IN PARTNERSHIP WITH:
Ecole des Ponts ParisTech

Université Paris-Est Marne-la-Vallée

Activity Report 2012

Team MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

RESEARCH CENTER
Paris - Rocquencourt

THEME
Stochastic Methods and Models

Table of contents

Team MATHRISK

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2. Overall Objectives

2.1. Introduction

MathRisk is a joint Inria project-team with ENPC (CERMICS Laboratory) and the University Paris-Est Marne la Vallée (UPEMLV, LAMA Laboratory), located in Rocquencourt and Marne la Vallée. https://www-rocq.inria.fr/mathfi/. This project is based on the former Mathfi project team.

Mathfi was founded in 2000, and was devoted to financial mathematics. The project was focused on advanced stochastic analysis and numerical techniques motivated by the development of increasingly complex financial products. Main applications concerned evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, and calibration of financial models. Special attention was paid to models with jumps, stochastic volatility models, asymmetry of information.

Crisis, deregulation, and impact on the research in finance.

The starting point of the development of modern finance theory is traditionally associated to the publication of the famous paper of Black and Scholes in 1973 [61]. Since then, in spite of sporadic crises, generally well overcome, financial markets have grown in a exponential manner. More and more complex exotic derivative products have appeared, on equities first, then on interest rates, and more recently on credit markets. The period between the end of the eighties and the crisis of 2008 can be qualified as the “golden age of financial mathematics”: finance became a quantitative industry, and financial mathematics programs flourished in top universities, involving seminal interplays between the worlds of finance and applied mathematics. During its 12 years existence, the Mathfi project team has extensively contributed to the development of modeling and computational methods for the pricing and hedging of increasingly complex financial products.

Since the crisis of 2008, there has been a critical reorientation of research priorities in quantitative finance with emphasis on risk. In 2008, the “subprime” crisis has questioned the very existence of some derivative products such as CDS (credit default swaps) or CDOs (collateralized debt obligations), which were accused to be responsible for the crisis. The nature of this crisis is profoundly different from the previous ones. It has negatively impacted the activity on the exotic products in general, - even on equity derivative markets-, and the interest in the modeling issues for these products. The perfect replication paradigm, at the origin of the success of the Black and Scholes model became unsound, in particular through the effects of the lack of liquidity. The interest of quantitative finance analysts and mathematicians shifted then to more realistic models taking into account the multidimensional feature and the incompleteness of the markets, but as such getting away from the “lost paradi(gm)” of perfect replication. These models are much more demanding numerically, and require the development of hedging risk measures, and decision procedures taking into account the illiquidity and various defaults.

Moreover, this crisis, and in particular the Lehman Brothers bankruptcy and its consequences, has underlined a systemic risk due to the strong interdependencies of financial institutions. The failure of one of them can cause a cascade of failures, thus affecting the global stability of the system. Better understanding of these interlinkage phenomena becomes crucial.

At the same time, independently from the subprime crisis, another phenomenon has appeared: deregulation in the organization of stock markets themselves. This has been encouraged by the Markets in Financial Instruments Directive (MIFID) which is effective since November, 1st 2007. This, together with the progress of the networks, and the fact that all the computers have now a high computation power, have induced arbitrage opportunities on the markets, by very short term trading, often performed by automatic trading. Using these high frequency trading possibilities, some speculating operators benefit from the large volatility of the markets. For example, the flash crash of May, 6 2010 has exhibited some perverse effects of these automatic speculating trading strategies. These phenomena are not well understood and the theme of high frequency trading needs to be explored.

To summarize, financial mathematics is facing the following new evolutions:

- the complete market modeling has become unsatisfactory to provide a realistic picture of the market and is replaced by incomplete and multidimensional models which lead to new modeling and numerical challenges.
- quantitative measures of risk coming from the markets, the hedging procedures, and the lack of liquidity are crucial for banks,
- uncontrolled systemic risks may cause planetary economic disasters, and require better understanding,
- deregulation of stock markets and its consequences lead to study high frequency trading.

The new project team MathRisk is designed to address these new issues, in particular dependence modeling, systemic risk, market microstructure modeling and risk measures. The research in modeling and numerical analysis remain active in this new context, motivated by new issues.

The MathRisk project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. https://www-rocq.inria.fr/mathfi/Premia/.

The MathRisk project is part of the Université Paris-Est **"Labex" BÉZOUT**.

2.2. Dependence modeling

Participants: Aurélien Alfonsi, Damien Lamberton, Bernard Lapeyre.

