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Project-Team FLUMINANCE

Fluid Flow Analysis, Description and Control from Image Sequences

RESEARCH CENTER
Rennes - Bretagne-Atlantique

THEME
Observation and Modeling for Environmental Sciences

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Project-Team FLUMINANCE

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2. Overall Objectives

2.1. Overall Objectives

The research group that we have entitled FLUMINANCE from a contraction between the words “Fluid” and “Luminance” is dedicated to the extraction of information on fluid flows from image sequences and to the development of tools for the analysis and control of these flows. The objectives of the group are at the frontiers of several important domains. The group aims at providing in the one hand image sequence methods devoted to the analysis and description of fluid flows and in the other hand physically consistent models and operational tools to extract meaningful features characterizing or describing the observed flow and enabling decisions or actions. Such a twofold goal is of major interest for the inspection, the analysis and the monitoring of complex fluid flows, but also for control purpose of specific flows involved in industrial problems. To reach these goals we will mainly rely on data assimilation strategies and on motion measurement techniques. From a methodological point of view, the techniques involved for image analysis are either stochastic or variational. One of the main originality of the the FLUMINANCE group is to combine cutting-edge researches on these methods with an ability to conduct proper intensive experimental validations on prototype flows mastered in laboratory. The scientific objectives decompose in three main themes:

- **Fluid flows characterization from images**

We aim here at providing accurate measurements and consistent analysis of complex fluid flows through image analysis techniques.The application domain ranges from industrial processes and experimental fluid mechanics to environmental and life sciences. This theme includes also the use of non-conventional imaging techniques such as Schlieren techniques, Shadowgraphs, holography. The objective will be here to go towards 3D dense velocity measurements.

- **Coupling dynamical model and image data**
  We focus here on the study, through image data, of complex and partially known fluid flows involving complex boundary conditions, multi-phase fluids, fluids and structures interaction problems. Our credo is that image analysis can provide sufficiently fine observations on small an medium scales to construct models which, applied at medium and large scale, account accurately for a wider range of the dynamics scales. The image data and a sound modeling of the dynamical uncertainty at the observation scale should allow us to reconstruct the observed flow and to provide efficient real flows (experimental or natural) based dynamical modeling. Our final goal will be to go towards a 3D reconstruction of real flows, or to operate large motion scales simulations that fit real world flow data and incorporate an appropriate uncertainty modeling.
- **Control and optimization of turbulent flows**
  We are interested on active control and more precisely on closed-loop control. The main idea is to extract reliable image features to act on the flow. This approach is well known in the robot control community, it is called visual servoing. More generally, it is a technique to control a dynamic system from image features. We plan to apply this approach on flows involved in various domains such as environment, transport, microfluidic, industrial chemistry, pharmacy, food industry, agriculture, etc.

3. Scientific Foundations

3.1. Fluid flow analysis and modeling

Turbulent fluid flows involved in environmental or industrial applications are complex. In fluid mechanics laboratories, canonical turbulent shear flows have been studied for many years and a relatively clear picture of their underlying structure exists. However, the direct applicability of these efforts to real relevant flows, which often occur in complex geometries and in the presence of multiple non canonical influences, like cross-shear, span wise non-uniform and thermal stratification, is still unknown. In addition, the turbulence can be characterized by Reynolds number ranging between 10^3 and 10^4, corresponding to transitional regime for which the use of classical turbulence models is limited.

In this context, we have performed research studies on turbulent shear flows of low velocities by tackling crucial topics of measurements, analysis and modeling of environmental and industrial flows in presence of non-canonical influences. This concerns more precisely the study of the interaction between a mixing layer and circular cylinder wake flow, the study of wake flow with span wise non uniformity, the study of mixing layer under the influence of thermal stratification and the study of mixing layer forced between non-uniform flows. The analysis of these flows has required the design of adequate dynamical models, using proper orthogonal decomposition and Galerkin projection. Understanding issues such as the mechanisms of heat and mass transfer involved in these shear flows provides meaningful information for the control of relevant engineering flows and the design of new technologies. To investigate more thoroughly these complex flows numerical and experimental tools have been designed. An immersed boundary method was proposed to mimic complex geometries into Direct Numerical simulation (DNS) and Large Eddy Simulations (LES) codes. A novel anemometer has been designed and implemented for the simultaneous measurement of velocity and temperature in air flows with a single hot-wire probe.

Mixing layer wake interaction

We have investigated the vortex shedding of a circular cylinder immersed in a plane turbulent mixing layer. For a centre span Reynolds number of 7500, the wake flow splits into three regions: a high-velocity wake, a low-velocity wake and a region of interaction in the middle span of the body. A strong unsteady secondary flow is observed, and explained with span wise base pressure gradients. Unexpected features are found for formation length and the base pressure along the span of the cylinder. In the high-velocity side, where the local Reynolds number is the highest, the formation length is longest. Based on the formation length measurements it was shown that as a function of the centre span Reynolds number, the wake flows behaves as circular cylinder in uniform flow. Three cells with a constant frequency with adjacent dislocations are observed. For each cell, a shedding mode was suggested. The relation of the secondary flow to the frequencies was examined. All the observations were analyzed by analogical reasoning with other flows. This pointed out the action of the secondary flow in the high-velocity side regarded as a wake interference mechanism.

Low order complex flow modeling

We have proposed improvements to the construction of low order dynamical systems (LODS) for incompressible turbulent external flows. The reduced model is obtained by means of a Proper Orthogonal Decomposition (POD) basis extracted through a truncated singular value decomposition of the flow auto-correlation matrix built from noisy PIV experimental velocity measurements. The POD modes are then used to formulate a reduced dynamical system that contains the main features of the flow. This low order dynamical system (LODS) is obtained through a Galerkin projection of the Navier-Stokes Equations on the POD basis. Usually, the resulting system of ordinary differential equations presents stability problems due to modes truncation and numerical uncertainties, especially when working on experimental data. The technique we proposed relies on an optimal control approach to estimate the dynamical system coefficients and its initial condition. This allows us to recover a reliable and stable spatio-temporal reconstruction of the large scales of the flow. The technique has been assessed on the near wake behind a cylinder observed through very noisy PIV measurement. It has been also evaluated for configurations involving a rotating cylinder.

Studies on complex 3D dynamical behavior resulting from the interaction between a plane mixing layer and the wake of a cylinder have been also investigated using POD representation, applied to data from two synchronized 2D PIV systems (Dual-plane PIV). This approach allowed us to construct a 3D-POD representation. An analysis of the correlations shows different length scales in the regions dominated by wake like structures and shear layer type structures [2]. In order to characterize the particular organization in the plane of symmetry, a Galerkin projection from a slice POD has been performed. This led to a low-dimensional dynamical system that allowed the analysis of the relationship between the dominant frequencies. This study led to a reconstruction of the dominant periodic motion suspected from previous studies [41]. This work allowed us to make a link between the three-dimensional organization and the secondary unsteady motion from the low velocity side to the high velocity side of the mixing layer, appearing in this highly 3D flow configuration.

Direct and Large Eddy simulations of complex flows

We have proposed a direct forcing method better suited to the use of compact finite difference schemes in Direct Numerical Simulation. The new forcing creates inside the body an artificial flow preserving the no-slip condition at the surface but reducing the step-like change of the velocity derivatives across the immersed boundary. This modification led to improve results both qualitatively and quantitatively for conventional and complex flow geometries [50].

Three-dimensional direct numerical simulations have been performed for vortex shedding behind cylinders. We focused in particular on cases for which the body diameter and the incoming flow involved span wise linear non-uniformity. Four configurations were considered: the shear flow, the tapered cylinder and their combinations, which gave rise namely to the adverse and aiding cases. In contrast with the observations of other investigators, these computations highlighted distinct vortical features between the shear case and the tapered case. In addition, it was observed that the shear case and the adverse case (respectively the tapered and aiding case), yielded similarities in flow topology. This phenomenon was explained by the span wise variations of the ratio of mean velocity and the cylinder diameter which seemed to govern these flows. Indeed, it was observed that large span wise variations of U/D seemed to enhance three-dimensionality, through the appearance of vortex-adhesions and dislocations. Span wise cellular pattern of vortex shedding were identified. Their modifications in cell size, junction position and number were correlated with the variation of U/D. In the Lee side of the obstacle a wavy secondary motion was identified. Induced secondary flow due to the bending of Karman vortices in the vicinity of vortex-adhesion and dislocations was suggested to explain this result [49].

