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Activity Report 2012

Project-Team GALAAD

Geometry, algebra, algorithms

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

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THEME
Algorithms, Certification, and Cryptography

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Project-Team GALAAD

Keywords: Algorithmic Geometry, Computer Algebra, Real Numbers

Galaad is a joint project with Laboratoire J.A. Dieudonné U.M.R. C.N.R.S. n^{O} 6621, University of Nice Sophia-Antipolis.

Creation of the Project-Team: April 01, 2002 .

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2. Overall Objectives

2.1. Overall Objectives

There is a shared vision that our day life environment will increasingly interact with a digital world, populated by captors, sensors, or devices used to simplify or improve some of our activities. Digital cameras, positioning systems, mobile phones, internet web interfaces are such typical examples which are nowadays completely standard tools. Interconnected with each other, these devices are producing, exchanging or processing digital data in order to interact with the physical world. Computing is becoming ubiquitous and this evolution raises new challenges to represent, analyze and transform this digital information.

From this perspective, geometry is playing an important role. There is a strong interaction between physical and digital worlds through geometric modeling and analysis. Understanding a physical phenomena can be done by analyzing numerical simulations on a digital representation of the geometry. Conversely developing digital geometry (as in Computer Aided Geometric Design – CAGD for short) is nowadays used to produce devices to overcome some physical difficulties (car, planes, ...). Obviously, geometry is not addressing directly problems related to storage or transmission of information, but it deals with structured and efficient representations of this information and methods to compute with these models.

Within this context, our research program aims at developing new and efficient methods for modeling geometry with algebraic representations. We don't see shapes just as set of points with simple neighbor information. In our investigations, we use richer algebraic models which provide structured and compact representation of the geometry, while being able to encode their important characteristic features.

The first challenge to be addressed is how to move from the digital world to an algebraic world. Our objective is to develop efficient methods which can transform digital data produced by cameras, laser scanners, observations or simulations into algebraic models involving few parameters. This is a way to structure the digital information and to further exploit its properties. This methodological investigations are connected with practical problems such as compression of data for exchange of geometric information, accurate description and simulation with manufactured objects, shape optimization in computer aided design, ...

A second challenge concerns operations and transformations on these algebraic representations. They require the development of dedicated techniques which fully exploit the algebraic characteristics of these representations. The theoretical foundations of our investigations are in algebraic geometry. This domain deals with the solutions of algebraic equations and its effective aspect concerns algorithms to compute and analyze them. It is an old, important and very active part of mathematics. Its combination with algorithmic developments for algebraic computation leads to new methods to treat effectively geometric problems. These investigations result in new contribution in commutative algebra, new algorithms in computer algebra, complexity analyses and/or software development for practical experimentation.

The third challenge is how to analyze and understand digital geometric data. In this approach, constructing algebraic representation and developing methods to compute with these models are the preliminary steps of our analysis process. The goal is to develop methods to extract some type of information we are searching from this data, such as topological descriptions, subdivisions in smooth components and adjacency relations, decomposition in irreducible components. The interplay between algebraic models and numerical computation is central in this activity. A main issue concerns the approximation of models and the certification of the computation.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is structured according to three broad topics:

1. **Algebraic representations for geometric modeling**.
2. **Algebraic algorithms for geometric computing**,
3. **Symbolic-numeric methods for analysis**,

3.2. Algebraic representations for geometric modeling

Compact, efficient and structured descriptions of shapes are required in many scientific computations in engineering, such as "Isogeometric" Finite Elements methods, point cloud fitting problems or implicit surfaces defined by convolution. Our objective is to investigate new algebraic representations (or improve the existing ones) together with their analysis and implementations.

We are investigating representations, based on semi-algebraic models. Such non-linear models are able to capture efficiently complexes shapes, using few data. However, they required specific methods to solve the underlying non-linear problems, which we are investigating.

Effective algebraic geometry is a natural framework for handling shape representations. This framework not only provides tools for modeling but it also allows to exploit rich geometric properties.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

The use of piecewise algebraic representations also raises problems of approximation and reconstruction, on which we are working on. In this direction, we are studying B-spline function spaces with specified regularity associated to domain partitions.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. We are interested in exploiting these invariant properties, to develop compact and adapted representations of shapes.

3.3. Algebraic algorithms for geometric computing

This topic is directly related to polynomial system solving and effective algebraic geometry. It is our core expertise and many of our works are contributing to this area.

Our goal is to develop algebraic algorithms to efficiently perform geometric operations such as computing the intersection or self-intersection locus of algebraic surface patches, offsets, envelopes of surfaces, ...

The underlying representations behind the geometric models we consider are often of algebraic type. Computing with such models raises algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

Geometric objects are often described in a parametric form. For performing efficiently on these objects, it can also be interesting to manipulate implicit representations. We consider particular projections techniques based on new resultant constructions or syzygies, which allow to transform parametric representations into implicit ones. These problems can be reformulated in terms of linear algebra. We investigate methods which exploit this matrix representation based on resultant constructions.

They involve structured matrices such as Hankel, Toeplitz, Bezoutian matrices or their generalization in several variables. We investigate algorithms that exploit their properties and their implications in solving polynomial equations.

We are also interested in the “effective” use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems.

We are also interested in subdivision methods, which are able to efficiently localise the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue while developing these methods is to analyse their practical and algorithmic behavior. Our aim is to obtain good complexity bounds and practical efficiency by exploiting the structure of the problem.

3.4. Symbolic numeric analysis

While treating practical problems, noisy data appear and incertitude has to be taken into account. The objective is to devise adapted techniques for analyzing the geometric properties of the algebraic models in this context.

