Activity Report 2012

Project-Team APICS

Analysis and Problems of Inverse type in Control and Signal processing

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Modeling, Optimization, and Control of Dynamic Systems

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Project-Team APICS

Keywords: System Analysis And Control, Nonlinear Control, Harmonic Analysis, Signal Processing

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2. Overall Objectives

2.1. Research Themes

The team develops constructive, function-theoretic approaches to inverse problems arising in modelling and design, in particular for electro-magnetic systems as well as in the analysis of certain classes of signals.

Data typically consist of measurements or desired behaviours. The general thread is to approximate them by families of solutions to the equations governing the underlying system. This leads us to consider various interpolation and approximation problems in classes of rational and meromorphic functions, harmonic gradients, or solutions to more general elliptic partial differential equations (PDE). A recurring difficulty is to control the singularities of the approximants.

The mathematical tools pertain to complex and harmonic analysis, approximation theory, potential theory, system theory, differential topology, optimization and computer algebra. Targeted applications include:

- identification and synthesis of analog microwave devices (filters, amplifiers),
- non-destructive control from field measurements in medical engineering (source recovery in magneto/electro-encephalography), paleomagnetism (determining the magnetization of rock samples), and nuclear engineering (plasma shaping in tokamaks).

In each case, the endeavour is to develop algorithms resulting in dedicated software.

2.2. International and industrial partners

- Collaboration under contract with Thales Alenia Space (Toulouse, Cannes, and Paris), CNES (Toulouse), XLim (Limoges), University of Bilbao (Universidad del País Vasco / Euskal Herriko Unibertsitatea, Spain).
- Regular contacts with research groups at UST (Villeneuve d'Asq), Universities of Bordeaux-I (Talence), Orléans (MAPMO), Pau (EPI commune Inria Magique-3D), Provence (Marseille, CMI), Nice (Lab. JAD), with CEA-IRFM (Cadarache), CWI (the Netherlands), MIT (Boston, USA) Michigan State University (East-Lansing, USA), Vanderbilt University (Nashville USA), Texas A&M University (College Station USA), State University of New-York (Albany, USA), University of Oregon (Eugene, USA), Politecnico di Milano (Milan, Italy), University of Trieste (Italy), RMC (Kingston, Canada), University of Leeds (UK), of Maastricht (The Netherlands), of Cork (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien (Austria), TFH-Berlin (Germany), ENIT (Tunis), KTH (Stockholm), University of Cyprus (Nicosia, Cyprus), University of Macau (Macau, China).
- The project is involved in the GDR-project AFHP (CNRS), in a EMS21-RTG NSF program (with MIT, Boston, and Vanderbilt University, Nashville, USA), in a LMS Grant with Leeds University (UK) and in a CSF program (with University of Cyprus).

3. Scientific Foundations

3.1. Introduction

Within the extensive field of inverse problems, much of the research by APICS deals with reconstructing solutions of classical elliptic PDEs from their boundary behaviour. Perhaps the most basic example of such a problem is harmonic identification of a stable linear dynamical system: the transfer-function f is holomorphic in the right half-pane, which means it satisfies there the Cauchy-Riemann equation $\overline{\partial} f = 0$, and in principle f can be recovered from its values on the imaginary axis, *e.g.* by Cauchy formula.

Practice is not nearly as simple, for f is only measured pointwise in the pass-band of the system which makes the problem ill-posed [69]. Moreover, the transfer function is usually sought in specific form, displaying the necessary physical parameters for control and design. For instance if f is rational of degree n, it satisfies $\overline{\partial} f = \sum_1^n a_j \delta_{z_j}$ where the z_j are its poles, and finding the domain of holomorphy (*i.e.* locating the z_j) amounts to solve a (degenerate) free-boundary inverse problem, this time on the left half-plane. To address these questions, the team has developed a two-step approach as follows.

Step 1: To determine a complete model, that is, one which is defined for every frequency, in a sufficiently flexible function class (*e.g.* Hardy spaces). This ill-posed issue requires regularization, for instance constraints on the behaviour at non-measured frequencies.

Step 2: To compute a reduced order model. This typically consists of rational approximation of the complete model obtained in step 1, or phase-shift thereof to account for delays. Derivation of the complete model is important to achieve stability of the reduced one.

Step 1 makes connection with extremal problems and analytic operator theory, see section 3.3.1. Step 2 involves optimization, and some Schur analysis to parametrize transfer matrices of given Mc-Millan degree when dealing with systems having several inputs and output, see section 3.3.2.2. It also makes contact with the topology of rational functions, to count critical points and to derive bounds, see section 3.3.2. Moreover, this step raises issues in approximation theory regarding the rate of convergence and whether the singularities of the approximant (*i.e.* its poles) converge to the singularities of the approximated function; this is where logarithmic potential theory becomes effective, see section 3.3.3.

Iterating the previous steps coupled with a sensitivity analysis yields a tuning procedure which was first demonstrated in [77] on resonant microwave filters.

Similar steps can be taken to approach design problems in frequency domain, replacing measured behaviour by desired behaviour. However, describing achievable responses from the design parameters at hand is generally cumbersome, and most constructive techniques rely on rather specific criteria adapted to the physics of the problem. This is especailly true of circuits and filters, whose design classically appeals to standard polynomial extremal problems and realization procedures from system theory [70], [55]. APICS is active in this field, where we introduced the use of Zolotarev-like problems for microwave multiband filter design. We currently favor interpolation techniques because of their transparency with respect to parameter use, see section 3.2.2.

In another connection, the example of harmonic identification quickly suggests a generalization of itself. Indeed, on identifying $\mathbb{C}$ with $\mathbb{R}^2$, holomorphic functions become conjugate-gradients of harmonic functions so that harmonic identification is, after all, a special case of a classical issue: to recover a harmonic function on a domain from partial knowledge of the Dirichlet-Neumann data; portion of the boundary where data are not available may be unknown, in which case we meet a free boundary problem. This framework for 2-D non-destructive control was first advocated in [59] and subsequently received considerable attention. This framework makes it clear how to state similar problems in higher dimensions and for more general operators than the Laplacian, provided solutions are essentially determined by the trace of their gradient on part of the boundary which is the case for elliptic equations [1] [79]. All these questions are particular instances of the so-called inverse potential problem, where a measure μ has to be recovered from knowledge of the gradient of its potential (*i.e.*, the field) on part of a hypersurface (a curve in 2-D) encompassing the support of μ. For Laplace's operator, potentials are logarithmic in 2-D and Newtonian in higher dimensions. For elliptic operators with non constant coefficients, the potential depends on the form of fundamental solutions and is less manageable because it is no longer of convolution type. In any case, by construction, the operator applied to the potential yields back the measure.

Inverse potential problems are severely indeterminate because infinitely many measures within an open set produce the same field outside this set [68]. In step 1 above we implicitly removed this indeterminacy by requiring that the measure be supported on the boundary (because we seek a function holomorphic throughout the right half space), and in step 2 by requiring, say, in case of rational approximation that the measure be discrete in the left half-plane. The same discreteness assumption prevails in 3-D inverse source problems. To recap, the gist of our approach is to approximate boundary data by (boundary traces of) fields arising from potentials of measures with specific support. Note this is different from standard approaches to inverse problems, where descent algorithms are applied to integration schemes of the direct problem; in such methods, it is the equation which gets approximated (in fact: discretized).

Along these lines, the team initiated the use of steps 1 and 2 above, along with singularity analysis, to approach issues of nondestructive control in 2 and 3-D [41] [6], [2]. We are currently engaged in two kinds of generalization, further described in section 3.2.1. The first one deals with non-constant conductivities, where Cauchy-Riemann equations for holomorphic functions are replaced by conjugate Beltrami equations for pseudo-holomorphic functions; there we seek applications to plasma confinement. The other one lies with inverse source problems for Laplace's equation in 3-D, where holomorphic functions are replaced by harmonic gradients, developing applications to EEG/MEG and inverse magnetization problems in paleomagnetism, see section 4.2

The main approximation-theoretic tools developed by APICS to get to grips with issues mentioned so far are outlined in section 3.3. In section 3.2 to come, we make more precise which problems are considered and for which applications.

3.2. Range of inverse problems

3.2.1. Elliptic partial differential equations (PDE)

Participants: Laurent Baratchart, Slah Chaabi, Juliette Leblond, Ana-Maria Nicu, Dmitry Ponomarev, Elodie Pozzi.

[1] There is a subtle difference here between dimension 2 and higher. Indeed, a function holomorphic on a plane domain is defined by its non-tangential limit on a boundary subset of positive linear measure, but there are non-constant harmonic functions in the 3-D ball, C^1 up to the boundary sphere, yet having vanishing gradient on a subset of positive measure of the sphere

This work is done in collaboration with Alexander Borichev (Univ. Provence).

Reconstructing Dirichlet-Neumann boundary conditions for a function harmonic in a plane domain when these are known on a strict subset E of the boundary, is equivalent to recover a holomorphic function in the domain from its boundary values on E. This is the problem raised on the half-plane in step 1 of section 3.1. It makes good sense in holomorphic Hardy spaces where functions are determined by their values on boundary subsets of positive linear measure, which is the framework for problem (P) in section 3.3.1. Such problems naturally arise in nondestructive testing of 2-D (or cylindical) materials from partial electrical measurements on the boundary. Indeed, the ratio between tangential and normal currents (so-called Robin coefficient) tells about corrosion of the material. Solving problem (P) where ψ is chosen to be the response of some uncorroded piece with identical shape allows one to approach such questions, and this was an initial application of holomorphic extremal problems to non-destructive control [56], [52].

A recent application by the team deals with non-constant conductivity over a doubly connected domain, E being the outer boundary. Measuring Dirichlet-Neumann data on E, we wanted to check whether the solution is constant on the inner boundary. We first had to define and study Hardy spaces of the conjugate Beltrami equation, of which the conductivity equation is the compatibility condition (just like Laplace's equation is the compatibility condition of the Cauchy-Riemann system). This was done in references [5] and [35]. Then, solving an obvious modification of problem (P) allows one to numerically check what we want. Further, the value of this extremal problem defines a criterion on inner boundaries, and subsequently a descent algorithm was set up to improve the initial boundary into one where the solution is closer to being constant, thereby trying to solve a free boundary problem..

