In [21], we developed an arbitrary order B-Spline Finite Element solver for the quasi-neutrality equation that is generally coupled to gyrokinetic Vlasov-solvers. Compared to the previous solver used in GYSELA which was spectral in the angular variable and second order Finite Differences in the radial variable this solver can be of high-order in both direction. This enables us for a given accuracy to decrease the number of grid points. Moreover, thanks to the periodicity in the angular variable and the tensor product structure of the problem, we could introduce a fast diagonalization method using a FFT such that the cost of the new solver only marginally depends on the order and is only slightly higher than the cost of the previously used method. Another important new algorithm introduced in this work is the decoupling of the parallel and transverse parts of the equation, by solving successively for the average value along the parallel direction and the remaining part.

In [28] we present an axisymmetric PIC code based on isogeometric analysis, which was initially the IsoPIC project (supported by CEA Gramat) of CEMRACS 2010. The goal of this study is to use it for solving the system of Vlasov-Maxwell equations. The idea is to develop an axisymmetric Finite Element PIC (Particle-In-Cell) code in which specific spline Finite Elements are used to solve the Maxwell equations (in 2D transverse electric mode) and the same spline functions serve as shape function for the particles. The computational domain itself is defined using splines or NURBS. We are in particular interested in the emission of electrons in a diode with hemispherical cathode (thanks to symmetry in θ direction, we can consider the 2D axisymmetric geometry).

6.3.2. Spline discrete differential forms

In [11], [36] we have developed the concept of spline discrete differential forms which can be used to discretized equations defined using the notions of exterior calculus. These have been applied for the numerical solution of the Maxwell and the Vlasov equations. Hodge operators either using a dual grid or a weak formulation are developed and commuting diagram properties are proved.

6.3.3. Drift-kinetic simulations

We implemented the conservative semi-Lagrangian method in the GYSELA code which is based on the classical backward semi-Lagrangian method which is not exactly conservative. We noticed that for the conservative method it is essential that the advection field remains numerically exactly divergence free in order to avoid numerical instabilities. In addition specific limiters for the conservative method were developed and comparison between the backward semi-Lagrangian method, the conservative semi-Lagrangian method with 1D splitting and the same method with an unsplit Finite Volume like formulation in the (r, θ) plane which provides a better conservation of volume [43].

6.3.4. Waterbag simulations

In [31] we apply the multi-water-bag model and the method of moments to the Vlasov-Poisson system in a case where the solution becomes multivalued. The motivation of this study is that the kinetic Vlasov-Poisson model is very expensive to solve numerically. It can be approximated by a multi-water-bag model in order to reduce the complexity. This model amounts to solve a set of Burgers equations, which can be done easily by finite volume methods. However, the physical solution can become multivalued (filamentation appears). In this case, shocks appear in the simulation and we lose information about the filaments. To catch them, we can use a moment method. We describe here the two models and present several numerical experiments.

A linear analysis code CYLGYR based on the gyrowaterbag model in cylindrical geometry has been developed. It enables, starting from a given equilibrium configuration, to obtain the whole set of modes that can exist. It was used to validate the linear phase of the previously developed non linear semi-lagrangian gyrowaterbag code in cylindrical geometry GMWB3D-SL [61]. Excellent agreement for the growth rates (eigenvalues) and the radial envelopes (eigenfunctions) has been obtained for global eigenmodes. On the other hand the linear code CYLGYR gives results in excellent agreement with those given by the linear kinetic code KINEZERO. We are now using as well the linear gyrowaterbag code CYLGYR and the non linear code GMWB3D-SL to compare the quasi-linear and non-linear fluxes and thus measure the validity of the quasi-linear approach for gyrokinetic turbulence. Such comparisons will also be performed with the gyrokinetic code GYSELA in cylindrical geometry.

