Activity Report 2013

Project-Team APICS

Analysis and Problems of Inverse type in Control and Signal processing

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Optimization and control of dynamic systems

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Project-Team APICS

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Creation of the Project-Team: 2005 January 01.

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2. Overall Objectives

2.1. Research Themes

The team develops constructive, function-theoretic approaches to inverse problems arising in modeling and design, in particular for electro-magnetic systems as well as in the analysis of certain classes of signals.

Data typically consist of measurements or desired behaviors. The general thread is to approximate them by families of solutions to the equations governing the underlying system. This leads us to consider various interpolation and approximation problems in classes of rational and meromorphic functions, harmonic gradients, or solutions to more general elliptic partial differential equations (PDE), in connection with inverse potential problems. A recurring difficulty is to control the singularities of the approximants.

The mathematical tools pertain to complex and harmonic analysis, approximation theory, potential theory, system theory, differential topology, optimization and computer algebra. Targeted applications include:

- identification and synthesis of analog microwave devices (filters, amplifiers),
- non-destructive control from field measurements in medical engineering (source recovery in magneto/electro-encephalography), paleomagnetism (determining the magnetization of rock samples), and nuclear engineering (plasma shaping in tokamaks).

In each case, the endeavor is to develop algorithms resulting in dedicated software.

3. Research Program

3.1. Introduction

Within the extensive field of inverse problems, much of the research by APICS deals with reconstructing solutions of classical elliptic PDEs from their boundary behavior. Perhaps the most basic example of such a problem is harmonic identification of a stable linear dynamical system: the transfer-function f is holomorphic in the right half-pane, which means it satisfies there the Cauchy-Riemann equation $\overline{\partial} f = 0$, and in principle f can be recovered from its values on the imaginary axis, *e.g.* by Cauchy formula.

Practice is not nearly as simple, for f is only measured pointwise in the pass-band of the system which makes the problem ill-posed [69]. Moreover, the transfer function is usually sought in specific form, displaying the necessary physical parameters for control and design. For instance if f is rational of degree n, it satisfies $\overline{\partial} f = \sum_1^n a_j \delta_{z_j}$ where the z_j are its poles, and finding the domain of holomorphy (*i.e.* locating the z_j) amounts to solve a (degenerate) free-boundary inverse problem, this time on the left half-plane. To address these questions, the team has developed a two-step approach as follows.

Step 1: To determine a complete model, that is, one which is defined for every frequency, in a sufficiently flexible function class (*e.g.* Hardy spaces). This ill-posed issue requires regularization, for instance constraints on the behavior at non-measured frequencies.

Step 2: To compute a reduced order model. This typically consists of rational approximation of the complete model obtained in step 1, or phase-shift thereof to account for delays. Derivation of the complete model is important to achieve stability of the reduced one.

Step 1 makes connection with extremal problems and analytic operator theory, see Section 3.3.1. Step 2 involves optimization, and some Schur analysis to parametrize transfer matrices of given Mc-Millan degree when dealing with systems having several inputs and outputs, see Section 3.3.2.2. It also makes contact with the topology of rational functions, to count critical points and to derive bounds, see Section 3.3.2. Moreover, this step raises issues in approximation theory regarding the rate of convergence and whether the singularities of the approximant (*i.e.* its poles) converge to the singularities of the approximated function; this is where logarithmic potential theory becomes effective, see Section 3.3.3.

Iterating the previous steps coupled with a sensitivity analysis yields a tuning procedure which was first demonstrated in [75] on resonant microwave filters.

Similar steps can be taken to approach design problems in the frequency domain, replacing measured behavior by desired behavior. However, describing achievable responses from the design parameters at hand is generally cumbersome, and most constructive techniques rely on rather specific criteria adapted to the physics of the problem. This is especially true of circuits and filters, whose design classically appeals to standard polynomial extremal problems and realization procedures from system theory [70], [59]. APICS is active in this field, where we introduced the use of Zolotarev-like problems for microwave multi-band filter design. We currently favor interpolation techniques because of their transparency with respect to parameter use, see Section 3.2.2.

In another connection, the example of harmonic identification quickly suggests a generalization of itself. Indeed, on identifying $\mathbb{C}$ with $\mathbb{R}^2$, holomorphic functions become conjugate-gradients of harmonic functions so that harmonic identification is, after all, a special case of a classical issue: to recover a harmonic function on a domain from partial knowledge of the Dirichlet-Neumann data; portion of the boundary where data are not available may be unknown, in which case we meet a free boundary problem. This framework for 2-D non-destructive control was first advocated in [62] and subsequently received considerable attention. It makes it clear how to state similar problems in higher dimensions and for more general operators than the Laplacian, provided solutions are essentially determined by the trace of their gradient on part of the boundary which is the case for elliptic equations [1] [78]. All these questions are particular instances of the so-called inverse

[1] There is a subtle difference here between dimension 2 and higher. Indeed, a function holomorphic on a plane domain is defined by its non-tangential limit on a boundary subset of positive linear measure, but there are non-constant harmonic functions in the 3-D ball, C^1 up to the boundary sphere, yet having vanishing gradient on a subset of positive measure of the sphere.

potential problem, where a measure μ has to be recovered from knowledge of the gradient of its potential (*i.e.*, the field) on part of a hypersurface (a curve in 2-D) encompassing the support of μ. For Laplace's operator, potentials are logarithmic in 2-D and Newtonian in higher dimensions. For elliptic operators with non constant coefficients, the potential depends on the form of fundamental solutions and is less manageable because it is no longer of convolution type. In any case, by construction, the operator applied to the potential yields back the measure.

Inverse potential problems are severely indeterminate because infinitely many measures within an open set produce the same field outside this set [68]. In step 1 above we implicitly removed this indeterminacy by requiring that the measure be supported on the boundary (because we seek a function holomorphic throughout the right half space), and in step 2 by requiring, say, in case of rational approximation that the measure be discrete in the left half-plane. The same discreteness assumption prevails in 3-D inverse source problems. To recap, the gist of our approach is to approximate boundary data by (boundary traces of) fields arising from potentials of measures with specific support. Note this is different from standard approaches to inverse problems, where descent algorithms are applied to integration schemes of the direct problem; in such methods, it is the equation which gets approximated (in fact: discretized).

Along these lines, the team initiated the use of steps 1 and 2 above, along with singularity analysis, to approach issues of nondestructive control in 2 and 3-D [44] [5], [2]. We are currently engaged in two kinds of generalization, further described in Section 3.2.1. The first one deals with non-constant conductivities, where Cauchy-Riemann equations for holomorphic functions are replaced by conjugate Beltrami equations for pseudo-holomorphic functions; there we seek applications to inverse free boundary problems such as plasma confinement in the vessel of a tokamak. The other one lies with inverse source problems for Laplace's equation in 3-D, where holomorphic functions are replaced by harmonic gradients, developing applications to EEG/MEG and inverse magnetization problems in paleomagnetism, see Section 4.2.

The main approximation-theoretic tools developed by APICS to get to grips with issues mentioned so far are outlined in Section 3.3. In Section 3.2 to come, we make more precise which problems are considered and for which applications.

3.2. Range of inverse problems

3.2.1. Elliptic partial differential equations (PDE)

Participants: Laurent Baratchart, Slah Chaabi, Sylvain Chevillard, Juliette Leblond, Dmitry Ponomarev, Elodie Pozzi.

This work has benefited from collaboration with Alexander Borichev (Aix-Marseille University).

Reconstructing Dirichlet-Neumann boundary conditions for a function harmonic in a plane domain when these are known on a strict subset E of the boundary, is equivalent to recover a holomorphic function in the domain from its boundary values on E. This is the problem raised on the half-plane in step 1 of Section 3.1. It makes good sense in holomorphic Hardy spaces where functions are determined by their values on boundary subsets of positive linear measure, which is the framework for Problem (P) in Section 3.3.1. Such problems naturally arise in nondestructive testing of 2-D (or cylindrical) materials from partial electrical measurements on the boundary. Indeed, the ratio between tangential and normal currents (so-called Robin coefficient) tells about corrosion of the material. Solving Problem (P) where ψ is chosen to be the response of some uncorroded piece with identical shape allows one to approach such questions, and this was an initial application of holomorphic extremal problems to non-destructive control [56], [60].

A recent application by the team deals with non-constant conductivity over a doubly connected domain, E being the outer boundary. Measuring Dirichlet-Neumann data on E, we want to quantify how constant the solution can be on the inner boundary. To this effect We define and study Hardy spaces of a conjugate Beltrami equation, of which the conductivity equation is the compatibility condition (just like Laplace's equation is the compatibility condition of the Cauchy-Riemann system). This is done in references [4] and [13]. Then, solving an obvious analog of Problem (P) allows one to numerically check what we want. Further, the value of this extremal problem defines a criterion on inner boundaries, and subsequently a descent algorithm was set up to improve the initial boundary into one where the solution is closer to being constant. This is a way to approach a free boundary problem.

When the domain is regarded as separating the edge of a tokamak's vessel from the plasma (rotational symmetry makes this a 2-D problem), the procedure just described suits plasma control from magnetic confinement. It was successfully applied in collaboration with CEA (the French nuclear agency) and the University of Nice (JAD Lab.) to data from *Tore Supra* [61]. This procedure is fast because no numerical integration of the underlying PDE is needed, as an explicit basis of solutions to the conjugate Beltrami equation in terms of Bessel functions was found in this case. Generalizing this approach in a more systematic manner into descent algorithms for boundary-value criteria using the gradient of a shape is an interesting perspective.

