

# **Activity Report 2011**

# **Project-Team POEMS**

Wave propagation: mathematical analysis and simulation

IN COLLABORATION WITH: Propagation des ondes : étude mathématique et simulation (POEMS)

RESEARCH CENTER Paris - Rocquencourt

**THEME** 

Computational models and simulation

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# 1. Members

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# 2. Overall Objectives

### 2.1. Introduction

The propagation of waves is one of the most common physical phenomena one can meet in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) and to the scale of the universe (electromagnetic waves, gravity waves), to the scale of the atom (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us.

The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in the various domains of physics and engineering science.

The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas justify together a research project in Scientific Computing entirely devoted to this theme.

The project POEMS is an UMR (Unité Mixte de Recherche) between CNRS, ENSTA and INRIA (UMR 7231). The general activity of the project is oriented toward the conception, the analysis, the numerical approximation, and the control of mathematical models for the description of wave propagation in mechanics, physics, and engineering sciences.

Beyond the general objective of contributing to the progress of the scientific knowledge, four goals can be ascribed to the project:

- the development of an expertise relative to various types of waves (acoustic, elastic, electromagnetic, gravity waves, ...) and in particular for their numerical simulation,
- the treatment of complex problems whose simulation is close enough to real life situations and industrial applications,
- the development of original mathematical and numerical techniques,
- the development of computational codes, in particular in collaboration with external partners (scientists from other disciplines, industry, state companies...)

# 2.2. Highlights

Among the significative scientific advances and successes of this year, that are illustrated by the finalization of several PhD theses, we would like to emphasize:

• The diversification and intensification of our research in the domain of ultra-sonic non destructive testing in the framework of a long term collaboration with CEA LIST. One spectacular concretization of this collaboration is the PhD thesis of S. Impériale about the modeling of piezoelectric sensors. A success in term of recognition of our activity in this fields is the acceptation of the European project.

Several spectacular advances in the mathematical understanding of electromagnetic wave propagation in metamaterials (in the more general sense of the term and the development of corresponding numerical methods. These progresses have been successfully recognized via the ANR Project METAMATH, whose Poems is coordinator, on the thematic of metamaterials, a major topic for Poems in the forthcoming years.

• A pioneering work on the full modelization by physical models of a concert piano via the PhD thesis of J. Chabassier. This is an exemplary success of a multi disciplinary collaboration with the Unity of Mechanics at ENSTA (A. Chaigne)

Let us also mention the arrival of two new CNRS researchers, Marc Bonnet (DR) and Stéphanie Chaillat (CR), which bring new competences in the domains of integral equations and inverse problems.

# 3. Scientific Foundations

## 3.1. Mathematical analysis and simulation of wave propagation

Our activity relies on the existence of mathematical models established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation (or a system of partial differential equations) of the hyperbolic type that are often (but not always) linear for most of the applications we are interested in. The prototype equation is the wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0,$$

which can be directly applied to acoustic waves but which also constitutes a simplified scalar model for other types of waves (This is why the development of new numerical methods often begins by their application to the wave equation). Of course, taking into account more realistic physics will enrich and complexify the basic models (presence of sources, boundary conditions, coupling of models, integro-differential or non linear terms,...)

It is classical to distinguish between two types of problems associated with these models: the time domain problems and the frequency domain (or time harmonic) problems. In the first case, the time is one of the variables of which the unknown solution depends and one has to face an evolution problem. In the second case (which rigorously makes sense only for linear problems), the dependence with respect to time is imposed a priori (via the source term for instance): the solution is supposed to be harmonic in time, proportional to  $e^{i\omega t}$ , where  $\omega>0$  denotes the pulsation (also commonly, but improperly, called the frequency). Therefore, the time dependence occurs only through this pulsation which is given a priori and plays the rôle of a parameter: the unknown is only a function of space variables. For instance, the wave equation leads to the Helmholtz wave equation (also called the reduced wave equation):

$$-c^2\Delta u - \omega^2 u = 0.$$

These two types of problems, although deduced from the same physical modelling, have very different mathematical properties and require the development of adapted numerical methods.

However, there is generally one common feature between the two problems: the existence of a dimension characteristic of the physical phenomenon: the wavelength. Intuitively, this dimension is the length along which the searched solution varies substantially. In the case of the propagation of a wave in an heterogeneous medium, it is necessary to speak of several wavelengths (the wavelength can vary from one medium to another). This quantity has a fundamental influence on the behavior of the solution and its knowledge will have a great influence on the choice of a numerical method.

Nowadays, the numerical techniques for solving the basic academic and industrial problems are well mastered. A lot of companies have at their disposal computational codes whose limits (in particular in terms of accuracy or robustness) are well known. However, the resolution of complex wave propagation problems close to real applications still poses (essentially open) problems which constitute a real challenge for applied mathematicians. A large part of research in mathematics applied to wave propagation problems is oriented towards the following goals:

- the conception of new numerical methods, more and more accurate and high performing.
- the treatment of more and more complex problems (non local models, non linear models, coupled systems, ...)
- the study of specific phenomena or features such as guided waves, resonances,...
- the development of approximate models in various situations,
- imaging techniques and inverse problems related to wave propagation.

# 4. Application Domains

### 4.1. Introduction

We are concerned with all application domains where linear wave problems arise: acoustics and elastodynamics (including fluid-structure interactions), electromagnetism and optics, and gravity water waves. We give in the sequel some details on each domain, pointing out our main motivations and collaborations.

# 4.2. Acoustics

As the acoustic propagation in a fluid at rest can be described by a scalar equation, it is generally considered by applied mathematicians as a simple preliminary step for more complicated (vectorial) models. However, several difficult questions concerning coupling problems have occupied our attention recently. Aeroacoustics, or more precisely, acoustic propagation in a moving compressible fluid, is for our team a new and very challenging topic, which gives rise to a lot of open questions, from the modeling (Euler equations, Galbrun equations) to the numerical approximation of such models (which poses new difficulties). Our works in this area are partially supported by EADS and Airbus. The typical objective is to reduce the noise radiated by Airbus planes. Vibroacoustics, which concerns the interaction between sound propagation and vibrations of thin structures, also raises up a lot of relevant research subjects.

Both applications (Aeroacoustics and Vibroacoustics) led us in particular to develop an academic research between volumis methods and integral equations in time domain.

Finally, A particularly attractive application concerns the simulation of musical instruments, whose objectives are both a better understanding of the behavior of existing instruments and an aid for the manufacturing of new instruments. The modeling and simulation of the timpani and of the guitar have been carried out in collaboration with A. Chaigne of ENSTA. We are currently on the piano.

# 4.3. Electromagnetism

This is a particularly important domain, first because of the very important technological applications but also because the treatment of Maxwell's equations is much more technically involved from the mathematical point of view that the scalar wave equation. Applied mathematics for electromagnetism during the last ten years have mainly concerned stealth technology, electromagnetic compatibility, design of optoelectronic microcomponents or smart materials. Stealth technology relies in particular on the conception and simulation of new absorbing materials (anisotropic, chiral, non-linear...). The simulation of antennas raises delicate questions related to the complexity of the geometry (in particular the presence of edges and corners). In optics, the development of the Mmcro and nano optics has made recently fantastic progress and the thematic of metamaterials (with negative index of refraction) opens new amazing applications. For all these reasons, we are developing an intense research in the following areas

- Highly accurate and hybrid numerical methods in collaboration with CEA (Gramat) and ONERA (Toulouse).
- Electromagmetic wave propagation in periodic media.
- Development of simplified approximate models by asymptotic analysis for various applications : boundary layers, thin coatings, thin domains, thin wires and cables, ...
- Mathematical and numerical questions linked to the modeling of metamaterials.

## 4.4. Elastodynamics

Wave propagation in solids is with no doubt, among the three fundamental domains that are acoustics, electromagnetism and elastodynamics, the one that poses the most significant difficulties from mathematical and numerical points of view.

Our activity on this topic began with applications in geophysics, which unfortunately has been forced to slow down in the middle of the 90's due to the disengagement of French oil companies in matter of research. However it has seen a most welcomed rebound through new academic problems (in parficular surface waves, perfectly matched layers techniques, inverse problems in wave guides) and industrial contacts, more precisely with CEA-LIST with which we have developed a long term collaboration in the domain of non destructive testing by ultrasounds. The most recent problems we have been dealing with in this domain concern elastic wave propagation in plates, the modeling of piezoelectric devices or elastic wave propagation in highly heterogeneous media.

# 5. Software

### 5.1. Introduction

We are led to develop two types of software. The first category is prototype softwares: various softwares are developed in the framework of specific research contracts (and sometimes sold to the contractor) or during PhD theses. They may be also contributions to already existing softwares developed by other institutions such as CEA, ONERA or EDF. The second category is advanced software which are intended to be developed, enriched and maintained over longer periods. Such softwares are devoted to help us for our research and/or promote our research. We have chosen to present here only our advanced softwares.

### 5.2. MELINA

This software has been developed under the leadership of D. Martin for several years in order to offer to the researchers a very efficient tool (in Fortran 77 and object oriented) for easily implementing finite element based original numerical methods for solving partial differential equations. It has specific and original potential in the domain of time harmonic wave problems (integral representations, spectral DtN conditions,...). Nowadays, it is fully functional in various application areas (acoustics and aeroacoustics, elastodynamics, electromagnetism, water waves). It is an open source software with on line documentation available at

### http://homepage.mac.com/danielmartin/melina/

The software is regularly used in about 10 research laboratories (in France and abroad) and number of research papers have published results obtained with MELINA (see the Web site). Moreover, every 2 years, a meeting is organized which combines a workshop which teaches new users with presentations by existing users.

During the last four years, apart from various local improvements of the code, new functionalities have been developed:

- Higher order finite elements (up to 10<sup>th</sup> order),
- Higher order quadrature formulae,
- DtN boundary conditions in 3D.

A new C++ version of the software is under development.

## 5.3. MONTJOIE

Montjoie is a software for the efficient and accurate wave propagation numerical modeling in both time dependent or time harmonic regimes in various domains of application: acoustics, aeroacoustics, elastodynamics and electromagnetism. It is based essentially on the use of hexahedral-dominant (including a small part of tetrahedra, pyramids and prisms) conforming meshes and continuous or discontinuous Galerkin approximations. The use of tensor product basis functions coupled to appropriate numerical quadrature techniques leads to important gains in both computing time and memory storage. Various techniques for treating unbounded domains have been incorporated: DtN maps, local absorbing conditions, integral representations and PMLs.

We have written an interface for the use of other libraries: SELDON, a C++ linear algebra library (interfaced with BLAS and LAPACK) used for iterative linear solvers, MUMPS, PASTIX and UMFPACK for sparse direct solvers, ARPACK for eigenvalue computation, METIS and SCOTCH for mesh decomposition. Except for trivial geometries, the mesh generation is not part of the code. It can be done with Modulef, Gmsh, Ghs3D or Cubit.

This code has been developed by Marc Duruflé during his PhD thesis (in 2006). Some other contributors have brought more specific enrichments to the code. The online documentation is available at :

http://montjoie.gforge.inria.fr/.