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many context, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate the geometric objects while controlling the quality of approximation. We investigate subdivision techniques based on the algebraic formulation of our problems which allow us to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

The problem of decomposition and factorisation is also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. A typical problem in this direction is the problem of approximate factorization, which requires to analyze perturbations of the data, which enables us to break up the problem.

4. Application Domains

4.1. Shape modeling

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, etc. Optimising these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship with the industrial contacts that we are developing in Computer Aided Geometric Design.

4.2. Shape processing

Many problems encountered in the application of computer sciences start from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where the geometry of distances plays a significant role, for example, in the reconstruction from NMR (Nuclear Magnetic Resonance) experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tend to be more and more easily used yield large set of data points from which one has to recover compact geometric model. We are working in collaboration with groups in agronomy on the problems of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems. Geometry is also highly involved in the numerical simulation of physical problems such as heat conduction, ship hull design, blades and turbines analysis, mechanical stress analysis. We apply our algebraic-geometric techniques in the isogeometric approach which uses the same (bspline) formalism to represent both the geometry and the solutions of partial differential equations on this geometry.

5. Software

5.1. Mathemagix, a free computer algebra environment

Participant: Bernard Mourrain.

http://www.mathemagix.org/

algebra, univariate polynomial, multivariate polynomial, matrices, series, fast algorithm, interpreter, compiler, hybrid software.

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments, based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on the embeding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

The project aims at building a bridge between symbolic computation and numerical analysis. It is structured by collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

- REALROOT: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms; exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterised, implicit with different type of coefficients; algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler AXEL.

Collaborators: Grégoire Lecerf, Joris van der Hoeven and Philippe Trébuchet.

5.2. Axel, a geometric modeler for algebraic objects

Participants: Anaïs Ducoffe, Bernard Mourrain, Meriadeg Perrinel.

http://axel.inria.fr.

computational algebraic geometry, curve, implicit equation, intersection, parameterisation, resolution, surface, singularity, topology

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The software contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

The developments related to isogeometric analysis have been integrated as dedicated plugins. Optimisation techniques and solvers for partial differential equations developed by R. Duvigneau (OPALE) have been connected.

A new version of the algebraic-geometric modelers is developed by Meriadeg Perinnel to connect it to the platform Dtk in order to provide a better modularity and a better interface to existing computation facilities and geometric rendering interface.

The package is distributed as binary packages for Linux as well as for MacOSX. It is hosted at Inria's gforge (http://gforge.inria.fr) and referenced by many leading software websites such as http://apple.com. The first version of the software has been downloaded more than 15000 times, since it is available.

Collaboration with Gang Xu (Hangzhou Dianzi University, China), Julien Wintz (Dream).

6. New Results

6.1. Algebraic representations for geometric modeling

6.1.1. Fitting ideals and multiple-points of surface parameterizations

Participants: Nicolàs Botbol, Laurent Busé.

Given a birational parameterization ϕ of an algebraic surface $\mathcal{S}$ in the projective space $\mathbb{P}^3$, the purpose of this ongoing work is to investigate the sets of points $D_k(\phi)$ on $\mathcal{S}$ whose preimage consists in k or more points, counting multiplicity. Our main result is an explicit description of these algebraic sets $D_k(\phi)$ in terms of Fitting ideals of some graded parts of a symmetric algebra associated to the parameterization ϕ.

This work is done in collaboration with Marc Chardin (University Pierre et Marie Curie).

6.1.2. Algebraic geometry tools for the study of entanglement: an application to spin squeezed states

Participant: Alessandra Bernardi.

In [18] a short review of Algebraic Geometry tools for the decomposition of tensors and polynomials is given from the point of view of applications to quantum and atomic physics. Examples of application to assemblies of indistinguishable two-level bosonic atoms are discussed using modern formulations of the classical Sylvester's algorithm for the decomposition of homogeneous polynomials in two variables. In particular, the symmetric rank and symmetric border rank of spin squeezed states is calculated as well as their Schrödinger-cat-like decomposition as the sum of macroscopically different coherent spin states; Fock states provide an example of states for which the symmetric rank and the symmetric border rank are different.

This is a joint work with I. Carusotto (University of Trento, Italy).

6.1.3. A partial stratification of secant varieties of Veronese varieties via curvilinear subschemes.

Participant: Alessandra Bernardi.

In [11] we give a partial quasi-stratification of the secant varieties of the order d Veronese variety $X_{m,d}$ of $\mathbb{P}^m$. It covers the set $\sigma_t(X_{m,d})^\dagger$ of all points lying on the linear span of curvilinear subschemes of $X_{m,d}$, but two quasi-strata may overlap. For low border rank, two different quasi-strata are disjoint and we compute the symmetric rank of their elements. Our tool is the Hilbert schemes of curvilinear subschemes of Veronese varieties. To get a stratification we attach to each $P \in \sigma_t(X_{m,d})^\dagger$ the minimal label of a quasi-stratum containing it.

This is a joint work with E. Ballico (University of Trento, Italy).

6.1.4. Decomposition of homogeneous polynomials with low rank.

Participant: Alessandra Bernardi.

Let F be a homogeneous polynomial of degree d in $m+1$ variables defined over an algebraically closed field of characteristic 0 and suppose that F belongs to the s-th secant variety of the d-uple Veronese embedding of $\mathbb{P}^m$ into $\mathbb{P}^{\binom{m+d}{d}-1}$ but that its minimal decomposition as a sum of d-th powers of linear forms $M_1, ..., M_r$ is $F = M_1^d + \cdots + M_r^d$ with $r > s$. In [12], we show that if $s + r \leq 2d + 1$ then such a decomposition of F can be split into two parts: one of them is made by linear forms that can be written using only two variables. The other part is uniquely determined once one has fixed the first part. We also obtain a uniqueness theorem for the minimal decomposition of F if r is at most d and a mild condition is satisfied.

