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Project-Team **NEUROMATHCOMP**

Mathematical and Computational Neuroscience

IN COLLABORATION WITH: Laboratoire d'Informatique de l'Ecole Normale Supérieure (LIENS), Laboratoire Jean-Alexandre Dieudonné (JAD)

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THEME
**Computational Medicine and Neuro-
sciences**

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Project-Team NEUROMATHCOMP

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2. Overall Objectives

2.1. Presentation

NEUROMATHCOMP focuses on the exploration of the brain from the mathematical and computational perspectives.

We want to unveil the principles that govern the functioning of neurons and assemblies thereof and to use our results to bridge the gap between biological and computational vision.

Our work is quite mathematical but we make heavy use of computers for numerical experiments and simulations. We have close ties with several top groups in biological neuroscience. We are pursuing the idea that the "unreasonable effectiveness of mathematics" can be brought, as it has been in physics, to bear on neuroscience.

Computational neuroscience attempts to build models of neurons at a variety of levels, microscopic, i.e., the single neuron, the minicolumn containing of the order of one hundred or so neurons, mesoscopic, i.e., the macrocolumn containing of the order of $10^4 - 10^5$ neurons, and macroscopic, i.e., a cortical area such as the primary visual area V1.

Modeling such assemblies of neurons and simulating their behaviour involves putting together a mixture of the most recent results in neurophysiology with such advanced mathematical methods as dynamic systems theory, bifurcation theory, probability theory, stochastic calculus, theoretical physics and statistics, as well as the use of simulation tools.

We conduct research in the following three main areas.

1. Modeling and simulating single neurons.
2. Modeling and simulating assemblies of neurons.
3. Visual perception modeling.

3. Scientific Foundations

3.1. Mean-field approaches

Modeling neural activity at scales integrating the effect of thousands of neurons is of central importance for several reasons. First, most imaging techniques are not able to measure individual neuron activity ("microscopic" scale), but are instead measuring mesoscopic effects resulting from the activity of several hundreds to several hundreds of thousands of neurons. Second, anatomical data recorded in the cortex reveal the existence of structures, such as the cortical columns, with a diameter of about $50\mu\text{m}$ to 1mm , containing of the order of one hundred to one hundred thousand neurons belonging to a few different species. The description of this collective dynamics requires models which are different from individual neurons models. In particular, when the number of neurons is large enough averaging effects appear, and the collective dynamics is well described by an effective mean-field, summarizing the effect of the interactions of a neuron with the other neurons, and depending on a few effective control parameters. This vision, inherited from statistical physics requires that the space scale be large enough to include a large number of microscopic components (here neurons) and small enough so that the region considered is homogeneous.

Our group is developing mathematical and numerical methods allowing on one hand to produce dynamic mean-field equations from the physiological characteristics of neural structure (neurons type, synapse type and anatomical connectivity between neurons populations), and on the other so simulate those equations. Our investigations have shown that the rigorous dynamics mean-field equations can have a quite more complex structure than the ones commonly used in the literature (e.g. Jansen-Rit, 95) as soon as realistic effects such as synaptic variability are taken into account. Our goal is to relate those theoretical results with experimental measurement, especially in the field of optical imaging. For this we are collaborating with the DYVA team at INT, Marseille.

3.2. Spike train statistics

The neuronal activity is manifested by the emission of action potentials (“spikes”) constituting spike trains. Those spike trains are usually not exactly reproducible when repeating the same experiment, even with a very good control ensuring that the experimental conditions have not changed. Therefore, researchers are seeking statistical regularities in order to provide an accurate model for spike train statistics. The spike trains statistics is assumed to be characterized by a hidden probability $\mu^{(h)}$ giving the probability of *spatio-temporal* spike patterns. A current goal in experimental analysis of spike trains is to approximate $\mu^{(h)}$ from data. A *model* is a probability distribution μ which approaches $\mu^{(h)}$. Typically, μ must predict the probability of spike events occurrence with a good accuracy.

In the simplest situation where one assumes $\mu^{(h)}$ to be invariant under time translation, the Gibbs distribution approach consists of fixing a set of quantities (observables O_k , $k = 1 \dots K$) whose average value C_k is computed from experimental spike trains. Then, one seeks a time-translation invariant probability μ satisfying $\mu(O_k) = C_k$, $k = 1 \dots K$, where $\mu(O_k)$ is the average value of O_k under μ . Additionally, one asks μ to maximise the entropy rate μ . Equivalently, introducing the function $\phi = \sum_{k=1}^K \lambda_k O_k$, where ϕ is called a Gibbs potential and λ_k a free parameters, one seeks a probability μ satisfying $h(\mu) + \mu(\phi) = \sup_{\nu \in M_{inv}} (h(\nu) + \nu(\phi))$. Here M_{inv} is the set of time-translation invariant probabilities on the set of spike trains. Under fairly general conditions, as far as the analysis of experimental spike trains is concerned, the probability realising the sup is unique. The quantity $P(\phi) = h(\mu) + \mu(\phi)$ is called the topological pressure. It matches in particular the property $\frac{\partial P(\phi)}{\partial \lambda_k} = \mu(O_k)$. This relations allows to tune the parameters λ_k so that $\mu(O_k) = C_k$. The process of computation of $P(\phi)$ and μ has been numerically implemented by our team together with the CORTEX INRIA team (Event Neural Assembly Simulation ENAS <http://enas.gforge.inria.fr/v3/>).

The notion of Gibbs distribution extends to the more general context of statistics which are not time translation invariant. A simple example is a probability distribution characterizing the trajectories of a non homogeneous Markov chain with strictly positive transition probabilities. This concept extends also to processes with an infinite memory (chains with complete connections). We have proven the existence and uniqueness of a Gibbs distribution of this last type, in several examples of neural networks models, submitted to time-dependent stimuli, and characterized some salient properties of the Gibbs distribution, in connection with neuronal dynamics and response to stimuli. Thus, Gibbs distributions seem to be a useful concept for the analysis of spike trains.

In this spirit, our group is, on one hand, producing analytical (and rigorous) results on statistics of spike trains in canonical neural network models (Integrate and Fire, conductance based). On the other hand we are using those results to resolve experimental questions and new algorithms for data treatments. We have developed a C++ library for spike train statistics based on Gibbs distributions analysis and freely available at <http://www-sop.inria.fr/neuromathcomp/public/index.shtml>. We are collaborating with several biologist groups involved in the analysis of retina spike trains (Centro de Neurociencia Valparaiso; Molecular Biology Lab, Princeton; Institut de la vision, Paris).

