IN PARTNERSHIP WITH:
CNRS

Université de Bordeaux

Activity Report 2012

Project-Team REALOPT

Reformulations based algorithms for Combinatorial Optimization

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB)

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
Optimization, Learning and Statistical Methods

Table of contents

Project-Team REALOPT

Keywords: Optimization, Combinatorial Optimization, Non Linear Programming, Operational Research

Creation of the Project-Team: January 01, 2009 .

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2. Overall Objectives

2.1. Overall Objectives

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decisions set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space. Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, Lagrangian decomposition methods to produce powerful relaxations, constraint programming to actively reduce the solution domain through logical implications, heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidate solutions, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique.

Building on complementary expertise, our team's overall goals are threefold:

Objective (i) To design tight formulations for specific problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. More broadly, to contribute to theoretical and methodological developments of exact approaches in combinatorial optimization, while extending the scope of applications.

Objective (ii) To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, constraint programming, combinatorial algorithms and graph theory, by developing "efficient" algorithms for specific mathematical models. To tackle large-scale real-life applications, providing provably good approximate solutions by combining exact methods and heuristics.

Objective (iii) To provide prototypes of specific model solvers and generic software tools that build on our research developments, writing proof-of-concept code, while making our research findings available to internal and external users.

2.2. Highlights of the Year

Our scientific output is marked by strong publications in prestigious journals such as Discrete Mathematics, Mathematical Programming, EURO Journal on Computational Optimization, INFORMS Journal on Computing, and Operations Research, for instance, with contributions ranging from theoretical and methodological to numerical and applied industrial problem solving. This is completed by conference invitations in China, Chili and Canada and proceedings in selective conferences.

Our methodology of combining an extended formulation approach with Dantzig-Wolfe decomposition and column generation, that is now published [21], is a great illustration of our team's threefold objective: it is a theoretically proved method, playing the complementary between exact optimization techniques, and leading to an computational edge in application solving. This methodology is a key tool for currently ongoing collaboration with EDF and Russian partners on railway applications.

The Samba project with our associated team in Brasil is picking up a new pace, with good progress on primal heuristics [27] [30] and stabilization techniques [23] [25] [29] [32]. In the coming year, short term visits will be completed by a one-month stay of Professor Uchoa, and a one-year-stay of his PhD student.

The composition of the team is going through rapid evolution: Gautier Stauffer, our Inria Chair, has been promoted as a Professor in Grenoble; Andrew Miller has returned to the US. Both positions have been republished in our thematic. We are currently building tighter links with CEPAGE by building closer work relations with Olivier Beaumont, Lionel Eyraud-Dubois, and Paul Renaud-Goud who share our methodologies, while emphasizing our expertise in the application domain of cloud computing.

The team has been integrated in the LaBEX CPU. The complete team participates in the WP5 "Network and Service Optimization". At the same time, there is a participation in the WP6 "Codes, Cryptologie, Algorithmique Arithmétique" with the proper methodology of the team.

3. Scientific Foundations

3.1. Introduction

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm* : (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a factional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM Ilog-CPLEX or FICO/Dash-Optimization's Xpress-mp). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is the study of non-linear models (non linearities are inherent in some engineering, economic and scientific applications) where solution techniques build on the best MIP approaches while demanding much more than simple extensions. Robust optimization is another area where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise mathematical programming (polyhedral approaches, Dantzig-Wolfe decomposition, non-linear integer programing, stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems.

3.2. Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [59]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if X is a mixed integer program: $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$ where $P = \{x \in \mathbb{R}^{n+p} : Ax \leq b\}$ with matrix $(A, b) \in \mathbb{Q}^{m \times (n+p+1)}$, then $conv(X)$ is a polyhedron that can be described in terms of linear constraints, i.e. it writes as $conv(X) = \{x \in \mathbb{R}^{n+p} : C\, x \leq d\}$ for some matrix $(C, d) \in \mathbb{Q}^{m' \times (n+p+1)}$ although the dimension m' is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem $\min\{cx : x \in X\}$ and solving the *separation problem* over the associated polyhedron $conv(X)$: if $\widetilde{x} \notin conv(X)$, find a linear inequality $\pi\, x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\widetilde{x}$. Hence, for NP-hard problems, one can not hope to get a compact description of $conv(X)$ nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of $conv(X)$ and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities $C\,x \leq d$ can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

