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**Centre d'expertise des risques,
de l'environnement, des
mobilités et de l'aménagement**

Activity Report 2013

Team ANGE

Numerical Analysis, Geophysics and Ecology

RESEARCH CENTER
Paris - Rocquencourt

THEME
**Earth, Environmental and Energy
Sciences**

Table of contents

1. Members	1
2. Overall Objectives	1
2.1. Presentation	1
2.2. Highlights of the Year	2
3. Research Program	2
3.1. Introduction	2
3.2. Geophysical flows – modelling and analysis	2
3.2.1. Multilayer approach	3
3.2.2. Non-hydrostatic models	3
3.3. Coupling	3
3.3.1. Analysis and numerical treatment	3
3.3.2. Data assimilation	4
4. Application Domains	4
4.1. Fluids with complex rheology	4
4.1.1. Arbitrary topography	4
4.1.2. Erosion and sedimentation	5
4.2. Ecology and sustainable energies	5
4.2.1. Hydrodynamics-biology coupling	5
4.2.1.1. Hydrodynamics-biology coupling for algae culture and biofuel production	5
4.2.1.2. Lacustrian ecosystems	5
4.2.2. Marine energies	6
5. Software and Platforms	6
6. New Results	6
6.1. Geophysical flows	6
6.1.1. A numerical scheme for the Saint-Venant equations	6
6.1.2. Two-phase flows	7
6.1.3. Non-hydrostatic models	7
6.1.4. Fluids with complex rheology	7
6.1.5. Dynamics of sedimentary river beds with stochastic fluctuations	7
6.2. Ecology and sustainable energies	8
6.3. Coupling methods	8
6.3.1. Data assimilation for conservation laws associated with kinetic description	8
6.3.2. Mach-parametrized flows	8
6.3.3. Error analysis in a coupling strategy	8
6.4. Software development and assessments	8
6.4.1. Analytical solutions for the incompressible Euler system	8
6.4.2. Software	9
7. Bilateral Contracts and Grants with Industry	9
7.1. Bilateral Contracts with Industry	9
7.2. Bilateral Grants with Industry	9
8. Partnerships and Cooperations	9
8.1. Regional Initiatives	9
8.1.1. Instabilities in Hydrodynamics (2011–2015)	9
8.1.2. Plasticity of geophysical flows and seismic emissions (2013–2016)	9
8.2. National Initiatives	10
8.2.1. GdR EGRIN (2013–2017)	10
8.2.2. Inria Project Lab “Algae in Silico”	10
8.2.3. ANR LANDQUAKE (2012–2016)	10
8.2.4. LRC Manon (2010–2014)	10

8.2.5. Structure Health Monitoring	10
8.2.6. ANR project HJnet (2013–2015)	10
8.2.7. Hydraulics for environment and sustainable development (HED ²)	11
8.3. European Initiatives	11
8.4. International Initiatives	11
8.5. International Research Visitors	11
9. Dissemination	11
9.1. Scientific Animation	11
9.2. Teaching - Supervision - Juries	12
9.2.1. Teaching	12
9.2.2. Supervision	12
9.2.3. Juries (PhD)	13
9.3. Popularization	14
10. Bibliography	14

Team ANGE

Keywords: Geophysics, Modeling, Numerical Methods, Simulation, Bioenergy

Formerly a part of the BANG team, the ANGE team was launched in 2012 and is aimed at focusing on geophysical flows in the framework of sustainable development. Members of the team are jointly hosted at Inria Rocquencourt and University Pierre et Marie Curie Paris 6 (Laboratory Jacques-Louis Lions).

Creation of the Team: 2012 November 01, updated into Project-Team: 2014 January 01.

1. Members

Research Scientists

Jacques Sainte-Marie [Team leader, CETMEF (ministry of ecology), Senior Researcher, HdR]
Yohan Penel [CETMEF (ministry of ecology), Researcher]

Faculty Members

Edwige Godlewski [Univ. Pierre et Marie Curie Paris 6, Professor, HdR]
Anne Mangeney [Univ. Denis Diderot Paris 7, Professor, partially funded by CETMEF, HdR]
Nicolas Seguin [Univ. Pierre et Marie Curie Paris 6, Associate Professor, HdR]

External Collaborators

Emmanuel Audusse [Univ. Paris 13, Associate Professor]
Cindy Guichard [Univ. Pierre et Marie Curie Paris 6, Associate Professor, from Oct 2013]
Marie-Odile Bristeau [Inria, Senior Researcher (retired)]
Nicole Goutal [EDF, Senior Researcher, HdR]

Engineer

Raouf Hamouda [Inria, IJD]

PhD Students

Nora Aïssiouene [granted by Inria (Cordi-S), from Nov 2013]
Anne-Céline Boulanger [granted by Univ. Pierre et Marie Curie Paris 6, until Nov 2013]
Dena Kazerani [granted by Univ. Pierre et Marie Curie Paris 6, from Mar 2013]
Philippe Ung [granted by Univ. Orléans, CNRS & EDF]

Administrative Assistant

Maryse Desnous [Inria]

2. Overall Objectives

2.1. Presentation

Among all aspects of geosciences, we mainly focus on gravity driven flows arising in many situations such as

- hazardous flows (flooding, rogue waves, landslides...),
- sustainable energies (hydrodynamics-biology coupling, biofuel production, marine energies...),
- risk management and land-use planning (morphodynamic evolutions, early warning systems...)

