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Activity Report 2011

Project-Team REALOPT

Reformulations and Algorithms for Combinatorial Optimization

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB)

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
Optimization, Learning and Statistical Methods

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Project-Team REALOPT

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1. Members

Research Scientist

Ruslan Sadykov [INRIA, Research Associate (CR1)]

Faculty Members

Andrew Miller [University Bordeaux 1, Professor (Pr), HdR]

Arnaud Pêcher [University Bordeaux 1, Professor (Pr), HdR]

Gautier Stauffer [University Bordeaux 1, Assistant Professor on an INRIA Chair of Excellence (MdC), HdR]

Pierre Pesneau [University Bordeaux 1, Assistant Professor (MdC)]

François Vanderbeck [University Bordeaux 1, Team Leader, Professor (Pr), HdR]

Cédric Joncour [University Bordeaux 1, until March 2011]

Technical Staff

Romain Leguay [INRIA, Associate Engineer (IJD) on ADT Hipad.]

PhD Students

Nastaran Rahmani [Cordi S, since 2011]

Nicolas Dupin [PhD part of a collaboration EDF-INRIA-DGA since 2011]

Administrative Assistant

Sylvie Embolla [INRIA]

2. Overall Objectives

2.1. Overall Objectives

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decisions set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space. Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, Lagrangian decomposition methods to produce powerful relaxations, constraint programming to actively reduce the solution domain through logical implications, heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidate solutions, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique.

Building on complementary expertise, our team's overall goals are threefold:

Objective (i) To design tight formulations for specific problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. More broadly, to contribute to theoretical and methodological developments of exact approaches in combinatorial optimization, while extending the scope of applications (in particular to encompass nonlinear models).

Objective (ii) To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, constraint programming, combinatorial algorithms and graph theory. To develop "efficient" algorithms for specific mathematical models and to tackle large-scale real-life applications, providing provably good approximate solutions by combining exact methods and heuristics.

Objective (iii) To provide prototypes of specific model solvers and generic software tools that build on our research developments, writing proof-of-concept code, while making our research findings available to internal and external users.

2.2. Highlights

In the follow-up of our participation to the 2010 ROADEF/EURO challenge on a production planning application at EDF, our team has embarqued this year on a research contract with EDF in collaboration with DOLPHIN. The project is quite ambitious. It requires producing a maintenance schedule for nuclear central that is robust to perturbation in maintenance duration, account for the stochastic electricity demand, and can be stably re-optimized dynamically as problem input are reviewed. In parallel we have develop a close colaboration with PUC-Rio and UFF in Brasil throught the associated team SAMBA: the project is focused on methodological developments for Branch-and-Price-and-Cut algrotihms, that shall translate into a state-of-the-art generic solver for decomposition based approaches in integer programming.

We keep up the momentum of strong papers being published in the best journals of the field (Mathematical Programming, Operations Research, Operations Research Letters, Optima, Discrete Applied Mathematics, INFORMS Journal on Computing, Computers and Operations Research, International Journal of Mathematics in Operational Research, Journal of Mathematical Modelling and Algorithms) and the most competitive conferences (such as ACM-SIAM Symposium on Discrete Algorithms (SODA)). We also contribute state-of-the-art review papers in in a reference book "Progress in Combinatorial Optimization" and received invitations as plenary speakers at international workshops.

On the team side, Cédric Joncour who completed his PhD with us in December 2010, was recruited as an assistant professor at the University of Le Havre. Gautier Stauffer obtained his HDR (Habilitation à Diriger des Recherches) for his work entitled "At Play With Combinatorial Optimization, Integer Programming and Polyhedra" in November [11].

3. Scientific Foundations

3.1. Introduction

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm* : (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a factional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM Ilog-CPLEX or FICO/Dash-Optimization's Xpress-mp). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is the study of non-linear models (non linearities are inherent in some engineering, economic and scientific applications) where solution techniques build on the best MIP approaches while demanding much more than simple extensions. Robust optimization is another area where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise mathematical programming (polyhedral approaches, Dantzig-Wolfe decomposition, non-linear integer programing, stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems.

3.2. Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [69]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if X is a mixed integer program: $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$ where $P = \{x \in \mathbb{R}^{n+p} : Ax \leq b\}$ with matrix $(A, b) \in \mathbb{Q}^{m\times(n+p+1)}$, then $conv(X)$ is a polyhedron that can be described in terms of linear constraints, i.e. it writes as $conv(X) = \{x \in \mathbb{R}^{n+p} : C\,x \leq d\}$ for some matrix $(C, d) \in \mathbb{Q}^{m'\times(n+p+1)}$ although the dimension m' is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem $\min\{cx : x \in X\}$ and solving the *separation problem* over the associated polyhedron $conv(X)$: if $\widetilde{x} \notin conv(X)$, find a linear inequality $\pi\, x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\widetilde{x}$. Hence, for NP-hard problems, one can not hope to get a compact description of $conv(X)$ nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of $conv(X)$ and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities $C\,x \leq d$ can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

3.3. Decomposition and reformulation approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called, *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, *cutting plane procedures* defined in the previous section reveal to be powerful. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

3.4. Constraint Programming (CP)

Constraint Programming focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems (minimizing the maximum of several variable values). Mixed Integer Programming (MIP), on the other hand, is effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables. Many problems belong to the intersection of these two classes. For example, some scheduling and timetabling problems are tightly constrained and have a sum-type objective. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

3.5. Mixed Integer NonLinear Programming (MINLP)

Many engineering, management, and scientific applications involve not only discrete decisions, but also nonlinear relationships that significantly affect the feasibility and optimality of solutions. MINLP problems combine the difficulties of MIP with the challenges of handling nonlinear functions. MINLP is one of the most flexible modeling paradigms available. However, solving such models is much more challenging: available softwares are not nearly as effective as standard softwares for linear MIP. The most powerful algorithms combine sophisticated methods that maintain outer linear programming approximation or convex relaxations with branch-and-bound enumeration; hence, the role of strong convex reformulations is crucial. The development of results for structured submodels are essential building blocks. Preprocessing and bound reduction (domain reduction logic similar to that used in CP) are quite important too. Finally, decomposition methods also permit to develop tight outer approximations.

3.6. Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

4. Application Domains

4.1. Introduction

Our group has tackled applications in logistics, transportation and routing [67], [66], [63], [65], in production planning [44], [84] and inventory control [63], [65], in network design and traffic routing [46], [54], [61], [89], [42], [55], [60], [72], [79], in cutting and placement problems [70], [71], [81], [82], [83], [85], and in scheduling [8], [74].

4.2. Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form “at least two paths link each pair of terminals”. We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [54]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today’s optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronic system installation cost. We developed and compared several decomposition approaches [91], [90], [89] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route [88]. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [61], [62].

We studied several time dependent formulations for the unit demand vehicle routing problem [48], [47]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We also worked on the p-median problem, applying the matching theory to develop an efficient algorithm in Y-free graphs and to provide a simple polyhedral characterization of the problem and therefore a simple linear formulation [78] simplifying results from Baiou and Barahona.

4.3. Packing and Covering Problems

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [21]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs. The paper which presents this work is being finalized.

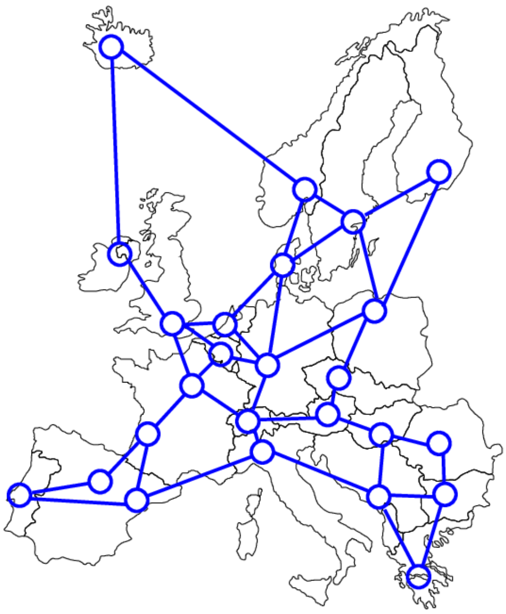

Figure 1. Design of a SDH/SONET european network where demands are multiplexed.

