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Activity Report 2011

Project-Team GALAAD

Geometry, algebra, algorithms

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

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THEME
Algorithms, Certification, and Cryptography

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Project-Team GALAAD

Keywords: Algorithmic Geometry, Computer Algebra, Real Numbers

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2. Overall Objectives

2.1. Overall Objectives

There is a shared vision that our day life environment will increasingly interact with a digital world, populated by captors, sensors, or devices used to simplify or improve some of our activities. Digital cameras, positioning systems, mobile phones, internet web interfaces are such typical examples which are nowadays completely standard tools. Interconnected with each other, these devices are producing, exchanging or processing digital data in order to interact with the physical world. Computing is becoming ubiquitous and this evolution raises new challenges to represent, analyze and transform this digital information.

From this perspective, geometry is playing an important role. There is a strong interaction between physical and digital worlds through geometric modeling and analysis. Understanding a physical phenomena can be done by analyzing numerical simulations on a digital representation of the geometry. Conversely developing digital geometry (as in Computer Aided Geometric Design) is nowadays used to produce devices to overcome some physical difficulties (car, planes, ...). Obviously, geometry is not addressing directly problems related to storage or transmission of information, but it deals with structured and efficient representations of this information and methods to compute with these models.

Within this context, our research program aims at developing new and efficient methods for modeling geometry with algebraic representations. We don't see shapes just as set of points with simple neighbor information. In our investigations, we use richer algebraic models which provide structured and compact representation of the geometry, while being able to encode their important characteristic features.

The first challenge to be addressed is how to move from the digital world to an algebraic world. Our objective is to develop efficient methods which can transform digital data produced by cameras, laser scanners, observations or simulations into algebraic models involving few parameters. This is a way to structure the digital information and to further exploit its properties. This methodological investigations are connected to practical problems such as compression of data for exchange of geometric information, accurate description and simulation with manufactured objects, shape optimization in computer aided design, ...

A second challenge concerns operations and transformations on these algebraic representations. They require the development of dedicated techniques which fully exploit the algebraic characteristics of these representations. The theoretical foundations of our investigations are in algebraic geometry. This domain deals with the solutions of algebraic equations and its effective aspect concerns algorithms to compute and analyze them. It is an old, important and very active part of mathematics. Its combination with algorithmic developments for algebraic computation leads to new methods to treat effectively geometric problems. These investigations result in new contribution in commutative algebra, new algorithms in computer algebra, complexity analyses and/or software development for practical experimentation.

The third challenge is how to analyze and understand digital geometric data. In this approach, constructing algebraic representation and developing methods to compute with these models are the preliminary steps of our analysis process. The goal is to develop methods to extract some type of information we are searching from this data, such as topological descriptions, subdivisions in smooth components and adjacency relations, decomposition in irreducible components. The interplay between algebraic models and numerical computation is central in this activity. A main issue concerns the approximation of models and the certification of the computation.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is structured according to three broad topics:

1. **Algebraic algorithms for geometric computing**,
2. **Symbolic-numeric methods for analysis**,
3. **Algebraic representations for geometric modeling**.

3.2. Algebraic algorithms for geometric computing

This topic is directly related to polynomial system solving and effective algebraic geometry. It is our core expertise and many of our works are contributing to this area.

Our goal is to develop algebraic algorithms to perform efficiently geometric operations such as computing the intersection or self-intersection locus of algebraic surface patches, offsets, envelopes of surfaces, ...

The underlying representation behind the geometric model we consider are often of algebraic type. Computing with such models raises algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

Geometric objects are often described in a parametric form. For performing efficiently on these objects, it can also be interesting to manipulate implicit representations. We consider particular projections techniques based on new resultant constructions or syzygies, which allow to transform parametric representations into implicit. These problems can be reformulated in terms of linear algebra. We investigate methods which exploit this matrix representation based on resultant constructions.

They involve structured matrices such as Hankel, Toeplitz, Bezoutian matrices or their generalization in several variables. We investigate algorithms that exploit their properties and their implications in solving polynomial equations.

We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems.

We are also interested in subdivision methods, which are able to localise efficiently the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue while developing these methods is to analyse their practical and algorithmic behavior. Our aim is to obtain good complexity bounds and practical efficiency by exploiting the structure of the problem.

3.3. Symbolic Numeric Analysis

While treating practical problems, noisy data appear and incertitude has to be taken into account. The objective is to devise adapted techniques for analyzing the geometric properties of the algebraic models in this context.

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many context, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate the geometric objects while controlling the quality of approximation. We investigate subdivision techniques based on the algebraic formulation of our problems which allow us to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

The problem of decomposition and factorisation is also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. Typical problem in this direction is the problem of approximate factorization, which requires to analyze perturbations of the data, which enables us to break up the problem.

3.4. Algebraic Representations for Geometric Modeling

Compact, efficient and structured descriptions of shapes are required in many scientific computations in engineering, such as “Isogeometric” Finite Elements methods, point cloud fitting problems or implicit surfaces defined by convolution. Our objective is to investigate new algebraic representations (or improve the existing ones) together with their analysis and implementations.