Three-dimensional versions of step 1 in Section 3.1 are also considered, namely to recover a harmonic function (up to a constant) in a ball or a half-space from partial knowledge of its gradient on the boundary. Such questions arise naturally in connection with neurosciences and medical imaging (electroencephalography, EEG) or in paleomagnetism (analysis of rocks magnetization) [2] [14], [18], see Section 6.1. They are not yet as developed as the 2-D case where the power of complex analysis is at work, but considerable progress was made over the last years through methods of harmonic analysis and operator theory.

The team is also concerned with non-destructive control problems of localizing defaults such as cracks, sources or occlusions in a planar or 3-dimensional domain, from boundary data (which may correspond to thermal, electrical, or magnetic measurements). These defaults can be expressed as a lack of analyticity of the solution of the associated Dirichlet-Neumann problem and we approach them using techniques of best rational or meromorphic approximation on the boundary of the object [3], [8], see Sections 3.3.2 and 4.2. In fact, the way singularities of the approximant relate to the singularities of the approximated function is an all-pervasive theme in approximation theory, and for appropriate classes of functions like those expressed as Cauchy integrals over certain extremal contours for the logarithmic potential, the location of the poles of a best rational approximant can be used as an estimator of the singularities of the approximated function (see Section 6.1). This circle of ideas is driving step 2 in Section 3.1.

A genuine 3-dimensional theory of approximation by discrete potentials, though, is still in its infancy.

3.2.2. Systems, transfer and scattering

Participants: Laurent Baratchart, Sylvain Chevillard, Sanda Lefteriu, Martine Olivi, Fabien Seyfert.

Through initial contacts with CNES, the French space agency, the team came to work on identification-for-tuning of microwave electromagnetic filters used in space telecommunications (see Section 4.5). The problem was to recover, from band-limited frequency measurements, the physical parameters of the device under examination. The latter consists of interconnected dual-mode resonant cavities with negligible loss, hence its scattering matrix is modeled by a 2×2 unitary-valued matrix function on the frequency line, say the imaginary axis to fix ideas. In the bandwidth around the resonant frequency, a modal approximation of the Helmholtz equation in the cavities shows that this matrix is approximately rational, of Mc-Millan degree twice the number of cavities.

This is where system theory enters the scene, through the so-called *realization* process mapping a rational transfer function in the frequency domain to a state-space representation of the underlying system of linear differential equations in the time domain. Specifically, realizing the scattering matrix allows one to construct a virtual electrical network, equivalent to the filter, the parameters of which mediate in between the frequency response and the geometric characteristics of the cavities (*i.e.* the tuning parameters).

Hardy spaces, in particular the Hilbert space H^2, provide a framework to transform this classical ill-posed issue into a series of regularized analytic and meromorphic approximation problems. The procedure sketched in Section 3.1 now goes as follows:

1. infer from the pointwise boundary data in the bandwidth a stable transfer function (*i.e.* one which is holomorphic in the right half-plane), that may be infinite dimensional (numerically: of high degree). This is done by solving in the Hardy space H^2 of the right half-plane a problem analogous to (P)

in Section 3.3.1, taking into account prior assumptions or knowledge on the decay of the response outside the bandwidth, see [19] for details.

2. From this stable model, a rational stable approximation of appropriate degree is computed. For this a descent method is used on the relatively compact manifold of inner matrices of given size and degree, using an original parametrization of stable transfer functions developed by the team [19].
3. From this rational model, realizations meeting certain constraints imposed by the technology in use are computed. These constraints typically come from the nature and coupling topology of the equivalent electrical network used to model the filter. This network is composed of resonators, coupled to each other by some specific coupling graph. Performing this realization step for given coupling topology can be recast, under appropriate compatibility conditions [7], as the problem of solving a zero-dimensional multivariate polynomial system. To tackle this problem in practice, we use Groebner basis techniques as well as continuation methods as implemented in the Dedale-HF software (see Section 5.4).

Let us also mention that extensions of classical coupling matrix theory to frequency-dependent (reactive) couplings have lately been carried-out [1] for wide-band design applications, although further study is needed to make them computationally effective.

Subsequently APICS started investigating issues pertaining to filter design rather than identification. Given the topology of the filter, a basic problem is to find the optimal response with respect to amplitude specifications in frequency domain bearing on rejection, transmission and group delay of scattering parameters. Generalizing the approach based on Chebyshev polynomials for single band filters, we recast the problem of multi-band response synthesis in terms of a generalization of classical Zolotarev min-max problem [34] for rational functions [10]. Thanks to quasi-convexity, the latter can be solved efficiently using iterative methods relying on linear programming. These are implemented in the software easy-FF (see Section 5.5).

Investigations by the team have extended to design and identification of more complex microwave devices, like multiplexers and routers, which connect several filters through wave guides. Schur analysis plays an important role here, which is no surprise since scattering matrices of passive systems are of Schur type (*i.e.* contractive in the stability region). The theory originates with the work of I. Schur [74], who devised a recursive test to check for contractivity of a holomorphic function in the disk. Generalizations thereof turned out to be very efficient to parametrize solutions to contractive interpolation problems subject to a well-known compatibility condition (positive definiteness of the so-called Pick matrix) [36]. Schur analysis became quite popular in electrical engineering, as the Schur recursion precisely describes how to chain two-port circuits.

Dwelling on this, members of the team contributed to differential parametrizations (atlases of charts) of lossless matrix functions [35][11], [9]. These are fundamental to our rational approximation software RARL2 (see Section 5.1). Schur analysis is also instrumental to approach de-embedding issues considered in Section 6.3, and provides further background to synthesis and matching problems for multiplexers. At the heart of the latter lies a variant of contractive interpolation with degree constraint introduced in [65].

We also mention the role played by multi-point Schur analysis in the team's investigation of spectral representation for certain non-stationary discrete stochastic processes [41], [39].

More recently, in collaboration with UPV (Bilbao), our attention was driven by CNES, to questions of stability relative to high-frequency amplifiers, see Section 7.2. Contrary to previously mentioned devices, these are *active* components. The amplifier can be linearized at a functioning point and admittances of the corresponding electrical network can be computed at various frequencies, using the so-called harmonic balance method. The goal is to check for stability of this linearized model. The latter is composed of lumped electrical elements namely inductors, capacitors, negative *and* positive reactors, transmission lines, and commanded current sources. Research so far focused on determining the algebraic structure of admittance functions, and setting up a function-theoretic framework to analyze them. In particular, much effort was put on realistic assumptions under which a stable/unstable decomposition can be claimed in $H^2 \oplus \overline{H^2}$ (see Section 6.4). Then, the unstable part of the elements under examination is rational and one can provide the designer with valuable estimates of stability using the general scheme sketched in Section 3.1.

3.3. Approximation of boundary data

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Martine Olivi, Dmitry Ponomarev, Elodie Pozzi, Fabien Seyfert.

The following people are collaborating with us on these topics: Bernard Hanzon (Univ. Cork, Ireland), Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris), Jonathan Partington (Univ. Leeds, UK), Ralf Peeters (Univ. Maastricht, NL), Edward Saff (Vanderbilt University, Nashville, USA), Herbert Stahl (TFH Berlin), Maxim Yattselev (Purdue Univ. at Indianapolis, USA).

3.3.1. Best constrained analytic approximation

In dimension 2, the prototypical problem to be solved in step 1 of Section 3.1 may be described as: given a domain $D \subset \mathbb{R}^2$, we want to recover a holomorphic function from its values on a subset of the boundary of D. Using conformal mapping, it is convenient for the discussion to normalize D. So, in the simply connected case, we fix D to be the unit disk with boundary the unit circle T. We denote by H^p the Hardy space of exponent p which is the closure of polynomials in the L^p-norm on the circle if $1 \leq p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which makes it possible to speak of (traces of) analytic functions on the boundary.

To find an analytic function in D approximately matching measured values f on a sub-arc K of T, we formulate a constrained best approximation problem as follows.

> (P) Let $1 \leq p \leq \infty$, K a sub-arc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behavior capturing *a priori* assumptions on the behavior of the model off K, while M is some admissible deviation from them. The value of p reflects the type of stability which is sought and how much one wants to smoothen the data. The choice of L^p classes is well-adapted to handling point-wise measurements.

To fix terminology we refer to (P) as a *bounded extremal problem*. As shown in [43], [45], [51], for $1 < p \leq \infty$, the solution to this convex infinite-dimensional optimization problem can be obtained upon iterating with respect to a Lagrange parameter the solution to spectral equations for some appropriate Hankel and Toeplitz operators. These equations in turn involve the solution to the standard extremal problem below [64]:

> (P_0) Let $1 \leq p \leq \infty$ and $\varphi \in L^p(T)$; find a function $g \in H^p$ such that $g - \varphi$ is of minimal norm in $L^p(T)$.

The case $p = 1$ of (P_0) is essentially open.

Various modifications of (P) have been studied in order to meet specific needs. For instance when dealing with loss-less transfer functions (see Section 4.5), one may want to express the constraint on $T \setminus K$ in a point-wise manner: $|g - \psi| \leq M$ a.e. on $T \setminus K$, see [47]. In this form, it comes close to (but still is different from) H^∞ frequency optimization methods for control [66], [73].