3.3. Synaptic Plasticity

Neural networks show amazing abilities for information storage and processing, and stimulus-dependent activity shaping, to evolve and adapt. These capabilities are mainly conditioned by plasticity mechanisms, and especially synaptic plasticity, inducing a mutual coupling between network structure and neuron dynamics. Synaptic plasticity occurs at many levels of organization and time scales in the nervous system (Bienenstock, Cooper, and Munroe, 1982). It is of course involved in memory and learning mechanisms, but it also alters excitability of brain area and regulates behavioural states (e.g. transition between sleep and wakeful activity). Therefore, understanding the effects of synaptic plasticity on neurons dynamics is a crucial challenge. On experimental grounds, different synaptic plasticity mechanisms have been exhibited from the Hebbian’s ones (Hebb, 1949) to Long Term Potentiation (LTP) and Long Term Depression (LTD), and more recently to Spike Time Dependent Plasticity (STDP) (Markram, Lubke, Frotscher, and Sakmann, 1997; Bi and Poo, 2001).

Synaptic plasticity implies that activity guides the way synapses evolve; but the resulting connectivity structure in turn can raise new dynamical regimes. This interaction becomes even more complex if the considered basic architecture is not feed-forward but includes recurrent synaptic links, like in cortical structures. Understanding this mutual coupling between dynamics and topology and its effects on the computations made by the network is a key problem in computational neuroscience.

Our group is developing mathematical and numerical methods to analyse this mutual interaction. Especially, we have shown that plasticity mechanisms, Hebbian-like or STDP, have strong effects on neuron dynamics complexity, such as dynamics complexity reduction, and spike statistics (convergence to a specific Gibbs distribution via a variational principle), resulting in a response-adaptation of the network to learned stimuli. Also, we are currently studying the conjugated effects of synaptic and intrinsic plasticity in collaboration with biologists in Jussieu, Paris.

3.4. Visual perception modeling

Visual perception is the result of complex dynamical interaction between cortical maps. Interestingly, in many situations, the dynamics of perceptual effects show striking similarities when observed at neuronal, psychophysical, and oculomotor levels. Based on the inter-relation and complementary insights given by those dynamics, we can investigate how basic mechanisms of dynamical cortical processing can be incorporated in dynamical models. This work requires to understand the behaviour of dynamical systems involving, e.g., the interaction between multiple areas, the role of synaptic plasticity, the effects of delays or slow-fast interaction, which are observed at the neuronal level. Thus, theoretical contributions can have a major impact on the development of visual perception models that are able to not only explain but also predict perceptual effects.

For example, our group is developing models of motion perception, in collaboration with Institut des Neurosciences de la Timone (Marseille, France). Our recurrent models of motion integration, involving V1 and MT cortical maps. Proposing a simple readout mechanism, we reproduce not only motion perception but also the dynamics of smooth pursuit eye movements on various line figures and gratings viewed through different apertures. Our models can also solve various contextual problems where extrinsic junctions should be eliminated, without relying on complex junction detectors or depth computation. We now investigate neural fields formulations which enable to consider further perceptual effects such as, e.g., multistability. Finally, one important goal of our contribution in this area is to formalize the comparison against visual performance and also to confront our results to computer vision when it is possible.

4. Software

4.1. Virtual Retina: A Large-Scale Simulator of Biological Retina

Participants: Bruno Cessac, Hassan Nasser, Pierre Kornprobst [correspondent], Adrien Wohrer [Group for Neural Theory - ENS].

Virtual Retina is a simulation software developed by Adrien Wohrer during his PhD [74], [73] that allows large-scale simulations of biologically-plausible retinas.

Virtual Retina has a variety of biological features implemented such as (i) spatio-temporal linear filter implementing the basic center/surround organization of retinal filtering, (ii) non-linear contrast gain control mechanism providing instantaneous adaptation to the local level of contrast; (iii) spike generation by one or several layers of ganglion cells paving the visual field.

Virtual Retina is under INRIA CeCill C open-source licence, so that one can download it, install it and run it on your own sequences. Virtual Retina also offers a web service (v 2.1), so that you may test directly the main software on user's own data, without any installation. This webservice was developed in collaboration with Nicolas Debeissat (engineer, 2002).

Virtual Retina continues its evolution thanks to the work done in our team by Bruno Cessac and Hassan Nasser who are interested in the analysis of the collective behavior of ganglion cells responses (see Section 5.1.1). To take this collective behavior into account, Virtual Retina needs to be extended since in its current version, ganglion cells are independent. Other evolutions of Virtual Retina are also investigated by external partners (see, e.g., [68]).

- IDDN number: IDDN.FR.001.210034.000.S.P.2007.000.31235
- Version: v 2.2.2 (September 2011)
- Link: <http://www-sop.inria.fr/neuromathcomp/public/software/virtualretina>

4.2. ABFilter: A Simulator Of V1 Simple and Complex Cells

Participants: Pierre Kornprobst [correspondant], Maria-Jose Escobar [Electronics Engineering Department of the Universidad Técnica Federico Santa María, Valparaíso, Chile.], Adrien Wohrer [Group for Neural Theory - ENS].

ABFilter is a C++-library that allows the implementation of spatiotemporal filtering in video sequences using filters proposed by Adelson and Bergen in [62].

Simple cells in V1 are characterized by linear receptive fields where the neuron response is a weighted linear combination of the input stimulus inside its receptive field. By combining two simple cells in a linear manner it is possible to get direction-selective cells. The direction-selectivity refers to the property of a neuron to respond to the direction of the stimulus motion. The way to model this selectivity is by obtaining receptive fields oriented in space and time. Some characteristics of V1 complex cells can be explained using a nonlinear combination of V1 simple cells as it has been proposed by, e.g., Adelson and Bergen in [62]. Implementing these cells properly is a difficult problem and this library offers the possibility to easily implement a V1 layer which can serve as an input to subsequent cortical areas such as MT (see, e.g., the architecture developed in [65]).

The ABFilter library is under a CeCill-C open-source license.

- IDDN.FR.001.280017.000.S.P.2011.000.31235
- Version: v 1.0 (May 2011)
- Download: <http://www-sop.inria.fr/neuromathcomp/public/software/abfilter-1.0.tar.gz>

4.3. MotionLib: A Neural-Fields Model for Motion Estimation

Participants: Pierre Kornprobst [correspondant], Emilien Tlapale.

MotionLib implements the neural field model of motion estimation described in [52], using the Python programming language.

Motion integration is the core of the model: It implements a two-layer model with feedbacks that selects and diffuses motion signals. The main structure has been implemented here, allowing contributors to enrich this model easily. Several tools are also provided to visualize and analyze the distributed velocity fields obtained by this approach.