6.3.5. Validation of the quasi-linear theory

We have developed and optimized a parallel semi-Lagrangian code for the numerical resolution of the Hamiltonian Vlasov-wave model in two phase-space dimensions. Using this code to perform a statistical study on a large number of runs of the system we have showed that the quasi-linear theory, whose aim is to justify the approximation of a self-coherent hamiltonian system like the Vlasov-Poisson model by diffusive self-coherent model of Fokker-Planck type, was valid in the strongly chaotic non linear regime of 1D electrostatic turbulence provided it is regarded from a statistical point of view [17].

6.4. Plasma-wall interactions - application to ELM modes on JET

Participants: Sever Hirstoaga, Giovanni Manfredi.

The aim of [24] was to model the effect of energetic charged particles (generated during violent events known as edge-localized modes) on the divertor plates of a tokamak. We thus have developed a 1D Vlasov-Poisson code with open boundaries for both ions and electrons. This work was already described in last year's report.

In the proceedings [33] we compare the numerical results previously obtained with the Eulerian code (in [24]) to those obtained with two different aproaches: a PIC code (developed at the University of Innsbruck) and a fluid code (developed at Culham, UK) which is based on a set of Branginskii-type equations. These comparisons show a very good agreement between Eulerian and PIC codes when computing the energy fluxes of both species, while the fluid code overestimates these fluxes. We also note that both the Eulerian and PIC codes yield similar results for the early burst of electrons, whereas the fluid code is not able to reproduce this effect, thus confirming its kinetic origin.

6.5. Full wave modeling of lower hybrid current drive in tokamaks

Participants: Pierre Bertrand, Simon Labrunie, Takashi Hattori, Jean Rodolphe Roche.

This work is performed in collaboration with Yves Peysson (DRFC, CEA Cadarrache).

The aim of this project is to develop a finite element numerical method for the full-wave simulation of electromagnetic wave propagation in aplasma. Full-wave calculations of the LH wave propagation is a challenging issue because of the short wave length with respect to the machine size. In the continuation of the works previously led in cylindrical geometry, a full toroidal description for an arbitrary poloidal cross-section of the plasma has been developed.

Since its wavelength λ at the LH frequency is very small compared to the machine size R, a conventional full wave description represents a considerable numerical effort. Therefore, the problem is addressed by an appropriate mathematical finite element technique, which incorporates naturally parallel processing capabilities. This is particularly important aspect when simulations for plasmas of large size must be considered. It is based on a mixed augmented variational (weak) formulation taking account of the divergence constraint and essential boundary conditions, which provides an original and efficient scheme to describe in a global manner both propagation and absorption of electromagnetic waves in plasmas.

With such a description, usual limitations of the conventional ray tracing related to the approximation $\lambda << \phi_B << R$, where ϕ_B is the size of the beam transverse to the rf power flow direction, may be overcome. Since conditions are corresponding to $\lambda << \phi_B \sim R$, the code under development may be considered as a WKB full wave, dielectric properties being local.

The domain considered is as near as possible of the cavity filled by a tokomak plasma. Toroidal coordinates are introduced. In our approach we consider Fourier decomposition in the angular coordinate to obtain stationary Maxwell equations in a cross-section of the tokamak cavity.

A finite element method is proposed for the simulation of time-harmonic electromagnetic waves in a plasma, which is an anisotropic medium. The approach chosen here is sometimes referred to as *full-wave modeling* in the literature: the original Maxwell's equations are used to obtain a second order equation for the time-harmonic electric field. These are written in a weak form using an augmented variational formulation (AVF), which takes into account the divergence. The variational formulation is then discretized using modified Taylor-Hood (nodal) elements.

During 2011 we introduced a new boundary condition in order to take account of the antenna and essential condition are considered in the code "FullWaveFEM" and new real case was considered.

6.6. Domain decomposition for the resolution of nonlinear equations

Participant: Jean Rodolphe Roche.

This a joint work with Noureddine Alaa, Professor at the Marrakech Cadi Ayyad University.

