Activity Report 2012

Project-Team GEOSTAT

Geometry and Statistics in acquisition data

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
Optimization, Learning and Statistical Methods

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Project-Team GEOSTAT

Keywords: Signal Processing, Multiscale Analysis, Complexity, Statistical Methods, Statistical Physics

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2. Overall Objectives

2.1. Overall objectives

singularity exponent A measure of the unpredictability around a point in a complex signal. Based on local reconstruction around a point, singularity exponents can be evaluated in different ways and in different contexts (e.g. non-localized, through the consideration of moments and structure fonctions, leading to singularity spectra). In GEOSTAT we study approaches corresponding to *far from equilibrium* hypothesis (e.g. microcanonical) leading to geometrically localized singulariy exponents.

LPE Local Predictability Exponent: another name for singularity exponents, that better underlines the relation with predictability.

Framework of reconstructible systems Complex systems whose acquisitions can be reconstructed from the knowledge of the geometrical sets that maximize statistical information content. Study of complex signals' compact representations associated to unpredictability.

MMF Microcanonical Multiscale Formalism.

Sparse representation The representation of a signal as a linear combination of elements taken in a dictionary, with the aim of finding the sparset possible one.

Optimal wavelet (OW). Wavelets whose associated multiresolution analysis optimizes inference along the scales in complex systems.

GEOSTAT is a research project in **nonlinear digital signal processing**, with the fundamental distinction that it considers the signals as the realizations of complex dynamic systems. The research in GEOSTAT encompasses nonlinear signal processing and the study of emergence in complex systems, with a strong emphasis on geometric approaches to complexity. Consequently, research in GEOSTAT is oriented towards the determination, in real signals, of quantities or phenomena that are known to play an important role both in the evolution of dynamical systems whose acquisitions are the signals under study, and in the compact representations of the signals themselves. Hence we first mention:

- Singularity exponents, also called Local Predictability Exponents or LPEs,
- how singularity exponents can be related to *sparse representations* with reconstruction formulae,
- comparison with embedding techniques, such as the one provided by the classical theorem of Takens [39], [31].
- Lyapunov exponents, how they are related to intermittency, large deviations and singularity exponents,
- various forms of entropies,
- multiresolution analysis, specifically when performed on the singularity exponents,
- the cascading properties of associated random variables,
- persistence along the scales, *optimal wavelets*,
- the determination of subsets where statistical information is maximized, their relation to reconstruction and compact representation,

and, above all, **the ways that lead to effective numerical and high precision determination of nonlinear characteristics in real signals**. The MMF (Multiscale Microcanonical Formalism) is one of the ways to partly unlock this type of analysis, most notably w.r.t. LPEs and reconstructible systems [8]. We presently concentrate our efforts on it, but GEOSTAT is intended to explore other ways [27]. Presently GEOSTAT explores new methods for analyzing and understanding complex signals in different applicative domains through the theoretical advances of the MMF, and the framework of **reconstructible systems** [40]. Derived from ideas in Statistical Physics, the methods developped in GEOSTAT provide new ways to relate and evaluate quantitatively the *local irregularity* in complex signals and systems, the statistical concepts of *information content* and *most informative subset*. That latter notion is developed through the notion of *transition front* and *Most Singular Manifold*. As a result, GEOSTAT is aimed at providing *radically new approaches* to the study of signals acquired from different complex systems (their analysis, their classification, the study of their dynamical properties etc.). The common characteristic of these signals, as required by *universality classes* [35] [36] [33], being the existence of a *multiscale organization* of the systems. For instance, the classical notion of *edge* or *border*, which is of multiscale nature, and whose importance is well known in Computer Vision and Image Processing, receives profound and rigorous new definitions, in relation with the more physical notion of *transition* and fits adequately to the case of chaotic data. The description is analogous to the modeling of states far from equilibrium, that is to say, there is no stationarity assumption. From this formalism we derive methods able to determine geometrically the most informative part in a signal, which also defines its global properties and allows for *compact representation* in the wake of known problematics addressed, for instance, in *time-frequency analysis*. In this way, the MMF allows the reconstruction, at any prescribed quality threshold, of a signal from its most informative subset, and is able to quantitatively evaluate key features in complex signals (unavailable with classical methods in Image or Signal Processing). It appears that the notion of *transition front* in a signal is much more complex than previously expected and, most importantly, related to multiscale notions encountered in the study of nonlinearity [37]. For instance, we give new insights to the computation of dynamical properties in complex signals, in particular in signals for which the classical tools for analyzing dynamics give poor results (such as, for example, correlation methods or optical flow for determining motion in turbulent datasets). The problematics in GEOSTAT can be summarized in the following items:

- the accurate determination in any n-dimensional complex signal of LPEs **at every point in the signal domain** [41][13].
- The geometrical determination and organization of *singular manifolds* associated to various transition fronts in complex signals, the study of their geometrical arrangement, and the relation of that arrangement with statistical properties or other global quantities associated to the signal, e.g. *cascading properties* [9].
- The study of the relationships between the dynamics in the signal and the distributions of LPEs [42][9].
- **Multiresolution analysis** and inference along the scales [9], [2].
- The study of the relationships between the distributions of LPEs and other formalisms associated to *predictability* in complex signals and systems, such as cascading variables, large deviations and Lyapunov exponents.
- The ability to compute *optimal wavelets* and relate such wavelets to the geometric arrangement of singular manifolds and cascading properties[3].
- The translation of *recognition*, *analysis* and *classification problems* in complex signals to simpler and more accurate determinations involving new operators acting on singular manifolds using the framework of reconstructible systems.

3. Scientific Foundations

3.1. Dynamics of complex systems

GEOSTAT is studying complex signals under the point of view of *nonlinear* methods, in the sense of *nonlinear physics* i.e. the methodologies developed to study complex systems, with a strong emphasis on multiresolution analysis. Linear methods in signal processing refer to the standard point of view under which operators are expressed by simple convolutions with impulse responses. Linear methods in signal processing are widely used, from least-square deconvolution methods in adaptive optics to source-filter models in speech processing. Linear methods do not unlock the multiscale structures and cascading variables of primarily importance as previewed by the physics of the phenomena. This is the reason why new approaches, such as DFA (Detrented Fluctuation Analysis), Time-frequency analysis, variations on curvelets [38] etc. have appeared during the last decades. One important result obtained in GEOSTAT is the effective use of multiresolution analysis associated to optimal inference along the scales of a complex system. The multiresolution analysis is performed on dimensionless quantities given by the *singularity exponents* which encode properly the geometrical structures associated to multiscale organization. This is applied successfully in the derivation of high resolution ocean dynamics, or the high resolution mapping of gazeous exchanges between the ocean and the atmosphere; the latter is of primary importance for a quantitative evaluation of global warming. Understanding the dynamics of complex systems is recognized as a new discipline, which makes use of theoretical and methodological foundations coming from nonlinear physics, the study of dynamical systems and many aspects of computer science. One of the challenges is related to the question of *emergence* in complex systems: large-scale effects measurable macroscopically from a system made of huge numbers of interactive agents [29], [26], [43], [34]. Some quantities related to nonlinearity, such as Lyapunov exponents, Kolmogorov-Sinai entropy etc. can be computed at least in the phase space [27]. Consequently, knowledge from acquisitions of complex systems (which include *complex signals*) could be obtained from information about the phase space. A result from F. Takens [39] about strange attractors in turbulence has motivated the determination of discrete dynamical systems associated to time series [31], and consequently the theoretical determination of nonlinear characteristics associated to complex acquisitions. Emergence phenomena can also be traced inside complex signals themselves, by trying to localize information content geometrically. Fundamentally, in the nonlinear analysis of complex signals there are broadly two approaches: characterization by attractors (embedding and bifurcation) and time-frequency, multiscale/multiresolution approaches. Time-frequency analysis [28] and multiscale/multiresolution are the subjects of intense research and are profoundly reshaping the analysis of complex signals by nonlinear approaches [25], [30]. In real situations, the phase space associated to the acquisition of a complex phenomenon is unknown. It is however possible to relate, inside the signal's domain, local predictability to local reconstruction and deduce from that singularity exponents (SEs) [8] [5]. The SEs are defined at any point in the signal's domain, they relate, but are different, to other kinds of exponents used in the nonlinear analysis of complex signals. We are working on their relation with:

- properties in universality classses,
- the geometric localization of multiscale properties in complex signals,
- cascading characteristics of physical variables,
- optimal wavelets and inference in multiresolution analysis.