We have designed a new algorithm for vertex packing (equivalently stable set) in claw-free graphs [68]. Previously the best known algorithm for this problem had a running time of $O(n^6)$ (with n the number of vertices in the graph) while our new algorithm runs in $O(n^4)$.

We studied a variant of the knapsack problem encountered in inventory routing problem [65]: we faced a multiple-class integer knapsack problem with setups [64] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory [59], [57] [14], [13]. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2d knapsack problem that uses our 2D packing feasibility check.

4.4. Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a truncated branch-and-price algorithm: periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem; the global planning of customer visits is generated by solving a master program. Confronted with the issue of symmetry in time, we used a state-space relaxation idea. Our algorithm provides solutions with reasonable deviation from optimality for large scale problems (260 customer sites, 60 time periods, 10 vehicles) coming from industry [15]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [67].

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [53] and exact algorithms [39]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [38]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [74]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [76]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [77], [75]. Finally, we participated in writing an invited survey in French on solution approaches for machine scheduling problems in general [40].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [37], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [36] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [41] provide demonstrably stronger formulations for some problem classes than any previously proposed.

We have been developing **robust optimization** models and methods to deal with a number of applications like the above in which uncertainty is involved. In [50], [49], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [43]. We considered train timetabling problems and their re-optimization after a perturbation in the network [52], [51]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

5. Software

5.1. BaPCod – a generic Branch-and-Price Code

Participants: Romain Leguay [Software Engineer], Pierre Pesneau, Ruslan Sadykov, François Vanderbeck [correspondant].

BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of a Dantzig-Wolfe reformulation technique. The reformulated problem is solved using a branch-and-price (column generation) algorithm. This software platform, made of C++ classes, offers a "*black-box*" implementation that does not require user input and is not application specific. The features are
(i) the automation of the Dantzig-Wolfe reformulation process (the user defines a mixed integer programming problem in terms of variables and constraints, identifies subproblems, and can provide the associated solvers if available, but he does not need to explicitly define the reformulation, the explicit form of the columns, their reduced cost, or the Lagrangian bounds).
(ii) a default column generation procedure with standard initialization and stabilization (it may offer a selection of solvers for the master) – the issue of stabilization is discussed in [1], and

(iii) a default branching scheme – recent progress has been made on the issue of generic branching scheme in [23].
(iv) default primal heuristics specially developed for use in a decomposition framework [58].
The prototype software was/is used as background solver for 5 PhD thesis. It also served as the framework for our comparative study in a INRIA collaborative research action [1]. It has been experimented by two of our industrial partners, Exeo Solutions (Bayonne), on an inventory routing problem, and Orange Lab (France Telecom, Paris) on network design problems, time tabling problme by EURODECISION and it is currently being tested by the University Paris 6 and EDF. The prototype also enables us to be very responsive in our industrial contact.

See also the web page https://wiki.bordeaux.inria.fr/realopt/pmwiki.php/Project/BaPCod.

- Version: 1

6. New Results

6.1. Theoretical and Methodological Developments

Participants: Cédric Joncour, Andrew Miller, Arnaud Pêcher, Pierre Pesneau, Ruslan Sadykov, Gautier Stauffer, François Vanderbeck.

We made progress in the developement of theory and algorithms in the area of “Reformulation and Decomposition Approaches for MIP”, “Mixed Integer Nonlinear Programming”, and “Polyhedral Combinatorics and Graph Theory”.

6.1.1. Column Generation for Extended Formulations

Working in an extended variable space allows one to develop tight reformulations for mixed integer programs. However, the size of the extended formulation grows rapidly too large for a direct treatment by a MIP-solver. Then, one can use projection tools to derive valid inequalities for the original formulation and implement a cutting plane approach. Or, one can approximate the reformulation, using techniques such as variable aggregation or by reformulating a submodel only. Such approaches result in outer approximation of the intended extended formulation. The alternative considered in [28], [25] is an inner approximation obtained by generating dynamically the variables of the extended formulation. It assumes that the extended formulation stems from a decomposition principle: a subproblem admits an extended formulation from which an extended formulation for the original problem can be derived. Then, one can implement column generation for the extended formulation of the original problem by transposing the equivalent procedure for the Dantzig-Wolfe reformulation. Pricing subproblem solutions are expressed in the variables of the extended formulation and added to the current restricted version of the extended formulation along with the subproblem constraints that are active for the subproblem solution.
Our paper [28], [25] revisits the column-and-row generation approach. Our purpose is to show light on this approach, to emphasize its wide applicability, and to present it with a new angle as a method that is natural when considering a problem reformulation based on any extended reformulation of a subproblem, whether it yields the subproblem integer hull or just an approximation of it. In the spirit of [80], column-and-row generation is viewed herein as a generalization of standard column generation, the latter being based on a specific subproblem extended formulation. This generic view not only highlights the scope of applicability of the method, but it also leads to a more general termination condition than the traditional reduced cost criteria and to theoretically stronger dual bounds (observing that solving the integer subproblems yields Lagrangian dual bounds that might be tighter than the extended formulation LP bound). We highlight a key motivation for working in the extended space: there arises natural recombinations of previously generated columns into new subproblem solutions, which results in an acceleration of the convergence. We point out that lifting the master program in the variable space of the extended formulation can be done while carrying pricing in the compact variable space of the original formulation, using any oracle.

With [28], [25], we establishe the validity of the column-and-row generation algorithm in a form that encompass all special cases of the literature. The analysis therein should help practitioners to evaluate whether this alternative procedure has potential to outperform classical column generation on a particular problem. Our numerical experiments highlight a key observation: lifting pricing problem solutions in the space of the extended formulation permits their recombination into new subproblem solutions and results in faster convergence.

6.1.2. Primal Heuristics for Branch-and-Price

Our goal is to exploit global optimization decomposition approaches to retrieve very good feasible solution to large scale problem. This required extending primal heuristic paradigms to the context of dynamic generation of the variables of the model. We highlight an important fact: such generic tools typically performs better than problem specific meta-heursitics, in terms of solution quality and computing times. Based on our application specific experience with these techniques [65], [67], [86], [87], and on a review of generic classes of column generation based primal heuristics, in [58], we are developping a full blown review of such techniques, completed with new methods and an extensive numerical study. This research is being carried on in collaboration with the membersof the associated team project, SAMBA.
Significant progress has been achieved in developing generic primal heuristics that made their way into commercial mixed integer programming (MIP) solvers. Extensions to the context of a column generation solution approach are considered by our team, in search for generic black-box primal heuristics for use in Branch-and-Price approaches. As the Dantzig-Wolfe reformulation is typically tighter than the original compact formulations, techniques based on rounding its linear programming solution have better chance to yield good primal solutions. The aggregated information built into the column definition and the price coordination mechanism provide a global view at the solution space that may be lacking in somewhat more “myopic” approaches based on compact formulations. However, the dynamic generation of variables requires specific adaptation of heuristic paradigms. We focus on “diving” methods and considered their combination with sub-MIPing, relaxation induced neighborhood search, and truncated backtracking using a Limited Discrepancy Search. These add-ons serves as local-search or diversification/intensification mechanisms. We also consider feasibility pump approaches. The methods are numerically tested on standard models such as Cutting Stock, Vertex Coloring, Generalized Assignment, Lot-Sizing, and Vehicle Routing problems.