- License IDDN.FR.001.210029.000.S.P.2011.000.31235
- Version: v 1.0 (October 2011)
- Download: <http://www-sop.inria.fr/neuromathcomp/public/software/motion.zip>

4.4. Event neural assembly Simulation

Participants: Frederic Alexandre [INRIA Cortex Nancy], Bruno Cessac [correspondent], Rodrigo Cofre Torres, Jeremy Fix [INRIA Cortex Nancy], Olivier Rochel [INRIA Cortex Nancy], Sélim Kraria, Olivier Marre, Hassan Nasser, Horacio Rostro-Gonzalez, Vivien Robinet, Thierry Viéville [INRIA Cortex Nancy], Juan-Carlos Vasquez.

Enas is a library providing numerical tools for the simulation of neural networks and the analysis of spike trains either coming from neural simulators or from biological experiments.

It is designed mainly as

- An existing simulator plug-in (e.g. MVASpike or other simulators via the NeuralEnsemble meta-simulation platform),
- Additional modules for computations with neural unit assembly on standard platforms (e.g. Python, Matlab or the Scilab platform).
- Original modules for the analysis of spike train statistics intended to be used by the neuroscientists community.

Achievements include:

- Spike trains statistical analysis via Gibbs distributions. They are based on the estimation of a parametric Gibbs potential optimally characterizing the statistics of empirical spike trains (by minimisation of the Kullback-Leibler divergence between the empirical measure and the Gibbs measure). From this, classical statistical indicators such as firing rate, correlations, higher order moments and statistical entropy are obtained. Also, the form of the Gibbs potential provides essential informations on the underlying neural network and its structure. This method does not only allows us to estimate the spikes statistics but also to compare different models, thus answering such questions about the neural code as: are correlations (or time synchrony or a given set of spike patterns, . . .) significant with respect to rate coding?
- Spiking network programming for exact event's sequence restitution;
- Discrete neural field parameters algorithmic adjustments and time-constrained event-based network simulation reconciling clock and event based simulation methods.

Compared to existing libraries Enas offers new computational methods taking into account time constraints in neural networks (such as memory effects), based on theoretical methods rooted in statistical physics and applied mathematics. The algorithms used are based on linear programming, nonlinear parameter estimations, statistical methods. The C/C++ code has been organized as "bean java" to ease its use by programmers non specialized in advanced object programming. As a consequence the code is distributed in the form of an include source for the lightest and the most universal integration into users codes. The standard algorithms are based on the best free libraries in the domain such as gsl <http://www.gnu.org/software/gsl>.

Event neural assembly simulation is developed in gForge. It is under CeCILL C licence
APP logiciel Enas: IDDN.FR.001.360008.000.S.P.2009.000.10600.

Its development as a friendly software designed for the neuroscience community is our next purpose (ADT proposal).

Website: <http://enas.gforge.inria.fr/>

5. New Results

5.1. Statistical analysis of spike trains

Modern advances in neurophysiology techniques, such as two-photons imaging of calcium signals or micro-electrode arrays (MEA) electro-physiology, have made it possible to observe simultaneously the activity of large assemblies of neurons. Such experimental recordings provide a great opportunity to unravel the underlying interactions of neural assemblies and to understand how neural populations dynamically encode information. The goal of the present project is to propose to the neuroscientists community statistical methods and numerical tools to analysing the statistics of action potentials (spike trains) obtained from MEA recordings. Our work is grounded on one hand on theoretical results on Gibbs distributions in neural

networks and the other hand on a C/C++ library of algorithms developed jointly with the CORTEX INRIA team, freely available at <http://enas.gforge.inria.fr/>. We have collaborations with several labs specialized in MEA recording from the retina: Centro Interdisciplinario de Neurociencia de Valparaiso, Universidad de Valparaiso, Chile <http://www.cinv.cl/>; Department of Molecular Biology and Princeton Neuroscience Institute, Princeton University, USA <http://www.princeton.edu/neuroscience/>; Institut de la Vision, Paris <http://www.institut-vision.org/>.

5.1.1. A discrete time neural network model with spiking neurons. Dynamics with noise.

Participant: Bruno Cessac [correspondent].

We provide rigorous and exact results characterizing the statistics of spike trains in a network of leaky Integrate-and-Fire neurons, where time is discrete and where neurons are submitted to noise, without restriction on the synaptic weights. We show the existence and uniqueness of an invariant measure of Gibbs type and discuss its properties. We also discuss Markovian approximations and relate them to the approaches currently used in computational neuroscience to analyse experimental spike trains statistics. This work has appeared in Journal of Mathematical Biology [17].

5.1.2. Statistics of spike trains in conductance-based neural networks: Rigorous results

Participant: Bruno Cessac [correspondent].

We consider a conductance-based neural network inspired by the generalized Integrate and Fire model introduced by Rudolph and Destexhe in 1996. We show the existence and uniqueness of a unique Gibbs distribution characterizing spike train statistics. The corresponding Gibbs potential is explicitly computed. These results hold in the presence of a time-dependent stimulus and apply therefore to non-stationary dynamics. This establishes a rigorous ground for the current investigations attempting to characterize real spike trains data with Gibbs distributions, such as the Ising-like distribution, using the maximal entropy principle. This work has appeared in Journal of Mathematical Neuroscience [18].

5.1.3. Spike Train Statistics from Empirical Facts to Theory: The Case of the Retina.

Participants: Bruno Cessac [correspondent], Adrian Palacios [Centro de Neurociencia, Valparaiso, Chile].

This work focuses on methods from statistical physics and probability theory allowing the analysis of spike trains in neural networks. Taking as an example the retina we present recent works attempting to understand how retina ganglion cells encode the information transmitted to the visual cortex via the optical nerve, by analyzing their spike train statistics. We compare the maximal entropy models used in the literature of retina spike train analysis to rigorous results establishing the exact form of spike train statistics in conductance-based Integrate-and-Fire neural networks. This work is submitted in “Mathematical Problems in Computational Biology and Biomedicine” Springer, [54].

5.1.4. Gibbs distribution analysis of temporal correlations structure in retina ganglion cells

Participants: Michael Berry II [Department of Molecular Biolog, Princeton University, USA], Bruno Cessac [correspondent], Olivier Marre, Adrian Palacios [Centro de Neurociencia, Valparaiso, Chile], Juan-Carlos Vasquez.