The analog of Problem (P) on an annulus, K being now the outer boundary, can be seen as a means to regularize a classical inverse problem occurring in nondestructive control, namely recovering a harmonic function on the inner boundary from Dirichlet-Neumann data on the outer boundary (see Sections 3.2.1, 4.2, 6.1.1, 6.2). It may serve as a tool to approach Bernoulli type problems where we are given data on the outer boundary and we *seek the inner boundary*, knowing it is a level curve of the flux. Then, the Lagrange parameter indicates which deformation should be applied on the inner contour in order to improve data fitting.

This is discussed in Sections 3.2.1 and 6.2 for more general equations than the Laplacian, namely isotropic conductivity equations of the form $\operatorname{div}(\sigma \nabla u) = 0$ where σ is non constant. In this case, the Hardy spaces in Problem (P) are those of a so-called conjugate or real Beltrami equation $\overline{\partial} f = \nu \overline{\partial f}$ [67], which were studied for $1 < p < \infty$ in [13], [4]. Expansions of solutions needed to constructively handle such issues have been carried out in [61].

Though originally considered in dimension 2, Problem (P) carries over naturally to higher dimensions where analytic functions get replaced by gradients of harmonic functions. Namely, given some open set $\Omega \subset \mathbb{R}^n$ and a $\mathbb{R}^n$-valued vector V field on an open subset O of the boundary of Ω, we seek a harmonic function in Ω whose gradient is close to V on O.

When Ω is a ball or a half-space, a convenient substitute of holomorphic Hardy spaces is provided by Stein-Weiss Hardy spaces of harmonic gradients [77]. Conformal maps are no longer available in $\mathbb{R}^n$ for $n > 2$ and other geometries have not been much studied so far. On the ball, the analog of Problem (P) is

> (P_1) Let $1 \leq p \leq \infty$ and $B \subset \mathbb{R}^n$ the unit ball. Fix O an open subset of the unit sphere $S \subset \mathbb{R}^n$. Let further $V \in L^p(O)$ and $W \in L^p(S \setminus O)$ be $\mathbb{R}^n$-valued vector fields, and $M > 0$; find a harmonic gradient $G \in H^p(B)$ such that $\|G - W\|_{L^p(S \setminus O)} \leq M$ and $G - V$ is of minimal norm in $L^p(O)$ under this constraint.

When $p = 2$, spherical harmonics offer a reasonable substitute to Fourier expansions and Problem (P_1) was solved in [2], together with its natural analog on a shell. The solution generalizes the Toeplitz operator approach to bounded extremal problems [43], and constructive aspects of the procedure (harmonic 3-D projection, Kelvin and Riesz transformation, spherical harmonics) were derived. An important ingredient is a refinement of the Hodge decomposition allowing us to express a $\mathbb{R}^n$-valued vector field in $L^p(S)$, $1 < p < \infty$, as the sum of a vector field in $H(B)$, a vector field in $H^p(\mathbb{R}^n \setminus \overline{B})$, and a tangential divergence free vector field. If $p = 1$ or $p = \infty$, L^p must be replaced respectively by the real Hardy space H^1 and the bounded mean oscillation space BMO, and H^∞ should be modified accordingly. This decomposition was fully discussed in [14] (for the case of the half-space) where it plays a fundamental role.

Problem (P_1) is under investigation in the case $p = \infty$, where even the case where $O = S$ is pending because a substitute of the Adamjan-Arov-Krein theory [71] is still to be built in dimension greater than 2.

Such problems arise in connection with source recovery in electro/magneto encephalography and paleomagnetism, as discussed in Sections 3.2.1 and 4.2.

3.3.2. Best meromorphic and rational approximation

The techniques explained in this section are used to solve step 2 in Section 3.2 *via* conformal mapping and subsequently are instrumental to approach inverse boundary value problems for Poisson equation $\Delta u = \mu$, where μ is some (unknown) distribution.

3.3.2.1. Scalar meromorphic and rational approximation

Let as before D designate the unit disk, and T the unit circle. We further put R_N for the set of rational functions with at most N poles in D, which allows us to define meromorphic functions in $L^p(T)$ as traces of functions in $H^p + R_N$.

A natural generalization of Problem (P_0) is:

> (P_N) Let $1 \leq p \leq \infty$, $N \geq 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Only for $p = \infty$ and continuous f it is known how to solve (P_N) in closed form. The unique solution is given by AAK theory (named after Adamjan, Arov and Krein), that connects the spectral decomposition of Hankel operators with best approximation in Hankel norm [71]. This theory allows one to express g_N in terms of the singular vectors of the Hankel operator with symbol f. The continuity of g_N as a function of f only holds for norms finer than uniform.

The case $p = 2$ is of special importance. In particular when $f \in \overline{H}^2$, the Hardy space of exponent 2 of the *complement* of D in the complex plane (by definition, $h(z)$ belongs to $\overline{H}^p$ if, and only if $h(1/z)$ belongs to H^p), then (P_N) reduces to rational approximation. Moreover, it turns out that the associated solution $g_N \in R_N$ has no pole outside D, hence it is a *stable* rational approximant to f. However, in contrast with the situation when $p = \infty$, this approximant may *not* be unique.

The former Miaou project (predecessor of APICS) has designed an adapted steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; until now it seems to be the only procedure meeting this property. Roughly speaking, it is a gradient algorithm that proceeds recursively with respect to the order N of the approximant, in a compact region of the parameter space [38]. Although it has proved effective in all applications carried out so far (see Sections 4.2, 4.5), it is not known whether the absolute minimum can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as is done by the RARL2 software, Section 5.1).

In order to establish global convergence results, APICS has undertaken a deeper study of the number and nature of critical points, in which tools from differential topology and operator theory team up with classical approximation theory. The main discovery is that the nature of the critical points (*e.g.*, local minima, saddle points...) depends on the decrease of the interpolation error to f as N increases [48]. Based on this, sufficient conditions have been developed for a local minimum to be unique. These conditions are hard to use in practice because they require strong estimates of the approximation error. These are often difficult to obtain for a given function, and are usually only valid for large N. Examples where uniqueness or asymptotic uniqueness has been proved this way include transfer functions of relaxation systems (*i.e.* Markov functions) [52] and more generally Cauchy integrals over hyperbolic geodesic arcs [54] and certain entire functions [50].

An analog to AAK theory has been carried out for $2 \leq p < \infty$ [51]. Although not computationally as powerful, it can be used to derive lower bounds [29] and to analyze the behavior of poles. When $1 \leq p < 2$, Problem (P_N) is still fairly open.

A common feature to all these problems is that critical point equations express non-Hermitian orthogonality relations for the denominator of the approximant. This makes connection with interpolation theory [55], [53] and is used in an essential manner to assess the behavior of the poles of the approximants to functions with branchpoint-type singularities, which is of particular interest for inverse source problems (*cf.* Sections 5.6 and 6.1).

In higher dimensions, the analog of Problem (P_N) is best approximation of a vector field with gradients of potentials generated by N point masses instead of meromorphic functions. This issue is by no means fully understood, and is an exciting line of research. It is connected with spectral properties of certain operators generalizing classical Toeplitz and Hankel ones, and to constructive approaches to so-called weak factorizations of div-curl type for real Hardy functions.

Certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus. Such questions have attracted significant attention of members of the team (see Section 4.5). For instance, convergence properties of multi-point Schur approximants, which are rational interpolants preserving contractivity of a function, were analyzed in [41]. Such approximants are useful in prediction theory of stochastic processes, but since they interpolate inside the domain of holomorphy they are of limited use in frequency design.

In another connection, the generalization to several arcs of classical Zolotarev problems [72] is an achievement by the team which is useful for multi-band synthesis [10]. Still, though the modulus of the response is the first concern in filter design, variation of the phase must nevertheless remain under control to avoid unacceptable distortion of the signal. This specific but important issue has less structure and was approached using constrained optimization; a dedicated code has been developed under contract with the CNES (see Section 5.5).

3.3.2.2. Matrix-valued rational approximation

Matrix-valued approximation is necessary for handling systems with several inputs and outputs, and it generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically. In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the degree.

The problem we consider is now: *let $\mathcal{F} \in (H^2)^{m\times l}$ and n an integer; find a rational matrix of size $m \times l$ without poles in the unit disk and of McMillan degree at most n which is nearest possible to $\mathcal{F}$ in $(H^2)^{m\times l}$.* Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The scalar approximation algorithm [38], mentioned in Section 3.3.2.1, generalizes to the matrix-valued situation [63]. The first difficulty here consists in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (*i.e.* matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n. The latter enter the picture in an essential manner as they play the role of the denominator in a fractional representation of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization). The set of inner matrices of given degree has the structure of a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (parametrization valid in a neighborhood of a point), and we must handle changes of charts in the course of the algorithm. Such parametrization can be obtained from interpolation theory and Schur type algorithms, the parameters being interpolation vectors or matrices ([35], [9], [11]). Some of them are particularly interesting to compute realizations and achieve filter synthesis ([9] [11]). For rational approximation software codes developed by the team, see Section 5.1.

Difficulties relative to multiple local minima naturally arise in the matrix-valued case as well, and deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The case of rational functions of sought degree or small perturbations thereof (the consistency problem) was solved in [49]. The case of matrix-valued Markov functions, the first example beyond rational functions, was treated in [37].