We are investigating representations, based on semi-algebraic models. Such non-linear models are able to capture efficiently complexes shapes, using few data. However, they required specific methods to handle and solve the underlying non-linear problems, which we are investigating.

Effective algebraic geometry is a naturally framework for handling such representations. The framework not only provides tools for modeling but also, it makes it possible to exploit the geometric properties of these algebraic varieties, in order to improve this modeling work.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

The use of piecewise algebraic representations also raises problems of approximation and reconstruction, on which we are working on. In this direction, we are studying B-spline function spaces with specified regularity associated to domain partitions.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. We are interested in exploiting these invariant properties, to develop compact and adapted representations of shapes.

4. Application Domains

4.1. Shape modeling

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, etc. Optimising these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts that we are developing in Computer Aided Geometric Design.

4.2. Shape processing

Many problems encounter in the application of computer sciences started from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where *Distance geometry* plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tends to be more and more easily used yield large set of data points from which one has to recover compact geometric model. We are working in collaboration with groups in agronomy on the problems of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems. Geometry is also highly involved in the numerical simulation of physical problems such as heat conduction, ship hull design, blades and turbines analysis, mechanical stress analysis. We apply our algebraic-geometric techniques in the isogeometric approach which use the same (bspline) formalism to represent both the geometry and the solutions of partial differential equations on this geometry.

5. Software

5.1. Mathemagix, a free computer algebra environment

Participants: Bernard Mourrain, Angelos Mantzaflaris.

http://www.mathemagix.org/

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments are based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on the embeding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

The project aims at building a bridge between symbolic computation and numerical analysis. It is structured by collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

- REALROOT: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms; exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterised, implicit with different type of coefficients; algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler AXEL.

Collaborators: Grégoire Lecerf, Joris van der Hoeven and Philippe Trébuchet.

5.2. Axel, a geometric modeler for algebraic objects

Participants: Angelos Mantzaflaris, Bernard Mourrain, Meriadeg Perrinel.

http://axel.inria.fr.

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The software contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

The developments related to isogeometric analysis in Exciting have been integrated as dedicated plugins. Optimisation techniques and solvers for partial differential equations developed by R. Duvigneau (OPALE) have been connected.

A new version of the algebraic-geometric modelers is developed by Meriadeg Perinnel to connect it to the platform Dtk in order to provide a better modularity and a better interface to existing computation facilities and geometric rendering interface.

The package is distributed as binary packages for Linux as well as for MacOSX. It is hosted at the INRIA's gforge (http://gforge.inria.fr) and referenced by many leading software websites such as http://apple.com. The first version of the software has been downloaded more than 15000 times, since it is available.

Collaboration with Gang Xu (Hangzhou Dianzi University, China), Julien Wintz (Dream).

5.3. Maple packages for differential algebra and algebraic invariants

Participant: Evelyne Hubert.

- The Maple package **diffalg** is a collection of routines to handle systems of polynomial differential equations and inequations. The functionalities include differential elimination, expansion of the solutions into formal power series and analysis of singular solutions. The underlying theory and terminology belongs to differential algebra.

  Collaborators: François Boulier and François Lemaire from University of Lille.
- The Maple **AIDA** package is a collection of routines to explore algebra of differential invariants: computation of generating sets of invariants, rewritings, syzygies, and their differential analogues. The package builds on the Maple libraries Groebner, Vessiot and diffalg.

  http://www-sop.inria.fr/members/Evelyne.Hubert/aida/.

6. New Results

6.1. Algebraic Algorithms for Geometric Computing

6.1.1. An Algebraic Approach to Continuous Collision Detection for Ellipsoids

Participant: Bernard Mourrain.

In [28], we present algebraic conditions for characterizing three configurations of two ellipsoids in R3 that are the most relevant to collision detection: separation, external touching and overlapping. These conditions are given in terms of explicit formulae expressed by the subresultant sequence of the characteristic polynomial of the two ellipsoids and its derivative. For any two ellipsoids, the signs of these formulae can easily be evaluated to classify their configuration. Furthermore, based on these algebraic conditions, an efficient method is developed for continuous collision detection for two moving ellipsoids under arbitrary motion.

This is a joint work with Xiaohong Jia, Yi-King Choi and Wenping Wang from the university of Hong Kong.

6.1.2. On Continued Fraction Expansion of Real Roots of Polynomial Systems, Complexity and Condition Numbers

Participants: Angelos Mantzaflaris, Bernard Mourrain.

In [29], we elaborate on a correspondence between the coeffcients of a multivariate polynomial represented in the Bernstein basis and in a tensor-monomial basis, which leads to homography representations of polynomial functions, that use only integer arithmetic (in contrast to Bernstein basis) and are feasible over unbounded regions. Then, we study an algorithm to split this representation and we obtain a subdivision scheme for the domain of multivariate polynomial functions. This implies a new algorithm for real root isolation, MCF, that generalizes the Continued Fraction (CF) algorithm of univariate polynomials. A partial extension of Vincent's Theorem for multivariate polynomials is presented, which allows us to prove the termination of the algorithm. Bounding functions, projection and preconditioning are employed to speed up the scheme. The resulting isolation boxes have optimized rational coordinates, corresponding to the first terms of the continued fraction expansion of the real roots. Finally, we present new complexity bounds for a simplified version of the algorithm in the bit complexity model, and also bounds in the real RAM model for a family of subdivision algorithms in terms of the real condition number of the system. Examples computed with our C++ implementation illustrate the practical aspects of our method.

