Activity Report 2012

Team GECO

Geometric Control Design

RESEARCH CENTER
Saclay - Île-de-France

THEME
Modeling, Optimization, and Control of Dynamic Systems

Table of contents

Team GECO

Keywords: Automatic Control, Nonlinear Control, Quantum Chemistry, System Analysis And Control, Tracking

Creation of the Team: May 01, 2011 .

1. Members

Research Scientists
Mario Sigalotti [Team leader, Junior Researcher Inria, HdR]
Ugo Boscain [Senior Researcher CNRS]
Jean-Paul Gauthier [Professor, on leave from 2/2012 to 7/2012]

External Collaborators
Grégoire Charlot [Associate Professor]
Yacine Chitour [Professor]
Frédéric Jean [Ens/Chercheur]
Paolo Mason [Junior Researcher CNRS]
Dominique Sugny [Associate Professor]
Emmanuel Trélat [Professor]

PhD Students
Moussa Gaye [since 10/2011, Fondation Hadamard scholarship]
Dario Prandi [since 10/2011, ERC scholarship]

Post-Doctoral Fellows
Davide Barilari [since 11/2011, ERC scholarship]
Ghassen Didri [since 9/2012, Inria scolarship]
Alexey Remizov [since 1/2012, Digitéo scholarship]

Administrative Assistant
Christelle Lievin

2. Overall Objectives

2.1. Highlights of the Year

Emmanuel Trélat obtained the Felix Klein Prize at the 6th European Congress of Mathematics in Krakow.

Motivations by the prize committee: Emmanuel Trélat receives the Felix Klein Prize for combining truly impressive and beautiful contributions in fine fundamental mathematics to understand and solve new problems in control of PDE's and ODE's (continuous, discrete and mixed problems), and above all for his studies on singular trajectories, with remarkable numerical methods and algorithms able to provide solutions to many industrial problems in real time, with substantial impact especially in the area of astronautics.

3. Scientific Foundations

3.1. Geometric control theory

The main research topic of the project-team will be **geometric control**, with a special focus on **control design**. The application areas that we target are control of quantum mechanical systems, neurogeometry and switched systems.

Geometric control theory provides a viewpoint and several tools, issued in particular from differential geometry, to tackle typical questions arising in the control framework: controllability, observability, stabilization, optimal control... [27], [62] The geometric control approach is particularly well suited for systems involving nonlinear and nonholonomic phenomena. We recall that nonholonomicity refers to the property of a velocity constraint that is not equivalent to a state constraint.

The expression **control design** refers here to all phases of the construction of a control law, in a mainly open-loop perspective: modeling, controllability analysis, output tracking, motion planning, simultaneous control algorithms, tracking algorithms, performance comparisons for control and tracking algorithms, simulation and implementation.

We recall that

- **controllability** denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law ;
- **output tracking** refers to a control strategy aiming at keeping the value of some functions of the state arbitrarily close to a prescribed time-dependent profile. A typical example is **configuration tracking** for a mechanical system, in which the controls act as forces and one prescribes the position variables along the trajectory, while the evolution of the momenta is free. One can think for instance at the lateral movement of a car-like vehicle: even if such a movement is unfeasible, it can be tracked with arbitrary precision by applying a suitable control strategy;
- **motion planning** is the expression usually denoting the algorithmic strategy for selecting one control law steering the system from a given initial state to an attainable final one;
- **simultaneous control** concerns algorithms that aim at driving the system from two different initial conditions, with the same control law and over the same time interval, towards two given final states (one can think, for instance, at some control action on a fluid whose goal is to steer simultaneously two floating bodies.) Clearly, the study of which pairs (or n-uples) of states can be simultaneously connected thanks to an admissible control requires an additional controllability analysis with respect to the plain controllability mentioned above.

At the core of control design is then the notion of motion planning. Among the motion planning methods, a preeminent role is played by those based on the Lie algebra associated with the control system ([83], [70], [76]), those exploiting the possible flatness of the system ([56]) and those based on the continuation method ([95]). Optimal control is clearly another method for choosing a control law connecting two states, although it generally introduces new computational and theoretical difficulties.

Control systems with special structure, which are very important for applications are those for which the controls appear linearly. When the controls are not bounded, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a smoothly varying norm (representing a cost of the control), the resulting geometrical structure is called *sub-Riemannian*. Sub-Riemannian geometry thus appears as the underlying geometry of the nonholonomic control systems, playing the same role as Euclidean geometry for linear systems. As such, its study is fundamental for control design. Moreover its importance goes far beyond control theory and is an active field of research both in differential geometry ([82]), geometric measure theory ([57], [31]) and hypoelliptic operator theory ([43]).

Other important classes of control systems are those modeling mechanical systems. The dynamics are naturally defined on the tangent or cotangent bundle of the configuration manifold, they have Lagrangian or Hamiltonian structure, and the controls act as forces. When the controls appear linearly, the resulting model can be seen somehow as a second-order sub-Riemannian structure (see [49]).

The control design topics presented above naturally extend to the case of distributed parameter control systems. The geometric approach to control systems governed by partial differential equations is a novel subject with great potential. It could complement purely analytical and numerical approaches, thanks to its more dynamical, qualitative and intrinsic point of view. An interesting example of this approach is the paper [28] about the controllability of Navier–Stokes equation by low forcing modes.

4. Application Domains

4.1. Quantum control

The issue of designing efficient transfers between different atomic or molecular levels is crucial in atomic and molecular physics, in particular because of its importance in those fields such as photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (NMR, control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing. This last application explicitly relies on the design of quantum gates, each of them being, in essence, an open loop control law devoted to a prescribed simultaneous control action. NMR is one of the most promising techniques for the implementation of a quantum computer.

Physically, the control action is realized by exciting the quantum system by means of one or several external fields, being them magnetic or electric fields. The resulting control problem has attracted increasing attention, especially among quantum physicists and chemists (see, for instance, [88], [93]). The rapid evolution of the domain is driven by a multitude of experiments getting more and more precise and complex (see the recent review [48]). Control strategies have been proposed and implemented, both on numerical simulations and on physical systems, but there is still a large gap to fill before getting a complete picture of the control properties of quantum systems. Control techniques should necessarily be innovative, in order to take into account the physical peculiarities of the model and the specific experimental constraints.

The area where the picture got clearer is given by finite dimensional linear closed models.

- **Finite dimensional** refers to the dimension of the space of wave functions, and, accordingly, to the finite number of energy levels.
- **Linear** means that the evolution of the system for a fixed (constant in time) value of the control is determined by a linear vector field.
- **Closed** refers to the fact that the systems are assumed to be totally disconnected from the environment, resulting in the conservation of the norm of the wave function.

The resulting model is well suited for describing spin systems and also arises naturally when infinite dimensional quantum systems of the type discussed below are replaced by their finite dimensional Galerkin approximations. Without seeking exhaustiveness, let us mention some of the issues that have been tackled for finite dimensional linear closed quantum systems:

- controllability [29],
- bounds on the controllability time [25],
- STIRAP processes [98],
- simultaneous control [71],
- optimal control ([67], [38], [50]),
- numerical simulations [77].

