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Activity Report 2012

# Project-Team ASPI

## Applications of interacting particle systems to statistics

IN COLLABORATION WITH: Institut de recherche mathématique de Rennes (IRMAR)

RESEARCH CENTER
**Rennes - Bretagne-Atlantique**

THEME
**Stochastic Methods and Models**

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# Project-Team ASPI

**Keywords:** Monte Carlo Methods, Markovian Model, Rare Events, Particle Filtering, Tracking, Data Assimilation

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# 2. Overall Objectives

## 2.1. Overall Objectives

The scientific objectives of ASPI are the design, analysis and implementation of interacting Monte Carlo methods, also known as particle methods, with focus on

- statistical inference in hidden Markov models and particle filtering,
- risk evaluation and simulation of rare events,
- global optimization.

The whole problematic is multidisciplinary, not only because of the many scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have already contributed to establish the foundations of the field

> target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods, etc.

Intuitively speaking, interacting Monte Carlo methods are sequential simulation methods, in which particles

- *explore* the state space by mimicking the evolution of an underlying random process,
- *learn* the environment by evaluating a fitness function,
- and *interact* so that only the most successful particles (in view of the value of the fitness function) are allowed to survive and to get offsprings at the next generation.

The effect of this mutation / selection mechanism is to automatically concentrate particles (i.e. the available computing power) in regions of interest of the state space. In the special case of particle filtering, which has numerous applications under the generic heading of positioning, navigation and tracking, in

> target tracking, computer vision, mobile robotics, wireless communications, ubiquitous computing and ambient intelligence, sensor networks, etc.,

each particle represents a possible hidden state, and is multiplied or terminated at the next generation on the basis of its consistency with the current observation, as quantified by the likelihood function. With these genetic–type algorithms, it becomes easy to efficiently combine a prior model of displacement with or without constraints, sensor–based measurements, and a base of reference measurements, for example in the form of a digital map (digital elevation map, attenuation map, etc.). In the most general case, particle methods provide approximations of Feynman–Kac distributions, a pathwise generalization of Gibbs–Boltzmann distributions, by means of the weighted empirical probability distribution associated with an interacting particle system, with applications that go far beyond filtering, in

> simulation of rare events, simulation of conditioned or constrained random variables, interacting MCMC methods, molecular simulation, etc.

The main applications currently considered are geolocalisation and tracking of mobile terminals, terrain–aided navigation, data fusion for indoor localisation, optimization of sensors location and activation, risk assessment in air traffic management, protection of digital documents.

# 3. Scientific Foundations

## 3.1. Interacting Monte Carlo methods and particle approximation of Feynman–Kac distributions

Monte Carlo methods are numerical methods that are widely used in situations where (i) a stochastic (usually Markovian) model is given for some underlying process, and (ii) some quantity of interest should be evaluated, that can be expressed in terms of the expected value of a functional of the process trajectory, which includes as an important special case the probability that a given event has occurred. Numerous examples can be found, e.g. in financial engineering (pricing of options and derivative securities) [50], in performance evaluation of communication networks (probability of buffer overflow), in statistics of hidden Markov models (state estimation, evaluation of contrast and score functions), etc. Very often in practice, no analytical expression is available for the quantity of interest, but it is possible to simulate trajectories of the underlying process. The idea behind Monte Carlo methods is to generate independent trajectories of this process or of an alternate instrumental process, and to build an approximation (estimator) of the quantity of interest in terms of the weighted empirical probability distribution associated with the resulting independent sample. By the law of large numbers, the above estimator converges as the size $N$ of the sample goes to infinity, with rate $1/\sqrt{N}$ and the asymptotic variance can be estimated using an appropriate central limit theorem. To reduce the variance of the estimator, many variance reduction techniques have been proposed. Still, running independent Monte Carlo simulations can lead to very poor results, because trajectories are generated *blindly*, and only afterwards are the corresponding weights evaluated. Some of the weights can happen to be negligible, in which case the corresponding trajectories are not going to contribute to the estimator, i.e. computing power has been wasted.

A recent and major breakthrough, has been the introduction of interacting Monte Carlo methods, also known as sequential Monte Carlo (SMC) methods, in which a whole (possibly weighted) sample, called *system of particles*, is propagated in time, where the particles

- *explore* the state space under the effect of a *mutation* mechanism which mimics the evolution of the underlying process,
- and are *replicated* or *terminated*, under the effect of a *selection* mechanism which automatically concentrates the particles, i.e. the available computing power, into regions of interest of the state space.

In full generality, the underlying process is a discrete–time Markov chain, whose state space can be

> finite, continuous, hybrid (continuous / discrete), graphical, constrained, time varying, pathwise, etc.,

the only condition being that it can easily be *simulated*.

In the special case of particle filtering, originally developed within the tracking community, the algorithms yield a numerical approximation of the optimal Bayesian filter, i.e. of the conditional probability distribution of the hidden state given the past observations, as a (possibly weighted) empirical probability distribution of the system of particles. In its simplest version, introduced in several different scientific communities under the name of *bootstrap filter* [52], *Monte Carlo filter* [57] or *condensation* (conditional density propagation) algorithm [54], and which historically has been the first algorithm to include a redistribution step, the selection mechanism is governed by the likelihood function: at each time step, a particle is more likely to survive and to replicate at the next generation if it is consistent with the current observation. The algorithms also provide as a by–product a numerical approximation of the likelihood function, and of many other contrast functions for parameter estimation in hidden Markov models, such as the prediction error or the conditional least–squares criterion.

Particle methods are currently being used in many scientific and engineering areas

> positioning, navigation, and tracking [53], [47], visual tracking [54], mobile robotics [48], [71], ubiquitous computing and ambient intelligence, sensor networks, risk evaluation and simulation of rare events [51], genetics, molecular simulation [49], etc.