This is a joint work with E. Tsigaridas, from the Department of Computer Science, University of Aarhus.

6.1.3. Matrix-based representations of rational hypersurfaces

Participants: Laurent Busé, Nicolas Botbol.

This ongoing work is related to matrix-based representations of rational hypersurfaces whose theoretical fundations has been recently developed by our team and several other authors in the context of the implicitization problem. Being given a parameterized curve or hypersurface, this method consists in building a matrix whose entries are typically linear forms in the variables of the ambient space and such that the ideal generated by the maximal minors of this matrix provide a good approximation of the original curve or hypersurface.

We aim to study and determine the geometric informations that are contained in a representation matrix. In particular, we are currently adressing the two following questions :

1) understand the extraneous components that are added by taking the initial Fitting ideal of a representation matrix, with respect to the original curve or surface. Indeed, these extraneous components appear because of the good behavior of Fitting ideals under change of bases. Therefore, one can expect that these extraneous components yields some geometric properties of a curve or surface as a member of a certain family. 2) examine the extraction of singularities from a representation matrix, similarly to the recent results on what is called "singular factors" for the case of rational curves.

6.1.4. The surface/surface intersection problem by means of matrix based representations

Participants: Laurent Busé, Thang Luu Ba.

Evaluating the intersection of two rational parameterized algebraic surfaces is an important problem in solid modeling. In this work, we made use of some generalized matrix based representations of parameterized surfaces in order to represent the intersection curve of two such surfaces as the zero set of a matrix determinant. As a consequence, we extended to a dramatically larger class of rational parameterized surfaces the applicability of a general approach to the surface/surface intersection problem due to J. Canny and D. Manocha. In this way, we obtained compact and efficient representations of intersection curves allowing to reduce some geometric operations on such curves to matrix operations using results from linear algebra.

See the preprint version at http://hal.inria.fr/inria-00620947/en/

6.2. Symbolic-Numeric Analysis

6.2.1. A Subdivision Method for Computing Nearest Gcd with Certification

Participants: André Galligo, Bernard Mourrain.

A new subdivision method for computing the nearest univariate gcd is described and analyzed in [24]. It is based on an exclusion test and an inclusion test. The exclusion test in a cell exploits Taylor expansion of the polynomial at the center of the cell. The inclusion test uses Smale's alpha-theorems to certify the existence and unicity of a solution in a cell. Under the condition of simple roots for the distance minimization problem, we analyze the complexity of the algorithm in terms of a condition number, which is the inverse of the distance to the set of degenerate systems. We report on some experimentation on representative examples to illustrate the behavior of the algorithm.

This is a joint work with Guillaume Chèze and Jean-Claude Yakoubsohn (University Paul-Sabatier, Toulouse).

6.2.2. An Adapted Version of the Bentley-Ottmann Algorithm for Invariants of Plane Curves Singularities

Participant: Bernard Mourrain.

In [34], we report on an adapted version of the Bentley-Ottmann algorithm for computing all the intersection points among the edges of the projection of a three-dimensional graph. This graph is given as a set of vertices together with their space Euclidean coordinates, and a set of edges connecting them. More precisely, the three-dimensional graph represents the approximation of a closed and smooth implicitly defined space algebraic curve, that allows us a simplified treatment of the events encountered in the Bentley-Ottmann algorithm. As applications, we use the adapted algorithm to compute invariants for each singularity of a plane complex algebraic curve, i.e. the Alexander polynomial, the Milnor number, the delta-invariant, etc.

This is a joint work with Madalina Hodorog and Joseph Schicho, from RICAM, Linz, Austria.

6.2.3. Virtual Roots of a Real Polynomial and Fractional Derivatives

Participant: André Galligo.

After the works of Gonzales-Vega, Lombardi, Mahé, and Coste, Lajous, Lombardi, Roy, we consider the virtual roots of a univariate polynomial f with real coefficients. Using fractional derivatives, we associate to f a bivariate polynomial $P_f(x,t)$ depending on the choice of an origin a, then two type of plan curves we call the FDcurve and stem of f. We show, in the generic case, how to locate the virtual roots of f on the Budan table and on each of these curves. The paper [32] is illustrated with examples and pictures computed with the computer algebra system Maple. It is a joint work with Daniel Bembe.

6.2.4. Computing monodromy via continuation methods on random Riemann surfaces

Participant: André Galligo.

In [25], we consider a Riemann surface X defined by a polynomial $f(x,y)$ of degree d, whose coefficients are chosen randomly. Hence, we can suppose that X is smooth, that the discriminant $\delta(x)$ of f has $d(d-1)$ simple roots Δ and that $\delta(0) \neq 0$, i.e. the corresponding fiber has d distinct points $\{y_1, \ldots, y_d\}$. When we lift a loop $0 \in \gamma \subset C-\Delta$ by a continuation method, we get d paths in X connecting $\{y_1, \ldots, y_d\}$, hence defining a permutation of that set. This is called monodromy.

