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Activity Report 2011

Project-Team TASC

Theory, Algorithms, and Systems for Constraints

IN COLLABORATION WITH: Laboratoire d'Informatique de Nantes Atlantique (LINA)

RESEARCH CENTER
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THEME
Programs, Verification and Proofs

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Project-Team TASC

Keywords: Constraints, Artificial Intelligence, Inference, Operational Research, Numerical Methods

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2. Overall Objectives

2.1. Objectives of the team

2.1.1. Origin and Current Situation

Constraint programming emerges in the eighties and develops at **the intersection of Artificial Intelligence and Operations Research**, of Computer Science and Mathematics. Multidisciplinary by nature it keeps on using knowledge from various topics such as discrete mathematics, theoretical computer science (graph theory, combinatorics, algorithmic, complexity), functional analysis and optimization, IT and software engineering. Constraint programming was identified in 1996 by the ACM as a *strategic topic for Computer Science*. The turn of the century has seen the development of optimization technology in the industry (with notably Ilog, IBM, Dash and more recently Microsoft, http://code.msdn.microsoft.com/solverfoundation, Google and Dynadec) and the corresponding scientific field, at the border of Constraint Programming, Mathematical Programming, Local Search and Numerical Analysis. Optimisation technology is now assisting public sector, companies and people to some extent for making decisions that use resources better and match specific requirements in an increasingly complex world. Indeed, computer aided decision and optimization is becoming one of the cornerstones for providing assistance to all kinds of human activities.

Today, with the preeminence of optimization technology in most industrial sectors, we argue that quick and ad hoc solutions, often used today, cannot support the long-term development of optimization technology and its broad diffusion. We also argue that there should be a much more direct link between mathematical results and their systematic reuse in the main fields of optimization technology.

2.1.2. General Challenges

In spite of its importance, computer aided decision and optimization suffers from a number of fundamental weaknesses that prevent from taking advantage of its full potential and hinder its progress and its capacity to deal with more and more complex situations. This can be mostly blamed on the diversity of actors, which are:

- Spread out in distinct scientific communities, each with its own focus:
  - On the one hand, computer science for providing languages, modelling tools and libraries. While focusing on providing flexible and powerful programming paradigm that can be easily deployed and maintained on modern architectures, it does not address the central question of how to come up in a systematic way with efficient methods for optimization and decision problems.
  - On the other hand, applied mathematics for the theory part. The focus is to come up with powerful abstractions that allow understanding the structure of a class of problems, independently of its practical and systematic uses in modern software components.
- Spread out in distinct technological communities, each independently pushing its own solving paradigm like constraint programming, linear and integer programming, continuous optimization, constraint-based local search (e.g., COMET). To some extent, most of these techniques exploit in different ways the same mathematical results, that are manually adapted to fit the main way to proceed of a given technology.

Thus, a first challenge encountered by constraint programming is the design of computer systems implementing **in a transparent way** effective solving techniques.

- Ideally, the user must be able to **describe his problem in a high level modelling language** without being concerned with the underlying solving mechanisms used. Such systems must also be independent both from any computer programming language and from any resolution engine.
- In order to assists user, systems must also offer **digital knowledge base in problem solving** that make available state of the art models and heuristics for large set of well identified problems.
- Lastly, the user must have the ability to interpret the returned solutions, in particular within the context of **over constrained problems where it is necessary to partly relax some constraints**, and that in the most realistic possible way.

A second challenge resides in the **speed of resolution especially in the context of large-scale data**. One has to adapt techniques such as constraint networks, graph algorithms, mathematical programming, meta-heuristics and to integrate them within the framework of constraint programming. This integration generates new questionings such as the design of incremental algorithms, the automatic decomposition or the automatic reformulation of problems.

Finally a third challenge deals with the use of constraint programming in the context of **complex industrial problems**, especially when both discrete and continuous aspects are present. Complexity has multiple causes such as:

- the combination of temporal and spatial aspects, of continuous and discrete aspects,
- the dynamic character of some phenomena inducing a modification of the constraints and data during time,
- the difficulty of expressing some physical constraints, e.g. load balancing and temporal stability,
- the necessary decomposition of large problems inducing significant solution performance losses.

2.2. Highlights

- Best student paper for M. Pelleau et al. at the 17th International Conference on Principles and Practice of Constraint Programming (CP 2011) for *Octogonal Domains for Continuous Constraints* (see [22]).
- Google Focused Research Awards (see item *Mathematical Optimization*) for providing explanation-based user-oriented features in constraint solvers given to N. Jussien.
- Constraint seeker available on line at http://www.emn.fr/z-info/sdemasse/gccat/ for identifying global constraints.

BEST PAPER AWARD :
[22] **17th International Conference on Principles and Practice of Constraint Programming (CP'11)**. M. PELLEAU, C. TRUCHET, F. BENHAMOU.