The main contributions of 2010 have been the following:

- parallelization of the interface with ARPACK,
- unitary tests for SELDON and Montjoie aiming at stabilizing the code,
- implementation of H(div) finite elements in 2D and 3D in collaboration with Morgane Bergot,
- optimization of the numerical simulation of the piano in collaboration with Juliette Chabassier (multithreading for the string nonlinear equations, improvement of the parallelization).

### 5.4. XLIFE++

During 2011, we performed a deep analysis of the two finite elements software developed by the lab (Melina and Monjoie) in order to propose a new software in C++ with extended capabilities and more integrated tools. The results of this analysis lead us to keep the general philosophy of Melina software (unified variational approach) but with major evolutions: integrated meshing tools, new variational description allowing FEM, BEM and DG formulation in an unified framework, global and local finite element computations, new approach to take into account essential boundary conditions and high performance computing skills (multithread and GPU computation). This new development will partly be supported by the Simposium european project dedicated to Non Destructive Testing tools (leader CEA/LIST, from september 2011 to august 2014) which requires some numerical simulation tools such as finite element library. It also a collaborative project with IRMAR (Rennes University). This new library, named xlife++ for eXtended LIbrary of Finite Element in C++, is an open source library (LGPL license) and its repository is the INRIA Gforge.

# 6. New Results

# 6.1. Numerical methods for time domain wave propagation

### 6.1.1. High Order Theta Scheme for the linear wave equation.

Participants: Juliette Chabassier, Sébastien Impériale.

We have pursued our work about a new class of high order implicit three time step schemes for semi-discretized wave equations of the form

$$\frac{d^2}{dt^2}u_h + K_h u_h = 0, \quad u_h(0) = u_{0,h}, \quad \frac{du_h}{dt}(0) = u_{1,h}, \tag{1}$$

where  $K_h$  is a symmetric positive definite matrix. For such problems, explicit three time step schemes generally show good performances but present two major drawbacks, in some situations, that have not yet been completely overcome:

- If the mesh presents elements of quite different sizes, the time step must be adapted to the smallest one because of the CFL condition.
- If the mass matrix is non diagonal or non block-diagonal, its inversion (at least one time per iteration)
  can lead to a dramatic over cost of the schemes, which is obviously not the case with implicit schemes
  for which a matrix has to be inverted in any case.

A natural way to avoid this restriction is to use local time stepping techniques which divides into two categories. First, the locally implicit technique, which is optimal in term of CFL restriction but "only" second order accurate in time, and requires the inversion of interface matrices. Second, the fully explicit local time stepping, as developed that enables to achieve high order time stepping but without (up to now) a full control over the CFL condition.

This is why we construct generalized implicit  $\theta$ -scheme using the modified equation approach to obtain 4th order approximation. These schemes introduce 2 degree of freedom  $\theta$  and  $\varphi$  and can be written under a general form as

$$\frac{u_h^{n+1} - 2u_h^n + u_h^{n-1}}{\Delta t^2} + K_h \left(\theta u_h^{n+1} + (1 - 2\theta) u_h^n + \theta u_h^{n-1}\right) 
+ \left(1 - \frac{\theta}{12}\right) K_h^2 \left(\varphi u_h^{n+1} + (1 - 2\varphi) u_h^n + \varphi u_h^{n-1}\right) = 0.$$

The parameters  $\theta$  and  $\varphi$  are chosen as functions of the time step through the resolution of an optimization problem: we consider that the time step is given and we try to obtain a scheme that minimizes the consistency errors still being stable. The limit problem when the time step tends is infinite gives an optimal unconditionally stable fourth order scheme. This work has been submitted for publication.

## 6.1.2. Discontinuous Galerkin Methods for wave equations

Participants: Patrick Joly, Antoine Tonnoir.

This has been the subject of the internship of A. Tonnoir and can be seen as a contribution to the mathematical analysis of coupled BEM / DG methods for time domain diffraction problems (see section 6.1.4). We did not pretend to make a real contribution to the field of the analysis of DG methods, but wanted to understand in sufficient detail the quite surprising observation that the non-centered (in space) schemes, provided by the use of non centered fluxes in the DG approach, leads to a better accuracy than the centered schemes issued for central fluxes! This has been done (for the simple 1D wave equation) from two points of view: the direct energy method and the Fourier analysis (or dispersion analysis) on regular grids.

### 6.1.3. Analysis of time domain boundary integral equations

Participants: Patrick Joly, Nicolas Le Guillarme.

This has been developed again as a part of the mathematical analysis of coupled BEM / DG methods for time domain diffraction problems (see section 6.1.4). With J. Rodríguez, we have revisited the analysis of the coercivity / continuity properties of space-time boundary integral associated to wave diffraction problem. Unlike the more traditional approach based on Laplace transform in time (cf the initial work by Bamberger-Ha Duong) we treat this question directly in the time domain by exploiting in a simple way the connection between integral equations and boundary value problems. This can be done in a very general setting but also particularized to the 1D case for which we got sharp estimates (internship of N. Le Guillarme). Furthermore, we can reinterpret the classical discretization by finite elements for retarded potentials as non conforming finite element methods. This allows to investigate in a new way the error analysis of time domain boundary element methods, which will be the subject of a future work.

# 6.1.4. Coupling Retarded Potentials and Discontinuous Galerkin Methods for time dependent wave propagation problems

Participant: Patrick Joly.

This topic is developed in collaboration with J. Rodriguez (Santiago de Compostela) in the framework of the contract ADNUMO with AIRBUS. Let us recall that our objective was to use time-domain integral equations (developed in particular at IMACS and EADS) as a tool for constructing transparent boundary conditions for wave problems in unbounded media. Our previous contribution of this topic concern the construction of an energy preserving method for the coupling space-time Galerkin approximation of the integral equations with discontinuous Galerkin finite elements and leap-frog time discretization for the numerical approximation of the equations inside the computational domain. For stability reasons, we had to use central fluxes. The drawback of central fluxes is that they do not lead to an optimal accuracy (see also the paragraph on the analysis of DG methods) which is traduced in practice by high frequency spurious oscillations.

Our objective this year was to look for another discretization approach that would overcome these problems. The approach we propose consists in playing on the time discretization procedure. For this we decompose the stiffness bilinear form as the sum of a conservative term corresponding to the use of central fluxes and a ("small") dissipative term due to off-centering and involving jumps across interfaces of the discrete solution. We propose a scheme which treats the conservative part of the equation in a centered way (leap-frog type) and the dissipative term in a non centered way (backward Euler type). Doing so, the overall accuracy in time of the scheme is preserved (with respect to the case of central fluxes) and the stability is maintained at the price of a (slightly) more constraining CFL conditions, which does not seem that much penalizing for the application. The stability is analyzed through the decay of an discrete energy. As a consequence, we can adapt the discretization of the coupling terms between integral and interior equations in order to preserve the stability of the fully coupled scheme under the same CFL condition. Numerical validations are in progress.

# 6.1.5. Solving the Homogeneous Isotropic Linear Elastodynamics Equations Using Potentials and Finite Elements.

Participants: Eliane Bécache, Aliénor Burel, Sébastien Impériale, Patrick Joly.

This topic is the subject of the first part oh th PhD thesis of A. Burel and has been developed for a part in collaboration with Marc Duruflé. Decomposing the displacement field into potentials is a well-known tool in elastodynamics, and it expresses the decoupling of the pressure wave and the shear wave inside a homogeneous isotropic media. Although this tool is classically used when searching for analytic solutions, it does not seem to have been exploited for numerical computation using finite elements for instance. However, this is a priori attracting since, contrary to a displacement field approach for instance, it allows to decouple the approximation of P and S waves and to adapt the discretization process (mesh size, order of elements) to the dynamics of each type of wave, which is a priori particularly interesting when S-waves propagate much more slowly than P-waves (soft materials such as rubber). The main difficulty is to cope with the coupling of

the different types of waves (the so-called conversion of modes) which occurs, due to wave reflections and transmissions, at interfaces between homogeneous media or at physical boundaries. The simplest situation where this phenomenon appears is the propagation of elastic waves in a homogeneous domain with clamped boundary. The main difficulty is to guarantee the stability of the treatment of the boundary condition. For this, we have proposed a variational formulation in which the stiffness bilinear form appears as a sum of a decoupled volumic bilinear form and a coupled surfacic bilinear form. This formulation is is compatible to an energy conservation result. After space discretization with finite elements spaces well adapted to each type of waves (using different meshes and / or polynomial degrees), we propose a discrete energy preserving numerical scheme, based on an explicit discretization of the volume terms and an implicit discretization of the surface terms. The resulting scheme is mainly explicit (only a sparse boundary linear system has to be solved at each time step) and stable under a CFL stability condition that is not affected by the presence of the boundary. An approach based on a modification of this scheme has been proposed for treating the free surface condition. This approach appears to give satisfactory results in the frequency domain (on the basis of numerical simulations) but fails in tim domain due to (apparently unconditional) instabilities.

### 6.1.6. Mathematical and numerical modeling of piezoelectric sensors.

Participants: Sébastien Impériale, Patrick Joly.

This research, which constitutes the subject of the PhD thesis of S. Imperiale (which will be defended in January 2012) is developed in the framework of a collaboration with CEA-LIST about the numerical modeling of non-destructive testing experiments using ultrasonic waves.

More precisely, we have concerned during the past three years by mathematical and numerical questions related to the simulation of non destructive testing experiments using piezoelectric devices. In particular, we have investigated the modeling of piezoelectric sensors that are used to generate and record ultrasonic waves in a solid material: such waves are typically used to investigate in a non invasive way the possible presence of defects in manufactured items. Such an issue has already been tackled in the engineering literature but not, to our knowledge, by way of rigorous applied mathematics. As we are arriving at the conclusion of this work, let us summarize our main contributions during these three years (these have been published in M2AN)

The full equations of piezoelectricity couple Maxwell's equations with linear elastodynamics equations which corresponds to a coupled hyperbolic system. This system presents quite different time scales due to the very large ratio between the speed of light and the sound speed, which makes it impossible to treat by a direct numerical approach. To overcome this problem, we have given a rigorous justification, via asymptotic analysis, of the so-called quasi-static approximation model in which the electric unknowns are reduced to a scalar electric potential: the reduced model appears as a coupled elliptic-hyperbolic system.

$$\begin{split} &\rho\partial_{tt}u-\mathbf{div}\ C\varepsilon(u)-\mathbf{div}\ \mathbf{e}\nabla\varphi=0 \qquad &\text{in }\ \Omega_S \quad \text{(the solid domain),} \\ &\nabla\cdot\epsilon\nabla\varphi-\nabla\cdot\mathbf{e}^T\varepsilon(u)=0 \qquad &\text{in }\ \mathbb{R}^3. \end{split}$$

- We have next justified the reduction of the computation of this electric potential to the piezoelectric
  parts of the computational domain, considering the large contrast of permittivity between piezoelectric materials and surrounding polymers. This relies again on a limit process: the ratio between
  permittivities is the small parameter.
- A particular attention has to be devoted to the modeling of the electric supply process: more precisely the nonlocal (in space) boundary conditions on the electrodes (used to model the emission and reception regimes), as well as the modeling of the coaxial cable connecting the sensor to the electric generator (see also section 6.4.1).