This is a joint work with E. Ballico (University of Trento, Italy).

6.1.5. Higher secant varieties of $\mathbb{P}^n \times \mathbb{P}^1$ embedded in bi-degree (a, b)

Participant: Alessandra Bernardi.

In [15], we compute the dimension of all the higher secant varieties to the Segre-Veronese embedding of $\mathbb{P}^n \times \mathbb{P}^1$ via the section of the sheaf $\mathcal{O}(a, b)$ for any $n, a, b \in \mathbb{Z}^+$. We relate this result to the Grassmann Defectivity of Veronese varieties and we classify all the Grassmann $(1, s-1)$-defective Veronese varieties.

This is a joint work with E. Ballico, M. V. Catalisano (University of Trento, Italy).

6.1.6. Symmetric tensor rank with a tangent vector: a generic uniqueness theorem

Participant: Alessandra Bernardi.

Let $X_{m,d} \subset \mathbb{P}^N$, $N := \binom{m+d}{m} - 1$, be the order d Veronese embedding of $\mathbb{P}^m$. Let $\tau(X_{m,d}) \subset \mathbb{P}^N$, be the tangent developable of $X_{m,d}$. For each integer $t \geq 2$ let $\tau(X_{m,d}, t) \subseteq \mathbb{P}^N$, be the join of $\tau(X_{m,d})$ and $t-2$ copies of $X_{m,d}$. In [13], we prove that if $m \geq 2$, $d \geq 7$ and $t \leq 1 + \lfloor \binom{m+d-2}{m} / (m+1) \rfloor$, then for a general $P \in \tau(X_{m,d}, t)$ there are uniquely determined $P_1, \cdots, P_{t-2} \in X_{m,d}$ and a unique tangent vector ν of $X_{m,d}$ such that P is in the linear span of $\nu \cup \{P_1, \cdots, P_{t-2}\}$. In other words, a degree d linear form f (a symmetric tensor T of order d) associated to P may be written as

$$f = L_{t-1}^{d-1} L_t + \sum_{i=1}^{t-2} L_i^d, \quad (T = v_{t-1}^{\otimes(d-1)} v_t + \sum_{i=1}^{t-2} v_i^{\otimes d})$$

with L_i linear forms on $\mathbb{P}^m$ (v_i vectors over a vector field of dimension $m+1$ respectively), $1 \leq i \leq t$, that are uniquely determined (up to a constant).

This is a joint work with E. Ballico (University of Trento, Italy).

6.1.7. General tensor decomposition, moment matrices and applications.

Participants: Alessandra Bernardi, Bernard Mourrain.

In [17] the tensor decomposition addressed may be seen as a generalisation of Singular Value Decomposition of matrices. We consider general multilinear and multihomogeneous tensors. We show how to reduce the problem to a truncated moment matrix problem and give a new criterion for flat extension of Quasi-Hankel matrices. We connect this criterion to the commutation characterisation of border bases. A new algorithm is described. It applies for general multihomogeneous tensors, extending the approach of J.J. Sylvester to binary forms. An example illustrates the algebraic operations involved in this approach and how the decomposition can be recovered from eigenvector computation.

This is a joint work with J. Brachat and P. Comon (i3S, CNRS).

6.1.8. On the cactus rank of cubic forms

Participant: Alessandra Bernardi.

In this work, we prove that the smallest degree of an apolar 0-dimensional scheme of a general cubic form in $n+1$ variables is at most $2n+2$, when $n \geq 8$, and therefore smaller than the rank of the form. For the general reducible cubic form the smallest degree of an apolar subscheme is $n+2$, while the rank is at least $2n$.

This is a joint work with K. Ranestad (University of Oslo, Norway) that will be published in 2013 in the Journal of Symbolic Computation. The preprint is available at http://hal.inria.fr/inria-00630456.

6.1.9. Tensor ranks on tangent developable of Segre varieties

Participant: Alessandra Bernardi.

In [14] we describe the stratification by tensor rank of the points belonging to the tangent developable of any Segre variety. We give algorithms to compute the rank and a decomposition of a tensor belonging to the secant variety of lines of any Segre variety. We prove Comon's conjecture on the rank of symmetric tensors for those tensors belonging to tangential varieties to Veronese varieties.

This is a joint work with E. Ballico (University of Trento, Italy).

6.1.10. On the dimension of spline spaces on planar T-meshes

Participant: Bernard Mourrain.

In [33], we analyze the space of bivariate functions that are piecewise polynomial of bi-degree $\leq (m, m')$ and of smoothness r along the interior edges of a planar T-mesh. We give new combinatorial lower and upper bounds for the dimension of this space by exploiting homological techniques. We relate this dimension to the weight of the maximal interior segments of the T-mesh, defined for an ordering of these maximal interior segments. We show that the lower and upper bounds coincide, for high enough degrees or for hierarchical T-meshes which are enough regular. We give a rule of subdivision to construct hierarchical T-meshes for which these lower and upper bounds coincide. Finally, we illustrate these results by analyzing spline spaces of small degrees and smoothness.

6.1.11. On the problem of instability in the dimension of a spline space over a T-mesh

Participant: Bernard Mourrain.

In [23], we discuss the problem of instability in the dimension of a spline space over a T-mesh. For bivariate spline spaces $S(5,5,3,3)$ and $S(4,4,2,2)$, the instability in the dimension is shown over certain types of T-meshes. This result could be considered as an attempt to answer the question of how large the polynomial degree (m, m') should be relative to the smoothness (r, r') to make the dimension of a spline space stable. We show in particular that the bound $m \geq 2r+1$ and $m' \geq 2r'+1$ are optimal.