We present a method to estimate Gibbs distributions with spatio-temporal constraints on spike trains statistics. We apply this method to spike trains recorded from ganglion cells of the salamander retina, in response to natural movies. Our analysis, restricted to a few neurons, performs more accurately than pairwise synchronization models (Ising) or the 1-time step Markov models (Marre et al. (2009)) to describe the statistics of spatio-temporal spike patterns and emphasizes the role of higher order spatio-temporal interactions This work has been accepted in Journal of Physiology, Paris [28] (in press).

5.1.5. A Markovian event-based framework for stochastic spiking neural networks

Participants: Olivier Faugeras, Jonathan Touboul.

In spiking neural networks, the information is conveyed by the spike times, that depend on the intrinsic dynamics of each neuron, the input they receive and on the connections between neurons. In this article we study the Markovian nature of the sequence of spike times in stochastic neural networks, and in particular the ability to deduce from a spike train the next spike time, and therefore produce a description of the network activity only based on the spike times regardless of the membrane potential process. To study this question in a rigorous manner, we introduce and study an event-based description of networks of noisy integrate-and-fire neurons, i.e. that is based on the computation of the spike times. We show that the firing times of the neurons in the networks constitute a Markov chain, whose transition probability is related to the probability distribution of the interspike interval of the neurons in the network. In the cases where the Markovian model can be developed, the transition probability is explicitly derived in such classical cases of neural networks as the linear integrate-and-fire neuron models with excitatory and inhibitory interactions, for different types of synapses, possibly featuring noisy synaptic integration, transmission delays and absolute and relative refractory period. This covers most of the cases that have been investigated in the event-based description of spiking deterministic neural networks.

This work has appeared in the Journal of Computational Neuroscience [26].

5.2. Coding by spikes

Our goal here is a better understanding of the extent to which computing and modeling with spiking neuron networks might be biologically plausible and computationally efficient. Based on a thorough characterization of the main constraints on spiking neural networks dynamics this has led us to propose new algorithms to infer the structure of the network from its spike trains and to propose an FPGA implementation of spiking neural networks.

5.2.1. Reverse-engineering of spiking neural networks parameters

Participants: Bruno Cessac [correspondent], Horacio Rostro-Gonzalez, Thierry Viéville [Cortex].

We consider the deterministic evolution of a time-discretized spiking network of neurons with connection weights having delays, modeled as a discretized neural network of the generalized integrate and fire (gIF) type. The purpose is to study a class of algorithmic methods allowing to calculate the proper parameters (synaptic weights) to reproduce exactly a given spike train generated by an hidden (unknown) neural network. This problem is linear (L) if the membrane potentials are observed and LP (Linear-Programming) if only spike times are observed, in the context of gIF models. The L or LP adjustment mechanism is local to each unit and has the same structure as an "Hebbian" rule. This paradigm is easily generalizable to the design of input-output spike train transformations. This means that we have a practical method to "program" a spiking network, i.e. find a set of parameters allowing us to exactly reproduce the network output, given an input

This work has been submitted in the Journal of Neural Engineering, 2011[25].

5.2.2. Development of FPGA-based efficient reconfigurable architectures for spiking neural networks

Participants: Bruno Cessac, Bernard Girau [INRIA Cortex], Horacio Rostro-Gonzalez, Cesar Torres-Huitzil [Information Technology Department, Polytechnic University of Victoria (UPV), Tamaulipas, Mexico], Thierry Viéville [Cortex, correspondent].

Spiking neural networks are able to perform very powerful computations with precise timed spikes. We are developing an FPGA (Field Programmable Gate Array) reconfigurable platform that enables the simulation of in silico models of spiking neural networks. Since the model is directly mapped into a FPGA device, the neural processing is accelerated and the time consumption reduced. We use VHDL and Handel-C to design the reconfigurable architecture of a discrete time Integrate-and-Fire model coded in CUDA, running on GPU.

This work has been accepted in Journal of Physiology, Paris [24].

5.2.3. Towards biologically inspired image coders

Participants: Marc Antonini [Laboratoire I3S, Sophia Antipolis, France], Pierre Kornprobst, Khaled Masmoudi [Laboratoire I3S, Sophia Antipolis, France].

In [51] we presented a novel bio-inspired and dynamic coding scheme for static images. Our coder aims at reproducing the main steps of the visual stimulus processing in the mammalian retina taking into account its time behavior. The main novelty of this work is to show how to exploit the time behavior of the retina cells to ensure, in a simple way, scalability and bit allocation. To do so, our main source of inspiration has been the biologically plausible retina model *Virtual Retina* described in Section 4.1. Following a similar structure, our model has two stages. The first stage is an image transform which is performed by the outer layers in the retina. Here it is modelled by filtering the image with a bank of difference of Gaussians with time-delays. The second stage is a time-dependent analog-to-digital conversion which is performed by the inner layers in the retina. Thanks to its conception, our coder enables scalability and bit allocation across time. Also, compared to the JPEG standards, our decoded images do not show annoying artefacts such as ringing and block effects. As a whole, this article shows how to capture the main properties of a biological system, here the retina, in order to design a new efficient coder.

5.3. Mean field methods

5.3.1. Noise-induced behaviors in neural mean field dynamics

Participants: Jonathan Touboul, Geoffroy Hermann, Olivier Faugeras.

The collective behavior of cortical neurons is strongly affected by the presence of noise at the level of individual cells. In order to study these phenomena in large-scale assemblies of neurons, we consider networks of firing-rate neurons with linear intrinsic dynamics and nonlinear coupling, belonging to a few types of cell populations and receiving noisy currents. Asymptotic equations as the number of neurons tends to infinity (mean field equations) are rigorously derived based on a probabilistic approach. These equations are implicit on the probability distribution of the solutions which generally makes their direct analysis difficult. However, in our case, the solutions are Gaussian, and their moments satisfy a closed system of nonlinear ordinary differential equations (ODEs), which are much easier to study than the original stochastic network equations, and the statistics of the empirical process uniformly converge towards the solutions of these ODEs. Based on this description, we analytically and numerically study the influence of noise on the collective behaviors, and compare these asymptotic regimes to simulations of the network. We observe that the mean field equations provide an accurate description of the solutions of the network equations for network sizes as small as a few hundreds of neurons. In particular, we observe that the level of noise in the system qualitatively modifies its collective behavior, producing for instance synchronized oscillations of the whole network, desynchronization of oscillating regimes, and stabilization or destabilization of stationary solutions. These results shed a new light on the role of noise in shaping collective dynamics of neurons, and gives us clues for understanding similar phenomena observed in biological networks.

This work has been accepted for publication in the SIAM Journal on Applied Dynamical Systems [72].

5.3.2. Mean Field description of and propagation of chaos in recurrent multipopulation networks of Hodgkin-Huxley and FitzHugh-Nagumo neurons

Participants: Javier Baladron, Diego Fasoli, Olivier Faugeras, Jonathan Touboul.