Here we present experimentations in Maple to get statistics on the distribution of transpositions corresponding to loops around each point of Δ. Multiplying families of "neighbor" transpositions, we construct permutations and the subgroups of the symmetric group they generate. This allows us to establish and study experimentally two conjectures on the distribution of these transpositions and on transitivity of the generated subgroups.

Assuming that these two conjectures are true, we develop tools allowing fast probabilistic algorithms for absolute multivariate polynomial factorization, under the hypothesis that the factors behave like random polynomials whose coefficients follow uniform distributions. It is a joint work with Adrien Poteaux (University of Lille).

6.3. Algebraic representations for geometric modeling

6.3.1. Multihomogeneous Polynomial Decomposition using Moment Matrices

Participants: Alessandra Bernardi, Jérôme Brachat, Bernard Mourrain.

In [33], we address the important problem of tensor decomposition which can be seen as a generalisation of Singular Value Decomposition for matrices. We consider general multilinear and multihomogeneous tensors. We show how to reduce the problem to a truncated moment matrix problem and we give a new criterion for flat extension of Quasi-Hankel matrices. We connect this criterion to the commutation characterization of border bases. A new algorithm is described: it applies for general multihomogeneous tensors, extending the approach of J.J. Sylvester on binary forms. An example illustrates the algebraic operations involved in this approach and how the decomposition can be recovered from eigenvector computation.

This is a joint work with Pierre Comon (I3S, CNRS).

6.3.2. *On the variety parametrizing completely decomposable polynomials.*

Participant: Alessandra Bernardi.

The purpose of the paper [15] is to relate the variety parameterizing completely decomposable homogeneous polynomials of degree d in $n+1$ variables on an algebraically closed field, called $\mathrm{Split}_d(\mathbb{P}^n)$, with the Grassmannian of $n-1$ dimensional projective subspaces of $\mathbb{P}^{n+d-1}$. We compute the dimension of some secant varieties to $\mathrm{Split}_d(\mathbb{P}^n)$ and find a counterexample to a conjecture that wanted its dimension related to the one of the secant variety to $\mathbb{G}(n-1, n+d-1)$. Moreover by using an invariant embedding of the Veronse variety into the Plücker space, then we are able to compute the intersection of $\mathbb{G}(n-1, n+d-1)$ with $\mathrm{Split}_d(\mathbb{P}^n)$, some of its secant variety, the tangential variety and the second osculating space to the Veronese variety.

This is a joint work with Enrique Arrondo (Universidad Complutense de Madrid, Spain)

6.3.3. *Computing symmetric rank for symmetric tensors.*

Participant: Alessandra Bernardi.

In [21] we consider the problem of determining the symmetric tensor rank for symmetric tensors with an algebraic geometry approach. We give algorithms for computing the symmetric rank for $2 \times ... \times 2$ tensors and for tensors of small border rank. From a geometric point of view, we describe the symmetric rank strata for some secant varieties of Veronese varieties.

This is a joint work with Alessandro Gimigliano and Monica Idà (Univesrità di Bologna, Italy).

6.3.4. *Higher secant varieties of $\mathbb{P}^n \times \mathbb{P}^m$ embedded in bi-degree $(1, d)$.*

Participant: Alessandra Bernardi.

Let $X^{(n,m)}_{(1,d)}$ denote the Segre-Veronese embedding of $\mathbb{P}^n \times \mathbb{P}^m$ via the sections of the sheaf $\mathcal{O}(1, d)$. In [20] we study the dimensions of higher secant varieties of $X^{(n,m)}_{(1,d)}$ and we prove that there is no defective s^{th} secant variety, except possibly for n values of s. Moreover when $\binom{m+d}{d}$ is multiple of $(m+n+1)$, the s^{th} secant variety of $X^{(n,m)}_{(1,d)}$ has the expected dimension for every s.

This is a joint work with Enrico Carlini (Politecnico di Torino, Italy, Maria Virginia Catalisano (Università di Genova, Italy).

6.3.5. *On the X-rank with respect to linear projections of projective varieties.*

Participant: Alessandra Bernardi.

In [17] we improve the known bound for the X-rank $R_X(P)$ of an element $P \in \mathbb{P}^N$ in the case in which $X \subset \mathbb{P}^n$ is a projective variety obtained as a linear projection from a general v-dimensional subspace $V \subset \mathbb{P}^{n+v}$. Then, if $X \subset \mathbb{P}^n$ is a curve obtained from a projection of a rational normal curve $C \subset \mathbb{P}^{n+1}$ from a point $O \subset \mathbb{P}^{n+1}$, we are able to describe the precise value of the X-rank for those points $P \in \mathbb{P}^n$ such that $R_X(P) \leq R_C(O) - 1$ and to improve the general result. Moreover we give a stratification, via the X-rank, of the osculating spaces to projective cuspidal projective curves X. Finally we give a description and a new bound of the X-rank of subspaces both in the general case and with respect to integral non-degenerate projective curves.

This is a joint work with Edoardo Ballico (Università di Trento, Italy).

6.3.6. Decomposition of homogeneous polynomials with low rank.

Participant: Alessandra Bernardi.