3.3. Decomposition and reformulation approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called, *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, *cutting plane procedures* defined in the previous section reveal to be powerful. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

3.4. Constraint Programming (CP)

Constraint Programming focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems (minimizing the maximum of several variable values). Mixed Integer Programming (MIP), on the other hand, is effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables. Many problems belong to the intersection of these two classes. For example, some scheduling and timetabling problems are tightly constrained and have a sum-type objective. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

3.5. Mixed Integer NonLinear Programming (MINLP)

Many engineering, management, and scientific applications involve not only discrete decisions, but also nonlinear relationships that significantly affect the feasibility and optimality of solutions. MINLP problems combine the difficulties of MIP with the challenges of handling nonlinear functions. MINLP is one of the most flexible modeling paradigms available. However, solving such models is much more challenging: available softwares are not nearly as effective as standard softwares for linear MIP. The most powerful algorithms combine sophisticated methods that maintain outer linear programming approximation or convex relaxations with branch-and-bound enumeration; hence, the role of strong convex reformulations is crucial. The development of results for structured sub-models are essential building blocks. Preprocessing and bound reduction (domain reduction logic similar to that used in CP) are quite important too. Finally, decomposition methods also permit to develop tight outer approximations.

3.6. Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

4. Application Domains

4.1. Introduction

Our group has tackled applications in logistics, transportation and routing [57], [56], [53], [55], in production planning [70] and inventory control [53], [55], in network design and traffic routing [39], [46], [51], [75], [36], [47], [61], [66], in cutting and placement problems [60], [67], [68], [69], [71], and in scheduling [7], [62], [34].

4.2. Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form "at least two paths link each pair of terminals". We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [46]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today's optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronic system installation cost. We developed and compared several decomposition approaches [77], [76], [75] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route [74]. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [51], [52].

We studied several time dependent formulations for the unit demand vehicle routing problem [41], [40]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We also worked on the p-median problem, applying the matching theory to develop an efficient algorithm in Y-free graphs and to provide a simple polyhedral characterization of the problem and therefore a simple linear formulation [65] simplifying results from Baiou and Barahona.

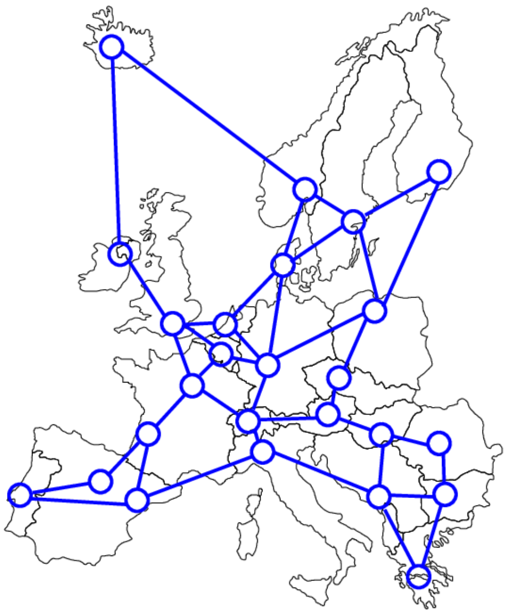

Figure 1. Design of a SDH/SONET european network where demands are multiplexed.

We considered the multi-commodity transportation problem. Applications of this problem arise in, for example, rail freight service design, "less than truckload" trucking, where goods should be delivered between different locations in a transportation network using various kinds of vehicles of large capacity. A particularity here is that, to be profitable, transportation of goods should be consolidated. This means that goods are not delivered directly from the origin to the destination, but transferred from one vehicle to another in intermediate locations. We proposed an original Mixed Integer Programming formulation for this problem which is suitable for resolution by a Branch-and-Price algorithm and intelligent primal heuristics based on it.

For the problem of routing freight railcars, we proposed two algorithmes based on the column generation approach. These algorithmes have been testes on a set of real-life instances coming from a Russian freight real transportation company. Our algorithmes have been faster on these instances than the current solution approach being used by the company.

4.3. Packing and Covering Problems

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [20]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs. The paper which presents this work is being finalized.

We have designed a new algorithm for vertex packing (equivalently stable set) in claw-free graphs [58]. Previously the best known algorithm for this problem had a running time of $O(n^6)$ (with n the number of vertices in the graph) while our new algorithm runs in $O(n^4)$.