The volatility is a key concept in modern mathematical finance, and an indicator of the market stability. Risk management and associated instruments depend strongly on the volatility, and volatility modeling has thus become a crucial issue in the finance industry. Of particular importance is the assets *dependence* modeling. The calibration of models for a single asset can now be well managed by banks but modeling of dependence is the bottleneck to efficiently aggregate such models. A typical issue is how to go from the individual evolution of each stock belonging to an index to the joint modeling of these stocks. In this perspective, we want to model stochastic volatility in a *multidimensional* framework. To handle these questions mathematically, we have to deal with stochastic differential equations that are defined on matrices in order to model either the instantaneous covariance or the instantaneous correlation between the assets. From a numerical point of view, such models are very demanding since the main indexes include generally more than thirty assets. It is therefore necessary to develop efficient numerical methods for pricing options and calibrating such models to market data. As a first application, modeling the dependence between assets allows us to better handle derivatives products on a basket. It would give also a way to price and hedge consistently single-asset and basket products. Besides, it can be a way to capture how the market estimates the dependence between assets. This could give some insights on how the market anticipates the systemic risk.

2.3. Liquidity risk

Participants: Aurélien Alfonsi, Anton Kolotaev, Marie-Claire Quenez, Agnès Sulem, Antonino Zanette.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the liquidity risk. Loosely speaking, liquidity risk is the risk that comes from the difficulty of selling (or buying) an asset. At the extreme, this may be the impossibility to sell an asset, which occured for "junk assets" during the subprime crisis. Hopefully, it is in general possible to sell assets, but this may have some cost. Let us be more precise. Usually, assets are quoted on a market with a Limit Order Book (LOB) that registers all the waiting limit buy and sell orders for this asset. The bid (resp. ask) price is the most expensive (resp. cheapest) waiting buy or sell order. If a trader wants to sell a single asset, he will sell it at the bid price. Instead, if he wants to sell a large quantity of assets, he will have to sell them at a lower price in order to match further waiting buy orders. This creates an extra cost, and raises important issues. From a short-term perspective (from few minutes to some days), this may be interesting to split the selling order and to focus on finding optimal selling strategies. This requires to model the market microstructure, i.e.

how the market reacts in a short time-scale to execution orders. From a long-term perspective (typically, one month or more), one has to understand how this cost modifies portfolio managing strategies (especially delta-hedging or optimal investment strategies). At this time-scale, there is no need to model precisely the market microstructure, but one has to specify how the liquidity costs aggregate.

2.3.1. Long term liquidity risk.

On a long-term perspective, illiquidity can be approached via various ways: transactions costs [48], [49], [60], [67], [74], [98], [94], delay in the execution of the trading orders [99], [97], [62], [30], trading constraints or restriction on the observation times (see e.g. [72] and references herein). As far as derivative products are concerned, one has to understand how delta-hedging strategies have to be modified. This has been considered for example by Cetin, Jarrow and Protter [96]. We plan to contribute on these various aspects of liquidity risk modeling and associated stochastic optimization problems. Let us mention here that the price impact generated by the trades of the investor is often neglected with a long-term perspective. This seems acceptable since the investor has time enough to trade slowly in order to eliminate its market impact. Instead, when the investor wants to make significant trades on a very short time horizon, it is crucial to take into account and to model how prices are modified by these trades. This question is addressed in the next paragraph on market microstructure.

2.3.2. Market microstructure.

The European directive MIFID has increased the competition between markets (NYSE-Euronext, Nasdaq, LSE and new competitors). As a consequence, the cost of posting buy or sell orders on markets has decreased, which has stimulated the growth of market makers. Market makers are posting simultaneously bid and ask orders on a same stock, and their profit comes from the bid-ask spread. Basically, their strategy is a "round-trip" (i.e. their position is unchanged between the beginning and the end of the day) that has generated a positive cash flow.

These new rules have also greatly stimulated research on market microstructure modeling. From a practitioner point of view, the main issue is to solve the so-called "optimal execution problem": given a deadline T, what is the optimal strategy to buy (or sell) a given amount of shares that achieves the minimal expected cost? For large amounts, it may be optimal to split the order into smaller ones. This is of course a crucial issue for brokers, but also market makers that are looking for the optimal round-trip.

Solving the optimal execution problem is not only an interesting mathematical challenge. It is also a mean to better understand market viability, high frequency arbitrage strategies and consequences of the competition between markets. For example when modeling the market microstructure, one would like to find conditions that allow or exclude round trips. Beyond this, even if round trips are excluded, it can happen that an optimal selling strategy is made with large intermediate buy trades, which is unlikely and may lead to market instability.

We are interested in finding synthetic market models in which we can describe and solve the optimal execution problem. A. Alfonsi and A. Schied (Mannheim University) [50] have already proposed a simple Limit Order Book model (LOB) in which an explicit solution can be found for the optimal execution problem. We are now interested in considering more sophisticated models that take into account realistic features of the market such as short memory or stochastic LOB. This is mid term objective. At a long term perspective one would like to bridge these models to the different agent behaviors, in order to understand the effect of the different quotation mechanisms (transaction costs for limit orders, tick size, etc.) on the market stability.