When the domain is regarded as separating the edge of a tokamak's vessel from the plasma (rotational symmetry makes this a 2-D problem), the procedure just described suits plasma control from magnetic confinement. It was successfully applied in collaboration with CEA (the French nuclear agency) and the University of Nice (JAD Lab.) to data from *Tore Supra* [58], see section 6.2. This procedure is fast because no numerical integration of the underlying PDE is needed, as an explicit basis of solutions to the conjugate Beltrami equation was found in this case.

Three-dimensional versions of step 1 in section 3.1 are also considered, namely to recover a harmonic function (up to a constant) in a ball or a half-space from partial knowledge of its gradient on the boundary. Such questions arise naturally in connection with neurosciences and medical imaging (electroencephalography, EEG) or in paleomagnetism (analysis of rocks magnetization) [2] [37], see section 6.1. They are not yet as developed as the 2-D case where the power of complex analysis is at work, but considerable progress was made over the last years through methods of harmonic analysis and operator theory.

The team is also concerned with non-destructive control problems of localizing defaults such as cracks, sources or occlusions in a planar or 3-dimensional domain, from boundary data (which may correspond to thermal, electrical, or magnetic measurements). These defaults can be expressed as a lack of analyticity of the solution of the associated Dirichlet-Neumann problem and we approach them using techniques of best rational or meromorphic approximation on the boundary of the object [4] [16], see sections 3.3.2 and 4.2. In fact, the way singularities of the approximant relate to the singularities of the approximated function is an all-pervasive theme in approximation theory, and for appropriate classes of functions the location of the poles of a best rational approximant can be used as an estimator of the singularities of the approximated function (see section 6.1). This circle of ideas is much in the spirit of step 2 in section 3.1.

A genuine 3-dimensional theory of approximation by discrete potentials, though, is still in its infancy.

3.2.2. Systems, transfer and scattering

Participants: Laurent Baratchart, Sylvain Chevillard, Sanda Lefteriu, Martine Olivi, Fabien Seyfert.

Through initial contacts with CNES, the French space agency, the team came to work on identification-for-tuning of microwave electromagnetic filters used in space telecommunications (see section 4.3). The problem was to recover, from band-limited frequency measurements, the physical parameters of the device under examination. The latter consists of interconnected dual-mode resonant cavities with negligible loss, hence its scattering matrix is modelled by a 2×2 unitary-valued matrix function on the frequency line, say the imaginary axis to fix ideas. In the bandwidth around the resonant frequency, a modal approximation of the Helmholtz equation in the cavities shows that this matrix is approximately rational, of Mc-Millan degree twice the number of cavities.

This is where system theory enters the scene, through the so-called *realization* process mapping a rational transfer function in the frequency domain to a state-space representation of the underlying system as a system of linear differential equations in the time domain. Specifically, realizing the scattering matrix allows one to construct a virtual electrical network, equivalent to the filter, the parameters of which mediate in between the frequency response and the geometric characteristics of the cavities (*i.e.* the tuning parameters).

Hardy spaces, and in particular the Hilbert space H^2, provide a framework to transform this classical ill-posed issue into a series of well-posed analytic and meromorphic approximation problems. The procedure sketched in section 3.1 now goes as follows:

1. infer from the pointwise boundary data in the bandwidth a stable transfer function (*i.e.* one which is holomorphic in the right half-plane), that may be infinite dimensional (numerically: of high degree). This is done by solving in the Hardy space H^2 of the right half-plane a problem analogous to (P) in section 3.3.1, taking into account prior knowledge on the decay of the response outside the bandwidth, see [18] for details.
2. From this stable model, a rational stable approximation of appropriate degree is computed. For this a descent method is used on the relatively compact manifold of inner matrices of given size and degree, using a novel parametrization of stable transfer functions [18].
3. From this rational model, realizations meeting certain constraints imposed by the technology in use are computed (see section 6.3). These constraints typically come from the nature and topology of the equivalent electrical network used to model the filter. This network is composed of resonators, coupled to each other by some specific coupling topology. Performing this realization step for given coupling topology can be recast, under appropriate compatibility conditions [8], as the problem of solving a zero-dimensional multivariate polynomial system. To tackle this problem in practice, we use Groebner basis techniques as well as continuation methods as implemented in the Dedale-HF software (5.4).

Let us also mention that extensions of classical coupling matrix theory to frequency-dependent (reactive) couplings have lately been carried-out [1] for wide-band design applications, but further study is needed to make them effective.

Subsequently APICS started investigating issues pertaining to filter design rather than identification. Given the topology of the filter, a basic problem is to find the optimal response with respect to amplitude specifications in frequency domain bearing on rejection, transmission and group delay of scattering parameters. Generalizing the approach based on Tchebychev polynomials for single band filters, we recast the problem of multi-band response synthesis in terms of a generalization of classical Zolotarev min-max problem [30] to rational functions [11]. Thanks to quasi-convexity, the latter can be solved efficiently using iterative methods relying on linear programming. These are implemented in the software easy-FF (see section 5.5).

Later, investigations by the team extended to design and identification of more complex microwave devices, like multiplexers and routers, which connect several filters through wave guides. Schur analysis plays an important role in such studies, which is no surprise since scattering matrices of passive systems are of Schur type (*i.e.* contractive in the stability region). The theory originates with the work of I. Schur [76], who devised a recursive test to check for contractivity of a holomorphic function in the disk. Generalizations thereof turned out to be very efficient to parametrize solutions to contractive interpolation problems subject to a well-known compatibility condition (positive definiteness of the so-called Pick matrix) [32]. Schur analysis became quite popular in electrical engineering, as the Schur recursion precisely describes how to chain two-port circuits.

Dwelling on this, members of the team contributed to differential parametrizations (atlases of charts) of lossless matrix functions to the theory [31][12], [10]. They are of fundamental use in our rational approximation software RARL2 (see section 5.1). Schur analysis is also instrumental to approach de-embedding issues considered in section 6.4, and provides further background to current studies by the team of synthesis and adaptation problems for multiplexers. At the heart of the latter lies a variant of contractive interpolation with degree constraint introduced in [62].

We also mention the role played by multipoint Schur analysis in the team's investigation of spectral representation for certain non-stationary discrete stochastic processes [3], [36].

Recently, in collaboration with UPV (Bilbao), our attention was driven by CNES, to questions of stability relative to high-frequency amplifiers, see section 7.2. Contrary to previously mentioned devices, these are *active* components. The amplifier can be linearized at a functioning point and admittances of the corresponding electrical network can be computed at various frequencies, using the so-called harmonic balance method. The goal is to check for stability of this linearised model. The latter is composed of lumped electrical elements namely inductors, capacitors, negative *and* positive reactors, transmission lines, and commanded current sources. Research so far focused on determining the algebraic structure of admittance functions, and setting up a function-theoretic framework to analyse them. In particular, much effort was put on realistic assumptions under which a stable/unstable decomposition can be claimed in $H^2 \oplus \overline{H^2}$ (see section 6.5). Under them, the unstable part of the elements under examination is rational and we expect to bring valuable estimates of stability to the designer using the general scheme in section 3.1.

3.3. Approximation of boundary data

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Martine Olivi, Dmitry Ponomarev, Elodie Pozzi, Fabien Seyfert.

The following people are collaborating with us on these topics: Bernard Hanzon (Univ. Cork, Ireland), Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris), Jonathan Partington (Univ. Leeds, UK), Ralf Peeters (Univ. Maastricht, NL), Edward Saff (Vanderbilt University, Nashville, USA), Herbert Stahl (TFH Berlin), Maxim Yattselev (Univ. Oregon at Eugene, USA).

3.3.1. Best constrained analytic approximation

In dimension 2, the prototypical problem to be solved in step 1 of section 3.1 may be described as: given a domain $D \subset \mathbb{R}^2$, we want to recover a holomorphic function from its values on a subset of the boundary of D. Using conformal mapping, it is convenient for the discussion to normalize D. So, in the simply connected case, we fix D to be the unit disk with boundary the unit circle T. We denote by H^p the Hardy space of exponent p which is the closure of polynomials in the L^p-norm on the circle if $1 \leq p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which makes it possible to speak of (traces of) analytic functions on the boundary.

To find an analytic function in D approximately matching measured values f on a sub-arc K of T, we formulate a constrained best approximation problem as follows.

> (P) Let $1 \leq p \leq \infty$, K a sub-arc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behaviour capturing *a priori* assumptions on the behaviour of the model off K, while M is some admissible deviation from them. The value of p reflects the type of stability which is sought and how much one wants to smoothen the data. The choice of L^p classes is well-adapted to handling pointwise measurements.

To fix terminology we refer to (P) as a *bounded extremal problem*. As shown in [40], [42], [47], for $1 < p \leq \infty$, the solution to this convex infinite-dimensional optimization problem can be obtained upon iterating with respect to a Lagrange parameter the solution to spectral equations for some appropriate Hankel and Toeplitz operators. These equations in turn involve the solution to the standard extremal problem below best approximation problem (P_0) below [61]:

(P_0) Let $1 \leq p \leq \infty$ and $\varphi \in L^p(T)$; find a function $g \in H^p$ such that $g - \varphi$ is of minimal norm in $L^p(T)$.

The case $p = 1$ of (P) is essentially open.

Various modifications of (P) have been studied in order to meet specific needs. For instance when dealing with loss-less transfer functions (see section 4.3), one may want to express the constraint on $T \setminus K$ in a pointwise manner: $|g - \psi| \leq M$ a.e. on $T \setminus K$, see [43]. In this form, it comes close to (but still is different from) H^∞ frequency optimization methods for control [64], [75].