We studied a variant of the knapsack problem encountered in inventory routing problem [55]: we faced a multiple-class integer knapsack problem with setups [54] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory [50], [48], [12], [11]. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2d knapsack problem that uses our 2D packing feasibility check.

4.4. Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a truncated branch-and-price algorithm: periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem; the global planning of customer visits is generated by solving a master program. Confronted with the issue of symmetry in time, we used a state-space relaxation idea. Our algorithm provides solutions with reasonable deviation from optimality for large scale problems (260 customer sites, 60 time periods, 10 vehicles) coming from industry [13]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [57].

Together with our partner company GAPSO from the associate team SAMBA, we are working on the equipment routing task scheduling problem [28] arising during port operations. In this problem, a set of tasks needs to be performed using equipments of different types with the objective of maximizing the weighted sum of performed tasks.

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [45] and exact algorithms [34]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [33]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [62]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [64]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [19], [63].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [14], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [14] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [35] provide demonstrably stronger formulations for some problem classes than any previously proposed.

We have been developing **robust optimization** models and methods to deal with a number of applications like the above in which uncertainty is involved. In [43], [42], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [37]. We considered train timetabling problems and their re-optimization after a perturbation in the network [44]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

5. Software

5.1. BaPCod – a generic Branch-and-Price Code

Participants: Romain Leguay [Software Engineer], Pierre Pesneau, Ruslan Sadykov, François Vanderbeck [correspondant].

BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of a Dantzig-Wolfe reformulation technique. The reformulated problem is solved using a branch-and-price (column generation) algorithm. This software platform, made of C++ classes, offers a "*black-box*" implementation that does not require user input and is not application specific. The features are
(i) the automation of the Dantzig-Wolfe reformulation process (the user defines a mixed integer programming problem in a pseudo modeling language, defining variables and constraints, identifying subproblems. He can provide subproblem solvers if available, but he does not need to explicitly define the reformulation, the explicit form of the columns, their reduced cost, or the Lagrangian bounds.
(ii) a default column generation procedure with standard initialization and stabilization [1], [23] [25] [29] [32] and
(iii) a default branching scheme that is generic to all applications [9],
(iv) default primal heuristics specially developed for use in a decomposition framework [49], [27], [30].
The prototype software was/is used as background solver for 5 PhD thesis. It also served as the framework for our comparative study in a Inria collaborative research action [1]. It has been experimented by two of our industrial partners, Exeo Solutions (Bayonne), on an inventory routing problem, and Orange Lab (France Telecom, Paris) on network design problems. The prototype also enables us to be very responsive in our industrial contact.

See also the web page https://wiki.bordeaux.inria.fr/realopt/pmwiki.php/Project/BaPCod.

6. New Results

6.1. Theoretical and Methodological Developments

Participants: Andrew Miller, Arnaud Pêcher, Pierre Pesneau, Ruslan Sadykov, Gautier Stauffer, François Vanderbeck.

We made progress in the development of theory and algorithms in the area of "Reformulation and Decomposition Approaches for MIP", "Mixed Integer Nonlinear Programming", and "Polyhedral Combinatorics and Graph Theory".

6.1.1. Column Generation for Extended Formulations

Working in an extended variable space allows one to develop tight reformulations for mixed integer programs. However, the size of the extended formulation grows rapidly too large for a direct treatment by a MIP-solver. Then, one can work with inner approximations defined and improved by generating dynamically variables and constraints. The alternative considered in [21] is an inner approximation obtained by generating dynamically the variables of the extended formulation. It assumes that the extended formulation stems from a decomposition principle. Then one can implement column generation for the extended formulation using Dantzig-Wolfe decomposition paradigm. Pricing subproblem solutions are expressed in the variables of the extended formulation and added to the current restricted version of the extended formulation along with the subproblem constraints that are active for the subproblem solutions.

Our paper [21] revisits the column-and-row generation approach, which is viewed herein as a generalization of standard column generation, the latter being based on a specific subproblem extended formulation. This generic view not only highlights the scope of applicability of the method, but it also leads to a more general termination condition than the traditional reduced cost criteria and to theoretically stronger dual bounds. We highlight a key benefit of the latter: lifting pricing problem solutions in the space of the extended formulation permits their recombination into new subproblem solutions and results in faster convergence.