3. Scientific Foundations

3.1. Overview

Basic research is guided by the challenges raised before: to classify and enrich the models, to automate reformulation and resolution, to dissociate declarative and procedural knowledge, to come up with theories and tools that can handle problems involving both continuous and discrete variables, to develop modelling tools and to come up with solving tools that scale well. On the one hand, **classification aspects** of this research are integrated within a knowledge base about combinatorial problem solving: the global constraint catalog (see http://www.emn.fr/x-info/sdemasse/gccat/index.html). On the other hand, **solving aspects** are capitalized within the constraint solving system CHOCO. Lastly, within the framework of its activities of valorisation, teaching and of partnership research, the team uses constraint programming for solving various concrete problems. The challenge is, on one side to increase the visibility of the constraints in the others disciplines of computer science, and on the other side to contribute to a broader diffusion of the constraint programming in the industry.

3.2. Fundamental Research Topics

This part presents the research topics investigated by the project:

- Global Constraints Classification, Reformulation and Filtering,
- Convergence between Discrete and Continuous,
- Dynamic, Interactive and over Constrained Problems,
- Solvers.

These research topics are in fact not independent. The work of the team thus frequently relates transverse aspects such as explained global constraints, Benders decomposition and explanations, flexible and dynamic constraints, linear models and relaxations of constraints.

3.2.1. Constraints Classification, Reformulation and Filtering

In this context our research is focused (a) first on identifying recurring combinatorial structures that can be used for modelling a large variety of optimization problems, and (b) exploit these combinatorial structures in order to come up with efficient algorithms in the different fields of optimization technology. The key idea for achieving point (b) is that many filtering algorithms both in the context of Constraint Programming, Mathematical Programming and Local Search can be interpreted as the maintenance of invariants on specific domains (e.g., graph, geometry). The systematic classification of global constraints and of their relaxation brings a synthetic view of the field. It establishes links between the properties of the concepts used to describe constraints and the properties of the constraints themselves. Together with SICS, the team develops and maintains *a catalog of global constraints*, which describes the semantics of more than 350 constraints, and proposes a unified mathematical model for expressing them. This model is based on graphs, automata and logic formulae and allows to derive filtering methods and automatic reformulation for each constraint in a unified way (see http://www.emn.fr/x-info/sdemasse/gccat/index.html). We consider hybrid methods (i.e., methods that involve more than one optimization technology such as constraint programming, mathematical programming or local search), to draw benefit from the respective advantages of the combined approaches. More fundamentally, the study of hybrid methods makes it possible to compare and connect strategies of resolution specific to each approach for then conceiving new strategies. Beside the works on classical, complete resolution techniques, we also investigate local search techniques from a mathematical point of view. These partly random algorithms have been proven very efficient in practice, although we have little theoretical knowledge on their behaviour, which often makes them problem-specific. Our research in that area is focused on a probabilistic model of local search techniques, from which we want to derive quantified information on their behaviour, in order to use this information directly when designing the algorithms and exploit their performances better. We also consider algorithms that maintain local and global consistencies, for more specific models. Having in mind the trade off between genericity and effectiveness, the effort is put on the efficiency of the algorithms with guarantee on the produced levels of filtering. This effort results in adapting existing techniques of resolution such as graph algorithms. For this purpose we identify necessary conditions of feasibility that can be evaluated by efficient incremental algorithms. Genericity is not neglected in these approaches: on the one hand the constraints we focus on are applicable in many contexts (for example, graph partitioning constraints can be used both in logistics and in phylogeny); on the other hand, this work led to study the portability of such constraints and their independence with specific solvers. This research orientation gathers various work such as strong local consistencies, graph partitioning constraints, geometrical constraints, and optimization and soft constraints. Within the perspective to deal with complex industrial problems, we currently develop meta constraints (e.g. *geost*) handling all together the issues of large-scale problems, dynamic constraints, combination of spatial and temporal dimensions, expression of business rules.

3.2.2. Convergence between Discrete and Continuous

Many industrial problems mix continuous and discrete aspects that respectively correspond to physical (e.g., the position, the speed of an object) and logical (e.g., the identifier, the nature of an object) elements. Typical examples of problems are for instance:

- *Geometrical placement problems* where one has to place in space a set of objects subject to various geometrical constraints (i.e., non-overlapping, distance). In this context, even if the positions of the objects are continuous, the structure of optimal configurations has a discrete nature.
- *Trajectory and mission planning problems* where one has to plan and synchronize the moves of several teams in order to achieve some common goal (i.e., fire fighting, coordination of search in the context of rescue missions, surveillance missions of restricted or large areas).
- *Localization problems in mobile robotic* where a robot has to plan alone (only with its own sensors) its trajectory. This kind of problematic occurs in situations where the GPS cannot be used (e.g., under water or Mars exploration) or when it is not precise enough (e.g., indoor surveillance, observation of contaminated sites).