Let F be a homogeneous polynomial of degree d in $m+1$ variables defined over an algebraically closed field of characteristic zero and suppose that F belongs to the s-th secant varieties of the standard Veronese variety $X_{m,d} \subset \mathbb{P}^{\binom{m+d}{d}-1}$ but that its minimal decomposition as a sum of d-th powers of linear forms $M_1, ..., M_r$ is $F = M_1^d + \cdots + M_r^d$ with $r > s$. In [16] we show that if $s + r \leq 2d + 1$ then such a decomposition of F can be split in two parts: one of them is made by linear forms that can be written using only two variables, the other part is uniquely determined once one has fixed the first part. We also obtain a uniqueness theorem for the minimal decomposition of F if the rank is at most d and a mild condition is satisfied.

This is a joint work with Edoardo Ballico (Università di Trento, Italy).

6.3.7. On the X-rank with respect to linearly normal curves.

Participant: Alessandra Bernardi.

In [18] we study the X-rank of points with respect to smooth linearly normal curves $X \subset \mathbb{P}^n$ of genus g and degree $n+g$.
We prove that, for such a curve X, under certain circumstances, the X-rank of a general point of X-border rank equal to s is less or equal than $n+1-s$.
In the particular case of $g = 2$ we give a complete description of the X-rank if $n = 3, 4$; while if $n \geq 5$ we study the X-rank of points belonging to the tangential variety of X.

This is a joint work with Edoardo Ballico (Università di Trento, Italy).

6.3.8. Symmetric tensor rank with a tangent vector: a generic uniqueness theorem

Participant: Alessandra Bernardi.

Let $X_{m,d} \subset \mathbb{P}^N$, $N := \binom{m+d}{m} - 1$, be the order d Veronese embedding of $\mathbb{P}^m$. Let $\tau(X_{m,d}) \subset \mathbb{P}^N$, be the tangent developable of $X_{m,d}$. For each integer $t \geq 2$ let $\tau(X_{m,d}, t) \subseteq \mathbb{P}^N$, be the join of $\tau(X_{m,d})$ and $t-2$ copies of $X_{m,d}$. In [19] we prove that if $m \geq 2$, $d \geq 7$ and $t \leq 1 + \lfloor \binom{m+d-2}{m} / (m+1) \rfloor$, then for a general $P \in \tau(X_{m,d}, t)$ there are uniquely determined $P_1, \cdots, P_{t-2} \in X_{m,d}$ and a unique tangent vector ν of $X_{m,d}$ such that P is in the linear span of $\nu \cup \{P_1, \cdots, P_{t-2}\}$, i.e. a degree d linear form f (a symmetric tensor T of order d) associated to P may be written as

$$f = L_{t-1}^{d-1} L_t + \sum_{i=1}^{t-2} L_i^d, \quad (T = v_{t-1}^{\otimes(d-1)} v_t + \sum_{i=1}^{t-2} v_i^{\otimes d})$$

with L_i linear forms on $\mathbb{P}^m$ (v_i vectors over a vector field of dimension $m+1$ respectively), $1 \leq i \leq t$, that are uniquely determined (up to a constant).

This is a joint work with Edoardo Ballico (Università di Trento, Italy).

6.3.9. Parametrization of computational domain in isogeometric analysis: methods and comparison

Participants: André Galligo, Bernard Mourrain.

Parameterization of computational domain plays an important role in isogeometric analysis as mesh generation in finite element analysis. In this paper, we investigate this problem in the 2D case, i.e, how to parametrize the computational domains by planar B-spline surface from the given CAD objects (four boundary planar B-spline curves). Firstly, two kinds of sufficient conditions for injective B-spline parameterization are derived with respect to the control points. Then we show how to find good parameterization of computational domain by solving a constraint optimization problem, in which the constraint condition is the injectivity sufficient conditions of planar B-spline parametrization, and the optimization term is the minimization of quadratic energy functions related to the first and second derivatives of planar B-spline parameterization. By using this method, the resulted parameterization has no self-intersections, and the isoparametric net has good uniformity and orthogonality. After introducing a posteriori error estimation for isogeometric analysis, we propose r-refinement method to optimize the parameterization by repositioning the inner control points such that the estimated error is minimized. Several examples are tested on isogeometric heat conduction problem to show the effectiveness of the proposed methods and the impact of the parameterization on the quality of the approximation solution. Comparison examples with known exact solutions are also presented. This joint work with Régis Duvigneau (EPI OPALE) and Gang Xu (Hangzhou Dianzi University, China) is published in [31].

6.3.10. Variational Harmonic Method for Parameterization of Computational Domain in 2D Isogeometric Analysis

Participants: André Galligo, Bernard Mourrain.

In isogeometric anlaysis, parameterization of computational domain has great effects as mesh generation in finite element analysis. In this paper, based on the concept of harmonic map from the computational domain to parametric domain, a variational approach is proposed to construct the parameterization of computational domain for 2D isogeometric analysis. Different from the previous elliptic mesh generation method in finite element analysis, the proposed method focus on isogeometric version, and converts the elliptic PDE into a nonlinear optimization problem. A regular term is integrated into the optimization formulation to achieve more uniform grid near convex (concave) parts of the boundary. Several examples are presented to show the efficiency of the proposed method.