The alternative approach taken in GEOSTAT is microscopical, or geometrical: the multiscale structures which have their "fingerprint" in complex signals are being isolated in a single realization of the complex system, i.e. using the data of the signal itself, as opposed to the consideration of grand ensembles or a wide set of realizations. This is much harder than the ergodic approaches, but it is possible because a reconstruction formula such as the one derived in [40] is local and reconstruction in the signal's domain is related to predictability.

Nonlinear signal processing is making use of quantities related to predictability. For instance the first Lyapunov exponent λ_1 is related, from Osedelec's theorem, to the limiting behaviour of the response, after a time t, to perturbation in the phase space $\log R_\tau(t)$:

$$\lambda_1 = \lim_{t\to\infty} \frac{1}{t} \langle \log R_\tau(t) \rangle \tag{1}$$

with $\langle\cdot\rangle$ being time average and R_τ the response to a perturbation [27]. More refined information is provided by the Kolmogorov-Sinai entropy:

$$h_{KS} = \lim_{\varepsilon\to 0} \lim_{t\to\infty} \frac{1}{t} \log N_{\text{eff}}(\varepsilon, t) \tag{2}$$

($N_{\text{eff}}(\varepsilon, t)$ is related to events which appear with very high probability in long time). In GEOSTAT our aim is to relate these classical quantities (among others) to the behaviour of SEs, which are defined by a limiting behaviour

$$\mu\left(\mathcal{B}_r(\mathbf{x})\right) \;=\; \alpha(\mathbf{x})\, r^{d+h(\mathbf{x})} \;+\; o\left(r^{d+h(\mathbf{x})}\right) \;\; (r \to 0) \tag{3}$$

(d: dimension of the signal's domain, μ: multiscale measure, typically whose density is the gradient's norm, $\mathcal{B}_r(\mathbf{x})$: ball of radius r centered at $\mathbf{x}$). For precise computation, SEs can be smoothly interpolated by projecting wavelets:

$$\mathcal{T}_\Psi \mu(\mathbf{x}, r) = \int_{\mathbb{R}^d} \mathrm{d}\mu(\mathbf{x}') \frac{1}{r^d} \Psi\left(\frac{\mathbf{x} - \mathbf{x}'}{r}\right) \tag{4}$$

(Ψ: mother wavelet, admissible or not). SEs are related to the framework of reconstructible systems, and consequently to predictability. They unlock the geometric localization of multiscale structures in a complex signal:

$$\mathcal{F}_h = \{\mathbf{x} \in \Omega \;\mid\; h(\mathbf{x}) = h\}, \tag{5}$$

(Ω: signal's domain) and are consequently in relation with *optimal wavelets*:

$$\mathcal{T}_\psi[\mathbf{s}](\mathbf{x},\mathbf{r_1}) = \zeta_{r_1/r_2}(\mathbf{x})\mathcal{T}_\psi[\mathbf{s}](\mathbf{x},\mathbf{r_2}) \tag{6}$$

($\mathbf{r_1} < \mathbf{r_2}$: two scales of observation, ζ: injection variable between the scales, ψ: optimal wavelet) and their **multiresolution analysis**. They are related to persistence along the scales and lead to multiresolution analysis whose coefficients verify

$$\alpha_s = \eta_1\alpha_f + \eta_2 \tag{7}$$

with α_s and α_f referrin to child and parent coefficients, η_1 and η_2 are random variables independent of α_s and α_f and also independent of each other.

In a first example we give some insight about the collaboration with LEGOS Dynbio team [1] about high-resolution ocean dynamics from microcanonical formulations in nonlinear complex signal analysis. LPEs relate to the geometric structures linked with the cascading properties of indefinitely divisible variables in turbulent flows. Cascading properties can be represented by optimal wavelets (OWs); this opens new and fascinating directions of research for the determination of ocean motion field at high spatial resolution. OWs in a microcanonical sense pave the way for the determination of the energy injection mechanisms between the scales. From this results a new method for the complete evaluation of oceanic motion field is introduced; it consists in propagating along the scales the norm and the orientation of ocean dynamics deduced at low spatial resolution (geostrophic from altimetry and a part of ageostrophic from wind stress products). Using this approach, there is no need to use several temporal occurrences. Instead, the proper determination of the turbulent cascading and energy injection mechanisms in oceanographic signals allows the determination of oceanic motion field at the SST or Ocean colour spatial resolution (pixel size: 4 kms). We use the Regional Ocean Modelling System (ROMS) to validate the results on simulated data and compare the motion fields obtained with other techniques. See figure 1.

In a second example, we show in figure 2 the highly promising results obtained in the application of nonlinear signal processing and multiscale techniques to the localization of heart fibrillation phenomenon acquired from a real patient and mapped over a reconstructed 3D surface of the heart. The notion of *source field*, defined in GEOSTAT from the computation of derivative measures related to the singularity exponents allows the localization of arythmic phenomena inside the heart [6].

Our last example is about speech. In speech analysis, we use the concept of the Most Singular Manifold (MSM) to localize critical events in domain of this signal. We show that in case of voiced speech signals, the MSM coincides with the instants of significant excitation of the vocal tract system. It is known that these major excitations occur when the glottis is closed, and hence, they are called the Glottal Closure Instants (GCI). We use the MSM to develop a reliable and noise robust GCI detection algorithm and we evaluate our algorithm using contemporaneous Electro-Glotto-Graph (EGG) recordings. See figure 3.

4. Application Domains

4.1. Application Domains

In GEOSTAT, the development of nonlinear methods for the study of complex systems and signals is conducted on four broad types of complex signals:

- Ocean dynamics and ocean/atmosphere interactions: generation of high-resolution maps from cascading properties and the determination of optimal wavelets[9], geostrophic or non-geostrophic-complex oceanic dynamics, mixing phenomena.

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[1] http://www.legos.obs-mip.fr/recherches/equipes/dynbio.