Other examples of the many applications of particle filtering can be found in the contributed volume [34] and in the special issue of *IEEE Transactions on Signal Processing* devoted to *Monte Carlo Methods for Statistical Signal Processing* in February 2002, where the tutorial paper [35] can be found, and in the textbook [67] devoted to applications in target tracking. Applications of sequential Monte Carlo methods to other areas, beyond signal and image processing, e.g. to genetics, can be found in [66]. A recent overview can also be found in [37].

Particle methods are very easy to implement, since it is sufficient in principle to simulate independent trajectories of the underlying process. The whole problematic is multidisciplinary, not only because of the already mentioned diversity of the scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have contributed to establish the foundations of the field

> target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods.

These algorithms can be interpreted as numerical approximation schemes for Feynman–Kac distributions, a pathwise generalization of Gibbs–Boltzmann distributions, in terms of the weighted empirical probability distribution associated with a system of particles. This abstract point of view [42], [40], has proved to be extremely fruitful in providing a very general framework to the design and analysis of numerical approximation schemes, based on systems of branching and / or interacting particles, for nonlinear dynamical systems with values in the space of probability distributions, associated with Feynman–Kac distributions. Many asymptotic results have been proved as the number $N$ of particles (sample size) goes to infinity, using techniques coming from applied probability (interacting particle systems, empirical processes [74]), see e.g. the survey article [42] or the recent textbook [40], and references therein

> convergence in $L^p$, convergence as empirical processes indexed by classes of functions, uniform convergence in time, see also [63], [64], central limit theorem, see also [59], propagation of chaos, large deviations principle, etc.

The objective here is to systematically study the impact of the many algorithmic variants on the convergence results.

## 3.2. Statistics of HMM

Hidden Markov models (HMM) form a special case of partially observed stochastic dynamical systems, in which the state of a Markov process (in discrete or continuous time, with finite or continuous state space) should be estimated from noisy observations. The conditional probability distribution of the hidden state given past observations is a well–known example of a normalized (nonlinear) Feynman–Kac distribution, see 3.1. These models are very flexible, because of the introduction of latent variables (non observed) which allows to model complex time dependent structures, to take constraints into account, etc. In addition, the underlying Markovian structure makes it possible to use numerical algorithms (particle filtering, Markov chain Monte Carlo methods (MCMC), etc.) which are computationally intensive but whose complexity is rather small. Hidden Markov models are widely used in various applied areas, such as speech recognition, alignment of biological sequences, tracking in complex environment, modeling and control of networks, digital communications, etc.

Beyond the recursive estimation of a hidden state from noisy observations, the problem arises of statistical inference of HMM with general state space [38], including estimation of model parameters, early monitoring and diagnosis of small changes in model parameters, etc.

**Large time asymptotics** A fruitful approach is the asymptotic study, when the observation time increases to infinity, of an extended Markov chain, whose state includes (i) the hidden state, (ii) the observation, (iii) the prediction filter (i.e. the conditional probability distribution of the hidden state given observations at all previous time instants), and possibly (iv) the derivative of the prediction filter with respect to the parameter. Indeed, it is easy to express the log–likelihood function, the conditional least–squares criterion, and many other clasical contrast processes, as well as their derivatives with respect to the parameter, as additive functionals of the extended Markov chain.

The following general approach has been proposed

- first, prove an exponential stability property (i.e. an exponential forgetting property of the initial condition) of the prediction filter and its derivative, for a misspecified model,
- from this, deduce a geometric ergodicity property and the existence of a unique invariant probability distribution for the extended Markov chain, hence a law of large numbers and a central limit theorem for a large class of contrast processes and their derivatives, and a local asymptotic normality property,
- finally, obtain the consistency (i.e. the convergence to the set of minima of the associated contrast function), and the asymptotic normality of a large class of minimum contrast estimators.

This programme has been completed in the case of a finite state space [7], and has been generalized [43] under an uniform minoration assumption for the Markov transition kernel, which typically does only hold when the state space is compact. Clearly, the whole approach relies on the existence of an exponential stability property of the prediction filter, and the main challenge currently is to get rid of this uniform minoration assumption for the Markov transition kernel [41], [64], so as to be able to consider more interesting situations, where the state space is noncompact.

**Small noise asymptotics** Another asymptotic approach can also be used, where it is rather easy to obtain interesting explicit results, in terms close to the language of nonlinear deterministic control theory [58]. Taking the simple example where the hidden state is the solution to an ordinary differential equation, or a nonlinear state model, and where the observations are subject to additive Gaussian white noise, this approach consists in assuming that covariances matrices of the state noise and of the observation noise go simultaneously to zero. If it is reasonable in many applications to consider that noise covariances are small, this asymptotic approach is less natural than the large time asymptotics, where it is enough (provided a suitable ergodicity assumption holds) to accumulate observations and to see the expected limit laws (law of large numbers, central limit theorem, etc.). In opposition, the expressions obtained in the limit (Kullback–Leibler divergence, Fisher information matrix, asymptotic covariance matrix, etc.) take here a much more explicit form than in the large time asymptotics.