6.1.3. Combining Bender’s and Dantzig-Wolfe Decomposition

In the follow-up of [56], [88], [89], [90], we are finalizing our work on the combination of Dantzig-Wolfe and Bender’s decomposition: Bender’s Master is solved by column generation [91]. The application we considered is a multi-layer network design model arising from a real-life telecommunication application where traffic routing decisions imply the installation of expensive nodal equipment. Customer requests come in the form of bandwidth reservations for a given origin destination pair. Bandwidth requirements are expressed as a multiple of nominal granularities. Each request must be single path routed. Grooming several requests on the same wavelength and multiplexing wavelengths in the same optical stream allow the packing of more traffic. However, each addition or withdrawal of a request from a wavelength requires optical to electrical conversion for which a specific portal equipment is needed. The objective is to minimize the number of such equipment. We deal with backbone optical networks, therefore with networks with a moderate number of nodes (14 to 20) but thousands of requests. Further difficulties arise from the symmetries in wavelength assignment and traffic loading. Traditional multi-commodity network flow approaches are not suited for this problem. Four alternative models relying on Dantzig-Wolfe and/or Benders’ decomposition are introduced and compared. The formulations are strengthened using symmetry breaking restrictions, variable domain reduction, zero-one decomposition of integer variables, and cutting planes. The resulting dual bounds are compared to the values of primal solutions obtained through hierarchical optimization and rounding procedures. For realistic size instances, our best approaches provide solutions with optimality gap of approximately 5% on average in around 2 hours of computing time.

6.1.4. Branching in Branch-and-Price: a generic scheme

Our innovative branching scheme, proposed for its compatible with the column generation procedure (it implies no structural modifications to the pricing problem) is now published in Mathematical Programming A [23]. The scheme proceeds by recursively partitioning the sub-problem solution set. Branching constraints are enforced in the pricing problem instead of being dualized in a Lagrangian way. The subproblem problem is solved by a limited number of calls to the provided solver. The scheme avoids the enumeration of symmetric solutions.

6.1.5. Strong Branching Inequalities for Convex Mixed Integer Nonlinear Programs

Strong branching is an effective branching technique that can significantly reduce the size of the branch-and-bound tree for solving Mixed Integer Nonlinear Programming (MINLP) problems. The focus of our paper [24] is to demonstrate how to effectively use discarded information from strong branching to strengthen relaxations of MINLP problems. Valid inequalities such as branching-based linearizations, various forms of disjunctive inequalities, and mixing-type inequalities are all discussed. The inequalities span a spectrum from those that require almost no extra effort to compute to those that require the solution of an additional linear program. In the end, we perform an extensive computational study to measure the impact of each of our proposed techniques. Computational results reveal that existing algorithms can be significantly improved by leveraging the information generated as a byproduct of strong branching in the form of valid inequalities.

6.1.6. Linear and Nonlinear Inequalities for a Nonseparable Quadratic Set

We described some integer-programming based approaches for finding strong inequalities for the convex hull of a quadratic mixed integer nonlinear set containing two integer variables that are linked by linear constraints. This study [31] was motivated by the fact that such sets appear can be defined by a convex quadratic program, and therefore strong inequalities for this set may help to strengthen the formulation of the original problem. Some of the inequalities we define for this set are linear, while others are nonlinear (specifically conic). The techniques used to define strong inequalities include not only ideas related to recent perspective reformulations of MINLPs, but also disjunctive and lifting arguments. Initial computational tests will be presented.

6.1.7. On the composition of convex envelopes for quadrilinear terms

Within the framework of the spatial Branch-and-Bound algorithm for solving Mixed-Integer Nonlinear Programs, different convex relaxations can be obtained for multilinear terms by applying associativity in different ways. The two groupings ((x1x2)x3)x4 and (x1x2x3)x4 of a quadrilinear term, for example, give rise to two different convex relaxations. In previous work, we proved that having fewer groupings of longer terms yields tighter convex relaxations. In this paper [35], we give an alternative proof of the same fact and perform a computational study to assess the impact of the tightened convex relaxation in a spatial Branch-and-Bound setting.

6.1.8. Stable sets in claw-free graphs

A *stable set* is a set of pairwise non adjacent vertices in a graph and a graph is *claw-free* when no vertex contains a stable set of size three in its neighborhood. Given weights on the vertices, the stable set problem (a NP-hard problem in general) consists in selecting a set of pairwise non adjacent vertices maximizing the sum of the selected weights. The stable set problem in claw-free graphs is a fundamental generalization of the classic matching problem that was shown to be polynomial by Minty in 1980 (G. Minty. *On maximal independent sets of vertices in claw-free graphs*. J. Combinatorial Theory B, 28:284-304 (1980)). However, in contrast with matching, the polyhedral structure (i.e. the integer hull of all stable sets in a claw-free graph) is not very well understood and thus providing a 'decent' linear description of this polytope has thus been a major open problem in our field.

We proposed a new algorithm to find a maximum weighted stable set in a claw-free graph [45] whose complexity is now drastically better than the original algorithm by Minty (n^3 versus n^6, where n is the number of vertices). We also provided a description of the polyhedra in an extended space (i.e. using additional artificial variables) and an *efficient procedure* to separate over the polytope in polynomial-time [27]. Beside those main contributions, we published another papers on the strongly minimal facets of the polytope [22].

We also published two survey papers on both the algorithmic and polyhedral aspects of the problem [32], [16].

6.1.9. Chvátal-Gomory rank of 0/1 polytopes

In [17], we study the Chvátal-Gomory rank of 0/1 polytopes. The Chvàtal-Gomory procedure is a generic cutting plane procedure to derive the integer hull of polyhedra, and the *rank* is the number of iterations needed. We revisited a classic framework by Chvátal, Cook and Hartmann (V. Chvtal, W. Cook, and M. Hartmann. *On cutting-plane proofs in combinatorial optimization*. Linear Algebra and its Applications, 114/115:455-499 (1989)) to prove lower bounds on the CG-rank and we made it more accessible (the original framework was hard to apply). It allowed us to give a very simple construction and to improve the lower bound on the rank of general 0/1 polytopes (the previous weaker lower bound relied on a sophisticated existence theorem by Erdös). This result is important as it shed some new light on a supposedly well understood procedure.

6.1.10. The Circular-Chromatic number

Another central contribution of our team concerns the chromatic number of a graph (the minimum number of independent stable sets needed to cover the graph). We proved that the chromatic number and the clique number of some superclasses of perfect graphs is computable in polynomial time [19], [18].

We investigated the circular-chromatic number. It is a well-studied refinement of the chromatic number of a graph (designed for problems with periodic solutions): the chromatic number of a graph is the integer ceiling of its circular-chromatic number. Xuding Zhu noticed in 2000 that circular cliques are the relevant circular counterpart of cliques, with respect to the circular chromatic number, thereby introducing circular-perfect graphs, a super-class of perfect graphs.

We proved that the chromatic number of circular-perfect graphs is computable in polynomial time [73], thereby extending Grötschel, Lovász and Schrijver's result to the whole family of circular-perfect graphs. We gave closed formulas for the Lovász Theta number of circular-cliques (previously, closed formulas were known for circular-cliques with clique number at most 3 only), and derived from them that the circular-chromatic number of circular-perfect graphs is computable in polynomial time [34].

6.2. Model Specific Developments and Applications

Participants: Cédric Joncour, Andrew Miller, Arnaud Pêcher, Pierre Pesneau, Ruslan Sadykov, Gautier Stauffer, Damien Trut, François Vanderbeck.

The models on which we made progress can be partitionned in three areas: "Packing and Covering Problems", "Network Design and Routing", and "Planning, Scheduling, and Logistic Problems".

6.2.1. Bin-Packing and Knapsack with Conflicts

The bin-packing problem consists in finding the minimum number of bin of fixed size one needs to pack a set of items of different sizes. We studied a generalization of this problem where items can be in conflicts and thus cannot be put together in the same bin. We show in [21] that the instances of the literature with 120 to 1000 items can be solved to optimality with a generic Branch-and-Price algorithm, such as our prototype BaPCod, within competitive computing time. Moreover, we solved to optimality all the 37 open instances. The approach involves generic primal heuristics, generic branching, but a specific pricing procedure.

The knapsack variant encountered in our bin packing problem resolution considers conflicts between items. This problem is quite difficult to solve compared to the usual knapsack problem. The latter is already NP-hard, but can be usually efficiently solved by dynamic programming. We have shown that when the conflict graph (the graph defining the conflicts between the items) is an interval graph, this generalization of the knapsack can also be solved quite efficiently by dynamic programming with the same complexity than the one to solve the common knapsack problem. For the case, when the conflict graph is arbitrary, we proposed a very efficient enumeration algorithm which outperforms the approaches used in the literature.