The interest of the approach is evaluated numerically on machine scheduling, bin packing, generalized assignment, and multi-echelon lot-sizing problems. We compare a direct handling of the extended formulation, a standard column generation approach, and the "column-and-row generation" procedure. The results illustrate the stabilization effect resulting from column disaggregation and recombinations that is shown to have a cumulative effect when used in combination with a standard stabilization technique.

6.1.2. Primal Heuristics for Branch-and-Price

Primal heuristics have become an essential component in mixed integer programming (MIP). Generic heuristic paradigms of the literature remain to be extended to the context of a column generation solution approach. Our goal is to derive black-box primal heuristics for use in Branch-and-Price approaches. This requires extending primal heuristic paradigms to the context of dynamic generation of the variables of the model. We highlight an important fact: such generic tools typically performs better than problem specific meta-heuristics, in terms of solution quality and computing times. Based on our application specific experience with these techniques [55], [57], [72], [73], and on a review of generic classes of column generation based primal heuristics, in [49], we are developing a full blown review of such techniques, completed with new methods and an extensive numerical study. This research is being carried on in collaboration with the members of the associated team project, SAMBA [27] [30].

As a Dantzig-Wolfe reformulation is typically tighter than the original compact formulation, techniques based on rounding its linear programming solution have better chance to yield good primal solutions. The aggregated information built into the column definition and the price coordination mechanism provide a global view at the solution space that may be lacking in somewhat more "myopic" approaches based on compact formulations. However, the dynamic generation of variables requires specific adaptation of heuristic paradigms. Our contribution [30] lies in proposing simple strategies to get around these technical issues. We initially concentrate on "diving" methods and consider their combination with "sub-MIPing", relaxation induced neighborhood search, truncated backtracking using a Limited Discrepancy Search. These add-ons serves as local-search or diversification/intensification mechanisms. The methods are numerically tested on standard models such as Cutting Stock, Vertex Coloring, Generalized Assignment, Lot-Sizing, and Vehicle Routing problems. We further extend this research by combining the "diving" method mentioned above with the "feasibility pump" approach [27]. We show how this combination can be implemented in a context of dynamically defined variables, and we report on numerically testing "feasibility pump" for cutting stock and generalized assignment problems.

6.1.3. Stabilization techniques for column generation

Within the SAMBA project, we are collaboratively studying techniques to accelerate the convergence of column generation algorithms [25]. This techniques exploit Lagrangian duality theory. By revisiting all the alternative approaches to solving the Lagrangian dual, we identify suitable combinations of paradigms.

We also bridge the gap with techniques used in the dual framework of cut generation that have their unexploited counterpart for column generation [32], [29]. Cutting plane algorithmic strategies translate into stabilization procedures for column generation. We establish the link between the in-out separation procedure and dual price smoothing techniques for column generation. In this framework, we develop generic convergence proofs and effective smoothing auto-regulating strategies that avoids the need for parameter tuning. We further improve performance of such stabilization by hybridization with an ascent method. This work might inspire novel cut separation strategies.

6.1.4. Stable sets in claw-free graphs

A *stable set* is a set of pairwise non adjacent vertices in a graph and a graph is *claw-free* when no vertex contains a stable set of size three in its neighborhood. Given weights on the vertices, the stable set problem (a NP-hard problem in general) consists in selecting a set of pairwise non adjacent vertices maximizing the sum of the selected weights. The stable set problem in claw-free graphs is a fundamental generalization of the classic matching problem that was shown to be polynomial by Minty in 1980 (G. Minty. *On maximal independent sets of vertices in claw-free graphs*. J. Combinatorial Theory B, 28:284-304 (1980)). However, in contrast with matching, the polyhedral structure (i.e. the integer hull of all stable sets in a claw-free graph) is not very well understood and thus providing a 'decent' linear description of this polytope has thus been a major open problem in our field.

We proposed a new algorithm to find a maximum weighted stable set in a claw-free graph [38] whose complexity is now drastically better than the original algorithm by Minty (n^3 versus n^6, where n is the number of vertices). We also provided a description of the polyhedra in an extended space (i.e. using additional artificial variables) and an *efficient procedure* to separate over the polytope in polynomial-time [26]. Beside those main contributions, we published another papers on the strongly minimal facets of the polytope.