The analog of problem (P) on an annulus, K being now the outer boundary, can be seen as a means to regularize a classical inverse problem occurring in nondestructive control, namely recovering a harmonic function on the inner boundary from Dirichlet-Neumann data on the outer boundary (see sections 3.2.1, 4.2, 6.1.1, 6.2). For $p = 2$ the solution is analysed in [66]. It may serve as a tool to approach Bernoulli type problems where we are given data on the outer boundary and we *seek the inner boundary*, knowing it is a level curve of the flux. Then, the Lagrange parameter indicates which deformation should be applied on the inner contour in order to improve data fitting.

This is discussed in sections 3.2.1 and 6.2 for more general equations than the Laplacian, namely isotropic conductivity equations of the form $\mathrm{div}(\sigma \nabla u) = 0$ where σ is non constant. In this case Hardy spaces in problem (P) become those of a so-called conjugate or real Beltrami equation [65], which are studied for $1 < p < \infty$ in [5], [35]. Expansions of solutions needed to constructively handle such issues have been carried out in [58].

Though originally considered in dimension 2, problem (P) carries over naturally to higher dimensions where analytic functions get replaced by gradients of harmonic functions. Namely, given some open set $\Omega \subset \mathbb{R}^n$ and a $\mathbb{R}^n$-valued vector V field on an open subset O of the boundary of Ω, we seek a harmonic function in Ω whose gradient is close to V on O.

When Ω is a ball or a half-space, a convenient substitute of holomorphic Hardy spaces is provided by Stein-Weiss Hardy spaces of harmonic gradients [78]. Conformal maps are no longer available in $\mathbb{R}^n$ for $n > 2$ and other geometries have not been much studied so far. On the ball, the analog of problem (P) is

$(P1)$ Let $1 \leq p \leq \infty$ and $B \subset \mathbb{R}^n$ the unit ball. Fix O an open subset of the unit sphere $S \subset \mathbb{R}^n$. Let further $V \in L^p(O)$ and $W \in L^p(S \setminus O)$ be $\mathbb{R}^n$-valued vector fields, and $M > 0$; find a harmonic gradient $G \in H^p(B)$ such that $\|G - W\|_{L^p(S \setminus O)} \leq M$ and $G - V$ is of minimal norm in $L^p(O)$ under this constraint.

When $p = 2$, spherical harmonics offer a reasonable substitute to Fourier expansions and problem $(P1)$ was solved in [2], together with its natural analog on a shell. The solution generalizes the Toeplitz operator approach to bounded extremal problems [40], and constructive aspects of the procedure (harmonic 3-D projection, Kelvin and Riesz transformation, spherical harmonics) were derived. An important ingredient is a refinement of the Hodge decomposition allowing us to express a $\mathbb{R}^n$-valued vector field in $L^p(S)$, $1 < p < \infty$, as the sum of a vector field in $H(B)$, a vector field in $H^p(\mathbb{R}^n \setminus \overline{B}$, and a tangential divergence free vector field. If $p = 1$ or $p = \infty$, L^p must be replaced respectively by the real Hardy space H^1 and the bounded mean oscillation space BMO, and H^∞ should be modified accordingly. This decomposition was fully discussed in [37] (for the case of the half-space) where it plays a fundamental role.

Problem $(P1)$ is still under investigation in the case $p = \infty$, where even the case where $O = S$ is pending because a substitute of the Adamjan-Arov-Krein theory [72] is still to be built in dimension greater than 2.

Such problems arise in connection with source recovery in electro/mgneto encephalography and paleomagnetism, as discussed in sections 3.2.1 and 4.2.

3.3.2. Best meromorphic and rational approximation

The techniques explained in this section are used to solve step 2 in section 3.2 *via* conformal mapping and subsequently instrumental to approach inverse boundary value problems for Poisson equation $\Delta u = \mu$, where μ is some (unknown) distribution.

3.3.2.1. Scalar meromorphic and rational approximation

Let as before D designate the unit disk, and T the unit circle. We further put R_N for the set of rational functions with at most N poles in D, which allows us to define the meromorphic functions in $L^p(T)$ as the traces of functions in $H^p + R_N$.

A natural generalization of problem (P_0) is:

> (P_N) Let $1 \leq p \leq \infty$, $N \geq 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Only for $p = \infty$ and continuous f it is known how to solve (P_N) in closed form. The unique solution is given by AAK theory (named after Adamjan, Arov and Krein), that connects the spectral decomposition of Hankel operators with best approximation in Hankel norm [72]. This theory allows one to express g_N in terms of the singular vectors of the Hankel operator with symbol f. The continuity of g_N as a function of f only holds for stronger norms than uniform.

The case $p = 2$ is of special importance. In particular when $f \in \overline{H}^2$, the Hardy space of exponent 2 of the *complement* of D in the complex plane (by definition, $h(z)$ belongs to $\overline{H}^p$ if, and only if $h(1/z)$ belongs to H^p), then (P_N) reduces to rational approximation. Moreover, it turns out that the associated solution $g_N \in R_N$ has no pole outside D, hence it is a *stable* rational approximant to f. However, in contrast with the situation when $p = \infty$, this approximant may *not* be unique.

The former Miaou project (predecessor of Apics) has designed an adapted steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; until now it seems to be the only procedure meeting this property. Roughly speaking, it is a gradient algorithm that proceeds recursively with respect to the order N of the approximant, in a compact region of the parameter space [34]. Although it has proved effective in all applications carried out so far (see sections 4.2, 4.3), it is not known whether the absolute minimum can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as is done by the RARL2 software, section 5.1).

In order to establish global convergence results, APICS has undertook a long-term study of the number and nature of critical points, in which tools from differential topology and operator theory team up with classical approximation theory. The main discovery is that the nature of the critical points (*e.g.*, local minima, saddles...) depends on the decrease of the interpolation error to f as N increases [44]. Based on this, sufficient conditions have been developed for a local minimum to be unique. These conditions are hard to use in practice because they require strong estimates of the approximation error. These are often difficult to obtain for a given function, and are usually only valid for large N. Examples where uniqueness or asymptotic uniqueness has been proved this way include transfer functions of relaxation systems (*i.e.* Markov functions) [48] and more generally Cauchy integrals over hyperbolic geodesic arcs [50] and certain entire functions [46].

An analog to AAK theory has been carried out for $2 \leq p < \infty$ [47]. Although not computationally as powerful, it can be used to derive lower bounds and helps analysing the behaviour of poles. When $1 \leq p < 2$, problem (P_N) is still fairly open.

A common feature to all these problems is that critical point equations express non-Hermitian orthogonality relations for the denominator of the approximant. This makes connection with interpolation theory [51][7] and is used in an essential manner to assess the behaviour of the poles of the approximants to functions with branchpoint-type singularities, which is of particular interest for inverse source problems (*cf.* sections 5.6 and 6.1).

In higher dimensions, the analog of problem (P_N) is the approximation of a vector field with gradients of potentials generated by N point masses instead of meromorphic functions. The issue is by no means understood at present, and is a major endeavour of future research problems.

Certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus. Such questions have become over years an increasingly significant part of the team's activity (see section 4.3). For instance, convergence properties of multipoint Schur approximants, which are rational interpolants preserving contractivity of a function, were analysed in [3]. Such approximants are useful in prediction theory of stochastic processes, but since they interpolate inside the domain of holomorphy they are of limited use in frequency design.

In another connection, the generalization to several arcs of classical Zolotarev problems [74] is an achievement by the team which is useful for multiband synthesis [11]. Still, though the modulus of the response is the first concern in filter design, variation of the phase must nevertheless remain under control to avoid unacceptable distortion of the signal. This specific but important issue has less structure and was approached using constrained optimization; a dedicated code has been developed under contract with the CNES (see section 5.5).

3.3.2.2. Matrix-valued rational approximation

Matrix-valued approximation is necessary for handling systems with several inputs and outputs, and it generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically. In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the degree.

The problem we want to consider reads: *Let $\mathcal{F} \in (H^2)^{m \times l}$ and n an integer; find a rational matrix of size $m \times l$ without poles in the unit disk and of McMillan degree at most n which is nearest possible to $\mathcal{F}$ in $(H^2)^{m \times l}$*. Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The scalar approximation algorithm [34], mentioned in section 3.3.2.1, generalizes to the matrix-valued situation [60]. The first difficulty here consists in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (*i.e.* matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n. The latter enter the picture in an essential manner as they play the role of the denominator in a fractional representation of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization). The set of inner matrices of given degree has the structure of a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (parametrization valid in a neighborhood of a point), and we must handle changes of charts in the course of the algorithm. Such parametrization can be obtained from interpolation theory and Schur type algorithms, the parameters being interpolation vectors or matrices ([31], [10], [12]). Some of them are particularly interesting to compute realizations and achieve filter synthesis ([10] [12]). Rational approximation software codes have been developed in the team (see sections 5.1).

Difficulties relative to multiple local minima naturally arise in the matrix-valued case as well, and deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The case of rational functions of sought degree or small perturbations thereof (the consistency problem) was solved in [45]. The case of matrix-valued Markov functions, the first example beyond rational functions, was treated in [33].

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.3.3. Behavior of poles of meromorphic approximants

Participant: Laurent Baratchart.

The following people collaborate with us on this subject: Herbert Stahl (TFH Berlin), Maxim Yattselev (Univ. Oregon at Eugene, USA).

We refer here to the behaviour of poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. If one normalizes the contour to be the unit circle T, we are back to the framework of section 3.3.2.1 and to problem (P_N); invariance of the problem under conformal mapping was established in [6]. Research so far has focused on functions whose singular set inside the contour is zero or one-dimensional.

Generally speaking, the behaviour of poles is particularly important in meromorphic approximation to obtain error rates as the degree goes large and to tackle constructive issues like uniqueness. As explained in section 3.2.1, we consider this issue in connection with approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities. The general theme is thus *how do the singularities of the approximant reflect those of the approximated function?* This approach to inverse problem for the 2-D Laplacian turns out to be attractive when singularities are zero- or one-dimensional (see section 4.2). It can be used as a computationally cheap initialization of more precise but heavier numerical optimizations.