6.2.2. Using graph theory for solving orthogonal knapsack problems

We investigated the orthogonal knapsack problem, with the help of graph theory. The multi-dimensional orthogonal packing problem (OPP) is defined as follows: given a set of items with rectangular shapes, the problem is to decide whether there is a non-overlapping packing of these items in a rectangular bin. The rotation of items is not allowed. A powerful caracterization of packing configurations by means of interval graphs was introduced by Fekete and Schepers using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* (whose complement admits a transitive orientation: each such orientation of the edges corresponds to a specific placement of the forms) for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs.

In [13], [14] [57], we give two new algorithms: the first one is based upon matrices with consecutive ones on each row as data structures and the second one uses so-called MPQ-trees, which were introduced by Korte and Möhring to recognize interval graphs. These two new algorithms are very efficient, as they outperform Fekete and Schepers' on most standard benchmarks.

6.2.3. Inventory routing and pickup-and-delivery problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. in [15], we considered the planning of single product pickups over time: each site accumulates stock at a deterministic rate; the stock is emptied on each visit. Our objective is to minimize a surrogate measure of routing cost while achieving some form of regional clustering by partitioning the sites between the vehicles. The fleet size is given but can potentially be reduced. Planning consists in assigning customers to vehicles in each time period, but the routing, i.e., the actual sequence in which vehicles visit customers, is considered as an "operational" decision. We developed a truncated branch-and-price algorithm. This exact optimization approach is combined with rounding and local search heuristics to yield both primal solutions and dual bounds that allow us to estimate the deviation from optimality of our solution. We were confronted with the issue of symmetry in time that naturally arises in building a cyclic schedule (cyclic permutations along the time axis define alternative solutions). Central to our approach is a state-space relaxation idea that allows us to avoid this drawback: the symmetry in time is eliminated by modelling an average behavior. Our algorithm provides solutions with reasonable deviation from optimality for large scale problems (260 customer sites, 60 time periods, 10 vehicles) coming from industry. The subproblem is interesting in its own right: it is a multiple-class integer knapsack problem with setups. Items are partitioned into classes whose use implies a setup cost and associated capacity consumption.

Through the internship of Damien Trut, we studyied the optimization problem consisted in the planning of the pick-up of full waste container and delivery of empty container at customer sites by simple vehicles that can carry a single container, or vehicles with a trailer attached that have a total capacity of 2 containers but require more time when handling containers. The model is a multi-period, multi-vehicle, pickup and delivery problem, with "many-to-many" multi commodity transfer requirements and transhipment nodes. In its short term variant, urgent order are coming online. We developed the prototype of a branch-and-price approach for this problem. The prototype was used by Exeo to convince their customer of the potential benefit of decision aid tools to automatically generate vehicle routes. Next, we shall be considering the dimensioning of a vehicle fleet and their allocation to cluster of collect points in a periodic solution (for a PhD project).

In collaboration with the group of M-C Speranza of the university of Brescia (Italy), we study the Vehicle Routing Problem with Discrete Split Deliveries (a customer demand can be partition in integer lot assigned to different vehicles). The development is done within BaPCod with specialized pricing and branching scheme.

6.2.4. Time-Dependent Travelling Salesman Problem and Resource Constrained Shortest Path

In [12] we present a new formulation for the Time-Dependent Travelling Salesman Problem (TDTSP). The main feature of our formulation is that it uses, as a subproblem, an exact description of the n-circuit problem. We present a new extended formulation that is based on using, for each node, a stronger subproblem, namely a n-circuit subproblem with the additional constraint that the corresponding node is not repeated in the circuit.

Although the new model has more variables and constraints than the model of Picard and Queyranne (1978), the results given from our computational experiments show that the linear programming relaxation of the new model gives, for many of the instances tested, gaps that are close to zero. We also provided a complete characterization of the feasible set of the corresponding linear programming relaxation in the space of the variables of the PQ model.
Following this work, we proposed an extended formulation in terms of the Asymmetric Travelling Salesman Problem (ATSP) in [33]. A tightening the linear programming relaxation is obtained by i) enhancing the subproblem arising in the standard multicommodity flow (MCF) model for the ATSP and then ii) by using modelling enhancement techniques. We compare the linear programming relaxation of the new formulation with the linear programming relaxation of the three compact and non-dominated formulations presented in Oncan et al. (2009). As a result of this comparison we present an updated classification of formulations for the asymmetric traveling salesman problem (ATSP).
In the intership of André Linhares, we studied the Resource Constrained Shortest Path Problem (RCSPP): we presented some of the state-of-the-art dynamic programming methods for solving the RCSPP in a unified manner, and we proposed some variants of these algorithms. We assessed the effectiveness of these algorithms through computational experiments.

6.2.5. Machine scheduling

The column-and-row generation method presented in [28], [25] is quite effective for the general machine scheduling problem. In our work [29], we show indeed that one of the most efficient approaches to solve this problem is to use time-indexed Integer Programming formulation, and to deal with its huge size by generating variables and constraints dynamically. The numerical results of [29], highlight the significant reduction in computing times that results from applying the column-and-row generation approach.

In [20], we coonsider the scheduling jobs in parallel, i.e., jobs can be executed on more than one processor at the same time. With the emergence of new production, communication and parallel computing system, the usual scheduling requirement that a job is executed only on one processor has become, in many cases, obsolete and unfounded. In this work, we consider the NP-hard problem of scheduling malleable jobs to minimize the total weighted completion time (or mean weighted flow time). For this problem, we introduce the class of "ascending" schedules in which, for each job, the number of machines assigned to it cannot decrease over time while this job is being processed. We prove that, under a natural assumption on the processing time functions of jobs, the set of ascending schedules is dominant for the problem. This result can be used to reduce the search space while looking for an optimal solution.

6.2.6. One warehouse multi-retailer problem

The *One-Warehouse Multi-retailer problem (OWMR)* is a very important NP-hard inventory control problem arising in the distribution of goods when one central warehouse is supplying a set of final retailers facing demand from customers. In [30], we provide a simple and fast 2-approximation algorithm for this problem (i.e. an algorithm ensuring a deviation by a factor at most two from the optimal solution). This result is both important in practice and in theory as it allows to approximate large real-world instances of the problem (we implemented this algorithm at IBM and it is within 10% of optimality in practice) and the techniques we developed appear to apply to more general settings. We are extending our results to other inventory control problems.

6.3. Software prototypes, Generic Developments and Specific Tools

Participants: Cédric Joncour, Romain LeGuay, Pierre Pesneau, Ruslan Sadykov, François Vanderbeck.

6.3.1. BaPCod - a generic branch-and-price code

The development of the prototype software platform is now supported by our junior engineer, Romanin Leguay, who started in September. He developped a new interface with the underlying MIP solver allowing multiple solvers to be called in the same run. The svn depository was re-organized in view of the increasing number of users to whom Romain offer precious support. Romain is currently redesigning parts of the code in the perspective of its parallelisation and is doing code profiling to identify bottlenecks.

The software platform BaPCod is continuously improved to include all the methodological features that arise from our research, in particular in our collaborative project with Brazil: SAMBA. BaPCod serves there as a proof-of-concept code and is useful for the transfer of knowledge between the parties, including the company GAPSO (a Brazilian spin-up launched by these academics).
We have two new institutional Beta users: EDF and Paris 6.

7. Contracts and Grants with Industry

7.1. Contract with EDF on maintenance planning

We are currently working on a project aiming to plan the energy production and the maintenance breaks for a set of nuclear power plants generating electricity. This problem has two different levels of decisions. The first one consist in determining, for a certain time horizon, when the different power plants will have to stop in order to perform a refueling and to decide the amount of this refueling. Given a set of scenarios defining variable levels of energy consumption, the second decision level aims to decide the quantity of power each plant will have to produce. The model that we are proposing combines issues of stochastic optimization (to handle demand scenarios), robust optimization (to account for variation in maintenance duration), and dynamic optimization (the maintenance of nuclear plants are programmed on a five year horizon, but the long term planning is review each month for adjustments due to perturbations.
This project is carried in collaboration between EDF R&D (OSIRIS lab) INRIA team Dolphin and Realopt. The research is the subject of the PhD thesis of Nicolas Dupin (DGA).