LES and experimental wake flow database
We contributed to the study of flow over a circular cylinder at Reynolds number $Re = 3900$. Although this classical flow is widely documented in the literature, especially for this precise Reynolds number, which leads to a sub critical flow regime, there is no consensus about the turbulence statistics immediately just behind the obstacle. This flow has been studied both numerically with Large Eddy Simulation and experimentally with Hot-Wire Anemometry and Particle Image Velocimetry. The numerical simulation has been performed using high-order schemes and the specific Immersed Boundary Method previously mentioned. We focused on turbulence statistics and power spectra in the near wake up to 10 diameters. Statistical estimation is shown to need large integration times increasing the computational cost and leading to an uncertainty of about 10% for most flow characteristics considered in this study. The present numerical and experimental results are found to be in good agreement with previous Large Eddy Simulation data. Our study has exhibited significant differences compared with the experimental data found in the literature. The obtained results attenuate previous numerical-experimental controversy for this type of flows [11].

Simultaneous velocity temperature measurements in turbulent flows
We have worked on the design of a novel anemometer for the simultaneous measurement of velocity and temperature in airflows with a single hot wire probe. The principle of periodically varying the overheat ratio of the wire has been selected and applied through a tunable electronic chain. Specific methods were developed for the calibration procedure and the signal processing. The accuracy of the measurements was assessed by means of Monte-Carlo simulations. Accurate results were provided for two types of turbulent non-isothermal flows, a coaxial heated jet and a low speed thermal mixing. The particular interest of the synchronization of the two measurements has been emphasized during the PhD thesis of T. Ndoye.

A new dynamic calibration technique has been developed for hot-wire probes. The technique permits, in a short time range, the combined calibration of velocity, temperature and direction calibration of single and multiple hot-wire probes. The calibration and measurements uncertainties were modeled, simulated and controlled, in order to reduce their estimated values.

3.2. Fluid motion analysis

Flow visualization has been a powerful tool to depict or to understand flow feature properties. Efforts to develop high-quality flow visualization techniques date back over a century. The analysis of the recorded images consisted firstly to a qualitative interpretation of the streak lines leading to an overall global insight into the flow properties but lacking quantitative details on important parameters such as velocity fields or turbulence intensities. Point measurement tools such as hot wire probes or Laser Doppler Velocimetry have typically provided these details. As these probes give information only at the point where they are placed, simultaneous evaluations at different points require to dispose a very large number of probes and the evaluation of unsteady field (most of the flows are unsteady) is almost unachievable with them.

In an effort to avoid the limitations of these probes, the Particle Image Velocimetry (PIV), a non-intrusive diagnostic technique, has been developed in the last two decades [40]. The PIV technique enables obtaining velocity fields by seeding the flow with particles (e.g. dye, smoke, particles) and observing the motion of these tracers. In computer vision, the estimation of the projection of the apparent motion of a 3D scene onto the image plane, refereed in the literature as optical-flow, is an intensive subject of researches since the 80's and the seminal work of B. Horn and B. Schunk [45]. Unlike to dense optical flow estimators, the former approach provides techniques that supply only sparse velocity fields. These methods have demonstrated to be robust and to provide accurate measurements for flows seeded with particles. These restrictions and their inherent discrete local nature limit too much their use and prevent any evolutions of these techniques towards the devising of methods supplying physically consistent results and small scale velocity measurements. It does not authorize also the use of scalar images exploited in numerous situations to visualize flows (image showing the diffusion of a scalar such as dye, pollutant, light index refraction, flurocein,...). At the opposite, variational techniques enable in a well-established mathematical framework to estimate spatially continuous velocity fields, which should allow more properly to go towards the measurement of smaller motion scales. As these methods are defined through PDE's systems they allow quite naturally including as constraints the kinematical and dynamical laws governing the observed fluid flows. Besides, within this framework it is also much easier to define characteristic features estimation on the basis of physically grounded data model that describes the relation linking the observed luminance function and some state variables of the observed flow. This route has demonstrated to be much more robust to scalar image. Several studies in this vein have strengthened our skills in this domain. All the following approaches have been either formulated within a statistical Markov Random Fields modeling or either within a variational framework. For a thorough description of these approaches see [7].

ICE data model and div-curl regularization This fluid motion estimator is constructed on a data model derived from the Integration of the Continuity Equation (ICE data model) [5] and includes a second order regularization scheme enabling to preserve blobs of divergence and curl. Intensive evaluations of this estimator on flow prototypes mastered in laboratory have shown that this estimator led to the same order of accuracy as the best PIV techniques but for an increase information density. This ability to get dense flow fields allowed us estimating proper vorticity or divergence maps without resorting to additional post-processing interpolation schemes.

Schlieren Image velocimetry We have addressed the problem of estimating the motion of fluid flows visualized with the Schlieren technique. Such an experimental visualization system is well known in fluid mechanics and it enables the visualization of unseeded flows. This technique authorizes the capture of phenomena that are impossible to visualize with particle seeding such as natural convection, phonation flow, breath flow and allows the setting of large scale experiments. Since the resulting images exhibit very low intensity contrasts, classical motion estimation methods based on the brightness constancy assumption (correlation-based approaches, optical flow methods) are completely inefficient. The global energy function we have defined for Schlieren images is composed of i) a specific data model accounting for the fact that the observed luminance is related to the gradient of the fluid density, and ii) a specific constrained div-curl regularization term. To date there exists no motion estimator allowing estimating accurately dense velocity fields on Schlieren images.

Low order fluid motion estimator This low-dimensional fluid motion estimator [6] is based on the Helmholtz decomposition, which consists in representing the velocity field as the sum of a divergence-free component and a curl-free one. In order to provide a low-dimensional solution, both components have been approximated using a discretization of the vorticity (curl of the velocity vector) and divergence maps through regularized Dirac measures [44]. The resulting so-called irrotational (resp. solenoidal) field is then represented by a linear combination of basis functions obtained by a convolution product of the Green kernel gradient and the vorticity map (resp. the divergence map). The coefficient values and the basis function parameters are obtained by minimizing a function formed by an integrated version of the mass conservation principle of fluid mechanics.

Potential functions estimation and finite mimetic differences We have studied a direct estimation approach of the flow potential functions (respectively the *stream* function and the *velocity* potential) from two consecutive images. The estimation has been defined on the basis of a high order regularization scheme and has been implemented through mimetic difference methods[12]. With these approaches the discretization preserves basic relationships of continuous vector analysis. Compared to previous discretization scheme based on auxiliary div-curl variables, the considered technique appeared to be numerically much more stable and led to an improve accuracy.

2D and 3D atmospheric motion layer estimation In this study, we have explored the problem of estimating mesoscales dynamics of atmospheric layers from satellite image sequences. Due to the intrinsic sparse 3-dimensional nature of clouds and to large occluded zones caused by the successive overlapping of cloud layers, the estimation of accurate layered dense motion fields is an intricate issue. Relying on a physically sound vertical decomposition of the atmosphere into layers, we have proposed two dense motion estimators for the extraction of multi-layer horizontal (2D) and 3D wind fields. These estimators are expressed as the minimization of a global function that includes a data-driven term and a spatio-temporal smoothness term. A robust data term relying on shallow-water mass conservation model has been proposed to fit sparse observations related to each layer. In the 3D case, the layers are interconnected through a term modeling mass exchanges at the layers surfaces frontiers [9].

A novel spatio-temporal regularizer derived from the shallow-water momentum conservation model has been considered to enforce temporal consistency of the solution along time. These constraints are combined with a robust second-order regularizer preserving divergent and vorticity structures of the flow. Besides, a two-level motion estimation scheme has been settled to overcome the limitations of the multiresolution incremental estimation scheme when capturing the dynamics of fine mesoscale structures. This alternative approach relies on the combination of correlation and optical-flow observations. An exhaustive evaluation of the novel method has been first performed on a scalar image sequence generated by Direct Numerical Simulation of a turbulent bi-dimensional flow. Based on qualitative experimental comparisons, the method has also been assessed on a Meteosat infrared image sequence.