- Concerning the numerical approximation, an energy preserving finite element / finite differences numerical scheme is developed. A specific procedure is used for treating the nonlocal boundary conditions on the electrodes. Unbounded media have been treated via PML techniques that are dealt with using an original mortar element technique (see section 6.3.1 for more details). Various numerical results in academic or more realistic situations have been obtained. For instance, we have been able to model how one can exploit multi-component devices (see figure 1) to produce well focused waves (see figure 2).
- A computational code issued from our research, named Ondomatic (12 000 lines in C++), has been implemented. This code uses the finite element library FEMME (15 000 lines in C++) that essentially relies upon the use of spectral finite element on hexahedral meshes.

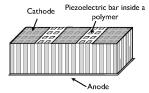


Figure 1.

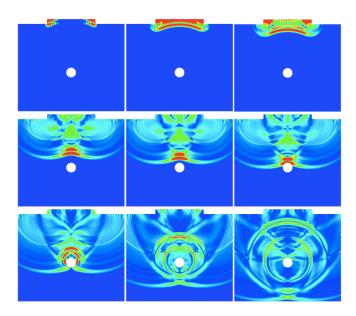


Figure 2.

## 6.1.7. Maxwell's equations in Lorentz materials

Participants: Patrick Ciarlet, Patrick Joly, Valentin Vinoles.

This is the time-domain counterpart of the research done at Poems about metamaterials (see also the section 6.2.7) and has been the subject of the internship of V. Vinoles. Lorentz materials are particular non dissipative dispersive materials which behave as metamaterials (with negative index of refraction) in some range of frequencies. Their constitutive laws (links between electric and magnetic fields and the related inductions) are described in terms of ordinary differential equations (harmonic oscillators). We have studied Maxwell's equations in such materials and in particular proposed an energy preserving space-time discretization scheme based on an extension of classical methods (mixed finite elements in space, leap-frog type schemes in time). This has been implemented in a code including PMLs for the treatment of unbounded domains. Various numerical experiments have been performed. They illustrate the spectacular dispersive effects of Lorentz materials and allow us to recover the expected focalisation phenomena by an interface between a metamaterial and a standard dielectric one. This code will be an essential tool for the further investigation of more theoretical (and challenging) issues such as the limiting amplitude principle in this context.

### 6.1.8. Evolution problems in perturbed infinite periodic media

Participants: Julien Coatléven, Sonia Fliss.

First, as a part of the PhD of J. Coatléven, based on the former method to solve linear evolution problems in locally perturbed infinite periodic strips through the construction of transparent boundary conditions, a method has been developed to solve its natural geometric extension, i.e. the case of a locally perturbed line defect in an infinite periodic media. The method is again based on a semi-discretization in time of the problem on the whole infinite periodic media, and a generalization of the Lippmann-Schwinger equation approach we have developed for the treatment of this kind of geometries but for harmonic wave problems. At each time step, the solution is written as the sum of the solution of a time step in the unperturbed line defect and a contribution of the perturbation due not only to the current time step but also to all time steps involved in the numerical scheme. This intrication of time steps requires a careful treatment of the Lippmann-Schwinger equation, and in particular of the source term. Using the Floquet-Bloch transform, the computation of all the quantities involved can be reduced to the resolution of a few time steps of linear evolution problems in locally perturbed infinite periodic strips, where we can use the former method. As in the harmonic case, the discretization of the inverse Floquet-Bloch transform is done using appropriate quadrature rules, whereas the space discretization requires classical finite elements. The theoretical basis as well as the numerical analysis of this method are well understood, and the method has been successfully tested numerically. For instance, it can be shown, and checked numerically, that in the particular case of the wave equation, if one uses enough quadrature points (depending on the length of the time interval), then this quadrature creates no error of approximation.

For parabolic problems set in locally perturbed periodic media, we have developed another approach to determine the time-domain DtN operator. The principle is to apply the Laplace Transform in time to the equation and use the construction of the DtN operator for stationary equations. The main difficulty is the computation of the inverse of the Laplace Transform, more precisely to understand how to deal with the unbounded interval of integration and the choice of the discretization of the laplace variable. To deal with the first difficulty for waveguide problem, we have studied the asymptotic behavior of the DtN operator in the laplace domain when the laplace variable tends to  $p_0 \pm \infty$ . To deal with the second difficulty, we have used the Z-Transformation and its properties. The numerical study is still in progress. This work enters in the framework of the ANR PRoject MicroWave (Sonia Fliss is an external collaborator), in collaboration with Karim Ramdani (Institut Elie Cartan de Nancy, UMR CNRS 7502), Christophe Besse and Ingrid Violet (Laboratoire Paul Painlevé, UMR CNRS 8524).

### 6.1.9. Modeling and numerical simulation of a piano.

Participants: Juliette Chabassier, Patrick Joly.

This work is developed in collaboration with Antoine Chaigne (UME, ENSTA). The purpose of this work the time domain modeling and numerical simulation of a piano. We aim at explaining the vibratory and acoustical behavior of the piano, by taking into account the main elements that contribute to sound production. The soundboard is modeled as a bidimensional thick, orthotropic, heterogeneous, frequency dependant damped plate, using Reissner Mindlin equations. The vibroacoustics equations allow the soundboard to radiate into the surrounding air, in which we wish to compute the complete acoustical field around the perfectly rigid rim. The soundboard is also coupled to the strings at the bridge, where they form a slight angle from horizontal. Each string is modeled by a one dimensional damped system of equations, taking into account not only the transversal waves excited by the hammer, but also the stiffness thanks to shear waves, as well as the longitudinal waves arising from geometric nonlinearities. The hammer is given an initial velocity that projects it towards a choir of strings, before being repelled. The interacting force is a nonlinear function of the hammer compression.

The final piano model that is discretized is a coupled system of partial differential equations, each of them exhibiting specific difficulties (nonlinear nature of the string system of equations, frequency dependant damping of the soundboard, great number of unknowns required for the acoustic propagation), in addition to couplings' inherent difficulties. On the one hand, numerical stability of the discrete scheme can be compromised by nonlinear and coupling terms. A very efficient way to guarantee this stability is to construct a numerical scheme which ensures the conservation (or dissipation) of a discrete equivalent of the continuous energy, across time steps. A major contribution of this work has been to develop energy preserving schemes for a class of nonlinear systems of equations, in which enters the string model. On the other hand, numerical efficiency and computation time reduction require that the unknowns of each problem's part, for which time discretization is specific, hence different, be updated separately. To achieve this artificial decoupling, adapted Schur complements are performed after Lagrange multipliers are introduced.

The potential of this time domain piano modeling is emphasized by realistic numerical simulations. Beyond greatly replicating the measurements, the program allows us to investigate the influence of physical phenomena (string stiffness or nonlinearity), geometry or materials on the general vibratory behavior of the piano, sound included. Spectral enrichment, << phantom partials >> and nonlinear precursors are clearly revealed when large playing amplitudes are involved, highlighting how this approach can help better understand how a piano works.

The main contributions of this year have been the following:

- to write the acoustic propagation as a first order system of equation, involving the physical sound pressure (as opposed to before, when its primitive had to be considered) and the acoustical velocity: this allowed us to artificially bound the computation domain with Perfectly Matched Layers.
- to decrease the numerical computation times via a massive parallelization of the code: parallel modal search for the soundboard, parallel dense matrix-vector product for the vibroacoustic equations in the modal basis, parallel resolution of the 3D acoustic propagation, multi-threaded computation of the strings' problem.
- to perform realistic computations, which have provided physically relevant numerical simulations (see Figure 3).

Several measurements have also been conducted on a grand piano in order to provide realistic values of parameters and calibrated data to compare simulation with.

# 6.2. Time-harmonic diffraction problems

### 6.2.1. Numerical computation of variational integral equation methods

Participants: Marc Lenoir, Nicolas Salles.

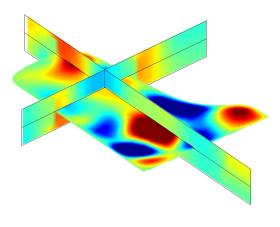


Figure 3.

The dramatic increase of the efficiency of the variational integral equation methods for the solution of scattering problems must not hide the difficulties remaining for an accurate numerical computation of some influence coefficients, especially when the panels are close and almost parallel.

The formulas have been extended to double layer potentials and, for self influence coefficients, to affine basis functions. Their efficiency for the solution of Maxwell equations has been proved in the framework of a collaboration with CERFACS. The redaction of a paper devoted to the case of parallel panels has been completed and submitted to SIAM J. Sci. Comp. Some preliminary work on the numerical integration of the regular part of the integrand has been undertaken in the context of an internship.

### 6.2.2. Fast Multipole Method for Viscoelastodynamics

Participants: Marc Bonnet, Stéphanie Chaillat.

This work is done in collaboration with Eva Grasso (LMS, Ecole Polytechnique) and Jean-François Semblat (IFSTTAR). We have extended the single- and multi-domain time-harmonic elastodynamic multi-level fast multipole BEM (Boundary Element Method) formulations to the case of weakly dissipative viscoelastic media [21]. The underlying boundary integral equation and fast multipole formulations are formally identical to that of elastodynamics, except that the wavenumbers are complex-valued due to attenuation. Attention was focused on evaluating the multipole decomposition of the visco-elastodynamic fundamental solution, involving complex-valued wavenumbers. As a result, a damping-dependent modification of the selection rule for the multipole truncation parameter was proposed and assessed on 3D single-region and multi-region visco-elastodynamic examples involving up to about  $3\,10^5$  boundary nodal unknowns.

# 6.2.3. Formulation and Fast Evaluation of the Multipole Expansions of the Elastic Half-Space Fundamental Solutions

Participants: Marc Bonnet, Stéphanie Chaillat.

This ongoing work is concerned with a formulation and computation algorithm for the elastodynamic Green's tensor for the traction-free half-space allowing its use within a Fast Multipole Boundary Element Method (FM-BEM). Due to the implicit satisfaction of the traction-free boundary condition achieved by the Green's tensor, discretization of (parts of) the free surface is no longer required. Unlike the full-space fundamental solution, the elastodynamic half-space Green's tensor cannot be expressed in terms of usual kernels such as  $e^{ikr}/r$  or 1/r. Its multipole expansion thus cannot be deduced from known expansions, and is formulated in this work using a spatial two-dimensional Fourier transform approach. The latter achieves the separation of variables which is required by the FMM. A key numerical issue, upon which current work is focused, is concerned with the definition of an efficient numerical quadrature for the evaluation of the inverse Fourier transform, whose integrand is both singular and oscillatory, as classical Gaussian quadratures would perform poorly, fail or require unacceptably large number of quadrature points.