This joint work with Régis Duvigneau (EPI OPALE) and Gang Xu (Hangzhou Dianzi University, China) is published in [36].

6.3.11. Warp-based Helical Implicit Primitives

Participant: Evelyne Hubert.

Implicit modeling with skeleton-based primitives has been limited up to now to planar skeletons elements, since no closed-form solution was found for convolution along more complex curves. We show that warping techniques can be adapted to efficiently generate convolution-like implicit primitives of varying radius along helices, a useful 3D skeleton found in a number of natural shapes. Depending on a single parameter of the helix, we warp it onto an arc of circle or onto a line segment. For those latter skeletons closed form convolutions are known for entire families of kernels. The new warps introduced preserve the circular shape of the normal cross section to the primitive.

This is joint work with Cédric Zanni and Marie-Paule Cani from the project-team EVASION (INRIA Grenoble Rhône-Alpes / LJK Laboratoire Jean Kuntzmann) which is publiched in [37].

6.4. National Initiatives

6.4.1. PlantScan3D

PlantScan3D is an ARC between coordinated by the EPI Virtual Plants (UMR DAP, INRIA-CIRAD, Montpellier), with the EPI Galaad (INRIA, Méditerranée) and Evasion (INRIA Rhône-Alpes, Grenoble).

A close collaboration between specialists in plant structures modelling, algebraic geometry, and 3D computer graphic is required to address plant structure reconstruction from laser scanned point clouds. Indeed it is required to take into account efficiently knowledge from topology and geometry to allow mapping and reconstruction of data despite noise, occlusions, and thinness of structure. The objective of the project is to provide as output a compact geometrical model that model smoothly branching point of tubular structure and organs (like leaves). At the end, this model should make it possible an interactive visualisation and automatize different measurement operators needed by biological partners.

More information available at http://plantscan3d.gforge.inria.fr/wiki.

6.4.2. GEOLMI

GEOLMI - Geometry and Algebra of Linear Matrix Inequalities with Systems Control Applications - is an ANR project working on topics related to the Geometry of determinantal varieties, positive polynomials, computational algebraic geometry, semidefinite programming and systems control applications

The partners are LAAS-CNRS, Univ. de Toulouse (coordinator), LJK-CNRS, Univ. Joseph Fourier de Grenoble; INRIA Sophia Antipolis Méditerranée; LIP6-CNRS Univ. Pierre et Marie Curie; Univ. de Pau et des Pays de l'Adour; IRMAR-CNRS, Univ. de Rennes.

More information available at http://homepages.laas.fr/henrion/geolmi

6.4.3. ANEMOS

ANEMOS - Advanced Numeric for ELMs : Modeling and Optimized Schemes - is an ANR project devoted to the numerical modelling study of such ELM control methods as Resonant Magnetic Perturbations (RMPs) and pellet ELM pacing both foreseen in ITER. The goals of the project are to improve understanding of the related physics and propose possible new strategies to improve effectiveness of ELM control techniques. The study of spline spaces for isogemetric finite element methods is proposed in this context.

The partners are IRFM, CEA, Cadarache; JAD, University of Nice - Sophia Antipolis; INRIA, Bacchus; Maison de la Simulation CEA-CNRS-INRIA-University of Orsay- University of Versailles St Quentin .

6.5. European Initiatives

6.5.1. FP7 Projects

6.5.1.1. TERRIFIC

Title: Towards Enhanced Integration of Design and Production in the Factory of the Future through Isogeometric Technologies

Type: COOPERATION (ICT)

Defi: PPP FoF: Digital factories: Manufactoring design and product lifecycle manage

Instrument: Specific Targeted Research Project (STREP)

Duration: September 2011 - August 2014

Coordinator: SINTEF, Oslo (Norway)

Others partners:

Alenia Aeronautica (Italy) INRIA Méditerranée (France); Jozef Kepler universitet, Linz (Austria) JOTNE, Oslo (Norway); MAGNA, Steyr (Austria); Missler Software (France); Siemens AG (Germany); Technische Universität Kaiserslautern (Germany); University of Pavia (Italy).

See also: http://terrific-project.eu

Abstract: The project aims at significant improvement of the interoperability of computational tools for the design, analysis and optimization of functional products. An isogeometric approach is applied for selected manufacturing application areas (cars, trains, aircraft) and for computer-aided machining. Computer Aided Design and numerical simulation algorithms are vital technologies in modern product development, yet they are today far from being seamlessly integrated. Their interoperability is severely disturbed by inconsistencies in the mathematical approaches used. Efficient feedback from analysis to CAD and iterative refinement of the analysis model is a feature of isogeometric analysis, and would be an essential improvement for computer-based design optimization and virtual product development. Our vision is to provide and disseminate tangible evidence of the performance of the isogeometric approach in comparison to traditional ones in four important application areas as well as addressing interoperability and other issues that necessarily arise in a large-scale industrial introduction of isogeometry.