2.4. Contagion modeling and systemic risk

Participants: Benjamin Jourdain, Agnès Sulem.

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these. Ideally, one would like to be able to find capital requirements (such as the one proposed by the Basel committee) that ensure that the probability of multiple defaults is below some level.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular R. Cont et al. [51] and A. Minca [84]) or mean field interaction models (Garnier-Papanicolaou-Yang [71]). The recent approach in [51] seems very promising. It describes the financial network approach as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. Distress propagation in a financial system may be modeled as an epidemics on this graph. In the case of incomplete information on the structure of the interbank network, cascade dynamics may be reduced to the evolution of a multi-dimensional Markov chain that corresponds to a sequential discovery of exposures and determines at any time the size of contagion. Little has been done so far on the *control* of such systems in order to reduce the systemic risk and we aim to contribute to this domain.

2.5. Stochastic analysis and numerical probability

2.5.1. Stochastic control

Participants: Vlad Bally, Jean-Philippe Chancelier, Marie-Claire Quenez, Agnès Sulem.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the default risk modeling,the interplay between information and performance, the model uncertainty and the associated robustness questions. We address these questions by further developing the theory of stochastic control in a broad sense, including stochastic optimization, nonlinear expectations, Malliavin calculus, stochastic differential games and various aspects of optimal stopping.

2.5.2. Simulation of stochastic differential equations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Vlad Bally, Damien Lamberton, Bernard Lapeyre, Jérôme Lelong, Céline Labart.

Effective numerical methods are crucial in the pricing and hedging of derivative securities. The need for more complex models leads to stochastic differential equations which cannot be solved explicitly, and the development of discretization techniques is essential in the treatment of these models. The project MathRisk addresses fundamental mathematical questions as well as numerical issues in the following (non exhaustive) list of topics: Multidimensional stochastic differential equations, High order discretisation schemes, Singular stochastic differential equations, Backward stochastic differential equations.

2.5.3. Monte-Carlo simulations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Mohamed Sbai, Vlad Bally, Bernard Lapeyre, Ahmed Kebaier, Céline Labart, Jérôme Lelong, Sidi-Mohamed Ould-Aly, Lokmane Abbas-Turki, Abdelkoddousse Ahida, Antonino Zanette, El Hadj Aly Dia.

Monte-Carlo methods is a very useful tool to evaluate prices especially for complex models or options. We carry on research on *adaptive variance reduction methods* and to use *Monte-Carlo methods for calibration* of advanced models.

This activity in the MathRisk team is strongly related to the development of the Premia software.

2.5.4. Optimal stopping

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton, Maxence Jeunesse, Ayech Bouselmi, Agnès Sulem, Marie-Claire Quenez.

The theory of American option pricing has been an incite for a number of research articles about optimal stopping. Our recent contributions in this field concern optimal stopping in models with jumps irregular obstacles, free boundary analysis, reflected BSDEs.

2.5.5. Malliavin calculus and applications in finance

Participants: Vlad Bally, Arturo Kohatsu-Higa, Agnès Sulem, Antonino Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [82]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [86], D. Bell [59] D. Ocone [88], B. Øksendal [101]) give expositions of the subject. See also V. Bally [55] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [87].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [83], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods.

We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be "smooth" in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequences of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M}E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [66] and [65] and the papers by Bally et al., Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [79].

L. Caramellino, A. Zanette and V. Bally have been concerned with the computation of conditional expectations using Integration by Parts formulas and applications to the numerical computation of the price and the Greeks (sensitivities) of American or Bermudean options. The aim of this research was to extend a paper of Reigner and Lions who treated the problem in dimension one to higher dimension - which represent the real challenge in this field. Significant results have been obtained up to dimension 5 [56] and the corresponding algorithms have been implemented in the Premia software.

Moreover, there is an increasing interest in considering jump components in the financial models, especially motivated by calibration reasons. Algorithms based on the integration by parts formulas have been developed in order to compute Greeks for options with discontinuous payoff (e.g. digital options). Several papers and two theses (M. Messaoud and M. Bavouzet defended in 2006) have been published on this topic and the corresponding algorithms have been implemented in Premia. Malliavin Calculus for jump type diffusions - and more general for random variables with localy smooth law - represents a large field of research, also for applications to credit risk problems.

The Malliavin calculus is also used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P. Imkeller in [79]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [79]).

A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. These SDEs can be interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals, as introduced by Russo and Valois [91]. This methodology has been applied for utility maximization with insiders.

2.6. Highlights of the Year

Creation of the Mathrisk Project Team.