For these multi-scale and multi-physics systems, the difficulty is often to isolate a reduced-size problem for which mathematical modelling and simulation can provide significant advances.

There exists a strong demand from scientists and engineers in fluid mechanics for models and numerical tools able to simulate not only the water depth and the velocity field but also the distribution and evolution of external quantities such as pollutants or biological species and the interaction between flows and structures (seashores, erosion processes...). The key point of the researches carried out within ANGE is to answer this demand by the development of efficient, robust and validated models and numerical tools.

2.2. Highlights of the Year

On the one hand, the ERC Consolidator Grant allocated to Anne Mangeney will enable cross-disciplinary works for the modelling of processes governing landslides. In the same spirit, the first Albert Tarantola workshop managed by A. Mangeney and J. Sainte-Marie held on September and aimed at promoting collaborations between mathematicians and geophysicists.

On the other hand, 2013 was dedicated to “Mathematics for Planet Earth” under the patronage of UNESCO. This international initiative consisted in highlighting the role played by mathematics in the modelling of processes that occur on earth including geophysics, biology and human sciences. The ANGE team got involved into this dynamic through the ARP “MathInTerre” from the French agency for research (ANR): scientific committee, organisation of dedicated workshops,...

3. Research Program

3.1. Introduction

The research activities carried out within the ANGE team strongly couple the development of methodological tools with applications to real-life problems and the transfer of numerical codes. Even if the present program is more problem-driven by challenging applications than methodology-driven, it is fundamental to justify mathematically results at each step.

The difficulties arising in gravity driven flow studies are threefold.

- Models and equations encountered in fluid mechanics (typically the free surface Navier-Stokes equations) are complex to analyze and solve.
- The underlying phenomena often take place over large domains with very heterogeneous length scales (size of the domain, mean depth, wave length,...) and distinct time scales, *e.g.* coastal erosion, propagation of a tsunami,...
- Last but not least, these problems are multi-physics with strong couplings and nonlinearities.

3.2. Geophysical flows – modelling and analysis

Hazardous flows are complex physical phenomena that can hardly be represented by shallow water type systems of partial differential equations (PDEs). In this domain, the research program is devoted to the derivation and analysis of reduced complexity models – compared to the Navier-Stokes equations – but relaxing the shallow water assumptions. The main purpose is then to obtain models adapted to the physical phenomena at stake and eventually to simulate them by means of robust and efficient numerical techniques.

Even if the resulting models do not strictly belong to the family of hyperbolic systems, they exhibit hyperbolic features: the analysis and discretization techniques we intend to develop have connections with those used for hyperbolic conservation laws. It is worth noticing that the need for robust and efficient numerical procedures is reinforced by the smallness of dissipative effects in geophysical models which therefore generate singular solutions and instabilities.

More precisely, the derivation of the Saint-Venant system from the Navier-Stokes equations is based on two main approximations – valid because of the shallow water assumption – namely

- the horizontal fluid velocity is well approximated by its mean along the vertical direction,
- the pressure is hydrostatic or equivalently the vertical acceleration of the fluid can be neglected compared to the gravitational effects.

As a consequence the objective is to get rid of these two assumptions, one after the other, in order to obtain models accurately approximating the incompressible Euler or Navier-Stokes equations.

3.2.1. Multilayer approach

As for the first assumption, *multi-layer* systems were proposed describing the flow as a superposition of Saint-Venant type systems [21], [25], [26]. Even if this approach has provided interesting results, it implies to consider each layer as isolated from its neighbours and this is a strong limitation. That is why we proposed a slightly different approach [22], [23] based on Galerkin type decomposition along the vertical axis of all variables and leading, both for the model and its discretization, to more accurate results.

A kinetic representation of our multilayer model allows to derive robust numerical schemes endowed with properties such as: consistency, conservativity, positivity, preservation of equilibria,...It is one of the major achievements of the team but it needs to be analyzed and extended in several directions namely:

- The convergence of the multilayer system towards the hydrostatic Euler system as the number of layers goes to infinity is a critical point. It is not fully satisfactory to have only formal estimates of the convergence and sharp estimates would enable to guess the optimal number of layers.
- The introduction of several source terms due for instance to Coriolis forces or extra terms from changes of coordinates seems necessary. Their inclusion should lead to substantial modifications of the numerical scheme.
- Its hyperbolicity has not yet been proved and conversely the possible loss of hyperbolicity cannot be characterized. Similarly, the hyperbolic feature is essential in the propagation and generation of waves.

3.2.2. Non-hydrostatic models

The hydrostatic assumption (*ii*) consists in neglecting the vertical acceleration of the fluid. It is considered valid for a large class of geophysical flows but is restrictive in various situations where the dispersive effects (like wave propagation) cannot be neglected. For instance, when a wave reaches the coast, bathymetry variations give a vertical acceleration to the fluid that strongly modifies the wave characteristics and especially its height.

When processing an asymptotic expansion (w.r.t. the aspect ratio for shallow water flows) into the Navier-Stokes equations, we obtain at the leading order the Saint-Venant system. Going one step further leads to a vertically averaged version of the Euler/Navier-Stokes equations integrating the non-hydrostatic terms. This model has several advantages:

- it admits an energy balance law (that is not the case for most of the models available in the literature),
- it reduces to the Saint-Venant system when the non-hydrostatic pressure term vanishes,
- it consists in a set of conservation laws with source terms,
- it does not contain high order derivatives.

The main challenge in the study of this model is the derivation of a robust and efficient numerical scheme endowed with properties such as: positivity, wet/dry interfaces treatment, consistency.