Beside numerical constraints that mix continuous and integer variables we also have global constraints that involve both type of variables. They typically correspond to graph problems (i.e., graph colouring, domination in a graph) where a graph is dynamically constructed with respect to geometrical and-or temporal constraints. In this context, the key challenge is avoiding decomposing the problem in a discrete and continuous parts as it is traditionally the case. As an illustrative example consider *the wireless network deployment problem*. On the one hand, the continuous part consists of finding out where to place a set of antenna subject to various geometrical constraints. On the other hand, by building an interference graph from the positions of the antenna, the discrete part consists of allocating frequencies to antenna in order to avoid interference. In the context of convergence between discrete and continuous variables, our goals are:

- First to identify and compare typical class of techniques that are used in the context of continuous and discrete solvers.
- To see how one can unify and/or generalize these techniques in order to handle in an integrated way continuous and discrete constraints within the same framework.

3.2.3. Dynamic, Interactive and over Constrained Problems

Some industrial applications are defined by a set of constraints which may change over time, for instance due to an interaction with the user. Many other industrial applications are over-constrained, that is, they are defined by set of constraints which are more or less important and cannot be all satisfied at the same time. Generic, dedicated and explanation-based techniques can be used to deal efficiently with such applications. Especially, these applications rely on the notion of *soft constraints* that are allowed to be (partially) violated. The generic concept that captures a wide variety of soft constraints is the violation measure, which is coupled with specific resolution techniques. Lastly, soft constraints allow to combinate the expressive power of global constraints with local search frameworks.

3.2.4. Solvers

Our theoretical work is systematically validated by concrete experimentations. We have in particular for that purpose the CHOCO constraint platform. The team develops and maintains CHOCO with the assistance of the laboratory e-lab of Bouygues (G. Rochart), the company Amadeus (F. Laburthe), and others researchers such as H. Cambazard (4C, INP Grenoble). The functionalities of CHOCO are gradually extended with the outcomes of our works: design of constraints, analysis and visualization of explanations, etc. The open source CHOCO library is downloaded on average 450 times each month since 2006. CHOCO is developed in line with the research direction of the team, in an open-minded scientific spirit. Contrarily to other solvers where the efficiency often relies on problem-specific algorithms, CHOCO aims at providing the users both with reusable techniques (based on an up-to-date implementation of the global constraint catalogue) and with a variety of tools to ease the use of these techniques (clear separation between model and resolution, event-based solver, management of the over-constrained problems, explanations, etc.). Since 2009 year, due to the hiring of G. Chabert, the team is also involved in the development of the continuous constraint solver IBEX. These developments led us to new research topics, suitable for the implementation of discrete and continuous constraint solving systems: portability of the constraints, management of explanations, incrementality and recalculation. They partially use aspect programming (in collaboration with the INRIA ASCOLA team). This work around the design and the development of solvers thus forms the fourth direction of basic research of the project.

4. Application Domains

4.1. Panorama

Constraint programming deals with the resolution of decision problems by means of rational, logical and computational techniques. Above all, constraint programming is founded on a clear distinction between, on the one hand the description of the constraints intervening in a problem, and on the other hand the techniques used for the resolution. The ability of constraint programming to handle in a flexible way heterogeneous constraints has raised the commercial interest for this paradigm in the early nighties. Among his fields of predilection, one finds traditional applications such as computer aided decision-making, scheduling, planning, placement, logistics or finance, as well as applications such as electronic circuits design (simulation, checking and test), DNA sequencing and phylogeny in biology, configuration of manufacturing products or web sites, formal verification of code. In 2011 the TASC team was involved in the following application domains:

- *Planning and replanning* in Data Centres (SelfXL project).
- *Packing complex shapes* in the context of a warehouse (NetWMS2 project).
- Building decision support system for *city development planning with evaluation of energy impacts* (SUSTAINS project).

5. Software

5.1. CHOCO

Participants: Nicolas Beldiceanu, Alexis De Clerq, Sophie Demassey, Jean-Guillaume Fages, Narendra Jussien [correspondant], Arnaud Letort, Xavier Lorca [correspondant], Arnaud Malapert, Julien Menana, Thierry Petit, Charles Prud'homme [correspondant].

CHOCO is a Java discrete constraints library integrating within a same system *explanations*, *soft constraints* and *global constraints* (90000 lines of source code). This year developments were focussing on the following aspects:

1. Providing a *complete solver independent specification of explanation algorithms*, data structure for encoding «nogoods» and treatment algorithms. A reference implementation is being made within the new version of our solver CHOCO.
2. Design and development of a dedicated languages to specify the propagation and the search heuristics of constraint solvers.
3. Providing efficient implementation of filtering algorithms for constraints such as *tree*, *increasing_sum*, *cumulative with resource overload*.
4. Providing an implementation of a probabilistic model for *alldifferent*.

N. Beldiceanu, A. De Clerq, S. Demassey, J.-G. Fages, N. Jussien, A. Letort, X. Lorca, A. Malapert, J. Menana, T. Petit and C. Prud'Homme have contributed in 2011. The link to the system and documentation is http://choco.emn.fr.