3. Application Domains

3.1. Application Domains

- Utility maximization in incomplete markets
- option pricing
- quantitative risk management
- systemic risk
- limit order books
- credit risk
- liquidity risk
- computational finance
- calibration

4. Software

4.1. PREMIA

Participants: Antonino Zanette, Mathrisk Research team, Agnès Sulem [correspondant].

Premia is a software designed for option pricing, hedging and financial model calibration. It is provided with it's C/C++ source code and an extensive scientific documentation. https://www-rocq.inria.fr/mathfi/Premia

The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is:

1. to be a powerful testing platform for comparing different numerical methods between each other;
2. to build a link between professional financial teams and academic researchers;
3. to provide a useful teaching support for Master and PhD students in mathematical finance.

- AMS: 91B28;65Cxx;65Fxx;65Lxx;65Pxx
- License: Licence Propriétaire (genuin license for the Consortium Premia)
- Type of human computer interaction: Console, interface in Nsp, Web interface
- OS/Middelware: Linux, Mac OS X, Windows
- APP: The development of Premia started in 1999 and 14 are released up to now and registered at the APP agency.
- Programming language: C/C++ librairie Gtk
- Documentation: the PNL library is interfaced via doxygen
- Size of the software: 11 Mbyte of code split into 275,000 lines. 93 Mbyte of PDF files of documentation
- Publications: [1] [64] [78] [89] [95], [47]

4.1.1. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. **Equity derivatives:**

   The following models are considered:

   Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

   For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods and Malliavin calculus based methods.

   Dynamic Hedging for Black-Scholes and jump models is available.

   Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. **Interest rate derivatives**

   The following models are considered:

   HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

   Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. **Credit derivatives: CDS, CDO**

Reduced form models and copula models are considered.

Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. **Hybrid products**

   PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. **Energy derivatives: swing options**

   Mean reverting and jump models are considered.

   Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

4.1.2. Premia design

Anton Kolotaev (ADT engineer), supervised by J. Lelong, has developed a web platform allowing online tests (https://quanto.inria.fr/premia/koPremia). This online version allow us to supply benchmarks both for professional R&D teams and academics in mathematical finance. This will considerably increase the impact and the visibility of the software. Up to now, to use the opensource version of the software, one has to download from Premia's website and install it on its own computer and this had become a brake on using Premia. Providing an online version of Premia is an original way of keeping up with the new standards of software usability without focusing too much on a dedicated solution per operation system.
To enable easy an advanced usage of Premia without being an advanced C or C++ programmer, we have started to implement Python bindings. The choice of Python has been quite obvious as Python has become over the past few years a standard cross–platform interpreted language for numerical problems.

Premia has managed to grow up over a period of more than a dozen years; this has been possible only because contributing an algorithm to Premia is subject to strict rules, which have become too stringent. To facilitate contributions, a standardized numerical library (PNL) has been developed under the LGPL since 2009, which offers a wide variety of high level numerical methods for dealing with linear algebra, numerical integration, optimization, random number generators, Fourier and Laplace transforms, and much more. Everyone who wishes to contribute is encouraged to base its code on PNL and providing such a unified numerical library has considerably eased the development of new algorithms which have become over the releases more and more sophisticated. An effort will be made to continue and stabilize the development of PNL.

4.1.3. Algorithms implemented in Premia in 2012

Premia 14 was delivered to the consortium members in March 2012. It contains the following new algorithms:

- **Interest Rate Derivatives**
  - An n-Dimensional Markov-functional Interest Rate Model
    L. Kaisajuntti J. Kennedy. Preprint 2008
  - Efficient log-Levy approximations for Levy-driven Libor model.
    A. Papapantoleon, J.Schoenmakers, D. Skovmand.
    Preprint 2111, TU Berlin.
- **Energy and Commodities**
  - Efficient pricing of Swing options in Lévy-driven models. O. Kudryavtsev, A. Zanette.
- **Credit Risk Derivatives**
  - Calibration in a local and stochastic intensity model. A. Alfonsi, C. Labart, J. Lelong
- **Equity Derivatives**
  - Forward Variance Dynamics: Bergomi's model revisited. S.M. Ould Aly
  - Volatility of Volatility Expansion for Bergomi's model. S.M. Ould Aly