### 6.2.4. Multiple scattering by small scatterers

**Participants:** Maxence Cassier, Christophe Hazard.

We consider the scattering of an acoustic time-harmonic wave by an arbitrary number of sound-soft obstacles located in a homogeneous medium. When the size of the obstacles is small compared with the wavelength, the numerical simulation of such a problem by classical methods (e.g., integral equation techniques or methods based on a Dirichlet-to Neumann map) can become highly time-consuming, particularly when the number of scatterers is large. In this case, the use of an asymptotic model may reduce considerably the numerical cost. Such a model was introduced by Foldy and Lax in the middle of the last century to study multiple isotropic scattering in a medium which contains randomly distributed small scatterers. Their asymptotic model is based on the fact that the scattered wave can be approximated by a wave emitted by point sources placed at the centers of the scatterers; the amplitudes of the sources are calculated by solving a linear system which represents the interactions between the scatterers. Nowadays, the Foldy–Lax model is still used in numerous physical and numerical applications to approximate the scattered wave in a deterministic media. But to the best of our knowledge, there was no mathematical justification of this asymptotic model. We have proposed such a justification which provides local error estimates for the two-dimensional problem in the case of circular obstacles. An article on this subject has been recently submitted to Wave Motion.

### 6.2.5. Harmonic wave propagation in locally perturbed infinite periodic media

Participants: Julien Coatléven, Sonia Fliss, Patrick Joly.

A part of the PhD of J. Coatléven consists in developing a method for solving harmonic wave problems with locally perturbed line defects in periodic media. For the treatment of these unbounded defects, which are structured apart from a local perturbation, a new approach has been developed, based on a perturbation principle. The solution is written as a the sum of a solution corresponding to the unperturbed line defect and a contribution of the local perturbation. This decomposition leads to a generalization of the so-called Lippmann-Schwinger equation, whose coefficients are computed through their Floquet-Bloch transform, which leads to solve wave-guide problems, these last problems being solved using the transparent boundary condition method developed during S. Fliss's PhD. The discretization of the inverse Floquet-Bloch transform is done using appropriate quadrature rules, whereas the space discretization requires classical finite elements. The theoretical basis as well as the numerical analysis of this method are well understood, and the method has been successfully tested numerically. In particular, the theoretical convergence estimates have been checked in practice, and the method has a satisfying behavior in limit cases not fully covered by the theory, such as the non-absorbing case.

Concerning the non absorbing case, the question of the limiting absorption principle has been treated for locally perturbed periodic media with particular assumptions. In this case, we are studying the behavior of the solution at large distance of the local perturbation.

### 6.2.6. Time harmonic aeroacoustics

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

We are still working on the numerical simulation of the acoustic scattering and radiation in presence of a mean flow. This is the object of the ANR project AEROSON, in collaboration with Florence Millot and Sébastien Pernet at CERFACS, Nolwenn Balin at EADS and Vincent Pagneux at the Laboratoire d'Acoustique de l'Université du Maine. The main recent improvements concern: the consideration of ducts with treated boundaries and the development of an alternative model to Galbrun's equation.

#### **Treated boundaries**

Our aim is to extend the time harmonic equation of Galbrun to take into account acoustically treated boundaries. Such boundaries are generally described by the Myers boundary condition. Since this condition is naturally expressed in terms of Galbrun's unknown, the displacement u, Galbrun's equation easily extends to treated boundaries. However we face a difficulty: the original equation of Galbrun leads to a non coercive problem. For rigid boundaries, considering an augmented variational formulation leads to well-posedness. But this approach does not work anymore for treated boundaries.

We have improved our understanding of this difficulty. We are now convinced that the Augmented Galbrun's equation combined with Myers condition leads to an ill-posed problem. More precisely source terms for which the solution of Galbrun's equation does not belong to standard functional spaces exist. Such source terms are very particular: located on the treated boundary and singular. This is why during the numerical validations performed at Cerfacs, for "standard" source terms ( $L^2$  functions compactly supported in the fluid) we did not get any problem.

To get a well-posed problem, Myers boundary condition, which just requires the normal displacement to belong to  $L^2$ , must be regularized. It can be achieved by requiring the tangential derivative of  $\mathbf{u} \cdot \mathbf{n}$  to belong to  $L^2$ . We have also understood that less regularity is sufficient to get a well-posed problem, as it is the case if the fluid is in contact of an elastic medium (such interface cannot be described by a Myers condition since it is necessarily non-local). In particular in the case of a uniform flow well-posedness is proved for a sufficiently slow flow.

### Alternative to Galbrun's model

We have kept on considering the model of Goldstein's equations, alternative to Galbrun's equation. The Goldstein's equations couples two unknowns: the velocity potential  $\varphi$  and a vectorial unknown  $\xi$ .  $\varphi$  satisfies a modified Helmholtz's equation with variable coefficients linked to the flow, in which  $\xi$  is added as a source term.  $\xi$  satisfies a transport equation coupled to the velocity potential. This new model facilitate the treatment of 3D problems since Galbrun's equation requires to introduce many unknowns. Moreover the vectorial unknown in Goldstein's formulation vanishes in the areas where the flow is potential which is interesting since realistic flows are mainly potential, the non-potential areas being located near the boundaries or behind obstacles.

As it is the case to calculate the vorticity  $\psi={\rm curl}{\bf u}$ , used to regularize Galbrun's equation, a Discontinuous Galerkin (DG) discretization is used to determine  $\xi$  and numerical simulations have been performed at Cerfacs. We have also developed an alternative method allowing to solve Goldstein's equations with simple Lagrange Finite Element and this was the object of Jean-Emmanuel Lauzet's intership. We have developed a method combining the Streamline Upwind Petrov Galerkin (SUPG) scheme to discretize the Goldstein's model with the introduction of PML to bound the calculation domain. To test the efficiency of the method, a non-potential flow has been determined analytically. It consists of a lid-cavity flow connected to a uniform flow in a duct. The viscous cavity flow is solution of the Stoke's equation and is determined in a rectangular domain by a modal method.

## 6.2.7. Modeling of meta-materials in electromagnetism

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Ciarlet, Lucas Chesnel.

A collaboration with Eric Chung (Chinese Univ. of Hong Kong) and Xavier Claeys (ISAE).

Meta-materials can be seen as particular media whose dielectric and/or magnetic constant are negative, at least for a certain range of frequency. This type of behavior can be obtained, for instance, with particular periodic structures. Of special interest is the transmission of an electromagnetic wave between two media with opposite sign dielectric and/or magnetic constants. As a matter of fact, applied mathematicians have to address challenging issues, both from the theoretical and the discretization points of view.

The first topic we considered a few years ago was: when is the (simplified) scalar model well-posed in the classical  $H^1$  framework? It turned out this issue could be solved with the help of the so-called T-coercivity framework. While numerically, we proved that the (simplified) scalar model could be solved efficiently by the most "naive" discretization, still using T-coercivity.

Recently, we have been able to provide sharp conditions for the T-coercivity to hold in general 2D and 3D geometries (with L. Chesnel), which involve explicit estimates in simplified geometries together with localization arguments. We then analyzed the discretization of the scalar problem with a classical,  $H^1$  conforming, finite element method, and proved the convergence under the same sharp conditions (with L. Chesnel). We also showed that the problem can be solved with the help of a Discontinuous Galerkin discretization, which allows one to approximate both the field and its gradient (with E. Chung).

Last (with L. Chesnel and X. Claeys), we investigated the case of a 2D corner which can be ill-posed (in the classical  $H^1$  framework). Using the Mellin transform, we showed that a radiation condition at the corner has to be imposed to restore well-posedness. Indeed there exists a wave which takes an infinite time to reach the corner: this "black hole" phenomenon is observed in other situations (elastic wedges for example).

As a second topic, we studied the transmission problem in a purely 3D electromagnetic setting from a theoretical point of view: to achieve well-posedness of this problem, we had to proceed in several steps, proving in particular that the space of electric fields is compactly embedded in  $L^2$ . For that, we had to assume that the interface is "sufficiently smooth", excluding in particular corners. With L. Chesnel, we have been able to remove this assumption, so that we can solve the problem around an interface with corners. It turns out the T-coercivity framework can be applied once more, under the same assumptions as for the scalar model. In the process, we recover more compact embedding results.

### 6.2.8. Numerical MicroLocal Analysis

Participants: Jean-David Benamou, Francis Collino, Simon Marmorat.

Numerical microlocal analysis of harmonic wavefields is based on a family of linear filters using Bessel functions and applied to wave data collected on a circle of fixed radius  $r_0$  around the observation point  $x_0$  where we want to estimate the Geometric Optics/ High Frequency components. The data can easily be reconstructed from more conventional line array or grid geometry. The output is an angular function presenting picks of amplitudes in the direction angles of rays.

The original NMLA algorithm relied on a local plane wave assumption for the data. For arbitrary waves, it meant linearization errors and accuracy limitations. Also, only the directions of the (multiple) rays are recovered but the traveltime and amplitudes are not reliably computed. We recently introduced a new "impedant" observable which allows to prove a stability theorem. Numerical results confirm that the new NMLA filter is robust to random and correlated noise.

Using asymptotic expansion on NMLA filtered point sources data, we designed a correction method for the angle which also estimates the wavefront curvature. It can be used to correct the linearization errors mentioned above and provides a second order correction in the Taylor approximation of the traveltime.

The parameters of the method (size of observation circle, discretization) are automatically optimized and a posteriori quantitative error on angles and curvature are available. Numerical studies validate the stability result and confirm the superior accuracy of the curvature corrected NMLA version over image processing methods.

When some bandwith is available we can also compute the traveltime. The amplitude remains polluted by phase errors. Its determination is still open.

## 6.3. Absorbing boundary conditions and absorbing layers

# 6.3.1. Perfectly matched transmission problem with absorbing layers: application to anisotropic acoustics

Participant: Sébastien Impériale.

This work has been carried out in collaboration with Edouard Demaldent from CEA-LIST. We have worked on an original approach to design perfectly matched layers (PML) for transient wave equations. This approach is based, first, on the introduction of a modified wave equation and, second, on the formulation of general "perfectly matched" transmission conditions for this equation. The stability of the transmission problem is discussed by way of the adaptation of a high frequency stability (necessary) condition, and we apply our approach to define PML suited for the anisotropic wave equation. A variational formulation of the problem is then developed. It includes a Lagrange multiplier at the interface between the physical and the absorbing domains to deal with the "perfectly matched" transmission conditions. We have carried out numerical results in 2D and 3D that first show the validity of our approach in term of stability and accuracy and the efficiency when using constant damping coefficients combined with high order elements. This work has been submitted for publication.

### 6.3.2. On the stability in PML corner domains

Participant: Eliane Bécache.

In collaboration with Andres Prieto, from the university of Santiago de Compostella.

We have finalized our work on the stability of the discretization of PMLs in the corners and submitted a paper (see preprint [24]).

### 6.3.3. High-order Absorbing Boundary Conditions for anisotropic elastodynamics

Participants: Daniel Baffet, Eliane Bécache.

This work is done in collaboration with Daniel Baffet, PhD student of Dan Givoli, at the Technion University in Haifa (Israel) and has started during a visit of Daniel at Poems.

The aim is to design new efficient and stable absorbing boundary conditions for anistropic materials. It is known that the anisotropy introduces a specific difficulty, for ABC as well as for PMLs, in particular for models which involve inverse modes, i.e. waves for which the phase velocity and the group velocity propagate in opposite directions (with respect to the boundary). This has given rise to specific treatment for scalar models but for anisotropic elastodynamics, there are some materials for which no satisfactory solution exist. for these materials however, we can design a low order boundary condition, which is proved to be stable via an energy estimate. We have started to investigate several ways to design higher-order boundary conditions. The main difficulty is to show whether these boundary conditions are stable or not...