5.2. IBEX

Participants: Gilles Chabert [correspondant], Rémi Douence.

IBEX (Interval-Based EXplorer) is a C++ library for solving nonlinear constraints over real numbers (25000 lines of source code). The main feature of Ibex is its ability to build solver/paver strategies declaratively through the contractor programming paradigm. Ibex includes a parser of the *QUIMPER* language (*QUick Interval Modeling and Programming in a bounded-ERror* context) and is currently used in several academic research labs.

G. Chabert and R. Douence (ASCOLA) have contributed in 2011 to the ongoing redesign of the architecture IBEX, the goal being to make it more flexible to cope with specific problems, and more easy to use. The link to the system and documentation is http://www.emn.fr/z-info/ibex/.

5.3. Global Constraint Catalog

Participants: Nicolas Beldiceanu [correspondant], Sophie Demassey, Mats Carlsson, Helmut Simonis.

The global constraint catalog presents and classifies global constraints and describes different aspects with meta data. It consist of

1. a pdf version that can be downloaded from http://www.emn.fr/z-info/sdemasse/gccat/ (at item *last working version*) containing 360 constraints, 3000 pages and 700 figures,
2. an on line version accessible from the previous address,
3. meta data describing the constraints (buton *PL* for each constraint, e.g., alldifferent.pl),
4. an online service (i.e, a *constraint seeker*) which provides a web interface to search for global constraints, given positive and negative ground examples.

This year developments were focussing on:

1. maintaining the catalogue,
2. deploying an on-line constraint seeker [16] (see http://seeker.mines-nantes.fr/ and http://4c.ucc.ie/~hsimonis/seekerhelp.html for explanation how to use),
3. providing the negation for constraints defined by automata (with and without counter),
4. defining properties of constraints arguments, and
5. providing modelling examples as well as points of interests and common misunderstanding for core constraints.

N. Beldiceanu, S. Demassey, M. Carlsson (SICS, Sweden) and H. Simonis (4C, Ireland) have contributed in 2011. The link to the global constraint catalog is http://www.emn.fr/z-info/sdemasse/gccat/.

6. New Results

6.1. Octagonal Domains for Continuous Constraints (continuous)

Participants: Marie Pelleau, Charlotte Truchet, Fréderic Benhamou.

Domains in Continuous Constraint Programming (CP) are generally represented with intervals whose n-ary Cartesian product (box) approximates the solution space. We propose a new representation for continuous variable domains based on octagons [28]. We generalize local consistency and split to this octagon representation, and we propose an octagonal-based branch and prune algorithm [22]. Experimental results in IBEX on the COCONUT benchmarks suite show promising performance improvements on several classical benchmarks.

The corresponding paper *Octagonal Domains for Continuous Constraints* got the *Best Student Paper Award* at the 17th International Conference on Principles and Practice of Constraint Programming (CP 2011) [22].

6.2. A Constraint Seeker (classification, constraints and application)

Participants: Nicolas Beldiceanu, Helmut Simonis.

We design a Constraint Seeker application which provides a web interface to search for global constraints in the global constraint catalog, given positive and negative ground examples. Based on the given instances the tool returns a ranked list of matching constraints, the rank indicating whether the constraint is likely to be the intended constraint of the user. A systematic evaluation is provided over the complete global constraint catalog.

The corresponding paper *A Constraint Seeker: Finding and Ranking Global Constraints from Examples* [16] was published at the 17th International Conference on Principles and Practice of Constraint Programming (CP 2011) and the corresponding tool is available as a web service at http://www.emn.fr/z-info/sdemasse/gccat/.

6.3. Soft Problems (modelling and filtering)

Participants: Thierry Petit, Alexis De Clerq, Nicolas Beldiceanu, Narendra Jussien.

1. **Side-constrained problems** We experimentally shown that *solving* some classes of over-constrained problems requires an efficient (global) propagation of side constraints on variables representing violations. This work completes our previous studies, which highlighted the interest of a variable-based representation of violations for sake of modelling.

The corresponding paper *Global Propagation of Side Constraints for Solving Over-constrained Problems* was published in the Annals of Operations Research journal [14].

2. **Soft cumulative scheduling** We proposed a new constraint for handling cumulative problems with exceeds of capacities, int the case where the time horizon is fixed and the capacity can vary over time. Sweep and Edge-finding algorithms for classical cumulative problems have been modified so as to provide a filtering algorithm for our constraint. Experiments shown that instances involving several hundreds of activities can be solved with our approach.

   The corresponding paper *Filtering Algorithms for Discrete Cumulative Problems with Over-loads of Resource* was published at the 17th International Conference on Principles and Practice of Constraint Programming (CP 2011), [17].

6.4. Constraints with costs (modelling and filtering)

Participants: Thierry Petit, Nicolas Beldiceanu.