Strongly problems of parabolic equations have received considerable attentions, and various forms of this problems have been proposed in the literature, especially in the area of reaction-diffusion equations with cross-diffusion, such problems arise from biological, chemical and physical systems. Various methods have been proposed in the mathematical literature to study the existence, uniqueness and compute numerical approximation of solutions for quasi-linear partial differential equation problems. This year our we develop a numerical method to solve periodic non linear parabolic equations based on domain decomposition and optimization interior points method, see [34].

6.7. Inverse problem governed by Maxwell equations

Participant: Jean Rodolphe Roche.

This work is performed in collaboration with Jose Herskovits Norman of UFRJ, Rio de Janeiro, Antonio André Novotny from the LNCC, Petropolis, both from Brazil and Alfredo Canelas from the University of the Republic, Montevideo, Uruguay.

The industrial technique of electromagnetic casting allows for contactless heating, shaping and controlling of chemical aggressive, hot melts. The main advantage over the conventional crucible shape forming is that the liquid metal does not come into contact with the crucible wall, so there is no danger of contamination. This is very important in the preparation of very pure specimens in metallurgical experiments, as even small traces of impurities, such as carbon and sulphur, can affect the physical properties of the sample. Industrial applications are, for example, electromagnetic shaping of aluminum ingots using soft-contact confinement of the liquid metal, electromagnetic shaping of components of aeronautical engines made of superalloy materials (Ni,Ti, ...), control of the structure solidification.

The electromagnetic casting is based on the repulsive forces that an electromagnetic field produces on the surface of a mass of liquid metal. In the presence of an induced electromagnetic field, the liquid metal changes its shape until an equilibrium relation between the electromagnetic pressure and the surface tension is satisfied. The direct problem in electromagnetic casting consists in determining the equilibrium shape of the liquid metal. In general, this problem can be solved either directly studying the equilibrium equation defined on the surface of the liquid metal, or minimizing an appropriate energy functional. The main advantage of this last method is that the resulting shapes are mechanically stable.

The inverse problem consists in determining the electric currents and the induced exterior field for which the liquid metal takes on a given desired shape. This is a very important problem that one needs to solve in order to define a process of electromagnetic liquid metal forming.

In a previous work we studied the inverse electromagnetic casting problem considering the case where the inductors are made of single solid-core wires with a negligible area of the cross-section. In a second paper we considered the more realistic case where each inductor is a set of bundled insulated strands. In both cases the number of inductors was fixed in advance. This year we aim to overcome this constraint, and look for configurations of inductors considering different topologies with the purpose of obtaining better results. In order to manage this new situation we introduce a new formulation for the inverse problem using a shape functional based on the Kohn-Vogelius criterion. A topology optimization procedure is defined by means of topological derivatives, see [18] and [30]. To take account the free boundary evolution we consider a new level set method adapted to topological first and second order asymtotic topological analysis.

6.8. Diffusion of knowledge and methods towards other fields

Participant: Emmanuel Frénod.

Methods, results and more generally knowledge produced within Calvi team have been applied to environmental sciences In [22] and [50] asymptotic methods initially designed for tokamak plasmas are applied to coastal ocean waters linked phenomena.

[38] deals with the concept of confinement of paralic ecosystems. It improves an existing model in order to account for tide oscillations in any kind of geometry such as a non-rectangular lagoons with a non-flat bottom. The model, that relies on PDEs is then implemented thanks to the finite element method. Numerical results confirm the feasibility of confinement studies thanks to the introduced model.

Methods for mass transfer modeling was applied in the haulage context in [49].

7. Partnerships and Cooperations

7.1. National Initiatives

7.1.1. CEA Projects

- FR FCM (CNRS Federation on Magnetic Confinement Fusion) project within Euratom-CEA association, Title:"Full wave modeling of lower hybrid current drive in tokamaks" The goal of this projet is to develop a full wave method to describe the dynamics of lower hybrid current drive problem in tokamaks.
- FR FCM (CNRS Federation on Magnetic Confinement Fusion) project within Euratom-CEA association, Title:"Numerical Methods for GYSELA", the goal is to help improving the numerical algorithms used by the GYSELA code developed at CEA Cadarache for the simulation of turbulence in magnetic fusion plasmas.