As regards crack detection or source recovery, the approach in question boils down to analysing the behaviour of best meromorphic approximants of a function with branch points. For piecewise analytic cracks, or in the case of sources, We were able to prove ([6], [38], [14]) that the poles of the approximants accumulate on some extremal contour of minimum weighted energy linkings the singular points of the crack, or the sources [41]. Moreover, the asymptotic density of the poles turns out to be the Green equilibrium distribution of this contour in D, hence puts heavy charge around the singular points (in particular at the endpoints) which are therefore well localized if one is able to approximate in sufficiently high degree (this is where the method could fail).

The case of two-dimensional singularities is still an outstanding open problem.

It is interesting that inverse source problems inside a sphere or an ellipsoid in 3-D can be attacked with the above 2-D techniques, as applied to planar sections (see section 6.1).

3.3.4. Miscellaneous

Participant: Sylvain Chevillard.

Sylvain Chevillard, joined team in November 2010. His coming resulted in APICS hosting a research activity in certified computing, centered around the software *Sollya* of which S. Chevillard is a co-author, see section 5.7. On the one hand, Sollya is an Inria software which still requires some tuning to a growing community of users. On the other hand, approximation-theoretic methods at work in Sollya are potentially useful for certified solutions to constrained analytic problems described in section 3.3.1. However, developing Solya is not a long-term objective of APICS.

4. Application Domains

4.1. Introduction

These domains are related to the problems described in sections 3.2.1 and 3.2.2. They are handled using the techniques described in section 3.3.

4.2. Inverse problems for elliptic PDE

Participants: Laurent Baratchart, Juliette Leblond, Ana-Maria Nicu, Dmitry Ponomarev.

This work is done in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena project-team.

Solving overdetermined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to extrapolate incomplete data (see section 3.2.1) is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG since the latter involves propagating the initial conditions through several layers of different conductivities, from the boundary down to the center of the domain where the singularities (*i.e.* the sources) lie. Actually, once propagated to the innermost sphere, it turns out that that traces of the boundary data on 2-D cross sections (disks) coincide with analytic functions in the slicing plane, that has branched singularities inside the disk [4]. These singularities are related to the actual location of the sources (namely, they reach in turn a maximum in modulus when the plane contains one of the sources). Hence, we are back to the 2-D framework of section 3.3.3 where approximately recovering these singularities can be performed using best rational approximation.

Numerical experiments gave very good results on simulated data and we are now proceeding with real experimental magneto-encephalographic data, see also sections 5.6 and 6.1. The PhD thesis of A.-M. Nicu [13] was concerned with these applications, see [16], in collaboration with the Athena team at Inria Sophia Antipolis, and neuroscience teams in partner-hospitals (hosp. Timone, Marseille).

Similar inverse potential problems appear naturally in magnetic reconstruction. A particular application, which is the object of a joint NSF project with Vanderbilt University and MIT, is to geophysics. There, the remanent magnetization of a rock is to be analysed to draw information on magnetic reversals and to reconstruct the rock history. Recently developed scanning magnetic microscopes measure the magnetic field down to very small scales in a "thin plate" geological sample at the Laboratory of planetary sciences at MIT, and the magnetization has to be recovered from the field measured on a plane located at small distance above the slab.

Mathematically speaking, EEG and magnetization inverse problems both amount to recover the (3-D valued) quantity m (primary current density in case of the brain or magnetization in case of a thin slab of rock) from measurements of the vector potential:

$$\int_{\Omega} \frac{\operatorname{div} m(x')\, dx'}{|x-x'|}\,,$$

outside the volume Ω of the object, from Maxwell's equations.

The team is also getting engaged in problems with variable conductivity governed by a 2-D conjugate-Beltrami equation, see [5], [58], [35]. The application we have in mind is to plasma confinement for thermonuclear fusion in a Tokamak, more precisely with the extrapolation of magnetic data on the boundary of the chamber from the outer boundary of the plasma, which is a level curve for the poloidal flux solving the original div-grad equation. Solving this inverse problem of Bernoulli type is of importance to determine the appropriate boundary conditions to be applied to the chamber in order to shape the plasma [54]. These issues are the topics of the PhD theses of S. Chaabi and D. Ponomarev [27], and of a joint collaboration with the Laboratoire J.-A. Dieudonné at the Univ. of Nice-SA (and the Inria team Castor), and the CMI-LATP at the Univ. of Aix-Marseille I (see section 6.2).

4.3. Identification and design of microwave devices

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

This work is done in collaboration with Stéphane Bila (XLim, Limoges) and Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

One of the best training grounds for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption works well in the considered range of frequency, and whose specifications are made in the frequency domain. This is the case of electromagnetic resonant systems which are of common use in telecommunications.

In space telecommunications (satellite transmissions), constraints specific to on-board technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study (of the Helmholtz equation) states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Figure 1. Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that 16 quantities must be optimized. Quantities such as the diameter and length of the cavities, or the width of the 11 slits are fixed during the design phase.

Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all cavities show the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 Ghz.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modelled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plugged on a resistor at one end and fed with some potential at the other end. We are then interested in the power which is transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the design, particularly the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearising near the central frequency, which is no longer conjugate symmetric (*i.e.* the underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80Mhz in the example).
- Solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, cf. section 3.3.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.
- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this example) via the Endymion or RARL2 software (cf. section 3.3.2.2).
- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

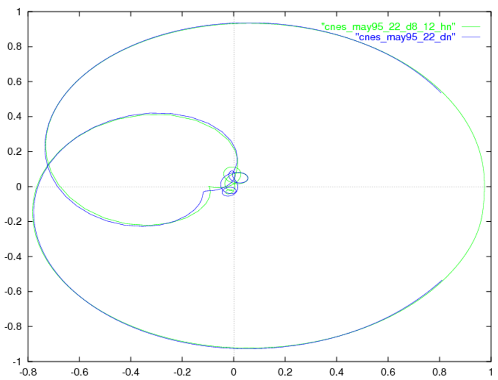

Figure 2. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}.

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 2). Non-physical couplings are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, useful for the synthesis of repeating devices.

The team also investigates problems relative to the design of optimal responses for microwave devices. The resolution of a quasi-convex Zolotarev problems was for example proposed, in order to derive guaranteed optimal multi-band filter's responses subject to modulus constraints [11]. This generalizes the classical single band design techniques based on Tchebychev polynomials and elliptic functions. These techniques rely on the fact that the modulus of the scattering parameters of a filters, say $|S_{1,2}|$, admits a simple expression in terms of the filtering function $D = |S_{1,1}|/|S_{1,2}|$ namely,

$$|S_{1,2}|^2 = \frac{1}{1+D^2}.$$

The filtering function appears to be the ratio of two polynomials p_1/p_2, the numerator of the reflection and transmission scattering factors, that can be chosen freely. The denominator q is obtained as the unique stable and unitary polynomial solving the classical Feldtkeller spectral equation:

$$qq^* = p_1 p_1^* + p_2 p_2^*.$$

The relative simplicity of the derivation of filter's responses under modulus constraints is due to this ability to "forget" about latter spectral equation, and express all design constraints on the filtering functions D. This no longer the case when considering the synthesis N-port devices for $N > 3$, like multiplexers, routers power dividers or when considering the synthesis of filters under matching conditions. The efficient derivation of multiplexers responses is one of the team's active recent research area, where technique based on constrained Nevanlinna-Pick interpolation problems are under study (see section 6.4.1).

5. Software

5.1. RARL2

Participant: Martine Olivi [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see section 3.3.2.2) http://www-sop.inria.fr/apics/RARL2/rarl2-eng.html.

The software RARL2 computes, from a given matrix-valued function in $\overline{H}^{2^{m\times l}}$, a local best rational approximant in the L^2 norm, which is *stable and of prescribed McMillan degree* (see section 3.3.2.2). It was initially developed in the context of linear (discrete-time) system theory and makes an heavy use of the classical concepts in this field. The matrix-valued function to be approximated can be viewed as the transfer function of a multivariable discrete-time stable system. RARL2 takes as input either:

- its internal realization,
- its first N Fourier coefficients,
- discretized (uniformly distributed) values on the circle. In this case, a least-square criterion is used instead of the L^2 norm.

It thus performs model reduction in case 1) and 2) and frequency data identification in case 3). In the case of band-limited frequency data, it could be necessary to infer the behavior of the system outside the bandwidth before performing rational approximation (see 3.2.2). An appropriate Moebius transformation allows to use the software for continuous-time systems as well.

The method is a steepest-descent algorithm. A parametrization of MIMO systems is used, which ensures that the stability constraint on the approximant is met. The implementation, in matlab, is based on state-space representations.

The number of local minima can be rather high so that the choice of an initial point for the optimization can play a crucial role. Two methods can be used: 1) An initialization with a best Hankel approximant. 2) An iterative research strategy on the degree of the local minima, similar in principle to that of Rarl2, increases the chance of obtaining the absolute minimum by generating, in a structured manner, several initial conditions.

RARL2 performs the rational approximation step in our applications to filter identification (see section 4.3) as well as sources or cracks recovery (see section 4.2). It was released to the universities of Delft, Maastricht, Cork and Brussels. The parametrization embodied in RARL2 was also used for a multi-objective control synthesis problem provided by ESTEC-ESA, The Netherlands. An extension of the software to the case of triple poles approximants is now available. It provides satisfactory results in the source recovery problem and it is used by FindSources3D (see section 5.6).

5.2. RGC

Participant: Fabien Seyfert [corresponding participant].

Status: A stable version is maintained.

This software is developed in collaboration with Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

The identification of filters modelled by an electrical circuit that was developed by the team (see section 4.3) led us to compute the electrical parameters of the underlying filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called "the arrow form" [53] which is of particular interest since it is unique for a given transfer function and is easily computed. The computation of this realization is the first step of RGC. Subsequently, if the target realization is not in arrow form, one can nevertheless show that it can be deduced from the arrow-form by a complex- orthogonal change of basis. In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has many local and global minima. In fact, there is not even uniqueness of the filter realization for a given geometry. Moreover, it is often relevant to know all solutions of the problem, because the designer is not even sure, in many cases, which one is being handled. The assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software provides no guarantee to obtain a single realization that satisfies the prescribed constraints. The software Dedale-HF (see section 5.4), which is the successor of RGC, solves with guarantees this constraint realization problem.