We derive the mean-field equations arising as the limit of a network of interacting spiking neurons, as the number of neurons goes to infinity. The neurons belong to a fixed number of populations and are represented either by the Hodgkin-Huxley model or by one of its simplified version, the FitzHugh-Nagumo model. The synapses between neurons are either electrical or chemical. The network is assumed to be fully connected. The maximum conductances vary randomly. Under the condition that all neurons initial conditions are drawn independently from the same law that depends only on the population they belong to, we prove that a propagation of chaos phenomenon takes place, namely that in the mean-field limit, any finite number

of neurons become independent and, within each population, have the same probability distribution. This probability distribution is solution of a set of implicit equations, either nonlinear stochastic differential equations resembling the McKean-Vlasov equations, or non-local partial differential equations resembling the McKean-Vlasov-Fokker-Planck equations. We prove the well-posedness of these equations, i.e. the existence and uniqueness of a solution. We also show the results of some preliminary numerical experiments that indicate that the mean-field equations are a good representation of the mean activity of a infinite size network, even for modest sizes. These experiment also indicate that the McKean-Vlasov-Fokker-Planck equations may be a good way to understand the mean-field dynamics through, e.g., a bifurcation analysis.

This work has been submitted for publication in the Journal of Mathematical Neuroscience [55].

5.3.3. *Three applications of GPU computing in neuroscience*

Participants: Javier Baladron, Olivier Faugeras.

GPUs are low cost highly parallel devices that are now not only used for graphics but also for numerical simulation. We present three applications of a computer system with multiple GPUs to the domain of theoretical neuroscience. The first application is to a continuous model of the primary visual area, the second to the simulation of a stochastic neural network, and the third to the computation of the probability distribution on the possible states of a network. In all three cases we show that the speed-up obtained by the use of GPUs has considerably helped answering a scientific or technological question.

This work has been accepted for publication in Computing in Science and Engineering [63].

5.4. Neural Fields

5.4.1. *Modelling the dynamics of contextual motion integration in the primate*

Participants: Heiko Neumann [Institute of Neural Information Processing, Ulm University, Ulm, Germany], Pierre Kornprobst, Guillaume Masson [Institut de Neurosciences de la Timone, UMR 6193, CNRS, Marseille, France], Emilien Tlapale.

The dynamics of motion integration show striking similarities when observed at neuronal, psychophysical, and oculomotor levels. Based on the inter-relation and complementary insights given by those dynamics, our goal is to investigate how basic mechanisms of dynamical cortical processing can be incorporated in a dynamical model to solve several aspects of 2D motion integration and segmentation.

Thanks to Emilien Tlapale PhD [13] (see also [16]), we have obtained the following results:

- We proposed a recurrent model of motion integration. Proposing a simple readout mechanism, we reproduced not only motion perception but also the dynamics of smooth pursuit eye movements on various line figures and gratings viewed through different apertures. Our model can also solve various contextual problems where extrinsic junctions should be eliminated, without relying on complex junction detectors or depth computation [71]. Finally, we have also shown how our model can be rewritten in the neural fields formalism (see [52] and the Software MotionLib), which has opened new perspectives as detailed in Section 5.4.2.
- We confronted our results to artificial and biological vision. To formalize the comparison against visual performance, we proposed a new evaluation methodology based on human visual performance by establishing a database of image sequences taken from biology and psychophysics literature [70], [69], [67]. We compared our results against the state of the art computer vision approaches and we found that our model also gives results comparable to recent computer vision approaches of motion estimation.

5.4.2. *Neural fields models for motion integration: Characterising the dynamics of multi-stable visual motion stimuli*

Participants: Olivier Faugeras, Pierre Kornprobst, Guillaume Masson [Institut de Neurosciences de la Timone, UMR 6193, CNRS, Marseille, France], Andrew Meso [Institut de Neurosciences de la Timone, UMR 6193, CNRS, Marseille, France], James Rankin, Emilien Tlapale, Romain Veltz.

In [57] we investigated the temporal dynamics of the neural processing of a multi-stable visual motion stimulus with two complementary approaches: psychophysical experiments and mathematical modelling. The so called “barber pole” stimulus is considered with an aperture configuration that supports horizontal (H), diagonal (D) or vertical (V) perceived directions for the same input. The phenomenon demonstrates an interesting variable and dynamic competition for perceptual dominance between underlying neural representations of the three directions. We probe the early processing from stimulus presentation to initial perceived direction (before perceptual reversals). Starting from a simplified neural fields model inspired from [13], we constructed a model of the necessary motion integration that shows a shift in perceptual dominance from D to either H or V with increasing duration. Further, the timing of this shift is shown to be controlled by a stimulus gain parameter analogous to contrast. In psychophysics experiments with concurrent eye movement recordings, observers report their perceived direction of motion for presentation durations between 0.1s and 0.5s. There is also a consistent transition in perceptual dominance from D to H/V as duration is increased. This trend, seen in both perceived direction decisions and eye movement patterns, is consistent with previous experiments using similar stimuli with an aperture configured for two (D/H) rather than three (D/H/V) states. The basic dynamic properties of the early transition from D to H/V are well predicted by the model. The experimental work additionally reveals asymmetric data patterns that guide adjustments to the model’s input equations. Observers have an H bias over V, which is also reflected in faster reaction times for H. In order to capture the bias between H and V a separate weighting is attributed to the local input corresponding to each state. The work presented forms a solid foundation for future experimental and modelling work investigating the longer term dynamics for which perceptual reversals are known to occur.

5.4.3. Analysis of a hyperbolic geometric model for visual texture perception

Participants: Pascal Chossat, Grégory Faye, Olivier Faugeras.

We study the neural field equations introduced by Chossat and Faugeras in [64] to model the representation and the processing of image edges and textures in the hypercolumns of the cortical area V1. The key entity, the structure tensor, intrinsically lives in a non-Euclidean, in effect hyperbolic, space. Its spatio-temporal behaviour is governed by nonlinear integro-differential equations defined on the Poincaré disc model of the two-dimensional hyperbolic space. Using methods from the theory of functional analysis we show the existence and uniqueness of a solution of these equations. In the case of stationary, i.e. time independent, solutions we perform a stability analysis which yields important results on their behavior. We also present an original study, based on non-Euclidean, hyperbolic, analysis, of a spatially localised bump solution in a limiting case. We illustrate our theoretical results with numerical simulations.

This work has been published in the Journal of Mathematical Neuroscience [21].

5.4.4. Bifurcation of Hyperbolic Planforms

Participants: Pascal Chossat, Grégory Faye, Olivier Faugeras.