- Robust Approximations for Pricing Asian Options and Volatility Swaps Under Stochastic Volatility. M. Forde, A.Jacquier *Applied Mathematical Finance, Volume 17 Issue 3 2010*
- Small-time asymptotics for implied volatility under the Heston model, M. Forde, A. Mijatovic, and Jaquier, A. *International Journal of Theoretical and Applied Finance, Volume 12, issue 6, 2009*
- Asymptotic formulae for implied volatility under the Heston model. Forde, F, Jacquier, A, and Mijatovic, A. *Proceedings of the Royal Society A, to appear*.
- A Mean-Reverting SDE on Correlation Matrices. A. Alfonsi, A.Ahdida
- Fast and Accurate Long Stepping Simulation of the Heston Stochastic Volatility Model J.H. Chan, M S. Joshi, Preprint
- High order discretization schemes for stochastic volatility models. B. Jourdain, M. Sbai. *Quantitative Finance, to appear*
- A Fourier-based Valuation Method for Bermudan and Barrier Options under Heston Model. F.Fang, C. W. Oosterlee. *SIAM J. Finan. Math. 2, 2011, 439-463.*
- Numerical methods and volatility models for valuing cliquet options.
H. Windcliff, P.A. Forsyth, K.R. Vetzal, *Applied Mathematical Finance 13 2006.*
- Pricing Discretely Monitored Asian Options by Maturity Randomization. G.Fusai, D. Marazzina, M. Marena. *SIAM Journal on Financial Mathematics, Vol. 2 2011*
- Wiener-Hopf techniques for Lookback options in Levy models. O. Kudryavtsev
- Computing VaR and AVar in Infinitely Divisible Distributions. Y.S. Kim, S. Rachev, M.S. Bianchi, F.J. Fabozzi. *Probability and Mathematical Statistics, Vol. 30, Fasc. 2 2010.*
- Analytical formulas for local volatility model with stochastic rates. E. Benhamou, E. Gobet, M. Miri 2009
- Nonparametric Variance Reduction Methods on Malliavin Calculus. B. Lapeyre, A. Turki *SIAM Journal on Financial Mathematics, to appear*
- Stochastic expansion for the pricing of call options with discrete dividends. P. Etore, E. Gobet. *Applied Mathematical Finance, to appear*
- American options in high dimension solving EDSR with penalization C. Labart, J. Lelong
- Static Hedging of Standard Options. P. Carr, L. Wu. Preprint.

The software Premia 14 has been deposited at the APP (Agence pour la Protection des Programmes) with the reference IDDN.FR.001.190010.011.S.C.2001.000.31000.

5. New Results

5.1. Dynamic risk measures and BSDEs with jumps

The standard approach of mathematical quantification of financial risk in terms of Value at Risk has serious deficiencies. This has motivated a systematic analysis of risk measures which satisfy some minimal requirements of coherence and consistency. The theory of risk measures has been first developed in [54] in the coherent case and then extended in various directions (convex, dynamic, law-invariant) (see e.g. [70], [68], [93], [69]). We are extending this theory, in particular in the case of markets with possible random jumps and model ambiguity, and investigate various types of optimization problems involving risk measures.

Mathematical techniques for the treatment of such problems are based on non linear expectations, backward stochastic differential equations (BSDEs), stochastic control, stochastic differential games.

In the Brownian case, links between dynamic risk measures and Backward Stochastic Differential Equations (BSDEs) have been established (see, among others, [57]). A. Sulem and M.-C. Quenez are exploring these links in the case of stochastic processes with jumps. To this purpose, we have recently extended some comparison theorems for BSDEs with jumps given in [90], and provided a representation theorem of convex dynamic risk measures induced by BSDEs with jumps (see [44]). Optimization of dynamic risk measures leads to stochastic differential games or to optimal control problems for coupled systems of forward-backward stochastic differential equations (FBSDEs). They can be studied by stochastic maximum principles [100] or by transforming them into controlled Backward Stochastic Partial Differential Equations (BSPDEs). We address these questions in collaboration with B. Øksendal (Oslo university) and T. Zhang (Manchester University).

The numerical study of (F)BSDEs with jumps is especially demanding in high dimensions and collaboration has started on these issues with J. Lelong (ENSIMAG) and C. Labart (Université de Savoie).

5.2. Stochastic Differential Games

In many situations, controls are chosen by several agents who interact in various ways. To handle such cases one may use the theory of SDGs. This applies to model uncertainty problems, which can be regarded as a zero-sum game between the agent and the "market" and risk minimization, with risk represented via dynamic risk measures. More general non-zero sum games, involving several players, possibly with asymmetric information or delay will be studied.

An interesting new application of the theory of stochastic differential games, is the issue of *Public Private Partnership* which is a mechanism for a community to outsource the construction of public equipment. The community agrees to pay a rent to the contractor in order to cover the depreciation of the equipment, the maintenance costs and the financial costs. We want to model such partnerships and to compute and compare Nash equilibria and Stackelberg equilibria when the community is the leader. We would also like to investigate whether the community aversion to debt may lead it to enter such a partnership even this is more costly than constructing and managing the equipment by itself.