6.1.5. The Circular-Chromatic number

Another central contribution of our team concerns the chromatic number of a graph (the minimum number of independent stable sets needed to cover the graph). We proved that the chromatic number and the clique number of some superclasses of perfect graphs is computable in polynomial time [17].

We investigated the circular-chromatic number. It is a well-studied refinement of the chromatic number of a graph (designed for problems with periodic solutions): the chromatic number of a graph is the integer ceiling of its circular-chromatic number. Xuding Zhu noticed in 2000 that circular cliques are the relevant circular counterpart of cliques, with respect to the circular chromatic number, thereby introducing circular-perfect graphs, a super-class of perfect graphs.

We proved that the clique and chromatic numbers of circular-perfect graphs is computable in polynomial time [16], thereby extending Grötschel, Lovász and Schrijver's result to the whole family of circular-perfect graphs. We gave closed formulas for the Lovász Theta number of circular-cliques (previously, closed formulas were known for circular-cliques with clique number at most 3 only), and derived from them that the circular-chromatic number of circular-perfect graphs is computable in polynomial time [24].

6.2. Model Specific Developments and Applications

Participants: Andrew Miller, Arnaud Pêcher, Pierre Pesneau, Ruslan Sadykov, Gautier Stauffer, François Vanderbeck.

The models on which we made progress can be partitioned in three areas: "Packing and Covering Problems", "Network Design and Routing", and "Planning, Scheduling, and Logistic Problems".

6.2.1. Bin-Packing with Conflicts

The bin-packing problem consists in finding the minimum number of bin of fixed size one needs to pack a set of items of different sizes. We studied a generalization of this problem where items can be in conflicts and thus cannot be put together in the same bin. We show in [20] that the instances of the literature with 120 to 1000 items can be solved to optimality with a generic Branch-and-Price algorithm, such as our prototype BaPCod, within competitive computing time. Moreover, we solved to optimality all the 37 open instances. The approach involves generic primal heuristics, generic branching, but a specific pricing procedure.

6.2.2. Using graph theory for solving orthogonal knapsack problems

We investigated the orthogonal knapsack problem, with the help of graph theory. The multi-dimensional orthogonal packing problem (OPP) is defined as follows: given a set of items with rectangular shapes, the problem is to decide whether there is a non-overlapping packing of these items in a rectangular bin. The rotation of items is not allowed. A powerful characterization of packing configurations by means of interval graphs was introduced by Fekete and Schepers using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* (whose complement admits a transitive orientation: each such orientation of the edges corresponds to a specific placement of the forms) for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs.

In [12], [11], we give two new algorithms: the first one is based upon matrices with consecutive ones on each row as data structures and the second one uses so-called MPQ-trees, which were introduced by Korte and Mohring to recognize interval graphs. These two new algorithms are very efficient, as they outperform Fekete and Schepers' on most standard benchmarks.

6.2.3. Inventory routing and logistics problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. in [13], we considered the planning of single product pickups over time: each site accumulates stock at a deterministic rate; the stock is emptied on each visit. Our objective is to minimize a surrogate measure of routing cost while achieving some form of regional clustering by partitioning the sites between the vehicles. The fleet size is given but can potentially be reduced. Planning consists in assigning customers to vehicles in each time period, but the routing, i.e., the actual sequence in which vehicles visit customers, is considered as an "operational" decision. We developed a truncated branch-and-price algorithm. This exact optimization approach is combined with rounding and local search heuristics to yield both primal solutions and dual bounds that allow us to estimate the deviation from optimality of our solution. We were confronted with the issue of symmetry in time that naturally arises in building a cyclic schedule (cyclic permutations along the time axis define alternative solutions). Central to our approach is a state-space relaxation idea that allows us to avoid this drawback: the symmetry in time is eliminated by modeling an average behavior. Our algorithm provides solutions with reasonable deviation from optimality for large scale problems (260 customer sites, 60 time periods, 10 vehicles) coming from industry. The subproblem is interesting in its own right: it is a multiple-class integer knapsack problem with setups. Items are partitioned into classes whose use implies a setup cost and associated capacity consumption.