The following results have been obtained using this approach

- the consistency of the maximum likelihood estimator (i.e. the convergence to the set $M$ of global minima of the Kullback–Leibler divergence), has been obtained using large deviations techniques, with an analytical approach [55],
- if the abovementioned set $M$ does not reduce to the true parameter value, i.e. if the model is not identifiable, it is still possible to describe precisely the asymptotic behavior of the estimators [56]: in the simple case where the state equation is a noise–free ordinary differential equation and using a Bayesian framework, it has been shown that (i) if the rank $r$ of the Fisher information matrix $I$ is constant in a neighborhood of the set $M$, then this set is a differentiable submanifold of codimension $r$, (ii) the posterior probability distribution of the parameter converges to a random probability distribution in the limit, supported by the manifold $M$, absolutely continuous w.r.t. the Lebesgue measure on $M$, with an explicit expression for the density, and (iii) the posterior probability distribution of the suitably normalized difference between the parameter and its projection on the manifold $M$, converges to a mixture of Gaussian probability distributions on the normal spaces to the manifold $M$, which generalized the usual asymptotic normality property,
- it has been shown [65] that (i) the parameter dependent probability distributions of the observations are locally asymptotically normal (LAN) [61], from which the asymptotic normality of the maximum likelihood estimator follows, with an explicit expression for the asymptotic covariance matrix, i.e. for the Fisher information matrix $I$, in terms of the Kalman filter associated with the linear tangent linear Gaussian model, and (ii) the score function (i.e. the derivative of the log–likelihood function w.r.t. the parameter), evaluated at the true value of the parameter and suitably normalized, converges to a Gaussian r.v. with zero mean and covariance matrix $I$.

## 3.3. Multilevel splitting for rare event simulation

*See 4.2, and 5.1, 5.6, 5.10 and 5.11.*

The estimation of the small probability of a rare but critical event, is a crucial issue in industrial areas such as

> nuclear power plants, food industry, telecommunication networks, finance and insurance industry, air traffic management, etc.

In such complex systems, analytical methods cannot be used, and naive Monte Carlo methods are clearly unefficient to estimate accurately very small probabilities. Besides importance sampling, an alternate widespread technique consists in multilevel splitting [60], where trajectories going towards the critical set are given offsprings, thus increasing the number of trajectories that eventually reach the critical set. As shown in [5], the Feynman–Kac formalism of 3.1 is well suited for the design and analysis of splitting algorithms for rare event simulation.

**Propagation of uncertainty** Multilevel splitting can be used in static situations. Here, the objective is to learn the probability distribution of an output random variable $Y = F(X)$, where the function $F$ is only defined pointwise for instance by a computer programme, and where the probability distribution of the input random variable $X$ is known and easy to simulate from. More specifically, the objective could be to compute the probability of the output random variable exceeding a threshold, or more generally to evaluate the cumulative distribution function of the output random variable for different output values. This problem is characterized by the lack of an analytical expression for the function, the computational cost of a single pointwise evaluation of the function, which means that the number of calls to the function should be limited as much as possible, and finally the complexity and / or unavailability of the source code of the computer programme, which makes any modification very difficult or even impossible, for instance to change the model as in importance sampling methods.

The key issue is to learn as fast as possible regions of the input space which contribute most to the computation of the target quantity. The proposed splitting methods consists in (i) introducing a sequence of intermediate regions in the input space, implicitly defined by exceeding an increasing sequence of thresholds or levels, (ii) counting the fraction of samples that reach a level given that the previous level has been reached already,

and (iii) improving the diversity of the selected samples, usually using an artificial Markovian dynamics. In this way, the algorithm learns

- the transition probability between successive levels, hence the probability of reaching each intermediate level,
- and the probability distribution of the input random variable, conditionned on the output variable reaching each intermediate level.

A further remark, is that this conditional probability distribution is precisely the optimal (zero variance) importance distribution needed to compute the probability of reaching the considered intermediate level.

**Rare event simulation** To be specific, consider a complex dynamical system modelled as a Markov process, whose state can possibly contain continuous components and finite components (mode, regime, etc.), and the objective is to compute the probability, hopefully very small, that a critical region of the state space is reached by the Markov process before a final time $T$, which can be deterministic and fixed, or random (for instance the time of return to a recurrent set, corresponding to a nominal behaviour).

The proposed splitting method consists in (i) introducing a decreasing sequence of intermediate, more and more critical, regions in the state space, (ii) counting the fraction of trajectories that reach an intermediate region before time $T$, given that the previous intermediate region has been reached before time $T$, and (iii) regenerating the population at each stage, through redistribution. In addition to the non–intrusive behaviour of the method, the splitting methods make it possible to learn the probability distribution of typical critical trajectories, which reach the critical region before final time $T$, an important feature that methods based on importance sampling usually miss. Many variants have been proposed, whether

- the branching rate (number of offsprings allocated to a successful trajectory) is fixed, which allows for depth–first exploration of the branching tree, but raises the issue of controlling the population size,
- the population size is fixed, which requires a breadth–first exploration of the branching tree, with random (multinomial) or deterministic allocation of offsprings, etc.

Just as in the static case, the algorithm learns

- the transition probability between successive levels, hence the probability of reaching each intermediate level,
- and the entrance probability distribution of the Markov process in each intermediate region.

Contributions have been given to

- minimizing the asymptotic variance, obtained through a central limit theorem, with respect to the shape of the intermediate regions (selection of the importance function), to the thresholds (levels), to the population size, etc.
- controlling the probability of extinction (when not even one trajectory reaches the next intermediate level),
- designing and studying variants suited for hybrid state space (resampling per mode, marginalization, mode aggregation),

and in the static case, to

- minimizing the asymptotic variance, obtained through a central limit theorem, with respect to intermediate levels, to the Metropolis kernel introduced in the mutation step, etc.

A related issue is global optimization. Indeed, the difficult problem of finding the set $M$ of global minima of a real–valued function $V$ can be replaced by the apparently simpler problem of sampling a population from a probability distribution depending on a small parameter, and asymptotically supported by the set $M$ as the small parameter goes to zero. The usual approach here is to use the cross–entropy method [68], [39], which relies on learning the optimal importance distribution within a prescribed parametric family. On the other hand, multilevel splitting methods could provide an alternate nonparametric approach to this problem.