Several of these results use suitable transformations or approximations (for instance the so-called rotating wave) to reformulate the finite-dimensional Schrödinger equation as a sub-Riemannian system. Open systems have also been the object of an intensive research activity (see, for instance, [30], [68], [89], [44]).

In the case where the state space is infinite dimensional, some optimal control results are known (see, for instance, [34], [45], [63], [35]). The controllability issue is less understood than in the finite dimensional setting, but several advances should be mentioned. First of all, it is known that one cannot expect exact controllability on the whole Hilbert sphere [97]. Moreover, it has been shown that a relevant model, the quantum oscillator, is not even approximately controllable [90], [80]. These negative results have been more recently completed by positive ones. In [36], [37] Beauchard and Coron obtained the first positive controllability result for a quantum particle in a 1D potential well. The result is highly nontrivial and is based on Coron's return method (see [52]). Exact controllability is proven to hold among regular enough wave functions. In particular, exact controllability among eigenfunctions of the uncontrolled Schrödinger operator can be achieved. Other important approximate controllability results have then been proved using Lyapunov methods [79], [84], [64]. While [79] studies a controlled Schrödinger equation in $\mathbb{R}$ for which the uncontrolled Schrödinger operator has mixed spectrum, [84], [64] deal mainly with general discrete-spectrum Schrödinger operators.

In all the positive results recalled in the previous paragraph, the quantum system is steered by a single external field. Different techniques can be applied in the case of two or more external fields, leading to additional controllability results [55], [41].

The picture is even less clear for nonlinear models, such as Gross–Pitaevski and Hartree–Fock equations. The obstructions to exact controllability, similar to the ones mentioned in the linear case, have been discussed in [61]. Optimal control approaches have also been considered [33], [46]. A comprehensive controllability analysis of such models is probably a long way away.

4.2. Neurophysiology

At the interface between neurosciences, mathematics, automatics and humanoid robotics, an entire new approach to neurophysiology is emerging. It arouses a strong interest in the four communities and its development requires a joint effort and the sharing of complementary tools.

A family of extremely interesting problems concerns the understanding of the mechanisms supervising some sensorial reactions or biomechanics actions such as image reconstruction by the primary visual cortex, eyes movement and body motion.

In order to study these phenomena, a promising approach consists in identifying the motion planning problems undertaken by the brain, through the analysis of the strategies that it applies when challenged by external inputs. The role of control is that of a language allowing to read and model neurological phenomena. The control algorithms would shed new light on the brain's geometric perception (the so-called neurogeometry [86]) and on the functional organization of the motor pathways.

- A challenging problem is that of the understanding of the mechanisms which are responsible for the process of image reconstruction in the primary visual cortex V1.

  The visual cortex areas composing V1 are notable for their complex spatial organization and their functional diversity. Understanding and describing their architecture requires sophisticated modeling tools. At the same time, the structure of the natural and artificial images used in visual psychophysics can be fully disclosed only using rather deep geometric concepts. The word "geometry" refers here to the internal geometry of the functional architecture of visual cortex areas (not to the geometry of the Euclidean external space). Differential geometry and analysis both play a fundamental role in the description of the structural characteristics of visual perception.

  A model of human perception based on a simplified description of the visual cortex V1, involving geometric objects typical of control theory and sub-Riemannian geometry, has been first proposed by Petitot ([87]) and then modified by Citti and Sarti ([51]). The model is based on experimental observations, and in particular on the fundamental work by Hubel and Wiesel [60] who received the Nobel prize in 1981.

  In this model, neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli arriving at a given point of the retina and oriented along a given direction. The retina is modeled by the real plane, while the directions at a given point are modeled by the projective line. The fiber bundle having as base the real plane and as fiber the projective line is called the *bundle of directions of the plane*.

  From the neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them sensitive to stimuli arriving at a given point, oriented along any direction. In the same hypercolumn, relative to a point of the plane, we also find neurons that are sensitive to other stimuli properties, such as colors. Therefore, in this model the visual cortex treats an image not as a planar object, but as a set of points in the bundle of directions of the plane. The reconstruction is then realized by minimizing the energy necessary to activate orientation columns among those which are not activated directly by the image. This gives rise to a sub-Riemannian problem on the bundle of directions of the plane.

- Another class of challenging problems concern the functional organization of the motor pathways.

  The interest in establishing a model of the motor pathways, at the same time mathematically rigorous and biologically plausible, comes from the possible spillovers in neurophysiology. It could help to design better control strategies for robots and artificial limbs, rendering them capable to move more progressively and smoothly and also to react to exterior perturbations in a flexible way. An underlying relevant societal goal (clearly beyond our domain of expertise) is to clarify the mechanisms of certain debilitating troubles such as cerebellar disease, chorea and Parkinson's disease.

  A key issue in order to establish a model of the motor pathways is to determine the criteria underlying the brain's choices. For instance, for the problem of human locomotion (see [32]), identifying such criteria would be crucial to understand the neural pathways implicated in the generation of locomotion trajectories.

  A nowadays widely accepted paradigm is that, among all possible movements, the accomplished ones satisfy suitable optimality criteria (see [96] for a review). One is then led to study an inverse optimal control problem: starting from a database of experimentally recorded movements, identify a cost function such that the corresponding optimal solutions are compatible with the observed behaviors.

  Different methods have been taken into account in the literature to tackle this kind of problems, for instance in the linear quadratic case [66] or for Markov processes [85]. However all these methods have been conceived for very specific systems and they are not suitable in the general case. Two approaches are possible to overcome this difficulty. The direct approach consists in choosing a cost function among a class of functions naturally adapted to the dynamics (such as energy functions) and to compare the solutions of the corresponding optimal control problem to the experimental data. In particular one needs to compute, numerically or analytically, the optimal trajectories and to choose suitable criteria (quantitative and qualitative) for the comparison with observed trajectories. The inverse approach consists in deriving the cost function from the qualitative analysis of the data.

4.3. Switched systems

Switched systems form a subclass of hybrid systems, which themselves constitute a key growth area in automation and communication technologies with a broad range of applications. Existing and emerging areas include automotive and transportation industry, energy management and factory automation. The notion of hybrid systems provides a framework adapted to the description of the heterogeneous aspects related to the interaction of continuous dynamics (physical system) and discrete/logical components.

The characterizing feature of switched systems is the collective aspect of the dynamics. A typical question is that of stability, in which one wants to determine whether a dynamical system whose evolution is influenced by a time-dependent signal is uniformly stable with respect to all signals in a fixed class ([73]).

The theory of finite-dimensional hybrid and switched systems has been the subject of intensive research in the last decade and a large number of diverse and challenging problems such as stabilizability, observability, optimal control and synchronization have been investigated (see for instance [94], [74]).