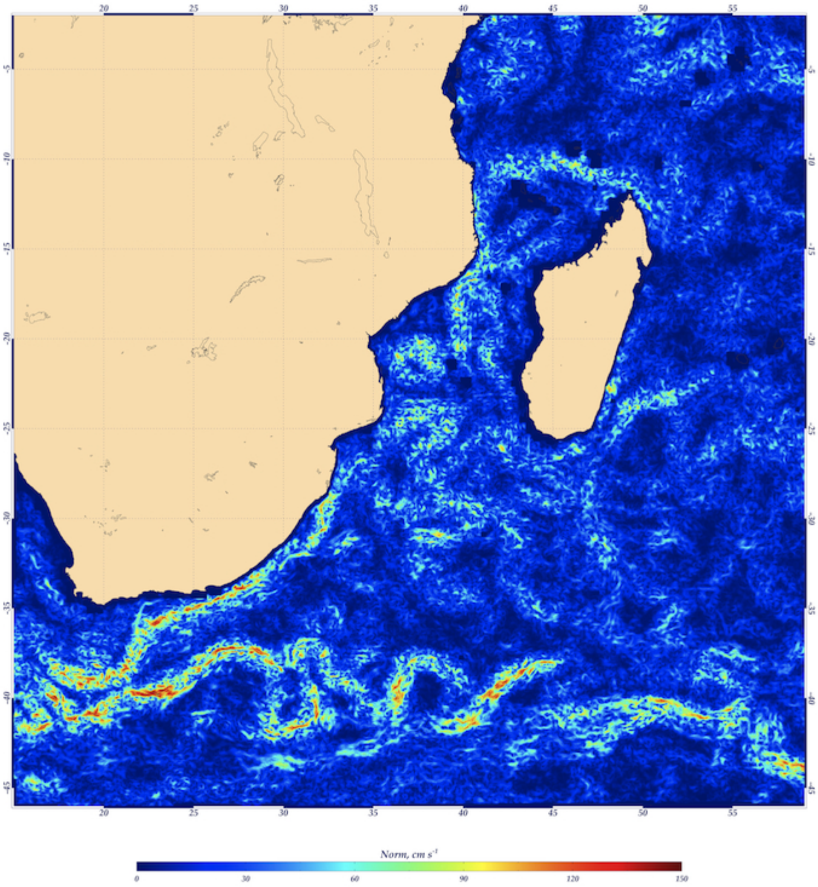

Figure 1. Visualization of the motion field computed at high spatial resolution (pixel size: 4kms) over a wide area around South Africa. The ocean dynamics is computed by propagating low resolution information coming from altimetry data (pixel size: 24 kms) along approximated optimal multiresolution analysis computed over the singularity exponents of Sea Surface Temperature data obtained from MODIS AQUA and OSTIA. Common work between GEOSTAT and DYNBIO (LEGOS, CNRS UMR 55 66, Toulouse).

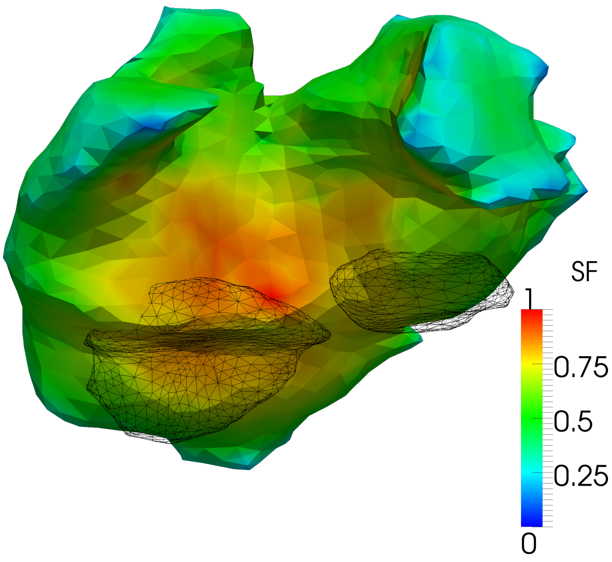

Figure 2. Result of the computation of a normalized source field over the 3D epicardial surface of the atria, from electric potential data acquired on a regular grid of electrodes placed on a patient's chest. There is a strong correlation between the red parts of the source field and the locations inside the heart where fibrillation occurs. Inputted data courtesy of IHU LIRYC.

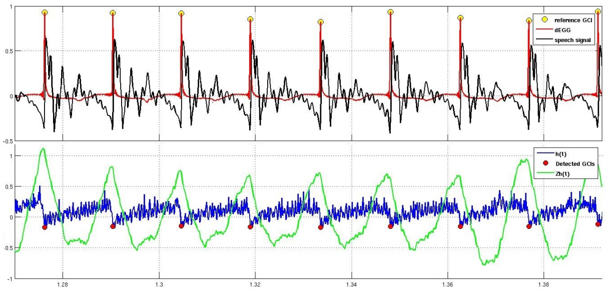

Figure 3. Top: A segment of a voiced speech signal (in black) along with the differentiated EGG (dEGG) recording (in red). Local maxima of dEGG shows the reference GCIs (yellow circles). Bottom: The singularity exponents (in blue) along with an auxiliary functional (in green) defined as $Zh(t) = \sum_{u=t-T_L}^{t-\delta t} h(u) - \sum_{u=t}^{t+T_L} h(u)$ *(*$h(t)$*: singularity exponent at* t*). In each positive half-period of* $Zh(t)$*, the minimum of singularity exponents is taken as the GCI (red circle).*

- Speech signal (analysis, recognition, classification)[1].
- Optimal wavelets for phase reconstruction in adaptive optics[2].
- Heartbeat signals, in cooperation with IHU LIRYC and Professor M. Haissaguerre (INSERM EA 2668 Electrophysiology and Cardiac Stimulation)[5] [35].

5. Software

5.1. FluidExponents

Participants: Denis Arrivault [correspondant], Hussein Yahia, Joel Sudre.

Denis Arrivault has joined the team for a complete refoundation, rewriting, generalization and diffusion of the FluidExponents software. FluidExponents is a software implementation of the MMF, presently written in Java, in a cooperative development mode on the Inria GForge, deposited at APP in 2010. The new software is presently in the phase of specification, and will be rewritten in C++, using existing libraries for data containers, mathematical computation and user interface. Denis Arrivault is recruited for a 24 month period on FluidExponents ADT.
During the new development, researchers still make use of the current version of the FluidExponents software written in Java, version number 0.8. Contact: denis.arrivault@inria.fr.

6. New Results

6.1. Multiresolution analysis and optimal inference for high resolution ocean dynamics and ocean/atmosphere fluxes

Participants: Hussein Yahia [correspondant], Véronique Garçon, Oriol Pont, Joel Sudre, Christine Provost, Antonio Turiel, Christoph Garbe, Claire Pottier, Boris Dewitte.

A $p_{CO_2}^{ocean}$ signal computed as an output from the ROMS coupled physical/biogeochemical simulation model possesses the characteristics of the presence of a multiscale organization, typical of turbulence, which can be evidenced by the computation of singularity spectra. The multiscale organization is related to the cascading properties of intensive variables acquired from the underlying system. We show how to perform inference along the scales in order to build higher resolution of $p_{CO_2}^{ocean}$ maps. Figure 4 illustrates clearly one of the main ideas implemented in this study: coherent structures of $p_{CO_2}^{ocean}$ and SST (Sea Surface Temperature) signals are related, and the LPEs, which are dimensionless quantities recording transition strengths in a signal, encode properly the multiscale transitions.

We perform a linear regression test:

$$\mathcal{S}(p_{CO_2}^{ocean})(x) = a(x)\mathcal{S}(SST)(x) + b(x)\mathcal{S}(CHLa)(x) + c(x) \tag{8}$$

with $\mathcal{S}(p_{CO_2}^{ocean})(x)$: LPE of $p_{CO_2}^{ocean}$ at x, $\mathcal{S}(SST)(x)$: LPE of SST at x, $\mathcal{S}(CHLa)(x)$: LPE of CHLa signal at x (CHLa: ocean colour data, corresponding to chlorophyl concentration). Tests are conducted over a period of 10 years on ROMS simulated data, with images corresponding to 128×128 pixels for the high resolution and 32×32 for the low resolution. There is one data every 10 days. In figure 5 we compare the functional dependencies of $p_{CO_2}^{ocean}$ vs. SST and CHLa with those of the corresponding LPEs: the original signals are physical variables of different dimensions, with complex undetermined functional dependencies. On the contrary, the dimensionless LPEs of these variables, which record the multiscale transitions, display clearly a much simpler dependency, approximated at satisfactory precision by a linear regression.