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.3.3. Behavior of poles of meromorphic approximants

Participant: Laurent Baratchart.

The following persons did collaborate with us on this subject: Herbert Stahl (TFH Berlin), Maxim Yattselev (Purdue Univ. at Indianapolis, USA).

We refer here to the behavior of poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. If one normalizes the contour to be the unit circle T, we are back to the framework of Section 3.3.2.1 and to Problem (P_N); invariance of the problem under conformal mapping was established in [5]. Research so far has focused on functions whose singular set inside the contour is zero or one-dimensional.

Generally speaking, the behavior of poles is particularly important in meromorphic approximation to obtain error rates as the degree goes large and to tackle constructive issues like uniqueness. As explained in Section 3.2.1, we consider this issue in connection with approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities. The general theme is thus *how do the singularities of the approximant reflect those of the approximated function?* This approach to inverse problem for the 2-D Laplacian turns out to be attractive when singularities are zero- or one-dimensional (see Section 4.2). It can be used as a computationally cheap initialization of more precise but heavier numerical optimizations.

As regards crack detection or source recovery, the approach in question boils down to analyzing the behavior of best meromorphic approximants of a function with branch points. For piecewise analytic cracks, or in the case of sources, we were able to prove ([5], [6], [40]), that the poles of the approximants accumulate on some extremal contour of minimum weighted energy linkings the singular points of the crack, or the sources [44]. Moreover, the asymptotic density of the poles turns out to be the Green equilibrium distribution of this contour in D, hence puts heavy charge around the singular points (in particular at the endpoints) which are therefore well localized if one is able to approximate in sufficiently high degree (this is where the method could fail).

The case of two-dimensional singularities is still an outstanding open problem.

It is interesting that inverse source problems inside a sphere or an ellipsoid in 3-D can be attacked with the above 2-D techniques, as applied to planar sections (see Section 6.1). This is at work in the software FindSources3D, see Section 5.6.

3.3.4. Miscellaneous

Participant: Sylvain Chevillard.

Sylvain Chevillard, joined team in November 2010. His coming resulted in APICS hosting a research activity in certified computing, centered on the software *Sollya* of which S. Chevillard is a co-author, see Section 5.7. On the one hand, Sollya is an Inria software which still requires some tuning to a growing community of users. On the other hand, approximation-theoretic methods at work in Sollya are potentially useful for certified solutions to constrained analytic problems described in Section 3.3.1. However, developing Sollya is not a long-term objective of APICS.

4. Application Domains

4.1. Introduction

These domains are naturally linked to the problems described in Sections 3.2.1 and 3.2.2. By and large, they split into a systems-and-circuits part and an inverse-source-and-boundary-problems part, united under a common umbrella of function-theoretic techniques described in Section 3.3.

4.2. Inverse source problems in EEG

Participants: Laurent Baratchart, Kateryna Bashtova, Juliette Leblond.

This work is done in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena Project-Team, and Jean-Paul Marmorat (Centre de mathématiques appliquées - CMA, École des Mines de Paris).

Solving overdetermined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to extrapolate incomplete data (see Section 3.2.1) is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG since the latter involves propagating the initial conditions through several layers of different conductivities, from the boundary down to the center of the domain where the singularities (*i.e.* the sources) lie. Then, once propagated to the innermost sphere, it turns out that that traces of the boundary data on 2-D cross sections (disks) coincide with analytic functions in the slicing plane, that has branched singularities inside the disk [3]. These singularities are related to the actual location of the sources (namely, they reach in turn a maximum in modulus when the plane contains one of the sources). Hence, we are back to the 2-D framework of Section 3.3.3 where approximately recovering these singularities can be performed using best rational approximation. The goal is to produce a fast but already good enough initial guess on the number and location of the sources in order to run heavier descent algorithms on the direct problem, which are more precise but computationally costly, and often fail to converge if not properly initialized.

Numerical experiments give very good results on simulated data and we are now engaged in the process of handling real experimental magneto-encephalographic data, see also Sections 5.6 and 6.1, in collaboration with the Athena team at Inria Sophia Antipolis, neuroscience teams in partner-hospitals (la Timone, Marseille), and the BESA company (Munich).

4.3. Inverse magnetization problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Dmitry Ponomarev.

Generally speaking, inverse potential problems similar to the one in Section 4.2 appear naturally in connection with systems governed by Maxwell's equation in the quasi-static approximation regime. In particular, they arise in magnetic reconstruction issues. A specific application is to geophysics, whose study led us to form an Inria Associate Team ("IMPINGE" for Inverse Magnetization Problems IN GEosciences) together with MIT and Vanderbilt University.

To set up the context, recall that the Earth's geomagnetic field is generated by convection of the liquid metallic core (geodynamo) and that rocks become magnetized by the ambient field as they are formed or after subsequent alteration. Their remanent magnetization provides records of past variations of the geodynamo, which is used to study important processes in Earth sciences like motion of tectonic plates and geomagnetic reversals. Rocks from Mars, the Moon, and asteroids also contain remanent magnetization which indicates the past presence of core dynamos. Magnetization in meteorites may even record fields produced by the young sun and the protoplanetary disk which may have played a key role in solar system formation.

For a long time, paleomagnetic techniques were only capable of analyzing bulk samples and compute their net magnetic moment. The development of SQUID microscopes has recently extended the spatial resolution to submillimeter scales, raising new physical and algorithmic challenges. This associate team aims at tackling them, experimenting with the SQUID microscope set up in the Paleomagnetism Laboratory of the department of Earth, Atmospheric and Planetary Sciences at MIT. Typically, pieces of rock are sanded down to a thin slab, and the magnetization has to be recovered from the field measured on a parallel plane at small distance above the slab.

Mathematically speaking, both inverse source problems for EEG from Section 4.2 and inverse magnetization problems described presently amount to recover the (3-D valued) quantity m (primary current density in case of the brain or magnetization in case of a thin slab of rock) from measurements of the vector potential:

$$\int_\Omega \frac{\operatorname{div} m(x')\, dx'}{|x-x'|}\,, \qquad (1)$$

outside the volume Ω of the object, from Maxwell's equations. The big difference is that the distribution m is located in a volume in the case of EEG, and on a plane in the case of rock magnetization. This results in quite different identifiability properties, see [14] and Section 6.1.2.

4.4. Free boundary problems

Participants: Laurent Baratchart, Juliette Leblond, Slah Chaabi.

The team has engaged in the study of problems with variable conductivity σ, governed by a 2-D equation of the form $\operatorname{div}(\sigma\nabla u) = 0$. Such equations are in one-to-one correspondence with real parts of solutions to conjugate-Beltrami equations $\overline{\partial} f = \nu\overline{\partial f}$, so that complex analysis is a tool to study them, see [4], [13], [28]. This research was prompted by issues in plasma confinement for thermonuclear fusion in a tokamak, more precisely with the extrapolation of magnetic data on the boundary of the chamber from the outer boundary of the plasma, which is a level curve for the poloidal flux solving the original div-grad equation. Solving this inverse problem of Bernoulli type is of importance to determine the appropriate boundary conditions to be applied to the chamber in order to shape the plasma [58]. This research was started in collaboration with CEA-IRFM (Cadarache) and the Laboratoire J.-A. Dieudonné at the Univ. of Nice-SA. Within the team, it is now expanding to cover Dirichlet-Neumann problems for larger classes of conductivities, *cf.* in particular, the PhD thesis of S. Chaabi [12], [28], jointly supervised with the CMI-LATP at the Aix-Marseille University. (see Section 6.2).

4.5. Identification and design of microwave devices

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

This work is done in collaboration with Stéphane Bila (XLIM, Limoges) and Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

One of the best training grounds for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption works well in the considered range of frequency, and whose specifications are made in the frequency domain. This is the case of electromagnetic resonant systems which are of common use in telecommunications.

In space telecommunications (satellite transmissions), constraints specific to on-board technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study of the Helmholtz equation states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Figure 1. Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that 16 quantities must be optimized. Quantities such as the diameter and length of the cavities, or the width of the 11 slits are fixed during the design phase.

Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all cavities show the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 GHz.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modeled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plugged on a resistor at one end and fed with some potential at the other end. We are then interested in the power which is transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the design, particularly the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.* the underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80MHz in the example).
- Solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, cf. Section 3.3.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.
- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this example) via the Endymion or RARL2 software (cf. Section 3.3.2.2).
- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

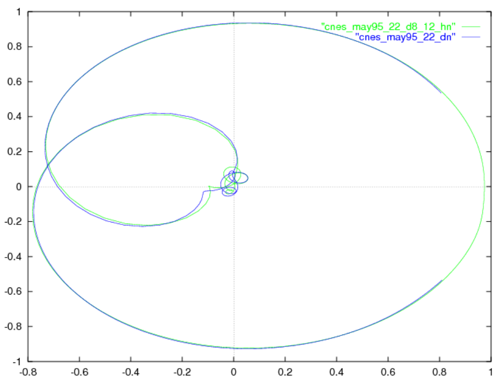

Figure 2. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}.

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 2). Non-physical couplings are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, useful for the synthesis of repeating devices.