5.3. PRESTO-HF

Participant: Fabien Seyfert [corresponding participant].

Status: Currently under development. A stable version is maintained.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for microwave filters http://www-sop.inria.fr/apics/personnel/Fabien.Seyfert/Presto_web_page/presto_pres.html. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single shot:

- determination of delay components caused by the access devices (automatic reference plane adjustment),
- automatic determination of an analytic completion, bounded in modulus for each channel,
- rational approximation of fixed McMillan degree,
- determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies on RARL2 (see section 5.1), a rational approximation engine developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Thales Alenia Space in Toulouse, Thales airborn systems and a license agreement has been recently negotiated with TAS-Espagna. XLim (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements are currently being considered with the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada).

5.4. Dedale-HF

Participant: Fabien Seyfert [corresponding participant].

Status: Currently under development. A stable version is maintained.

Dedale-HF is a software dedicated to solve exhaustively the coupling matrix synthesis problem in reasonable time for the users of the filtering community. For a given coupling topology, the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of given filter characteristics (see section 6.3). Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements (see Figure 3).

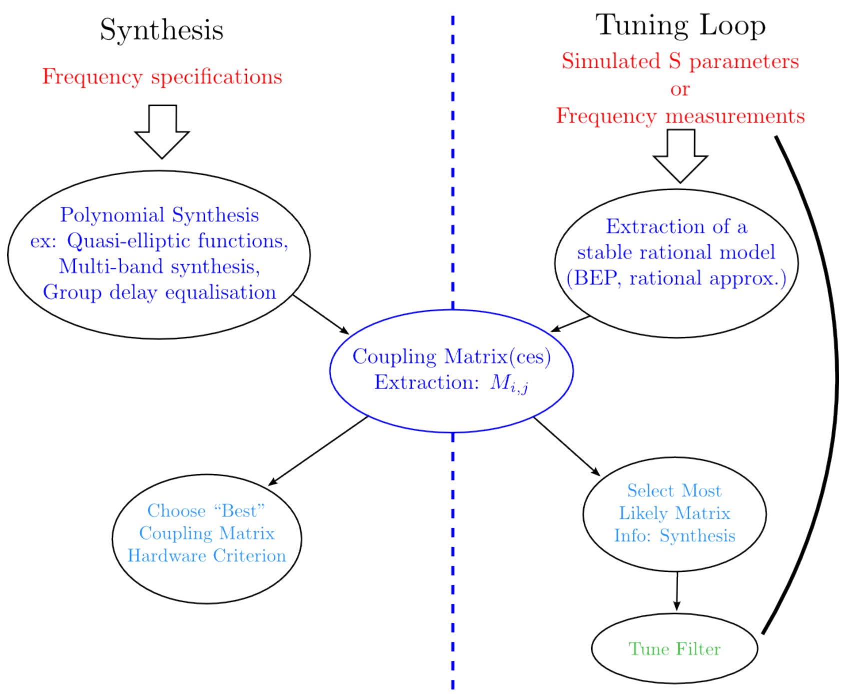

Figure 3. Overall scheme of the design and tuning process of a microwave filter.

Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. problem of the user, *i.e.* the one corresponding to user-specified filter characteristics. The reference files are computed off-line using Groebner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such a continuation technique combined with an efficient implementation of the integrator produces a drastic reduction, by a factor of 20, of the computational time.

Access to the database and integrator code is done via the web on http://www-sop.inria.fr/apics/Dedale/WebPages. The software is free of charge for academic research purposes: a registration is however needed in order to access full functionality. Up to now 90 users have registered world wide (mainly: Europe, U.S.A, Canada and China) and 4000 reference files have been downloaded.

A license of this software has been sold end 2011, to TAS-Espagna to tune filter, with topologies with multiple solutions. The usage of Dedale-HF is here considered together with Presto-HF.

5.5. easyFF

Participant: Fabien Seyfert.

Status: A stable version is maintained.

This software has been developed by Vincent Lunot (Taiwan Univ.) during his Ph.d. He still continues to maintain it.

EasyFF is a software dedicated to the computation of complex, and in particular multi-band, filtering functions. The software takes as input, specifications on the modulus of the scattering matrix (transmission and rejection), the filter's order and the number of transmission zeros. The output is an "optimal" filtering characteristic in the sense that it is the solution of an associated min-max Zolotarev problem. Computations are based on a Remez-type algorithm (if transmission zeros are fixed) or on linear programming techniques if transmission zeros are part of the optimization [11].

5.6. FindSources3D

Participant: Juliette Leblond [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena EPI, and with Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

FindSources3D is a software dedicated to source recovery for the inverse EEG problem, in 3-layer spherical settings, from pointwise data (see http://www-sop.inria.fr/apics/FindSources3D/). Through the algorithm described in [16] and section 4.2, it makes use of the software RARL2 (section 5.1) for the rational approximation step in plane sections. The data transmission preliminary step ("cortical mapping") is solved using boundary element methods through the software OpenMEEG (its CorticalMapping features) developed by the Athena Team (see http://www-sop.inria.fr/athena/software/OpenMEEG/). A first release of FindSources3D is now available, which will be demonstrated and distributed within the medical teams we are in contact with (see Figure 4, CeCILL license, APP version 1.0: IDDN.FR.001.45009.S.A.2009.000.10000).

5.7. Sollya

Participant: Sylvain Chevillard [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Christoph Lauter (LIP6) and Mioara Joldeş (Uppsala University, Sweden).

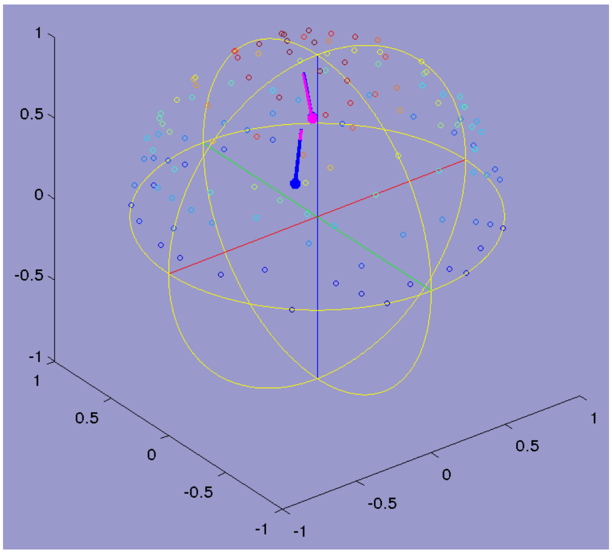

Figure 4. Potential values at electrodes on a sphere (scalp), recovered 2 sources (FindSources3D).

Sollya is an interactive tool where the developers of mathematical floating-point libraries (libm) can experiment before actually developing code. The environment is safe with respect to floating-point errors, *i.e.* the user precisely knows when rounding errors or approximation errors happen, and rigorous bounds are always provided for these errors.

Amongst other features, it offers a fast Remez algorithm for computing polynomial approximations of real functions and also an algorithm for finding good polynomial approximants with floating-point coefficients to any real function. It also provides algorithms for the certification of numerical codes, such as Taylor Models, interval arithmetic or certified supremum norms.

It is available as a free software under the CeCILL-C license at http://sollya.gforge.inria.fr/.

6. New Results

6.1. Source recovery problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Ana-Maria Nicu.

The works presented here are done in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena EPI, with Doug Hardin and Edward Saff from Vanderbilt University (Nashville, USA), and with Abderrazek Karoui (Univ. Bizerte, Tunisie) and Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

This section in dedicated to inverse problems for 3-D Poisson-Laplace equations. Though the geometrical settings differ in the 2 sections below, the characterization of silent sources (that give rise to a vanishing potential at measurement points) is a common problem to both which has been recently achieved, see [37],[29], [39].These are sums of (distributional) derivatives of Sobolev functions vanishing on the boundary.

6.1.1. Application to EEG

In 3-D, functional or clinical active regions in the cortex are often represented by pointwise sources that have to be localized from measurements on the scalp of a potential satisfying a Laplace equation (EEG, electroencephalography). In the work [4] it was shown how to proceed via best rational approximation on a sequence of 2-D disks cut along the inner sphere, for the case where there are at most 2 sources. A milestone in a long-haul research on the behaviour of poles of best rational approximants of fixed degree to functions with branch points has been reached this year [14], which shows that the technique carries over to finitely many sources (see section 4.2). In this connection, a dedicated software "FindSources3D" (see section 5.6) has been developed, in collaboration with the team Athena [16], [26].

Further, it appears that in the rational approximation step of these schemes, *multiple* poles possess a nice behaviour with respect to the branched singularities. This is due to the very basic physical assumptions on the model (for EEG data, one should consider *triple* poles). Though numerically observed in [16], there is no mathematical justification so far why these multiple poles have such strong accumulation properties, which remains an intriguing observation.

Issues of robust interpolation on the sphere from incomplete pointwise data are also under study in order to improve numerical accuracy of our reconstruction schemes. Spherical harmonics, Slepian bases and related special functions are of special interest (thesis of A.-M. Nicu [13], [67]), while other techniques should be considered as well.

Also, magnetic data from MEG (magneto-encephalography) will soon become available, which should enhance the accuracy of source recovery algorithms.

It turns out that discretization issues in geophysics can also be approached by these approximation techniques. Namely, in geodesy or for GPS computations, one may need to get a best discrete approximation of the gravitational potential on the Earth's surface, from partial data collected there. This is the topic of a beginning collaboration with a physicist colleague (IGN, LAREG, geodesy). Related geometrical issues (finding out the geoid, level surface of the gravitational potential) are worthy of consideration as well.