### 6.3.4. Dirichlet to Neumann map with overlap for waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Geoffrey Martinache, Antoine Tonnoir.

For scattering problems in acoustic waveguides, a usual approach consists in restricting the computation to a bounded domain containing the sources and the perturbations, using transparent boundary conditions on the artificial boundaries. These conditions are written by using the so-called Dirichlet-to-Neumann maps which can be expressed thanks to a modal decomposition.

An iterative solution of the related linear system can be seen as a domain decomposition formulation without overlapping, where one domain is the bounded region and the other one is infinite. This iterative method does not converge necessarily. A classical idea is to consider a domain decomposition method with overlapping. In this work, we find the equivalent of this method in terms of a new Dirichlet-to-Neumann operator which links the trace of the solution on a section of the waveguide to the normal trace on a different one. This operator can also be expressed analytically via a modal decomposition. Its main advantage is that, because of the overlapping, it becomes compact and this is exactly why we think an iterative resolution has more chance to converge. Other advantages will appear with the elasticity application. Indeed, in the formulation of the transparent boundary condition without overlapping, appears a lagrange multiplier which makes the resolution more costly. This additional unknown will be avoided with an overlap.

For now, the theory is done for the scalar acoustic waveguide and the method has been implemented in the Melina code. The extension to the elastic case is in progress.

### 6.3.5. An alternative to DTN maps in waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Guillaume Legendre.

We are interested by the treatment of the radiation condition at infinity for the numerical solution of a problem set in an unbounded waveguide. We have proposed an alternative to the classical approach involving a modal expression of Dirichlet-to-Neumann (DtN) operators. This new method is particularly simple to implement since it only requires to solve several times a boundary value problem with local boundary conditions. In the case of a an acoustic waveguide, we have proved that the corresponding approximate solution is comparable in accuracy to the one obtained by truncating the infinite series in the DtN maps. The number of linear systems to invert, which has to be greater than the number of propagative guided modes, can be significantly reduced by combining the approach with the perfectly matched layer (PML) technique. It works even in elastic waveguides, despite existence of the so-called backward waves which are known to make the PMLs inefficient, when used alone.

# 6.4. Waveguides, resonances, and scattering theory

## 6.4.1. Modelling of non-homogeneous lossy coaxial cable for time domain simulation.

Participants: Sébastien Impériale, Patrick Joly.

In this work, we focus on the time-domain simulation of the propagation of electromagnetic waves in non-homogeneous lossy coaxial cables. This question has been motivated by our collaboration with CEA-LIST about the numerical modeling of non-destructive testing (for the detection of cracks in metallic bodies for instance) by ultra-sounds and more precisely the modeling of piezo-electric transducers (see section 6.1.6). The complete description of such devices often requires an accurate modeling of the supply process, which includes the propagation of the electric current along coaxial cables. This question appears as an independent sub-problem.

The main characteristic of such coaxial cables is that their transverse directions are very small with respect to their length as well as the wavelength. As a consequence, one would like to use a simplified 1D model as an effective (or homogenized) model for electromagnetic propagation. In this work, we construct and justify rigorously such a model by way of an asymptotic analysis of time harmonic 3D Maxwell's equations in such a structure. The effective model appears as a generalized wave equation with additional time convolution terms that take into account electric and magnetic losses. By this way, we justify and extend some models proposed in the electrical engineering literature, in particular the well-known telegraphist's equation. The properties of our limit model in time domain has been analyzed and a stable discretization process has been proposed. Numerical simulation in academic simulations exhibit some exotic phenomena of "dispersive dissipation".

Our further investigations of the subject will be developed in the framework of the new ANR project SODDA in collaboration with the team Sysiphe (M. Sorine).

### 6.4.2. Study of lineic defect in periodic media

Participant: Sonia Fliss.

This works deals with one dimensional infinite perturbation - namely line defects - in periodic media. In optics, such defects are created to construct an (open) waveguide that concentrates light. The existence and the computation of the eigenmodes is a crucial issue. This is related to a selfadjoint eigenvalue problem associated to a PDE in an unbounded domain (in the directions orthogonal to the line defect), which makes both the analysis and the computations more complex. Using a Dirichlet-to-Neumann (DtN) approach, we show that this problem is equivalent to one set on a small neighborhood of the defect. In opposition to existing methods, this method is exact but there is a price to be paid: the reduction of the problem leads to a nonlinear eigenvalue problem of a fixed point nature. An article presenting the method and its properties is being written, the numerical study is in progress in collaboration with Kersten Schmidt and Dirk Klindworth from the Technische Universität Berlin.

Our further investigations of the subject will be developed in the framework of a new DGA project.

# 6.4.3. A new approach for the numerical computation of non linear modes of vibrating systems Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

A collaboration with Cyril Touzé and François Blanc (Unité de Mécanique, ENSTA). The simulation of vibrations of large amplitude of thin plates or shells requires the expensive solution of a non-linear finite element model. The main objective of the proposed study is to develop a reliable numerical method which reduces drastically the number of degrees of freedom. The main idea is the use of the so-called non-linear modes to project the dynamics on invariant subspaces, in order to generate accurate reduced-order models. Cyril Touzé from the Unité de Mécanique of ENSTA has derived an asymptotic method of calculation of the non-linear modes for both conservative and damped systems. But the asymptotically computed solution remains accurate only for moderate amplitudes. This motivates the present study which consists in developing a numerical method for the computation of the non-linear modes, without any asymptotic assumption. This is the object of a collaboration with Cyril Touzé, and new results have been obtained during the post-doc of François Blanc in the Unité de Mécanique of ENSTA. The partial differential equations defining the invariant manifold of the non-linear mode are seen as a vectorial transport problem: the variables are the amplitude and the phase  $(a, \varphi)$  where the phase  $\varphi$  plays the role of the time. In the case of conservative systems, a finite difference scheme is used and an iterative algorithm is written, to take into account the  $2\pi$ -periodicity in  $\varphi$  which is seen as a constraint. An adjoint state approach has been introduced to evaluate the gradient of the coast function. The method has been validated in a simple example with two degrees of freedom. Good agreement with an alternative method, the continuation of periodic solutions method, has been found. Currently the method is extended to the case of damped systems. The main difficulty is that, due to a change of variables, the  $2\pi$ -periodicity does not hold anymore and new constraints more complicated to implement must be considered. Numerical implementation is still under progress.

# 6.5. Asymptotic methods and approximate models

### 6.5.1. Effective boundary conditions for thin periodic coatings

Participants: Mathieu Chamaillard, Patrick Joly.

This topic is developed in collaboration with H. Haddar (DEFI, INRIA Saclay) can be seen as a continuation of the PhD thesis of B. Delourme (see the activity report of last year) on effective transmission conditions for thin rough interfaces. On this last subject, the mathematical analysis of such transmission conditions in the (difficult) case of 3D Maxwell's equations has been completed and submitted for publication.

We are now coming back to the more traditional issue of effective or approximate boundary conditions for simulating thin periodic coatings at the surface of a diffractive obstacle. This subject has already been more widely investigated, in particular in France (see the works by Achdou, Ammari and their collaborators for instance). However, we attack, with the PhD thesis of M. Chamaillard, supported by a DGA/INRIA scholarship, various new aspects of the problem, namely:

- the treatment of surfaces of general geometry,
- the use of non standard materials, such as ferromagnetic materials, for the coating,
- the construction of higher order impedance conditions.

This is motivated by various recent progress in the domain of stealth technology and we hope to develop a collaboration in this domain with CEA-DAM (CESTA and Le Ripault).

### 6.5.2. Elastic wave propagation in strongly heterogeneous media

Participants: Patrick Joly, Simon Marmorat.

This subject enters our long term collaboration with CEA-LIST on the development on numerical methods for time-domain non destructive testing experiments using ultra-sounds. This is also the subject of the PhD thesis of Simon Marmorat. Our objective is to develop an efficient numerical approach for the propagation of elastic waves in a medium which is made of many small inclusions / heterogeneities embedded in a smooth (or piecewise smooth) background medium, without any particular assumption (such as periodicity) on the spatial distribution of these heterogeneities. Our idea is to exploit the smallness of the inclusions (with respect to the wavelength in the background medium) to derive a simplified approximate model in which each inclusion would de described by very few parameters (functions of time) coupled to the displacement field in background medium for which we could use a computational mesh that ignores the presence of the heterogeneities. For deriving such a model, we intend to use and adapt the asymptotic methods previously developed at Poems (such as matched asymptotic expansions).

### 6.5.3. Approximate models in aeroacoustics

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Joly, Guillaume Legendre, Ricardo Weder.

This topic concerns the 2D acoustic propagation in presence of a mean flow, modeled for instance by the Galbrun equation. We had previously derived effective boundary conditions taking into account the boundary layers of the mean flow near a rigid or treated boundary. These boundary conditions are in general non local with respect to the normal coordinate inside the boundary layer. However when the Mach profile in the shear layer is piecewise linear, the condition can be replaced by a system of 1D advection equations, which are coupled with the Galbrun equation in the 2D domain. We have derived a variational formulation for this model, in time-harmonic regime and for the case of a rigid boundary. This formulation has been implemented in the Melina code: the first results are promising and the validation, by comparison to the solution obtained by a full discretization of the shear layer, is in progress.

# 6.6. Imaging and inverse problems

### 6.6.1. Sampling methods in waveguides

Participants: Laurent Bourgeois, Eric Lunéville, Alexandre Routier.

We have derived a modal formulation of sampling methods (both the Linear Sampling Method and the Factorization Method) in an acoustic waveguide when the obstacle to recover is a set of cracks. This was the subject of the Master internship of Alexandre Routier. For such particular obstacle, we have analyzed the importance of the test function we introduce in the sampling method if we a priori know the type of boundary condition we have on the lips of the crack, both from the theoretical and the numerical point of view. Besides we have proved that in our modal formulation, the Factorization Method is applicable by using the same data as those used in the Linear Sampling Method, which is a novelty as concerns sampling methods in waveguides. The Linear Sampling Method has been extended to the elastic case, for which the usual obstacle is a set of traction free cracks. This makes the choice of the test function crucial, and we have emphasized such fact on numerical examples.

### 6.6.2. Inverse scattering with generalized impedance boundary conditions

Participants: Laurent Bourgeois, Nicolas Chaulet.

This work is a collaboration between POEMS and DEFI projects (more precisely Houssem Haddar) and constitutes the subject of the PhD thesis of N. Chaulet. We are concerned with the identification of some obstacle and some Generalized Impedance Boundary Conditions (GIBC) on the boundary of such obstacle from far field measurements. The GIBCs are approximate models for thin coatings or corrugated surfaces. During this last year, we have completed the computation of the partial derivatives of the far field with respect to the unknowns, among which is the boundary of the obstacle, and we have implemented many numerical experiments. In particular, we have shown the efficiency of the method consisting in approximating a perfect conductor which is coated with a thin dielectric layer of variable width by a second order GIBC in order to retrieve the obstacle, as well as the refraction index and the width of the layer.

### 6.6.3. Detection of targets using time-reversal

Participants: Maxence Cassier, Patrick Joly, Christophe Hazard.