Motivated by a model for the perception of textures by the visual cortex in primates, we analyze the bifurcation of periodic patterns for nonlinear equations describing the state of a system defined on the space of structure tensors, when these equations are further invariant with respect to the isometries of this space. We show that the problem reduces to a bifurcation problem in the hyperbolic plane D (Poincaré disc). We make use of the concept of a periodic lattice in D to further reduce the problem to one on a compact Riemann surface D/Γ , where Γ is a cocompact, torsion-free Fuchsian group. The knowledge of the symmetry group of this surface allows us to use the machinery of equivariant bifurcation theory. Solutions which generically bifurcate are called “H-planforms”, by analogy with the “planforms” introduced for pattern formation in Euclidean space. This concept is applied to the case of an octagonal periodic pattern, where we are able to classify all possible H-planforms satisfying the hypotheses of the Equivariant Branching Lemma. These patterns are, however, not straightforward to compute, even numerically, and in the last section we describe a method for computation illustrated with a selection of images of octagonal H-planforms.

This work has been published in the Journal of Nonlinear Science [19].

5.4.5. *Bifurcation diagrams and heteroclinic networks of octagonal H-planforms*

Participants: Grégory Faye, Pascal Chossat [correspondent].

This paper completes the classification of bifurcation diagrams for H-planforms in the Poincaré disc \mathbb{D} whose fundamental domain is a regular octagon. An H-planform is a steady solution of a PDE or integro-differential equation in \mathbb{D} , which is invariant under the action of a lattice subgroup Γ of $U(1, 1)$, the group of isometries of \mathbb{D} . In our case Γ generates a tiling of \mathbb{D} with regular octagons. This problem was introduced in [19] as an example of spontaneous pattern formation in a model of image feature detection by the visual cortex where the features are assumed to be represented in the space of structure tensors. Under "generic" assumptions the bifurcation problem reduces to an ODE which is invariant by an irreducible representation of the group of automorphisms \mathcal{G} of the compact Riemann surface \mathbb{D}/Γ . The irreducible representations of \mathcal{G} have dimension one, two, three and four. The bifurcation diagrams for the representations of dimension less than four have already been described and correspond to already well known group actions. In the present work we compute the bifurcation diagrams for the remaining three irreducible representations of dimension four, thus completing the classification. In one of these cases, there is generic bifurcation of a heteroclinic network connecting equilibria with two different orbit types.

This work has been accepted for publication in the Journal of Nonlinear Science [22].

5.4.6. *Hopf bifurcation curves in neural field networks with space-dependent delays*

Participant: Romain Veltz.

We give an analytical parametrization of the curves of purely imaginary eigenvalues in the delay-parameter plane of the linearized neural field network equations with spacedependent delays. In order to determine if the rightmost eigenvalue is purely imaginary, we have to compute a finite number of such curves; the number of curves is bounded by a constant for which we give an expression. The Hopf bifurcation curve lies on these curves.

This work has appeared in the Comptes Rendus Mathématiques de l'Académie des Sciences [30].

5.4.7. *Stability of the stationary solutions of neural field equations with propagation delays*

Participants: Olivier Faugeras, Romain Veltz.

We consider neural field equations with space-dependent delays. Neural fields are continuous assemblies of mesoscopic models arising when modeling macroscopic parts of the brain. They are modeled by nonlinear integro-differential equations. We rigorously prove, for the first time to our knowledge, sufficient conditions for the stability of their stationary solutions. We use two methods 1) the computation of the eigenvalues of the linear operator defined by the linearized equations and 2) the formulation of the problem as a fixed point problem. The first method involves tools of functional analysis and yields a new estimate of the semigroup of the previous linear operator using the eigenvalues of its infinitesimal generator. It yields a sufficient condition for stability which is independent of the characteristics of the delays. The second method allows us to find new sufficient conditions for the stability of stationary solutions which depend upon the values of the delays. These conditions are very easy to evaluate numerically. We illustrate the conservativeness of the bounds with a comparison with numerical simulation.

This work has appeared in the Journal of Mathematical Neuroscience [29].

5.4.8. *Neural Mass Activity, Bifurcations and Epilepsy*

Participants: Patrick Chauvel [INSERM U751, Marseille, Assistance Publique-Hopitaux de Marseille Tim-one, and Université Aix-Marseille, Marseille], Olivier Faugeras, Jonathan Touboul, Fabrice Wendling [INSERM, U642, Rennes].

We propose a general framework for studying neural mass models defined by ordinary differential equations. By studying the bifurcations of the solutions to these equations and their sensitivity to noise we establish an important relation, similar to a dictionary, between their behaviors and normal and pathological, especially epileptic, cortical patterns of activity. We then apply this framework to the analysis of two models that feature most phenomena of interest, the Jansen and Rit model, and the slightly more complex model recently proposed by Wendling and Chauvel. This model-based approach allows to test various neurophysiological hypotheses on the origin of pathological cortical behaviors and to investigate the effect of medication. We also study the effects of the stochastic nature of the inputs which gives us clues about the origins of such important phenomena as interictal spikes, inter-ictal bursts and fast onset activity, that are of particular relevance in epilepsy.

This work has appeared in *Neural Computation* [27].

6. Partnerships and Cooperations

6.1. European Initiatives

6.1.1. FP7 Projects

6.1.1.1. BRAINSCALES

Title: BrainScaleS: Brain-inspired multiscale computation in neuromorphic hybrid systems

Type: COOPERATION (ICT)

Defi: Brain-inspired multiscale computation in neuromorphic hybrid systems

Instrument: Integrated Project (IP)

Duration: January 2011 - December 2014

Coordinator: Universitaet Ruprecht- Karls Heidelberg (Germany)

See also: <http://brainscales.kip.uni-heidelberg.de/>

Abstract: The BrainScaleS project aims at understanding function and interaction of multiple spatial and temporal scales in brain information processing. The fundamentally new approach of BrainScaleS lies in the in-vivo biological experimentation and computational analysis. Spatial scales range from individual neurons over larger neuron populations to entire functional brain areas. Temporal scales range from milliseconds relevant for event based plasticity mechanisms to hours or days relevant for learning and development. In the project generic theoretical principles will be extracted to enable an artificial synthesis of cortical-like cognitive skills. Both, numerical simulations on petaflop supercomputers and a fundamentally different non-von Neumann hardware architecture will be employed for this purpose. Neurobiological data from the early perceptual visual and somatosensory systems will be combined with data from specifically targeted higher cortical areas. Functional databases as well as novel project-specific experimental tools and protocols will be developed and used. New theoretical concepts and methods will be developed for understanding the computational role of the complex multi-scale dynamics of neural systems in-vivo. Innovative in-vivo experiments will be carried out to guide this analytical understanding. Multiscale architectures will be synthesized into a non-von Neumann computing device realised in custom designed electronic hardware. The proposed Hybrid Multiscale Computing Facility (HMF) combines microscopic neuromorphic physical model circuits with numerically calculated mesoscopic and macroscopic functional units and a virtual environment providing sensory, decision-making and motor interfaces. The project also plans to employ petaflop supercomputing to obtain new insights into the specific properties of the different hardware architectures. A set of demonstration experiments will link multiscale analysis of biological systems with functionally and architecturally equivalent synthetic systems and offer the possibility for quantitative statements on the validity of theories bridging multiple scales.