It has to be noticed that even if the non-hydrostatic model looks like an extension of the Saint-Venant system, most of the known techniques used in the hydrostatic case are not efficient as we recover strong difficulties encountered in incompressible fluid mechanics due to the extra pressure term. These difficulties are reinforced by the absence of viscous/dissipative terms.

It is important to point out that the modelling and efficient simulations of non-hydrostatic models allow to answer important and various questions such as:

- accurate description of propagation waves (tsunamis, rogue waves),
- accurate representation of the dispersive effects when a wave reaches the coast,
- wave reflection and roughness in harbors, design of seashores.

3.3. Coupling

3.3.1. Analysis and numerical treatment

The coupling of models and numerical codes is an acute problem encountered in practice by many engineers. E. Godlewski and N. Seguin have recently proposed neat techniques for the coupling of hyperbolic systems and numerical codes.

For hyperbolic systems, finite volume methods are often used with explicit time discretization. When the source terms, typically viscosity and friction, have small influence compared to the hyperbolic part, fractional time steps are suitable. This no longer holds when non trivial equilibria between advection and dissipative terms occur and the concept of Asymptotic-Preserving (AP) methods has been proposed to study these difficulties. AP methods make a breakthrough in the numerical resolution of asymptotic perturbations of partial-differential equations.

Another strategy in the quest for a better balance between accuracy and efficiency is the adaptation of models. Indeed, the systems of partial differential equations we consider result from a hierarchy of simplifying assumptions. However, some of these hypotheses may turn out to be irrelevant locally. The adaptation of models thus consists in determining areas where a simplified model (*e.g.* shallow water type) is valid and where it is not. In the latter case, we may go back to the “parent” model (*e.g.* Euler) in the corresponding area. This implies to know how to handle the coupling between the aforementioned models from both theoretical and numerical points of view. In particular, the numerical treatment of transmission conditions is a key point.

Coupling problems also arise within the fluid when it contains pollutants, density variations or biological species. In such situations, reaction terms interact with advection effects and need sophisticated treatment for a more complete description.

3.3.2. Data assimilation

Data assimilation consists in a coupling between a model and observation measurements. Developing robust data assimilation methods for hyperbolic-type conservation laws is a challenging subject. Those PDEs indeed show no dissipation effects and the input of additional information in the model equations may introduce errors that propagate and create shocks. We have recently proposed a new approach based on the kinetic description of the conservation law. Hence, data assimilation is carried out at the kinetic level, using a Luenberger observer. Assimilation then resumes to the handling of a BGK type equation. The advantage of this framework is that we deal with a single “linear” equation instead of a nonlinear system and it is easy to recover the macroscopic variables. We are able to prove the convergence of the model towards the data in case of complete observations in space and time.

This work is done in collaboration with the M3DISIM Inria project-team. M. Doumic and B. Perthame (BANG) also participate.

4. Application Domains

4.1. Fluids with complex rheology

Whereas the viscous effects can often be neglected in water flows, they have to be taken into account in situations such as avalanches, debris flows, pyroclastic flows, erosion processes,...*i.e.* when the fluid rheology becomes more complex. Gravity driven granular flows consist of solid particles commonly mixed with an interstitial lighter fluid (liquid or gas) that may interact with the grains and decrease the intensity of their contacts, thus reducing energy dissipation and favoring propagation. Examples include subaerial or subaqueous rock avalanches (*e.g.* landslides).

As mentioned above, the main issue is to propose models of reduced complexity, suitable for scientific computing and endowed with stability properties (continuous and/or discrete). In addition, models and their numerical approximations have to be confronted with experimental data, as analytical solutions are hardly accessible for these problems/models. A. Mangeney (IPGP) and N. Goutal (EDF) may provide useful data.

4.1.1. Arbitrary topography

Most shallow water type models are derived under the assumption of small/ smooth bottom variations whereas in practice the topography along which the flow (avalanche, debris flow,...) occurs can be quite steep and rough. An improved Saint-Venant system, due to Savage-Hutter, and valid for large slopes and small slope variations

has been proposed. A new model relaxing all restrictions upon the topography has been proposed for shallow water flows by Bouchut *et al.* [24], [27]. The extension of this work to the case of models with distributed velocities along the vertical axis is an important objective with many applications (landslides, avalanches,...).

4.1.2. Erosion and sedimentation

The sediment transport modelling is of major interest in terms of applications. It also raises interesting issues from a numerical aspect. This is an example of coupling between the flow and another phenomenon, namely the deformation of the bottom of the basin that can be carried out either by bed load where the sediment has its own velocity or suspended load in which the particles are mostly driven by the flow. This phenomenon involves different time scales and nonlinear retroactions; hence the need for accurate mechanical models and very robust numerical methods. In collaboration with industrial partners (EDF-LNHE), the team already works on the improvement of numerical methods for existing (mostly empirical) models but our aim is also to propose new (quite) simple models that contain important features and satisfy some basic mechanical requirements. The extension of our 3D models to the transport of weighted particles can also be here of great interest.

4.2. Ecology and sustainable energies

Sustainable development and environment preservation have a growing importance and scientists have to address difficult issues such as: management of water resources, renewable energy production, biogeochemistry of oceans, resilience of society w.r.t. hazardous flows,...

4.2.1. Hydrodynamics-biology coupling

Nowadays, simulations of the hydrodynamic regime of a river, a lake or an estuary, are not restricted to the determination of the water depth and the fluid velocity. They have to predict the distribution and evolution of external quantities such as pollutants, biological species or sediment concentration.