3.3. Data assimilation and Tracking of characteristic fluid features

Classical motion estimation techniques usually proceed on pairs of two successive images, and do not enforce temporal consistency. This often induces an estimation drift which is essentially due to the fact that motion estimation is formulated as a local process in time. No adequate physical dynamics law, or conservation law, related to the observed flow, is taken into account over long time intervals by the usual motion estimators. The estimation of an unknown state variable trajectory on the basis of specified dynamical laws and some incomplete and noisy measurements of the variable of interest can be either conducted through optimal control techniques or through stochastic filtering approach. These two frameworks have their own advantages and deficiencies. We rely indifferently on both approaches.

Stochastic filtering for fluid motion tracking We have proposed a recursive Bayesian filter for tracking velocity fields of fluid flows. The filter combines an Îto diffusion process associated to 2D vorticity-velocity formulation of Navier-Stokes equation and discrete image error reconstruction measurements. In contrast to usual filters, designed for visual tracking problem, our filter combines a continuous law for the description of the vorticity evolution with discrete image measurements. We resort to a Monte-Carlo approximation based on particle filtering. The designed tracker provides a robust and consistent estimation of instantaneous motion fields along the whole image sequence. In order to handle a state space of reasonable dimension for the stochastic filtering problem, the motion field is represented as a combination of adapted basis functions. The basis functions are derived from a mollification of Biot-Savart integral and a discretization of the vorticity and divergence maps of the fluid vector field. The output of such a tracking is a set of motion fields along the whole time range of the image sequence. As the time discretization is much finer than the frame rate, the method provides consistent motion interpolation between consecutive frames. In order to reduce further the dimensionality of the associated state space when we are facing a large number of motion basis functions, we have explored a new dimensional reduction approach based on dynamical systems theory. The study of the stable and unstable directions of the continuous dynamics enables to construct an adaptive dimension reduction procedure. It consists in sampling only in the unstable directions, while the stable ones are treated deterministically [6].

When the likelihood of the measurement can be modeled as Gaussian law, we have also investigated the use of so-called ensemble Kalman filtering for fluid tracking problems. This kind of filters introduced for the analysis of geophysical fluids is based on the Kalman filter update equation. Nevertheless, unlike traditional Kalman filtering setting, the covariances of the estimation errors, required to compute the Kalman gain, rely on an ensemble of forecasts. Such a process gives rise to a Monte Carlo approximation for a family of stochastic non linear filters enabling to handle state spaces of large dimension. We have recently proposed an extension of this technique that combines sequential importance sampling and the propagation law of ensemble kalman filter. This technique leads to ensemble Kalman filter with an improve efficiency. This appears to be a generalization of the optimal importance sampling strategy we proposed in the context of partial conditional Gaussian trackers [1].

Variational assimilation technique
We investigated the use of variational framework for the tracking from image sequence of features belonging to high dimensional spaces. This framework relies on optimal control principles as developed in environmental sciences to analyze geophysical flows [46], [47]. Within the PhD of Nicolas Papadakis [10], we have first devised a data assimilation technique for the tracking of closed curves and their associated motion fields. The proposed approach enables a continuous tracking along an image sequence of both a deformable curve and its associated velocity field. Such an approach has been formalized through the minimization of a global spatio-temporal continuous cost functional, with respect to a set of variables representing the curve and its related motion field. The resulting minimization sequence consists in a forward integration of an evolution law followed by a backward integration of an adjoint evolution model. The latter pde includes a term related to the discrepancy between the state variables evolution law and discrete noisy measurements of the system. The closed curves are represented through implicit surface modeling [48], whereas the motion is described either by a vector field or through vorticity and divergence maps according to the type of targeted application. The efficiency of the approach has been demonstrated on two types of image sequences showing deformable objects and fluid motions.

More recently assimilation technique for the direct estimation of atmospheric wind field from pressure images have been proposed [4]. These techniques rely on a brightness variation model of the intensity function. They do not include anymore motion measurements provided by external motion estimators. The resulting estimator allows us to recover accurate fluid motion fields and enables tracking dense vorticity maps along an image sequence.

3.4. Visual servoing

Nowadays, visual servoing is a widely used technique is robot control. It consists in using data provided by a vision sensor for controlling the motions of a robot [43]. Various sensors can be considered such as perspective cameras, omnidirectional cameras, 2D ultrasound probes or even virtual cameras. In fact, this technique is historically embedded in the larger domain of sensor-based control [51] so that other sensors than vision sensors can be properly used. On the other hand, this approach was first dedicated to robot arms control. Today, much more complex system can be considered like humanoid robots, cars, submarines, airships, helicopters, aircrafts. Therefore, visual servoing is now seen as a powerful approach to control the state of dynamic systems.

Classically, to achieve a visual servoing task, a set of visual features $\mathbf{s}$ has to be selected from visual measurements $\mathbf{m}$ extracted from the image. A control law is then designed so that these visual features reach a desired value $\mathbf{s}^*$ related to the desired state of the system. The control principle is thus to regulate to zero the error vector $\mathbf{e} = \mathbf{s} - \mathbf{s}^*$. To build the control law, the knowledge of the so-called *interaction matrix* $\mathbf{L_s}$ is usually required. This matrix links the time variation of $\mathbf{s}$ to the camera instantaneous velocity $\mathbf{v}$

$$\dot{\mathbf{s}} = \mathbf{L_s}\,\mathbf{v} + \frac{\partial \mathbf{s}}{\partial t} \tag{1}$$

where the term $\frac{\partial \mathbf{s}}{\partial t}$ describes the non-stationary behavior of $\mathbf{s}$. Typically, if we try to ensure an exponential decoupled decrease of the error signal and if we consider the camera velocity as the input of the robot controller, the control law writes as follow

$$\mathbf{v} = -\lambda \widehat{\mathbf{L_s}}^{+} \mathbf{e} - \widehat{\mathbf{L_s}}^{+} \widehat{\frac{\partial \mathbf{e}}{\partial t}} \tag{2}$$

with λ a proportional gain that has to be tuned to minimize the time-to-convergence, $\widehat{\mathbf{L_s}}^+$ the pseudo-inverse of a model or an approximation of $\mathbf{L_s}$ and $\widehat{\frac{\partial \mathbf{e}}{\partial t}}$ an estimation of $\frac{\partial \mathbf{e}}{\partial t}$.

The behavior of the closed-loop system is then obtained, from (2), by expressing the time variation of the error $\mathbf{e}$

$$\dot{\mathbf{e}} = -\lambda \mathbf{L_s} \widehat{\mathbf{L_s}}^+ \mathbf{e} - \mathbf{L_s} \widehat{\mathbf{L_e}}^+ \widehat{\frac{\partial \mathbf{e}}{\partial t}} + \frac{\partial \mathbf{e}}{\partial t}. \tag{3}$$

As can be seen, visual servoing explicitly relies on the choice of the visual features $\mathbf{s}$ and then on the related interaction matrix; that is the key point of this approach. Indeed, this choice must be performed very carefully. Especially, an isomorphism between the camera pose and the visual features is required to ensure that the convergence of the control law will lead to the desired state of the system. An optimal choice would result in finding visual features leading to a diagonal and constant interaction matrix and, consequently, to a linear decoupled system for which the control problem is well known. Thereafter, the isomorphism as well as the global stability would be guaranteed. In addition, since the interaction matrix would present no more nonlinearities, a suitable robot trajectory would be ensured.

However, finding such visual features is a very complex problem and it is still an open issue. Basically, this problem consists in building the visual features $\mathbf{s}$ from the nonlinear visual measurements $\mathbf{m}$ so that the interaction matrix related to $\mathbf{s}$ becomes diagonal and constant or, at least, as simple as possible.