6.1.2. Magnetization issues

Magnetic sources localization from observations of the field away from the support of the magnetization is an issue under investigation in a joint effort with the Math. department of Vanderbilt University and the Earth Sciences department at MIT. The goal is to recover the magnetic properties of rock samples (*e.g.* meteorites or stalactites) from fine field measurements close to the sample that can nowadays be obtained using SQUIDs (supraconducting coil devices).

The magnetization operator is the Riesz potential of the divergence of the magnetization. The problem of recovering a thin plate magnetization distribution from measurements of the field in a plane above the sample lead us to an analysis of the kernel of this operator, which we characterized in various function and distribution spaces (arbitrary compactly supported distributions or derivatives of bounded functions). For this purpose, we introduced a generalization of the Hodge decomposition in terms of Riesz transforms and showed that a thin plate magnetization is "silent" (i.e. in the kernel) if the normal component is zero and the tangential component is divergence free. In particular, we show that a unidirectional non-trivial magnetization with compact support cannot be silent. The same is true for bidirectional magnetizations if at least one of the directions is nontangential. We also proved that any magnetization is equivalent to a unidirectional. We did introduce notions of being silent from above and silent from below, which are in general distinct. These results have been reported in a paper to appear [37].

We currently work on Fourier based inversion techniques for unidirectional magnetizations, and Figures 5, 6, 7 and 8 show an example of reconstruction. A joint paper with our collaborators from VU and MIT is being written on this topic.

For more general magnetizations, the severe ill-posedness of reconstruction challenges discrete Fourier methods, one of the main problems being the truncation of the observations outside the range of the SQUID measurements. We look forward to develop extrapolation techniques in the spirit of step 1 in section 3.1.

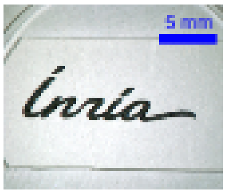

Figure 5. Inria's logo were printed on a piece of paper. The ink of the letters "In" were magnetized along a direction D_1. The ink of the letters "ria" were magnetized along another direction D_2 (almost orthogonal to D_1).

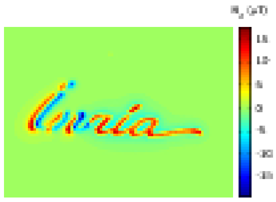

Figure 6. The Z-component of the magnetic field generated by the sample is measured by a SQUID microscope. The measure is performed 200µm above the sample.

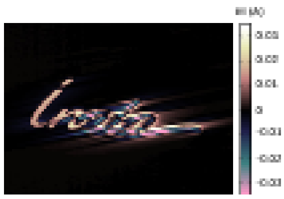

Figure 7. The field measured in Figure 6 is inversed, assuming that the sample is unidimensionally magnetized along the direction D_1. The letters "In" are fairly well recovered while the rest of the letters is blurred (because the hypothesis about the direction of magnetization is false for "ria").

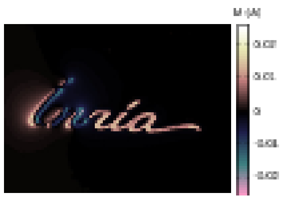

Figure 8. The field measured in Figure 6 is inversed, assuming that the sample is unidimensionally magnetized along the direction D_2. The letters "ria" are fairly well recovered while the rest of the letters is blured (because the hypothesis about the direction of magnetization is false for "In").

6.2. Boundary value problems, generalized Hardy classes

Participants: Laurent Baratchart, Slah Chaabi, Juliette Leblond, Dmitry Ponomarev.

This work has been performed in collaboration with Yannick Fischer from the Magique3D EPI (Inria Bordeaux, Pau).

In collaboration with the CMI-LATP (University Aix-Marseille I), the team considers 2-D diffusion processes with variable conductivity. In particular its complexified version, the so-called *conjugate* or *real Beltrami equation*, was investigated. In the case of a smooth domain, and for Lipschitz conductivity, we analyzed the Dirichlet problem for solutions in Sobolev and then in Hardy classes [5].

Their traces merely lie in L^p $(1 < p < \infty)$ of the boundary, a space which is suitable for identification from pointwise measurements. Again these traces turn out to be dense on strict subsets of the boundary. This allows us to state Cauchy problems as bounded extremal issues in L^p classes of generalized analytic functions, in a reminiscent manner of what was done for analytic functions as discussed in section 3.3.1.

We generalized the construction to finitely connected Dini-smooth domains and $W^{1,q}$-smooth conductivities, with $q > 2$ [35]. The case of an annular geometry is the relevant one for the application to plasma shaping mentioned below [58], [35]. The application that initially motivated this work came from free boundary problems in plasma confinement (in tokamaks) for thermonuclear fusion. This work was initiated in collaboration with the Laboratoire J. Dieudonné (University of Nice).

In the transversal section of a tokamak (which is a disk if the vessel is idealized into a torus), the so-called poloidal flux is subject to some conductivity equation outside the plasma volume for some simple explicit smooth conductivity function, while the boundary of the plasma (in the Tore Supra tokamak) is a level line of this flux [54]. Related magnetic measurements are available on the chamber, which furnish incomplete boundary data from which one wants to recover the inner (plasma) boundary. This free boundary problem (of Bernoulli type) can be handled through the solutions of a family of bounded extremal problems in generalized Hardy classes of solutions to real Beltrami equations, in the annular framework [35].

In the particular case at hand, the conductivity is $1/x$ and the domain is an annulus embedded in the right half-plane. We obtained a basis of solutions (exponentials times Legendre functions) upon separating variables in toroidal coordinates. This provides a computational setting to solve the extremal problems mentioned before, and was the topic of the PhD thesis of Y. Fischer [58], [27]. In the most recent tokamaks, like Jet or ITER, an interesting feature of the level curves of the poloidal flux is the occurrence of a cusp (a saddle point of the poloidal flux, called an X point), and it is desirable to shape the plasma according to a level line passing through this X point for physical reasons related to the efficiency of the energy transfer. We established well-posedness of the Dirichlet problem in weighted L^p classes for harmonic measure on piecewise smooth domains without cusps, thereby laying ground for such a study. This issue is next in line, now that the present approach has been validated numerically on Tore Supra data, and the topic of the PhD thesis of D. Ponomarev.

The PhD work of S. Chaabi is devoted to further aspects of Dirichlet problems for the conjugate Beltrami equation. On the one hand, a method based on Foka's approach to boundary value problems, which uses Lax pairs and solves for a Riemann-Hilbert problem, has been devised to compute in semi explicit form solutions to Dirichlet and Neumann problems for the conductivity equation satisfied by the poloidal flux. Also, for more general conductivities, namely bounded below and lying in $W^{1,s}$ with $s \geq 2$, parameterization of solutions to Dirichlet problems on the disk by Hardy function was achieved through Bers-Nirenberg factorization. Note the conductivity may be unbounded when $s = 2$, which is completely new. Two papers are being prepared reporting on these topics.

Finally, note that the conductivity equation can be expressed like a static Schrödinger equation, for smooth enough conductivity coefficients. This provides a link with the following results recently set up by D. Ponomarev, who recently join the team for his PhD. A description of laser beam propagation in photopolymers can be crudely approximated by a stationary two-dimensional model of wave propagation in a medium with negligible change of refractive index. In such setting, Helmholtz equation is approximated by a linear Schrödinger equation with one of spatial coordinates being an evolutionary variable. Explicit comparison of the solutions in the whole half-space allows to establish global justification of the Schrodinger model for sufficiently smooth pulses [73]. This phenomenon can also be described by a nonstationary model that relies on the spatial nonlinear Schrödinger (NLS) equation with the time-dependent refractive index. A toy problem is considered in [71], when the rate of change of refractive index is proportional to the squared amplitude of the electric field and the spatial domain is a plane. The NLS approximation is derived from a 2-D quasi-linear wave equation, for small time intervals and smooth initial data. Numerical simulations illustrate the approximation result in the 1-D case.

6.3. Circuit realisations of filter responses: determination of canonical forms and exhaustive computations of constrained realisations

Participant: Fabien Seyfert.

This work has been done in collaboration with Smain Amari (Royal Military College, Kingston, Canada), Jean Charles Faugère (SALSA EPI, Inria Rocquencourt), Giuseppe Macchiarella (Politecnico di Milano, Milan, Italy), Uwe Rosenberg (Design and Project Engineering, Osterholz-Scharmbeck, Germany) and Matteo Oldoni (Politecnico di Milano, Milan, Italy).

We continued our work on the circuit realizations of filters' responses with mixed type (inductive or capacitive) coupling elements and constrained topologies [1]. For inline circuits, methods based on sequential extractions of electrical elements are best suited due to their computational simplicity. On the other hand, for circuits with no inline topology ,such methods are inefficient while algebraic methods (based on a Groebner basis) can be used, but at high computational cost. In order to tackle large order circuits, our approach is to decompose them into connected inline sections, which can be directly realized by extraction techniques, and into complex sections, where algebraic methods are needed for realization. In order to do this, we started studying the synthesis of filter responses by means of circuits with reactive non-resonating nodes (dangling resonators) [22]. Links of this topic with Potapov's factorization of J-inner functions are currently being investigated.

In this connection, sensitivity analysis of the electrical response of a filter with respect to the electrical parameters of the underlying circuit has been published in collaboration with the University of Cartagena and ESA [20]. We essentially proved that the total electrical sensitivity of a filters' response does not depend on the coupling topology of the underlying circuit: the latter however controls the distribution of this sensitivity within each resonator.

6.4. Synthesis of compact multiplexers and de-embedding of multiplexers

Participants: Martine Olivi, Sanda Lefteriu, Fabien Seyfert.

This work has been done in collaboration with Stéphane Bila (Xlim, Limoges, France), Hussein Ezzedin (Xlim, Limoges, France), Damien Pacaud (Thales Alenia Space, Toulouse, France), Giuseppe Macchiarella (Politecnico di Milano, Milan, Italy, and Matteo Oldoni (Politecnico di Milano, Milan, Italy).