This topic concerns the studies started last year about time-reversal in the context of Maxence Cassier's thesis. The main question is to generate a time-dependent wave that focuses on one given scatterer not only in space, but also in time. Our recent works concern two items. On one hand, we have proposed a way to construct such a focusing wave which does not require an a priori knowledge of the location of the obstacle. This wave is represented by a suitable superposition of the eigenvectors of the so-called time-reversal operator in the frequency domain. Numerical results show the focusing properties of such a wave. On the other hand, we try to understand how to translate the physical idea of "focusing" into mathematical terms. We have proposed an energy criterion which can be used in numerical experiments in order to evaluate the quality of the focus. The question is to relate such a criterion with the construction of the above mentioned focusing wave. Works on this topic are in progress.

### 6.6.4. Interior transmission problem

Participants: Anne-Sophie Bonnet-Ben Dhia, Lucas Chesnel.

This work is a collaboration with Houssem Haddar from the DEFI project. The interior transmission problem plays an important role in the inverse scattering theory for inhomogeneous media. In particular, it arises when one is interested in the reconstruction of an inclusion embedded in a background medium from multi-static measurements of diffracted fields at a given frequency. Physically, it is important to prove that, for a given frequency, there are no waves which do not scatter. Mathematically, this last property boils down to say that the frequency is not a transmission eigenvalue, that is, an eigenvalue of the interior transmission problem. An important issue is to prove that transmission eigenvalues form at most a discrete set with infinity as the only accumulation point. This is not trivial because the operator associated with this problem exhibits a sign changing in its principal part and its study is not standard. Using the T-coercivity approach, we proved the discreteness under relatively weak assumptions. In particular, the simple technique we proposed allows to treat cases, which were not covered by existing methods, where the difference between the inclusion index and the background index changes sign.

### 6.6.5. Flaw identification using elastodynamic topological derivative

Participant: Marc Bonnet.

In collaboration with Cédric Bellis (Columbia Univ. USA), Bojan Guzina (Univ. of Minnesota, USA). The concept of topological derivative (TD) quantifies the perturbation induced to a given cost functional by the nucleation of an infinitesimal flaw in a reference defect-free body, and may serve as a flaw indicator function. In this work, the TD is derived for three-dimensional crack identification exploiting over-determined transient elastodynamic boundary data. This entails in particular the derivation of the relevant polarization tensor, here given for infinitesimal trial cracks in homogeneous or bi-material elastic bodies. Simple and efficient adjoint-state based formulations are used for computational efficiency, allowing to compute the TD field for arbitrarily shaped elastic solids. The latter is then used as an indicator function for the spatial location of the sought

crack(s). This approach, which allows a qualitative reconstruction of cracks in terms of their location but also their orientation (utilizing the fact that the polarization tensor depends on the normal to the trial small crack), has been implemented within a conventional FEM platform. A standard Newmark unconditionally-stable time-marching scheme is used for simulating data, and for computing the free and adjoint solutions used in the evaluation of the TD field. Extensive 3D time-domain numerical experiments for the detection of cracks buried either in a homogeneous pipe-like structure or on the interface between two sandwiched plates highlight its usefulness and performance. The application of TD to flaw identification has thus far rested upon a heuristic basis. Its justification in limiting situations such as the Born approximation is currently being investigated.

## 6.6.6. Topological derivative in anisotropic elasticity

Participant: Marc Bonnet.

In collaboration with Gabriel Delgado (CMAP, Ecole Polytechnique). This work addresses the current lack of a comprehensive treatment of the topological derivative for anisotropic elasticity, by addressing the case where both the background material and the trial small inhomogeneity have arbitrary anisotropic elastic properties. Accordingly, a formula for the topological derivative of any cost functional defined in terms of regular volume or surface densities depending on the displacement is established, by combining small-inhomogeneity asymptotics and the adjoint solution approach. The latter feature makes the proposed result simple to implement and computationally efficient. Both three-dimensional and plane-strain settings are covered; they differ mostly on details pertaining to the elastic moment tensor. This result achieves a direct generalization to the fully anisotropic case of previously-known formulations for isotropic elasticity. Moreover, the main properties of the EMT, a critical feature of any elastic topological derivative formula, are studied for the fully anisotropic case, generalizing available results on the isotropic case. Finally, further generality is achieved by also deriving the topological derivative of strain energy-based cost functionals, which depend on the displacement gradient. This case, seldom addressed so far, requires a specific, and separate, treatment. Applications of these results include topology optimization of composite structures (a topic currently pursued by G. Delgado) or flaw identification using experimental data from nondestructive testing.

# 6.6.7. Energy functionals for elastic medium reconstruction using transient data Participant: Marc Bonnet.

In collaboration with Wilkins Aquino (Cornell Univ., USA).

Energy-based misfit cost functionals, known in mechanics as error in constitutive relation (ECR) functionals, are known since a long time to be well suited to (electrostatic, elastic,...) medium reconstruction. In this ongoing work, a transient elastodynamic version of this methodology is developed, with emphasis on its applicability to large time-domain finite element modeling of the forward problem. The formulation involves coupled transient forward and adjoint solutions, a fact which greatly hinders large-scale computations. A computational approach combining an iterative treatment of the coupled problem and the adjoint to the discrete Newmark time-stepping scheme is found to perform well on large FE models, making the time-domain ECR functional a worthwhile tool for medium identification.

### 6.6.8. Accelerated boundary element method for diffuse optical imaging

Participant: Marc Bonnet.

In collaboration with Simon Arridge and Josias Elisee (University College London, UK). Numerical methods for calculating forward models of light propagation in tissue are extensively used in diffuse optical imaging (DOI). DOI involves a Helmholtz-type PDE with a complex-valued wavenumber. It requires modelling large optical regions whose parameters are known and piecewise constant. The boundary element method (BEM) answers this need and avoids the detailed interior meshing of these regions. The single-level Fast Multipole Method has been applied for solving the DOI governing equation, allowing substantial reduction of computational costs. The enhanced practicability of the BEM in DOI was demonstrated through test examples on single-layer problems, where two-digit reduction factors on solution time are achieved, and on a high-resolution version of a three-layered neonate's head.

# 6.6.9. High-Velocity Estimates and Inverse Scattering for Quantum N-Body Systems with Stark Effect

Participants: Ricardo Weder, Gerardo Daniel Valencia.

In an N-body quantum system with a constant electric field, by inverse scattering, we uniquely reconstruct pair potentials, belonging to the optimal class of short-range potentials and long-range potentials, from the high-velocity limit of the Dollard scattering operator. We give a reconstruction formula with an error term.

# 6.6.10. Small-Energy Analysis for the Matrix Schrödinger Operator on the Half-Line

Participant: Ricardo Weder.

In collaboration with Tuncay Aktosun (University of Texas Arlington) and Martin Klaus (Virginia Tech). The matrix Schrödinger equation with a selfadjoint matrix potential is considered on the half line with the most general selfadjoint boundary condition at the origin. When the matrix potential is integrable and has a first moment, it is shown that the corresponding scattering matrix is continuous at zero energy. An explicit formula is provided for the scattering matrix at zero energy. The small-energy asymptotics are established also for the related Jost matrix, its inverse, and various other quantities relevant to the corresponding direct and inverse scattering problems.

# 6.7. Other topics

# 6.7.1. Fast non-overlapping Schwarz domain decomposition methods for the neutron diffusion equation

Participant: Patrick Ciarlet.

A collaboration with Erell Jamelot (CEA Saclay/DEN). Investigating numerically the steady state of a nuclear core reactor can be very expensive, in terms of memory storage and computational time. In order to address both requirements, one can use a domain decomposition method, which is then implemented on a parallel computer.

We model the problem using a mixed approach, which involves a scalar flux and a vector current. The equivalent variational formulation is then discretized with the help of Raviart-Thomas-Nédélec finite elements.

The domain decomposition method is based on the Schwarz iterative algorithm with Robin interface conditions to handle communications. This method is analyzed from the continuous to the discrete point of views: well-posedness, convergence of the finite element method, optimality of the parameter appearing in the Robin interface condition and algorithms. Numerical experiments carried out on realistic 3D configurations using the APOLLO3©code (of CEA/DEN) show the parallel efficiency of the algorithm.

# 6.7.2. Equivalent local boundary conditions for the Monge Kantorovitch Mass Transport problem

Participant: Jean-David Benamou.

This work is done in collaboration with Adam Oberman Britanny Froese from Simon Fraser University, Vancouver. In the last 20 years, the Monge Kantorovich Optimal Transport problem (OTP) and its relationship with Partial Differential Equations (PDE) experienced a spectacular research revival (the Fields medal was awarded to Cedric Villani in 2010 partly for his contributions to OTP). Applications appeared in fields as diverse as meteorology, medical image processing, astronomy and economy. This new area offers numerical challenges which go beyond current knowledge in the field. Novel computational tools are needed. The OTP can also be reformulated as a Monge-Ampere (MA) PDE with non standard/non local boundary conditions (BC). We would like to use the new and efficient wide-stencil Finite Difference MA solver developed by Oberman and Froese but we do not know how to deal with the OTP equivalent BCs. We pursue the investigation of the two following innovative strategies: 1. Iteratively construct Neumann BCs such that the solution of the MA equation satisfies the OTP BC in the limit. 2. Merge the data in all of space and design simplified asymptotic local BCs at infinity which can be used to formulate local transparent BC on a truncated domain.

### 6.7.3. Topological Effects in Quantum Mechanics and High-Velocity Estimates

Participant: Ricardo Weder.

This work is done in collaboration with Miguel Ballesteros. High-velocity -or high-energy- estimates for scattering solutions to the Schrödinger equation are important for many reasons. For example, in topological effects in quantum mechanics, where the space accessible to the particles has a non-trivial topology, like, for example, in the celebrated magnetic Aharonov-Bohm effect, where an electron is constrained to be on the exterior of a torus that contains a magnetic flux inside. Here, the solution acquires a phase if the electron travels inside the hole of the magnet and, on the contrary, it acquires no phase if the particle travels outside the hole. We obtain precise high-velocity estimates for the scattering solutions, that prove that quantum mechanics actually predicts the existence of the magnetic Aharonov-Bohm effect, under the conditions of the celebrated Tonomura et al. experiments. Moreover, in the case of the electric Aharonov-Bohm effect, we provide precise conditions for the validity of the Aharonov-Bohm Ansatz and we give a rigorous proof that quantum mechanics predicts the existence of this effect.

# 6.7.4. Entanglement Creation in Low-Energy Scattering

Participant: Ricardo Weder.

We study the entanglement creation in the low-energy scattering of two particles of mass,  $m_1, m_2$  in three dimensions. We consider a general class of interaction potentials that are not required to be spherically symmetric. The incoming asymptotic state, before the collision, is a product of two normalized Gaussian states with the same variance,  $\sigma$ , and opposite mean momentum. After the scattering the particles are in the outgoing asymptotic state that is not a product state. We take as a measure of the entanglement created by the collision the purity of one of the particles in the outgoing asymptotic state. In the incoming asymptotic state the purity is one. We provide a rigorous explicit computation, with error bound, of the leading order of the purity at low-energy. The leading order depends strongly in the difference of the masses. The entanglement takes its minimum when the masses are equal, and it increases rapidly with the difference of the masses. It is quite remarkable that the anisotropy of the potential plays no role, on spite of the fact that entanglement is a second order effect.