This is a joint work with Berdinsky Dmitry, Oh Min-Jae and Kim Taewan (Department of Naval Architecture and Ocean Engineering Seoul National University, South Korea).

6.1.12. Homological techniques for the analysis of the dimension of triangular spline spaces

Participant: Bernard Mourrain.

The spline space $C^r_k(\Delta)$ attached to a subdivided domain Δ of $\mathbb{R}^d$ is the vector space of functions of class C^r which are polynomials of degree $\leq k$ on each piece of this subdivision. Classical splines on planar rectangular grids play an important role in Computer Aided Geometric Design, and spline spaces over arbitrary subdivisions of planar domains are now considered for isogeometric analysis applications. In [34], we address the problem of determining the dimension of the space of bivariate splines $C^r_k(\Delta)$ for a triangulated region Δ in the plane. Using the homological introduced by Billera (1988), we number the vertices and establish a formula for an upper bound on the dimension. There is no restriction on the ordering and we obtain more accurate approximations to the dimension than previous methods. Furthermore, in certain cases even an exact value can be found. The construction makes it also possible to get a short proof for the dimension formula when $k \geq 4r+1$, and the same method we use in this proof yields the dimension straightaway for many other cases.

This is a joint work with Nelly Villamizar (CMA, University of Oslo, Norway).

6.1.13. Analysis-suitable volume parameterization of multi-block computational domain in isogeometric applications

Participants: Bernard Mourrain, André Galligo.

Parameterization of computational domain is a key step in isogeometric analysis just as mesh generation is in finite element analysis. In [36], we study the volume parameterization problem of multi-block computational domain in isogeometric version, i.e., how to generate analysis-suitable parameterization of the multi-block computational domain bounded by B-spline surfaces. Firstly, we show how to find good volume parameterization of single-block computational domain by solving a constraint optimization problem, in which the constraint condition is the injectivity sufficient conditions of B-spline volume parametrization, and the optimization term is the minimization of quadratic energy functions related to the first and second derivatives of B-spline volume parameterization. By using this method, the resulted volume parameterization has no self-intersections, and the isoparametric structure has good uniformity and orthogonality. Then we extend this method to the multi-block case, in which the continuity condition between the neighbor B-spline volume should be added to the constraint term. The effectiveness of the proposed method is illustrated by several examples based on three-dimensional heat conduction problem.

This is a joint work with Régis Duvigneau (Inria, EPI OPALE) and Xu Gang (College of computer - Hangzhou Dianzi University, China).

6.1.14. A new error assessment method in isogeometric analysis of 2D heat conduction problems

Participants: Bernard Mourrain, André Galligo.

In [35], we propose a new error assessment method for isogeometric analysis of 2D heat conduction problems. A posteriori error estimation is obtained by resolving the isogeometric analysis problem with several k-refinement steps. The main feature of the proposed method is that the resulted error estimation surface has a B-spline form, according to the main idea of isogeometric analysis. Though the error estimation method is expensive, it can be used as an error assessment method for isogeometric analysis. Two comparison examples are presented to show the efficiency of the proposed method.

This is a joint work with Régis Duvigneau (Inria, EPI OPALE) and Xu Gang (College of computer - Hangzhou Dianzi University, China).

6.1.15. On the cut-off phenomenon for the transitivity of randomly generated subgroups

Participant: André Galligo.

Consider $K \geq 2$ independent copies of the random walk on the symmetric group S_N starting from the identity and generated by the products of either independent uniform transpositions or independent uniform neighbor transpositions. At any time $n \in \mathbb{N}$, let G_n be the subgroup of S_N generated by the K positions of the chains. In the uniform transposition model, we prove in [28] that there is a cut-off phenomenon at time $N \ln(N)/(2K)$ for the non-existence of fixed point of G_n and for the transitivity of G_n, thus showing that these properties occur before the chains have reached equilibrium. In the uniform neighbor transposition model, a transition for the non-existence of a fixed point of G_n appears at time of order $N^{1+\frac{2}{K}}$ (at least for $K \geq 3$), but there is no cut-off phenomenon. In the latter model, we recover a cut-off phenomenon for the non-existence of a fixed point at a time proportional to N by allowing the number K to be proportional to $\ln(N)$. The main tools of the proofs are spectral analysis and coupling techniques.

This is a joint work with Laurent Miclo (University of Toulouse).

6.2. Algebraic algorithms for geometric computing

6.2.1. On the isotopic meshing of an algebraic implicit surface

Participant: Bernard Mourrain.

In [22], we present a new and complete algorithm for computing the topology of an algebraic surface given by a squarefree polynomial in $\mathbb{Q}[X, Y, Z]$. Our algorithm involves only subresultant computations and entirely relies on rational manipulation, which makes it direct to implement. We extend the work in [15], on the topology of non-reduced algebraic space curves, and apply it to the polar curve or apparent contour of the surface S. We exploit simple algebraic criterion to certify the pseudo-genericity and genericity position of the surface. This gives us rational parametrizations of the components of the polar curve, which are used to lift the topology of the projection of the polar curve. We deduce the connection of the two-dimensional components above the cell defined by the projection of the polar curve. A complexity analysis of the algorithm is provided leading to a bound in $\widetilde{\mathcal{O}}_B(d^{15}\tau)$ for the complexity of the computation of the topology of an implicit algebraic surface defined by integer coefficients polynomial of degree d and coefficients size τ . Examples illustrate the implementation in Mathemagix of this first complete code for certified topology of algebraic surfaces.

This is a joint work with Daouda Niang Diatta, Olivier Ruatta (XLIM, University of Limoges).

6.2.2. Moment matrices, border basis and real radical computation

Participant: Bernard Mourrain.