The question of stability, in particular, because of its relevance for applications, has spurred a rich literature. Important contributions concern the notion of common Lyapunov function: when there exists a Lyapunov function that decays along all possible modes of the system (that is, for every possible constant value of the signal), then the system is uniformly asymptotically stable. Conversely, if the system is stable uniformly with respect to all signals switching in an arbitrary way, then a common Lyapunov function exists [75]. In the *linear* finite-dimensional case, the existence of a common Lyapunov function is actually equivalent to the global uniform exponential stability of the system [81] and, provided that the admissible modes are finitely many, the Lyapunov function can be taken polyhedral or polynomial [39], [40], [53]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see [54] and references therein). Algebraic approaches to prove the stability of switched systems under arbitrary switching, not relying on Lyapunov techniques, have been proposed in [72], [26].

Other interesting issues concerning the stability of switched systems arise when, instead of considering arbitrary switching, one restricts the class of admissible signals, by imposing, for instance, a dwell time constraint [59].

Another rich area of research concerns discrete-time switched systems, where new intriguing phenomena appear, preventing the algebraic characterization of stability even for small dimensions of the state space [69]. It is known that, in this context, stability cannot be tested on periodic signals alone [42].

Finally, let us mention that little is known about infinite-dimensional switched system, with the exception of some results on uniform asymptotic stability ([78], [91], [92]) and some recent papers on optimal control ([58], [99]).

5. Software

5.1. IRHD

We developed a first version of a software for reconstruction of corrupted and damaged images, named IRHD (for Image Reconstruction via Hypoelliptic Diffusion). One of the main features of the algorithm on which the software is based is that we don't use any information about the location and character of the corrupted places; this allows us to work with real non-academic images. Another important advantage is that this method is massively parallelizable; this allows to work with sufficiently large images. Theoretical background of the presented method is based on the model of geometry of vision due to Petitot, Citti and Sarti. The main step is numerical solution of the equation of 3D hypoelliptic diffusion. IRHD is based on Fortran. Alexey Remizov is in charge with the development of the software, in collaboration with Ugo Boscain and Jean-Paul Gauthier.

6. New Results

6.1. New results: geometric control

We start by presenting some results on the design of motion planning and tracking algorithms.

- In [10] we present an iterative steering algorithm for nonholonomic systems (also called driftless control-affine systems) and we prove its global convergence under the sole assumption that the Lie Algebraic Rank Condition (LARC) holds true everywhere. That algorithm is an extension of the one introduced in [65] for regular systems. The first novelty here consists in the explicit algebraic construction, starting from the original control system, of a lifted control system which is regular. The second contribution of the paper is an exact motion planning method for nilpotent systems, which makes use of sinusoidal control laws and which is a generalization of the algorithm described in [83] for chained-form systems.
- [6] and [5] are about motion planning for kinematic systems, and more particularly ε-approximations of non-admissible trajectories by admissible ones. This is done in a certain optimal sense. The resolution of this motion planing problem is showcased through the thorough treatment of the ball with a trailer kinematic system, which is a non-holonomic system with flag of type $(2,3,5,6)$.

Application-oriented results about motion planning are contained in [15]. The paper proposes in particular a strategy for providing Unmanned Aerial Vehicles with a certain degree of autonomy, via autonomous planification/replanification strategies.

Let us list some new results in sub-Riemannian geometry and hypoellitpic diffusion.

- In [1] we study the Radon-Nikodym derivative of the spherical Hausdorff measure with respect to a smooth volume for a regular sub-Riemannian manifold . We prove that this is the volume of the unit ball in the nilpotent approximation and it is always a continuous function. We then prove that up to dimension 4 it is smooth, while starting from dimension 5, in corank 1 case, it is C^3 (and C^4 on every smooth curve) but in general not C^5. These results answer to a question addressed by Montgomery about the relation between two intrinsic volumes that can be defined in a sub-Riemannian manifold, namely the Popp and the Hausdorff volume. If the nilpotent approximation depends on the point (that may happen starting from dimension 5), then they are not proportional, in general.
- In [9] we study the Laplace–Beltrami operator on generalized Riemannian structures on orientable surfaces for which a local orthonormal frame is given by a pair of vector fields that can become collinear. Under the assumption that the structure is 2-step Lie bracket generating, we prove that the Laplace–Beltrami operator is essentially self-adjoint and has discrete spectrum. As a consequence, a quantum particle cannot cross the singular set (i.e., the set where the vector fields become collinear) and the heat cannot flow through the singularity.
- For an equiregular sub-Riemannian manifold M, Popp's volume is a smooth volume which is canonically associated with the sub-Riemannian structure, and it is a natural generalization of the Riemannian one. In [4] we prove a general formula for Popp's volume, written in terms of a frame adapted to the sub-Riemannian distribution. As a first application of this result, we prove an explicit formula for the canonical sub-Laplacian, namely the one associated with Popp's volume. Finally, we discuss sub-Riemannian isometries, and we prove that they preserve Popp's volume. We also show that, under some hypotheses on the action of the isometry group of M, Popp's volume is essentially the unique volume with such a property.
- In [21], for a sub-Riemannian manifold provided with a smooth volume, we relate the small time asymptotics of the heat kernel at a point y of the cut locus from x with roughly "how much" y is conjugate to x. This is done under the hypothesis that all minimizers connecting x to y are strongly normal, i.e. all pieces of the trajectory are not abnormal. Our result is a refinement of the one of Leandre $4t\log p_t(x,y) \to -d^2(x,y)$ for $t \to 0$, in which only the leading exponential term is detected. Our results are obtained by extending an idea of Molchanov from the Riemannian to the sub-Riemannian case, and some details we get appear to be new even in the Riemannian context. These results permit us to obtain properties of the sub-Riemannian distance starting from those of the heat kernel and vice versa. For the Grushin plane endowed with the Euclidean volume we get the expansion $p_t(x,y) \sim t^{-5/4}\exp\left(-d^2(x,y)/4t\right)$ where y is reached from a Riemannian point x by a minimizing geodesic which is conjugate at y. In [22] we investigate the small time heat kernel asymptotics on the cut locus on the class of two-spheres of revolution, which is the simplest class of 2-dimensional Riemannian manifolds different from the sphere with nontrivial cut-conjugate locus. We determine the degeneracy of the exponential map near a cut-conjugate point and present the consequences of this result to the small time heat kernel asymptotics at this point. These results give a first example where the minimal degeneration of the asymptotic expansion at the cut locus is attained.
- In [24] we studied normal forms for 2-dimensional almost-Riemannian structures. The latter are generalized Riemannian structures on surfaces for which a local orthonormal frame is given by a Lie bracket generating pair of vector fields that can become collinear. Generically, there are three types of points: Riemannian points where the two vector fields are linearly independent, Grushin points where the two vector fields are collinear but their Lie bracket is not, and tangency points where the two vector fields and their Lie bracket are collinear and the missing direction is obtained with one more bracket. In [24] we consider the problem of finding normal forms and functional invariants at each type of point. We also require that functional invariants are complete, in the sense that they permit to recognize locally isometric structures. The problem happens to be equivalent to the one of finding a smooth canonical parameterized curve passing through the point and being transversal to the distribution. For Riemannian points such that the gradient of the Gaussian curvature K is

different from zero, we use the level set of K as support of the parameterized curve. For Riemannian points such that the gradient of the curvature vanishes (and under additional generic conditions), we use a curve which is found by looking for crests and valleys of the curvature. For Grushin points we use the set where the vector fields are parallel. Tangency points are the most complicated to deal with. The cut locus from the tangency point is not a good candidate as canonical parameterized curve since it is known to be non-smooth. Thus, we analyse the cut locus from the singular set and we prove that it is not smooth either. A good candidate happens to be a curve which is found by looking for crests and valleys of the Gaussian curvature. We prove that the support of such a curve is uniquely determined and has a canonical parametrization.