1. **Distribution of costs** We presented a new cardinality constraint dedicated to sequences of totally ordered cost. This constraint is useful to impose a precise (fair) distribution of the values taken by the cost variables in a given sequence, for instance in a bin packing with safety rules or in cumulative scheduling with overloads of resource. We came up with a generalized arc-consistency filtering algorithm, whose time complexity is linear in the sum of the number of variables and the number of values in the union of their domains.

   The corresponding paper *the Ordered Distribute Constraint* was published in the International Journal on Artificial Intelligence Tools [15].

2. **The objective sum constraint.** Constraint toolkits generally provide a sum constraint whose propagation is poor to solve optimization problems. Therefore, solving real-world problems requires to develop ad-hoc techniques for handling sums, based on the particular properties of each problem. We proposed a generic technique which improves the standard sum constraint by exploiting the propagation of a set of constraints defined on the variables involved in a sum.

   The corresponding paper *The Objective Sum Constraint* was published in the 8th International Conference on Integration of Artificial Intelligence and Operations Research Techniques in Constraint Programming for Combinatorial Optimization Problems (CPAIOR 2011) [25].

3. **the increasing sum constraint** Given a sequence of variables X of length n, we consider the *increasing sum* constraint, which imposes the variables of X to be sorted in non strictly order, and that the sum of the variables of X is equal to s. We propose an linear time bound-consistency algorithm for *increasing sum*. This work is related to problems with variable symmetries, when some of the symmetric variables are involved in sum constraints.

   The paper *A Theta(n) Bound-Consistency Algorithm for the Increasing Sum Constraint* was published at the 17th International Conference on Principles and Practice of Constraint Programming (CP 2011), [24].

These works were all done in collaboration with J.-C. Régin (Univ. Nice).

6.5. Efficient Filtering Algorithms for Generic Constraints (filtering)

Participants: Nicolas Beldiceanu, Xavier Lorca, Thierry Petit.

Counting constraints We identified a family of counting constraints for which performing a complete filtering is a tractable problem. We provided a generalized arc-consistency algorithm and its specialization to some well-known global constraints. For some of them the obtained time complexity is linear in the sum of domain sizes, which improve or equals the best known results in the literature.

The corresponding paper *A Generalized Arc-Consistency Algorithm for a Class of Counting Constraints* was published at the 22th International Joint Conference on Artificial Intelligence (IJCAI'11) [23].

6.6. Efficient Filtering Algorithms and Heuristics for Dedicated Constraints (filtering)

Participants: Jean-Guillaume Fages, Xavier Lorca, Arnaud Malapert, Narendra Jussien.

1. **Revisiting the *tree* constraint** We revisit the *tree* constraint introduced at CPAIOR 2005 in [35] which partitions the nodes of a n-nodes, m-arcs directed graph into a set of node-disjoint anti-arborescences for which only certain nodes can be tree roots. We introduce in a new filtering algorithm that enforces generalized arc-consistency in O(n+m) time while the original filtering algorithm reaches O(nm) time. This result allows to tackle larger scale problems involving graph partitioning in CHOCO.

   The corresponding paper *Revisiting the tree Constraint* was published at the 17th International Conference on Principles and Practice of Constraint Programming (CP 2011), [18].

2. **An Optimal Constraint Programming Approach to the Open-Shop Problem** We present an optimal constraint programming approach for the Open-Shop problem, which integrates recent constraint propagation and branching techniques with new upper bound heuristics for the Open-Shop problem. Randomized restart policies combined with nogood recording allow to search diversification and learning from restarts. This approach closed all remaining problems of the Brucker et al. and Guéret and Prins benchmarks with cpu times that are orders of magnitude lower than the best known metaheuristics.

   The corresponding paper *An Optimal Constraint Programming Approach to the Open-Shop Problem* was published in the INFORMS Journal on Computing [13]. This work was done in collaboration with H. Cambazard (4C, INP Grenoble), C. Guéret (IRCCyN), A. Langevin (Ecole Polytechnique Montréal) and L.-M. Rousseau (Ecole Polytechnique Montréal).

6.7. Explanations for Contraint Programming (solver)

Participants: Narendra Jussien, Charles Prud'homme.

Constraint programming, despite its numerous successes in practice, suffers from not being really user-friendly when used by software engineers. Indeed, when faced with a *no solution* message from a constraint solver, it is hard yet impossible to identify the cause of this message: is it from a bad modelling, an ill-written constraint, a bug in the solver, ... Explanations for constraint programming have addressed this issue but are not yet widely used in the CP community. Recent work in the field tend to demonstrate that providing explanation-based user-oriented features can be done quite easily in modern constraint solvers. The objective of this line of work is to specify an user-oriented explanation-module for flexible solver architectures. This line of work is financed through a Google focused research grant. First results provide a complete solver independent specification of explanation algorithms, data structure for encoding nogoods and treatment algorithms. A reference implementation is being made within the new version of our solver CHOCO.

6.8. Bin repacking (constraints and application)

Participants: Sophie Demassey, Xavier Lorca, Fabien Hermenier.