6.4.1. Synthesis of compact multiplexers

We focused our research on multiplexer with a star topology. These are comprised of a central N-port junction, and of filters plugged on all but common ports (see Figure 9). A possible approach to synthesis of the multiplexer's response is to postulate that each filter channel has to match the multiplexer at n_k frequencies (n_k being the order of the filter) while rejecting the energy at m_k other frequencies (m_k being the order the transmission polynomial of the filter). The desired synthesis can then be cast into computing of a collection of filter's responses matching the energy as prescribed and rejecting it at specified frequencies when plugged simultaneously on the junction. Whether such a collection exists is one of the main open issues facing co-integration of systems in electronics. Investigating the latter led us to consider the simpler problem of matching a filter, on a frequency-varying load, while rejecting energy at fixed specified frequencies. If the order of the filter is n this amounts to fix a given transmission polynomial r and to solve for a unitary polynomial p meeting interpolation conditions of the form:

$$j = 1..n, \qquad \frac{p}{q}(w_j) = \gamma_j$$

where q is the unique monic Hurwitz polynomial satisfying the Feldtkeller equation

$$qq^* = pp^* + rr^*.$$

This problem can be seen as an extended Nevanlinna-Pick interpolation problem, which was considered in [62] when the interpolation frequencies lie in the *open* left half-plane. We conjecture that existence and uniqueness of the solution still holds in our case, where interpolation takes place on the boundary, provided r has no roots on the imaginary axis. Numerical experiments based on continuation techniques tend to corroborate our belief: efforts now focus on a mathematical proof. The derived numerical tools have already been used to successfully to design multiplexer's responses in collaboration with CNES and Xlim, thereby initiating a collaboration with Xlim on co-integration of filters and antennas.

6.4.2. De-embedding of multiplexers

Let S be the measured scattering matrix of a multiplexer composed of a N-port junction with response T and $N-1$ filters with responses $F_1, \cdots F_{N-1}$ as plotted on Figure 9. The de-embedding question we raise is the following: given S and T, is it possible to retrieve the F_k's ? The answer to this question depends of course of the admissible class of filters. For the simplest case where no assumption is made (except reciprocity), we showed that the problem has a unique solution for $N > 3$ and for generic T, while for $N = 2$ the solution space at each frequency point has real dimension 2. This redundancy can be explained by the existence of "ghost" or "silent" components that can hide behind the junction: when being chained to the junction these components do not affect its response. We also experienced that the generic behaviour for $N > 3$ is rather theoretical, as usual junctions are often made of chained T-junctions: in this non generic case (which is rather generic in practice !) some "silent" component still exists for $N > 3$. Additional hypotheses, such as rationality with prescribed degree for F_k, are currently being studied and already yielded results for the case $N = 3$ [21].

This work is pursued in collaboration with Thales Alenia Space, Politecnico di Milano, Xlim and CNES in particular within the contract CNES-Inria on compact N-port synthesis (see section 7.1).

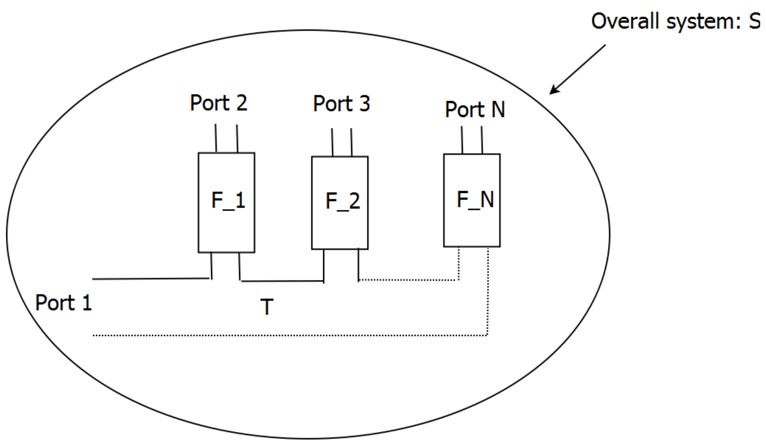

Figure 9. Multiplexer made of a junction T and filtering devices $F_1, F_2 \cdots F_N$

6.5. Detection of the instability of amplifiers

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

This work is conducted in collaboration with Jean-Baptiste Pomet from the McTao team. It is a continuation of a collaboration with CNES and the University of Bilbao.The goal is to help developing amplifiers, in particular to detect instability at an early stage of the design.

Currently, Electrical Engineers from the University of Bilbao, under contract with CNES (the French Space Agency), use heuristics to diagnose instability before the circuit is physically implemented. We intend to set up a rigorously founded algorithm, based on properties of transfer functions of such amplifiers which belong to particular classes of analytic functions.

In non-degenerate cases, non-linear electrical components can be replaced by their first order approximation when studying stability to small perturbations. Using this approximation, diodes appear as perfect negative resistors and transistors as perfect current sources controlled by the voltages at certain points of the circuit.

In 2011, we had proved that the class of transfer functions which can be realized with such ideal components and standard passive components (resistors, selfs, capacitors and transmission lines) is rather large since it contains all rational functions in the variable and in the exponentials thereof.

In 2012, we focused on the kind of instabilities that these ideal systems can exhibit. We showed that a circuit can be unstable, although it has no pole in the right half-plane. This remains true even if a high resistor is put in parallel of the circuit, which is rather unusual. This pathological example is unrealistic, though, because it assumes that non-linear elements continue to provide gain even at very high frequencies. In practice, small capacitive and inductive effects (negligible at moderate frequencies) make these components passive for very high frequencies. Under this simple assumption, we proved that the class of transfer functions of realistic circuits is much smaller than in previous situation. In fact, a realistic circuit is unstable if and only if it has poles in the right half-plane. Moreover, there can only be finitely many of them. An article is currently being written on the subject.

6.6. Best constrained analytic approximation

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Dmitry Ponomarev, Elodie Pozzi.

This work is performed in collaboration with Jonathan Partington (Univ. Leeds, UK).

Continuing effort is being paid by the team to carry over the solution to bounded extremal problems of section 3.3.1 to various settings. We mentioned already in section 6.2 the extension to 2-D diffusion equations with variable conductivity for the determination of free boundaries in plasma control and the development of a generalized Hardy class theory. We also investigate the ordinary Laplacian in $\mathbb{R}^3$, where targeted applications are to data transmission step for source detection in electro/magneto-encephalography (EEG/MEG, see section 6.1).

Still, questions about the behaviour of solutions to the standard bounded extremal problems (P) of section 3.3.1 deserve attention. We realized this year that Slepian functions are eigenfunctions of truncated Toeplitz operators in 2-D. This can be used to quantify robustness properties of our resolution schemes in H^2 and to establish error estimates, see [25]. Moreover we considered additional interpolation constraints [28], as a simplified but already interesting issue, before getting at extremal problems for generalized analytic functions in annular non-smooth domains. The latter arise in the context of plasma shaping in tokamaks like ITER, and will be the subject of the PhD thesis of D. Ponomarev.

In another connection, weighted composition operators on Lebesgue, Sobolev, and Hardy spaces appear in changes of variables while expressing conformal equivalence of plane domains. A universality property related to the existence of invariant subspaces for these important classes of operators has been established in [19]. Additional density properties also allow one to handle some of their dynamical aspects (like cyclicity).

6.7. Rational Approximation for fitting Non-Negative EPT densities

Participants: Martine Olivi, Fabien Seyfert.

This work has been done in collaboration with Bernard Hanzon and Conor Sexton from Univ. Cork.

The problem is to fit a probability density function on a large set of financial data. The model class is the set of non-negative EPT (Exponential-Polynomials-Trigonometric) functions which provides a useful framework for probabilistic calculation as illustrated in the link http://www.2-ept.com/2-ept-literature.html. Moreover, an EPT function can alternatively be interpreted as the impulse response of a continuous time stable system whose Laplace transform is a rational transfer function. This interpretation allows us to approach this problem using approximation tools developed by the team. The very context brings up a classical, as yet essentially unsolved difficulty in rational approximation, namely preservation of positivity. This is known to be a hard issue. Our work, initiated in 2011, resulted this year in an improved approach for checking non-negativity of an EPT function. These results have been presented at the 16th IFAC Conference on System Identification [23]. The proposed method was demonstrated on the positive daily Dow Jones Industrial Average (DJIA) log returns over 80 years.

6.8. Rational and meromorphic approximation

Participant: Laurent Baratchart.

This work has been done in collaboration with Herbert Stahl (TFH Berlin) and Maxim Yattselev (Univ. Oregon at Eugene, USA).

We completed and published this year the proof of an important result in approximation theory, namely the counting measure of poles of best H^2 approximants (more generally: of critical points) of degree n to a function analytically continuable, except over finitely many branchpoints lying outside the unit disk, converges to the Green equilibrium distribution of the compact set of minimal Green capacity outside of which the function is single valued [14]. The proof requires showing existence and uniqueness of a compact set of minimal weighted logarithmic capacity in a field, outside of which the function is single-valued. Structure of this contour, along with error estimates, also come out of the proof. The result is in fact valid for functions that are Cauchy integrals of Dini-smooth functions on such a contour. We rely in addition on asymptotic interpolation estimates from [63].

This result warrants source recovery techniques used in section 6.1.1.

We also studied partial realizations, or equivalently Padé approximants to transfer functions with branchpoints. Identification techniques based on partial realizations of a stable infinite-dimensional transfer function are known to often provide unstable models, but the question as to whether this is due to noise or to intrinsic instability was not clear. In the case of 4 branchpoints, expressing the computation of Padé approximants in terms of the solution to a Riemann-Hilbert problem on the Riemann surface of the function, we proved that the pole behaviour generically shows deterministic chaos [49].

6.9. Tools for numerically guaranteed computations

Participant: Sylvain Chevillard.

The overall and long-term goal is to enhance the quality of numerical computations. The progress made during year 2012 is the following:

- Publication of a work about the implementation of functions erf and erfc in multiple precision and with correct rounding [15]. It corresponds to a work initially begun in the Arénaire team and finished in the Caramel team. The goal of this work is to show on a representative example the different steps of the rigorous implementation of a function in multiple precision arithmetic (choice of a series approximating the function, choice of the truncation rank and working precision used for the computation, roundoff analysis, etc.). The steps are described in such a way that they can easily be reproduced by someone who would like to implement another function. Moreover, it is showed that the process is very regular, which suggests that it (or at least large parts of it) could be automated.