5.3. Optimal control of Stochastic Partial Differential equations (SPDEs)

SPDEs appear in the modeling of a number of situations: for example, in dynamic pollution models, in financial models involving interest rate derivatives, in systemic risk modeling. The research issues include optimal control of SPDEs and nonlinear filtering theory, stochastic control of forward-backward systems of SPDEs with imperfect and/or asymmetric information, optimal stochastic control of mean-field systems of SPDEs. We have started to study singular control of SPDEs. We plan to give a method for solving optimal control problems for general, possibly non-Markovian systems of FBSDEs by means of BSPDEs with jumps and associated comparison theorems.

5.4. Optimal stopping

Our research on optimal stopping problems covers the analysis of free boundaries in optimal stopping problems for multidimensional stochastic processes with jumps (Thesis of A. Bouselmi, supervised by D. Lamberton). Numerical issues are also be investigated (Monte Carlo methods, quantization methods, methods based on Malliavin calculus). Even in diffusion models, a realistic dividend modeling introduces jumps in the dynamics : at the dividend dates the spot value of the stock undergoes a jump equal to minus the dividend amount. We plan to take into account this feature in optimal stopping problems (Thesis of M. Jeunesse, supervised by B. Jourdain).

The pricing of American options with irregular payoff such as, for instance, binary options, leads to challenging mathematical problems. Some theoretical properties of optimal stopping problems with irregular payoffs have already been obtained. We now plan to focus on the Markovian case by using viscosity solutions and numerical analysis techniques.

In [45], we study optimal stopping problems for (non necessarily) convex dynamic risk measures induced by BSDEs with jumps and establish their connections with *Reflected* BSDEs with jumps. Such problems are related to optimal stopping for non linear expectations, which has been recently studied by [58] in the convex case only. We also address the case of model ambiguity and its relation with mixed control/optimal stopping problems.

5.5. Analysis of stochastic processes with jumps

The use of stochastic processes with jumps in financial modeling has been constantly increasing in the recent years. Simulation of these processes raises specific difficulties. A PhD thesis (V. Rabiet, adviser: V. Bally) has started on regularity properties of the law of multi-dimensional processes with jumps and on sensitivity analysis of derivative products with singular payoffs in such models.

5.6. Monte-Carlo methods

5.6.1. Adaptive variance reduction methods.

Stochastic algorithms [52], [53], [80], [63], [81], [27] or, more recently, direct stochastic optimization [73] proved to be a promising path to automatic variance reduction methods. Direct stochastic optimization techniques are easier to use in practice, avoiding completely any manual tuning needed for stochastic algorithms. This method is well understood (see [73]) only in the Gaussian case and for regular functions. We plan to extend the algorithms and prove rigorous results in non Gaussian cases with financial applications in view for jumps models (see [77], [76], [75]).

5.6.2. Monte-Carlo methods for calibration.

The interest for models combining local and stochastic volatility has been growing recently. Indeed, the local volatility model is not rich enough to efficiently deal with complex derivatives. A popular model is the so called Heston model, in which the volatility process solves a square-root stochastic differential equation (just as in the Cox-Ingersoll-Ross model for interest rate modeling). The thesis of L. Abbas-Turki [12](advisers: D. Lamberton and B. Lapeyre) concentrates on the multi-dimensional Heston model. For these models, numerical aspects are very demanding and we plan to use Monte-Carlo methods using advanced parallel devices (GPU clusters,...) both for price computations and calibration procedures. The thesis of Abbas-Turki is supported by the *Pôle de Compétitivité Finance Innovation* within the consortium *CrediNext*.

5.7. Systemic Risk

We extend the model in [51] in two major ways: First, study the optimal intervention strategy by a lender of last resort that would minimize the size of contagion under budget constraints. Second, allow our model not to be constrained to a single type of financial distress and model jointly insolvency and illiquidity. The interplay of these two mechanisms yields a more potent type of contagion than just the mechanical balance-sheet insolvency type of contagion [85], [92]. In [35], we have started to tackle these issues. This study can be enriched in many different manners.

Benjamin Jourdain and Agnès Sulem have organized a CEA-EDF-Inria school (70 participants) on the issues of Systemic risk and quantitative risk management in October 15-17 2012. (http://bit.ly/finance_inria). A special issue on "Systemic Risk" of the journal *Statistics and Risk Modeling* with B. Jourdain and A. Sulem as guest editors will be published in 2013.

6. Bilateral Contracts and Grants with Industry

6.1. Bilateral Contracts with Industry

PREMIA consortium, presently composed of Crédit Agricole CIB, Société Générale, Natixis, and Pricing Partners.

7. Partnerships and Cooperations

7.1. National Initiatives

7.1.1. ANR

ANR ANR-08-BLAN, Program: Big'MC (Issues in large scale Monte Carlo). (2009-2012).

Partners ENST, ENPC, University Paris-Dauphine.

7.1.2. Competitivity Clusters

Pôle Finance Innovation.

Project "Credinext" on credit risk derivatives (2009-2012).