7.1.2. ANR projects

- GYPSI project (2010–2014), https://sites.google.com/site/anrgypsi/: coordinator Philippe Ghendrih (CEA Cadarache), other participants, University of Marseille, Universities of Strasbourg and Nancy (CALVI project-team). The aim is to understand the physics of turbulence in magnetically confined plasma using numerical simulation.

7.1.3. INRIA initiatives

- Large scale Initiative FUSION (2009–2012), http://www-math.u-strasbg.fr/ae_fusion: Modeling and numerical simulation of magnetic fusion plasmas in view of the ITER project.

7.2. European Initiatives

7.2.1. Major European Organizations with which you have followed Collaborations

Claus-Dieter Munz: University of Stuttgart, IAG (Germany)

Development of particle in cell methods for the Vlasov-Maxwell equations

7.3. International Initiatives

7.3.1. Participation In International Programs

J. R. Roche participated in the CAPES-COFECUB binational project with the COPPE-Federal University of Rio de Janeiro and the National Laboratory of Scientific Computing of Brazil.

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Invitations at conferences and summer schools

- Emmanuel Frénod (http://www-labsticc.univ-ubs.fr/~frenod/) was invited to give lectures in
  - Cemracs 2011 Summer School on "Two Scale Convergence" at Cirm, Marseille in July 2011 (http://smai.emath.fr/cemracs/cemracs11/),
  - Fusion Summer School on the "Gyro-Kinetic Approximation" in Paris in September 2011(http://www.ljll.math.upmc.fr/charles/fusion/index.html).
- Eric Sonnendrücker gave invited talks
  - Lawrence Berkeley National Laboratory (USA), March 16, "Semi-Lagrangian methods for the Vlasov equation",
  - LawrenceLivermore National Laboratory (USA), March 17, "Gyrokinetic simulations",
  - i4energy seminar, University of California at Berkeley, March 18, "High performance computing challenges for magnetic fusion"
  - International Conference on the Numerical Simulation of Plasmas, Long Branch (USA), September 6-9, "Semi-Lagrangian methods for the Vlasov equation",
  - mini-course of 6 hours at ICERM, Brown university (USA), September 12-16, "Vlasov simulations",
  - Vlasov Workshop, ICERM, Brown university (USA), September 19-23, "gyrokinetic simulations",
  - NELIA workshop, Santiago de Compostella (Spain), October 25-28, "Arbitrary order discrete differential forms for Maxwell's equations",
  - Case studies, ETH Zurich (Switzerland), November 24, "Modelling and simulation of particle accelerators".

8.1.2. Administrative duties

- Emmanuel Frénod, Michael Gutnic, Michel Mehrenberger, Eric Sonnendrücker
  - were member of selection committee for an Assistant Professor of Applied Maths position at Strasbourg.
- Eric Sonnendrücker
  - was a member of the AERES committee of M2P2 laboratory in Marseille
  - was a member of the AERES committee of mathematics laboratory of Université Technologique de Compiègne
  - was a member of CNU section 26 (applied mathematics)
  - is a member of the scientific committee of CIRM (Centre International de Rencontres Mathématiques) at Luminy,
  - was a member of a selection committee for a Professor position in Applied Mathematics in Marseille
  - was a member of a selection committee for a Professor position in Applied Mathematics in Lyon.

8.1.3. Organization of conferences

- Michael Gutnic and Philippe Helluy were co-organizers of CEMRACS 2011 at Luminy (France).
- Eric Sonnendrücker was a member of the scientific committee for the Fusion Summer School, Paris, September 26-30.