The team also investigates problems relative to the design of optimal responses for microwave devices. The resolution of a quasi-convex Zolotarev problems was for example proposed, in order to derive guaranteed optimal multi-band filter's responses subject to modulus constraints [10]. This generalizes the classical single band design techniques based on Chebyshev polynomials and elliptic functions. These techniques rely on the fact that the modulus of the scattering parameters of a filters, say $|S_{1,2}|$, admits a simple expression in terms of the filtering function $D = |S_{1,1}|/|S_{1,2}|$ namely,

$$|S_{1,2}|^2 = \frac{1}{1+D^2}.$$

The filtering function appears to be the ratio of two polynomials p_1/p_2, the numerator of the reflection and transmission scattering factors, that can be chosen freely. The denominator q is obtained as the unique stable and unitary polynomial solving the classical Feldtkeller spectral equation:

$$qq^* = p_1p_1^* + p_2p_2^*.$$

The relative simplicity of the derivation of a filter's response under modulus constraints is due to the possibility of "forgetting" about Feldtkeller's equation, and express all design constraints in terms of the filtering function D. This no longer the case when considering the synthesis N-port devices for $N > 3$, like multiplexers, routers power dividers or when considering the synthesis of filters under matching conditions. The efficient derivation of multiplexers responses is one of the team's active recent research area, where techniques based on constrained Nevanlinna-Pick interpolation problems are being considered (see Section 6.3.1).

5. Software and Platforms

5.1. RARL2

Participant: Martine Olivi [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see Section 3.3.2.2) http://www-sop.inria.fr/apics/RARL2/rarl2.html.

The software RARL2 computes, from a given matrix-valued function in $\overline{H}^{2\,m\times l}$, a local best rational approximant in the L^2 norm, which is *stable and of prescribed McMillan degree* (see Section 3.3.2.2). It was initially developed in the context of linear (discrete-time) system theory and makes an heavy use of the classical concepts in this field. The matrix-valued function to be approximated can be viewed as the transfer function of a multivariable discrete-time stable system. RARL2 takes as input either:

- its internal realization,
- its first N Fourier coefficients,
- discretized (uniformly distributed) values on the circle. In this case, a least-square criterion is used instead of the L^2 norm.

It thus performs model reduction in case 1) and 2) and frequency data identification in case 3). In the case of band-limited frequency data, it could be necessary to infer the behavior of the system outside the bandwidth before performing rational approximation (see Section 3.2.2). An appropriate Möbius transformation allows to use the software for continuous-time systems as well.

The method is a steepest-descent algorithm. A parametrization of MIMO systems is used, which ensures that the stability constraint on the approximant is met. The implementation, in Matlab, is based on state-space representations.

The number of local minima can be rather high so that the choice of an initial point for the optimization can play a crucial role. Two methods can be used: 1) An initialization with a best Hankel approximant. 2) An iterative research strategy on the degree of the local minima, similar in principle to that of RARL2, increases the chance of obtaining the absolute minimum by generating, in a structured manner, several initial conditions.

RARL2 performs the rational approximation step in our applications to filter identification (see Section 4.5) as well as sources or cracks recovery (see Section 4.2). It was released to the universities of Delft, Maastricht, Cork and Brussels. The parametrization embodied in RARL2 was also used for a multi-objective control synthesis problem provided by ESTEC-ESA, The Netherlands. An extension of the software to the case of triple poles approximants is now available. It provides satisfactory results in the source recovery problem and it is used by FindSources3D (see Section 5.6).

5.2. RGC

Participant: Fabien Seyfert [corresponding participant].

Status: A stable version is maintained.

This software is developed in collaboration with Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

The identification of filters modeled by an electrical circuit that was developed by the team (see Section 4.5) led us to compute the electrical parameters of the underlying filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called “the arrow form” [57] which is of particular interest since it is unique for a given transfer function and is easily computed. The computation of this realization is the first step of RGC. Subsequently, if the target realization is not in arrow form, one can nevertheless show that it can be deduced from the arrow-form by a complex- orthogonal change of basis. In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has many local and global minima. In fact, there is not even uniqueness of the filter realization for a given geometry. Moreover, it is often relevant to know all solutions of the problem, because the designer is not even sure, in many cases, which one is being handled. The assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software provides no guarantee to obtain a single realization that satisfies the prescribed constraints. The software Dedale-HF (see Section 5.4), which is the successor of RGC, solves with guarantees this constraint realization problem.

5.3. PRESTO-HF

Participant: Fabien Seyfert [corresponding participant].

Status: Currently under development. A stable version is maintained.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for microwave filters http://www-sop.inria.fr/apics/Presto-HF. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single shot:

- determination of delay components caused by the access devices (automatic reference plane adjustment),
- automatic determination of an analytic completion, bounded in modulus for each channel,
- rational approximation of fixed McMillan degree,
- determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies on RARL2 (see Section 5.1), a rational approximation engine developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Thales Alenia Space in Toulouse, Thales airborn systems and a license agreement has been recently negotiated with TAS-Espagna. XLIM (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements are currently being considered with the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada).

5.4. Dedale-HF

Participant: Fabien Seyfert [corresponding participant].

Status: Currently under development. A stable version is maintained.

Dedale-HF is a software dedicated to solve exhaustively the coupling matrix synthesis problem in reasonable time for the users of the filtering community. For a given coupling topology, the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of given filter characteristics. Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements (see Figure 3).

Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. problem of the user, *i.e.* the one corresponding to user-specified filter characteristics. The reference files are computed off-line using Groebner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such a continuation technique combined with an efficient implementation of the integrator produces a drastic reduction, by a factor of 20, of the computational time.

Access to the database and integrator code is done via the web on http://www-sop.inria.fr/apics/Dedale/WebPages. The software is free of charge for academic research purposes: a registration is however needed in order to access full functionality. Up to now 90 users have registered world wide (mainly: Europe, U.S.A, Canada and China) and 4000 reference files have been downloaded.

A license of this software has been sold end of 2011, to TAS-Espagna, in order for it to tune filters with topologies having multiple solutions. The use of Dedale-HF is here coupled with that of Presto-HF.

5.5. easyFF

Participant: Fabien Seyfert.

Status: A stable version is maintained.

This software has been developed by Vincent Lunot (Taiwan Univ.) during his PhD. He still continues to maintain it.

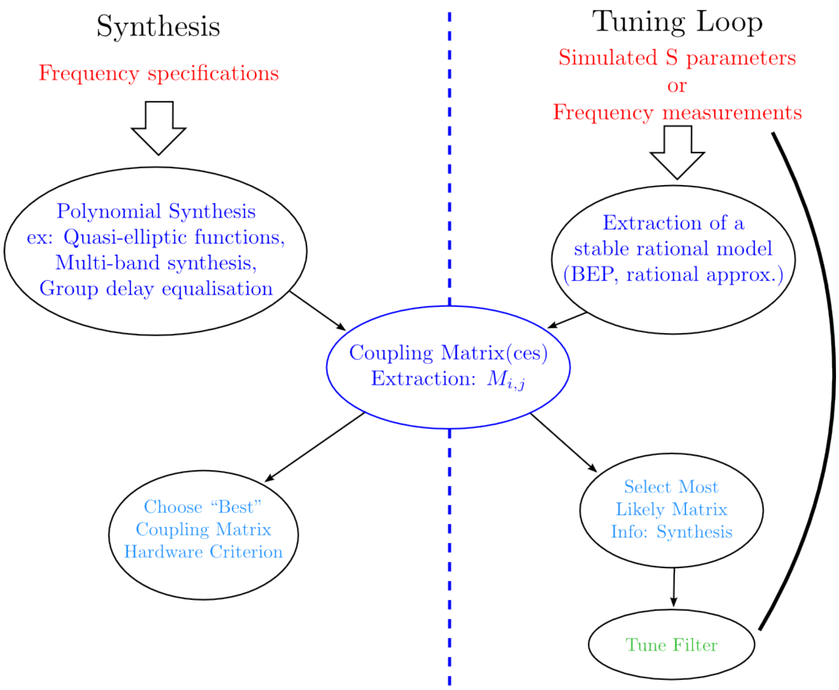

Figure 3. Overall scheme of the design and tuning process of a microwave filter.

EasyFF is a software dedicated to the computation of complex, and in particular multi-band, filtering functions. The software takes as input, specifications on the modulus of the scattering matrix (transmission and rejection), the filter's order and the number of transmission zeros. The output is an "optimal" filtering characteristic in the sense that it is the solution of an associated min-max Zolotarev problem. Computations are based on a Remez-type algorithm (if transmission zeros are fixed) or on linear programming techniques if transmission zeros are part of the optimization [10].

5.6. FindSources3D

Participant: Juliette Leblond [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena Project-Team, and with Jean-Paul Marmorat (Centre de mathématiques appliquées - CMA, École des Mines de Paris).

FindSources3D [2] is a software dedicated to source recovery for the inverse EEG problem, in 3-layer spherical settings, from point-wise data (see http://www-sop.inria.fr/apics/FindSources3D/). Through the algorithm described in [8] and Section 4.2, it makes use of the software RARL2 (Section 5.1) for the rational approximation step in plane sections. The data transmission preliminary step ("cortical mapping") is solved using boundary element methods through the software OpenMEEG (its CorticalMapping features) developed by the Athena Team (see http://www-sop.inria.fr/athena/software/OpenMEEG/). A new release of FindSources3D is now available, which is being demonstrated and distributed to the medical team we maintain contact with (hosp. la Timone, Marseille). A further release is currently under development, due to the strong interest for this software by the German firm BESA GmbH (see http://www.besa.de/), involved in EEG software for research and clinical applications, and a deeper collaboration with this company has been started this year. Figure 4 shows the good results of a two sources distribution recovered by FindSources3D from potential values at electrodes on a sphere (scalp) generated by BESA's simulator, and then back to a more realistic head geometry. There, the achieved localization error is small enough, and FindSources3D provides suitable initial guess to heavier dedicated recovery tools, along with an estimation of the number of sources which may be incorporated to the software as an additional functionality (at the moment, the user is still involved in this estimation). Taking into account several time instants will be considered next.