A datacenter can be viewed as a dynamic bin packing system where servers host applications with varying resource requirements and varying relative placement constraints. When those needs are no longer satisfied, the system has to be reconfigured. Virtualization allows to distribute applications into Virtual Machines (VMs) to ease their manipulation. In particular, a VM can be freely migrated without disrupting its service, temporarily consuming resources both on its origin and destination.

We introduce the Bin Repacking Scheduling Problem in this context. This problem is to find a final packing and to schedule the transitions from a given initial packing, accordingly to new resource and placement requirements, while minimizing the average transition completion time. Our CP-based approach uses CHOCO and is implemented into Entropy, an autonomous VM manager which detects reconfiguration needs, generates and solves the CP model, then applies the computed decision. CP provides the awaited flexibility to handle heterogeneous placement constraints and the ability to manage large datacenters with up to 2000 servers and 10000 VMs.

The corresponding paper *Bin-Repacking Scheduling in Virtualized Datacenters* was published at the 17th International Conference on Principles and Practice of Constraint Programming (CP 2011), [19].

6.9. A Global Constraint for Multi-Agent Localization (constraints and application)

Participants: Gilles Chabert, Sophie Demassey.

This work has been initiated in the context of the Angels research project, in which G. Chabert has been involved during two years. The idea was to provide a new method for inter-localizing a group of autonomous underwater robots, traditional Kalman-based methods being inadequate in this context due to the highly nonlinear models derived from the sensing technology (electric fish robots).

We proposed, through a rough discretization of the signals, to consider the problem as a whole and under a combinatorial form. The level of the signal is basically associated to a cardinality of surrounding objects. This led to a global constraint, namely a conjunction of *among* constraint with interval value domains and in multiple dimension (objects are variables with several components).

Conjunction of *among* constraints had been already studied but not in the case of interval value domains. We therefore conducted a theoretical study and proved that the problem was tractable in the one-dimensional case, but not in higher dimension. We have also investigated different decompositions and filtering algorithms. This work is submitted to CPAIOR 2012. An INRIA research report has also been issued in June 2011, where the robotics aspects are described.

7. Contracts and Grants with Industry

7.1. Contracts

7.1.1. Ligéro

Participant: Sophie Demassey.

Title: Ligéro.
Duration: 2009-2012.
Type: Regional research group
Budget: PhD founded by the project.
Others partners: LISA, IRCCyN (team SLP), LERIA (team MOA), LINA (team OPTI).

The goal of the project is to create an internationally visible regional research group putting together the key actors in the domain of Operations Research in the Pays de la Loire region.

7.1.2. CPER

Participant: Charles Prud'Homme.

Title: CPER.
Duration: 2010-2014.
Type: Regional research group.
Budget: 250000 Euros.
Others partners: EMN (team ATLANMOD), EMN (team ASCOLA), IRCCyN (team SLP).

Develop, promote and build up an eco-system around free software in the Pays de la Loire region. The TASC team is involved in the maintenance and development of the free constraint programming platform CHOCO.

7.1.3. UNIT

Participant: Nicolas Beldiceanu.

Title: UNIT.
Duration: 2011.
Type: Developing teaching material.
Budget: 5000 Euros.
Others partners: EMN (CAPE).

Pedagogical material and software for learning constraints programming for non experts (integrated within the global constraint catalog).

7.1.4. FUI SUSTAINS

Participants: Charlotte Truchet, Bruno Belin.

Title: SUSTAINS.
Duration: 2010-2015.
Type: FUI.
Budget: 151400 Euros.
Others partners: Artefacto, Artelys, Areva TA, EPAMarne, LIMSI.

The SUSTAINS project (*Constraint-based Prototyping of Urban Environments*) aims at building decision support system for city development planning with evaluation of energy impacts. The project is focussed on spatial allocation of typical units such as industrial areas, commercial areas and leaving areas with their respective appropriate infrastructure. Its integrates sustainability, transport and energy concerns.

7.1.5. ANR BOOLE

Participants: Jérémie du Boisberranger, Xavier Lorca, Charlotte Truchet.

Title: BOOLE.
Duration: 2010-2015.
Type: open research program.
Budget: founding a PhD student and travels.
Others partners: Univ. de Versailles Saint-Quentin, Univ. Caen, Univ. Paris 8, Univ. Aix-Marseille, Univ. Paris Nord, Univ. Paris 11, ENS Paris.
Défi: Probabilistic method for combinatorial problems.

The work of TASC focuses on the use of probabilistic methods to avoid waking systematically global constraints for nothing. The goal is to provide probabilistic models for the consistency of global constraints such as *alldifferent* or *nvalue*. We compute the probability of a constraint to be still consistent after fixing one of its variables and provide an approximation that can be computed in constant time. The PhD of J. du Boisberranger is co-supervised with D. Gardy from Univ. de Versailles Saint-Quentin.

7.1.6. ANR SelfXL

Participants: Nicolas Beldiceanu, Sophie Demassey, Xavier Lorca, Arnaud Letort.