8. Partnerships and Cooperations

8.1. International Initiatives

8.1.1. ANR Gratel

André Raspaud launched in 2005 a fruitful cooperation with the Department of Applied Mathematics of the Sun Yat-Sen University of Kaohsiung, Taiwan. This gave rise to an international ANR project funded for three years (January 2010 - December 2013), that is managed by Arnaud Pêcher. The scientific priority theme is “Telecommunications”, a well-known key application area of graph theory. The aim is to tackle especially wireless communications problems, with the help of graph colorings and polyhedral graph theory. Currently, Sagnik Sen (PhD student of E. Sopena, A. Pêcher, A. Raspaud) benefits from a scholarship on this ANR.

8.1.2. INRIA Associate Team: SAMBA

Title: Synergies for Ameliorations and Mastering of Branch-and-price Algorithms

INRIA principal investigator: François Vanderbeck

International Partner:

Institution: Pontificia Universidade Catolica do Rio de Janeiro (Brazil)

Laboratory: ATD-Lab

Researcher: Marcus Poggi, Artur Pessoa, and Eduardo Uchoa

Duration: 2011 - 2013

See also: https://wiki.bordeaux.inria.fr/realopt/pmwiki.php/Project/Samba

The so-called Dantzig-Wolfe decomposition approach has not yet made its way into general purpose solvers for Mixed Integer Programming (MIP). Despite its proved efficiency, the use of the method is currently restricted to specific applications and requires ad-hoc algorithms developed by experts. Our project is to develop general purpose algorithms to make this method generic. We shall focus in particular on (i) preprocessing procedures, (ii) warm-starting, (iii) stabilization (to improve convergence), (iv) strategies for combining cut and column generation, and (v) primal heuristics. The project builds on the accumulated experience of both the Brazilian and the French teams that have done pioneering work in tackling complex applications and deriving generic solution strategies using this decomposition approach. The new algorithms shall be implemented and tested in the software platform BaPCod. (BaPCod is a generic Branch-and-Price code developed by ReAlOpt as a C++ library that is build as a layer above MIP solvers.) Hence, the collaborative research on methodological developments should lead to, as a bi-product, a Version 2 of BaPCod as a state-of-the-art Branch-and-Price-and-Cut Solver. This prototype should (i) serve as proof-of-concept code for the research planned in this project and beyond, (ii) enable us to achieve new benchmark results on key problems, (iii) provide incentive for the use of the method by non experts, (iv) leverage technology transfer to industry.

8.1.3. Visits of International Scientists

8.1.3.1. Short term Visitors

- Mathieu Van Vyve, CORE and LSM, Université catholique de Louvain, Belgium.
- Claudia d'Ambrosio, DEIS, Università di Bologna, Italy.
- Marcus Poggi, Departamento de Informatica, PUC-Rio, Brazil.
- Artur Pessoa, LOGIS, the Universidade Federal Fluminense, Brazil.
- Eduardo Uchoa, LOGIS, the Universidade Federal Fluminense, Brazil.

8.1.3.2. Internships

- Internship IMB–Ecole Polytechnique: André Linhares: "Dynamic Programming Algorithms for the Resource Constrained Shortest Path Problem", April - August 2011, F. Vanderbeck, R. Sadykov
- Master internship: Damien Trut: "Collecting Containers: operationnal planning", March - September 2011, F. Vanderbeck, R. Sadykov

9. Dissemination

9.1. Animation of the scientific community

9.1.1. Organization of workshops

- Gauttier Stauffer has co-organized the 2011 Cargèse Worshop on Combinatorial Optimization : http://www.math.u-bordeaux.fr/~gstauffer/Cargese_workshop/Overview.html.
- Arnaud Pêcher has co-organized the 2011 Bordeaux Workshop on identifying code. Within the GDR Informatique Mathématique, he is co-responsible of the working group Graphes, since 2010.

9.1.2. Invitations to conferences

- Arnaud Pêcher was invited by the National Taiwan University, Taipei fopr the 2011 Workshop on Graph Theory. His presentation was entitled ”The circular chromatic number of circular-perfect graphs is polytime”.

- Gautier Stauffer was invited to give a talk at the NII Shonan meeting on Graph Algorithm and Combinatorial Optimization (http://www.nii.ac.jp/shonan/seminar01/). His presentation was entitled “An algorithmic decomposition of claw-free graphs leading to an O(n^3)-algorithm for the weighted stable set problem”.
- Gautier Stauffer was invited speaker at theand at SODA 2011 (http://www.siam.org/meetings/da11/). His presentation was entitled “A Simple and Fast 2-approximation for the One-warehouse Multi-retailers Problem”.
- Gautier Stauffer was invited to give a plenary session at LAGOS 2011 (http://www-2.dc.uba.ar/lagos2011/). His presentation was entitled “A history of the stable set polytope of claw-free graphs”.
- François Vanderbeck was invited speaker at the *workshop in Mixed Integer Programming MIP2011*, University of Waterloo, Canada, June 2011 (http://www.math.uwaterloo.ca/~mip2011/). His presentation was entitled “Column Generation for Extended Formulations: experimental report”.
- François Vanderbeck was invited speaker at the *workshop on Integer Programming Down & Under*, NewCastle University, Australia, July 2011 (http://carma.newcastle.edu.au/nuor/ipdu/). His presentation was entitled “Column Generation for Extended Formulations: stabilization by column recombinations”.

9.1.3. Teaching

Licence “DUT Informatique”, University of Bordeaux, France:

‘Système Unix”, 36h, L2 (Arnaud Pêcher)

“Algorithmique”, 12h, L2 (Arnaud Pêcher)

“Programmation Objet”, 32h, L2 (Arnaud Pêcher)

“Java Avancé”, 16h, L2 (Arnaud Pêcher)

“Programmation Objet”, 68h, L1 (Arnaud Pêcher)

Licence of mathematics, University of Bordeaux, France:

“ modèles et méthodes d’optimisation”, 15 hetd, L2, (Cédric Joncour)

Master “Ingénierie Mathématique, Statistiques, et Economic”: “Optimisation dans les graphes”, University of Bordeaux, France.

“Modèles de Flot et Combinatoire”, 45 hetd, M1 (Gautier Stauffer)

“Optimisation dans les graphes”, 15h, M1, (Arnaud Pêcher)

“optimisation combinatoire”, 30 hetd, M1, ((Andrew Miller, Francois Vanderbeck)

“outils et logiciels d’optimisation”, 14 hetd, M1, (Andrew Miller)

“Programmation linéaire”, 60 hetd, M1, (Andrew Miller, Pierre Pesneau)

“Programmation entière”, 40 hetd, M2, (Francois Vanderbeck)

“Gestion des Opérations et planification de la production”, 30 hetd, M1, (Andrew Miller)

“Programmation Orientée Objet”, 30 hetd, M2, (Pierre Pesneau)

“Introduction à la Programmation par Contraintes”, 30 HETD, M2, (Ruslan Sadykov).

Other Masters:

Master “MIAGE”: “Services Web”, 27h, M2, University of Bordeaux, France (Arnaud Pêcher)

Enseirb, Computer Sciences: “Recherche Opérationnelle”, 30 hetd, 2eme année, Institut Polytechnique de Bordeaux, France (Pierre Pesneau, Francois Vanderbeck)

CNAM Aquitaine, "Modélisation, optimisation, complexite et algorithmes" I et II, 25 hetd (niveau II) + 34 HETD (niveau I), M1, France (Ruslan Sadykov).

PhD & HdR :

HdR : Gautier Stauffer, “At Play With Combinatorial Optimization, Integer Programming and Polyhedra”, University of Bordeaux, November 28, 2011 [11].