4.2.1.1. Hydrodynamics-biology coupling for algae culture and biofuel production

The potential of micro-algae as a source of biofuel and as a technological solution for CO₂ fixation is the subject of intense academic and industrial research. Large-scale production of micro-algae has potential for biofuel applications owing to the high productivity that can be attained in high-rate raceway ponds.

One of the key challenges in the production of micro-algae is to maximize algae growth with respect to the exogenous energy that must be used (paddlewheel, pumps,...). There is a large number of parameters that need to be optimized (characteristics of the biological species, raceway shape, stirring provided by the paddlewheel); consequently our strategy is to develop efficient models and numerical tools to reproduce the flow induced by the paddlewheel and the evolution of the biological species within this flow. Here, mathematical models can greatly help us reduce experimental costs.

Owing to the high heterogeneity of raceways due to gradients of temperature, light intensity and nutrient availability through water height, we cannot use depth-averaged models. We adopt instead more accurate multilayer models that have recently been proposed.

It is clear however that many complex physical phenomena have to be added to our model, such as the effect of sunlight on water temperature/ density, evaporation and external forcing (wind).

4.2.1.2. Lacustrine ecosystems

Many problems previously mentioned also arise in larger scale systems like lakes. Hydrodynamics of lakes is mainly governed by atmospheric forcing terms: wind, temperature variations,...

If the interactions between hydrodynamics and biology are known via laboratory experiments, it is more difficult to predict the evolution – especially for the biological quantities – in a real and heterogeneous system. The objective is to model and reproduce the hydrodynamics modifications due to forcing term variations (in time and space). We are typically interested in phenomena such as eutrophication, development of harmful bacteria (cyanobacteria) and upwelling phenomena.

4.2.2. Marine energies

One of the booming lines of business is the field of renewable and decarbonated energies. In particular in the marine realm, several processes have been proposed in order to produce electricity thanks to the recovering of wave, tidal and current energies. We may mention water-turbines, buoys turning variations of the water height into electricity or turbines motioned by currents. Although these processes produce an amount of energy which is less substantial than in thermal or nuclear power plants, they have smaller dimensions and can be set up more easily.

The fluid energy has a kinetic and potential part. The buoys use the potential energy whereas the turbines are activated by currents. To become economically relevant, these systems need to be optimized (shape, position, durability, ...) in order to improve their productivity. This is a complex and original issue which requires efficient numerical tools.

Some processes are currently running. However, they have not been studied from an optimization point of view. While for the construction of a harbour, the goal is to minimize swell, in our framework we intend to maximize the wave energy. A key-point is the optimization of the bathymetry in a given geometrical domain which influences the swell and thus the effectiveness of processes. Optimization involving fluid mechanics is quite complex. Although such an approach seems innovative, it clearly requires the development of methodological tools. In a second step, experiments will be necessary for the validation.

5. Software and Platforms

5.1. FRESHKISS

Although the Saint-Venant system is the cornerstone of flow modelling in geosciences, this does not mean that the transfer of the efficient dedicated simulation tools is achieved in the geoscience community.

ANGE collaborates with scientists, laboratories and companies that are interested in scientific advances which makes the valuation and the transfer of results easier.

The development of robust and efficient numerical tools has been a strong point of the activities within the BANG project-team. ANGE aims at pursuing this effort as most publications of the team members contain both modelling and simulation/validation aspects. For the simulation of the free surface Navier-Stokes equations, numerical tools have been developed namely FRESHKISS2D¹ and FRESHKISS3D. These tools are used by several scientists typically in the BIOCORE Inria project-team, at EDF and in public research laboratories.

FRESHKISS3D is a numerical code solving the 3D hydrostatic and incompressible Navier-Stokes equations with variable density. This code was initially dedicated to research activities within the team but we now aim at turning it into a numerical tool being used by non-mathematicians. Indeed, there is a demand in research laboratories and companies to use this tool. A young engineer (R. Hamouda) has been hired (ADT In@lgae funded by Inria) and its assignment is to improve/enrich the code and to make it user-friendly. Notice that FRESHKISS3D is used for teaching (master students in geosciences) at university Denis Diderot Paris 7 and IPGP.

6. New Results

6.1. Geophysical flows

6.1.1. A numerical scheme for the Saint-Venant equations

Participants: Emmanuel Audusse, Christophe Chalons [Univ. Versailles], Philippe Ung.

¹FRESHKISS: FREe Surface Hydrodynamics using KInetic SchemeS

In order to improve the numerical simulations of the shallow-water equations, one has to face three important issues related to the well-balanced, positivity and entropy-preserving properties, as well as the ability to handle vacuum states. In that purpose, we propose a Godunov-type method based on the design of a three-wave Approximate Riemann Solver (ARS) which satisfies all aforementioned properties.

6.1.2. *Two-phase flows*

Participants: Frédéric Coquel [CNRS], Jean-Marc Hérard [EDF], Khaled Saleh [IRSN], Nicolas Seguin.

After having developed numerical schemes for models of compressible two-phase flows [17], [19], we have proven some fundamental properties of these systems: symmetrizability and (non strict) convexity of the entropy [18]. This enables us now to address the well-posedness of these models when the relaxation terms are included.

6.1.3. *Non-hydrostatic models*

Participants: Marie-Odile Bristeau, Dena Kazerani, Anne Mangeney, Jacques Sainte-Marie, Nicolas Seguin.