6.2. New results: quantum control

New results have been obtained for the control of the bilinear Schrödinger equation.

- In [16] we obtained a sufficient condition for approximate controllability of the bilinear discrete-spectrum Schrödinger equation exploiting the use of more than one control. The controllability result extends to simultaneous controllability, approximate controllability in H^s, and tracking in modulus. The result is more general than those present in the literature even in the case of one control and permits to treat situations in which the spectrum of the uncontrolled operator is very degenerate (e.g. multiple eigenvalues or presence of equal gaps among eigenvalues). These results are applied to the case of a rotating polar linear molecule in the space, driven by three external fields. A remarkable property of this model is the presence of infinitely many degeneracies and resonances in the spectrum preventing the application of the results in the literature.
- In [19] we present a constructive method to control the bilinear Schrödinger equation by means of two or three controlled external fields. The method is based on adiabatic techniques and works if the spectrum of the Hamiltonian admits eigenvalue intersections, with respect to variations of the controls, and if the latter are conical. We provide sharp estimates of the relation between the error and the controllability time.
- In [18] we consider the minimum time population transfer problem for a two level quantum system driven by two external fields with bounded amplitude. The controls are modeled as real functions and we do not use the Rotating Wave Approximation. After projection on the Bloch sphere, we tackle the time-optimal control problem with techniques of optimal synthesis on 2-D manifolds. Based on the Pontryagin Maximum Principle, we characterize a restricted set of candidate optimal trajectories. Properties on this set, crucial for complete optimal synthesis, are illustrated by numerical simulations. Furthermore, when the two controls have the same bound and this bound is small with respect to the difference of the two energy levels, we get a complete optimal synthesis up to a small neighborhood of the antipodal point of the starting point.

6.3. New results: neurophysiology

- In [17] we study the global properties of an optimal control model of geometry of vision due to Petitot, Citti and Sarti. In particular, we consider the problem of minimizing $\int_0^L \sqrt{\xi^2 + K^2(s)}\, ds$ for a planar curve having fixed initial and final positions and directions. The total length L is free. Here s is the variable of arclength parametrization, $K(s)$ is the curvature of the curve and $\xi > 0$ a parameter. The main feature of the problem is that, if for a certain choice of boundary conditions there exists a minimizer, then this minimizer is smooth and has no cusp. However, not for all choices of boundary conditions there is a global minimizer. We study existence of local and global minimizers for this problem. We prove that if for a certain choice of boundary conditions there is no global minimizer, then there is neither a local minimizer nor a stationary curve (geodesic). We give properties of the set of boundary conditions for which there exists a solution to the problem. Finally, we present numerical computations of this set.

- In [2] we studied the general problem of reconstructing the cost from the observation of trajectories, in a problem of optimal control. It is motivated by the problem of determining what is the cost minimized in human locomotion. This applied question is very similar to the following applied problem, concerning HALE drones: one would like them to decide by themselves for their trajectories, and to behave at least as a good human pilot. These starting points are the reasons for the particular classes of control systems and of costs under consideration. To summarize, our conclusion is that in general, inside these classes, three experiments visiting the same values of the control are needed to reconstruct the cost, and two experiments are in general not enough. The method is constructive. The proof of these results is mostly based upon the Thom's transversality theory.

6.4. New results: switched systems

- In [12] we study the phenomenon of polynomial instability of switched systems. Stability properties for continuous-time linear switched systems are at first determined by the (largest) Lyapunov exponent associated with the system, which is the analogue of the joint spectral radius for the discrete-time case. We provided a characterization of marginally unstable systems, i.e., systems for which the Lyapunov exponent is equal to zero and such that there exists an unbounded trajectory. We also analyzed the asymptotic behavior of their trajectories. Our main contribution consists in pointing out a resonance phenomenon associated with marginal instability. In the course of our study, we derived an upper bound of the state at time t, which is polynomial in t and whose degree is computed from the resonance structure of the system. We also derived analogous results for discrete-time linear switched systems.
- The paper [13] is concerned with the stability of planar linear singularly perturbed switched systems of the type $\dot{x}(t) = \sigma(t)A_1^\epsilon x(t) + (1-\sigma(t))A_2^\epsilon x(t)$, where $\sigma : [0,+\infty) \to \{0,1\}$, A_1^ϵ and A_2^ϵ are real matrices which represent singularly perturbed modes. By ϵ we denote here the parameter of singular perturbation. We propose a characterization of the stability properties of such singularly perturbed switched systems based on the results given in [47]. More generally, we study transitions as ϵ varies and we restrict their number and nature. Finally, we compare the results obtained in this way with the Tikhonov-type results for differential inclusions obtained in the literature.

7. Partnerships and Cooperations

7.1. Regional Initiatives

- **Digitéo project CONGEO.** CONGEO (2009–2013) is financed by Digitéo in the framework of the DIM *Logiciels et systèmes complexes*. It focuses on the neurophysiology applications. U. Boscain, Y. Chitour (leader), F. Jean and P. Mason are part of the project.

7.2. National Initiatives

- **ANR project GCM.** The project ANR GCM (*programme blanc*, 2009–13) involves the great majority of GECO's members (permanent and external). It focuses on various theoretical aspects of geometric control and on quantum control. It is coordinated by J.-P. Gauthier.
- **ANR ArHyCo.** The project ANR ArHyCo (*programme ARPEGE*, 2009–12) is about switched systems. It is coordinated by J. Daafouz. The first theme of the ANR, on stability of switched systems, is lead by M. Sigalotti.

7.3. European Initiatives

7.3.1. FP7 Projects

Program: ERC Starting Grant

Project acronym: GeCoMethods

Project title: Geometric Control Methods for the Heat and Schroedinger Equations

Duration: 1/5/2010 - 1/5/2015

Coordinator: Ugo Boscain

Abstract: The aim of this project is to study certain PDEs for which geometric control techniques open new horizons. More precisely we plan to exploit the relation between the sub-Riemannian distance and the properties of the kernel of the corresponding hypoelliptic heat equation and to study controllability properties of the Schroedinger equation.

All subjects studied in this project are applications-driven: the problem of controllability of the Schroedinger equation has direct applications in Laser spectroscopy and in Nuclear Magnetic Resonance; the problem of nonisotropic diffusion has applications in cognitive neuroscience (in particular for models of human vision).

Participants. Main collaborator: Mario Sigalotti. Other members of the team: Andrei Agrachev, Riccardo Adami, Thomas Chambrion, Grégoire Charlot, Yacine Chitour, Jean-Paul Gauthier, Frédéric Jean.

7.4. International Initiatives

7.4.1. Inria International Partners

SISSA (Scuola Internazionale Superiore di Studi Avanzati), Trieste, Italy.