## 3.4. Nearest neighbor estimates

This additional topic was not present in the initial list of objectives, and has emerged only recently.

In pattern recognition and statistical learning, also known as machine learning, nearest neighbor (NN) algorithms are amongst the simplest but also very powerful algorithms available. Basically, given a training set of data, i.e. an $N$–sample of i.i.d. object–feature pairs, with real–valued features, the question is how to generalize, that is how to guess the feature associated with any new object. To achieve this, one chooses some integer $k$ smaller than $N$, and takes the mean–value of the $k$ features associated with the $k$ objects that are nearest to the new object, for some given metric.

In general, there is no way to guess exactly the value of the feature associated with the new object, and the minimal error that can be done is that of the Bayes estimator, which cannot be computed by lack of knowledge of the distribution of the object–feature pair, but the Bayes estimator can be useful to characterize the strength of the method. So the best that can be expected is that the NN estimator converges, say when the sample size $N$ grows, to the Bayes estimator. This is what has been proved in great generality by Stone [69] for the mean square convergence, provided that the object is a finite–dimensional random variable, the feature is a square–integrable random variable, and the ratio $k/N$ goes to 0. Nearest neighbor estimator is not the only local averaging estimator with this property, but it is arguably the simplest.

The asymptotic behavior when the sample size grows is well understood in finite dimension, but the situation is radically different in general infinite dimensional spaces, when the objects to be classified are functions, images, etc.

**Nearest neighbor classification in infinite dimension** In finite dimension, the $k$–nearest neighbor classifier is universally consistent, i.e. its probability of error converges to the Bayes risk as $N$ goes to infinity, whatever the joint probability distribution of the pair, provided that the ratio $k/N$ goes to zero. Unfortunately, this result is no longer valid in general metric spaces, and the objective is to find out reasonable sufficient conditions for the weak consistency to hold. Even in finite dimension, there are exotic distances such that the nearest neighbor does not even get closer (in the sense of the distance) to the point of interest, and the state space needs to be complete for the metric, which is the first condition. Some regularity on the regression function is required next. Clearly, continuity is too strong because it is not required in finite dimension, and a weaker form of regularity is assumed. The following consistency result has been obtained: if the metric space is separable and if some Besicovich condition holds, then the nearest neighbor classifier is weakly consistent. Note that the Besicovich condition is always fulfilled in finite dimensional vector spaces (this result is called the Besicovich theorem), and that a counterexample [3] can be given in an infinite dimensional space with a Gaussian measure (in this case, the nearest neighbor classifier is clearly nonconsistent). Finally, a simple example has been found which verifies the Besicovich condition with a noncontinuous regression function.

**Rates of convergence of the functional $k$–nearest neighbor estimator** Motivated by a broad range of potential applications, such as regression on curves, rates of convergence of the $k$–nearest neighbor estimator of the regression function, based on $N$ independent copies of the object–feature pair, have been investigated when the object is in a suitable ball in some functional space. Using compact embedding theory, explicit and general finite sample bounds can be obtained for the expected squared difference between the $k$–nearest neighbor estimator and the Bayes regression function, in a very general setting. The results have also been particularized to classical function spaces such as Sobolev spaces, Besov spaces and reproducing kernel Hilbert spaces. The rates obtained are genuine nonparametric convergence rates, and up to our knowledge the first of their kind for $k$–nearest neighbor regression.

This emerging topic has produced several theoretical advances [1], [2] in collaboration with Gérard Biau (université Pierre et Marie Curie, ENS Paris and EPI CLASSIC, Inria Paris—Rocquencourt), and a possible target application domain has been identified in the statistical analysis of recommendation systems, that would be a source of interesting problems.

# 4. Application Domains

## 4.1. Localisation, navigation and tracking

*See 5.9.*

Among the many application domains of particle methods, or interacting Monte Carlo methods, ASPI has decided to focus on applications in localisation (or positioning), navigation and tracking [53], [47], which already covers a very broad spectrum of application domains. The objective here is to estimate the position (and also velocity, attitude, etc.) of a mobile object, from the combination of different sources of information, including

- a prior dynamical model of typical evolutions of the mobile, such as inertial estimates and prior model for inertial errors,
- measurements provided by sensors,
- and possibly a digital map providing some useful feature (terrain altitude, power attenuation, etc.) at each possible position.

In some applications, another useful source of information is provided by

- a map of constrained admissible displacements, for instance in the form of an indoor building map,

which particle methods can easily handle (map-matching). This Bayesian dynamical estimation problem is also called filtering, and its numerical implementation using particle methods, known as particle filtering, has been introduced by the target tracking community [52], [67], which has already contributed to many of the most interesting algorithmic improvements and is still very active, and has found applications in

> target tracking, integrated navigation, points and / or objects tracking in video sequences, mobile robotics, wireless communications, ubiquitous computing and ambient intelligence, sensor networks, etc.

ASPI is contributing (or has contributed recently) to several applications of particle filtering in positioning, navigation and tracking, such as geolocalisation and tracking in a wireless network, terrain–aided navigation, and data fusion for indoor localisation.

## 4.2. Rare event simulation

*See 3.3, and 5.1, 5.6, 5.10 and 5.11.*

Another application domain of particle methods, or interacting Monte Carlo methods, that ASPI has decided to focus on is the estimation of the small probability of a rare but critical event, in complex dynamical systems. This is a crucial issue in industrial areas such as

> nuclear power plants, food industry, telecommunication networks, finance and insurance industry, air traffic management, etc.

In such complex systems, analytical methods cannot be used, and naive Monte Carlo methods are clearly unefficient to estimate accurately very small probabilities. Besides importance sampling, an alternate widespread technique consists in multilevel splitting [60], where trajectories going towards the critical set are given offsprings, thus increasing the number of trajectories that eventually reach the critical set. This approach not only makes it possible to estimate the probability of the rare event, but also provides realizations of the random trajectory, given that it reaches the critical set, i.e. provides realizations of typical critical trajectories, an important feature that methods based on importance sampling usually miss.