6.5.1.2. EXCITING

Title: Exact geometry simulation for optimized design of vehicles and vessels

Type: FP7-CP-SST-2007-RTD-1-218536, COOPERATION (TRANSPORTS)

Instrument: Specific Targeted Research Project (STREP)

Duration: October 2008 - September 2011

Coordinator: Jozef Kepler universitet, Linz (Austria)

Others partners:

SINTEF, Oslo (Norway); Siemens AG (Germany); National Technical University of Athens (Greece); Hellenic Register of Shipping (Greece); University of Technology, Munich (Germany); INRIA Méditerranée (France); VA Tech Hydro (Austria); Det Norske Veritas AS (Norway).

See also: http://exciting-project.eu/

Abstract: This project focuses on computational tools for the optimized design of functional free-form surfaces. Specific applications are ship hulls and propellers in naval engineering and car components, frames, and turbochargers in the automotive and railway transportation industries. The objective is to base the corresponding computational tools on the same exact representation of the geometry. This should lead to huge benefits for the entire chain of design, simulation, optimization, and life cycle management, including a new class of computational tools for fluid dynamics and solid mechanics, simulations for vehicles and vessels based. This seamless integration of CAD and FEM will have direct applications in product design, simulation and optimization of core components of vehicles and vessels.

6.5.1.3. SAGA

Title: ShApe, Geometry and Algebra, 2008-2012

Type: FP7-PEOPLE-2007-1-1-ITN.

Instrument: Initial Training Network (ITN)

Duration: November 2008 - October 2012

Coordinator: SINTEF (Norway)

Others partners: University of Oslo (Norway); Johannes Kepler Universitaet Linz (Austria); Universidad de Cantabria, Santander (Spain); Vilniaus Universitetas (Lithuany); National and Kapodistrian University of Athens (Greece); INRIA Méditerranée (France); GraphiTech (Italy); Kongsberg SIM GmbH (Austria); Missler Software (France);

See also: http://saga-network.eu/.

Abstract: The project aims at promoting the interaction between Geometric Modeling and Real Algebraic Geometry and, in general, at strengthening interdisciplinary and inter-sectorial research and development concerning CAD/CAM. Its objective is also to train a new generation of researchers familiar with both academic and industry viewpoints, while supporting the cooperation among the partners and with other interested collaborators in Europe.

6.5.1.4. DECONSTRUCT

Title:Decomposition of Structured Tensors, Algorithms and Characterization.

Type: PEOPLE (FP7-PEOPLE-2009-IEF)

Instrument: Marie Curie Intra-European Fellowships for Career Development (IEF)

Duration: November 2010 - November 2012

Coordinator: INRIA (France)

Others partners: No.

See also: http://www-sop.inria.fr/teams/galaad/joomla/index.php/international-collaborations-147/172-deconstruct.html

Abstract: Tensors play a wide role in numerous application areas as Signal Processing for Telecommunications, Arithmetic Complexity or Data Analysis. In some applications tensors may be completely symmetric, or symmetric only in some modes, or may not be symmetric. In most of these applications, the decomposition of a tensor into a sum of rank-1 terms is relevant, since tensors of interest have a reduced rank. Most of them are structured i.e. they are either symmetric or enjoy some index-invariance. Lastly, they are often real, which raises open problems concerning the existence and calculation of the decompositions. These issues build the basic bricks of the research program we propose. The classes of tensors described above have a geometric translation in terms of classical algebraic varieties: Segre, Veronese, Segre-Veronese varieties and Grassmannians and their secant varieties. A complete description of equations for those secant varieties and their dimensions is still not known (only dimensions of secant varieties to Veronsean are classified), although they have been studied by algebraic and differential geometers and algebraists for a long period up to now. The aim of this research project is:

- to attack both the description of the ideal of those secant varieties and their dimensions, starting from low dimensions and low degrees,
- to propose algorithms able to compute the rank of structured tensors.

6.6. International Initiatives

6.6.1. Visits of International Scientists

Annie Cuyt and Wen-Shin Lee (University of Antwerpen, Belgium) visited from January 24th to February 2nd to initiate a collaboration on the topic of shape from moments.

George Labahn (University of Waterloo, Canada) visited February 14th-18th and October 3rd-8th to collaborate with Evelyne Hubert on scaling invariants and their application to symmetry reduction of dynamical system (with parameters).

Mark Hickman (University of Canterbury, New Zealand) visited from March to June, as part of his sabbatical year, to collaborate with Evelyne Hubert on the topic of integral and moment invariants and their applications in computer vision.

Nelly Villamizar (University of Oslo) visited us from March 15 to May 15, to collaborate with B. Mourrain on splines spaces, for her secondement in the context of the ITN Marie-Curie SAGA.

Nguyen Tuan Thien (JKU, LINZ) visited us from March to May, to collaborate with B. Mourrain and A. Galligo on parameterization problems in isogeometric analysis, for his secondement in the context of the ITN Marie-Curie SAGA.

6.6.2. Participation In International Programs

6.6.2.1. PAI STAR South Corea collaboration

Participants: Laurent Busé, André Galligo, Evelyne Hubert, Angelos Mantzaflaris, Bernard Mourrain.

The objective of this collaboration is to conduct research in algebraic techniques for solving geometric modeling problems. More specially, we are interested in developing efficient and robust methods to solve non-linear constraints which appear in geometric computation. These methods will be used in applications such as shape design and reconstruction for solving interpolation or approximation problems. A typical area in which we will apply our methods is ship design. Experimentation and validation will lead to open source software implementation.