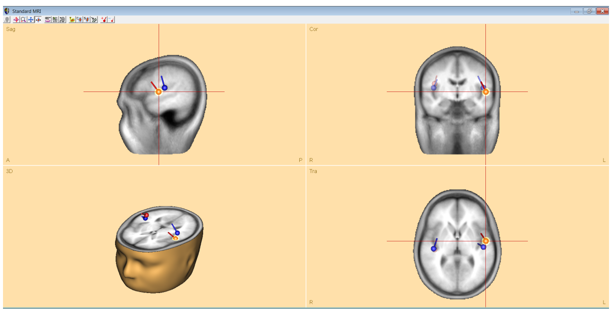

Figure 4. Recovered 2 sources by FindSources3D (courtesy of BESA).

5.7. Sollya

Participant: Sylvain Chevillard [corresponding participant].

[2] CeCILL license, APP version 2.0 (2012): IDDN.FR.001.45009.001.S.A.2009.000.10000

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Christoph Lauter (LIP6) and Mioara Joldeş (LAAS).

Sollya is an interactive tool where the developers of mathematical floating-point libraries (libm) can experiment before actually developing code. The environment is safe with respect to floating-point errors, *i.e.* the user precisely knows when rounding errors or approximation errors happen, and rigorous bounds are always provided for these errors.

Among other features, it offers a fast Remez algorithm for computing polynomial approximations of real functions and also an algorithm for finding good polynomial approximants with floating-point coefficients to any real function. It also provides algorithms for the certification of numerical codes, such as Taylor Models, interval arithmetic or certified supremum norms.

It is available as a free software under the CeCILL-C license at http://sollya.gforge.inria.fr/.

6. New Results

6.1. Source recovery problems

Participants: Laurent Baratchart, Kateryna Bashtova, Sylvain Chevillard, Juliette Leblond, Dmitry Ponomarev.

This section is concerned with inverse problems for 3-D Poisson-Laplace equations. Though the geometrical settings differ in the 2 sections below, the characterization of silent sources (that give rise to a vanishing potential at measurement points) is one of the common problems to both which has been recently achieved in the magnetization setup, see [14].

6.1.1. Application to EEG

This work is conducted in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena Project-Team, and with Jean-Paul Marmorat (Centre de mathématiques appliquées - CMA, École des Mines de Paris).

In 3-D, functional or clinical active regions in the cortex are often modeled by point-wise sources that have to be localized from measurements on the scalp of a potential satisfying a Laplace equation (EEG, electroencephalography). In the work [3] it was shown how to proceed via best rational approximation on a sequence of 2-D disks cut along the inner sphere, for the case where there are at most 2 sources. Last year, a milestone was reached in the research on the behavior of poles in best rational approximants of fixed degree to functions with branch points [6], to the effect that the technique carries over to finitely many sources (see Section 4.2).

In this connection, a dedicated software "FindSources3D" is being developed, in collaboration with the team Athena and the CMA. We took on this year algorithmic developments, prompted by recent and promising contacts with the firm BESA (see Section 5.6), namely automatic detection of the number of sources (which is left to the user at the moment) and simultaneous processing of data from several time instants. It appears that in the rational approximation step, *multiple* poles possess a nice behavior with respect to branched singularities. This is due to the very physical assumptions on the model (for EEG data, one should consider *triple* poles). Though numerically observed in [8], there is no mathematical justification so far why multiple poles generate such strong accumulation of the poles of the approximants. This intriguing property, however, is definitely helping source recovery. It is used in order to automatically estimate the "most plausible" number of sources (numerically: up to 2, at the moment).

In connection with the work [14] related to inverse magnetization issues (see Section 6.1.2), the characterization of silent sources for EEG has been carried out [42]. These are sums of (distributional) derivatives of Sobolev functions vanishing on the boundary.

In a near future, magnetic data from MEG (magneto-encephalography) will become available along with EEG data; indeed, it is now possible to use simultaneously corresponding measurement devices, in order to measure both electrical and magnetic fields. This should enhance the accuracy of our source recovery algorithms.

Let us mention that discretization issues in geophysics can also be approached by such techniques. Namely, in geodesy or for GPS computations, one is led to seek a discrete approximation of the gravitational potential on the Earth's surface, from partial data collected there. This is the topic of a beginning collaboration with physicist colleagues (IGN, LAREG, geodesy). Related geometrical issues (finding out the geoid, level surface of the gravitational potential) are worthy of consideration as well.

6.1.2. Magnetization issues

This work is carried out in the framework of the "équipe associée Inria" IMPINGE, comprising Eduardo Andrade Lima and Benjamin Weiss from the Earth Sciences department at MIT (Boston, USA) and Douglas Hardin and Edward Saff from the Mathematics department at Vanderbilt University (Nashville, USA),

Localizing magnetic sources from measurements of the magnetic field away from the support of the magnetization is the fundamental issue under investigation by IMPINGE The goal is to determine magnetic properties of rock samples (*e.g.* meteorites or stalactites) from fine field measurements close to the sample that can nowadays be obtained using SQUIDs (supraconducting coil devices). Currently, rock samples are cut into thin slabs and the magnetization distribution is considered to lie in a plane, which makes for a somewhat less indeterminate framework than EEG as regards inverse problems because "less" magnetizations can produce the same field (for the slab has no inner volume).

The magnetization operator is the Riesz potential of the divergence of the magnetization, see (1). Last year, the problem of recovering a thin plate magnetization distribution from measurements of the field in a plane above the sample led us to an analysis of the kernel of this operator, which we characterized in various functional and distributional spaces [14]. Using a generalization of the Hodge decomposition, we were able to describe all magnetizations equivalent to a given one. Here, equivalent means that the magnetizations generate the same field from above and from below if, say, the slab is horizontal. When magnetizations have bounded support, which is the case for rock samples, we proved that magnetizations equivalent from above are also equivalent from below, but this is no longer true for unbounded supports. In fact, even for unidirectional magnetizations, uniqueness of a magnetization generating a given field depends on the boundedness of the support, as we proved that *any* magnetization is equivalent from above to a unidirectional one (with infinite support in general). This helps explaining why methods in the Fourier domain (which essentially loose track of the support information) do encounter problems. It also shows that information on the support must be used in a crucial way to solve the problem.

This year, we produced a fast inversion scheme for magnetic field maps of unidirectional planar geological magnetization with discrete support located on a regular grid, based on discrete Fourier transform [18]. Figures 5, 6, 7 and 8 show an example of reconstruction. As the just mentioned article shows, the Fourier approach is computationally attractive but undergoes aliasing phenomena that tend to offset its efficiency. In particular, estimating the total moment of the magnetization sample seems to require data extrapolation techniques which are to take place in the space domain. This is why we have started to study regularization schemes based on truncation of the support in connection with singular values analysis of the discretized problem.

In a joint effort by all members of IMPINGE, we set up a heuristics to recover dipolar magnetizations, using a discrete least square criterion. At the moment, it is solved by a singular value decomposition procedure of the magnetization-to-field operator, along with a regularization technique based on truncation of the support. Preliminary experiments on synthetic data give quite accurate results to recover the net moment of a sample, see the preliminary document http://www-sop.inria.fr/apics/IMPINGE/Documents/NotesSyntheticExample.pdf. We also ran the procedure on real data (measurements of the field generated by Lunar spherules) for which the net moment can be estimated by other methods. The net moment thus recovered matches well the expected moment.

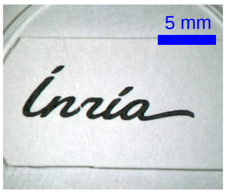

Figure 5. Inria's logo were printed on a piece of paper. The ink of the letters "In" were magnetized along a direction D_1. The ink of the letters "ria" were magnetized along another direction D_2 (almost orthogonal to D_1).

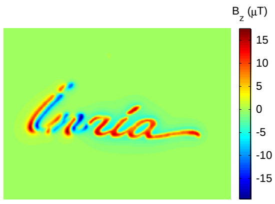

Figure 6. The Z-component of the magnetic field generated by the sample is measured by a SQUID microscope. The measure is performed 200µm above the sample.

Figure 7. The field measured in Figure 6 is inversed, assuming that the sample is uni-dimensionally magnetized along the direction D_1. The letters "In" are fairly well recovered while the rest of the letters is blurred (because the hypothesis about the direction of magnetization is false for "ria").

Figure 8. The field measured in Figure 6 is inversed, assuming that the sample is uni-dimensionally magnetized along the direction D_2. The letters "ria" are fairly well recovered while the rest of the letters is blurred (because the hypothesis about the direction of magnetization is false for "In").