On the other hand, a robust extraction, matching (between the initial and desired measurements) and real-time spatio-temporal tracking (between successive measurements) have to be ensure but have proved to be a complex task, as testified by the abundant literature on the subject. Nevertheless, this image process is, to date, a necessary step and often considered as one of the bottlenecks of the expansion of visual servoing. That is why more and more non geometric visual measurements are proposed [3].

3.5. Sparse Representations and Bayesian model selection

Sparse representation methods aim at finding representations of a signal with a small number of components taken from an over-complete dictionary of elementary functions or vectors. Sparse representation are of interest in a number of applications in Physics and signal processing. In particular, they provide a simple characterization of certain families of signals encountered in practice. For example, smooth signals can be shown to have a sparse representation in over-complete Fourier or wavelet dictionaries. More recently, it has been emphasized in [42] that the solutions of certain differential equations (*e.g.,* diffusion or transport equation) have a sparse representation in dictionaries made up of curvelets.

Finding the sparse representation of a signal typically requires to solve an under-determined system of equations under the constraint that the solution is composed of the minimum number of non-zero elements. Unfortunately, this problem is known to be NP-hard and sub-optimal procedures have to be devised to find practical solutions. Among the various algorithms that find approximate solutions, let us mention for example the matching pursuit, orthogonal matching pursuit or basis pursuit algorithms.

Choosing appropriate models and fixing hyper-parameters is a tricky and often hidden process in optic-flow estimation. Most of the motion estimators proposed so far have generally to rely on successive trials and a empirical strategy for fixing the hyper parameters values and choosing the adequate model. Besides of its computational inefficiency, this strategy may produce catastrophic estimate without any relevant feedback for the end-user, especially when motions are difficult to apprehend as for instance for complex deformations or non-conventional imagery. Imposing hard values to these parameters may also yield poor results when the lighting conditions or the underlying motions differ from those the system has been calibrated with. At the extreme, the estimate may be either too smooth or at the opposite non-existent strong motion discontinuities.

Bayesian model selection offers an attractive solution to this problem. The Bayesian paradigm implicitly requires the definition of several competing observation and prior *probabilistic* model(s). The observation model relates the motion of the physical system to the spatial and temporal variations of the image intensity. The prior models define the spatio-temporal constraints that the motion have to satisfy. Considering these competing models, the Bayesian theory provides methodologies to select the best models under objective performance criterion (minimum probability of error, minimum mean square error, etc). Moreover, due to the generality of this problem, numerous algorithms and approximations exist in the literature to implement efficient and effective practical solutions: Monte-Carlo integration, mean-field and Laplace approximations, EM algorithm, graphical models, etc.

4. Application Domains

4.1. Introduction

By designing new approaches for the analysis of fluid-image sequences the FLUMINANCE group aims at contributing to several application domains of great interest for the community and in which the analysis of complex fluid flows plays a central role. The group focuses mainly on two broad application domains:

- Environmental sciences;
- Experimental fluid mechanics and industrial flows.

4.2. Environmental sciences

The first huge application domain concerns all the sciences that aim at observing the biosphere evolution such as meteorology, climatology or oceanography but also remote sensing study for the monitoring of meteorological events or human activities consequences. For all these domains image analysis is a practical and unique tool to *observe, detect, measure, characterize or analyze* the evolution of physical parameters over a large domain. The design of generic image processing technique for all these domains might offer practical software tools to measure precisely the evolution of fluid flows for weather forecasting or climatology studies. It might also offer possibilities of closed surveillance of human and natural activities in sensible areas such as forests, river edges, and valley in order to monitor pollution, floods or fire. The need in terms of local weather forecasting, risk prevention, or local climate change is becoming crucial for our tomorrow's life. At a more local scale, image sensors may also be of major utility to analyze precisely the effect of air curtains for safe packaging in agro-industrial.

4.3. Experimental fluid mechanics and industrial flows

In the domain of **experimental fluid mechanics**, the visualization of fluid flows plays a major role, especially for turbulence study since high frequency imaging has been made currently available. Together with analysis of turbulence at different scales, one of the major goals pursued at the moment by lot of scientists and engineers consists in studying the ability to manipulate a flow to induce a desired change. This is of huge technological importance to enhance or inhibit mixing in shear flows, improve energetic efficiency or control the physical effects of strain and stresses. This is for instance of particular interest for:

- military applications, for example to limit the infra-red signatures of fighter aircraft;
- aeronautics and transportation, to limit fuel consumption by controlling drag and lift effects of turbulence and boundary layer behavior;
- industrial applications, for example to monitor flowing, melting, mixing or swelling of processed materials, or preserve manufactured products from contamination by airborne pollutants, or in industrial chemistry to increase chemical reactions by acting on turbulence phenomena.

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5. Software

5.1. DenseMotion software - Estimation of 2D dense motion fields

Participants: Thomas Corpetti, Patrick Héas, Etienne Mémin.

This code allows the computation from two consecutive images of a dense motion field. The estimator is expressed as a global energy function minimization. The code enables the choice of different data model and different regularization functional depending on the targeted application. Generic motion estimator for video sequences or dedicated motion estimator for fluid flows can be specified. This estimator allows in addition the users to specify additional correlation based matching measurements. It enables also the inclusion of a temporal smoothing prior relying on a velocity vorticity formulation of the Navier-Stoke equation for Fluid motion analysis applications. The different variants of this code correspond to research studies that have been published in IEEE transaction on Pattern Analysis and machine Intelligence, Experiments in Fluids, IEEE transaction on Image Processing, IEEE transaction on Geo-Science end Remote Sensing. The binary of this code can be freely downloaded on the FLUID web site http://fluid.irisa.fr.

5.2. 2DLayeredMotion software - Estimation of 2D independent mesoscale layered atmospheric motion fields

Participants: Patrick Héas, Etienne Mémin.

This software enables to estimate a stack of 2D horizontal wind fields corresponding to a mesoscale dynamics of atmospheric pressure layers. This estimator is formulated as the minimization of a global energy function. It relies on a vertical decomposition of the atmosphere into pressure layers. This estimator uses pressure data and classification clouds maps and top of clouds pressure maps (or infra-red images). All these images are routinely supplied by the EUMETSAT consortium which handles the Meteosat and MSG satellite data distribution. The energy function relies on a data model built from the integration of the mass conservation on each layer. The estimator also includes a simplified and filtered shallow water dynamical model as temporal smoother and second-order div-curl spatial regularizer. The estimator may also incorporate correlation-based vector fields as additional observations. These correlation vectors are also routinely provided by the Eumetsat consortium. This code corresponds to research studies published in IEEE transaction on Geo-Science and Remote Sensing. It can be freely downloaded on the FLUID web site http://fluid.irisa.fr.

5.3. 3DLayeredMotion software - Estimation of 3D interconnected layered atmospheric motion fields

Participants: Patrick Héas, Etienne Mémin.

This software extends the previous 2D version. It allows (for the first time to our knowledge) the recovery of 3D wind fields from satellite image sequences. As with the previous techniques, the atmosphere is decomposed into a stack of pressure layers. The estimation relies also on pressure data and classification clouds maps and top of clouds pressure maps. In order to recover the 3D missing velocity information, physical knowledge on 3D mass exchanges between layers has been introduced in the data model. The corresponding data model appears to be a generalization of the previous data model constructed from a vertical integration of the continuity equation. This research study has been recently accepted for publication in IEEE trans. on Geo-Science and Remote Sensing. A detailed description of the technique can be found in an Inria research report. The binary of this code can be freely downloaded on the FLUID web site http://fluid.irisa.fr.

5.4. Low-Order-Motion - Estimation of low order representation of fluid motion

Participants: Anne Cuzol, Etienne Mémin.

This code enables the estimation of a low order representation of a fluid motion field from two consecutive images.The fluid motion representation is obtained using a discretization of the vorticity and divergence maps through regularized Dirac measure. The irrotational and solenoidal components of the motion fields are expressed as linear combinations of basis functions obtained through the Biot-Savart law. The coefficient values and the basis function parameters are obtained as the minimizer of a functional relying on an intensity variation model obtained from an integrated version of the mass conservation principle of fluid mechanics. Different versions of this estimation are available. The code which includes a Matlab user interface can be downloaded on the FLUID web site http://fluid.irisa.fr. This program corresponds to a research study that has been published in the International Journal on computer Vision.