PhD in progress: Petru Valicov, "Algorithmes de graphes pour les problémes d'ordonnancement", September 2009, A. Pêcher, M. Montassier, E. Sopena

PhD in progress: Sagnik Sen, "Graphes et télécommunications", January 2011, E. Sopena, A. Pêcher, A. Raspaud.

PhD in progress: Nastaran Rahmani, "Planning and Routing via decomposition approaches", April 2011, R. Sadykov, F. Vanderbeck

PhD in progress: Nicolas Dupin, "Scheduling Maintenance of Nuclear Plants in Power Production Planning", Mai 2011, A. Miller, R. Sadykov, E. Talbi, F. Vanderbeck.

9.1.4. Administrative Responsabilities

Each member of the team is quite involved in teaching in the thematic specialties of the project, including in the research track of the Masters in applied mathematics or computer science and an Operations Research Track in the computer science department of the Engineering school ENSEIRB-MATMECA. Moreover, we are largely implied in the organization of the curriculum:

- Arnaud Pêcher was the head of IUT Computer Science's special year, since 2010.
- Andrew Miller was the head of Applied Mathematics Department, since 2010.
- Francois Vanderbeck has succeeded to Andrew Miller as the head of the Master Speciality in Operations Research.

10. Bibliography

Major publications by the team in recent years

[1] O. Briant, C. Lemaréchal, P. Meurdesoif, S. Michel, N. Perrot, F. Vanderbeck. *Comparison of Bundle and Classical Column Generation*, in "Mathematical Programming", 2008, vol. 113, n^{o} 2, p. 299–344.

[2] M. Constantino, A. Miller, M. Van Vyve. *Mixing MIR Inequalities with Two Divisible Coefficients*, in "Mathematical Programming, Series A", 2010, vol. 123, p. 451-483 [*DOI :* 10.1007/s10107-009-0266-9], http://hal.inria.fr/hal-00387098/en.

[3] S. Coulonges, A. Pêcher, A. Wagler. *Characterizing and bounding the imperfection ratio for some classes of graphs*, in "Mathematical Programming, Series A", 2009, vol. 118, n^{o} 1, p. 37-46, http://hal.archives-ouvertes.fr/hal-00308150/en/.

[4] F. Eisenbrand, G. Oriolo, G. Stauffer, P. Ventura. *The stable set polytope of quasi-line graphs*, in "Combinatorica", 2008, vol. 28, n^{o} 1, p. 45-67.

[5] L. Gouveia, P. Pesneau. *On extended formulations for the precedence constrained asymmetric traveling salesman problem*, in "Networks", 2006, vol. 48, n^{o} 6, p. 77–89.

[6] Y. Guan, A. Miller. *Polynomial-Time Algorithms for Stochastic Uncapacitated Lot-Sizing Problems*, in "Operations Research", 2008, vol. 56, n^{o} 2, p. 1172–1183.

[7] A. PÊCHER, A. WAGLER. *Almost all webs are not rank-perfect*, in "Mathematical Programming", 2006, vol. 105, n^{o} 2-3, p. 311–328.

[8] R. SADYKOV, L. A. WOLSEY. *Integer Programming and Constraint Programming in Solving a Multimachine Assignment Scheduling Problem with Deadlines and Release Dates*, in "INFORMS Journal on Computing", 1 2006, vol. 18, n^{o} 2, p. 209–217.

[9] F. VANDERBECK. *On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm*, in "Operations Research", 2000, vol. 48, n^{o} 1, p. 111–128.

[10] F. VANDERBECK. *Computational Study of a Column Generation algorithm for Bin Packing and Cutting Stock problems*, in "Mathematical Programming, Ser. A", 1999, vol. 86, p. 565–594.

Publications of the year

Doctoral Dissertations and Habilitation Theses

[11] G. STAUFFER. *Excursions en Optimisation Combinatoire, Programmation Entiere et Polyedres.*, Université Sciences et Technologies - Bordeaux I, November 2011, HDR, http://tel.archives-ouvertes.fr/tel-00653059.

Articles in International Peer-Reviewed Journal

[12] M. T. GODINHO, L. GOUVEIA, P. PESNEAU. *Natural and Extended formulations for the Time-Dependent Traveling Salesman Problem*, in "Discrete Applied Mathematics", November 2011 [*DOI :* 10.1016/J.DAM.2011.11.019], http://hal.inria.fr/hal-00648451/en.

[13] C. JONCOUR, A. PÊCHER. *Consecutive ones matrices for multi-dimensional orthogonal packing problems*, in "Journal of Mathematical Modelling and Algorithms", 2011 [*DOI :* 10.1007/S10852-011-9167-Z], http://hal.inria.fr/hal-00652574/en.

[14] C. JONCOUR, A. PÊCHER, P. VALICOV. *MPQ-trees for the orthogonal packing problem*, in "Journal of Mathematical Modelling and Algorithms", August 2011, A paraitre, To appear in Journal of Mathematical Modelling and Algorithms [*DOI :* 10.1007/S10852-011-9159-Z], http://hal.inria.fr/hal-00611528/en.

[15] S. MICHEL, F. VANDERBECK. *A Column Generation based Tactical Planning Method for Inventory Routing*, in "Operations Research", 2011, http://hal.inria.fr/inria-00169311/en.

[16] G. ORIOLO, G. STAUFFER, P. VENTURA. *Stable set in claw-free graphs: recent achievement and future challenges*, in "Optima Mathematical Optimization Society Newsletter", August 2011, n^{o} 86, http://hal.inria.fr/hal-00648013.

[17] S. POKUTTA, G. STAUFFER. *Lower bounds for the Chvátal-Gomory closure in the 0/1 cube*, in "Operations Research Letters", January 2011, http://hal.inria.fr/inria-00539038.

[18] A. PÊCHER, A. WAGLER. *Computing the clique number of a-perfect graphs in polynomial time*, in "Electronic Notes in Discrete Mathematics", September 2011, p. 705-710, http://hal.archives-ouvertes.fr/hal-00592080.

[19] A. PÊCHER, A. WAGLER. *Polynomial time computability of some graph parameters for superclasses of perfect graphs*, in "International Journal of Mathematics in Operational Research", 2011, http://hal.inria.fr/inria-00560144/en.

[20] R. SADYKOV. *A dominant class of schedules for malleable jobs in the problem to minimise the total weighted completion time*, in "Computers and Operations Research", 2011, vol. To appear [*DOI :* 10.1016/J.COR.2011.02.023], http://hal.inria.fr/inria-00539875/en.

[21] R. SADYKOV, F. VANDERBECK. *Bin Packing with conflicts: a generic branch-and-price algorithm*, in "INFORMS Journal on Computing", 2011, vol. To appear, http://hal.inria.fr/inria-00539869.

[22] G. STAUFFER. *The strongly minimal facets of the stable set polytope of quasi-line graphs*, in "Operations Research Letters", March 2011, http://hal.inria.fr/inria-00463669/en.

[23] F. VANDERBECK. *Branching in Branch-and-Price: a Generic Scheme*, in "Mathematical Programming, Series A", 2011, vol. 130, p. 249-294 [*DOI :* 10.1007/S10107-009-0334-1], http://hal.inria.fr/inria-00311274.

Articles in Non Peer-Reviewed Journal

[24] M. KILIÇ, J. LINDEROTH, J. LUEDTKE, A. MILLER. *Strong Branching Inequalities for Convex Mixed Integer Nonlinear Programs*, in "Optimization Online", April 2011, http://www.optimization-online.org/DB_HTML/2011/09/3150.html.

[25] R. SADYKOV, F. VANDERBECK. *Column Generation for Extended Formulations*, in "Optimization Online", July 2011, http://hal.inria.fr/hal-00661758.

Invited Conferences

[26] A. PÊCHER. *The circular chromatic number of circular-perfect graphs is polytime*, in "2011 Workshop on Graph Theory", Taipei, Taiwan, Province Of China, March 2011, http://hal.inria.fr/hal-00652773/en.