Sector of Functional Analysis and Applications, Geometric Control group. Coordinator: Andrei A. Agrachev.

We collaborate with the Geometric Control group at SISSA mainly on subjects related with sub-Riemannian geometry. Thanks partly to our collaboration, SISSA has established an official research partnership with École Polytechnique.

7.4.2. Participation In International Programs

- Laboratoire Euro Maghrébin de Mathématiques et de leurs Interactions (LEM2I)
  http://www.lem2i.cnrs.fr/
- GDRE Control of Partial Differential Equations (CONEDP)
  http://www.ceremade.dauphine.fr/~glass/GDRE/

7.5. International Research Visitors

7.5.1. Visits of International Scientists

Gianluca Panati visited GECO from 18/6 to 18/7 (thanks to an invitation by École Polytechinque). He worked on the control of spin-boson systems in collaboration with U. Boscain, P. Mason and M. Sigalotti.

7.5.1.1. Internships

Guilherme MAZANTI (from Jul 2012 until Nov 2012)

Subject: Persistent excitation with bounded variation & arbitrary rate of stabilization

Institution: University of São Paulo (Brazil)

8. Dissemination

8.1. Scientific Animation

8.1.1. Conference organization

- "INDAM meeting on Geometric Control and sub-Riemannian Geometry", Cortona, Italy, May 21 - 25, 2012 (co-organizers: D. Barilari, U. Boscain, D. Prandi, M. Sigalotti).
- "Workshop Architecture Hybride et Contraintes", Paris, June 4th-5th 2012 (co-organizer: M. Sigalotti)

8.1.2. Editorial activity

- U. Boscain is Associate Editor of Journal of Dynamical and Control Systems, ESAIM Control, Optimisation and Calculus of Variations, Mathematical Control and Related Fields. He is also referee for Journal of Differential equations, AIMS Book series: Applied mathematics, SIAM J. Control Optim., Automatica, Rendiconti dei Lincei, Matematica ed Applicazioni, Physica A...and for the conferences ACC, CDC, MTNS...
- M. Sigalotti is Associate Editor of Journal of Dynamical and Control Systems. He is also referee for IEEE TAC, SIAM J. Control Optim., Automatica, MathSciNet, Journal of Functional Analysis...and for the conferences CDC, ACC, IFAC...

8.2. Teaching - Supervision - Juries

8.2.1. Prizes

- Emmanuel Trélat obtained the Felix Klein Prize (see Section "Highlights of the year").
- Jacek Jendrej won the *Prix du Centre de Recherche, Promotion 2009* by École Polytechnique for his stage made in 2012 under the supervision of Davide Barilari.

8.2.2. Supervision

PhD & HdR

PhD in progress: Dario Prandi, "Geometric control and PDEs", 1/9/2011, supervisors: Ugo Boscain, Mario Sigalotti.

PhD in progress: Moussa Gaye, "Some problems of geometric analysis in almost-Riemannian geometry and of stability of switching systems", 1/9/2011, supervisors: Ugo Boscain, Paolo Mason.

8.2.3. Juries

- U. Boscain was *rapporteur* of the PhD thesis of O. Cots (ICB, Dijon), defended the 20/11/2012.
- M. Sigalotti was *supervisor* of the PhD thesis of F. El Hachemi (CRAN, Nancy), defended the 5/12/2012.

8.3. Popularization

- D. Sugny cosigned a popularization paper with M. Lapert, Y. Zhang, M. Janich and S. J. Glaser for the "Actualités scientifiques du CNRS" (october 2012) on the optimal control in IMR (see [20]).
- M. Sigalotti was a speaker at the seminar Unithé of the Saclay CRI on November 2012. Title of the seminar: "Copier les algos du cerveau".

9. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

[1] A. A. AGRACHEV, D. BARILARI, U. BOSCAIN. *On the Hausdorff volume in sub-Riemannian geometry*, in "Calculus of Variations and Partial Differential Equations", 2012, vol. 43, n^{o} 3-4, p. 355–388, http://dx.doi.org/10.1007/s00526-011-0414-y.

[2] A. AJAMI, J.-P. GAUTHIER, T. MAILLOT, U. SERRES. *How humans fly*, in "ESAIM: Control, Optimisation and Calculus of Variations", 2013.

[3] D. BARILARI, U. BOSCAIN, J.-P. GAUTHIER. *On 2-step, corank 2, nilpotent sub-Riemannian metrics*, in "SIAM J. Control Optim.", 2012, vol. 50, n^{o} 1, p. 559–582, http://dx.doi.org/10.1137/110835700.

[4] D. BARILARI, L. RIZZI. *A formula for Popp's volume in sub-Riemannian geometry*, in "Analysis and Geometry in Metric Spaces", 2013, http://arxiv.org/pdf/1211.2325v1.pdf.

[5] N. BOIZOT, J.-P. GAUTHIER. *Motion Planning for Kinematic Systems*, in "IEEE Transaction on Automatic Control", 2013.

[6] N. BOIZOT, J.-P. GAUTHIER. *On the motion planning of the ball with a trailer*, in "Math. control and related fields", 2013.

[7] U. BOSCAIN, M. CAPONIGRO, T. CHAMBRION, M. SIGALOTTI. *A weak spectral condition for the controllability of the bilinear Schrödinger equation with application to the control of a rotating planar molecule*, in "Comm. Math. Phys.", 2012, vol. 311, n^{o} 2, p. 423–455, http://dx.doi.org/10.1007/s00220-012-1441-z.

[8] U. BOSCAIN, F. CHITTARO, P. MASON, M. SIGALOTTI. *Adiabatic control of the Schrödinger equation via conical intersections of the eigenvalues*, in "IEEE Trans. Automat. Control", 2012, vol. 57, p. 1970–1983.

[9] U. BOSCAIN, C. LAURENT. *The Laplace–Beltrami operator in almost-Riemannian geometry*, in "Ann. Inst. Fourier", 2013.

[10] Y. CHITOUR, F. JEAN, R. LONG. *A global steering method for nonholonomic systems*, in "Journal of Differential Equations", 2013, http://dx.doi.org/10.1016/j.jde.2012.11.012.

[11] Y. CHITOUR, F. JEAN, P. MASON. *Optimal control models of goal-oriented human locomotion*, in "SIAM J. Control Optim.", 2012, vol. 50, n^{o} 1, p. 147–170, http://dx.doi.org/10.1137/100799344.

[12] Y. CHITOUR, P. MASON, M. SIGALOTTI. *On the marginal instability of linear switched systems*, in "Systems & Control Letters", 2012, vol. 61, n^{o} 6, p. 747–757, http://dx.doi.org/10.1016/j.sysconle.2012.04.005.

[13] F. EL HACHEMI, M. SIGALOTTI, J. DAAFOUZ. *Stability analysis of singularly perturbed switched linear systems*, in "IEEE Trans. Automat. Control", 2012, vol. 57, p. 2116–2121.