6. New Results

6.1. Fluid motion estimation

6.1.1. Stochastic uncertainty models for motion estimation

Participants: Thomas Corpetti, Etienne Mémin.

In this study we have proposed a stochastic formulation of the brightness consistency used principally in motion estimation problems. In this formalization the image luminance is modeled as a continuous function transported by a flow known only up to some uncertainties. Stochastic calculus then enables to built conservation principles which take into account the motion uncertainties. These uncertainties defined either from isotropic or anisotropic models can be estimated jointly to the motion estimates. Such a formulation besides providing estimates of the velocity field and of its associated uncertainties allows us to define a natural linear scale space multiresolution framework. The corresponding estimator, implemented within a local least squares approach, has shown to improve significantly the results of the corresponding deterministic estimator (Lucas and Kanade estimator). This fast local motion estimator has been shown to provide results that are of the same order of accuracy than state-of-the-art dense fluid flow motion estimator for particle images. The uncertainties estimated provide a useful piece of information in the context of data assimilation. This ability has been exploited to define multiscale data assimilation filtering schemes. These works have been recently published in *IEEE trans. on Image Processing* and in Numerical Mathematics: Theory, Methods and Applications [16], [18]. We intend to pursue this formalization to define dense motion estimators that allow us handling, in the same way, luminance conservation under motion uncertainty principles. An efficient GP-GPU implementation of the local estimator is also targeted.

6.1.2. 3D flows reconstruction from image data

Participants: Ioana Barbu, Cédric Herzet, Etienne Mémin.

Our work focuses on the design of new tools for the problem of 3D reconstruction of a turbulent flow motion. This task includes both the study of physically-sound models on the observations and the fluid motion, and the design of low-complexity and accurate estimation algorithms. On the one hand, state-of-the-art methodologies such as "sparse representations" will be investigated for the characterization of the observation and fluid motion models. Sparse representations are well-suited to the representation of signals with very few coefficients and offer therefore advantages in terms of computational and storage complexity. On the other hand, the estimation problem will be placed into a probabilistic Bayesian framework. This will allow the use of state-of-the-art inference tools to effectively exploit the strong time-dependence of the fluid motion. In particular, we will investigate the use of "ensemble Kalman" filter to devise low-complexity sequential estimation algorithms.

At the beginning of Ioana Barbu's PhD, we concentrated our efforts on the problem of reconstructing the particle positions from several two-dimensional images. Our approach is based on the exploitation of a particular family of sparse representation algorithms, namely the so-called "pursuit algorithms". Indeed, the pursuit procedures generally allow a good trade-off between performance and complexity. Hence, we have performed a thorough study comparing the reconstruction performance and the complexity of different state-of-the-art algorithms to that achieved with pursuit algorithms. This work has led to the publication of two conference papers in experimental fluid mechanics.

This year, our work has focused on: i) the estimation of the 3D velocity field of the fluid flow from the reconstructed volumes of particles; ii) the design of new methodologies allowing to jointly estimate the volume of particles and the velocity field from the received image data. More particularly, we have implemented a motion estimator generalizing the local Lucas-Kanade's procedure to a 3D problem. A potential strength of the proposed approach is the possibility to consider a fully parallel (and therefore very fast) implementation. On the other hand, we have started investigating the problem of jointly estimating the volumes of particles and the velocity field. Our approach is based on the combination of sparse representation algorithms and "Lucas-Kanade"-like motion estimation methods. We are about testing the proposed approach on experimental data in order to assess its performance in practical scenarios of fluid mechanics. We also intend to collaborate with the group of Fulvio Scarano at TU Delft to assess and compare our method on experimental 3D data.

6.1.3. Motion estimation techniques for turbulent fluid flows

Participants: Patrick Héas, Dominique Heitz, Cédric Herzet, Etienne Mémin.

Based on physical laws describing the multi-scale structure of turbulent flows, this study concerns the proposition of smoothing functional for the estimation of homogeneous turbulent flow velocity fields from an image sequence. This smoothing is achieved by imposing some scale invariance property between histograms of motion increments computed at different scales. By reformulating this problem from a Bayesian perspective, an algorithm is proposed to jointly estimate motion, regularization hyper-parameters, and to select the most likely physical prior among a set of models. Hyper-parameter and model inference is conducted by likelihood maximization, obtained by marginalizing out non-Gaussian motion variables. The Bayesian estimator is assessed on several image sequences depicting synthetic and real turbulent fluid flows. Results obtained with the proposed approach in the context of fully developped turbulence improve significantly the results of state of the art fluid flow dedicated motion estimators. This series of works, which have been done in close collaboration with P. Minnini (University of Buenos Aires), have been published in several journals [21], [22], [23].

6.1.4. Wavelet basis for multi-scale motion estimation

Participants: Pierre Dérian, Patrick Héas, Cédric Herzet, Souleymane Kadri Harouna, Etienne Mémin.

This work describes the implementation of a simple wavelet-based optical-flow motion estimator dedicated to the recovery of fluid motion. The wavelet representation of the unknown velocity

field is considered. This scale-space representation, associated to a simple gradient-based optimization algorithm, sets up a natural multiscale/multigrid optimization framework for the optical flow estimation that can be combined to more traditional incremental multiresolution approaches. Moreover, a very simple closure mechanism, approximating locally the solution by high-order polynomials, is provided by truncating the wavelet basis at intermediate scales. This offers a very interesting alternative to traditional Particle Image Velocimetry techniques. As another alternative to this medium-scale estimator, we explored strategies to define estimation at finer scales. These strategies rely on the encoding of high-order smoothing functional on appropriate wavelet basis. Divergence-free bi-othogonal wavelet bases enable to further nicely enforce volume preserving motion field. Numerical results on several examples have demonstrated the relevance of the method for divergence free-2D flows. These studies have been published in the journal of Numerical Mathematics: Theory, Methods and Applications [19] and in the journal of Computer Vision [24]. The extension to 3D flows would be an interesting perspective.

6.1.5. Wavelet-based divergence-free fBm prior: application to turbulent flow estimation

Participant: Patrick Héas.

This work is concerned with the estimation of turbulent flows from the observation of an image sequence. From a Bayesian perspective, we propose to study divergence-free isotropic fractional Brownian motion (fBm) as a prior model for instantaneous turbulent velocity fields. These priors are self-similar stochastic processes, which characterize accurately second-order statistics of velocity fields in incompressible isotropic turbulence. Although, these models belong to a well-identified family of rotation invariant regularizers, there is a lack of effective algorithms in the literature to deal in practice with their fractional nature. To respond to this problem, we propose to decompose fBms on well-chosen wavelet bases. As a first alternative, we propose to design wavelets as whitening filters for divergence-free isotropic fBms, which are correlated both in space and scale. The second alternative is to use a divergence-free wavelet basis, which will take implicitly into account the divergence-free constraint arising form the physics.

6.1.6. Sparse-representation algorithms

Participant: Cédric Herzet.

The paradigm of sparse representations is a rather new concept which appears to be central in many field of signal processing. In particular, in the field of fluid motion estimation, sparse representation appears to be potentially useful at several levels: i) it provides a relevant model for the characterization of the velocity field in some scenarios; ii) it turns out to be central in the recovering of volumes of particles in the 3D Tomo-PIV problem. In these contexts, the dimensionality of the problem can be very large and the use of sparse-representation algorithms allowing for a good trade-off between complexity and effectiveness is needed.

This year, we have therefore pursued our study of efficient sparse decomposition algorithms. In particular, we have extended our work addressing the problem of finding good sparse representations into a probabilistic framework. First, we have proposed a new family of pursuit algorithms able to take into account any type of dependence (e.g. spatial or temporal) between the atoms of the sparse decomposition. This work has led to the publication of a paper in the proceedings of the international conference IEEE ICASSP 2012.