6.2.4. Scheduling

Cross docking terminals allow companies to reduce storage and transportation costs in a supply chain. At these terminals, products of different types from incoming trucks are unloaded, sorted, and loaded to outgoing trucks for delivery. In [19], we focus on the operational activities at a cross docking terminal with two doors: one for incoming trucks and another one for outgoing trucks. We consider the truck scheduling problem with the objective to minimize the storage usage during the product transfer inside the terminal. Our interest in this problem is mainly theoretical. We show that it is NP-hard in the strong sense even if there are only two product types. For a special case with fixed subsequences of incoming and outgoing trucks, we propose a dynamic programming algorithm, which is the first polynomial algorithm for this case. The results of numerical tests of the algorithm on randomly generated instances are also presented.

In [18], we consider the scheduling jobs in parallel, i.e., jobs can be executed on more than one processor at the same time. With the emergence of new production, communication and parallel computing system, the usual scheduling requirement that a job is executed only on one processor has become, in many cases, obsolete and unfounded. In this work, we consider the NP-hard problem of scheduling malleable jobs to minimize the total weighted completion time (or mean weighted flow time). For this problem, we introduce the class of "ascending" schedules in which, for each job, the number of machines assigned to it cannot decrease over time while this job is being processed. We prove that, under a natural assumption on the processing time functions of jobs, the set of ascending schedules is dominant for the problem. This result can be used to reduce the search space while looking for an optimal solution.

Currently, we are working on a scheduling application at a port. For this application, an equipment routing task scheduling problem [28] has been formulated, where a set of tasks needs to be performed. Tasks require equipment of different types. A particularity of the problem is that an equipment needs to be moved to the actual locations of tasks which use this equipment. So, there are both scheduling and routing decisions are to be taken simultaneously.

6.2.5. One warehouse multi-retailer problem

The *One-Warehouse Multi-retailer problem (OWMR)* is a very important NP-hard inventory control problem arising in the distribution of goods when one central warehouse is supplying a set of final retailers facing demand from customers. In [22], we provide a simple and fast 2-approximation algorithm for this problem (i.e. an algorithm ensuring a deviation by a factor at most two from the optimal solution). This result is both important in practice and in theory as it allows to approximate large real-world instances of the problem (we implemented this algorithm at IBM and it is within 10% of optimality in practice) and the techniques we developed appear to apply to more general settings. We are extending our results to other inventory control problems.

6.3. Software prototypes, Generic Developments and Specific Tools

Participants: Romain LeGuay, Pierre Pesneau, Ruslan Sadykov, François Vanderbeck.

6.3.1. BaPCod - a generic branch-and-price code

The development of the prototype software platform is supported by our junior engineer, Romain Leguay. He developed a new interface with the underlying MIP solver allowing multiple solvers to be called in the same run. He then re-organized the svn depository and a web distribution platform in view of the increasing number if users to whom Romain offers precious support. Romain has then redesigned parts of the code in the perspective of its parallelization and contributed to designing a pseudo modeling language for a friendly user interface. The emphasis is currently on enhancing the code performance in particular through rapid access data structure. Romain also participates to the setting up of stabilization and preprocessing algorithms.

The software platform BaPCod is continuously improved to include all the methodological features that arise from our research, in particular in our collaborative project with Brazil: SAMBA. BaPCod serves there as a proof-of-concept code and is useful for the transfer of knowledge between the parties, including the company GAPSO (a Brazilian spin-up launched by these academics).

7. Bilateral Contracts and Grants with Industry

7.1. Contract with EDF on maintenance planning

We are currently working on a project aiming to plan the energy production and the maintenance breaks for a set of nuclear power plants generating electricity. This problem has two different levels of decisions. The first one consist in determining, for a certain time horizon, when the different power plants will have to stop in order to perform maintenance and to decide the amount of fuel to be reloaded. The second decision level aims to decide the quantity of power each plant will have to produce for each probabilistic scenario on the energy consumption and the duration of maintenance. The model that we are considering combines issues of stochastic optimization (to handle demand scenarios), robust optimization (to account for variation in maintenance duration), and dynamic optimization (the maintenance of nuclear plants are programmed on a five year horizon, but the long term planning is review each month for adjustments due to perturbations.

This project is carried in collaboration between EDF R&D (OSIRIS lab) Inria team Dolphin and Realopt. The research is the subject of the PhD thesis of Nicolas Dupin (DGA).