The demonstration experiments will also explore non-von Neumann computing outside the realm of brain-science. BrainScaleS will establish close links with the EU Brain-i-Nets and the Blue Brain project at the EPFL Lausanne. The consortium consists of a core group of 10 partners with 13 individual groups. Together with other projects and groups the BrainScaleS consortium plans to make important contributions to the preparation of a future FET flagship project. This project will address the understanding and exploitation of information processing in the human brain as one of the major intellectual challenges of humanity with vast potential applications.

This project started on January 1st, 2011 and is funded for four years.

6.1.1.2. FACETS-ITN

Title: FACETS-ITN

Instrument: Initial Training Network (ITN)

Duration: September 2009 - August 2013

Coordinator: Universität Heidelberg- Ruprecht-Karls (Germany)

See also: <http://facets.kip.uni-heidelberg.de/ITN/index.html>

This 'Marie-Curie Initial Training Network' (funded by the EU) involves 15 groups at European Research Universities, Research Centers and Industrial Partners in 6 countries. It funds two PhD students in the NeuroMathComp group. Website: <http://facets.kip.uni-heidelberg.de/ITN/index.html>

6.1.1.3. SEARISE

Title: SEARISE

Defi: Smart Eyes, Attending and Recognizing Instances of Salient Events

Duration: March 2008 - February 2011

Coordinator: Fraunhofer Institute for Applied Information Technology FIT (Germany)

Other partners:

Institution: Ulm University (Germany)

Laboratory: Department of Neural Information Processing

Researcher: Heiko Neumann

See also: <http://www.searise.eu/web/doku.php>

Abstract: The SEARISE project developed a trinocular active cognitive vision system, the Smart-Eyes, for detection, tracking and categorization of salient events and behaviours. Inspired by the human visual system, a cyclopean camera performs wide range monitoring of the visual field while active binocular stereo cameras will fixate and track salient objects, mimicking a focus of attention that switches between different interesting locations. The core of this artificial cognitive visual system is a dynamic hierarchical neural architecture – a computational model of visual processing in the brain. Information processing in Smart-Eyes is highly efficient due to a multi-scale design: Controlled by the cortically plausible neural model, the active cameras provide a multi-scale video record of salient events. The processing self-organizes to adapt to scale variations and to assign the majority of computational resources to the informative parts of the scene. The Smart-Eyes system has been tested in real-life scenarios featuring the activity of people in different scales. In a long-range distance scenario, the system analysed crowd behaviour of sport fans in a football arena. In a short range scenario, the system analysed the behaviour of small groups of people and single individuals.

6.1.2. Collaborations in European Programs, except FP7

6.1.2.1. ERC NerVi

Program: ERC IDEAS

Project acronym: NerVi

Project title: From single neurons to visual perception

Duration: January 2009 - December 2013

Coordinator: Olivier Faugeras

Abstract: The project is to develop a formal model of information representation and processing in the part of the neocortex that is mostly concerned with visual information. This model will open new horizons in a well-principled way in the fields of artificial and biological vision as well as in computational neuroscience. Specifically the goal is to develop a universally accepted formal framework for describing complex, distributed and hierarchical processes capable of processing seamlessly a continuous flow of images. This framework features notably computational units operating at several spatiotemporal scales on stochastic data arising from natural images. Mean-field theory and stochastic calculus are used to harness the fundamental stochastic nature of the data, functional analysis and bifurcation theory to map the complexity of the behaviours of these assemblies of units. In the absence of such foundations, the development of an understanding of visual information processing in man and machines could be greatly hindered. Although the proposal addresses fundamental problems, its goal is to serve as the basis for ground-breaking future computational development for managing visual data and as a theoretical framework for a scientific understanding of biological vision.

6.2. International Initiatives

6.2.1. INRIA Associate Teams

6.2.1.1. CORTINA

Title: Retina neural network coding

principal investigator: Frédéric Alexandre (INRIA Cortex)

International Partner (for NeuroMathComp):

Institution: University of Valparaiso (Chile)

Laboratory: Centro Interdisciplinario de Neurociencia de Valparaiso

Researcher: Adrian PALACIOS

International Partner:

Institution: UTFSM Valparaiso (Chile)

Laboratory: Direccion General de Investigacion y Postgrado

Researcher: Maria-Jose ESCOBAR

Duration: 2011 - 2013

See also: <http://cortex.loria.fr/Projects/Cortina>

Much progress has been made in the last decades in understanding the basic organization and function of the nervous system in general. Contributions to this end have come from various domains including computational neuroscience and numerical science of the information in general. The goal of this associate team is to combine our complementary expertise, from experimental biology and mathematical models (U de Valparaiso and U Federico Santa-Maria) to computational neuroscience (CORTEX and NEUROMATHCOMP), in order to develop numerical tools for the study and characterization of neural coding and related sensory-motor loops. Recording and modeling spike trains from the retina neural network, an accessible part of the brain, is a difficult task that our partnership

can address, what constitute an excellent and unique opportunity to work together sharing our experience and to focus in developing computational tools for methodological innovations. To understand How the neural spike coding from natural image sequences works we are addressing the following issues: How visual signals are coded at earlier steps in the case of natural vision? What are their functions? What are the computational "coding" principles explaining (in artificial or biological system) the statistical properties of natural images? We wish to advance our actual knowledge in natural and artificial visual signals processing and apply it to the field of education; to foster better capacities for learning and memory; sensory prosthesis design, to will help unpaired sensory persons to sense the world and physical rehabilitation, among others. In the context of the cooperation between the INRIA and Chile we propose to develop new neural decoding algorithms that are transverse to several field and applications.