[14] F. HANTE, M. SIGALOTTI, M. TUCSNAK. *On conditions for asymptotic stability of dissipative infinite-dimensional systems with intermittent damping*, in "Journal of Differential Equations", May 2012, vol. 252, n^{o} 10, p. 5569-5593 [*DOI :* 10.1016/J.JDE.2012.01.037], http://hal.inria.fr/inria-00616474.

International Conferences with Proceedings

[15] A. AJAMI, J.-F. BALMAT, J.-P. GAUTHIER, T. MAILLOT. *Path Planning and Ground Control Station Simulator for UAV*, in "Proceedings of the 2013 IEEE aerospace conference", 2013.

[16] U. BOSCAIN, M. CAPONIGRO, M. SIGALOTTI. *Controllability of the bilinear Schrödinger equation with several controls and application to a 3D molecule*, in "Proceedings of the 51th IEEE Conference on Decision and Control", 2012.

[17] U. BOSCAIN, R. DUITS, F. ROSSI, Y. SACHKOV. *Curve cuspless reconstruction via sub-Riemannian geometry*, in "Proceedings of the 51th IEEE Conference on Decision and Control", 2012, http://arxiv.org/abs/1203.3089.

[18] U. BOSCAIN, F. GRÖNBERG, R. LONG, H. RABITZ. *Time minimal trajectories for two-level quantum systems with two bounded controls*, in "Proceedings of the 51th IEEE Conference on Decision and Control", 2012, http://arxiv.org/abs/1211.0666.

[19] F. CHITTARO, P. MASON, U. BOSCAIN, M. SIGALOTTI. *Controllability of the Schroedinger equation via adiabatic methods and conical intersections of the eigenvalues*, in "Proceedings of the 51th IEEE Conference on Decision and Control", 2012.

Scientific Popularization

[20] M. LAPERT, Y. ZHANG, M. JANICH, S. J. GLASER, D. SUGNY. *Une nouvelle métode pour optimiser le contraste en imagerie médicale*, in "Actualités scientifiques du CNRS", 2012.

Other Publications

[21] D. BARILARI, U. BOSCAIN, R. W. NEEL. *Small time heat kernel asymptotics at the sub-Riemannian cut locus*, 2012, http://hal.inria.fr/hal-00687651.

[22] D. BARILARI, J. JENDREJ. *Small time heat kernel asymptotics at the cut locus on two-spheres of revolution*, http://arxiv.org/pdf/1211.1811v1.pdf.

[23] U. BOSCAIN, M. CAPONIGRO, M. SIGALOTTI. *Controllability of the bilinear Schrödinger equation with several controls and application to a 3D molecule*, http://hal.inria.fr/hal-00691706.

[24] U. BOSCAIN, G. CHARLOT, R. GHEZZI. *Normal forms and invariants for 2-dimensional almost-Riemannian structures*, 2012, http://arxiv.org/abs/1008.5036.

References in notes

[25] A. A. AGRACHEV, T. CHAMBRION. *An estimation of the controllability time for single-input systems on compact Lie groups*, in "ESAIM Control Optim. Calc. Var.", 2006, vol. 12, n^{o} 3, p. 409–441.

[26] A. A. AGRACHEV, D. LIBERZON. *Lie-algebraic stability criteria for switched systems*, in "SIAM J. Control Optim.", 2001, vol. 40, n^{o} 1, p. 253–269, http://dx.doi.org/10.1137/S0363012999365704.

[27] A. A. AGRACHEV, Y. L. SACHKOV. *Control theory from the geometric viewpoint*, Encyclopaedia of Mathematical Sciences, Springer-Verlag, Berlin, 2004, vol. 87, xiv+412, Control Theory and Optimization, II.

[28] A. A. AGRACHEV, A. V. SARYCHEV. *Navier-Stokes equations: controllability by means of low modes forcing*, in "J. Math. Fluid Mech.", 2005, vol. 7, n^{o} 1, p. 108–152, http://dx.doi.org/10.1007/s00021-004-0110-1.

[29] F. ALBERTINI, D. D'ALESSANDRO. *Notions of controllability for bilinear multilevel quantum systems*, in "IEEE Trans. Automat. Control", 2003, vol. 48, n^o 8, p. 1399–1403.

[30] C. ALTAFINI. *Controllability properties for finite dimensional quantum Markovian master equations*, in "J. Math. Phys.", 2003, vol. 44, n^o 6, p. 2357–2372.

[31] L. AMBROSIO, P. TILLI. *Topics on analysis in metric spaces*, Oxford Lecture Series in Mathematics and its Applications, Oxford University Press, Oxford, 2004, vol. 25, viii+133.

[32] G. ARECHAVALETA, J.-P. LAUMOND, H. HICHEUR, A. BERTHOZ. *An optimality principle governing human locomotion*, in "IEEE Trans. on Robotics", 2008, vol. 24, n^o 1.

[33] L. BAUDOUIN. *A bilinear optimal control problem applied to a time dependent Hartree-Fock equation coupled with classical nuclear dynamics*, in "Port. Math. (N.S.)", 2006, vol. 63, n^o 3, p. 293–325.

[34] L. BAUDOUIN, O. KAVIAN, J.-P. PUEL. *Regularity for a Schrödinger equation with singular potentials and application to bilinear optimal control*, in "J. Differential Equations", 2005, vol. 216, n^o 1, p. 188–222.

[35] L. BAUDOUIN, J. SALOMON. *Constructive solution of a bilinear optimal control problem for a Schrödinger equation*, in "Systems Control Lett.", 2008, vol. 57, n^o 6, p. 453–464, http://dx.doi.org/10.1016/j.sysconle.2007.11.002.

[36] K. BEAUCHARD. *Local controllability of a 1-D Schrödinger equation*, in "J. Math. Pures Appl. (9)", 2005, vol. 84, n^o 7, p. 851–956.

[37] K. BEAUCHARD, J.-M. CORON. *Controllability of a quantum particle in a moving potential well*, in "J. Funct. Anal.", 2006, vol. 232, n^o 2, p. 328–389.

[38] M. BELHADJ, J. SALOMON, G. TURINICI. *A stable toolkit method in quantum control*, in "J. Phys. A", 2008, vol. 41, n^o 36, 362001, 10, http://dx.doi.org/10.1088/1751-8113/41/36/362001.

[39] F. BLANCHINI. *Nonquadratic Lyapunov functions for robust control*, in "Automatica J. IFAC", 1995, vol. 31, n^o 3, p. 451–461, http://dx.doi.org/10.1016/0005-1098(94)00133-4.

[40] F. BLANCHINI, S. MIANI. *A new class of universal Lyapunov functions for the control of uncertain linear systems*, in "IEEE Trans. Automat. Control", 1999, vol. 44, n^o 3, p. 641–647, http://dx.doi.org/10.1109/9.751368.

[41] A. M. BLOCH, R. W. BROCKETT, C. RANGAN. *Finite Controllability of Infinite-Dimensional Quantum Systems*, in "IEEE Trans. Automat. Control", 2010.

[42] V. D. BLONDEL, J. THEYS, A. A. VLADIMIROV. *An elementary counterexample to the finiteness conjecture*, in "SIAM J. Matrix Anal. Appl.", 2003, vol. 24, n^o 4, p. 963–970, http://dx.doi.org/10.1137/S0895479801397846.