ASPI is contributing (or has contributed recently) to several applications of multilevel splitting for rare event simulation, such as risk assessment in air traffic management, detection in sensor networks, and protection of digital documents.

# 5. New Results

## 5.1. On the length of one–dimensional reactive paths

**Participants:** Frédéric Cérou, Arnaud Guyader, Florent Malrieu.

*See 3.3 and 4.2.*

This is a collaboration with Tony Lelièvre (ENPC).

Motivated by some numerical observations on molecular dynamics simulations, we analyze metastable trajectories in a very simple setting, namely paths generated by a one-dimensional overdamped Langevin equation for a double well potential. More precisely, we are interested in so–called reactive paths, namely trajectories which leave definitely one well and reach the other one. The aim of [32] is to precisely analyze the distribution of the lengths of reactive paths in the limit of small temperature, and to compare the theoretical results to numerical results obtained by a Monte Carlo method, namely the multi–level splitting approach.

## 5.2. Long time behavior of piecewise–deterministic Markov processes

**Participant:** Florent Malrieu.

This is a collaboration with Michel Benaïm (université de Neuchâtel), Stéphane Le Borgne (IRMAR) and Pierre–André Zitt (université de Marne–la–Vallée).

### *5.2.1. Quantitative ergodicity for some switched dynamical systems*

We provide quantitative bounds for the long time behavior of a class of piecewise deterministic Markov processes with state space $R^d \times E$ where $E$ is a finite set. The continuous component evolves according to a smooth vector field that switches at the jump times of the discrete coordinate. The jump rates may depend on the whole position of the process. Under regularity assumptions on the jump rates and stability conditions for the vector fields we provide explicit exponential upper bounds for the convergence to equilibrium in terms of Wasserstein distances [13]. As an example, we obtain convergence results for a stochastic version of the Morris–Lecar model of neurobiology.

### *5.2.2. On the stability of planar randomly switched systems*

Consider the random process $(X_t)$ solution of $dX_t/dt = A(I_t)X_t$ where $(I_t)$ is a Markov process on $\{0,1\}$ and $A_0$ and $A_1$ are real Hurwitz matrices on $R^2$. Assuming that there exists $\lambda \in (0,1)$ such that $(1-\lambda)A_0 + \lambda A_1$ has a positive eigenvalue, we establish that the norm of $X_t$ may converge to 0 or infinity, depending on the the jump rate of the process $I$. An application to product of random matrices is studied. The paper [29] can be viewed as a probabilistic counterpart of the paper [36] by Baldé, Boscain and Mason.

### *5.2.3. Qualitative properties of certain piecewise deterministic Markov processes*

We study a class of piecewise deterministic Markov processes with state space $R^m \times E$ where $E$ is a finite set. The continous component evolves according to a smooth vector field that it switched at the jump times of the discrete coordinate. The jump rates may depend on the whole position of the process. Working under the general assumption that the process stays in a compact set, we detail a possible construction of the process and characterize its support, in terms of the solutions set of a differential inclusion. We establish results on the long time behaviour of the process, in relation to a certain set of accessible points, which is shown to be strongly linked to the support of invariant measures. Under Hörmander–type bracket conditions, we prove that there exists a unique invariant measure and that the processes converges to equilibrium in total variation. Finally we give examples where the bracket condition does not hold, and where there may be one or many invariant measures, depending on the jump rates between the flows [30].

## 5.3. Quantitative long time behavior of an ergodic variant of the telegraph process

**Participant:** Florent Malrieu.

This is a collaboration with Joaquin Fontbona (University of Chile) and Hélène Guérin (IRMAR).

Motivated by stability questions on piecewise deterministic Markov models of bacterial chemotaxis, we study the long time behavior of a variant of the classic telegraph process having a non–constant jump rate that induces a drift towards the origin. We compute its invariant law and show exponential ergodicity, obtaining a quantitative control of the total variation distance to equilibrium at each instant of time. These results [15] rely on an exact description of the excursions of the process away from the origin and on the explicit construction of an original coalescent coupling for both velocity and position. Sharpness of the obtained convergence rate is discussed.

## 5.4. Total variation estimates for the TCP process

**Participant:** Florent Malrieu.

This is a collaboration with Jean-Baptiste Bardet (université de Rouen), Alejandra Christen (University of Chile), Arnaud Guillin (université de Clermont–Ferrand), and Pierre–André Zitt (université de Marne–la–Vallée).

The TCP window size process appears in the modeling of the famous Transmission Control Protocol used for data transmission over the Internet. This continuous time Markov process takes its values in $[0, \infty)$, is ergodic and irreversible. The sample paths are piecewise linear deterministic and the whole randomness of the dynamics comes from the jump mechanism. The aim of [28] is to provide quantitative estimates for the exponential convergence to equilibrium, in terms of the total variation and Wasserstein distances.

## 5.5. Convergence results for approximate Bayesian computation

**Participants:** Frédéric Cérou, Arnaud Guyader.

This is a collaboration with Gérard Biau (ENS and université Pierre et Marie Curie).

Approximate Bayesian computation (ABC for short) is a family of computational techniques which offer an almost automated solution in situations where evaluation of the posterior likelihood is computationally prohibitive, or whenever suitable likelihoods are not available. In [31], we analyze the procedure from the point of view of k-nearest neighbor theory and explore the statistical properties of its outputs. We discuss in particular some asymptotic features of the genuine conditional density estimate associated with ABC, which is a new interesting hybrid between a k-nearest neighbor and a kernel method. These are among the very few results on the convergence of ABC, and our assumptions on the underlying probability distribution are minimal.