In [32], we describe new methods to compute the radical (resp. real radical) of an ideal, assuming its complex (resp. real) variety is finite. The aim is to combine approaches for solving a system of polynomial equations with dual methods which involve moment matrices and semi-definite programming. While border basis algorithms are efficient and numerically stable for computing complex roots, algorithms based on moment matrices allow the incorporation of additional polynomials, e.g., to restrict the computation to real roots or to eliminate multiple solutions. The proposed algorithm can be used to compute a border basis of the input ideal and, as opposed to other approaches, it can also compute the quotient structure of the (real) radical ideal directly, i.e., without prior algebraic techniques such as Gröbner bases. It thus combines the strengths of existing algorithms and provides a unified treatment for the computation of border bases for the ideal, the radical ideal and the real radical ideal.

This is a joint work with Jean-Bernard Lasserre (LAAS, Toulouse), Monique Laurent (CWI, Amsterdam, Netherland), Philipp Rostalski (University of California, Berkeley, US) Philippe Trébuchet (APR, LIP6, Paris).

6.2.3. On the computation of matrices of traces and radicals of ideals

Participant: Bernard Mourrain.

Let $f_1, ..., f_s \in \mathbb{K}[x_1, ..., x_m]$ be a system of polynomials generating a zero-dimensional ideal I, where $\mathbb{K}$ is an arbitrary algebraically closed field. In [31], we study the computation of "matrices of traces" for the factor algebra $\mathcal{A} := \mathbb{C}[x_1, ..., x_m]/I$, i.e. matrices with entries which are trace functions of the roots of I. Such matrices of traces in turn allow us to compute a system of multiplication matrices $\{M_{x_i} | i = 1, ..., m\}$ of the radical $\sqrt{I}$. We first propose a method using Macaulay type resultant matrices of $f_1, ..., f_s$ and a polynomial J to compute moment matrices, and in particular matrices of traces for $\mathcal{A}$. Here J is a polynomial generalizing the Jacobian. We prove bounds on the degrees needed for the Macaulay matrix in the case when I has finitely many projective roots in $\mathbb{P}^m_{\mathbb{C}}$. We also extend previous results which work only for the case where $\mathcal{A}$ is Gorenstein to the non-Gorenstein case. The second proposed method uses Bezoutian matrices to compute matrices of traces of $\mathcal{A}$. Here we need the assumption that $s = m$ and $f_1, ..., f_m$ define an affine complete intersection. This second method also works if we have higher dimensional components at infinity. A new explicit description of the generators of $\sqrt{I}$ are given in terms of Bezoutians.

This is a joint work with Itnuit Janovitz-Freireich (Departamento de Matemáticas, Mexico), Lajos Ronayi (Hungarian Academy of Sciences and Budapest University of Technology and Economics, Budapest), Agnes Szanto (Department of Computer Science, North Carolina State University, US).

6.2.4. Border basis representation of a general quotient algebra

Participant: Bernard Mourrain.

In [40], we generalized the construction of border bases to non-zero dimensional ideals for normal forms compatible with the degree, tackling the remaining obstacle for a general application of border basis methods. First, we give conditions to have a border basis up to a given degree. Next, we describe a new stopping criterion to determine when the reduction with respect to the leading terms is a normal form. This test based on the persistence and regularity theorems of Gotzmann yields a new algorithm for computing a border basis of any ideal, which proceeds incrementally degree by degree until its regularity. We detail it, prove its correctness, present its implementation and report some experimentations which illustrate its practical good behavior.

This is a joint work with Philippe Trébuchet (APR, LIP6, Paris).

6.2.5. Voronoï diagrams of algebraic distance fields

Participant: Bernard Mourrain.

In [25], we design and implement an efficient and certified algorithm for the computation of Voronoi Diagrams (VD's) constrained to a given domain. Our framework is general and applicable to any VD-type where the distance field is given explicitly or implicitly by a polynomial, notably the anisotropic VD or VD's of non-punctual sites. We use the Bernstein form of polynomials and DeCasteljau's algorithm to subdivide the initial domain and isolate bisector, or domains that contain a Voronoi vertex. The efficiency of our algorithm is due to a filtering process, based on bounding the field over the subdivided domains. This allows us to exclude functions (thus sites) that do not contribute locally to the lower envelope of the lifted diagram. The output is a polygonal description of each Voronoi cell, within any user-defined precision, isotopic to the exact VD. Correctness of the result is implied by the certified approximations of bisector branches, which are computed by existing methods for handling algebraic curves. First experiments with our C++ implementation, based on double precision arithmetic, demonstrate the adaptability of the algorithm.

This is a joint work with Ioannis Emiris (ERGA, National Kapodistrian University of Athens, Greece), Angelos Mantzaflaris (RICAM, Austrian Academy of Sciences, Austria).

6.2.6. Rational invariants of scalings from Hermite normal forms

Participant: Evelyne Hubert.

Scalings form a class of group actions that have both theoretical and practical importance. A scaling is accurately described by an integer matrix. In [39] tools from linear algebra are exploited to compute a minimal generating set of rational invariants, trivial rewriting and rational sections for such a group action. The primary tools used are Hermite normal forms and their unimodular multipliers. With the same line of ideas, a complete solution to the scaling symmetry reduction of a polynomial system is also presented.

This is joint work with George Labahn (University of Waterloo, Canada).

6.2.7. Scaling invariants and symmetry reduction of dynamical systems

Participant: Evelyne Hubert.

The motivation for this subject is to offer an algorithmic scheme for reducing the number of parameters in physical, chemical or biological models. This comes as a special case of a symmetry reduction scheme that can be fully realized by linear algebra over the integers. See http://hal.inria.fr/hal-00668882. We provide there the algebraic determination of the scaling symmetry of a dynamical system and an complete explicit symmetry reduction scheme with polynomial complexity.