Exploiting further this probabilistic framework, we have then considered the design of structured soft pursuit algorithms. In particular, instead of making hard decisions on the support of the sparse representation and the amplitude of the non-zero coefficients, our soft procedures iteratively update probability on the latter values. The proposed algorithms are designed within the framework of the mean-field approximations and resort to the so-called variational Bayes EM algorithm to implement an efficient minimization of a Kullback-Leibler criterion. On the other hand, the proposed methodologies can handle "structured" sparse representations, that is, sparse decompositions where some dependence exists between the non-zero elements of the support. The prior model on the support of the sparse decomposition is based on a Boltzmann machine which encompasses as particular cases many type of dependence (Markov chain, Ising model, tree-like structure, etc). This work has been published in the journal IEEE Trans. on Signal Processing in 2012.

6.2. Tracking and data assimilation

6.2.1. Stochastic filtering for fluid motion tracking

Participants: Sébastien Béyou, Anne Cuzol, Etienne Mémin.

We investigated the study of a recursive Bayesian filter for tracking velocity fields of fluid flows. The filter combines an Ito diffusion process associated to 2D vorticity-velocity formulation of Navier-Stokes equation and discrete image error reconstruction measurements. In contrast to usual filters designed for visual tracking problems, our filter combines a continuous law for the description of the vorticity evolution with discrete image measurements. We resort to a Monte-Carlo approximation based on particle filtering. The designed tracker provides a robust and consistent estimation of instantaneous motion fields along the whole image sequence.

When the likelihood of the measurements can be modeled as a Gaussian law, we have also investigated the use of the so-called ensemble Kalman filtering for fluid tracking problems. This kind of filters introduced for the analysis of geophysical fluids is based on the Kalman filter update equation. Nevertheless, unlike traditional Kalman filtering setting, the covariances of the estimation errors, required to compute the so-called Kalman gain, relies on an ensemble of forecasts. Such a process gives rise to a Monte-Carlo approximation for a family of non-linear stochastic filters enabling to handle state spaces of large dimension. We have recently proposed an extension of this technique that combines sequential importance sampling and the propagation law of an ensemble Kalman filter. This technique leads to an ensemble Kalman filter with an improved efficiency. We have in particular investigated the introduction of a nonlinear direct image measurement operator within this ensemble Kalman scheme. This modification of the filter provides very good results on 2D numerical and experimental flows even in the presence of strong noises. We are currently assessing its application to oceanic satellite images for the recovering of ocean streams. We are also studying the impact on the stochastic dynamics of turbulent noise defined as auto-similar Gaussian random fields and the introduction within an incremental ensemble analysis scheme of multiscale motion measurements. This work has been recently accepted for publication in the Tellus A journal [17].

6.2.2. Reduced-order models for flows representation from image data

Participants: Cédric Herzet, Etienne Mémin, Véronique Souchaud.

One of the possibilities to neglect the influence of some degrees of freedom over the main characteristics of a flow consists in representing it as a sum of K orthonormal spatial basis functions weighted with temporal coefficients. To determine the basis function of this expansion, one of the usual approaches relies on the Karhunen-Loeve decomposition (refered to as proper orthogonal decomposition – POD – in the fluid mechanics domain). In practice, the spatial basis functions, also called modes, are the eigenvectors of an empirical auto-correlation matrix which is built from “snapshots" of the considered physical process.

In this axis of work we focus on the case where one does not have a direct access to snapshots of the considered physical process. Instead, the POD has to be built from the partial and noisy observation of the physical phenomenon of interest. Instances of such scenarios include situations where real instantaneous vector-field snapshots are estimated from a sequence of *images*. We have been working on several approaches dealing with such a new paradigm. A first approach consists in extending standard penalized motion-estimation algorithms to the case where the sought velocity field is constrained to span a low-dimensional subspace. In particular, we have considered scenarios where the standard optical flow constraint (OFC) is no longer statisfied and one has therefore to resort to a Discrete Finite Difference (DFD) model. The non-linearity of the latter leads to several practical issues that we have addressed this year. We are currently assessing the performance of the proposed method on experimental data in order to validate its relevance in practical scenarios. In a second approach we have studied two variational data assimilation techniques for the estimation of low order dynamical models for fluid flows. Both methods are built from optimal control recipes and rely on POD representation associated to Galerkin projection of the Navier Stokes equations. The proposed techniques differ in the control variables they involve. The first one introduces a weak dynamical model defined only up to an additional uncertainty time dependent function whereas the second one, handles a strong dynamical constraint in which the coefficients of the dynamical system constitute the control variables. Both choices correspond to different approximations of the relation between the reduced basis on which is expressed the motion feld and the basis components that have been neglected in the reduced order model construction. The techniques have been assessed on numerical data and for real experimental conditions with noisy Image Velocimetry data. This work has been published in the Journal of Computational Physics [15]. In collaboration with the University of Buenos Aires, we have also explored, a method that combines Proper Orthogonal Decomposition with a spectral technique to analyze and extract reduced order models of flows from time resolved data of velocity fields. This methodology, relying on the eigenfunctions of the Koopman operator, is specifically adapted to flows with quasi periodic orbits in the phase space. The technique is particularly suited to cases requiring a discretization with a high spatial and temporal resolution. The proposed analysis enables to decompose the flow dynamics into modes that oscillate at a single frequency. For each modes an energy content and a spatial structure can be put in correspondence. This approach has been assessed for a wake flow behind a cylinder at Reynolds number 3900 and has been recently accepted under minor revisions condition to the journal of Theoretical and Computational Fluid Dynamics.

6.2.3. Optimal control techniques for the coupling of large scale dynamical systems and image data

Participants: Dominique Heitz, Etienne Mémin, Cordelia Robinson, Yin Yang.

This work aims at investigating the use of optimal control techniques for the coupling of Large Eddies Simulation (LES) techniques and 2D image data. The objective is to reconstruct a 3D flow from a set of simultaneous time resolved 2D image sequences visualizing the flow on a set of 2D plans enlightened with laser sheets. This approach will be experimented on shear layer flows and on wake flows generated on the wind tunnel of Irstea Rennes. Within this study we wish also to explore techniques to enrich large-scale dynamical models by the introduction of uncertainty terms or through the definition of subgrid models from the image data. This research theme is related to the issue of turbulence characterization from image sequences. Instead of predefined turbulence models, we aim here at tuning from the data the value of coefficients involved in traditional LES subgrid models or in longer-term goal to learn empirical subgrid models directly from image data. An accurate modeling of this term is essential for Large Eddies Simulation as it models all the non resolved motion scales and their interactions with the large scales.

We have pursued the first investigations on a 4DVar assimilation technique, integrating PIV data and Direct Numerical Simulation (DNS), to reconstruct two-dimensional turbulent flows. The problem we are dealing with consists in recovering a flow obeying Navier-Stokes equations, given some noisy and possibly incomplete PIV measurements of the flow. By modifying the initial and inflow conditions of the system, the proposed method reconstructs the flow on the basis of a DNS model and noisy measurements. The technique has been evaluated in the wake of a circular cylinder. It denoises the measurements and increases the spatiotemporal resolution of PIV time series. These results have been conditionally accepted for publication in Journal of Computational Physics. Along the same line of studies we have started to investigate the 3D case. The goal consists here to reconstruct a 3D flow from a set of simultaneous time resolved 2D images of planar sections of the 3D volume. This work is mainly conducted within the PhD of Cordelia Robinson. The development of the variational assimilation code has been initiated within a collaboration with A. Gronskis, S. Laizé (lecturer, Imperial College, UK) and Eric Lamballais (institut P' Poitiers).

6.2.4. Free surface flows reconstruction and tracking

Participants: Benoît Combes, Dominique Heitz, Etienne Mémin, Cordelia Robinson, Yin Yang.