Title: SelfXL.
Duration: 2009-2011.
Type: embedded systems and large infrastructures research program.
Budget: founding for half a PhD.
Others partners: ASCOLA.

Flexible and efficient tools for complex-large scale autonomic systems. TASC contributes for handling bin packing and bin repacking problems with side constraints derived from migration modes of virtual machines between servers. Constraints based models and CHOCO based solvers are developed for this purpose. The work was done with F. Hermenier and J.-M. Menaud.

7.1.7. ANR NetWMS2

Participants: Nicolas Beldiceanu, Gilles Chabert.

Title: Networked Warehouse Management Systems 2: packing with complex shapes.

Duration: 2011-2014.

Type: cosinus research program, **new project**.

Budget: 189909 Euros.

Others partners: KLS Optim and CONTRAINTES (INRIA Rocquencourt).

This project builds on the former European FP6 Net-WMS Strep project that has shown that constraint-based optimisation techniques can considerably improve industrial practice for box packing problems, while identifying hard instances that cannot be solved optimally, especially in industrial 3D packing problems with rotations, the needs for dealing with more complex shapes (e.g. wheels, silencers) involving continuous values. This project aims at generalising the geometric kernel *geost* for handling non-overlapping constraints for complex two and three dimensional curved shapes as well as domain specific heuristics. This will be done within the continuous solver IBEX, where discrete variables will be added for handling polymorphism (i.e., the fact that an object can take one shape out of a finite set of given shapes).

7.1.8. ANR INFRA-JVM

Participants: Xavier Lorca, Charles Prud'Homme.

Title: Towards a Java Virtual Machine for pervasive computing.

Duration: 2011-2013.

Type: **new project**.

Budget: 78000 Euros.

Others partners: Univ. Paris 6 (REGAL team), LaBRI (LSR team), IRISA (TRISKELL).

The INFRA-JVM project will investigate how to enhance the design of Java virtual machines with new functionalities to better manage resources, namely resource reservation, scheduling policies, and resource optimization at the middleware level. TASC is concerned with this later aspect. The performance of CHOCO will be improved using the memory snapshot mechanism that will be developed.

7.2. Grants

7.2.1. Google Focused Research Awards

Participants: Narendra Jussien, Charles Prud'Homme.

Title: Non intrusive explanations.

Duration: 2011.

Type: **new grant**.

Budget: 75000 Euros.

Constraint programming, despite its numerous successes in practice, suffers from not being really user-friendly when used by software engineers. Explanations for constraint programming have addressed this issue but are not yet widely used in the CP community. The objective of our work is to specify an user-oriented explanation-module for flexible solver architectures and provide a reference implementation within the new version of our solver CHOCO. This line of work will be founded in 2012 by the CNRS (one year of engineer).

8. Partnerships and Cooperations

8.1. Regional Initiatives

The goal of Ligéro is to create an internationally visible regional research group putting together the key actors in the domain of Operations Research in the Pays de la Loire region.

8.2. National Initiatives

- Cooperation with J.-C. Régin from Univ. Nice on efficient filtering algorithms (3 papers in 2011).
- Cooperation with A. Miné from ENS Paris on abstract domains by M. Pelleau and C. Truchet (2 visits in Paris).

8.3. European Initiatives

- Cooperation with P. Van Hentenryck from Univ. Brown (USA) for the supervision of the PhD of M. Pelleau (1 visit in Nantes).
- Cooperation with P. Flener from Univ. Uppsala (Sweden) on automata, invited (3 visits in Uppsala, 1 visit in Nantes).
- Cooperation with H. Simonis from 4C (Ireland) on the constraint and model seekers (2 visits in Cork, 1 visit in Nantes, 2 papers in 2011).
- Cooperation with M. Carlsson from SICS (Sweden) on the global constraint catalog (negation of automata with and without counters) (1 visit in Uppsala, update of the global constraint catalog in September 2011).

9. Dissemination

9.1. Animation of the scientific community

- Nicolas Beldiceanu:
  - Member of the program committee of CP 2011.
  - Co-chair of CPAIOR 2012.
  - Reviewer of the thesis of Nadjib Lazaar (Univ. Rennes, December 5).
  - Reviewer for IJCAI 2011 and ECAI 2012.
  - Reviewer in 2011 for Annals of Operations Research and Constraint Programming Letter.
- Sophie Demassey:
  - Member of the program committees of CPAIOR 2011, MELO 2011 and of the doctoral program committee of CP 2011.
  - Reviewer for Journal of Scheduling, Annals of Operations Research and Constraints.
  - Member of the PhD committee of J. Menana (Univ. Nantes, October 28).
- Narendra Jussien:
  - Member of the program committees of MODELS 2011, ACM-SAC 2011 and LION 2011.
  - Co-chair of CPAIOR 2012.
  - Reviewer in 2011 for Journal of Scheduling, Artificial Intelligence, and Constraints.