This shows that the technique we use to reduce the support, which is based on thresholding contributions of dipoles to the observations, is capable of eliminating some nearly silent dipole distributions which flaw the singular value analysis. In order to better understand the geometric nature of such distributions, and thus affirm theoretical bases to the above mentioned heuristics, we raised the question of determining an eigenbasis for the positive self adjoint operator mapping a L^2 magnetization on a rectangle to the field it generates on a rectangle parallel to the initial one. Once ordered according to decreasing eigenvalues, such a basis should retain "as much information as possible" granted the order of truncation.

This is not such an easy problem and currently, in the framework of the PhD thesis of D. Ponomarev, we investigate a simplified two-dimensional analog, defined via convolution of a function on a segment with the Poisson kernel of the upper half-plane and then restriction to a parallel segment in that half-plane. Surprisingly perhaps, this issue was apparently not considered in spite of its natural character and the fact that it makes contact with classical spectral theory. Specifically, it amounts to spectral representation of certain compressed Toeplitz operators with exponential-of-modulus symbols. Beyond the bibliographical research needed to understand the status of this question, only preliminary results have been attained so far.

6.2. Boundary value problems

Participants: Laurent Baratchart, Slah Chaabi, Sylvain Chevillard, Juliette Leblond, Dmitry Ponomarev, Elodie Pozzi.

This work was the occasion of collaborations with Alexander Borichev (Aix-Marseille University), Jonathan Partington (Univ. Leeds, UK), and Emmanuel Russ (Univ. Grenoble, IJF).

6.2.1. Generalized Hardy classes

As we mentioned in Section 4.4 2-D diffusion equations of the form $\mathrm{div}(\sigma\nabla u) = 0$ with real non-negative valued conductivity σ can be viewed as compatibility relations for the so-called conjugate Beltrami equation: $\overline{\partial} f = \nu\overline{\partial f}$ with $\nu = (1-\sigma)/(1+\sigma)$ [4]. Thus, the conjugate Beltrami equation is a means to replace the initial second order diffusion equation by a first order system of two real equations, merged into a single complex one. Hardy spaces under study here are those of this conjugate Beltrami equation: they are comprised of solutions to that equation in the considered domain whose L^p means over curves tending to the boundary of the domain remain bounded. They will for example replace holomorphic Hardy spaces in Problem (P) when dealing with non-constant (isotropic) conductivity. Their traces merely lie in L^p ($1 < p < \infty$), which is suitable for identification from point-wise measurements, and turn out to be dense on strict subsets of the boundary. This allows one to state Cauchy problems as bounded extremal issues in L^p classes of generalized analytic functions, in a reminiscent manner of what was done for analytic functions as discussed in Section 3.3.1.

The study of such Hardy spaces for Lipschitz σ was reduced in [4] to that of spaces of pseudo-holomorphic functions with bounded coefficients, which were apparently first considered on the disk by S. Klimentov. Typical results here are that solution factorize as $e^s F$, where F is a holomorphic Hardy function while s is in the Sobolev space $W^{1,r}$ for all $r < \infty$ (Bers factorization), and the analog to the M. Riesz theorem which amounts to solvability of the Dirichlet problem for the initial conductivity equation with L^p boundary data for all $p \in (1,\infty)$. Over the last two years, the case of $W^{1,q}$ conductivities over finitely connected domains, $q > 2$, has been carried out in [13] [61].

In 2013, completing a study begun last year in the framework of the PhD of S. Chaabi, we established similar results in the case where $\log\sigma$ lies in $W^{1,2}$, which corresponds to the critical exponent in Vekua's theory of pseudo-holomorphic functions. This is completely new, and apparently the first example of a solvable Dirichlet problem with L^p boundary data where the conductivity can be both unbounded an vanishing at some places. Accordingly, solutions may also be unbounded inside the domain of the equation, that is, the maximum principle no longer holds. The proof develops a refinement of the Bers factorization based on Muckenhoupt weights and on an original multiplier theorem for $\log W^{1,2}$ functions. A paper on this topic has been submitted [28].

The PhD work of S. Chaabi (defended December 2) contains further work on the Weinstein equation and certain generalizations thereof. This equation results from 2-D projection of Laplace's equation in the presence of rotation symmetry in 3-D. In particular, it is the equation governing the free boundary problem of plasma confinement in the plane section of a tokamak. A method dwelling on Fokas's approach to elliptic boundary value problems has been developed which uses Lax pairs and solves for a Riemann-Hilbert problem on a Riemann surface. It was used to devise semi-explicit forms of solutions to Dirichlet and Neumann problems for the conductivity equation satisfied by the poloidal flux.

In another connection, the conductivity equation can also be regarded as a static Schrödinger equation for smooth coefficients. In particular, a description of laser beam propagation in photopolymers can be crudely approximated by a stationary two-dimensional model of wave propagation in a medium with negligible change of refractive index. In this setting, Helmholtz equation is approximated by a linear Schrödinger equation with one spatial coordinate as evolutionary variable. This phenomenon can be described by a non-stationary model that relies on a spatial nonlinear Schrödinger (NLS) equation with time-dependent refractive index. A model problem has been considered in [20], when the rate of change of refractive index is proportional to the squared amplitude of the electric field and the spatial domain is a plane.

We have also studied composition operators on generalized Hardy spaces in the framework of [13]. In the work [32] submitted for publication, we provide necessary and/or sufficient conditions on the composition map, depending on the geometry of the domains, ensuring that these operators are bounded, invertible, isometric or compact.

6.2.2. Best constrained analytic approximation

Several questions about the behavior of solutions to the bounded extremal problem (P) of Section 3.3.1 have been considered. For instance, truncated Toeplitz operators have been studied in [17], that can be used to quantify robustness properties of our resolution schemes in H^2 and to establish error estimates. Moreover we considered additional interpolation constraints on the disk in Problem (P), and derived new stability estimates for the solution [46]. Such interpolation constraints arise naturally in inverse boundary problems like plasma shaping in last generation tokamaks, where some measurements are performed inside the chamber 4.4. Of course the version studied so far is much simplified, as it must be carried over to non-constant conductivities and annular geometries.

6.3. Synthesis of compact multiplexers and de-embedding of multiplexers

Participants: Martine Olivi, Sanda Lefteriu, Fabien Seyfert.

This work has been done in collaboration with Stéphane Bila (XLIM, Limoges, France), Hussein Ezzedin (XLIM, Limoges, France), Damien Pacaud (Thales Alenia Space, Toulouse, France), Giuseppe Macchiarella (Politecnico di Milano, Milan, Italy), and Matteo Oldoni (Siae Microelettronica, Milan, Italy).

6.3.1. Synthesis of compact multiplexers

We focused our research on multiplexer with a star topology. These are comprised of a central N-port junction, and of filters plugged on all but common ports (see Figure 9). A possible approach to synthesis of the multiplexer's response is to postulate that each filter channel has to match the multiplexer at n_k frequencies (n_k being the order of the filter) while rejecting the energy at m_k other frequencies (m_k being the order the transmission polynomial of the filter). The desired synthesis can then be cast into computing of a collection of filter's responses matching the energy as prescribed and rejecting it at specified frequencies when plugged simultaneously on the junction. Whether such a collection exists is one of the main open issues facing co-integration of systems in electronics. Investigating the latter led us to consider the simpler problem of matching a filter, on a frequency-varying load, while rejecting energy at fixed specified frequencies. If the order of the filter is n this amounts to fix a given transmission polynomial r and to solve for a unitary polynomial p meeting interpolation conditions of the form:

$$j = 1 \cdots n, \qquad \frac{p}{q}(w_j) = \gamma_j, \ \ |\gamma_j| < 1$$

where q is the unique monic Hurwitz polynomial satisfying the Feldtkeller equation

$$qq^* = pp^* + rr^*.$$

This problem can be seen as an extended Nevanlinna-Pick interpolation problem, which was considered in [65] when the interpolation frequencies lie in the *open* left half-plane. Last year we conjectured the existence and uniqueness of a solution, which were eventually proved true this year when r has no roots on the imaginary axis. We already communicated on the subject (9.1), and a scientific report as well as an article are being written on this result [30]. The proof relies on the local invertibility of an evaluation map that is established using a differential argument and the structure of particular Pick matrices. The case where r has zeros on the imaginary axis is of great interest, and though existence then holds again uniqueness is still not well-understood: it is conjectured that under minor restrictions on the localization of the $\gamma_k' s$ (typically off an algebraic subvariety) the main results still hold.

This research lies at heart of our collaboration with CNES on multiplexer synthesis and the core of the starting ANR project COCORAM on co-integration of filters and antennas (see Section 8.1.1).

6.3.2. De-embedding of multiplexers

Let S be the external scattering parameters of a multiplexer composed of a N-port junction with response T and $N-1$ filters with responses $F_1, \cdots F_{N-1}$ as plotted on Figure 9. The de-embbeding problem concerns the recovery of the F_k and can be considered under different hypotheses. Last year we studied the de-embedding problem where S and T are known [76] but no particular structure on the F_k is assumed. It was shown that for a generic junction T and for $N > 3$ the de-embedding problem has a unique solution. It was however observed that in practice the junction's response is far from being generic (as it is usually obtained by assembly of smaller T-junctions) which renders the problem extremely sensitive to measurement noise. It was also noticed that in practical applications, scattering measurements of the junction are hardly available.