## 5.6. Soft level splitting for rare event estimation

**Participants:** Frédéric Cérou, Arnaud Guyader.

*See 3.3 and 4.2.*

This is a collaboration with Nicolas Hengartner (Los Alamos).

It is well established now that one can use adaptive splitting levels to compute the conditional probabilities of nested sets. To get an efficient algorithm, the probability of a set given the previous one should be always the same, which is approximately achieved adaptively by using the empirical cdf (cumulative distribution function) of the scores. The way to proceed is to fix a probability of success $p_0$, and then choose the $p_0$ quantile of the current scores. Here we investigate whether, by using the whole cdf, and not only one quantile, we can design an algorithm with better performance. The main trick is a transformation to have a sample of exponential variables. This would require the knowledge of the cdf of the cost, which is obviously unavailable, but we can replace it by the empirical cdf of the sample at the previous level. The complete theoretical study of this algorithm is still to be done, but we have illustrated by some examples that it can lead to significantly better results than the standard splitting procedure with the same number of intermediate levels.

## 5.7. Decoding fingerprints using the Markov chain Monte Carlo method

**Participants:** Frédéric Cérou, Arnaud Guyader.

This is a collaboration with Teddy Furon (Inria Rennes, project–team TEXMEX).

The paper [22] proposes a new fingerprinting decoder based on the Markov chain Monte Carlo (MCMC) method. A Gibbs sampler generates groups of users according to the posterior probability that these users could have forged the sequence extracted from the pirated content. The marginal probability that a given user pertains to the collusion is then estimated by a Monte Carlo method. The users having the biggest empirical marginal probabilities are accused. This MCMC method can decode any type of fingerprinting codes. This paper is in the spirit of the *learn and match* decoding strategy: it assumes that the collusion attack belongs to a family of models. The expectation–maximization algorithm estimates the parameters of the collusion model from the extracted sequence. This part of the algorithm is described for the binary Tardos code and with the exploitation of the soft outputs of the watermarking decoder. The experimental body considers some extreme setups where the fingerprinting code lengths are very small. It reveals that the weak link of our approach is the estimation part. This is a clear warning to the *learn and match* decoding strategy.

## 5.8. Iterative isotone regression

**Participants:** Arnaud Guyader, Nicolas Jégou.

This is a collaboration with Nicolas Hengartner (Los Alamos) and Eric Matzner–Løber (université de Rennes 2), and with Alexander B. Németh (Babeş Bolyai University) and Sándor Z. Németh (University of Birmingham).

The current collaboration on nonparametric regression focuses on a novel nonparametric regression technique that applies ideas borrowed from iterative bias reduction to estimating functions of bounded variations. This work has emerged from the joint supervision of Nicolas Jégou's PhD thesis by Arnaud Guyader, Nick Hengartner and Eric Matzner-Løber.

A geometric approach has been investigated, as an extension of some ideas developed in the thesis. The current work [33] proposes and analyzes a novel method for estimating a univariate regression function of bounded variation. The underpinning idea is to combine two classical tools in nonparametric statistics, namely isotonic regression and the estimation of additive models. A geometrical interpretation enables us to link this iterative method with Von Neumann's algorithm. Moreover, making a connection with the general property of isotonicity of projection onto convex cones, we derive another equivalent algorithm and go further in the analysis. As iterating the algorithm leads to overfitting, several practical stopping criteria are also presented and discussed.

## 5.9. Detection issues in track–before–detect

**Participants:** François Le Gland, Alexandre Lepoutre.

*See 4.1.*

This is a collaboration with Olivier Rabaste (ONERA Palaiseau).

Track–before–detect refers to situations where the target SNR is so low that it is practically impossible to detect the presence of a target, using a simple thresholding rule. In such situations, the solution is to keep all the information available in the raw radar data and to address directly the tracking problem, using a particle filter with a binary Markov variable that models the presence or absence of the target. The choice of the proposal distribution is crucial here, and an efficient particle filter is proposed [24] that is based on a relevant proposal distribution built from detection and estimation considerations, that aims at extracting all the available information from the measurements. The proposed filter leads to a dramatically improved performance as compared with particle filters based on the classical proposal distribution, both in terms of detection and estimation. A further improvement, in terms of detection performance, is to model the problem as a quickest change detection problem [70] in a Bayesian framework. In this context, the posterior distribution of the first time of appearance of the target is a mixture where each component represents the hypothesis that the target appeared at a given time. The posterior distribution is intractable in practice, and it is proposed [23] to approximate each component of the mixture by a particle filter. It turns out that the mixture weights can be computed recursively in terms of quantities that are provided by the different particle filters. The overall filter yields good performance as compared with classical particle filters for track–before–detect.

## 5.10. Estimation of conflict probability

**Participants:** François Le Gland, Damien–Barthélémy Jacquemart.

*See 3.3 and 4.2.*

This is a collaboration with Jérôme Morio (ONERA Palaiseau).

In [16], the conflict probability between aircraft in uncontrolled airspace is estimated using the importance splitting method, and this algorithm is applied on realistic situations of aircraft conflict. The current work aims at designing efficient intermediate regions at a reasonnable computational cost, or alternatively at introducing weights to compensate for a simple but suboptimal design of the intermediate regions.

## 5.11. Minimum volume set for a rare event

**Participants:** François Le Gland, Rudy Pastel.

*See 3.3 and 4.2.*

This is a collaboration with Jérôme Morio (ONERA Palaiseau).

The paper [19] first reviews the principle of minimum volume set estimation of a given probability level for a multidimensional density, a strategy that provides a sound solution to the multidimensional quantile issue. It then describes an importance sampling algorithm that is suitable for this kind of estimation problems, and provides simulation results for the estimation of the impact zone of a space launcher. The current work aims at designing an importance splitting method that would be more efficient for extreme quantiles.