Figure 4. Local Predictability Exponents (LPEs) of ROMS-simulated $p^{ocean}_{CO_2}$ signal (left) and of corresponding SST (Sea Surface Temperature) generated signal (right). Transitions are are visually and quantitatively correlated, although not the same.

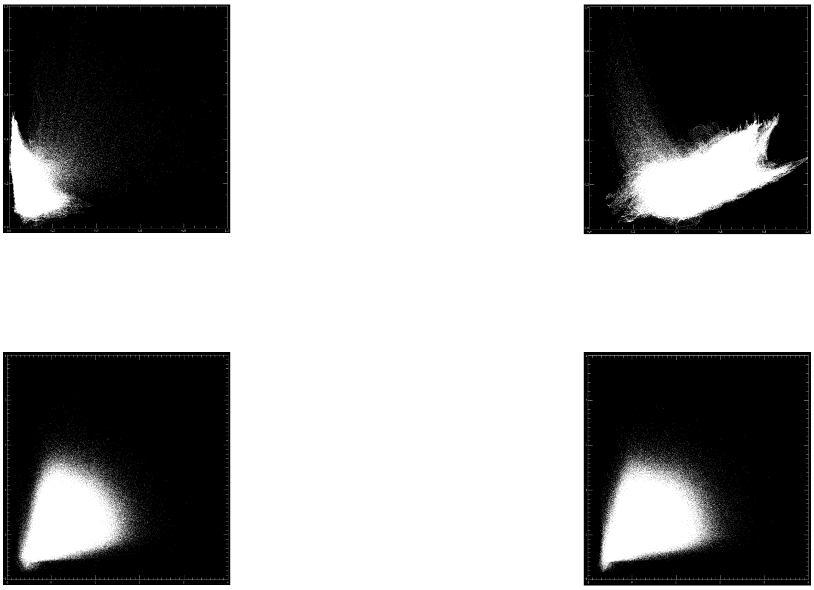

Figure 5. Pictures indicating the nature of the functional dependencies of $p^{ocean}_{CO_2}$ vs. CHLa (top left), of $p^{ocean}_{CO_2}$ vs. SST (top right), of $\mathbb{S}(p^{ocean}_{CO_2})$ vs $\mathbb{S}(CHLa)$ (bottom left) and of $\mathbb{S}(p^{ocean}_{CO_2})$ vs $\mathbb{S}(SST)$ (bottom right). The dependencies are computed on a 10-year ROMS simulation dataset, with a time frequency of one every 10 days.

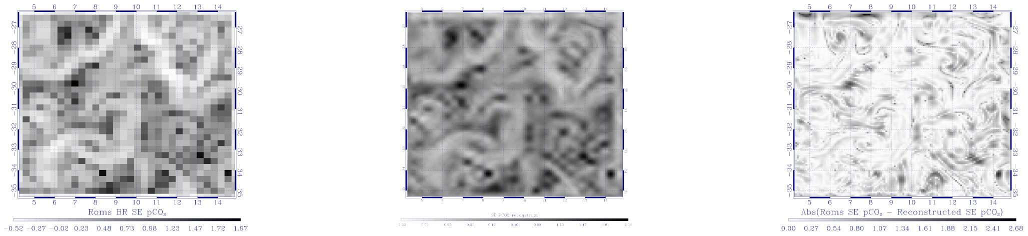

Figure 6. Left : the low resolution version of LPEs for $\mathcal{S}(p_{CO_2}^{ocean})$. Middle: result of the reconstruction. Right: absolute difference map between the ROMS generated high resolution LPEs and the reconstructed.

We prove the feasibility of a reconstruction by computing the high resolution LPEs $\mathcal{S}(p_{CO_2}^{ocean})(x)$ from their low resolution counterparts and an effective multiresolution analysis, using only an approximation of the optimal wavelet in the form of a Battle-Lemarié 3-31 mother wavelet. We show in figure 6 the results obtained by inference along the scales. The reconstructed LPEs of $p_{CO_2}^{ocean}$ are in good correspondence with the original high resolution signal.

- Related publications: [15], [16], [24], [14].

6.2. Singularity analysis and reconstructible systems

Participants: Oriol Pont [correspondant], Hussein Yahia, Antonio Turiel.

The local singularity exponents of a signal are directly related to the distribution of information in it. This fact implies that accurate evaluation of such exponents opens the door to signal reconstruction and characterisation of the dynamical parameters of the process originating the signal. Many practical implications arise in a context of digital signal processing, since the information on singularity exponents is usable for compact encoding, reconstruction and inference. The evaluation of singularity exponents in a digital context is not straightforward and requires the calculation of the Unpredictable Point Manifold of the signal. In this work, we present an algorithm for estimating the values of singularity exponents at every point of a digital signal of any dimension. We show that the key ingredient for robust and accurate reconstructibility performance lies on the definition of multiscale measures in the sense that they encode the degree of singularity and the local predictability at the same time. See figure 7.

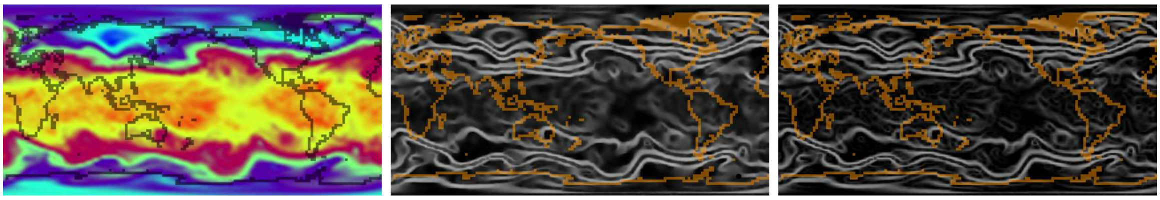

Figure 7. Left: 876576th hour slice of ERA-40 artificially rescaled 4x with bicubic interpolation for the purpose of clarity of illustration. Middle: singularity exponents calculated only in the space. Some rescaling artefacts visibly appear but without significant disturbance of the fine structure details. Right: singularity exponents calculated in the space-time domain. Notice the increased degree of detail when the temporal information is taken into account.

- Related publication: [13].

6.3. Multiscale analysis of the heart electric potential: describing atrial fibrillation

Participants: Oriol Pont [correspondant], Hussein Yahia, Rémi Dubois.

The cardiac electrical activity is a complex system, for which nonlinear signal-processing is required to characterize it properly. In this context, an analysis in terms of singularity exponents is shown to provide compact and meaningful descriptors of the structure and dynamics. In particular, singularity components reconstruct the epicardial electric potential maps of human atria, inverse-mapped from surface potentials; such approach describe sinus-rhythm dynamics as well as atrial flutter and atrial fibrillation. See figure 2.

- Related publications: [12], [20], [23].

6.4. Edges, transitions and criticality

Participants: Suman Maji [correspondant], Hussein Yahia.