The objective is to derive a model corresponding to a depth averaged version of the incompressible Euler equations with free surface. We have already contributed to this subject but the obtained results extend previous ones [29] in several directions:

- the derivation of the model is more rigorous and follows the entropy-based moment closures proposed in [28],
- the properties of the model and especially its connections with Green-Nagdhi model have been investigated,
- a family of analytical solutions for the proposed model have been obtained.

These analytical solutions emphasize the non-hydrostatic effects appearing for large slope variations.

6.1.4. *Fluids with complex rheology*

Participants: Anne Mangeney, Jacques Sainte-Marie.

We have been able

- to develop detection, characterization and localisation methods applicable to the seismic signals generated by rockfalls and thus to analyse the spatio-temporal change of rockfall localisation and properties during several years, making it possible to show how rockfalls can be used as a precursor of volcanic activity,
- to propose an empirical “universal” law describing friction weakening in landslides over a broad range of volumes and geological contexts,
- to propose a new debris flow model with an energy balance,
- show the existence of a slow propagation phase in granular flows, playing a key role in their dynamics and in erosion processes.

6.1.5. *Dynamics of sedimentary river beds with stochastic fluctuations*

Participants: Emmanuel Audusse, Philippe Ung.

The Exner equation is a coarse model for the dynamics of sedimentary river beds, derived using both many heuristics and empirism. Though, it is also quite practical for hydraulic engineering applications, and efficient enough in numerous situations. Our goal in this work is to improve the model by including some effects that have been neglected so far in the heuristics. In particular, inline with other current research directions in the field, we study the possibility of introducing some stochasticity in the model. To this end, we suggest to numerically experiment some recently proposed variations of the Exner equation based on the introduction of stochastic fluctuations within the standard formulation.

This project has been the subject of a study during the 2013 session of the CEMRACS.

6.2. Ecology and sustainable energies

6.2.1. *Hydrodynamic-biology coupling*

Participants: Olivier Bernard [Inria BIOCORE], Anne-Céline Boulanger, Marie-Odile Bristeau, Raouf Hamouda, Jacques Sainte-Marie.

An important part of our research activity is built around a biological and industrial problem: the simulation of the coupling of hydrodynamics and biology in the context of industrial microalgae culture in outdoor raceways. The numerical modelling is addressed with the use of a multilayer vertical discretization of hydrostatic Navier-Stokes equations coupled with a light sensitive Droop model. Numerically, kinetic schemes allow for the development of efficient, positivity preserving, well balanced and entropy satisfying schemes. Simulations are carried out in 2D and 3D [1]. From a practical point of view, this model is capable of accounting for the utility of a paddlewheel and exhibits Lagrangian trajectories underwent by algae. Hence providing hints on the light history of algae in the pond, which is a key information to biologists, since it enables them to adapt their phytoplankton growth models to those particular, non natural conditions.

6.3. Coupling methods

6.3.1. *Data assimilation for conservation laws associated with kinetic description*

Participants: Anne-Céline Boulanger, Philippe Moireau [Inria M3DISIM], Jacques Sainte-Marie.

In order to take advantage of the kinetic description of conservation laws already used for the building of efficient schemes, an innovative data assimilation method for hyperbolic balance laws based in a Luenberger observer on the kinetic equation is developed. It provides a nice theoretical framework for scalar conservation laws, for which we study the cases of complete observations, partial observations in space, in time, and noisy observations. As far as systems are concerned, we focus on the Saint-Venant system, which is hyperbolic, nonlinear and has a topographic source term. We build an observer based only on water depths measurements. Numerical simulations are provided in the case of scalar laws and systems, in one and two dimensions, which validate the efficiency of the method [14].

6.3.2. *Mach-parametrized flows*

Participants: Stéphane Dellacherie [CEA], Bruno Després [UPMC Paris 6], Yohan Penel.

In order to enrich the modelling of fluid flows, we investigate in this paper a coupling between two models dedicated to distinct regimes. More precisely, we focus on the influence of the Mach number as the low Mach case is known to induce theoretical and numerical issues in a compressible framework. A moving interface is introduced to separate a compressible model (Euler with source term) and its low Mach counterpart through relevant transmission conditions. A global steady state for the coupled problem is exhibited. Numerical simulations are then performed to highlight the influence of the coupling by means of a robust numerical strategy [20].

6.3.3. *Error analysis in a coupling strategy*

Participants: Clément Cancès [UPMC Paris 6], Frédéric Coquel [CNRS], Edwige Godlewski, Hélène Mathis [Univ. Nantes], Nicolas Seguin.

We have proposed in a simplified framework an error analysis for an adaptive method which automatically selects the optimal model to use, the choice being between a reference model and an associated simplified one, see [15]. In particular, we are able to balance the thickness of the coupling buffer zone with the threshold on the modelling error which appears when introducing the coarse model.

6.4. Software development and assessments

6.4.1. *Analytical solutions for the incompressible Euler system*

Participants: Anne-Céline Boulanger, Marie-Odile Bristeau, Jacques Sainte-Marie.

We have proposed in [5] a large set of analytical solutions (FRESH-ASSESS) for the hydrostatic incompressible Euler system in 2d and 3d. These solutions mainly concern free surface flows but partially free surface flows are also considered. These analytical solutions can be especially useful for the validation of numerical schemes.