6.2.2. Participation In International Programs

6.2.2.1. KEOPS

Title: Algorithms for modeling the visual system: From natural vision to numerical applications.

principal investigator: Thierry Viéville (Cortex)

International Partner for NeuroMathComp:

Institution: University of Valparaiso (Chile)

Laboratory: Centro Interdisciplinario de Neurociencia de Valparaiso

Researcher: Adrian PALACIOS

International Partner:

Institution: UTFSM Valparaiso (Chile)

Laboratory: Direccion General de Investigacion y Postgrado

Researcher: Maria-Jose ESCOBAR

Duration: 2011 - 2013

See also: <http://cortex.loria.fr/Research/Keops>

KEOpS attempts to study and model the non-standard behavior of retinal (ganglion cells) sensors observed in natural scenarios. KEOpS also attempts to incorporate the resulting models into real engineering applications as new dynamical early-visual modules. The retina, an accessible part of the brain, is a unique model for studying the neural coding principles for natural scenarios. A recent study proposes that some visual functions (e.g. movement, orientation, anticipatory temporal prediction, contrast), thought to be the exclusive duty of higher brain centers, are actually carried at the retina level. The anatomical and physiological segregation of visual scenes into spatial, temporal and chromatic channels begins at the retina through the action of local neural networks. However, how the precise articulation of this neural network contributes to local solutions and global perception necessary to resolve natural task remains in general a mystery. KEOpS thus attempts to study the complexity of retinal ganglion cells (the output to the brain) behaviors observed in natural scenarios² and to apply this result to artificial visual systems. We revisit both the retinal neural coding information sent to the brain, and at the same time, the development of new engineering applications inspired by the understanding of such neural encoding mechanisms. We develop an innovative formalism that takes the real (natural) complexity of retinal responses into account. We also develop new dynamical early-visual modules necessary to solve visual problems task.

7. Dissemination

7.1. Animation of the scientific community

Bruno Cessac is a reviewer for the CONYCIT (Chile) and COFECUB (Brasil) program and for the journals *Physica D*, *Nonlinearity*, *Chaos*, *Journal of Statistical Physics*, *IEEE Transaction in Neural Networks*, *Journal of Mathematical Biology*, *Journal of Computational Neuroscience*. He is in charge of internships organisation in the Master of Computational Biology, Nice.

Pascal Chossat is deputy scientific director of INSMI, the newly created mathematics institute of CNRS, in charge of the international relations of CNRS in this field. He is the coordinator of a geographic EraNet (EC program) named New Indigo for the development of scientific networks between European member states and India.

Olivier Faugeras is a member of the French Academy of Sciences and the French Academy of Technology. He is on the Editorial board of the *International Journal of Computer Vision (IJCV)*. He is the co-editor in chief of the *Journal of Mathematical neuroscience*, Springer: Website: <http://www.mathematical-neuroscience.com/> He is a member of the Institut Thématique Multi-organismes Neurosciences, Sciences cognitives, Neurologie, Psychiatrie. He is a member of the ERC PE1 panel. He co-organized a thematic semester at CIRM, Luminy, France, October-December 2011, on “Theoretical, Mathematical and Computational Neuroscience” which featured four one-week workshops and six one-week courses. For more information see Website: <http://www-sop.inria.fr/manifestations/SemesterCirm/>.

Pierre Kornprobst has been the coordinator of the MSc 2 in Computational Biology and Biomedicine (Université Nice Sophia Antipolis) until September 2011, together with Frédéric Cazals (Website: <http://cbb.unice.fr>).

7.2. Teaching

- Licence 1 : Grégory Faye, *Mathematics for Biology*, 50h, L1, Université Nice Sophia Antipolis, France.
- Licence 3 : Hassan Nasser, *Electronique numérique*, 36h, L3, Université de Nice Sophia-Antipolis, France
- License 3 : Hassan Nasser, *Microprocesseurs*, 28h, L3, Université de Nice Sophia-Antipolis, France
- License 3 : Mathieu Galtier, *Mathématiques*, Ecole des Mines, France.
- Master: Bruno Cessac, *Neuronal dynamics*, 36 hours, Master of Computational Biology and Biomedicine, Université Nice Sophia Antipolis, France.
- Master: Olivier Faugeras, *Mathematical Methods for Neuroscience*, 27h, M2, ENS Paris, France. 15h were taught by Grégory Faye.

PhD & HdR

PhD : Mathieu Galtier, “A mathematical approach to unsupervised learning in recurrent neural networks”, December 2011, supervised by Olivier Faugeras [11].

PhD : Horacio Rostro, “Computing with spikes, architecture, properties and implementation of emerging paradigms”, Université Nice Sophia Antipolis, January 2011, supervised by Bruno Cessac and Thierry Viéville [12].

PhD : Emilien Tlapale, “Modelling the dynamics of contextual motion integration in the primate”, Université Nice Sophia Antipolis, January 2011, supervised by Pierre Kornprobst and Guillaume S. Masson [13].

PhD : Juan-Carlos Vasquez, “Analyzing the neural code, mathematical and computational properties of spiking neural networks”, Université Nice Sophia Antipolis, March 2011, supervised by Bruno Cessac and Thierry Viéville [14].

PhD : Romain Veltz, *Nonlinear analysis methods in neural field models*, Univ Paris Est ED MSTIC, December 16th 2011, supervised by O. Faugeras [15].

PhD: Geoffroy Hermann, “Some mean field equations in neuroscience”, 08/01/08, supervised by Olivier Faugeras and Jonathan Touboul. January 2012 [66].

PhD in progress: Rodrigo Cofre-Torres, «Statistics of spike trains and neuronal structures», 2014, supervised by Bruno Cessac [60].

PhD in progress: Grégory Faye, “Symmetry breaking and pattern formation in neural field equations”, 2012, supervised by P. Chossat and O. Faugeras.

PhD in progress: Diego Fasoli, “Mean-field theory of realistic spiking neurons”, Université de Nice Sophia-Antipolis, defence planned in 2013, supervised by Olivier Faugeras.

PhD in progress: Javier Baladron, “Parallel implementations of mean field and neural field equations”, Université de Nice Sophia-Antipolis, defence planned in 2013, supervised by Olivier Faugeras.

PhD in progress: Hassan Nasser, «Reproducing and anticipating retinal responses», defence planned in 2013, supervised by Bruno Cessac .

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Publications of the year

Doctoral Dissertations and Habilitation Theses

- [11] M. GALTIER. *A mathematical approach to unsupervised learning in recurrent neural networks*, ParisTech, December 2011.
- [12] H. ROSTRO-GONZALEZ. *Computing with spikes, architecture, properties and implementation of emerging paradigms*, EDSTIC, 2011.
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- [14] J.-C. VASQUEZ. *Analyzing the neural code, mathematical and computational properties of spiking neural networks*, EDSTIC, 2011.
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