In this work, various notions of edges encountered in digital image processing are reviewed in terms of compact representation (or completion). We show that critical exponents defined in Statistical Physics lead to a much more coherent definition of edges, consistent across the scales in acquisitions of natural phenomena, such as high resolution natural images or turbulent acquisitions. Edges belong to the multiscale hierarchy of an underlying dynamics, they are understood from a statistical perspective well adapted to fit the case of natural images. Numerical computation methods for the evaluation of critical exponents in the non-ergodic case are recalled, which apply for the vast majority of natural images. We study the framework of reconstructible systems in a microcanonical formulation, show how it redefines edge completion, and how it can be used to evaluate and assess quantitatively the adequation of edges as candidates for compact representations. We study with particular attention the case of turbulent data, in which edges in the classical sense are particularly challenged. Tests are conducted and evaluated on a standard database for natural images. We test the newly introduced compact representation as an ideal candidate for evaluating turbulent cascading properties of complex images, and we show better reconstruction performance than the classical tested methods. See figure 8.

6.5. Reconstruction of Optical phase from acquired sub-image gradients

Participants: Suman Maji [correspondant], Hussein Yahia, Thierry Fusco.

Turbulence in the Earth's atmosphere leads to a distortion in the planar wavefront from outer space resulting in a phase error. This phase error is responsible for the refractive blurring of images accounting to the loss in spatial resolution power of ground based telescopes. The common mechanism used to remove phase error from incoming wavefront is Adaptive Optics (AO). In AO systems, an estimate of the phase error is obtained from the gradient measurements of the wavefront collected by a Hartmann-Shack (HS) sensor. The correction estimate is then passed through a servo-control loop to a deformable mirror which compensates for the loss in resolution power. In this work, we propose a new approach to reconstructing the phase error from the HS gradient measurements using the MMF. We also validate the results using standard validation techniques in Adaptive Optics (log power spectrum, structure functions). See figure 9.

- Related publications: [18], [19].

6.6. Discriminative learning for Automatic speaker recognition

Participants: Reda Jourani [correspondant], Khalid Daoudi, Régine André-Obrecht, Driss Aboutajdine.

We continued our work aiming at developing efficient versions of Large Margin Gaussian Mixture Models (LM-GMM) for speaker identification. We developed a new and efficient learning algorithm and evaluated it on NIST-SRE'2006 data. The results show that, combined with the channel compesentation technique SFA, this new algorithm outperforms the state-of-the-art discriminative method GMM-supervectors SVM combined with NAP compensatation.

- Related publication: [10].

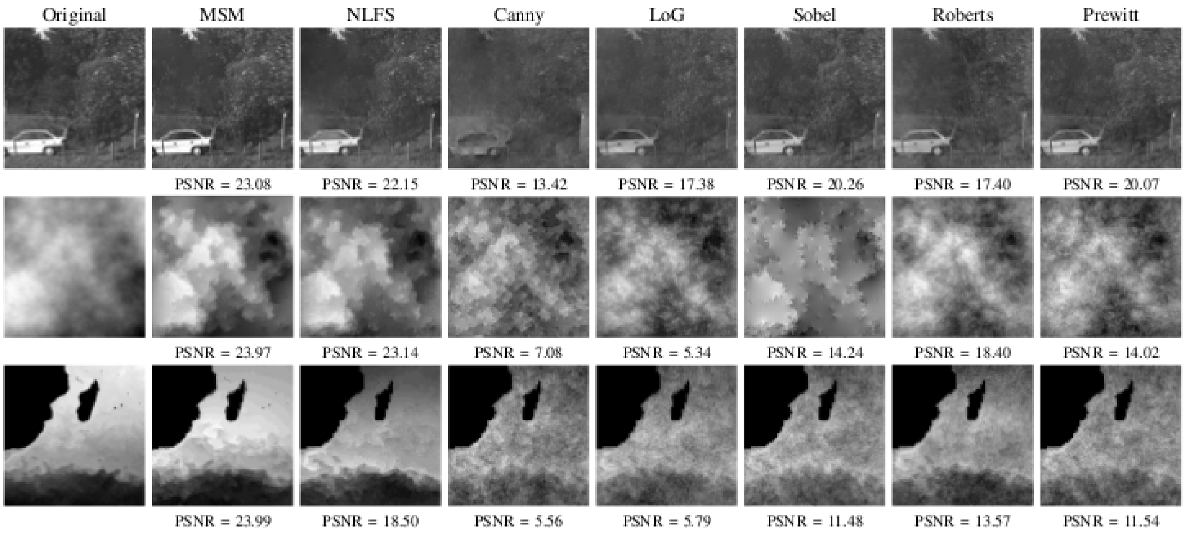

Figure 8. From left to right in each line: an original input image, and the reconstruction performed on the outputs resulting from various edge detection algorithms, showing the superiority of edge pixels computed from the Microcanonical Multiscale Formalism (column MSM). Note that NLFS [32], which is based on nonlinear filtering, performs the best after MSM.

6.7. Speech Analysis

Participants: Vahid Khanagha [correspondant], Khalid Daoudi, Hussein Yahia, Oriol Pont.

- Development of a GCI detection algorithm (Vahid Khanagha, Khalid Daoudi, Hussein Yahia). According to the aerodynamic theory of voicing, the excitation source for voiced speech sounds is represented as glottal pulses, which to a first approximation, can be considered to occur at discrete instants of time. This major excitation usually coincides with the Glottal Closure Instants (the GCIs). The precise detection of GCIs has found many applications in speech technology: accurate estimation of vocal tract system, pitch marking of speech for pitch synchronous speech processing algorithms, conversion of pitch and duration of speech recordings, prosody modification and synthesis. We use the MMF for detection of these physically important instants. To do so, we study the correspondence of the Most Singular Manifold with the physical production mechanism of the speech signal and we show that this subset can be used for GCI detection. We show that, in clean speech, our algorithm has similar performance to recent methods and, in noisy speech, it significantly outperforms state-of-the-art methods. Indeed, as our algorithm is based on both time domain and inter-scale smoothings, it provides higher robustness against many types of noises. In the mean-time, the high geometrical resolution of singularity exponents prevents the accuracy to be compromised. Moreover, the algorithm extracts GCIs directly from the speech signal and does not rely on any model of the speech signal (such as the autoregressive model in linear predictive analysis). See figure 10.
- Development of an efficient algorithm for sparse Linear Prediction Analysis (Vahid Khanagha, Khalid Daoudi). We address the problem of sparse Linear Prediction (LP) analysis, which involves the estimation of vocal tract model such that the corresponding LP residuals are as sparse as possible: for voiced sounds, one desires the residual to be zero all the time, except for few impulses at GCIs. Sparse Linear Prediction Analysis (LPA) problem has recently got much scientific attention and its classical solutions suffer from computational and algorithmic complexties. We introduce a simple

Table 1: Evaluation of the reconstructed phase under different levels of noise.

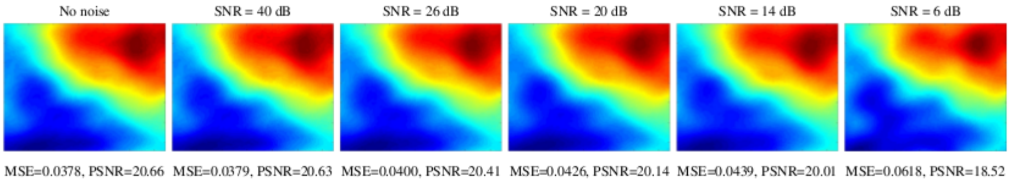

Table 2: Evaluation of the reconstructed phase using log power spectrum (**row 1**) and atmospheric structure functions (**row 2**).