[43] A. BONFIGLIOLI, E. LANCONELLI, F. UGUZZONI. *Stratified Lie groups and potential theory for their sub-Laplacians*, Springer Monographs in Mathematics, Springer, Berlin, 2007, xxvi+800.

[44] B. Bonnard, D. Sugny. *Time-minimal control of dissipative two-level quantum systems: the integrable case*, in "SIAM J. Control Optim.", 2009, vol. 48, n° 3, p. 1289–1308, http://dx.doi.org/10.1137/080717043.

[45] A. Borzì, E. Decker. *Analysis of a leap-frog pseudospectral scheme for the Schrödinger equation*, in "J. Comput. Appl. Math.", 2006, vol. 193, n° 1, p. 65–88.

[46] A. Borzì, U. Hohenester. *Multigrid optimization schemes for solving Bose-Einstein condensate control problems*, in "SIAM J. Sci. Comput.", 2008, vol. 30, n° 1, p. 441–462, http://dx.doi.org/10.1137/070686135.

[47] U. Boscain. *Stability of planar switched systems: the linear single input case*, in "SIAM J. Control Optim.", 2002, vol. 41, n° 1, p. 89–112, http://dx.doi.org/10.1137/S0363012900382837.

[48] C. Brif, R. Chakrabarti, H. Rabitz. *Control of quantum phenomena: Past, present, and future*, Advances in Chemical Physics, S. A. Rice (ed), Wiley, New York, 2010.

[49] F. Bullo, A. D. Lewis. *Geometric control of mechanical systems*, Texts in Applied Mathematics, Springer-Verlag, New York, 2005, vol. 49, xxiv+726, Modeling, analysis, and design for simple mechanical control systems.

[50] R. Cabrera, H. Rabitz. *The landscape of quantum transitions driven by single-qubit unitary transformations with implications for entanglement*, in "J. Phys. A", 2009, vol. 42, n° 27, 275303, 9, http://dx.doi.org/10.1088/1751-8113/42/27/275303.

[51] G. Citti, A. Sarti. *A cortical based model of perceptual completion in the roto-translation space*, in "J. Math. Imaging Vision", 2006, vol. 24, n° 3, p. 307–326, http://dx.doi.org/10.1007/s10851-005-3630-2.

[52] J.-M. Coron. *Control and nonlinearity*, Mathematical Surveys and Monographs, American Mathematical Society, Providence, RI, 2007, vol. 136, xiv+426.

[53] W. P. Dayawansa, C. F. Martin. *A converse Lyapunov theorem for a class of dynamical systems which undergo switching*, in "IEEE Trans. Automat. Control", 1999, vol. 44, n° 4, p. 751–760, http://dx.doi.org/10.1109/9.754812.

[54] L. El Ghaoui, S.-I. Niculescu. *Robust decision problems in engineering: a linear matrix inequality approach*, in "Advances in linear matrix inequality methods in control", Philadelphia, PA, Adv. Des. Control, SIAM, Philadelphia, PA, 2000, vol. 2, p. xviii, 3–37.

[55] S. Ervedoza, J.-P. Puel. *Approximate controllability for a system of Schrödinger equations modeling a single trapped ion*, in "Ann. Inst. H. Poincaré Anal. Non Linéaire", 2009, vol. 26, p. 2111–2136.

[56] M. Fliess, J. Lévine, P. Martin, P. Rouchon. *Flatness and defect of non-linear systems: introductory theory and examples*, in "Internat. J. Control", 1995, vol. 61, n° 6, p. 1327–1361, http://dx.doi.org/10.1080/00207179508921959.

[57] B. Franchi, R. Serapioni, F. Serra Cassano. *Regular hypersurfaces, intrinsic perimeter and implicit function theorem in Carnot groups*, in "Comm. Anal. Geom.", 2003, vol. 11, n° 5, p. 909–944.

[58] M. GUGAT. *Optimal switching boundary control of a string to rest in finite time*, in "ZAMM Z. Angew. Math. Mech.", 2008, vol. 88, n^{o} 4, p. 283–305.

[59] J. HESPANHA, S. MORSE. *Stability of switched systems with average dwell-time*, in "Proceedings of the 38th IEEE Conference on Decision and Control, CDC 1999, Phoenix, AZ, USA", 1999, p. 2655–2660.

[60] D. HUBEL, T. WIESEL. *Brain and Visual Perception: The Story of a 25-Year Collaboration*, Oxford University Press, Oxford, 2004.

[61] R. ILLNER, H. LANGE, H. TEISMANN. *Limitations on the control of Schrödinger equations*, in "ESAIM Control Optim. Calc. Var.", 2006, vol. 12, n^{o} 4, p. 615–635, http://dx.doi.org/10.1051/cocv:2006014.

[62] A. ISIDORI. *Nonlinear control systems*, Communications and Control Engineering Series, Second, Springer-Verlag, Berlin, 1989, xii+479, An introduction.

[63] K. ITO, K. KUNISCH. *Optimal bilinear control of an abstract Schrödinger equation*, in "SIAM J. Control Optim.", 2007, vol. 46, n^{o} 1, p. 274–287.

[64] K. ITO, K. KUNISCH. *Asymptotic properties of feedback solutions for a class of quantum control problems*, in "SIAM J. Control Optim.", 2009, vol. 48, n^{o} 4, p. 2323–2343, http://dx.doi.org/10.1137/080720784.

[65] F. JEAN, G. ORIOLO, M. VENDITTELLI. *A Globally Convergent Steering Algorithm for Regular Nonholonomic Systems*, in "Proceedings of 44th IEEE CDC-ECC'05, Sevilla, Spain", 2005.

[66] R. KALMAN. *When is a linear control system optimal?*, in "ASME Transactions, Journal of Basic Engineering", 1964, vol. 86, p. 51–60.

[67] N. KHANEJA, S. J. GLASER, R. W. BROCKETT. *Sub-Riemannian geometry and time optimal control of three spin systems: quantum gates and coherence transfer*, in "Phys. Rev. A (3)", 2002, vol. 65, n^{o} 3, part A, 032301, 11.

[68] N. KHANEJA, B. LUY, S. J. GLASER. *Boundary of quantum evolution under decoherence*, in "Proc. Natl. Acad. Sci. USA", 2003, vol. 100, n^{o} 23, p. 13162–13166, http://dx.doi.org/10.1073/pnas.2134111100.

[69] V. S. KOZYAKIN. *Algebraic unsolvability of a problem on the absolute stability of desynchronized systems*, in "Avtomat. i Telemekh.", 1990, p. 41–47.

[70] G. LAFFERRIERE, H. J. SUSSMANN. *A differential geometry approach to motion planning*, in "Nonholonomic Motion Planning (Z. Li and J. F. Canny, editors)", Kluwer Academic Publishers, 1993, p. 235-270.

[71] J.-S. LI, N. KHANEJA. *Ensemble control of Bloch equations*, in "IEEE Trans. Automat. Control", 2009, vol. 54, n^{o} 3, p. 528–536, http://dx.doi.org/10.1109/TAC.2009.2012983.