8. Partnerships and Cooperations

8.1. Regional Initiatives

Region Aquitaine is supporting a post-doc in our team. Jinil Han has been recruited to contribute to our team effort to develop efficient decomposition based approaches to real-life combinatorial optimization problems. Jinil's research aims at enhancing performance of such approach and prepare the way to high performance computing through parallelization. Jinil's mission extends to problem solving that serves both as a motivation and an proof-of-concept. Jinil has contributed to warm-starting the methods and to convergence acceleration through stabilization techniques [23].

8.2. National Initiatives

8.2.1. CNRS

Pierre Pesneau has got a grant from the OR research group from CNRS to finance mission between Bordeaux and Paris within the context of a collaboration with University Paris 6 (P. Fouilhoux) and University Paris 13 (S. Borne, R. Grappe, M. Lacroix). This collaboration aims to study polyhedral properties and algorithmic aspects to the problem of connected graph partitioning.

8.3. International Initiatives

8.3.1. ANR Gratel

André Raspaud launched in 2005 a fruitful cooperation with the Department of Applied Mathematics of the Sun Yat-Sen University of Kaohsiung, Taiwan. This gave rise to an international ANR project funded for three years (January 2010 - December 2013), that is managed by Arnaud Pêcher and André Raspaud. The scientific priority theme is "Telecommunications", a well-known key application area of graph theory. The aim is to tackle especially wireless communications problems, with the help of graph colorings and polyhedral graph theory. Currently, Sagnik Sen (PhD student of E. Sopena, A. Pêcher, A. Raspaud) benefits from a scholarship on this ANR.

8.3.2. Inria Associate Teams

8.3.2.1. SAMBA

Title: Combinatorial optimization problems

Inria principal investigator: François Vanderbeck

International Partner (Institution - Laboratory - Researcher):

Pontificia Universidade Catolica do Rio de Janeiro (Brazil) - ATD-Lab - Marcus Poggi

Duration: 2011 - 2013

See also: https://wiki.bordeaux.inria.fr/realopt/pmwiki.php/Project/Samba

The so-called Dantzig-Wolfe decomposition approach has not yet made its way into general purpose solvers for Mixed Integer Programming (MIP). Despite its proved efficiency, the use of the method is currently restricted to specific applications and requires ad-hoc algorithms developed by experts. Our project is to develop general purpose algorithms to make this method generic. We shall focus in particular on (i) preprocessing procedures, (ii) warm-starting, (iii) stabilization (to improve convergence), (iv) strategies for combining cut and column generation, and (v) primal heuristics. The project builds on the accumulated experience of both the Brazilian and the French teams that have done pioneering work in tackling complex applications and deriving generic solution strategies using this decomposition approach. The new algorithms are implemented and tested in the software platform BaPCod. Hence, the collaborative research on methodological developments should lead to, as a bi-product, a Version 2 of BaPCod as a state-of-the-art Branch-and-Price-and-Cut Solver. This prototype should (i) serve as proof-of-concept code for the research planned in this project and beyond, (ii) enable us to achieve new benchmark results on key problems,(iii) provide incentive for the use of the method by non experts, (iv) leverage technology transfer to industry.

8.4. International Research Visitors

8.4.1. Visits of International Scientists

8.4.1.1. Short term Visitors

- Artur Pessoa, LOGIS, the Universidade Federal Fluminense (UFF), Brazil.
- Oriol Serra, Universitat Politècnica de Catalunya, Spain
- Eduardo Uchoa, LOGIS, the Universidade Federal Fluminense, Brazil.

8.4.1.2. Internships

Diego PECIN, from Pontificade Universitat Catholica (PUC-Rio) from Feb 2012 until Mar 2012

Subject: Comparative study of column generation stabilization techniques

Institution: Federal University of Rio de Janeiro (Brazil)

Alexey KARPYCHEV (from Sep 2012 until Dec 2012)

Subject: Multi-commodity transportation problem with application to the freight service design

Institution: Moscow Institute of Physics and Technology (Russia)

8.4.2. Visits to International Teams

Pierre Pesneau was invited one week (Mars 5th-9th, 2012) by Luis Gouveia (Universidade de Lisboa) to work on time-dependent formulations for the capacitated vehicle routing problem.

Ruslan Sadykov and Francois Vanderbeck have both spend a two-week visit in our associated team at PUC-Rio and UFF in Brazil in March 2012.