This is joint work with George Labahn (University of Waterloo, Canada).

6.2.8. A computational approach to the discriminant of homogeneous polynomials

Participant: Laurent Busé.

In this work, the discriminant of homogeneous polynomials is studied in two particular cases: a single homogeneous polynomial and a collection of $n-1$ homogeneous polynomials in n variables. In these two cases, the discriminant is defined over a large class of coefficient rings by means of the resultant. Many formal properties and computational rules are provided and the geometric interpretation of the discriminant is investigated over a general coefficient ring, typically a domain.

This work is done in collaboration with Jean-Pierre Jouanolou (University of Strasbourg). A preprint is available at http://hal.inria.fr/hal-00747930/en/.

6.2.9. Intersection between rational curves and surfaces by means of matrix representations

Participant: Laurent Busé.

In [37], we propose a survey of matrix representations for parameterized curves and surfaces. Illustrations of the properties of these representations are given for intersection problems. In particular, we focus on the ray/surface intersection which is an important step in ray-tracing algorithms.

6.2.10. A root isolation algorithm for sparse univariate polynomials

Participant: André Galligo.

In [38], we consider a univariate polynomial f with real coefficients having a high degree N but a rather small number $d+1$ of monomials, with $d \ll N$. Such a sparse polynomial has a number of real roots smaller or equal to d. Our target is to find for each real root of f an interval isolating this root from the others. The usual subdivision methods, relying either on Sturm sequences or Moebius transform followed by Descartes's rule of signs, destruct the sparse structure. Our approach relies on the generalized Budan-Fourier theorem of Coste, Lajous, Lombardi, Roy and the techniques developed in some previous works of Galligo. To such a f is associated a set of $d+1$ $\mathbb{F}$-derivatives. The Budan-Fourier function $V_f(x)$ counts the sign changes in the sequence of $\mathbb{F}$-derivatives of f evaluated at x. The values at which this function jumps are called the $\mathbb{F}$-virtual roots of f. These include the real roots of f. We also consider the augmented $\mathbb{F}$-virtual roots of f and introduce a genericity property which eases our study. We present a real root isolation method and an algorithm which has been implemented in Maple. We rely on an improved generalized Budan-Fourier count applied to both the input polynomial and its reciprocal, together with Newton like approximation steps.

This is a joint work with Maria Emilia Alonso (University of Madrid).

6.2.11. Deformation of roots of polynomials via fractional derivatives

Participant: André Galligo.

In [26], we first recall the main features of Fractional calculus. In the expression of fractional derivatives of a real polynomial $f(x)$, we view the order of differentiation q as a new indeterminate; then we define a new bivariate polynomial $Pf(x,q)$. For $0 \leq q \leq 1$, $Pf(x,q)$ defines a homotopy between the polynomials $f(x)$ and $xf'(x)$. Iterating this construction, we associate to $f(x)$ a plane spline curve, called the stem of f. Stems of classic random polynomials exhibits intriguing patterns; moreover in the complex plane $Pf(x,q)$ creates an unexpected correspondence between the complex roots and the critical points of $f(x)$. We propose 3 conjectures to describe and explain these phenomena. Illustrations are provided relying on the Computer Algebra System Maple.

7. Partnerships and Cooperations

7.1. National Initiatives

7.1.1. GEOLMI

GEOLMI - Geometry and Algebra of Linear Matrix Inequalities with Systems Control Applications - is an ANR project working on topics related to the Geometry of determinantal varieties, positive polynomials, computational algebraic geometry, semidefinite programming and systems control applications.

The partners are LAAS-CNRS, Univ. de Toulouse (coordinator), LJK-CNRS, Univ. Joseph Fourier de Grenoble; Inria Sophia Antipolis Méditerranée; LIP6-CNRS Univ. Pierre et Marie Curie; Univ. de Pau et des Pays de l'Adour; IRMAR-CNRS, Univ. de Rennes.

More information available at http://homepages.laas.fr/henrion/geolmi.

7.1.2. ANEMOS

ANEMOS - Advanced Numeric for ELMs : Modeling and Optimized Schemes - is an ANR project devoted to the numerical modelling study of such ELM control methods as Resonant Magnetic Perturbations (RMPs) and pellet ELM pacing both foreseen in ITER. The goals of the project are to improve understanding of the related physics and propose possible new strategies to improve effectiveness of ELM control techniques. The study of spline spaces for isogemetric finite element methods is proposed in this context.

The partners are IRFM, CEA, Cadarache; JAD, University of Nice - Sophia Antipolis; Inria, Bacchus; Maison de la Simulation CEA-CNRS-Inria-University of Orsay- University of Versailles St Quentin .

7.2. European Initiatives

7.2.1. FP7 Projects

7.2.1.1. TERRIFIC

Title: Towards Enhanced Integration of Design and Production in the Factory of the Future through Isogeometric Technologies

Type: COOPERATION (ICT)

Defi: PPP FoF: Digital factories: Manufactoring design and product lifecycle manage

Instrument: Specific Targeted Research Project (STREP)

Duration: September 2011 - August 2014

Coordinator: SINTEF, Oslo (Norway)

Others partners:

Alenia Aeronautica (Italy); Inria Méditerranée (France); Jozef Kepler universitet, Linz (Austria); JOTNE, Oslo (Norway); MAGNA, Steyr (Austria); Missler Software (France); Siemens AG (Germany); Technische Universität Kaiserslautern (Germany); University of Pavia (Italy).