International Conferences with Proceedings

[27] Y. FAENZA, G. ORIOLO, G. STAUFFER. *Separating stable sets in claw-free graphs via Padberg-Rao and compact linear programs*, in "ACM-SIAM Symposium on Discrete Algorithms", Kyoto, Japon, January 2012, http://hal.inria.fr/hal-00648011.

[28] R. SADYKOV, F. VANDERBECK. *Column Generation for Extended Formulations*, in "6th Latin-American Algorithms, Graphs and Optimization Symposium", Bariloche, Argentina, Electronic Notes in Discrete Mathematics, Elsvier, 2011, vol. 37, p. 357-362 [*DOI :* 10.1016/J.ENDM.2011.05.061], http://hal.inria.fr/inria-00539870/en.

[29] R. SADYKOV, F. VANDERBECK. *Machine scheduling by column-and-row generation on the time-indexed formulation*, in "10th International Workshop on Models and Algorithms for Planning and Scheduling Problems", Nymburk, Czech Republic, 2011, p. 55-57, http://hal.inria.fr/hal-00649184/en.

[30] G. STAUFFER, G. MASSONNET, C. RAPINE, J.-P. GAYON. *A simple and fast 2-approximation algorithm for the one warehouse multi-retailer problem*, in "Proceedings of the ACM-SIAM Symposium on Discrete Algorithms (SODA) 2011", San Francisco, United States, SIAM, January 2011, http://hal.inria.fr/inria-00539042/en.

Conferences without Proceedings

[31] J. HYEMIN, J. LINDEROTH, A. MILLER. *Linear and Nonlinear Inequalities for a Nonseparable Quadratic Set*, in "SIAM Conference on Optimization", 2011, http://www.siam.org/meetings/op11/program.php.

Scientific Books (or Scientific Book chapters)

[32] Y. FAENZA, G. ORIOLO, G. STAUFFER, P. VENTURA. *Stable Sets in Claw-free Graphs : A Journey Through Algorithms and Polytopes*, in "Progress in Combinatorial Optimization.", A. R. MAHJOUB (editor), Wiley, October 2011, http://hal.inria.fr/hal-00648014/en.

[33] M. T. GODINHO, L. GOUVEIA, P. PESNEAU. *On a Time-Dependent Formulation and an Updated Classification of ATSP Formulations*, in "Progress in Combinatorial Optimization", A. R. MAHJOUB (editor), Wiley, October 2011, http://hal.inria.fr/hal-00648457/en.

Other Publications

[34] C. BACHOC, A. PÊCHER, A. THIERY. *On the theta number of powers of cycle graphs*, 2011, 17 pages, http://hal.inria.fr/hal-00572897/en.

[35] P. BELOTTI, S. CAFIERI, J. LEE, L. LIBERTI, A. MILLER. *On the composition of convex envelopes for quadrilinear terms*, 2011, 15 pages.

References in notes

[36] K. AKARTUNALI, A. J. MILLER. *A Computational Analysis of Lower Bounds for Big Bucket Production Planning Problems*, 2009, http://hal.archives-ouvertes.fr/hal-00387105/en/.

[37] K. AKARTUNALI, A. MILLER. *A heuristic approach for big bucket multi-level production planning problems*, in "European Journal of Operational Research", 2009, p. 396-411, http://hal.archives-ouvertes.fr/hal-00387052/en/.

[38] P. BAPTISTE, R. SADYKOV. *On Scheduling a Single Machine to Minimize a Piecewise Linear Objective Function : A Compact MIP Formulation*, in "Naval Research Logistics / Naval Research Logistics An International Journal", 2009, vol. 56, n^{o} 6, p. 487–502, http://hal.inria.fr/inria-00387012/en/.

[39] P. BAPTISTE, R. SADYKOV. *Time Indexed Formulations for Scheduling Chains on a Single Machine: An Application to Airborne Radars*, in "European Journal of Operational Research", 2009, http://hal.inria.fr/inria-00339639/en/.

[40] P. BAPTISTE, R. SADYKOV, S. DAVID. *Planification, ordonnancement : résolution de problèmes disjonctifs*, in "Gestion de la complexité et de l'information dans les grands systèmes critiques", A. APPRIOU (editor), CNRS Éditions, 2009, http://hal.inria.fr/inria-00440278/en/.

[41] M. CONSTANTINO, A. MILLER, M. VAN VYVE. *Mixing MIR Inequalities with Two Divisible Coefficients*, in "Mathematical Programming, Series A", 2009, p. 1–1, http://hal.archives-ouvertes.fr/hal-00387098/en/.

[42] G. DAHL, D. HUYGENS, A. R. MAHJOUB, P. PESNEAU. *On the k-edge disjoint 2-hop-constrained paths polytope*, in "Operations Research Letters", 2005, vol. 34, p. 577–582.

[43] B. DENTON, A. MILLER, H. BALASUBRAMANIAN, T. HUSCHKA. *Optimal Allocation of Surgery Blocks to Operating Rooms Under Uncertainty*, in "Operations Research", 2009, p. 1–1, http://hal.archives-ouvertes.fr/hal-00386469/en/.

[44] L. DUBOST, R. GONZALEZ, C. LEMARÉCHAL. *A primal-proximal heuristic applied to the French unit-commitment problem*, in "Mathematical Programming", 2005, vol. 104, n^o 1, p. 129–152.

[45] Y. FAENZA, G. ORIOLO, G. STAUFFER. *An Algorithmic Decomposition of Claw-free Graphs Leading to an O(n^3)-algorithm for the Weighted Stable Set Problem*, in "Proceedings of the ACM-SIAM Symposium on Discrete Algorithms (SODA) 2011", United States San Francisco, SIAM, September 2010, http://hal.inria.fr/inria-00463671/en.

[46] B. FORTZ, A. R. MAHJOUB, S. T. MCCORMICK, P. PESNEAU. *Two-edge connected subgraphs with bounded rings: polyhedral results and branch-and-cut*, in "Mathematical Programming", 2006, vol. 105, n^o 1, p. 85–111.

[47] M. T. GODINHO, L. GOUVEIA, T. L. MAGNANTI, P. PESNEAU, J. PIRES. *On a Time-Dependent Model for the Unit Demand Vehicle Routing Problem*, Centro de Investigacao Operacional da Universidade de Lisboa, 2007, n^o 11-2007.

[48] M. T. GODINHO, L. GOUVEIA, T. L. MAGNANTI, P. PESNEAU, J. PIRES. *On Time-Dependent Model for Unit Demand Vehicle Routing Problems*, in "International Conference on Network Optimization, INOC", International Network Optimization Conference (INOC), 2007.

[49] Y. GUAN, S. AHMED, A. MILLER, G. NEMHAUSER. *On formulations of the stochastic uncapacitated lot-sizing problem*, in "Operations Research Letters", 2006, vol. 34, p. 241-250.

[50] Y. GUAN, S. AHMED, G. NEMHAUSER, A. MILLER. *A branch-and-cut algorithm for the stochastic uncapacitated lot-sizing problem*, in "Mathematical Programming", 2006, vol. 105, p. 55-84.

[51] L. GÉLY, G. DESSAGNE, P. PESNEAU, F. VANDERBECK. *A multi scalable model based on a connexity graph representation*, in "11th International Conference on Computer Design and Operation in the Railway and Other Transit Systems COMPRAIL'08", Toledo, Spain, September 2008.

[52] L. GÉLY. *Real-time train scheduling at SNCF*, in "1st Workshop on Robust Planning and Rescheduling in Railways", ARRIVAL meeting on Robust planning and Rescheduling in Railways, April 2007.

[53] Y. HENDEL, R. SADYKOV. *Timing problem for scheduling an airborne radar*, in "Proceedings of the 11th International Workshop on Project Management and Scheduling", Istanbul, Turkey, April 2008, p. 132-135.

[54] D. HUYGENS, M. LABBÉ, A. R. MAHJOUB, P. PESNEAU. *The two-edge connected hop-constrained network design problem: Valid inequalities and branch-and-cut*, in "Networks", 2007, vol. 49, n^o 1, p. 116-133.