Characterizing a free-surface flow (space and time-dependent velocity and geometry) given observations/measures at successive times is an ubiquitous problem in fluid mechanic and in hydrology. Observations can consist of e.g. measurements of velocity, or like in this work of measurements of the geometry of the free-surface. Indeed, recently developed depth/range sensors allow to capture directly a rough 3D geometry of surfaces with high space and time resolution. We have investigated the performance of the Kinect and have shown that it is likely to capture temporal sequences of depth observations of wave-like surfaces with wavelengths and amplitudes sufficiently small to characterize medium/large scale flows. Several data assimilation methods have been experimented and compared to estimate both time dependent geometry and displacement field associated to a free-surface flow from a temporal sequence of Kinect data. This study have been conducted on synthetic and real-world data. It has been presented to a data assimilation conference [35]. Finally, we explored the application of such techniques to hydrological applications. These results are currently considered for submission to Journal of Hydrology.

6.2.5. Stochastic filtering technique for the tracking of closed curves

Participants: Christophe Avenel, Etienne Mémin.

We have proposed a filtering methodology for the visual tracking of closed curves. Opposite to works of the literature related to this issue, we consider here a curve dynamical model based on a continuous time evolution law with different noise models. This led us to define three different stochastic differential equations that capture the uncertainty relative to curve motions. This new approach provides a natural understanding of classical level-set dynamics in terms of such uncertainties. These evolution laws have been combined with various color and motion measurements to define probabilistic state-space models whose associated Bayesian filters can be handled with particle filters. This ongoing work will be continued within extensive curve tracking experiments and extended to the tracking of other very high dimensional entities such as vector fields and surfaces. This work, which corresponds to the PhD thesis of Christoph Avenel has been presented in several conferences and has been submitted to two different journals. Let us note that it has also led to a fruitful collaboration with MeteoFrance [30]

6.2.6. *Sequential smoothing for fluid motion*

Participants: Anne Cuzol, Etienne Mémin.

In parallel to the construction of stochastic filtering techniques for fluid motions, we have proposed a new sequential smoothing method within a Monte-Carlo framework. This smoothing aims at reducing the temporal discontinuities induced by the sequential assimilation of discrete time data into continuous time dynamical models. The time step between observations can indeed be long in environmental applications for instance, and much longer than the time step used to discretize the model equations. While the filtering aims at estimating the state of the system at observations times in an optimal way, the objective of the smoothing is to improve the estimation of the hidden state between observation times. The method is based on a Monte-Carlo approximation of the filtering and smoothing distributions, and relies on a simulation technique of conditional diffusions. The proposed smoother can be applied to general non linear and multidimensional models. It has been applied to a turbulent flow in a high-dimensional context, in order to smooth the filtering results obtained from a particle filter with a proposal density built from an Ensemble Kalman procedure. This conditional simulation framework can also be used for filtering problem with low measurement noise. This has been explored through a collaboration with Jean-Louis Marchand (LORIA) in the context of vorticity tracking from image data.

6.2.7. *Stochastic fluid flow dynamics under uncertainty*

Participant: Etienne Mémin.

In this research axis we aim at devising stochastic Eulerian expressions for the description of fluid flow evolution laws incorporating uncertainty on the particles location. Such an uncertainty modeled through the introduction of a random term, allows taking into account approximations or truncation effects performed within the dynamics analytical constitution steps. This includes for instance the modeling of unresolved scales interaction in large eddies simulation (LES) or in Reynolds average numerical simulation (RANS), but also uncertainties attached to non-uniform grid discretization. This model is mainly based on a stochastic version of the Reynolds transport theorem. Within this framework various simple expressions of the mean drift component can be exhibited for different models of the random field carrying the uncertainties we have on the flow. We aim at using such a formalization within image-based data assimilation framework and to derive appropriate stochastic versions of geophysical flow dynamical modeling.

6.2.8. *Variational assimilation of images for large scale fluid flow dynamics with uncertainty*

Participants: Souleymane Kadri Harouna, Etienne Mémin.

In this work we explore the assimilation of a large scale representation of the flow dynamics with image data provided at a finer resolution. The velocity field at large scales is described as a regular smooth components whereas the complement component is a highly oscillating random velocity field defined on the image grid but living at all the scales. Following this route we have started to assess the performance of a variational assimilation technique with direct image data observation. Preliminary encouraging results obtained for a wavelet-based 2D Navier Stokes implementation and images of a passive scalar transported by the flow have been obtained. Large-scale simulation under uncertainty for the 3D viscous Taylor-Green vortex flow have been carried out and show promising results of the approach.

6.3. Analysis and modeling of turbulent flows

6.3.1. *Hot-wire anemometry at low velocities*

Participant: Dominique Heitz.

A new dynamical calibration technique has been developed for hot-wire probes. The technique permits, in a short time range, the combined calibration of velocity, temperature and direction calibration of single and multiple hot-wire probes. The calibration and measurements uncertainties were modeled, simulated and controlled, in order to reduce their estimated values. Based on a market study the french patent application has been extended this year to a Patent Cooperation Treaty (PCT) application.

6.3.2. Numerical and experimental image and flow database

Participant: Dominique Heitz.

The goal was to design a database for the evaluation of the different techniques developed in the Fluminance group. The main challenge was to enlarge a database mainly based on two-dimensional flows, with three-dimensional turbulent flows. New synthetic image sequences based on homogeneous isotropic turbulence and on circular cylinder wake have been provided. These images have been completed with real image sequences based on wake and mixing layers flows. This new database provides different realistic conditions to analyse the performance of the methods: time steps between images, level of noise, Reynolds number, large-scale images.

6.4. Visual servoing approach for fluid flow control

6.4.1. Fully exploitation of the controlled degrees of freedom of the 2D plane Poiseuille flow

Participants: Christophe Collewet, Xuan Quy Dao.

This works concerns the Phd of Xuan-Quy Dao and can be seen as an extension of the works carried out by Romeo Tatsambon during its post-doc position. Indeed, during this post-doc we proved that our vision-based approach overcomes the traditional approaches. Nevertheless, to compare our method with the literature, we used a traditional control law, the LQR control law. However, we can fully exploit the capabilities of visual servoing techniques by designing a more efficient control law than the LQR one. This has been done this year. We have validated our approach to the problem of minimizing the drag of the 2D plane Poiseuille flow. An important issue was also to ensure that, during the process of drag reduction, the kinetic energy density will not grow. This is of great importance since it is well known that the controlled flow may become turbulent when this kinetic energy density is growing. To cope with this problem we have proposed to design a control law based on partitioned visual servo. Indeed, following this way, we are able to simultaneously minimize the drag AND the kinetic energy density in contrast to the existing approaches. This work has been accepted to the "American control conference (ACC'12)", to the "Conférence internationale francophone d'automatique (CIFA'12)" and to the "6th AIAA Flow Control Conference". We have also explored an approach based on eigenstructure placement to ensure a strict decrease of the kinetic energy density. Another approach has also been explored, it tends to decouple all the controlled degrees of freedom of the system so that an exponential decoupled decrease of each component of the state vector is obtained. The great advantage of this approach is that all quantities depending on the state vector (like the drag) are also exponential decreasing functions.

6.4.2. Control behind a backward-facing step

Participant: Christophe Collewet.

Instead of setting up an experimental closed loop control problem for the plane Poiseuille flow with temporal perturbations, which is theoretically based on an unrealistic infinite channel, we explore in this axis of work a closed-loop control of a flow behind a backward step. The control is expressed through the visual servoing formalism and fast velocity measurements in the recirculation zone. This work is performed in the context of the PhD thesis of Nicolas Gautier from PMMH-ESCPI. This thesis is co-supervised with Jean-Luc Aider (CNRS/PMMH-ESPCI).

7. Partnerships and Cooperations

7.1. Regional Initiatives

7.1.1. Britanny concil ARED IMAGEO:

Participants: Cédric Herzet, Etienne Mémin, Véronique Souchaud.

duration 36 months. This project of the Britanny concil, which finances the PhD thesis of Véronique Souchaud, aims at studying methods for the estimation of reduced order modeling of fluid flows evolution laws from image sequences. The goal consists here at defining the estimation of a reduced basis describing the flow evolution as a motion estimation problem.

7.2. National Initiatives

7.2.1. *ANR-COSINUS PREVASSEMBLE: Ensemble methods for assimilation of observaions and for prevision in Meteorology and Oceanography*

Participants: Sébastien Béyou, Anne Cuzol, Etienne Mémin.

duration 36 months.