## 6.7.5. Open Scattering Channels in Manifolds with Horns

Participant: Ricardo Weder.

This work is done in collaboration with Olaf Post (Humboldt University, Berlin) and Rainer Hempel (Mount Allison University, Sackville New Brunswick). In the framework of time-dependent geometric scattering theory, we study the existence and completeness of the wave operators for perturbations of the Riemannian metric for the Laplacian on a complete manifold of dimension n. The smallness condition for the perturbation is expressed in purely geometric terms using the harmonic radius; therefore, the size of the perturbation can be controlled in terms of local bounds on the radius of injectivity and the Ricci-curvature, and no global assumption is needed. As an application of these ideas we obtain a stability result for the scattering matrix. As a consequence we find that a scattering channel which interacts with other channels preserves this property under small perturbations.

# 7. Contracts and Grants with Industry

## 7.1. Contract POEMS-CEA-LIST-1: CASSIS PROJECT

Participant: Gary Cohen.

Start: 03/25/2008, End: 06/18/2011. Administrator: INRIA.

G. Cohen participates to Project CASSIS headed by the LIST laboratory of CEA and funded by the EADS Fundation which started in June 2008. This project aims to simulate elastic waves in thin layered anisotropic media for non-destructive testing. In collaboration with E. Demaldent, G. Cohen must provide a code based on spectral element methods to model these waves.

### 7.2. Contract POEMS-CEA-LIST-2

Participant: Anne-Sophie Bonnet-Ben Dhia.

Start: 09/01/2010, End: 07/31/2013. Administrator: ENSTA.

This contract is about the scattering of elastic waves by a stiffener in an anisotropic plate.

### 7.3. Contract POEMS-CEA-LIST-3

Participants: Laurent Bourgeois, Eric Lunéville.

Start: 10/01/2011, End: 09/30/2012. Administrator: ENSTA.

This contract is about the linear sampling methods for elastic waveguides.

### 7.4. Contract POEMS-CEA-LIST-DIGITEO

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Antoine Tonnoir.

Start: 10/01/2011, End: 09/30/2014. Administrator: ENSTA.

This contract is about the scattering of elastic waves by a local defects in an anisotropic plate. It consists on the funding of Antoine Tonnoir's Phd.

### 7.5. Contract POEMS-DGA

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Patrick Joly.

Start: 09/01/2011, End: 12/31/2012. Administrator: ENSTA.

This contract is about the waveguide in photonic crystals: we want to develop new mathematical and numerical tools for the characterization, the study and the computation of the guided modes in photonic crystals.

# 7.6. Contract POEMS-ONERA-CE Gramat: DIGATOP PROJECT

Participant: Gary Cohen.

Start: 06/16/2009, End: 12/15/2011. Administrator: INRIA.

In collaboration with ONERA-DEMR, G. Cohen participates to the FEMGD project funded by CEG (Centre d'Études de Gramat), which started in 2004. This project is devoted to the construction of a software using spectral discontinuous Galerkin methods for Maxwell's equations.

### 7.7. Contract POEMS-CE Gramat: NADEGE PROJECT

Participants: Gary Cohen, Alexandre Sinding.

Start: 03/06/2009, End: 03/06/2012. Administrator: INRIA.

In collaboration with ONERA-DEMR, G. Cohen participates with A. Sinding to the NADEGE project funded by CEG (Centre d'Études de Gramat). This project is devoted to the construction of a software based on FEMGD for solving Vlasov-Maxwell's equations by a PIC method.

## 7.8. Contract POEMS-Airbus: ADNUMO PROJECT

Participant: Patrick Joly.

Start: 01/01/2011, End: 12/31/2011. Administrator: INRIA.

This contract is about the hybridation of time domain numerical techniques in aeroacoustics (Linearized Euler equations).

# 8. Partnerships and Cooperations

## 8.1. National Initiatives

- GDR Ultrasons: this GDR, which regroups more than regroup 15 academic and industrial research laboratories in Acoustics and Applied Mathematics working on nondestructive testing. It has been renoveled this year with the participation of Great Britain.
- ANR project AEROSON: Simulation numérique du rayonnement sonore dans des géométries complexes en présence d'écoulements réalistes

Partners: EADS-IW, CERFACS, Laboratoire d'Acoutique de l'Université du Maine.

Start: 10/01/2009, End: 09/30/2012. Administrator: CNRS. Coordinator: Jean-François Mercier.

• ANR project PROCOMEDIA: Propagation d'ondes en milieux complexes

Partners: ESPCI, Laboratoire d'Acoutique de l'Université du Maine, Departamento de Fisica de la Universidad de Chile.

Start: 04/01/2011, End: 03/30/2014. Administrator: CNRS. Coordinator for POEMS: Jean-François Mercier.

 ANR project METAMATH: modélisation mathématique et numérique pour la propagation des ondes en présence de métamatériaux. Partners: EPI DEFI (INRIA Saclay), IMATH-Université de Toulon, DMIA-ISAE.

Start: 12/01/2011, End: 11/30/2015. Administrator: INRIA. Coordinator: Sonia Fliss.

## 8.2. European Initiatives

### 8.2.1. FP7 Projet

8.2.1.1. SIMPOSIUM

Title:Simulation Platform for Non Destructive Evaluation of Structures and Materials

Type: COOPERATION (ICT)

Defi: PPP FoF: Digital factories: Manufactoring design and product lifecycle manage

Instrument: Integrated Project (IP)

Duration: September 2011 - August 2014 Coordinator: CEA (Philippe Lecestre) (France)

Others partners:SERCO LIMITED (UNITED KINGDOM), SIMULAYT LTD (UNITED KINGDOM), SKF SVERIGE AB (SWEDEN), UNIVERSITA DEGLI STUDI DI NAPOLI FEDERICO II (ITALY), UNIVERSITA DEGLI STUDI DI CASSINO (ITALY), VOLKSWAGEN AG (GERMANY), ARCELORMITTAL MAIZIÈRES RESEARCH SA (FRANCE), EXTENDE (FRANCE), EUROPEAN AERONAUTIC DEFENCE AND SPACE COMPANY EADS FRANCE SAS (FRANCE), IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE (UNITED KINGDOM), SAARSCHMIEDE GMBH FREIFORMSCHMIEDE\* (GERMANY), KATHOLIEKE UNIVERSITEIT LEUVEN (BELGIUM), FRAUNHOFER-GESELLSCHAFT ZUR FOERDERUNG DER ANGEWANDTEN FORSCHUNG E.V (GERMANY).

 $See \ also: http://cordis.europa.eu/fetch?CALLER=PROJ\_ICT\&ACTION=D\&CAT=PROJ\&RCN=99653$ 

### 8.3. International Initiatives

### 8.3.1. Visits of International Scientists

- Ricardo Weder, Professor at the Universidad Nacional Autónoma de Mexico.
- Gerardo Daniel Valencia, Professor at the Universidad Nacional Autónoma de Mexico.

# 9. Dissemination

# 9.1. Animation of the scientific community

- A. S. Bonnet-Ben Dhia is the Head of the Electromagnetism Group at CERFACS (Toulouse)
- A. S. Bonnet-Ben Dhia is in charge of the relations between l'ENSTA and the Master "Dynamique des Structures et des Systèmes Couplés (Responsible : Etienne Balmes)".
- A. S. Bonnet-Ben Dhia is presidente of the "Conseil scientifique de l'Institut des sciences de l'ingénierie et des systèmes (INSIS-CNRS)".
- M. Bonnet is associate editor of European Journal of Mechanics A/Solids (since Jan. 2008).
- M. Bonnet is associate editor of Engineering Analyses with Boundary Elements (since July 2011).
- M. Bonnet is on the editorial board of *Inverse Problems*.
- M. Bonnet is on the editorial board of *Computational Mechanics*.
- P. Ciarlet is an editor of DEA (Differential Equations and Applications) since July 2008
- G. Cohen is a scientific expert of ONERA.
- P. Joly is a member of the scientific committee of CEA-DAM.
- P. Joly is a member of the Hiring Committee of Ecole Polytechnique in Applied Mathematics.
- P. Joly is a member of the Post Docs Commission of INRIA Rocquencourt.
- P. Joly is a member of the Scientific Committee of the Seminar in Applied Mathematics of College de France (P. L. Lions).
- P. Joly is an editor of the journal Mathematical Modeling and Numerical Analysis.
- P. Joly is a member of the Book Series Scientific Computing of Springer Verlag.
- P. Joly is an expert for the MRIS (Mission pour l'Innovation et la Recherche Scientifique) of DGA (Direction Générale de l'Armement)
- P. Joly is a scientific expert for the "Fondation de Recherche pour l'Aéronautique et l'Espace" in the thematic "Mathématiques Appliquées au domaine de l'Aéronautique et Espace".
- M. Lenoir is a member of the Commission de Spécialistes of CNAM.
- M. Lenoir is in charge of Master of Modelling and Simulation at INSTN.
- E. Lunéville is the Head of UMA (Unité de Mathématiques Appliquées) at ENSTA.
- The Project organizes the monthly Seminar Poems (Coordinators: A. Burel, N. Chaulet)

## 9.2. Teaching

- Eliane Bécache
  - Compléments sur la méthode des éléments finis, ENSTA, Paris, (2nd year).
  - Cours sur les PML, Collège Polytechnique, Paris.
- Anne-Sophie Bonnet-Ben Dhia
  - Outils élémentaires d'analyse pour les EDP, ENSTA, Paris (1st year).
  - Propagation dans les guides d'ondes, ENSTA, Paris (3rd year).
  - Théorie spectrale des opérateurs autoadjoints et applications aux guides optiques, EN-STA, Paris (2nd year).
  - Propagation des ondes, Ecole Centrale de Paris (M2).
- Marc Bonnet

- Problèmes inverses, Master TACS (ENS Cachan) et DSMSC (Centrale Paris).
- Méthodes intégrales, Master TACS (ENS Cachan).
- Equations intégrales et multipôles rapides, Ecole doctorale MODES (Univ. Paris Est, Marne la Vallée).

### • Laurent Bourgeois

- Outils élémentaires pour l'analyse des EDP, ENSTA, Paris (1st year).
- Fonctions de la variable complexe, ENSTA, Paris (2nd year)

### • Aliénor Burel

- Probabilités, IUT d'informatique, Université Paris-Sud XI, Orsay (2nd year).
- Analyse, IUT d'informatique, Université Paris-Sud XI, Orsay (1st year).

### Maxence Cassier

- Système dynamique: Stabilité et Commande, ENSTA, Paris (1st year).
- Introduction à MATLAB, ENSTA, Paris (1st year).
- Fonction de variable complexe, ENSTA, Paris (2nd year)
- Tutorat pour élèves en difficulté en mathématiques appliquées, ENSTA, Paris (1st year).

### • Stéphanie Chaillat

Introduction à la discrétisation des équations aux dérivées partielles, ENSTA, Paris (1st year).