## 5.12. Laplace and sequential Monte Carlo methods in Bayesian filtering

**Participants:** François Le Gland, Paul Bui–Quang.

This is a collaboration with Christian Musso (ONERA Palaiseau).

The Laplace method is a deterministic technique to approximate integrals, and it has been widely used in Bayesian statistics, e.g. to compute posterior means and variances [72]. The approximation is consistent as the observations sample size goes to infinity or as the observation noise intensity goes to zero, and the main condition to apply the method is that the model should be identifiable. The aim of [21] is to combine SMC methods and the Laplace method in order to better approximate the posterior density in nonlinear Bayesian filtering. At each stage of the proposed algorithm, a first approximate density is build from the current population of particles, then an accurate estimate of the posterior mean and covariance matrix is obtained using the Laplace method, and these estimates are used to shift and rescale the population of particles. Overall, this procedure could be interpreted as another design of an importance distribution that takes the observations into account. The current work aims at using the Laplace method to cope with *weight degeneracy* in particle filtering, a phenomenon that typically occurs when the observation noise is small, which is precisely the situation where the Laplace method is efficient.

## 5.13. Wind–wave modelling

**Participant:** Valérie Monbet.

This is a collaboration with Pierre Ailliot (UBO).

Climate change will bring large changes to the mean climate, and especially to climate extremes, over the coming decades. Computationally expensive global climate model (GCM) projections provide good information about future mean changes. Computationally efficient, yet physically consistent, statistical models of weather variables (stochastic weather generators) allow us to explore the frequency and severity of weather and climate events in much greater detail. When deployed as a complement to GCMs, stochastic weather generators provide a much richer picture of the future, allowing us to better understand, evaluate and manage future weather and climate risks, especially for renewal energy. In this context we are developing a space time model for wind fields in the North–East Atlantic, based on a conditionally transformed Gaussian state space model.

### 5.14. Sequential data assimilation: ensemble Kalman filter vs. particle filter

**Participants:** François Le Gland, Valérie Monbet.

Surprisingly, very little was known about the asymptotic behaviour of the ensemble Kalman filter [44], [45], [46], whereas on the other hand, the asymptotic behaviour of many different classes of particle filters is well understood, as the number of particles goes to infinity. Interpreting the ensemble elements as a population of particles with mean–field interactions, and not only as an instrumental device producing an estimation of the hidden state as the ensemble mean value, it has been possible to prove the convergence of the ensemble Kalman filter, with a rate of order $1/\sqrt{N}$, as the number $N$ of ensemble elements increases to infinity [62]. In addition, the limit of the empirical distribution of the ensemble elements has been exhibited, which differs from the usual Bayesian filter. The next step has been to prove (by induction) the asymptotic normality of the estimation error, i.e. to prove a central limit theorem for the ensemble Kalman filter.

# 6. Partnerships and Cooperations

## 6.1. National initiatives

### *6.1.1. Ensemble methods for prediction and data assimilation (PREVASSEMBLE) — ANR Conception et Simulation*

**Participants:** François Le Gland, Valérie Monbet.

*See 5.14.*

*Inria contract ALLOC 3767 — January 2009 to December 2012.*

This ANR project is coordinated by École Normale Supérieure, Paris. The other partner is Météo–France. This is a collaboration with Étienne Mémin and Anne Cuzol (Inria Rennes Bretagne Atlantique, project–team FLUMINANCE).

The contribution of ASPI to this project is to continue the comparison of sequential data assimilation methods initiated in [73], [62], such as the ensemble Kalman filter (EnKF) and the weighted ensemble Kalman filter (WEnKF), with particle filters. This comparison has been made on the basis of asymptotic variances, as the ensemble or sample size goes to infinity, and also on the impact of dimension on small sample behavior.

The consortium has organized the international conference on *Ensemble Methods in Geophysical Sciences*, held in Toulouse in November 2012.

# 7. Dissemination

## 7.1. Scientific animation

Arnaud Guyader and Frédéric Cérou have co–organized the workshop on *Computation of Transition Trajectories and Rare Events in Non-Equilibrium Systems*, held in Lyon in June 2012.

Arnaud Guyader has organized the session on *Rare Events Simulation* at *Journées MAS de la SMAI*, held in Clermont–Ferrand in August 2012. He has also co–organized the 2012 edition of the *Journées de Statistiques Rennaises*, held in Rennes in October 2012. He is the co–author of a book [27] on the statistical software R.

François Le Gland was a member of the scientific and organizing committees for the international conference on *Ensemble Methods in Geophysical Sciences*, held in Toulouse in November 2012, an event organized within the ANR project PREVASSEMBLE.

François Le Gland has been a member of the committee for the PhD thesis of Cyrille Dubarry (université Pierre et Marie Curie, advisor: Éric Moulines) and he as been a reviewer for the PhD theses of Romain Leroux (université de Poitiers, advisors: Ludovic Chatellier and Laurent David), Virgile Caron (université Pierre et Marie Curie, advisor: Michel Broniatowski), and Thierry Dumont (université Paris–Sud, advisor: Elisabeth Gassiat).

Florent Malrieu has co–organized the 2012 edition of *Journées de probabilités*, held in Roscoff in June 2012.

Valérie Monbet has co–organized the first international workshop on *Stochastic Weather Generators*, held in Roscoff in May 2012. It gathered 30 participants from France, UK, USA and New-Zealand. Most major teams working on WGs were present. The latest developments were presented, thus providing an up–to–date and almost comprehensive snapshot of the state–of–the art.

François Le Gland is a member of the "conseil d'UFR" of the department of mathematics of université de Rennes 1.