[55] D. HUYGENS, A. R. MAHJOUB, P. PESNEAU. *Two edge-disjoint hop-constrained paths and polyhedra*, in "SIAM Journal of Discrete Mathematics", 2004, vol. 18, n^o 2, p. 287–312.

[56] B. JAUMARD, F. VANDERBECK, B. VIGNAC. *A Nested Decomposition Approach to an Optical Network Design Problem*, in "9th INFORMS Telecommunications Conference", Washington, March 2008.

[57] C. JONCOUR. *Problèmes de placement 2D et application à l ordonnancement : modélisation par la théorie des graphes et approches de programmation mathématique*, December 2010.

[58] C. JONCOUR, S. MICHEL, R. SADYKOV, D. SVERDLOV, F. VANDERBECK. *Column Generation based Primal Heuristics*, in "Submitted to ISCO 2010", 2010.

[59] C. JONCOUR, A. PÊCHER, P. PESNEAU, F. VANDERBECK. *Mathematical programming formulations for the orthogonal 2d knapsack problem*, in "Livre des résumé du 9ème Congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision", February 2008, p. 255–256, http://hal.archives-ouvertes.fr/hal-00307152/en/.

[60] H. KERIVIN, A. MAHJOUB. *On Survivable Network Polyhedra*, in "Discrete Mathematics", 2005, vol. 290, p. 183–210.

[61] P. MEURDESOIF, P. PESNEAU, F. VANDERBECK. *Meter installation for monitoring network traffic*, in "International Conference on Network Optimization, INOC", International Network Optimization Conference (INOC), 2007.

[62] P. MEURDESOIF, P. PESNEAU, F. VANDERBECK. *A Branch-and-Cut algorithm to optimize sensor installation in a network*, in "Graph and Optimization Meeting GOM2008", France Saint-Maximin, 2008.

[63] S. MICHEL. *Optimisation des tournées de véhicules combinées à la gestion de stock*, Université Bordeaux 1, Dec 2006.

[64] S. MICHEL, N. PERROT, F. VANDERBECK. *Knapsack Problems with Setups*, in "European Journal of Operational Research", 2009, vol. 196, p. 909-918, http://hal.inria.fr/inria-00232782/en/.

[65] S. MICHEL, F. VANDERBECK. *A Column Generation based Tactical Planning Method for Inventory Routing*, INRIA, 2008, http://hal.inria.fr/inria-00169311/en/.

[66] M. MOURGAYA, F. VANDERBECK. *The Periodic Vehicle Routing Problem: classification and heuristic*, in "RAIRO-OR", April-June 2006, vol. 40, n^{o} 2, p. 169–194.

[67] M. MOURGAYA, F. VANDERBECK. *Column generation based heuristic for tactical planning in multi period vehicle routing*, in "European Journal of Operational Research", 2007, vol. 183, n^{o} 3, p. 1028-1041.

[68] G. ORIOLO, U. PIETROPAOLI, G. STAUFFER. *A new algorithm for the maximum weighted stable set problem in claw-free graphs*, in "Lecture notes in computer science. Proceedings of the 13th IPCO Conference", 2008, http://hal.inria.fr/inria-00442281.

[69] M. PADBERG, G. RINALDI. *A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems*, in "SIAM Review", 1991, vol. 33, n^{o} 1, p. 60–100.

[70] N. PERROT. *Advanced IP column generation strategies for the cutting stock stock problem and its variants*, University Bordeaux 1, 2005.

[71] N. PERROT, F. VANDERBECK. *Industrial Cutting Stock Problem*, in "Collection Sciences, Technologie Informatique", PRESSES UNIVERSITAIRES DE VALENCIENNES (editor), ROADEF 2006, 2006.

[72] A. PÊCHER, A. K. WAGLER. *Almost all webs are not rank-perfect*, in "Mathematical Programming", 2006, vol. 105, p. 311–328.

[73] A. PÊCHER, A. WAGLER. *Clique and chromatic number of circular-perfect graphs*, in "Electronic Notes in Discrete Mathematics", Hammamet, Tunisie, February 2010, vol. 36, p. 199-206, http://hal.archives-ouvertes.fr/hal-00453400.

[74] R. SADYKOV. *A branch-and-check algorithm for minimizing the sum of the weights of the late jobs on a single machine with release dates*, in "European Journal of Operations Research", 2008, vol. 189, n^{o} 3, p. 1284–1304 [*DOI :* 10.1016/J.EJOR.2006.06.078].

[75] R. SADYKOV. *A polynomial algorithm for a simple scheduling problem at cross docking terminals*, INRIA, 2009, RR-7054, http://hal.inria.fr/inria-00412519/en/.

[76] R. SADYKOV. *On scheduling malleable jobs to minimise the total weighted completion time*, in "13th IFAC Symposium on Information Control Problems in Manufacturing", Russie Moscow, 2009, http://hal.inria.fr/inria-00339646/en/.

[77] R. SADYKOV. *A polynomial algorithm for a simple scheduling problem at cross docking terminals*, in "Project Management and Scheduling", France Tours, 2010, p. 345-348, http://hal.inria.fr/inria-00539843/en.

[78] G. STAUFFER. *The p-median Polytope of Y-free Graphs: An Application of the Matching Theory*, in "Operations Research Letters", 2008, http://hal.inria.fr/inria-00442282.

[79] A. SUTTER, F. VANDERBECK, L. WOLSEY. *Optimal Placement of Add/Drop Multiplexers: Heuristic and Exact Algorithms*, in "Operations Research", 1998, vol. 46, n^{o} 5, p. 719–728.

[80] F. VANDERBECK, M. SAVELSBERGH. *A generic view of Dantzig-Wolfe decomposition in mixed integer programming*, in "Operations Research Letters", 2006, vol. 34, n^{o} 3, p. 296–306.

[81] F. VANDERBECK. *Exact algorithm for minimising the number of setups in the one-dimensional cutting stock problem*, in "Operations Research", 2000, vol. 48, n^{o} 5, p. 915–926.

[82] F. VANDERBECK. *A nested decomposition approach to a 3-stage 2-dimensional cutting stock problem*, in "Management Science", 2001, vol. 47, n^{o} 2, p. 864–879.

[83] F. VANDERBECK. *Extending Dantzig's Bound to the Bounded Multiple-class Knapsack Problem*, in "Mathematical Programming, Ser. A", 2002, vol. 94, n^{o} 1, p. 125–136.

[84] F. VANDERBECK. *Lot-sizing with start-up times*, in "Management Science", 1998, vol. 44, n^{o} 10, p. 1409–1425.

[85] F. VANDERBECK. *Computational study of a column generation algorithm for bin packing and cutting stock problems*, in "Mathematical Programming, Ser. A", 1999, vol. 86, p. 565–594.

[86] B. VIGNAC, B. JAUMARD, F. VANDERBECK. *An Hierarchical Optimization approach to Optical Network Design where Traffic Grooming and Routing is Solved by Column Generation*, 2008, submitted to INOC 2009.

[87] B. VIGNAC, B. JAUMARD, F. VANDERBECK. *Hierarchical Optimization Procedure for Traffic Grooming in WDM Optical Networks*, 2008, submitted to IEEE conference ONDM2009.

[88] B. VIGNAC, B. JAUMARD, F. VANDERBECK. *Hierarchical Heuristic for the GRWA Problem in WDM Networks with Delay Constraints*, INRIA, 2009, http://hal.inria.fr/inria-00415513/en/.

[89] B. VIGNAC, F. VANDERBECK, B. JAUMARD. *Nested Decomposition Approach to an Optical Network Design Problem*, INRIA, 2009, http://hal.inria.fr/inria-00415500/en/.

[90] B. VIGNAC, F. VANDERBECK, B. JAUMARD. *Reformulation and Decomposition Approaches for Traffic Routing in Optical Networks*, INRIA, 2009, http://www.math.u-bordeaux.fr/~fv/papers/grwaWP.pdf, http://hal.inria.fr/inria-00392256/en/.

[91] B. VIGNAC. *Résolution d'un problème de groupage dans le réseaux optiques maillés*, Université de Montréal, January 2010.