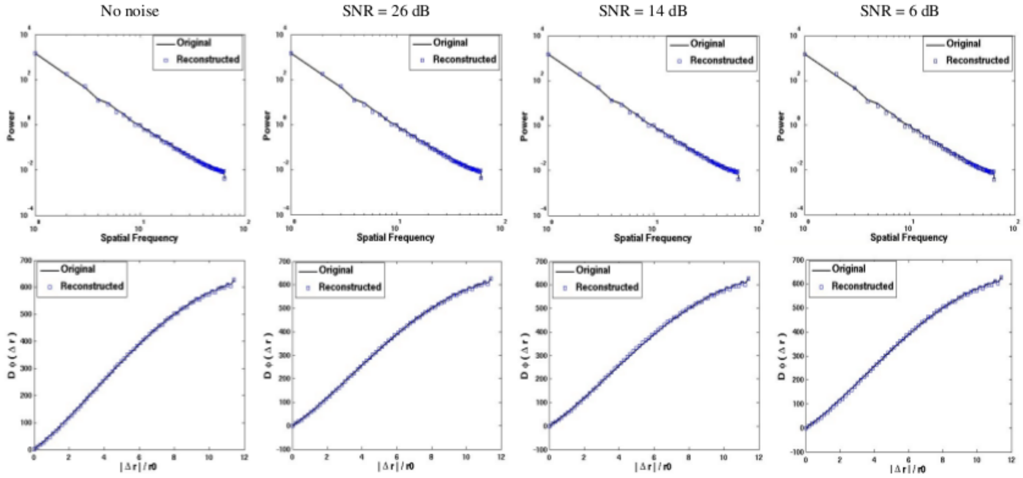

Figure 9. Results showing the robustness of the multiscale phase reconstruction algorithm for Adaptive Optics (AO) under various conditions of noise.

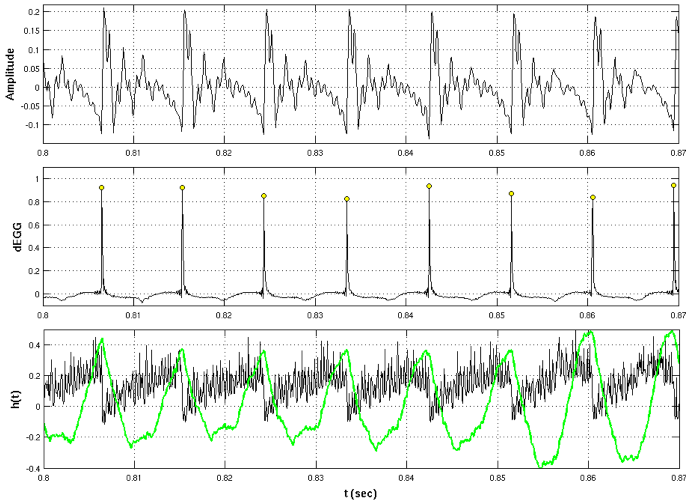

Figure 10. Top: a voiced segment of the speech signal taken from KED database. Middle: the differenced Electro-Glotto-Graph signal which serves for extraction of reference GCI points. The peaks are marked with yellow circles as the reference GCIs. Bottom: singularity exponents are shown by black color and an auxiliary functional showing changes in DC level of exponents is shown in green. The local minima of singularity exponents within each positive half-period of the auxiliary functional are taken as GCIs.

closed-form solution in this chapter which is based on the minimization of weighted l_2-norm of residuals. The weighting function plays the most important role in our solution in maintaining the sparsity of the resulting residuals. We use our MSM-based GCI detector to extract from the speech signal itself, the points having the potential of attaining largest norms of residuals and then we construct the weighting function such that the prediction error is relaxed on these points. Consequently, the weighted l_2-norm objective function can be efficiently minimized by the solution of normal equations of liner least squares problem. The choice of our MSM-based GCI detector is particularly justified, considering the fact that most of the successful GCI detection methods actually use LP residuals for their detection and hence, they cannot be used for constraining the LP problem. Our algorithm is completely independent of any model that might be assumed for speech signal. We will see that when compared to classical techniques, our simple algorithm provides better sparseness properties and does not suffer from usual instabilities. We also present an experiment to show how such sparse solution may result in more realistic estimates of the vocal tract by decoupling of the contributions of the excitation source from that of the vocal tract filter. See figure 11.

- Multi-pulse estimation of speech excitation source (Vahid Khanagha, Khalid Daoudi). In the GCI detector algorithm, the cardinality of MSM was restricted to one sample per pitch period. We then proceed to study the significance of MSMs of higher cardinalities, in the framework of multi-pulse estimation of voiced sound excitation source. Multi-pulse source coding has been widely used and studied within the framework of Linear Predictive Coding (LPC). It consists in finding a sparse representation of the excitation source (or residual) which yields a source-filter reconstruction with high perceptual quality. The MultiPulse Excitation (MPE) method is the first and one of the most popular techniques to achieve this goal. MPE provides a sparse excitation sequence through an iterative Analysis-by-Synthesis procedure to find the position and amplitudes of the excitation source in two stages: first the location of pulses are estimated one at a time by minimization of perceptually wieghted reconstruction error. In the second stage, the amplitude of these pulses are jointly re-optimized to find the optimal pulse values. Using the MSM, we propose a novel approach to find the locations of the multi-pulse sequence that approximates the speech source excitation. We consider locations of MSM points as the locations of excitation impulses and then, the amplitude of these

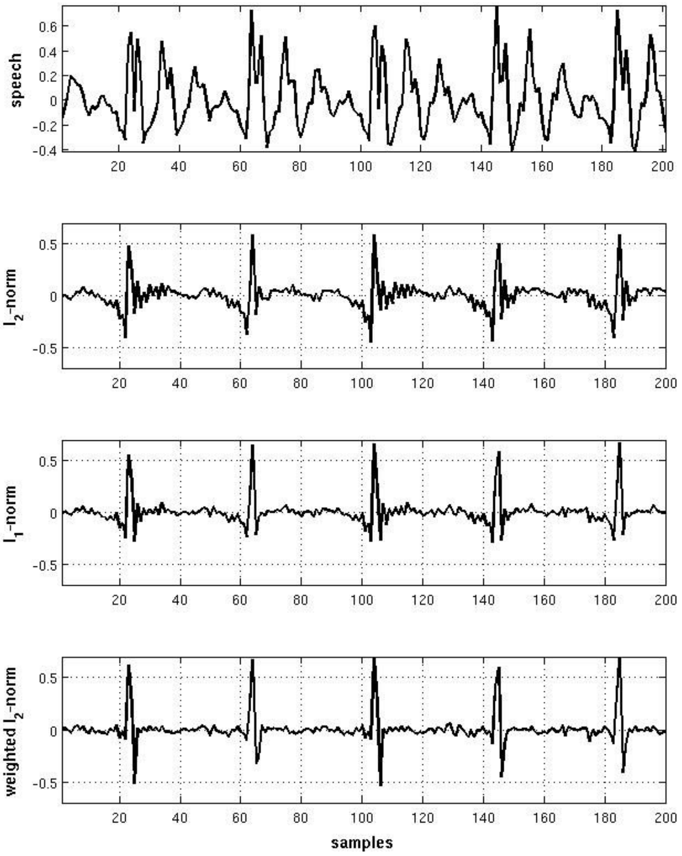

Figure 11. The residuals of the LP analysis obtained from different optimization strategies.

impulses are computed using the second stage of the classical MPE coder by minimization of the spectrally weighted mean squared error of reconstruction. The multi pulse sequence is then fed to the classical LPC synthesizer to reconstruct speech. Our algorithm is more efficient than classical methods, while providing the same level of perceptual quality as the classical MPE method. See figure 12.