Florent Malrieu is a member of the "conseil" of IRMAR (institut de recherche mathématiques de Rennes, UMR 6625).

Valérie Monbet is a member of the "comité de direction" and of the "conseil" of IRMAR (institut de recherche mathématiques de Rennes, UMR 6625). She is also the director of the master on statistics and econometry at université de Rennes 1.

## 7.2. Teaching

Arnaud Guyader is a member of the committee of "oraux blancs d'agrégation de mathématiques" for ENS Cachan at Ker Lann.

François Le Gland gives a course on Kalman filtering and hidden Markov models, at université de Rennes 1, within the master SISEA (signal, image, systèmes embarqués, automatique, école doctorale MATISSE), a 3rd year course on Bayesian filtering and particle approximation, at ENSTA (école nationale supérieure de techniques avancées), Paris, within the systems and control module, a 3rd year course on linear and nonlinear filtering, at ENSAI (école nationale de la statistique et de l'analyse de l'information), Ker Lann, within the statistical engineering track, and a 3rd year course on hidden Markov models, at Télécom Bretagne, Brest. He has also organized a thematic school on particle filtering, proposed as a complementary scientific training to PhD students of école doctorale MATISSE.

Florent Malrieu teaches in the probability and statistics track of the training programme for "agrégation de mathématiques" at université de Rennes 1.

Valérie Monbet gives several courses on data analysis, on time series and hidden Markov models, and on mathematical statistics, all at université de Rennes 1 within the master on statistics and econometrics.

## 7.3. PhD and habilitation theses

Arnaud Guyader has been supervising one PhD student

- Nicolas Jégou, title: *Régression isotonique itérée*, defense in November 2012, co–direction: Nick Hengartner (Los Alamos) and Éric Matzner–Løber (université de Rennes 2).

Valérie Monbet is currently supervising one PhD student

- Julie Bessac, provisional title: *Space time modelling of wind fields*, started in October 2011, co–direction : Pierre Ailliot (université de Bretagne Occidentale),

and she is a member of the PhD thesis committe of

- Jérôme Weiss, provisional title: *Modelling of extreme storm surge series*, funding : CIFRE grant with EDF R&D, direction : Michel Benoît (Laboratoire d'Hydraulique Saint-Venant).

François Le Gland has been supervising one PhD student

- Rudy Pastel, title: *Estimation of rare event probabilities and extreme quantiles. Applications in the aerospace domain*, defense in February 2012, funding: ONERA grant, co–direction: Jérôme Morio (ONERA, Palaiseau).

and he is currently supervising three PhD students

- Paul Bui–Quang, provisional title: *The Laplace method for particle filtering*, started in October 2009, expected defense in 2013, funding: ONERA grant, co–direction: Christian Musso (ONERA, Palaiseau).
- Alexandre Lepoutre, provisional title: *Detection issues in track–before–detect*, started in October 2010, funding: ONERA grant, co–direction: Olivier Rabaste (ONERA, Palaiseau).
- Damien Jacquemart, provisional title: *Rare event methods for the estimation of collision risk*, started in October 2011, funding: DGA / ONERA grant, co–direction: Jérôme Morio (ONERA, Palaiseau).

## 7.4. Participation in workshops, seminars, lectures, etc.

In addition to presentations with a publication in the proceedings, which are listed at the end of the document in the bibliography, members of ASPI have also given the following presentations.

Arnaud Guyader has been invited to give a talk on adaptive multilevel splitting for rare event estimation in a static case, at the workshop on *Sequential Monte Carlo Methods and Efficient Simulation in Finance*, held at École Polytechnique in October 2012, and a talk on Monte Carlo methods for rare event simulation, at the *Rencontres Statistiques Lyonnaises*, held in Lyon in October 2012. He has given a talk on the nonparametric analysis of the ABC algorithm and a talk on iterative isotone regression, at the *44èmes Journées de Statistique*, held in Brussels in May 2012, and a talk on soft level splitting for rare event estimation, at the *9th International Workshop on Rare Event Simulation*, held in Trondheim in June 2012.

François Le Gland has given a talk on adaptive resampling in sequential Monte Carlo methods, at the CRiSM workshop on *Recent Advances in Sequential Monte Carlo*, held at the University of Warwick in September 2012, and a talk on large sample asymptotics of the ensemble Kalman filter, at the workshop on *Data Assimilation*, held at the University of Oxford in September 2012, and at the international conference on *Ensemble Methods in Geophysical Sciences*, held at the Météo–France center in Toulouse in November 2012.

Florent Malrieu has given a three–hour mini–course on the long time asymptotics of piecewise–deterministic Markov models, in the workshop on *Piecewise–Deterministic Markov Processes*, held in Marne–la–Vallée in March 2012. He has been an invited speaker at the ERGONUM workshop on *Probabilistic Analysis of Large Time Systems*, held in Sophia–Antipolis in June 2012, and at the EPSRC workshop *At the Frontier of Analysis and Probability*, held in Warwick in September 2012. He has been invited to give seminar talks on the long time behaviour of the TCP process in Marseilles in January 2012 and in Paris–Nanterre in May 2012, and on the long time behaviour of some piecewise deterministic Markov processes in Tours in October 2012, in Montpellier and in Toulouse in November 2012.

## 7.5. Visits and invitations

Arnaud Guyader has been invited by Nicolas Hengartner to visit Los Alamos National Laboratories in April 2012.

François Le Gland has been invited by Arunabha Bagchi to visit the department of applied mathematics of the University of Twente in Enschede and the technical business unit on radar engineering at Thalès Nederland in Hengelo in December 2012, and he has given there a talk on rare event simulation in stochastic hybrid systems, a talk on Laplace and SMC methods in Bayesian filtering, and a talk on detection issues in track–before–detect.

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