6.4.2. Software

Several tasks have been achieved in the FRESHKISS3D software (§ 5.1):

- First tests with a uniform pressure before moving to the variable case;
- Rethinking of the C++ code with an object-oriented rewriting which provides a better memory management;
- Automatic boundary conditions handling in the case of a fluid/solid transition;
- New computations of the particule trajectories when they leave out the domain by means of directional interpolation procedures;
- Achievement of the 2nd-order space accuracy;
- Taking into account the wind.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

The team is engaged in two industrial contracts:

- **La compagnie du vent (subsidiary of GDF-Suez)** The team is intended to provide simulations of hydrodynamics in salt marshes This contract is shared with the BIOCORE Inria project-team and comes to 20.000 euros.
- **SAUR** Discussions have been engaged in 2013 and might lead to a research contract in 2014. This project would rely on the optimization of hydrodynamics in a lagoon in order to depollute it.

7.2. Bilateral Grants with Industry

The PhD thesis of P. Ung is financed by CNRS, by AMIES (French agency for mathematics in interaction with companies and the society) and by GeoHyd (now a part of ANTEA-group) whose mission is the management of integrated natural resources. The PhD comprises simulations of concrete cases by means of the EDF software Telemac.

8. Partnerships and Cooperations

8.1. Regional Initiatives

8.1.1. *Instabilities in Hydrodynamics (2011–2015)*

Participant: Nicolas Seguin.

The Emergence project (Ville de Paris and FSMP) “Instabilities in Hydrodynamics” is related to theoretical, applied, and numerical mathematics for the study of hydrodynamical turbulence phenomena. The postdoc of Aude Bernard-Champmartin is held within this project.

8.1.2. *Plasticity of geophysical flows and seismic emissions (2013–2016)*

Participant: Anne Mangeney.

This project is funded by Sorbonne Paris Cité (80.000 euros) and is a collaboration between IPGP and Univ. Paris 13.

8.2. National Initiatives

8.2.1. *GdR EGRIN (2013–2017)*

Participant: Jacques Sainte-Marie.

EGRIN stands for Gravity-driven flows and natural hazards. J. Sainte-Marie is the head of the scientific committee of this CNRS research group. As such, J. Sainte-Marie participated to the consortium of the prospective think tank “Mathematics and the complexity of the system Earth” launched by the French agency for research in the framework of the UNESCO year “Mathematics of Planet Earth”.

8.2.2. *Inria Project Lab “Algae in Silico”*

Participants: Anne-Céline Boulanger, Marie-Odile Bristeau, Raouf Hamouda, Jacques Sainte-Marie.

The team is involved in the GreenStars project (“Investissement d’avenir”) which is a collaboration between academic institutions (INRA, Inria, Univ. Pierre et Marie Curie Paris 6, ...) and the industrial world.² The main purpose of GreenStars is to lay the foundations for the entire sector, from energy generation to waste recycling and production of compounds of interest. GreenStars also plans to play a long-term role in this field by training technicians, engineers and researchers. In order to structure and support the contributions of Inria in this domain, an Inria Project Lab called “Algae in Silico” has been funded.

The PhD thesis of A.-C. Boulanger was a part of this project. Likewise, the ADT In@lgae was launched in this framework in collaboration with the BIOCORE Inria project-team and enabled the recruitment of R. Hamouda as a young engineer.

8.2.3. *ANR LANDQUAKE (2012–2016)*

Participant: Anne Mangeney.

Within the ANR domain “Mathematics and Interfaces”, this ANR project (between Univ. Paris-Est – LAMA, Univ. Denis Diderot Paris 7 – IPGP, Univ. Nantes – LPGN, Univ. Strasbourg EOST, 180.000 euros) deals with the mathematical and numerical modelling of landslides and generated seismic waves.

8.2.4. *LRC Manon (2010–2014)*

Participants: Edwige Godlewski, Yohan Penel, Nicolas Seguin.

CEA and Laboratory Jacques-Louis Lions launched a collaboration 4 years ago. Studies are carried out about compressible two-phase flows and model coupling, for instance in the case of an asymptotic hierarchy of models.

8.2.5. *Structure Health Monitoring*

Participant: Nicolas Seguin.

This collaboration with the Ifsttar also comprises Inria researchers from the I4S team. The goal is to provide efficient numerical tools to take into account the impact of the flows around the structures. The most challenging part of this project concerns the off-shore wind turbines and the understanding of the ice formation on the structure.

8.2.6. *ANR project HJnet (2013–2015)*

Participant: Edwige Godlewski.

²among which are: Air Liquide, ACRI, Alfa Laval, Algaestream, Algenics, Algu’Innov, Bioalgotral, EADS, Eco-Solution, Envolure, Fermentalg, Greensea, IDEE Aquaculture, La Compagnie du Vent-GDF Suez, Microphyt, Naskeo Environnement, Ondalys, Peugeot Citroën Automobiles, Rhodia, Roquette, Sofiprotéol, Soliance, Solvay, Suez Environnement, TIA, TOTAL, Véolia Environnement.

This research project consists in studying Hamilton-Jacobi equations on networks, and more generally on heterogeneous structures. This theoretical problem has several potential applications, in particular to traffic flow theory.

8.2.7. *Hydraulics for environment and sustainable development (HED²)*

The scientific group (GIS in French), to which Inria is a partner, brings together scientists and engineers involved in hydraulics, risk management and sustainable development. ANGE belongs to this group. On the one hand, the team can be provided with experimental measurements (erosion, long waves, fluid structure interactions,...) thanks to this collaboration; on the other hand, the GIS can favor the transfer of numerical tools and scientific results.