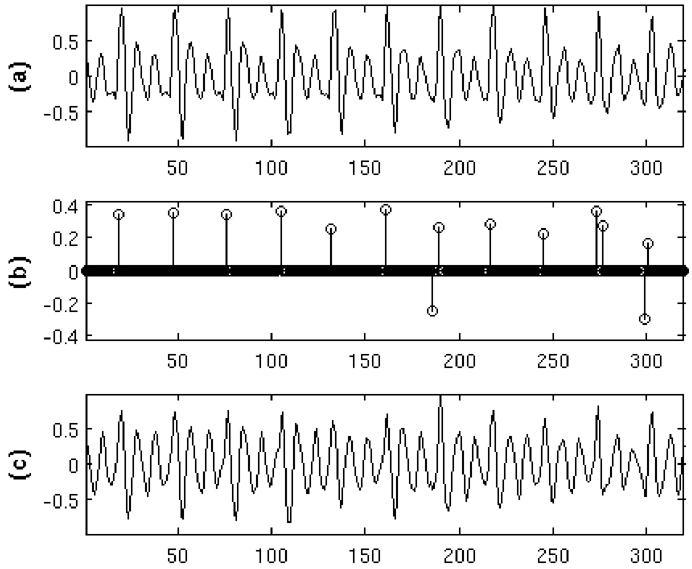

Figure 12. (a) a 40 ms segment of stationary voiced speech, (b) the MSM excitation sequence using 7 pulses per 20 ms and (c) the corresponding reconstructed signal.

- Speech representation based local singularity analysis (Vahid Khanagha, Khalid Daoudi, Hussein Yahia, Oriol Pont). Precise estimation of singularity exponents unlocks the determination a collection of points inside the complex signal which are considered as the least predictable points (the MSM). This leads to the associated compact representation and reconstruction. This work presents the very first steps in establishing the links between the MSM and the speech signal. To do so, we make slight modifications to the formalism so as to adapt it to the particularities of the speech signal. Indeed, the complex intertwining of different dynamics in speech (added to purely turbulent descriptions) suggests the definition of appropriate multi-scale functionals that might influence the evaluation of SEs, hence resulting in a more parsimonious MSM. We present a study that comforts these observations: we show that an alternative multi-scale functional does lead to a more parsimonious MSM from which the whole speech signal can be reconstructed with good perceptual quality. As MSM is composed of a collection of irregularly spaced samples, we use a classical method for the interpolation of irregularly spaced samples, called the Sauer-Allebach algorithm, to reconstruct the speech signal from its MSM. We show that by using this generic algorithm [and even by slight violation of its conditions] high quality speech reconstruction can still be achieved from a MSM of low cardinality. This shows that the MSM formed using the new multi-scale functional we define, indeed can give access to a subset of potentially interesting points in the domain of speech signal. Finally, in order to show the potential of this parsimonious representation in practical speech processing applications, we quantize and encode the MSM so as to develop a waveform coder. See figure 13.
- Related publications: [10], [17], [11].

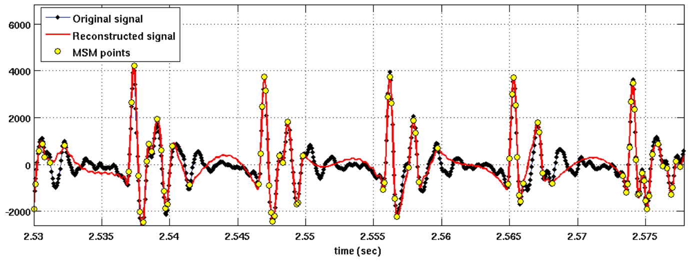

Figure 13. Waveforms of the original signal and the reconstructed signal. Samples belonging to MSM are marked with yellow circles.

6.8. Reconstruction and gradient-based video editing

Participants: Hicham Badri [correspondant], Hussein Yahia, Driss Aboutajdine.

Figure 14. From left to right: original image and examples of non-photoralistic rendering.

Gradient-domain methods have become a standard for many computational photography applications including object cloning, panorama stitching and non-photorealistic rendering. Integration from a vector field is required to perform gradient-domain-based applications and this operation must be fast enough for interactive editing. The most popular way to perform this integration is known as the Poisson equation and requires solving a large linear system that becomes more costly as the region of interest becomes larger. We propose to use an FFT-based solution and the framework of reconstructible systems instead of performing interactive local/global editing in the gradient domain on the CPU/GPU for both images and videos. See figures 14, 15.

- Related publication: [21].

7. Partnerships and Cooperations

7.1. Regional Initiatives

- OPTAD project. Title: *Méthodes multiéchelles pour l'optique adaptative et les données d'astronomie"*, with Conseil Régional Région Aquitaine. Duration: 2010-2013.
- Convention CRA 20111602015 on speech processing, with Conseil Régional Région Aquitaine (2011-2014) (funding, equipement and Speech databases).
- DIAFIL project, cofunded by Conseil Régional Région Aquitaine and IHU LYRIC. Title: *Méthodes non-linéaires pour le diagnostic et la prévention de la fibrillation ventriculaire*.

Figure 15. Top left: original image. Top right: object removal with FFT-reconstruction algorithm. Botton left: object removal with MVC (Mean Value Coordinates) algorithm. Bottom right: object removal by numerical solving of Poisson equation.

7.2. National Initiatives

- HIRESUBCOLOR, OSTST-CNES-NASA program. Partners: DYNBIO (LEGOS UMR CNRS 5565), LOCEAN, ICM-CSIC. Title: *Multiscale methods for the evaluation of high resolution ocean surface velocities and subsurface dynamics from ocean color, SST and altimetry*. We obtained a 1 year prolongation in 2012 from CNES. Coordinator: H. Yahia. Abstract: nonlinear signal processing methods for high resolution mapping of ocean dynamics. Duration: 2008-2012.
- FIBAUR ARC: *Fibrillation auriculaire: approches nouvelles pour l'analyse des signaux complexes du rythme cardiaque*. Inria ARC, duration: 2011-2012. Partners: GEOSTAT, INSERM EA3668, SIGMA team (ESPCI).

7.3. International Initiatives

- CRSNG Canadian program. Title: *Profilage à partir des données hétérogènes du Web pour la cybercriminalité*. Partners: Concordia University, University of Sherbrooke, E-Profile Compagny, S. d. Quebec, GEOSTAT (Inria). Coordinator: Concordia University. Duration: 2011-2014. Abstract: use of various complex signals for cybersecurity.

7.4. European Initiatives

- OCEANFLUX project, ESA (European Space Agency), Program: Support to Sicence Element ESRIN/AO/1-6668/11/I-AM, fund: E/0029-01-L. Partners: IWR (University of Heidelberg, Germany), GEOSTAT (Inria, France) , KIT (Karlsruher Institu fur Technologie, Germany), LEGOS (CNRS DR14, France), IRD (France), University Paul Sabatier (France). Duration: 2011-2013. Abstract: Mapping at high spatial resolution of GHGs exchange flux between ocean and atmosphere using model outputs and nonlinear techniques in signal processing. Coordinator: C. Garbe, Interdiscplinary Center for Scientific Computing (IWR), University of Heidelberg.
- PHC Volubilis. Title: *Study of upwelling in the Moroccan coast by satellite imaging*. Partners: GEOSTAT, Rabat University, CRTS. French coordinator: K. Daoudi. Abstract: multiscale methods for the characterization of coastal upwelling from remote sensing data. Duration: 2010-2012.