8.2. Teaching

Nicolas Besse

Licence: Complex analysis 18h, L3, UHP Licence: Analysis 80H L2, Université Henri Poincaré, France

Licence: Integration and differential form, 35h, L1, Université Henri Poincaré, France

Licence: Colles d'analyse 16h, L1, Université Henri Poincaré, France

Master: Plasma modelling and numerical methods 30h, M2, Université Henri Poincaré, France

Sever Hirstoaga

Analyse numérique, 70 h, L3, l'UFR Physique et Ingénierie, Université de Strasbourg, France

Simon Labrunie

Licence : Mathématiques générales, 160h, L1, Université Henri Poincaré Nancy, France

Licence : Mathématiques générales, 48h, L2, Université Henri Poincaré Nancy, France

Vladimir Latocha

Découverte des mathématiques, 81 h, L1, Nancy université, France

Méthodologie de l'exposé écrit et oral, 30 heures, L1, Nancy université, France

Analyse numérique, 12 h, L3, Nancy université, France

Algorithmique, 52h, L3, Nancy université, France

Analyse complexe, 10 h, L3, ENSEM, France

Mathématiques PACES UE4, 15h, L1, Nancy université, France

Livre pour la première année de santé, à paraître : Mathématiques UE4, éd. Ellipses, à paraître en décembre 2011.

Michel Mehrenberger

Licence : Optimisation non lineaire, 54h, L3, Université de Strasbourg, France

Licence : Méthodes d'Analyse Numérique, 39h, L3, ENSIIE (ecole d'ingenieur, antenne de Strasbourg), France

Licence : Analyse Numérique, 72h, L2, Université de Strasbourg, France

Licence : Calcul Formel et Simulation Numérique, 18h, L2, Université de Strasbourg, France

Licence : Compléments d'Analyse, 30h , L3, Université de Strasbourg

Master : Spectral Analysis, 30h, M1, Université de Strasbourg, France

Jean Roche

Licence : Analyse, 170h, L2, Université Henri Poincaré Nancy, France

Master : optimisation, 24h, M1, Université Henri Poincaré Nancy, France

Master : Décompostion de domaines, 24h, M2, Université Henri Poincaré Nancy, France

Eric Sonnendrücker

Master : Projet EDP, 30h, M2, Université de Strasbourg, France

Master : Calcul Scientifique pour l'agrégation, 60h, M2, Université de Strasbourg, France

Master : Méthodes numériques pour les EDP, 65h, M1, Université de Strasbourg, France

Master et Doctorat : Numerical methods for the Vlasov-Maxwell equations 36h, M2+D, ETH Zürich, Switzerland.

PhD & HdR:

HdR : Nicolas Crouseilles, Contributions à la simulation numérique des modèles de Vlasov en physique des plasmas, Université de Strasbourg, 14 janvier 2011

PhD : Ahmed Ratnani, L'analyse isogéométrique dans la physique des plasmas et l'électromagnétisme, Université de Strasbourg, 7 octobre 2011, Nicolas Crouseilles et Eric Sonnendrücker

PhD : Aurore Back, Etude théorique et numérique des équations de Vlasov-Maxwell dans le formalisme covariant, Université de Strasbourg, 7 novembre 2011, Advisors: Emmanuel Frénod et Eric Sonnendrücker

PhD in progress : Céline Caldini, Collisions dans les modèles gyrocinétiques, septembre 2010, Advisor: Mihai Bostan,

PhD in progress : Anaïs Crestetto, Méthode des moments pour les équations cinétiques, Advisor: Philippe Helluy,

PhD in progress : Mohammed Ghattassi, développement d'une méthode de résolution des équations de transfert radiatif couplées avec une équation de diffusion en deux et trois dimensions d?espace par des méthodes inspirées de la résolution des équations de Vlassov Poisson, septembre 2011, Advisor: Jean Roche.

PhD in progress : Mathieu Lutz, Etude théorique et numérique de l'approximation gyrocinétique, septembre 2010, Advisors: Emmanuel Frénod et Eric Sonnendrücker

PhD in progress : Sandrine Marchal, septembre 2006, Domain decomposition methods to solve a system of hyperbolic equations. Advisors: Simon Labrunie and Jean Rodolphe Roche.

PhD in progress : Christophe Steiner, Méthodes numériques pour l'équation de Vlasov, Advisors: Nicolas Crouseilles and Michel Mehrenberger

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