### Nicolas Chaulet

- Equations différentielles et introduction à l'automatique, ENSTA, Paris (1st year)
- Optimisation quadratique et analyse convexe, ENSTA, Paris (1st year)
- Méthode des éléments finis, ENSTA, Paris (2nd year)

### Lucas Chesnel

- Méthode des éléments finis, ENSTA, Paris (2nd year)
- Introduction au calcul scientifique, ENSTA, Paris (2nd year)
- Les équations de Maxwell et leur discrétisation, ENSTA, Paris (3rd year), and Master "Modeling and Simulation" (M2)

### Patrick Ciarlet

- Méthode des éléments finis, ENSTA, Paris (2nd year)
- Parallélisme et calcul réparti, ENSTA (3rd year), and Master "Modeling and Simulation"
   (M2)
- Les équations de Maxwell et leur discrétisation, ENSTA, Paris (3rd year), and Master "Modeling and Simulation" (M2)
- Practical tools to solve indefinite problems, Master, Facultade de Matemáticas, USC, Santiago de Compostela.

## • Julien Coatléven

Introduction à la discrétisation des équations aux dérivées partielles, ENSTA, Paris (1st year)

### • Gary Cohen

 Méthodes numériques pour les équations des ondes, Master 2, Université de Paris-Dauphine

### Sonia Fliss

- Méthode des éléments finis, ENSTA, Paris (2nd year)
- Programmation scientifique et simulation numérique, ENSTA, Paris (2nd year)
- Introduction à la discrétisation des équations aux dérivées partielles, ENSTA, Paris (1st year).

### Christophe Hazard

- Outils élémentaires d'analyse pour les EDP, ENSTA, Paris (1st year)
- Théorie spectrale des opérateurs autoadjoints et applications aux guides optiques, EN-STA, Paris (2nd year).

### Sébastien Impériale

- Introduction à la discrétisation des équations aux dérivées partielles, ENSTA, Paris (1st year).
- Programmation scientifique et simulation numérique, ENSTA, Paris (2nd year)
- Discrétisation de systèmes hyperboliques symétriques par Galerkin discontinu, Collège polytechnique.

### Patrick Joly

- Introduction à la discrétisation des équations aux dérivées partielles, ENSTA, Paris (1st year).
- Outils élémentaires d'analyse pour les EDP, ENSTA, Paris (1st year).

#### Marc Lenoir

- Fonctions de variable complexe, ENSTA, Paris (2nd year).
- Equations intégrales, ENSTA, Paris (3rd year).

### • Eric Lunéville

- Introduction au calcul scientifique, ENSTA, Paris (2nd year)
- Programmation scientifique et simulation numérique, ENSTA, Paris (2nd year)
- Propagation dans les guides d'ondes, , ENSTA, Paris (2rd year)

### • Jean-François Mercier

- Outils élémentaires d'analyse pour les EDP, ENSTA, Paris (1st year).
- Fonctions de variable complexe, ENSTA, Paris (2nd year).
- Fluides compressibles, ENSTA, Paris (2nd year).
- Théorie spectrale des opérateurs autoadjoints et application aux guides optiques, ENSTA, Paris (2nd year).

### Nicolas Salles

- Analyse et séries de Fourier, Université Paris XI Orsay (L2)
- Systèmes Linéaires (Matlab), Université Paris XI Orsay (L3)
- Calcul scientifique, Université Paris XI Orsay (L3)

# 9.3. Participation in Conferences, Workshops and Seminars

#### • Eliane Bécache

Some contributions to wave propagation problems in unbounded domains, WAM, Heraklion, May 2-5, 2011

### • Anne-Sophie Bonnet-Ben Dhia

- Asymptotic modeling of boundary layers in aeroacoustics, 17th AIAA/CEAS Aeroacoustics Conference (32nd AIAA Aeroacoustics Conference), Portland, mai 2011.
- Time harmonic maxwell equations with shifting coefficients: mathematical and numerical aspects, Saint-Jacques de Compostelle (Espagne), octobre 2011.

#### Marc Bonnet

- 3D time-domain topological sensitivity for wave-based crack imaging in elastic solids,
   Applied Inverse Problems 2011, College Station, USA, mai 2011.
- Wave-based crack imaging in elastic solids using 3D time-domain topological sensitivity, 10th International Conference on Mathematical and Numerical Aspects of Waves, Vancouver, July, 2011.
- Topological derivative of elastodynamic energy functionals, Workshop on Inverse Problems and Applications, Ecole Polytechnique, sept. 2011.

### • Jean-David Benamou

- Numerical Microlocal Analysis, Advances in Viscosity Solutions Nom de la conference 1, Banff Research Centre (Canada), February 2011.
- Numerical Microlocal Analysis, 10th International Conference on Mathematical and Numerical Aspects of Waves, Vancouver, July, 2011.

### • Laurent Bourgeois

Imaging an elastic waveguide with the Linear Sampling Method by using the Lamb modes,
 Applied Inverse Problems, College Station (USA), may 2011

### Maxence Cassier

 Multiple Acoustic Scattering by small obstacles in two dimensions, 10th International Conference on Mathematical and Numerical Aspects of Waves, Vancouver, July, 2011.

### • Juliette Chabassier

- Modeling and numerical simulation of a grand piano, 10th International Conference on Mathematical and Numerical Aspects of Waves, Vancouver, July, 2011.
- Introduction of high order  $\theta$  schemes for the linear wave equation, ICIAM conference, Vancouver, July, 2011.
- Modeling and numerical simulation of a piano, Acoustical Society of America 161st Meeting, Seattle, USA, May 2011

### • Stéphanie Chaillat

- FaIMS: Un algorithme rapide pour un problème inverse en diffraction acoustique, Séminaire de l'équipe projet DEFI, INRIA Saclay, France, March 2011.
- Algorithme rapide et adaptatif pour le problème inverse de milieu pour l'équation des ondes scalaire avec fréquences et sources multiples, 10eme Colloque National en Calcul des Structures, Giens, May 2011.
- Fast Multipole Method for 3-D elastodynamic and viscoelastodynamic boundary integral equations, Séminaire de l'équipe Analyse Numérique de l'IRMAR, Université Rennes 1, France, June 2011.

 Fast Multipole Method for 3-D elastodynamic and viscoelastodynamic boundary integral equations, Seminar of the Department of Mathematics, Parma University, Italy, June 2011.

 A fast and adaptive algorithm for the inverse medium problem with multiple frequencies and multiple sources for the 3-D elastodynamic equation, 8th International Conference on Structural Dynamics (Eurodyn 2011), Leuven, Belgium, July 2011.

#### Nicolas Chaulet

- Simultaneous reconstruction of an obstacle and its Generalized Impedance Boundary Condition, ICIAM conference, Vancouver, July 2011.
- Stability results for the identification of generalized impedance boundary coeficients, 10th International Conference on Mathematical and Numerical Aspects of Waves, Vancouver, July, 2011.

#### Lucas Chesnel

- Radiation condition for a non-smooth interface between a dielectric and a metamaterial, Numerical Electromagnetics & Industrial Applications (NELIA 2011), Santiago de Compostela, October, 2011.
- Radiation condition for a non-smooth interface between a dielectric and a metamaterial, 10th International Conference on Mathematical and Numerical Aspects of Waves, Vancouver, July, 2011.
- Scalar waves transmission problems with sign-changing coefficients, ICIAM conference, Vancouver, July, 2011.
- Problèmes de propagation d'ondes dans un milieu diélectrique métamatériau Aspects mathématiques, Colloque National Métamatériaux 2011, Orsay, March, 2011.

### Patrick Ciarlet

 Scalar transmission problems between dielectrics and metamaterials, 10th International Conference on Mathematical and Numerical Aspects of Waves, Vancouver, July, 2011.

### Sonia Fliss

- Vers des méthodes de décomposition de domaines pour des problèmes de guides d'ondes non classiques, 5ème Colloque sur les Tendances des Applications Mathématiques en Tunisie, Algérie, Maroc (TAMTAM), Sousse (Tunisie), 23-26 April 2011.
- Exact boundary conditions, wave propagation and periodic media, Minisymposia Modern Numerical Methods for waves, ICIAM conference, Vancouver, July, 2011.
- A DtN approach for the exact computation of guided moded in a photonic crystal waveguides, 10th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Vancouver, July 2011.

### • Christophe Hazard

- Using time reversal for space-time focusing of acoustic waves, 5ème Colloque sur les Tendances des Applications Mathématiques en Tunisie, Algérie, Maroc (TAMTAM), Sousse (Tunisie), 23-26 April 2011.
- A uniqueness property for the scattering by a junction of open waveguides, 10th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Vancouver, July 2011.

### • Sébastien Impériale

 High Order Theta Scheme for the linear wave equation., ICIAM conference, Vancouver, July, 2011. Mathematical and numerical modelling of piezoelectric sensors., 10th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Vancouver, July 2011.

### Patrick Joly

- Galerkin approach to retarded potentials (II) Complements and application to transparent conditions, Workshop "Time Domain Boundary Integral Equations", Max Planck Institute for Mathematics in the Sciences, Leipzig, Mai 2011
- Propagation of acoustic waves in fractal networks, 10th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Vancouver, July 2011.
- Mathematical and numerical modeling of piezoelectric sensors for non destructive testing,
   Workshop "Modern Techniques in the Numerical Solution of Partial Differential Equations",
   University of Crete, Heraklion, September 2011
- Mathematical and numerical modeling of piezoelectric sensors for non destructive testing, NELIA 2011, Santiago de Compostela, October 2011
- Mathematical modelling of electromagnetic waves in heterogeneous lossy coaxial cables,
   4th International Conference SCPDE, Hong Kong Baptist University, Hong Kong, December 2011

### • Jean-François Mercier

- Time harmonic acoustic scattering in presence of a shear flow and an impedance condition, Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier and Florence Millot, 10th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Vancouver, July 2011.
- Time harmonic acoustic scattering in presence of a shear flow and a Myers impedance condition, Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier and Florence Millot, 17th AIAA/CEAS Aeroacoustics Conference (32nd AIAA Aeroacoustics Conference), American Institute of Aeronautics and Astronautics, Portland, may 2011

### Nicolas Salles

 A New Treatment of Singularities in Galerkin Boundary Integral Equations, Marc Lenoir, Nicolas Salles, 10th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Vancouver, July 2011.

### Ricardo Weder

- Aeroacústica en Capas a la Frontera, Coloquio del Instituto Universitario Aeronáutico, Córdoba, Argentina, 28.10.2011.
- Creación de Entrelazamiento en Colisiones a Bajas Energías, Seminario del Instituto de Física Rosario, Rosario, Argentina, 24.10.2011.

# 10. Bibliography

# **Publications of the year**

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- [11] L. BOURGEOIS, F. LE LOUER, E. LUNÉVILLE. On the use of Lamb modes in the linear sampling method for elastic waveguides, in "Inverse Problems", 2011, vol. 27, 27.
- [12] L. CHESNEL, P. CIARLET. Compact imbeddings in electromagnetism with interfaces between classical materials and meta-materials, in "SIAM J. Math. Anal.", 2011, vol. 43, p. 2150–2169.
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### **National Conferences with Proceeding**

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### **Research Reports**

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- [24] E. BÉCACHE, A. PRIETO. Remarks on the stability of Cartesian PMLs in corners, INRIA, May 2011, n<sup>o</sup> RR-7620, http://hal.inria.fr/inria-00593182/en.
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