Collaborators from Seoul National University: Tae-Wan Kim, Sharma Rajiv, Hur Seok, Yeong-hwa Seo.

Tae-Wan Kimm visited INRIA-GALAAD from April 17 to April 23.

7. Dissemination

7.1. Animation of the scientific community

Laurent Busé

- organized in collaboration with Carlos D'Andrea (University of Barcelona, Spain) and Chris Peterson (University of Colorado) the workshop *Computational Algebraic Geometry* during the conference *Foudations of Computational Mathematics* that was held in Budapest in July. See http://www-sop.inria.fr/teams/galaad/conf/FOCM11/focm.html.
- was member of the Ph.D. committee of Thang Luu Ba, July 12, University of Nice Sophia Antipolis and Elimane Ba, December 12, University of Nice.

Evelyne Hubert

- organized in collaboration with George Labahn (University of Waterloo, Ontario) and Michael Singer (North Carolina State University) the workshop *Symbolic Analysis* during the conference *Foudations of Computational Mathematics* that was held in Budapest in July. See http://www.damtp.cam.ac.uk/user/na/FoCM11.
- is the co-editor, with Alicia Dickenstein (University of Buenos Aires, Argentina), Sandra di Rocco (Royal Institute of Technology, Stockholm, Sweden) and Joseph Schicho (Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria) of the special issue of the Journal of Symbolic Computation that follows the MEGA (Effective Methods in Algebraic Geometry).
- is associate editor of the Journal of Symbolic Computation (http://www.journals.elsevier.com/journal-of-symbolic-computation).
- is part of the Advisory Board of the MEGA conference series and acted as program committee member for the 2011 edition.

Bernard Mourrain

- organized in collaboration with Sri Wahyuni the CIMPA-UNESCO-MICINN-INDONESIA School on Non-linear Computational Geometry, at the Universitas Gadjah Mada, Yogyakarta, Indonesia, July 18-29,
- served as referee for the journal of Symbolic Computation, Computer Aided Design, Computer Aided Geometric Design, Linear Algebra and Applications,
- is associate editor of the Journal of Symbolic Computation (http://www.journals.elsevier.com/journal-of-symbolic-computation).

- is member of the Advisory Board of the MEGA conference series and acted as program committee member for the 2011 edition.
- was member of the program committee of the conference Symbolic-Numeric Computation in San Jose, USA; GD/SPM 2011 in Miami, USA; SIGGRAPH Asia 2011 in Hong Kong, China,
- was member of the HDR of Laurent Busé, April 29, University of Nice Sophia Antipolis, of the Ph.D. committee of Jérôme Brachat, July 4, University of Nice Sophia Antipolis, Thang Luu Ba, July 12, University of Nice Sophia Antipolis, Angelos Mantzaflaris, October 3, University of Nice Sophia Antipolis, Christophe Mouilleron, November 4, LIP, ENS LYON, Madalina Hodorog, November 28, JKU, Linz,
- is chair of the local INRIA Committee for Courses and Conferences.

7.2. Teaching

Laurent Busé teaching activities:

Master : Courbes et Surfaces, 42h ETD, first year of Master MAM4 of the EPU, university of Nice, France.

Bernard Mourrain teaching activities:

Master 2: *Computational Algebra for Real Geometry* 30h ETD, Master 2 of Mathematics, Lab. J.A. Dieudonné.

7.2.1. Defended Habilitation thesis

Laurent Busé, Représentations matricielles en théorie de l'élimination et applications à la géométrie, April 29$^{\text{th}}$ 2011, University of Nice.

7.2.2. Defended Ph.D. thesis

Elimane Ba, Résultant déterminantiel et applications, December 12$^{\text{th}}$ 2011, Université de Nice. Advisor: Mohamed Elkadi.

Jérôme Brachat, *Dualité effective pour la résolution d'équations polynomiales*, July 4$^{\text{th}}$ 2011, University of Nice. Advisor: Bernard Mourrain.

Thang Luu Ba, Représentation matricielle implicite de courbes et surfaces algébriques et applications, July 4$^{\text{th}}$ 2011, Université de Nice. Advisor: Laurent Busé and André Galligo.

Angelos Mantzaflaris, *Robust algebraic methods for geometric computations*, October 3$^{\text{rd}}$ 2011, University of Nice. Advisor: Bernard Mourrain

7.2.3. Ph.D. thesis in progress

Marta Abril Bucero, Moment matrices, real algebraic geometry and polynomial optimization. Advisor: Bernard Mourrain.

Mathieu Collowald, Integral representation of shapes for feature conservation or extraction. Advisors: Evelyne Hubert and Bernard Mourrain.

Abdallah Lachaal, Geometric computing with procedural models and applications. Advisor: Bernard Mourrain.

7.2.4. Internship

J. Dehais, *Reconstruction of branching structure from point clouds*, 5 months, M2, ARC PlantScan3D.

J. Proust, *Modélisation et optimisation de carènes*, 5 months, M1.

8. Bibliography

Major publications by the team in recent years

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