8.3. European Initiatives

8.3.1. *ERC Consolidator Grant (2013–2018)*

Participant: Anne Mangeney.

The project SLIDEQUAKES about detection and understanding of landslides by observing and modelling gravitational flows and generated earthquakes has been funded by the European Research Council (2.000.000 euros).

8.4. International Initiatives

8.4.1. *Informal International Partners*

The team has developed strong relations with researchers from Spanish universities, in particular with Carlos Pares (Malaga), Enrique Fernandez-Nieto and Tomas Chacon Rebollo (Sevilla). They have an expertise in complex flows, including variable density flows, erosion, non-hydrostatic effects, ...

8.5. International Research Visitors

Enrique Fernandez-Nieto and Gladys Narbona-Reina (Univ. Sevilla) were hosted for 1 month by A. Mangeney's team at IPGP.

9. Dissemination

9.1. Scientific Animation

11/01/13: Y. Penel was involved in the organisation of the second Forum for Jobs in Mathematics held at CNAM Paris.

27-31/05/13: Y. Penel was a member of the organising committee of the biannual congress of the French society for applied and industrial mathematics which gathered more than 400 mathematicians.

09/09/13: A. Mangeney and J. Sainte-Marie organised a workshop Mathematics & Geophysics in the framework of the Tarantola action proposed by Univ. Pierre et Marie Curie Paris 6 (Laboratoire Jacques-Louis Lions) and Univ. Denis Diderot Paris 7 (Institut de Physique du Globe).

E. Godlewski is also a member of the board of AMIES (see § 7.2).

A. Mangeney is a member of the scientific committees of the Institut de Physique du Globe de Paris (Univ. Paris 7), of Observatory of Côte d'Azur, of the Bureau de Recherches Géologiques et Minières and of the CNRS Institut National des Sciences de l'Univers "Natural Hazards". She is also a member of the management committee of the IPGP center of parallel computing and data processing.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master's degree E. Godlewski and J. Sainte-Marie, (*Not so*) *Hyperbolic models for complex flows. Application to sustainable energies*, nombre d'heures en équivalent TD, 20 hours (lectures), M2, Univ. Pierre et Marie Curie Paris 6

Engineering school E. Audusse, *Finite Elements*, 30 hours (practical works), 2nd year (~ M1), SupGalilee

Engineering school E. Audusse, *Optimisation*, 27 hours (exercise sessions), 2nd year (~ M1), SupGalilee

Engineering school E. Audusse, *Introduction to scientific computing*, 36 hours (lectures), (~ L2), SupGalilee

Engineering school Y. Penel, *Partial Differential Equations*, 15 hours (exercise sessions), 1st year (~ L3), Ecole Centrale Paris

Engineering school Y. Penel, *Numerical Analysis*, 10 hours (lectures) + 15 hours (exercise sessions), 2nd year (~ M1), EFREI

We also mention that E. Godlewski is the head of the “Mathematics for Industry” M.Sc. program of Univ. Pierre et Marie Curie Paris 6. E. Audusse is the deputy director of the “Applied Mathematics and Scientific Computing” program of the SupGalilee engineering school.

9.2.2. Supervision

HdR Nicole Goutal, *Modélisation et simulation des écoulements à surface libre pour les eaux continentales*, Univ. Paris Est Marne-la-Vallée, 06/12/13

PhD Anne-Céline Boulanger, *Modélisation, simulation et assimilation de données autour d'un problème de couplage hydrodynamique-biologie*, Univ. Pierre et Marie Curie Paris 6, supervised by J. Sainte-Marie (in collaboration with B. Perthame), 13/09/13

PhD Saïda Sari, *Modélisation mathématique et numérique de transport de sédiment dans les écoulements d'eau en zone côtière*, Univ. Paris 13, supervised by E. Audusse (in collaboration with F. Benkhaldoun), 08/07/13

M2 internship Dena Kazerani, *Ondes de gravité et effets non-hydrostatiques*, Univ. Pierre et Marie Curie Paris 6, supervised by J. Sainte-Marie, Spring 2013

M2 internship Clément Mifsud, *Méthodes variationnelles et hyperboliques appliquées aux systèmes mécaniques sous contrainte*, Univ. Pierre et Marie Curie Paris 6, supervised by N. Seguin (in collaboration with J.-F. Babadjian and B. Després), Spring 2013

PostDoc in progress Aude Bernard-Champmartin, *Stabilité locale et montée en ordre pour la reconstruction de quantités volumes finis sur maillages coniques non structurés*, Univ. Pierre et Marie Curie Paris 6 (CEA grant), supervised by N. Seguin (in collaboration with P. Hoch), from Nov. 2013

PostDoc in progress Minh Lê, Univ. Paris 13 (IRSN grant), supervised by E. Audusse (in collaboration with F. Benkhaldoun and M. Bourgeois), from June 2013

PostDoc in progress Clara Levy, *Spatio-temporal analysis of gravitational activity using generated seismic signals*, Institut de Physique du Globe (Univ. Paris 7), supervised by A. Mangeney, from 2012

PhD in progress Nora Aïssiouene, *Derivation and analysis of a non-hydrostatic Shallow water type model*, Univ. Pierre et Marie Curie Paris 6 (Inria grant), supervised by E. Godlewski and J. Sainte-Marie, from Nov. 2013

PhD in progress Tim Borikov, *Physical processes at play in Martian landslides*, Institut de Physique du Globe (Univ. Paris 7), supervised by A. Mangeney (in collaboration with D. Mège), from 2012