7.5. International Research Visitors

7.5.1. Visits of International Scientists

Max Little (MIT Media Lab Human Dynamics Group, Visiting Senior Research Associate, Oxford Complex Systems) has made one month visit at GEOSTAT. He made a presentation to Inria BSO: *A global functional minimization approach to nonlinear signal processing* on Thursday, April 5th.

7.5.1.1. Internships

Hicham Badri (from Mar 2012 until Aug 2012)

Subject: Computer graphics effects from the framework of reconstructible systems

Institution: Université Mohamed V Agdal - Faculté des Sciences de Rabat (Morocco)

Nicolas Vinuesa (from October 1st 2012 until April 31 2013)

Subject: Biologically realistic coding efficiency in auditory cortex vs wavelet analysis

Institution: Universidad Nacional de Rosario, Facultad de Ciencias Exactas, Agrimensura Y Ingineria, Rosaria, Argentina.

8. Dissemination

8.1. Scientific Animation

- H. Yahia is a member of the editorial board of Elsevier's journal *Digital Signal Processing* (http://www.journals.elsevier.com/digital-signal-processing/editorial-board).
- H. Yahia is a member of the editorial board of *Frontiers in fractal physiology* (http://www.frontiersin.org/Fractal_Physiology/editorialboard).
- H. Yahia is a member of CNU's section 61 (CNU: *Conseil National des Universités*).

8.2. Teaching - Supervision - Juries

8.2.1. Teaching

Master : K. Daoudi was invited by the Moroccan CNRST within the FINCOME'2012 program (http://www.fincome.cnrst.ma/) to give a 20 hours lecture on speech processing at the Master2 InfoTelecom of the faculty of sciences, Rabat (http://www.fsr.ac.ma/MIT/).

8.2.2. Supervision

PhD : R. Jourani, title: *reconnaissance automatique du locuteur par GMM à grande marge*, co-supervised between University Paul Sabatier (Toulouse, France) and Rabat-Agdal University (Morroco), defended September 6th, 2012, supervisors: K. Daoudi and R. André-Obrecht.

PhD in progress : V. Khanagha, title: *novel multiscale methods for nonlinear speech analysis using the Microcanonical Multiscale Formalism*, PhD started in 2009, supervisors: H. Yahia and K. Daoudi, to de defended on January 16th, 2013.

PhD in progress : S. Maji, title: *méthodes multiéchelles en traitement du signal pour l'optique adaptative*, PhD started in 2010, supervisor: H. Yahia.

PhD in progress : H. Badri , title: *sparse representation and gradient manipulation: application to multidimensional signals, natural and synthetic*, PhD started in 2012, supervisors: H. Yahia, D. Aboutajdine.

PhD in progress : A. Tamim , title: *image procesing for the segmentation and temporal evolution of moroccan upwelling*, PhD started in 2010, supervisors: K. Daoudi, D. Aboutajdine, H. Yahia.

8.2.3. Juries

- H. Yahia was a member of Mr. Binbin Xu's PhD jury. The PhD was defended on July 11th, 2012, at Université de Bourgogne. Title: *étude de la dynamique des ondes spirales à l'échelle cellulaire par modèles expérimental et numérique*. The jury was composed of: Professor O. Meste, Dr. H. Yahia, Professors V. Kazantzev, M. Nadi, J.-M. Bilbault, S. Binczak, Dr. G. Laurent and Dr. S. Jaquir.
- H. Yahia and K. Daoudi were members of H. Badri's master internship jury. The defence took place on October, 13th, 2012, at Rabat University, Morroco.

8.3. Diffusion

- H. Yahia was an invited speaker at the EGU (European Geophysical Association) General Assembly, held in Vienna, Austria, from April 22th to April 27th, 2012. Session NP3.1 ("Nonlinear, scaling and Complex Physical and Biogeophysical Processes in the Atmosphere and Ocean") [14].
- K. Daoudi was invited from April 11th to April 22th, 2012, by Concordia University (Montreal, Canada), for a visit to Concordia and Sherbrooke universities. K. Daoudi has given a talk at Concordia on April 16th.
- K. Daoudi was invited from September 13th to September 15th, 2012, by the Speech Group at Microsoft Research (Redmond, USA) and has given a talk on September 14th on the subject of nonlinear signal processing for speech.
- H. Yahia participated to the CNU session held in Saint Malo, France, on January, 23th, 24th, 2012.

- H. Yahia was invited by F. Schmidt, head of the LOG (Laboratoire d'Océanologie et de Géosciences, UMR CNRS 8187 and Université du Littoral), to make a lecture on the subject: *structure multiéchelle des signaux complexes et circulation océanique*, on June, 29th, 2012.

9. Bibliography

Major publications by the team in recent years

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[2] S. MAJI, O. PONT, H. YAHIA, J. SUDRE. *Inferring Information across Scales in Acquired Complex Signals*, in "European Conference on Complex Systems, ECCS'12, Brussels, The Complex Systems Society, Springer LNCS (number under way)", 2012.

[3] O. PONT, A. TURIEL, C. PEREZ-VICENTE. *On optimal wavelet bases for the realization of microcanonical cascade processes*, in "International Journal of Wavelets Multiresolution and Information Processing", 2011, vol. 9, p. 35-61, http://hal.inria.fr/inria-00582372.

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Publications of the year

Articles in International Peer-Reviewed Journals

[10] R. JOURANI, K. DAOUDI, R. ANDRÉ-OBRECHT, D. ABOUTAJDINE. *Discriminative speaker recognition using Large Margin GMM*, in "Neural Computing and Applications", 2012, http://hal.inria.fr/hal-00750385.

[11] V. KHANAGHA, K. DAOUDI. *An Efficient Solution to Sparse Linear Prediction Analysis of Speech*, in "EURASIP Journal on Audio, Speech, and Music Processing, Special Issue on Sparse Modeling for Speech and Audio Processing", 2012, http://hal.inria.fr/hal-00709168.

[12] O. PONT, M. HAÏSSAGUERRE, H. YAHIA, N. DERVAL, M. HOCINI. *Microcanonical processing methodology for ECG and intracardial potential: application to atrial fibrillation*, in "Transactions on Mass-Data Analysis of Images and Signals", 2012, vol. 4, n^{o} 1, http://hal.inria.fr/hal-00668550.

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Invited Conferences

[14] H. YAHIA, J. SUDRE, V. GARÇON, C. POTTIER. *High-Resolution Ocean Dynamics from Microcanonical Formulations in Nonlinear Complex Signal Analysis*, in "European Geosciences Union General Assembly (EGU)", Vienna, Austria, 2012, Session NP3.1 "Nonlinear, scaling and Complex Physical and Biogeophysical Processes in the Atmosphere and Ocean", http://hal.inria.fr/hal-00762869.

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[15] C. GARBE, A. BUTZ, I. DADOU, B. DEWITTE, V. GARÇON, S. ILLIG, A. PAULMIER, J. SUDRE, H. YAHIA. *Climatically-Active Gases In The Eastern Boundary Upwelling And Oxygen Minimum Zone (Omz) Systems*, in "Geoscience and Remote Sensing Symposium (IGARSS), 2012 IEEE International", Munich, Germany, 2012, http://hal.inria.fr/hal-00760423.

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