  - Director of the series *Operations Research and Constraint Programming* from ISTE/Wiley.
  - Member of the PhD committees of A. Malapert (Univ. Nantes, September 9) and J. Menana (Univ. Nantes, October 28).
- X. Lorca:
  - Member of the program committee of JFPC 2011.
  - Publication of the book Tree-based Graph Partitioning Constraint by X. Lorca, Wiley, June 2011, ISBN: 9781848213036.
- Thierry Petit:
  - Member of the program committee of CPAIOR 2011.
  - Co-chair CPAIOR 2012.
  - Reviewer in 2011 for the Journal of Artificial Intelligence Research.
- Charlotte Truchet:
  - Member of AFPC board in 2011.
  - Member of the program committee of JFPC 2011.
  - Publication of the book Constraint Programming in Music, Edited by C. Truchet, Univ. Nantes and France G. Assayag, IRCAM-CNRS, France Wiley, April 2011, ISBN: 9781848212886.

9.2. Teaching

- Nicolas Beldiceanu:
  - Head of the INRIA TASC team and LINA TASC team.
  - 141 hours teaching in 2010-2011 in all years at EMN, in GIPAD, major in CS for decision support of the Master in Engineering at EMN and in the International MSc in Computer science ORO (Univ. Nantes and EMN).
  - In Charge of the Logic Programming course (EMN) and the Constraint Programming course (ORO).
  - PhD advisor of ongoing thesis of A. Letort and A. De Clerq.
- Fréderic Benhamou:
  - Head of the Computer Science department at Univ. Nantes and of the CNRS ICT Research Institute ATLANSTIC.
  - PhD advisor of ongoing thesis of M. Pelleau.
- Gilles Chabert:
  - 135 hours teaching in 2010-2011 in the first three years at EMN.
- Sophie Demassey:
  - 158 hours teaching in 2010-2011 in all years at EMN, in GIPAD (major in CS for decision support of the Master in Engineering at EMN), and in the International MSc in Computer science ORO (Univ. Nantes, EMN).
  - In charge of the GIPAD (major in CS for decision support of the Master in Engineering at EMN).
  - PhD advisor of ongoing thesis of A. Merel and X. Libeaut.

  - Scientific advisor of the defended PhD thesis of J. Menana (*Automata for Constraint Modelling and Solving*, PhD Defense October 28, 2011, Univ. Nantes with the following committee: J.-C. Régin - Professor (Univ. Nice), L.-M. Rousseau - Professor (Ecole Polytechnique Montréal), P. Boizumault - Professor (Univ. Caen), B. Rottembourg (EURODECISION), N. Jussien, - Professor (EMN, PhD Advisor), S. Demassey, - Assistant Professor, (EMN, Scientific Advisor).)

- Narendra Jussien:
  - Head of the computer science department at EMN.
  - 75 hours teaching in 2010-2011 in the first year at EMN, the option GOPL (EMN), the option decision at Polytechnique Nantes.
  - In charge of the *Introduction to Computer Science* course.
  - PhD advisor of the ongoing thesis of A. De Clerq and C. Prud'homme.
  - PhD advisor of the defended thesis of J. Menana (see item S. Demassey for the committee).
  - PhD advisor of the defended thesis of A. Malapert (*Shop and Batch Scheduling with Constraints*, PhD Defense September 9, 2011, Univ. Nantes with the following committee: N. Jussien - Professor (EMN, PhD Advisor), L.-M. Rousseau - Professor (Ecole Polytechnique Montréal, PhD co-advisor), G. Pesant - Professor (Ecole Polytechnique Montréal), C. Artigues - Professor (LAAS Toulouse), C. Guéret - Assistant Professor (EMN), A. Langevin - Professor (Ecole Polytechnique Montréal).)
- Xavier Lorca:
  - 176 hours teaching in 2010-2011 in all years at EMN.
  - In charge of the *Data base* course.
  - In charge of the GIPAD (major in CS for decision support of the Master in Engineering at EMN) since September 2011.
  - Scientific advisor of the ongoing PhD thesis of J.-G. Fages, J. du Boisberranger and C. Prud'homme.
- Thierry Petit:
  - Vice team leader of the LINA TASC team since July 2011.
  - 148 hours teaching in 2010-2011 in all years at EMN plus projects.
  - In charge of the 2nd year promotion in Computer Science at EMN.
  - In charge of the *Data Structure* and *Constraint-based Scheduling* courses at EMN.
  - Scientific advisor of the ongoing PhD thesis of A. De Clerq.
- Charlotte Truchet:
  - On leave from Univ. Nantes (at INRIA).
  - Vice team leader of the LINA TASC team until July 2011.
  - Scientific advisor of the ongoing PhD thesis of M. Pelleau, J. du Boisberranger and B. Belin.
  - Advisor of the master thesis of B. Belin.

10. Bibliography

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[12] J. MENANA. *Automata for Constraint Modelling and Solving*, Université de Nantes, Nantes, France, october 2011.

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