See also: http://terrific-project.eu

Abstract: The project aims at significant improvement of the interoperability of computational tools for the design, analysis and optimization of functional products. An isogeometric approach is applied for selected manufacturing application areas (cars, trains, aircrafts) and for computer-aided machining. Computer Aided Design (CAD) and numerical simulation algorithms are vital technologies in modern product development, yet they are today far from being seamlessly integrated. Their interoperability is severely disturbed by inconsistencies in the mathematical approaches used. Efficient feedback from analysis to CAD and iterative refinement of the analysis model is a feature of isogeometric analysis, and would be an essential improvement for computer-based design optimization and virtual product development. Our vision is to provide and disseminate tangible evidence of the performance of the isogeometric approach in comparison to traditional ones in four important application areas as well as addressing interoperability and other issues that necessarily arise in a large-scale industrial introduction of isogeometry.

7.2.1.2. EXCITING

Title: Exact geometry simulation for optimized design of vehicles and vessels

Type: FP7-CP-SST-2007-RTD-1-218536, COOPERATION (TRANSPORTS)

Instrument: Specific Targeted Research Project (STREP)

Duration: October 2008 - April 2012

Coordinator: Jozef Kepler universitet, Linz (Austria)

Others partners:

SINTEF, Oslo (Norway); Siemens AG (Germany); National Technical University of Athens (Greece); Hellenic Register of Shipping (Greece); University of Technology, Munich (Germany); Inria Méditerranée (France); VA Tech Hydro (Austria); Det Norske Veritas AS (Norway).

See also: http://exciting-project.eu/

Abstract: This project focuses on computational tools for the optimized design of functional freeform surfaces. Specific applications are ship hulls and propellers in naval engineering and car components, frames, and turbochargers in the automotive and railway transportation industries. The objective is to base the corresponding computational tools on the same exact representation of the geometry. This should lead to huge benefits for the entire chain of design, simulation, optimization, and life cycle management, including a new class of computational tools for fluid dynamics and solid mechanics, simulations for vehicles and vessels based. This seamless integration of CAD and FEM will have direct applications in product design, simulation and optimization of core components of vehicles and vessels.

7.2.1.3. SAGA

Title: ShApe, Geometry and Algebra, 2008-2012

Type: FP7-PEOPLE-2007-1-1-ITN.

Instrument: Initial Training Network (ITN)

Duration: November 2008 - October 2012

Coordinator: SINTEF (Norway)

Others partners: University of Oslo (Norway); Johannes Kepler Universitaet Linz (Austria); Universidad de Cantabria, Santander (Spain); Vilniaus Universitetas (Lithuany); National and Kapodistrian University of Athens (Greece); Inria Méditerranée (France); GraphiTech (Italy); Kongsberg SIM GmbH (Austria); Missler Software (France);

See also: http://saga-network.eu/

Abstract: The project aims at promoting the interaction between Geometric Modeling and Real Algebraic Geometry and, in general, at strengthening interdisciplinary and inter-sectorial research and development concerning CAD/CAM. Its objective is also to train a new generation of researchers familiar with both academic and industry viewpoints, while supporting the cooperation among the partners and with other interested collaborators in Europe.

7.2.1.4. DECONSTRUCT

Title:Decomposition of Structured Tensors, Algorithms and Characterization.

Type: PEOPLE (FP7-PEOPLE-2009-IEF)

Instrument: Marie Curie Intra-European Fellowships for Career Development (IEF)

Duration: November 2010 - November 2012

Coordinator: Inria (France)

Others partners: No.

See also: http://www-sop.inria.fr/teams/galaad/joomla/index.php/international-collaborations-147/172-deconstruct.html

Abstract: Tensors play a wide role in numerous application areas as Signal Processing for Telecommunications, Arithmetic Complexity or Data Analysis. In some applications tensors may be completely symmetric, or symmetric only in some modes, or may not be symmetric. In most of these applications, the decomposition of a tensor into a sum of rank-1 terms is relevant, since tensors of interest have a reduced rank. Most of them are structured, i.e., they are either symmetric or enjoy some index-invariance. Lastly, they are often real, which raises open problems concerning the existence and calculation of the decompositions. These issues build the basic bricks of the research program we propose. The classes of tensors described above have a geometric translation in terms of classical algebraic varieties: Segre, Veronese, Segre-Veronese varieties and Grassmannians and their secant varieties. A complete description of equations for those secant varieties and their dimensions is still not known (only dimensions of secant varieties to Veronsean are classified), although they have been studied by algebraic and differential geometers and algebraists for a long period up to now. The aim of this research project is:

- To attack both the description of the ideal of those secant varieties and their dimensions, starting from low dimensions and low degrees.
- To propose algorithms able to compute the rank of structured tensors.

7.2.2. Collaborations in European Programs, except FP7

7.2.2.1. PHC TOURNESOL FL

Program: Tournesol

Project acronym: PHC TOURNESOL FL 2012 - 26409SH

Project title: Extracting multidimensional shapes

Duration: January 2012 - December 2013

Coordinator: E. Hubert (Inria), A. Cuyt (Universiteit Antwerpen)

Other partners: Inria Sophia-Antipolis (France); Universiteit Antwerpen (Belgium)

Abstract: We are working on the shape-from-moments problem : from measurement-like data, reconstructing a desired object. Since many years, this problem has been solved and optimized in the 2D-case thanks to use of complex numbers. Thanks to a new formula, we want to stay in the real domain in order to generalize this problem to multidimensional shapes - in particular 3D-shapes. For more details about our project Tournesol : http://www-sop.inria.fr/teams/galaad/joomla/index.php/international-collaborations-147/173-tournesol.html. For more details about the program Tournesol : http://www.campusfrance.org/fr/tournesol-communaute-francaise.

7.3. International Initiatives

7.3.1. Participation In International Programs

7.3.1.1. CNRS-NSFC collaboration with Hangzhou Dianzi University

Contact in China: Xu Gang, College of computer - Hangzhou Dianzi University.