[72] D. LIBERZON, J. P. HESPANHA, A. S. MORSE. *Stability of switched systems: a Lie-algebraic condition*, in "Systems Control Lett.", 1999, vol. 37, n^{o} 3, p. 117–122, http://dx.doi.org/10.1016/S0167-6911(99)00012-2.

[73] D. LIBERZON. *Switching in systems and control*, Systems & Control: Foundations & Applications, Birkhäuser Boston Inc., Boston, MA, 2003, xiv+233.

[74] H. LIN, P. J. ANTSAKLIS. *Stability and stabilizability of switched linear systems: a survey of recent results*, in "IEEE Trans. Automat. Control", 2009, vol. 54, n$^{\mathrm{o}}$ 2, p. 308–322, http://dx.doi.org/10.1109/TAC.2008.2012009.

[75] Y. LIN, E. D. SONTAG, Y. WANG. *A smooth converse Lyapunov theorem for robust stability*, in "SIAM J. Control Optim.", 1996, vol. 34, n$^{\mathrm{o}}$ 1, p. 124–160, http://dx.doi.org/10.1137/S0363012993259981.

[76] W. LIU. *Averaging theorems for highly oscillatory differential equations and iterated Lie brackets*, in "SIAM J. Control Optim.", 1997, vol. 35, n$^{\mathrm{o}}$ 6, p. 1989–2020, http://dx.doi.org/10.1137/S0363012994268667.

[77] Y. MADAY, J. SALOMON, G. TURINICI. *Monotonic parareal control for quantum systems*, in "SIAM J. Numer. Anal.", 2007, vol. 45, n$^{\mathrm{o}}$ 6, p. 2468–2482, http://dx.doi.org/10.1137/050647086.

[78] A. N. MICHEL, Y. SUN, A. P. MOLCHANOV. *Stability analysis of discountinuous dynamical systems determined by semigroups*, in "IEEE Trans. Automat. Control", 2005, vol. 50, n$^{\mathrm{o}}$ 9, p. 1277–1290, http://dx.doi.org/10.1109/TAC.2005.854582.

[79] M. MIRRAHIMI. *Lyapunov control of a particle in a finite quantum potential well*, in "Proceedings of the 45th IEEE Conference on Decision and Control", 2006.

[80] M. MIRRAHIMI, P. ROUCHON. *Controllability of quantum harmonic oscillators*, in "IEEE Trans. Automat. Control", 2004, vol. 49, n$^{\mathrm{o}}$ 5, p. 745–747.

[81] A. P. MOLCHANOV, Y. S. PYATNITSKIY. *Criteria of asymptotic stability of differential and difference inclusions encountered in control theory*, in "Systems Control Lett.", 1989, vol. 13, n$^{\mathrm{o}}$ 1, p. 59–64, http://dx.doi.org/10.1016/0167-6911(89)90021-2.

[82] R. MONTGOMERY. *A tour of subriemannian geometries, their geodesics and applications*, Mathematical Surveys and Monographs, American Mathematical Society, Providence, RI, 2002, vol. 91, xx+259.

[83] R. M. MURRAY, S. S. SASTRY. *Nonholonomic motion planning: steering using sinusoids*, in "IEEE Trans. Automat. Control", 1993, vol. 38, n$^{\mathrm{o}}$ 5, p. 700–716, http://dx.doi.org/10.1109/9.277235.

[84] V. NERSESYAN. *Growth of Sobolev norms and controllability of the Schrödinger equation*, in "Comm. Math. Phys.", 2009, vol. 290, n$^{\mathrm{o}}$ 1, p. 371–387.

[85] A. Y. NG, S. RUSSELL. *Algorithms for Inverse Reinforcement Learning*, in "Proc. 17th International Conf. on Machine Learning", 2000, p. 663–670.

[86] J. PETITOT. *Neurogéomètrie de la vision. Modèles mathématiques et physiques des architectures fonctionnelles*, Les Éditions de l'École Polythecnique, 2008.

[87] J. PETITOT, Y. TONDUT. *Vers une neurogéométrie. Fibrations corticales, structures de contact et contours subjectifs modaux*, in "Math. Inform. Sci. Humaines", 1999, n$^{\mathrm{o}}$ 145, p. 5–101.

[88] H. RABITZ, H. DE VIVIE-RIEDLE, R. MOTZKUS, K. KOMPA. *Wither the future of controlling quantum phenomena?*, in "SCIENCE", 2000, vol. 288, p. 824–828.

[89] D. ROSSINI, T. CALARCO, V. GIOVANNETTI, S. MONTANGERO, R. FAZIO. *Decoherence by engineered quantum baths*, in "J. Phys. A", 2007, vol. 40, n^{o} 28, p. 8033–8040, http://dx.doi.org/10.1088/1751-8113/40/28/S12.

[90] P. ROUCHON. *Control of a quantum particle in a moving potential well*, in "Lagrangian and Hamiltonian methods for nonlinear control 2003", Laxenburg, IFAC, Laxenburg, 2003, p. 287–290.

[91] A. SASANE. *Stability of switching infinite-dimensional systems*, in "Automatica J. IFAC", 2005, vol. 41, n^{o} 1, p. 75–78, http://dx.doi.org/10.1016/j.automatica.2004.07.013.

[92] A. SAURABH, M. H. FALK, M. B. ALEXANDRE. *Stability analysis of linear hyperbolic systems with switching parameters and boundary conditions*, in "Proceedings of the 47th IEEE Conference on Decision and Control, CDC 2008, December 9-11, 2008, Cancún, Mexico", 2008, p. 2081–2086.

[93] M. SHAPIRO, P. BRUMER. *Principles of the Quantum Control of Molecular Processes*, Principles of the Quantum Control of Molecular Processes, pp. 250. Wiley-VCH, February 2003.

[94] R. SHORTEN, F. WIRTH, O. MASON, K. WULFF, C. KING. *Stability criteria for switched and hybrid systems*, in "SIAM Rev.", 2007, vol. 49, n^{o} 4, p. 545–592, http://dx.doi.org/10.1137/05063516X.

[95] H. J. SUSSMANN. *A continuation method for nonholonomic path finding*, in "Proceedings of the 32th IEEE Conference on Decision and Control, CDC 1993, Piscataway, NJ, USA", 1993, p. 2718–2723.

[96] E. TODOROV. *12*, in "Optimal control theory", Bayesian Brain: Probabilistic Approaches to Neural Coding, Doya K (ed), 2006, p. 269–298.

[97] G. TURINICI. *On the controllability of bilinear quantum systems*, in "Mathematical models and methods for ab initio Quantum Chemistry", M. DEFRANCESCHI, C. LE BRIS (editors), Lecture Notes in Chemistry, Springer, 2000, vol. 74.

[98] L. YATSENKO, S. GUÉRIN, H. JAUSLIN. *Topology of adiabatic passage*, in "Phys. Rev. A", 2002, vol. 65, 043407, 7.

[99] E. ZUAZUA. *Switching controls*, in "Journal of the European Mathematical Society", 2011, vol. 13, n^{o} 1, p. 85–117.