The purpose of this project is to further study ensemble methods -, and to develop their use for both assimilation of observations and prediction. Among the specific questions to be studied are the theory of Particle Filters and Ensemble Kalman Filters, the possibility of taking temporal correlation into account in ensemble assimilation, the precise assessment of what can and cannot be achieved in ensemble prediction, and the objective validation of ensemble methods.

The partners of this project are Laboratoire de Météorologie Dynamique/ENS (leader), Météo-France and three Inria groups (ALEA, ASPI, FLUMINANCE).

7.2.2. *ANR SYSCOMM MSDAG: MultiScale Data Assimilation in Geophysics*

Participants: Pierre Dérian, Patrick Héas, Dominique Heitz, Cédric Herzet, Etienne Mémin.

duration 36 months.

Changing scale is a well-known topic in physics (geophysics, fluid mechanics and turbulence, theoretical and statistical physics, mechanics, porous media, etc.) It has lead to the creation of powerful sophisticated mathematical tools: renormalization, homogenization, etc. These ideas are also used in numerical analysis (the so-called multigrid approach) for solving efficiently partial differential equations. Data assimilation in geophysics is a set of methods that allows to combine optimally numerical models in large spaces with large dataset of observations. At the confluence of these two topics, the goal of this project is to study how to embed the change of scales (a multiscale point of view) issue into the framework of geophysical data assimilation, which is a largely unexplored subject.

The partners of this 3 years project are the CEREA/ CLIME Inria group (leader), the LSCE/CEA, the Inria groups MOISE and FLUMINANCE.

7.2.3. *ANR SYSCOMM GeoFluids:*

Participants: Patrick Héas, Dominique Heitz, Souleymane Kadri Harouna, Etienne Mémin, Véronique Souchaud.

duration 48 months.

The project Geo-FLUIDS focuses on the specification of tools to analyse geophysical fluid flows from image sequences. Geo-FLUIDS aims at providing image-based methods using physically consistent models to extract meaningful features describing the observed flow and to unveil the dynamical properties of this flow. The main targeted application domains concern Oceanography and Meteorology . The project consortium gathers the Inria research groups: FLUMINANCE (leader), CLIME, IPSO, and MOISE. The group of the “Laboratoire de Météorologie Dynamique” located at the ENS Paris, the IFREMER-CERSAT group located at Brest and the METEOFRANCE GMAP group in Toulouse.

7.3. International Initiatives

7.3.1. Inria Associate Teams

7.3.1.1. HURACAN

Title: Analysis and control of fluid flows from image sequences

Inria principal investigator: Etienne Memin

International Partners (Institution - Laboratory - Researcher):

IRSTEA (France)

University of Buenos Aires (Argentina)

Duration: 2010 - 2012

See also: http://huracan.inria.fr

The HURACAN associated team is centered on the analysis and the control of fluid flows from image sequences. The research objectives of this team are organized into two distinct work axes. The first one aims at defining and studying visual servoing techniques for fluid flows control. In addition to the definition of efficient visual servoing schemes this axis of work gathers research issues related to fluid flows velocity measurement from images and to flows excitation through plasma actuators. The second research axis focuses on the coupling between large scales representations of geo-physical flows and image data. More precisely, it aims at studying means to define directly from the image sequences the small scales terms of the dynamics. This research axis includes the study of coupling models and data defined at different scales, problems of multiscale velocities estimation respecting turbulence phenomenological laws and issues of experimental validation.

7.3.2. Participation In International Programs

STIC AmSud project "Physics-based modeling of voice production" leaded by D. Sciamarella CNRS/LFD-FIUBA.

This project is an interdisciplinary project with researchers spanning from aeroacoustics to physiology, and from computational physics to phonetics. It aims at studying the mechanisms of human voice production system for applications ranging from man-machine communication to medical diagnosis.

8. Dissemination

8.1. Scientific Animation

Christophe Collewet

- Member of the irstea evaluation committee
- Member of the ecotechnologies department committee

Patrick Héas

- Seminar Image group GREYC, December 2012
- Reviewing in IEEE trans. on Geosciences and Remote Sensing (TGRS) and in IEEE trans. on Image Processing (IP)

Dominique Heitz

- Member of IRSTEA "Comité directeur des Systèmes d'Information"
- Member of IRSTEA "Comité Technique Spécial"
- Responsible of the IRSTEA ACTA Team

Cédric Herzet

- Organization commitee "Journées "Apprentissage et Parcimonie" October 2012
- Technical program committees of ICASSP 2013 and SPARS 2013
- project reviewer for the "Fond National de la Recherche Scientique" (FNRS), Belgique

Etienne Mémin

- Invited conference "Conf'luences" IMFT, Toulouse, April 2012.
- Invited conference "Journée LEFE/MANU de l'INSU", ENS Paris, March 2012
- Invited conference "Workshop on Tolopology in Fluid flow visualization", Pisa, June 2012
- Seminar LFD-FIUBA, Buenos Aires, November 2012
- Reviewer of FET Open projects
- Reviewer for NWO (Netherlands Organisation for Scientific Research) Vici projects
- Associate editor of the Journal of Computer Vision
- Associate editor of the journal of Image and Vision Computing
- Scientific committee and Organisation committee of the International Conference on Ensemble Methods in Geophysical Sciences, Toulouse, November 2012
- Reviewing for Tellus-A, IEEE Im. Proc., IEEE trans. Pat. Anal. Mach. Intel. , Int. Journ. Comp. Vis., Im. Vis. Comp., Exp in Fluids, ECCV'12
- Responsible of the "Commission Développement Technologique" Inria-IRISA Rennes
- member of the "Commission Personnel" Inria-IRISA Rennes

8.2. Teaching - Supervision - Juries

8.2.1. Teaching

Licence : Dominique Heitz, Mécanique des fluides, 30h, niveau L2 INSA Rennes

Master : Patrick Héas, Modélisation Statistique des Images, Mastere SISEA, 15h , niveau M2, Université de Rennes 1.

Master : Dominique Heitz, Mécanique des fluides, 25h, niveau M1, Dep GMA INSA Rennes

Master : Cedric Herzet, Analyse de données, Mastere de Statistiques et Econométrie, 10h, niveau M1, Université de Rennes I

Master : Etienne Mémin, Analyse du mouvement, Mastere Informatique, 15h, niveau M2, Université de Rennes 1.

Master : Etienne Mémin, Vision par ordinateur , 15h, niveau M2, ESIR Université de Rennes 1.

8.2.2. Supervision

PhD & HdR :

PhD : Christophe Avenel, Suivi de courbes libres fermées déformables par processus stochastiques, Université de Rennes I, 08/12/2011, Etienne Mémin et Patrick Pérez

PhD : Pierre Dérian, Estimation multi-échelle de mouvements de Fluides turbulents dans des séquences d'images,Université de Rennes I, 07/11/2012, Patrick Héas et Etienne Mémin

PhD in progress : Ioana Barbu, Estimation volumique de mouvement fluides /'a partir de séquence d'images, 01/11/2010, Cédric Herzet et Etienne Mémin

PhD in progress : Sébastien Béyou, Filtre d'ensemble pour l'assimilation de données images, 01/11/2010, Francois Le Gland et Etienne Mémin

PhD in progress : Qui Dao, Commande des écoulements fluides par asservissement visuel, 01/10/2010, Christophe Collewet

PhD in progress : Cordelia Robinson, Assimilation de données images dans un modèle LES : application à la reconstruction d'écoulements turbulents tridimensionnels , 01/11/2011, Dominique Heitz et Etienne Mémin

PhD in progress : Yin Yang , Assimilation d'images par techniques variationnelle ensembliste , 01/11/2011, Etienne Mémin

PhD in progress : Véronique Souchaud, Estimation directe de décomposition orthogonales propres à partir de séquences d'images, 01/11/2009, Cédric Herzet et Etienne Mémin

8.2.3. Juries

Etienne Mémin

- Président du Jury de th/'ese de Pierre Alain, Université de Bretagne Sud, mai 2011.
- Examinateur Jury HDR d'Arthur Vidard, Université J. Fourier, decembre 2012.

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