Partners: Thomson Reuters, Lunalogic, Pricing Partners, Ecole Polytechnique, Inria, ENPC, Université Paris-Est Marne la Vallée.

(Several PhD and Postdoc grants)

7.2. European Initiatives

Eurostars Program "Transparency in Financial Markets" (OSEO grant) (Postdoc grants).

7.3. International Research Visitors

7.3.1. Visits of International Scientists

Emmanuella Rosazza Gianin, Bococca Milano University , January 2012

Peter Forsyth, Waterloo university Canada, July 2012

7.3.2. Internships

- Roxana Dumitrescu, Master 2 , University Paris-Dauphine
- Jiang Pu , Ecole Polytechnique, 3rd year

8. Dissemination

8.1. Scientific Animation

8.1.1. Collective responsabilities

- A. Alfonsi: Co-organizer of the working group seminar of MathRisk "Méthodes stochastiques et finance".
- D. Lamberton:
  1. "Associate Editor" of *Mathematical Finance*, co-editor of *ESAIM P&S*.
  2. In charge of the master program "Mathématiques et Applications" (Universities of Marne-la-Vallée, Créteil and Evry, and Ecole Nationale des Ponts et Chaussées).
  3. Vice-président recherche, Université Paris-Est Marne-la-Vallée.
- A. Sulem:
  1. Associate editor of:
     * *SIAM Journal on Financial Mathematics (SIFIN)* (since its creation in 2008)
     * *International Journal of Stochastic Analysis (IJSA)* (since 2009)
     * *Journal of Mathematical Analysis and Applications (JMAA)*(since 2011)

2. Jury for assitant professor position in financial mathematics and numerical probability, Laboratoire de probabilités Université Paris VII, 2012.

- B. Jourdain and A. Sulem: Organisation of a school CEA EDF Inria "Systemic Risk and Quantitative Risk management", Octobre 15-17 2012, http://bit.ly/finance_inria

8.1.2. *Invitations and participation in conferences*

- A. Sulem

  - Plenary conferences

    – Main speaker, "Probability & Finance" Final Conference of the Research Project PRIN 2008 Pescara, Italy, September 2012 http://www.dec.unich.it/convsem/2012-09-10/?home

    – Cornell University, ORIE Colloqium, April 10 2012 http://www.orie.cornell.edu/news/seminars/

  - invited conferences

    – "PDE and Mathematical Finance V", Stockholm, June 10-14, 2013.

    – Symposium "Commodities, Energy Markets & Equilibrium" , 4th Conference of the Financial Mathematics and Engineering (FME) SIAG (SIAM Activity Group). Minneapolis, Minnesota, July 9-11, 2012. http://www.siam.org/meetings/fm12/

    – Universite Evry seminar, Juin 2012

    – Semester program on Stochastic Analysis and Applications at the Centre Interfacultaire Bernoulli, Ecole Polytechnique Fédérale de Lausanne, January-June 2012.

8.2. Teaching - Supervision - Juries

8.2.1. *Teaching*

- A. Alfonsi:
  1. “Modéliser, Programmer et Simuler”, second year course at the Ecole des Ponts.
  2. “Calibration, Volatilité Locale et Stochastique”, third-year course at ENSTA (Master with Paris I).
  3. “Traitement des données de marché : aspects statistiques et calibration”, lecture for the Master at UPEMLV.
  4. “Mesures de risque”, Master course of UPEMLV and Paris VI.
- V. Bally:
  1. Master 2 of the University Marne la Vallée:

     -Malliavin Calculus and numerical applications in fiance

     - Probabilistic methods for risk analysis.

     -Taux d’itérêt
- B. Jourdain :
  1. Course "Probability theory and statistics", first year ENPC
  2. Course "Introduction to probability theory", 1st year, Ecole Polytechnique
  3. Course "Stochastic numerical methods", 3rd year, Ecole Polytechnique
  4. projects in finance and numerical methods, 3rd year, Ecole Polytechnique
- B. Jourdain, B. Lapeyre: course "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, University of Marne-la-Vallée
- J.-F. Delmas, B. Jourdain: course "Jump processes with applications to energy markets", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée

- D. Lamberton:
  1. Second year of Licence de mathématiques (probability), Université Paris-Est Marne-la-Vallée.
  2. Master course “Calcul stochastique et applications en finance", Université Paris-Est Marne-la-Vallée.
- A. Sulem:
  1. Master course, Université Paris IX-Dauphine, Département MIDO (Mathématiques et Informatique de la Décision et des Organisations), Master MASEF, 21 h., *Méthodes numériques en Finance*
  2. Master of Mathematics, Université du Luxembourg, 15h, 2012. *Numerical Methods in Finance*.