PhD in progress Maxime Farin, *Analysis of the seismic signal generated by laboratory granular flows*, Institut de Physique du Globe (Univ. Paris 7), supervised by A. Mangeney (in collaboration with R. Toussaint and J. de Rosny), from 2011

PhD in progress Khalil Haddaoui, *Couplage interfacial de systèmes hyperboliques. Application aux écoulements diphasiques*, Univ. Pierre et Marie Curie Paris 6 (ONERA grant), supervised by E. Godlewski (in collaboration with F. Coquel and F. Renac), from Oct. 2012

PhD in progress Dena Kazerani, *Simulation et modélisation de problèmes à frontière libre*, Univ. Pierre et Marie Curie Paris 6, supervised by N. Seguin (in collaboration with P. Frey and C. Audiard), from Oct. 2013

PhD in progress Jannes Kinscher, *Analysis of seismicity in quarries*, Institut de Physique du Globe (Univ. Paris 7), supervised by A. Mangeney (in collaboration with P. Bernard and I. Contrucci), from 2011

PhD in progress Clément Mifsud, *Analyse et approximation des systèmes de Friedrichs : application à la modélisation de l'élastoplasticité*, Univ. Pierre et Marie Curie Paris 6, supervised by N. Seguin (in collaboration with J.-F. Babadjian and B. Després), from Oct. 2013

PhD in progress Laurent Moretti, *Modelling of seismic waves generated by landslides*, Institut de Physique du Globe (Univ. Paris 7), supervised by A. Mangeney (in collaboration with Y. Capdeville), from 2010

PhD in progress Amandine Sergeant-Boy, *Detection and characterization of seismic sources generated by glaciers: numerical modelling and analysis of seismic waves*, Institut de Physique du Globe (Univ. Paris 7), supervised by A. Mangeney (in collaboration with J.-P. Montagner, E. Stutzmann and O. Castelnaud), from 2013

PhD in progress Philippe Ung, *Simulation, modélisation et analyse numérique pour le transport sédimentaire*, Univ. Orléans (EDF-CNRS grant), supervised by E. Audusse (in collaboration with S. Cordier), from Nov. 2012

9.2.3. Juries (PhD)

12/03/13 N. Seguin (referee) : Céline Sarazin-Desbois (Univ. Nantes, *Méthodes numériques pour des systèmes hyperboliques avec terme source provenant de physiques complexes autour du rayonnement*)

04/06/13 N Seguin (referee) : Marie Martin (Univ. Nice, *Modélisations fluides pour les plasmas de fusion : approximation par éléments finis C^1 de Bell*)

03/07/13 N. Seguin (co-supervisor) : Magali Tournus (UPMC Paris 6, *Modèles d'échanges ioniques dans le rein. Théorie, Analyse asymptotique et Applications numériques*)

08/07/13 E. Audusse (co-supervisor) : Saïda Sari (Univ. Paris 13, *Modélisation mathématique et numérique de transport de sédiment dans les écoulements d'eau en zone côtière*)

13/09/13 E. Godlewski, J. Sainte-Marie (supervisor), N. Seguin : Anne-Céline Boulanger (UPMC Paris 6, *Modélisation, simulation et assimilation de données autour d'un problème de couplage hydrodynamique-biologie*)

28/10/13 E. Audusse, E. Godlewski (referee) : Manel Tayachi (UJF Grenoble, *Couplage de modèles de dimensions hétérogènes et application en hydrodynamique*)

26/11/13 E. Godlewski : Sophie Gérald (UPMC Paris 6, *Méthode de Galerkin Discontinue et intégrations explicites-implicites en temps basées sur un découplage des degrés de liberté. Applications au système des équations de Navier-Stokes*)

13/12/13 E. Godlewski (president) : François McKee (Univ. Nantes, *Etude et mise à l'échelle des écoulements diphasiques en milieux poreux hétérogènes par une approche d'optimisation*)

19/12/13 J. Sainte-Marie (referee) : Christelle Lusso (univ. Paris-Est, *Modélisation numérique des écoulements gravitaires viscoplastiques avec transition fluide/solide*)

22/01/14 J. Sainte-Marie (referee) : David Benoit (Univ. Paris-Est, *Divers problèmes théoriques et numériques liés à la simulation de fluides non newtoniens*)

9.3. Popularization

08/01/13 J. Sainte-Marie together with O. Bernard (Inria BIOCORE) and B. Sialve (Nasceo) published a short article named “Un zeste de mathématiques pour les biocarburants de demain” on the french Mathematics for Planet Earth website.

30/05-02/06/13 N. Seguin participated to the “Salon de la culture et des jeux mathématiques” at the UPMC stand. This forum aims at explaining the job of researcher in Mathematics to pupils and high school students.

10-11/10/13 E. Audusse gave a talk for high school students at “Savante Banlieue”.

10-12/10/13 E. Audusse, E. Godlewski, R. Hamouda, Y. Penel and J. Sainte-Marie run the Inria stand at the 2013 edition of the “Fête de la Science”. They presented animations of past tsunamis to a general public.

2013 E. Audusse intervened in a secondary school (Pontault Combault) for “Maths en Jeans”.

2013–2014 E. Audusse, R. Hamouda and J. Sainte-Marie implemented a software (“Tsunamath”) for simulating tsunamis in the context of a worldwide exhibition for Mathematics of Planet Earth. The first stage will be held in Berlin in March 2014.

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Articles in International Peer-Reviewed Journals

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