9. Dissemination

9.1. Scientific Animation

9.1.1. Organization of workshops

- Arnaud Pêcher was in the organizing committee of the Bordeaux Graph Workshop 2012.
- Pierre Pesneau is member of the organizing committee of the working group "Polyhedra and Combinatorial Optimization" affiliated to the French operations research society (ROADEF) and the operations research group of CNRS. The purpose of this working group is to promote the field of polyhedra in the research domain of combinatorial optimization. To this aim, the group organizes every even years a biennial international symposium on combinatorial optimization (ISCO 2010 in Tunisia, ISCO 2012 in Greece, ISCO 2014 will be held in Portugal) and every odd years the national Polyhedra and Combinatorial Optimization Days. Both of them are preceded by a doctoral spring school.

9.1.2. Invitations to conferences

- Arnaud Pêcher, *How unique is Lovász's theta function?*, 2012 International Conference on Graph Theory, Combinatorics and Applications, Jinhua, Chine, 2012
- Francois Vanderbeck had an “invited talk” at the *International Workshop on Column Generation*, Montréal 2012. He was also an *Invited speaker* at the *Workshop on Integer Programming*, Valparaiso, Chile, March 2012.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master “Ingénierie Mathématique, Statistiques, et Economique”: “Optimisation dans les graphes”, University of Bordeaux, France.

“Modèles de Flot et Combinatoire”, 45 hetd, M1 (Gautier Stauffer)

“Optimisation dans les graphes”, 15h, M1, (Arnaud Pêcher)

“Optimisation combinatoire”, 30 hetd, M1, (Andrew Miller, Francois Vanderbeck)

“Outils et logiciels d’optimisation”, 14 hetd, M1, (Andrew Miller)

“Programmation linéaire”, 60 hetd, M1, (Andrew Miller, Pierre Pesneau)

“Programmation linéaire”, 30 hetd, M1 (Pierre Pesneau)

“Introduction à la Programmation en Nombres Entiers”, 15 hetd, M1 (Pierre Pesneau)

“Programmation Orienté Objet”, 15 hetd, M1 (Pierre Pesneau)

“Introduction à la Programmation par Contraintes”, 30 hetd, M2, (Ruslan Sadykov).

“Gestion des opérations et Planification de la Production”, 30 hetd, M2, (Ruslan Sadykov).

Other Masters:

Enseirb, Computer Sciences: “Recherche Opérationnelle”, 37 hetd, 2nd year, Institut Polytechnique de Bordeaux, France (Pierre Pesneau, Francois Vanderbeck)

9.2.2. Supervision

PhD: Petru Valicov, “Algorithmes de graphes pour les problèmes d’ordonnancement”, Juillet 2012, A. Pêcher, M. Montassier, E. Sopena

PhD in progress: Sagnik Sen, “Graphes et télécommunications”, January 2011, E. Sopena, A. Pêcher, A. Raspaud.

PhD in progress: Nastaran Rahmani, “Planning and Routing via decomposition approaches”, April 2011, R. Sadykov, F. Vanderbeck

PhD in progress: Nicolas Dupin, “Scheduling Maintenance of Nuclear Plants in Power Production Planning”, Mai 2011, A. Miller, R. Sadykov, E. Talbi, F. Vanderbeck.

9.2.3. Juries

- Francois Vanderbeck was a member of the PhD jury of Sergey Kovalev, at Ecole des mines de St Etienne.

9.2.4. Administrative Responsabilities

Each member of the team is quite involved in teaching in the thematic specialties of the project, including in the research track of the Masters in applied mathematics or computer science and an Operations Research Track in the computer science department of the Engineering school ENSEIRB-MATMECA. Moreover, we are largely implied in the organization of the curriculum:

- Arnaud Pêcher was the head of IUT Computer Science’s special year, since 2010.
- Francois Vanderbeck has succeeded to Andrew Miller as the head of the Master Speciality in Operations Research. He is aslo a member of the council of the laboratory of mathematics of Bordeaux (IMB) and its scientific committee.
- Pierre Pesneau is elected (since March 2011) to the council of the laboratory of mathematics of Bordeaux (IMB) and (since March 2012) to the council of the department of Mathematics and Computer Science of the University of Bordeaux.
- Ruslan Sadykov is elected (since September 2012) to the research council of the laboratory of mathematics of Bordeaux (IMB).

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