8.2.2. Supervision

- HdR :

  Aurélien Alfonsi, *Discrétisation de processus et modélisation en finance*, December 14 2012 , Ecole des Ponts Université Paris Est
- PhD :

  Lokmane Abbas Turki. *Calcul parallèle pour les problèmes linéaires, non-linéaires et linéaires inverses en finance.* PhD thesis, Université Paris-Est, September 21, 2012,
  this thesis was funded by Credinext.
  Advisers: D. Lamberton and B. Lapeyre,
  Current Position: Postdoc, Humbold University, Berlin
- PhD in progress
  - José Infante Acevedo: (from Oct. 2009). Half of this thesis is dedicated to *liquidity risk and limit order books modelling* . Adviser: A. Alfonsi.
  - Pierre Blanc: *Modeling the price impact of limit and market orders*. Adviser: A. Alfonsi.
  - Ayech Bouselmi: (3nd year, started in October 2009). Allocataire de recherche, Université Paris-Est. *Lévy processes and multi-dimensional models in finance*. Adviser: D. Lamberton.
  - Roxana Dumitrescu: started October 2012, *Gestion de risques sous contraintes de portefeuille*. Fondation Sciences Mathématiques de Paris grant, Inria and Université Paris-Dauphine, Inria adviser: A. Sulem.
  - Jing Chen: (Shandong University grant), Inria *Non Markovian Stochastic Control and Backward SDEs*, Adviser: A. Sulem .
  - Maxence Jeunesse: (started in November 2009), *Study of some numerical methods in finance*. Adviser: B. Jourdain and J.-Ph. Chancelier, chair "Risques financiers" grant.
  - Jyda Mint Moustapha: (started in november 2012), IFSTTAR, *Etude et caractérisation de pelotons de véhicules sur des routes à forte circulation*. Advisers: D. Daucher and B. Jourdain.
  - Ernesto Palidda: ENPC and Crédit Lyonnais GRO. *Multi-dimensional stochastic volatility for Interest Rates*. Adviser: B. Lapeyre.
  - Paola Pigato: (started November 2012). UPEMLV and University of Pisa, *Calcul de Malliavin*. Adviser: V. Bally.
  - Clément Rey: ENPC and UPMLV. *Weak error analysis of discretization schemes for some financial processes*. Advisers: A. Alfonsi and V. Bally.
  - Julien Reygner: (started in september 2011), IPEF, ENPC. *Convergence à l’équilibre de processus stochastiques*. Advisers: L. Zambotti and B. Jourdain.

– Victor Rabiet: (started 01/10/2009). ENS Cachan and UPEMLV : *Régularité du semi-groupe pour des équations stochastiques avec sauts*. Adviser: V. Bally.

8.2.3. Juries of PhD

- Agnes Sulem
  – Carmine De Franco, Laboratoire de Probabilités et Modeles Aléatoires (LPMA), Université Paris VII, 29 Juin 2012.
  – Lebovits Joachim, Ecole Centrale de Paris, 25 janvier 2012: Stochastic calculus with respect to multi-fractional Brownian motion and applications to finance
- Damien Lamberton
  – Carmine De Franco, Laboratoire de Probabilités et Modeles Aléatoires (LPMA), Université Paris VII, 29 Juin 2012.

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Major publications by the team in recent years

[1] *Numerical Methods implemented in the Premia Software*, 2009, Bankers, Markets, Investors, Introduction by Agnès Sulem and A. Zanette.

[2] A. ALFONSI, A. FRUTH, A. SCHIED. *Optimal execution strategies in limit order books with general shape functions*, in "Quantitative Finance", 2009, vol. 10, n^{o} 2, p. 143-157, DOI:10.1080/14697680802595700.

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Doctoral Dissertations and Habilitation Theses

[12] L. ABBAS-TURKI. *Calcul parallèle pour les problèmes linéaires, non-linéaires et linéaires inverses en finance*, Université Paris-Est, Marne la Vallée, September 21 2012, (Credinext grant).

[13] A. ALFONSI. *Discrétisation de processus et modélisation en finance*, Université Paris-Est, Ecole des Ponts, December 14 2012, Habilitation à Diriger des Recherches.

Articles in International Peer-Reviewed Journals

[14] A. ALFONSI, A. SCHIED, A. SLYNKO. *Order Book Resilience, Price Manipulation, and the Positive Portfolio Problem,*, in "SIAM J. Finan. Math.", 2012, vol. 3, p. 511-533.

[15] V. BALLY, M. CABALLERO, B. FERNANDEZ, N. EL-KAROUI. *Reflected BSDE's, PDE's and Variational Inequalities*, in "Bernouilli", 2012, accepted for publication.

[16] V. BALLY, L. CARAMELLINO. *Positivity and lower bounds for the density of Wiener functionals*, in "Potential Analysis", 2012, Accepted, http://arxiv.org/abs/1004.5269.

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