Participants in France: André Galligo, Bernard Mourrain, R. Duvigneau, B. Nkonga.

Abstract: CAD/CAE technology plays an important role in advanced manufacture, and the seamless integration of CAD/CAE is a difficult and important problem. The current CAD/CAE workflow can be classified into three steps: Computer-aided design, finite element analysis (FEA) and shape optimization. From the above workflow in CAD/CAE, the main gap of the geometric data is from the analysis step. Isogeometric analysis (IGA) can be employed to overcome the gap between CAD and finite element analysis by using the same geometric representation based on NURBS for the design and analysis tasks. In this collaboration, we studied the following problems: (1) Parameterization of computational domain for IGA methods, in particular generation of volume parameterization from CAD surface models. (2) IGA on complicated geometry and topology.

7.4. International Research Visitors

7.4.1. Visits of International Scientists

Wen-Shin Lee and Annie Cuyt (University of Antwerp, Belgium) visited on April 23-27 and December 10-22 in the context of the TOURNESOL project .

Nelly Villamizar (University of Oslo, Norway) visited us from March 28 to May 15, to collaborate with B. Mourrain on splines spaces, in the context of the ITN Marie-Curie SAGA.

Ibrahim Adamou (University of Cantabria, Spain) visited us from September 30 to October 8 to collaborate with B. Mourrain on Voronoï diagrams of half-lines and robust geometric computation, for his secondement in the context of the ITN Marie-Curie SAGA.

Gang Xu visited Inria and the university of Nice from November 1 to November 8 in the context of the CNRS-NSFC collaboration program.

Xiao-Shan Gao and Jingsan Chen (Chinese Academy of Science, Beijing) visited from July 18 to July 20.

George Labahn (University of Waterloo, Canada) visited from July 16 to July 22 to explore new collaboration topics with Evelyne Hubert.

8. Dissemination

8.1. Scientific Animation

Laurent Busé

- was a member of the program committee of the 2012 ISSAC international conference,
- is a member of the committee of "agrégation de mathématiques".

Evelyne Hubert

- is an associate editor of the Journal of Symbolic Computation (http://www.journals.elsevier.com/journal-of-symbolic-computation),
- is part of the Advisory Board of the MEGA conference series and co-edited this year a special issue of the Journal of Symbolic Computation,
- is part of the local Inria Committee for Invited Professors and Postodoctoral Fellows and was elected to the new hiring committee at the mathematics department of the University of Nice (CPRH 25-26-60).

Bernard Mourrain

- is an associate editor of the Journal of Symbolic Computation (http://www.journals.elsevier.com/journal-of-symbolic-computation),
- is member of the Advisory Board of the MEGA conference series,
- is chair of the local Inria Committee for Courses and Conferences.

8.2. Teaching - Supervision - Juries

8.2.1. Teaching

Laurent Busé teaching activities:

Master : *Courbes et Surfaces*, 42h ETD, first year of Master MAM4 of the EPU, University of Nice.

Master MDFI: *Elimination theory*, 13h, University of Luminy, Marseille.

Mathieu Collowald teaching activities:

L1 MASS : Statistics (Exercices), 20h, University of Nice-Sophia Antipolis.

L1 I : Algebra (Exercices), 44h, University of Nice-Sophia Antipolis.

André Galligo teaching activities:

Master : *Introduction to random polynomials and matrices*, University of Buenos Aires, Argentina, 20h.

Bernard Mourrain teaching activities:

Master 2: *Computational Algebra for Real Geometry* 30h ETD, Master 2 of Mathematics, Lab. J.A. Dieudonné.

Master MDFI: *Elimination theory*, 15h, University of Luminy, Marseille.

8.2.2. Supervision

8.2.2.1. Defended Habilitation thesis

Evelyne Hubert was awarded her habilitation in mathematics from the University of Nice in September 2012. The Jury consisted of Elizabeth Mansfield (University of Kent, UK), Michael Singer (North Carolina State University, USA), Emilio Musso (Politecnico de Turino, Italy), Pascal Chossat (University of Nice), Bruno Salvy (Inria Paris), Bernard Mourrain (Inria Méditerranée).

8.2.2.2. Ph.D. thesis in progress

Marta Abril Bucero, Moment matrices, real algebraic geometry and polynomial optimization. Advisor: Bernard Mourrain.

Mathieu Collowald, Integral representation of shapes for feature conservation or extraction. Advisors: Evelyne Hubert and Bernard Mourrain.

Abdallah Lachaal, Geometric computing with procedural models and applications. Advisor: Bernard Mourrain.

8.2.2.3. Internship

Valentin Michelet, *Intersection courbe/surface au moyen des représentations matricielles* , 3 months, Master1.

Baptiste Mourrain, *A dynamic web application*, 1 month, first year of engineer school.

Zora Abdoul Kaid, *Isoparametrisation of multi-patch surfaces*, 3 months, MAM4, Master1.

Rémy Abdeljalil, *Adaptative approximation of point sets by T-splines surfaces*, 3 months, MAM4, Master1.

8.2.3. Juries

Bernard Mourrain was member of the HDR of Evelyne Hubert, of the Ph.D. committee of R. Lebreton "Contribution à l'algorithmique détendue et à la résolution des systèmes polynomiaux", École Polytechnique (11/12/2012), of the PhD of C. Preusakarn "Reconstructing of Plant Architecture from 3D Laser Scanner Data", University of Montepellier 2, (19/12/2012).

André Galligo was a member of the PhD of Santiago Laplagne, "Algoritmos de Algebra Conmutativa en Anillos de Polinomios" University of Buenos Aires (Argentina), Avpil 2012, of the PhD of Nelly Villamizar, "Algebraic Geometry of Splines". University of Oslo (Norway), December 2012.

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