- In the same field of multiple precision arithmetic, and with Marc Mezzarobba (Aric team), we proposed an algorithm for the efficient evaluation of the Airy $\mathrm{Ai}(x)$ function when x is moderately large [57]. Again, this work deals with a representative example, with the idea of trying to automate the process as a future work. The Taylor series of the Airy Ai function (as many others such as, e.g., Bessel functions or erf) is ill-conditioned when x is not small. To overcome this difficulty, we extend a method by Gawronski, Müller and Reinhard, known to solve the issue in the case of the error function erf. We rewrite $\mathrm{Ai}(x)$ as $G(x)/F(x)$ where F and G are two functions with well-conditioned series. However, the coefficients of G turn out to obey a three-terms ill-conditioned recurrence. We evaluate this recurrence using Miller's backward algorithm with a rigorous error analysis.
- Finally, a more general endeavor is to develop a tool that helps developers of libms in their task. This is performed by the software Sollya [2], developed in collaboration with C. Lauter (Université Pierre et Marie Curie) and M. Joldeş (Uppsala University). During year 2012, a large effort has been made in view of the release of version 4.0 (to come in 2013). This effort (of about 400 commits in the svn repository of the project) is mainly intended to provide a library version of Sollya, as well as a robust test suite for the tool. As a matter of course, it allowed us to detect and fix a dozen of bugs.

7. Bilateral Contracts and Grants with Industry

7.1. Contract CNES-Inria-Xlim

Contract (reference Inria: 7066, CNES: 127 197/00) involving CNES, XLim and Inria, focuses on the development of synthesis procedures for N-ports microwave devices. The objective is here to derive analytical procedures for the design of multiplexers and routers as opposed to the classical "black box optimization" which is usually employed in this field (for $N \geq 3$).

7.2. Contract CNES-Inria-UPV/EHU

Contract (reference CNES: RS10/TG-0001-019) involving CNES, University of Bilbao (UPV/EHU) and Inria whose objective is to set up a methodology for testing the stability of amplifying devices. The work at Inria concerns the design of frequency optimization techniques to identify the linearized response and analyze the linear periodic components.

8. Partnerships and Cooperations

8.1. European Initiatives

8.1.1. Collaborations with Major European Organizations

APICS is part of the European Research Network on System Identification (ERNSI) since 1992.

Subject: System identification concerns the construction, estimation and validation of mathematical models of dynamical physical or engineering phenomena from experimental data.

8.2. International Initiatives

8.2.1. Inria International Partners

LMS grant, support of collaborative research with Leeds Univ., U.K., School of Mathematics (no. 41130, 2012).

[2] http://sollya.gforge.inria.fr/

PHC Utique CMCU, cooperation France-Tunisia (no. 10G 1503, led by Univ. Orléans, MAPMO).

NSF CMG collaborative research grant DMS/0934630, "Imaging magnetization distributions in geological samples", with Vanderbilt University and the MIT (USA).

Cyprus NF grant "Orthogonal polynomials in the complex plane: distribution of zeros, strong asymptotics and shape reconstruction."

8.3. International Research Visitors

8.3.1. Visits of International Scientists

- Smain Amari (RMC Ontario).
- Bernard Hanzon (Univ. Cork, External Collaborator).
- Tahar Moumni (Univ. Bizerte, Tunisia).
- Jonathan R. Partington (Univ. Leeds, U.K., External Collaborator).
- Vladimir Peller (Michigan state Univ. at East Lansing)
- Yves Rolain (Vrije Universiteit Brussels).
- Nikos Stylianopoulos (Univ. of Cyprus).

8.3.2. Internships

Shubham KUMAR (from May 2012 until Sep 2012)

Subject: Mathematical methods for multiplexers study

Institution: IIT Delhi (India)

Dmitry Ponomarev (from Jun 2012 until Aug 2012)

Subject: Constrained optimization with prescribed values on the disk

Pre-doctoral trainee

Rahul PRAKASH (from May 2012 until Sep 2012)

Subject: Mathematical methods for multiplexers study

Institution: IIT Delhi (India)

Xuan Zhang (from May 2012 until Sep 2012)

Subject: Groebner basis methods for multiplexers study

Institution: Polytech'Nice

Jie Zhou (from May 2012 until Aug 2012)

Subject: A Hardy-Hodge Decomposition on the 2D Sphere

Institution: Ecole des Mines de Nancy

8.4. External collaborators of the team

The following people are external collaborators of the team:

- Smain Amari [RMC (Royal Military College), Kingston, Canada, since October].
- Ben Hanzon [Univ. Cork, Ireland, since October].
- Mohamed Jaoua [French Univ. of Egypt].
- Jean-Paul Marmorat [Centre de mathématiques appliquées (CMA), École des Mines de Paris].
- Jonathan Partington [Univ. Leeds, UK].
- Edward Saff [Vanderbilt University, Nashville, USA].

9. Dissemination

9.1. Scientific Animation

L. Baratchart, S. Chevillard and J. Leblond gave communications at the Workshop on Inverse Magnetization Problems, Nashville, USA (Apr.).

L. Baratchart and J. Leblond gave communications at PICOF, Conference Problèmes Inverses, Contrôle et Optimisation de Formes, Palaiseau, France (Apr.).

L. Baratchart gave invited talks at the Workshop on Potential Theory and Applications, Szeged, Hungary (June), and at SIGMA 2012, CIRM-Luminy (Nov). He gave a talk at the Conférence en l'honneur de Gauthier Sallet, Saint Louis du Sénégal (Dec.). He was a colloquim speaker at the State University of New York, Albany, USA (October) and at the University of Oregon, USA (Oct.).

S. Chevillard gave a talk at the ERNSI 2012 conference in Maastricht (Netherlands). He reviewed an article for the Journal of Symbolic Computation.

J. Leblond was invited to give a talk at the following conferences: Conference Control & Inverse Problems for PDE (CIPPDE), Santiago, Chili (Jan.), Workshop Control of Fluid-Structure Systems & Inverse Problems, Toulouse, France (Jun.), International Conference on Constructive Complex Approximation, Lille, France (Jun.), Joint Congress of the French & Vietnamese Math. Soc. (VMS-SMF), Hué, Vietnam (Aug.), Congress on Numerical MEthods & MOdelisation (MEMO), Tunis, Tunisie (Dec.). She also gave communications at the seminars of the School of Mathematics, Univ. Leeds, U.K. (Feb.), of the Institut de Mathématiques de Bordeaux (IMB, Univ. Bordeaux, Mar.), of the Department of Math. & Geosciences, Univ. Trieste, Italy (Oct.), and at the 2nd Nice Physical Day ("Journées de la Physique de Nice"), Nice (Dec.).

M. Olivi was co-organizer (with B. Hanzon and R. Peeters) of an invited session "model reduction/approximation" at the 16th IFAC Symposium on System Identification, Brussels, July 2012.

D. Ponomarev presented a poster [27] at the 2nd PhD Event in Fusion Science and Engineering, Pont-a-Mousson (Oct.).

E. Pozzi gave several communications at seminars at Univ. of Besançon, Grenoble (Jan.), Bordeaux, Orléans (Mar.), Marseille, Lille (Apr.)

F. Seyfert was invited to give a talk at the European Microwave Week 2012, Workshop on Advances of N-port networks for Space Application, Amsterdam, Netherlands.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Licence: E. Pozzi, Numerical algorithmics, 26h ETD (from Sep.), L3, Computer Sciences, Polytech'Nice, Univ. Nice Sopia Antipolis, France.

9.2.2. Supervision

PhD: A.-M. Nicu, Approximation et représentation des fonctions sur la sphère. Applications aux problèmes inverses de la géodésie et l'imagerie médicale [13]. Univ. Nice Sophia Antipolis, ED STIC, Feb. 2012 (advisor: J. Leblond).

PhD in progress: S. Chaabi, Boundary value problems for pseudo-holomorphic functions, since Nov. 2008 (advisors: L. Baratchart and A. Borichev).

PhD in progress: D. Ponomarev, Inverse problems for planar conductivity and Schrödinger PDEs, since Nov. 2012 (advisors: J. Leblond, L. Baratchart).

9.2.3. Juries

- J. Leblond (advisor) and M. Olivi (examinator) were members of the PhD jury of A.-M. Nicu (Univ. Nice Sophia Antipolis, Feb.) [13].
- J. Leblond was a member (reviewer) of the PhD jury of N. Chaulet (Ecole Polytechnique, Nov.).
- L. Baratchart was the head of the PhD jury of Matteo Santacesaria (Ecole Polytechnique, Dec.).

9.3. Popularization

- M. Olivi is co-president with I. Castellani of the Committee MASTIC (Commission d'Animation, de Médiation et d'Animation Scientifique) https://project.inria.fr/mastic/. She is responsible for Scientific Mediation.
- J. Leblond and E. Pozzi are members of this committee.
- E. Pozzi participated to the "Filles et mathématiques" day, Avignon, Nov.

9.4. Community services

L. Baratchart is a member of the Editorial Boards of *Constructive Methods and Function Theory* and *Complex Analysis and Operator Theory*. He is Inria's representative at the « conseil scientifique » of the Univ. Provence (Aix-Marseille).

S. Chevillard is representative at the « comité de centre » and at the « comité des projets » (Research Center Inria-Sophia).

J. Leblond is an elected member of the "Conseil Scientifique" of Inria. Together with C. Calvet from Human Resources, she is in charge of the mission "Conseil et soutien aux chercheurs" within the Research Centre, and she participated to the working group BEAT ("Bien Être Au Travail").

M. Olivi is a member of the CSD (Comité de Suivi Doctoral) of the Research Center. She is responsible for scientific mediation.

F. Seyfert is a member of CUMIR at InRIA Sophia-Antipolis-Méditerrannée.

10. Bibliography

Major publications by the team in recent years

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