It was therefore natural to consider following de-embedding problem. Given S the external scattering measurement of the multiplexer, and under the assumptions:

- the F_k are rational of known McMillan degree,
- the coupling geometry of their circuital realization is known,

what can be said about the filter's responses ? It was shown that under the above hypotheses, in particular with no a priori knowledge of T, the filter's responses are identifiable up to a constant chain matrix chained at their second port (nearest to the junction) [24]. It was also shown that this uncertainty bears only on the resonant frequency of the last cavity of each filter, as well as on their output coupling. Most of the filters' important parameters can therefore be recovered. The approach is constructive and relies on rational approximation of certain external scattering parameters, and on an extraction procedure similar to Darlington's synthesis for filters. Software developments have been pursued to implement the latter and practical studies are under way with data furnished by Thales Alenia Space and by Siae Microelettronica. A medium term objective is to extend the Presto-HF (5.3) software to de-embedding problems for multiplexers and more general multi-ports.

This work is pursued in collaboration with Thales Alenia Space, Siae Microelettronica, XLIM and CNES in particular under contract with CNES on compact N-port synthesis (see Section 7.1).

6.4. Detection of the instability of amplifiers

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

This work is conducted in collaboration with Jean-Baptiste Pomet from the McTao team. It is a continuation of a collaboration with CNES and the University of Bilbao.The goal is to help developing amplifiers, in particular to detect instability at an early stage of the design.

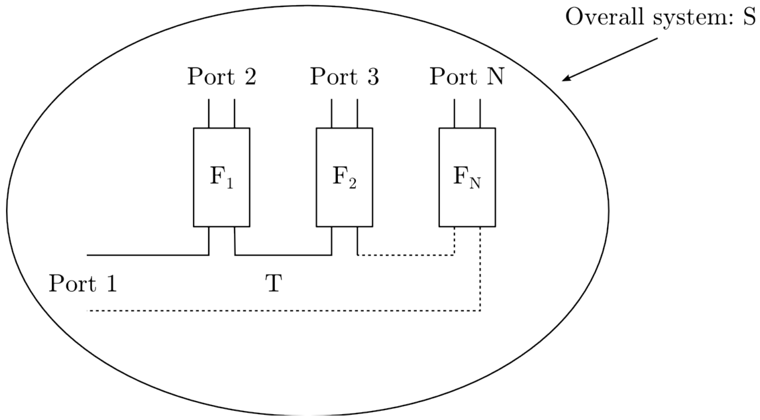

Figure 9. Multiplexer made of a junction T and filtering devices $F_1, F_2 \cdots F_N$

Currently, electrical engineers from the University of Bilbao, under contract with CNES (the French Space Agency), use heuristics to diagnose instability before the circuit is physically implemented. We intend to set up a rigorously founded algorithm, based on properties of transfer functions of such amplifiers which belong to particular classes of analytic functions.

In non-degenerate cases, non-linear electrical components can be replaced by their first order approximation when studying stability to small perturbations. Using this approximation, diodes appear as perfect negative resistors and transistors as perfect current sources controlled by the voltages at certain points of the circuit.

In previous years, we had proved that the class of transfer functions which can be realized with such ideal components and standard passive components (resistors, selfs, capacitors and transmission lines) is rather large since it contains all rational functions in the variable and in the exponentials thereof. This makes possible to design circuits that are unstable, although they have no pole in the right half-plane. This remains true even if a high resistor is put in parallel of the circuit, which is rather unusual. These pathological examples are unrealistic, though, because they assume that non-linear elements continue to provide gain even at very high frequencies. In practice, small capacitive and inductive effects (negligible at moderate frequencies) make these components passive for very high frequencies.

In 2013, we showed that under this simple assumption that there are small inductive and capacitive effects in active components, the class of transfer functions of realistic circuits is much smaller than in previous situation. Our main result is that a realistic circuit is unstable if and only if it has poles in the right half-plane. Moreover, there can only be finitely many of them. Besides this result, we also generalized our description of the class of transfer functions achievable with ideal components, to include the case of transmission lines with loss. An article is currently being written on this subject.

6.5. Rational and meromorphic approximation

Participants: Laurent Baratchart, Sylvain Chevillard.

This work has been done in collaboration with Herbert Stahl (Beuth-Hochsch.), Maxim Yattselev (Purdue Univ. at Indianapolis, USA), Tao Qian (Univ. Macao).

We published last year an important result in approximation theory, namely the counting measure of poles of best H^2 approximants of degree n to a function analytically continuable, except over finitely many branchpoints lying outside the unit disk, converges to the Green equilibrium distribution of the compact set of minimal Green capacity outside of which the function is single valued [6] (see also [21]). This result warrants source recovery techniques used in Section 6.1.1. We considered this year a similar problem for best uniform meromorphic approximants on the unit circle (so-called AAK approximants after Adamjan, Arov and Krein), in the case where the function may have poles and essential singularities. The technical difficulties are considerable, and though a line of attack has been adopted we presently struggle with the proof.

We also studied partial realizations, or equivalently Padé approximants to transfer functions with branchpoints. Identification techniques based on partial realizations of a stable infinite-dimensional transfer function are known to often provide unstable models, but the question as to whether this is due to noise or to intrinsic instability was not clear. This year, we published a paper showing that, in the case of 4 branchpoints, the pole behavior generically has deterministic chaos to it [15].

We also considered the issue of lower bounds in rational approximation. Prompted by renewed interest for linearizing techniques such as vector fitting in the identification community, we studied linearized errors in light of the topological approach in [51], to find that, when properly normalized, they give rise to lower bounds in L^2 rational approximation. Moreover, these make contact with AAK theory which furnishes more, easily computable lower bounds. This is an interesting finding, for lower bounds are usually difficult to get in approximation and though quite helpful to get an appraisal of what can be hoped for in modeling. Dwelling on this, we established for the first time lower bounds in L^2 rational approximation to some badly L^∞ approximable functions (Blaschke products) and showed equivalence, up to a constant, of best L^2 and L^∞ approximation to functions with branchpoints (such as those appearing in inverse source problems for EEG, see Section 6.1.1). An article on this subject is currently submitted for publication in the Journal of Approximation Theory [29].

6.6. Tools for numerically guaranteed computations

Participant: Sylvain Chevillard.

The overall and long-term goal is to enhance the quality of numerical computations. The progress made during year 2013 is the following:

- Publication of a work with Marc Mezzarobba (who was with Aric project-team at that time, and who is now with LIP6) about the efficient evaluation of the Airy $\mathrm{Ai}(x)$ function when x is moderately large [22]. The Taylor series of the Airy Ai function (as many others such as, e.g., Bessel functions or erf) is ill-conditioned when x is not small. To overcome this difficulty, we extend a method by Gawronski, Müller and Reinhard, known to solve the issue in the case of the error function erf. We rewrite $\mathrm{Ai}(x)$ as $G(x)/F(x)$ where F and G are two functions with well-conditioned series. However, the coefficients of G turn out to obey a three-terms ill-conditioned recurrence. We evaluate this recurrence using Miller's backward algorithm with a rigorous error analysis. Function Ai is an example, but ideally the process could be automated to handle some appropriate class of functions in a future work.
- A more general endeavor is to develop a tool that helps developers of libms in their task. This is performed by the software Sollya [3], developed in collaboration with C. Lauter (Université Pierre et Marie Curie) and M. Joldeş (LAAS). In 2013, we released version version 4.0 (in May) and 4.1 (in November) of Sollya. Among other things these releases make available to the user all features of Sollya as a C library. They also introduce the possibility of computing Chebyshev models, and a generalization of Remez algorithm allowing the user to compute a L^∞ best approximation of a real-valued function on a bounded real interval by any linear combination of given functions.

7. Bilateral Contracts and Grants with Industry

7.1. Contract CNES-Inria-XLIM

[3] http://sollya.gforge.inria.fr/

Contract (reference Inria: 7066, CNES: 127 197/00) involving CNES, XLIM and Inria, focuses on the development of synthesis procedures for N-ports microwave devices. The objective is here to derive analytical procedures for the design of multiplexers and routers as opposed to the classical "black box optimization" which is usually employed in this field (for $N \geq 3$). Emphasis at the moment bears on so-called "star-topologies".

7.2. Contract CNES-Inria-UPV/EHU

Contract (reference CNES: RS10/TG-0001-019) involving CNES, University of Bilbao (UPV/EHU) and Inria whose objective is to set up a methodology for testing the stability of amplifying devices. The work at Inria concerns the design of frequency optimization techniques to identify the linearized response and analyze the linear periodic components.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

The ANR (Astrid) project COCORAM (Co-design et co-intégration de réseaux d'antennes actives multi-bandes pour systèmes de radionavigation par satellite) has been accepted and will officially start January 2014. We are associated in this project with three other teams from XLIM (Limoges University), specialized respectively on filters, antennas and amplifiers. The core idea of the project is to work on the co-integration of various microwave devices in the context of GPS satellite systems and in particular for us to work on matching problems (see Section 6.3.1).

8.2. European Initiatives

8.2.1. Collaborations with Major European Organizations

APICS is part of the European Research Network on System Identification (ERNSI) since 1992.

Subject: System identification concerns the construction, estimation and validation of mathematical models of dynamical physical or engineering phenomena from experimental data.

8.3. International Initiatives

8.3.1. Inria Associate Teams

8.3.1.1. IMPINGE

Title: Inverse Magnetization Problems IN GEosciences.

Inria principal investigator: Laurent Baratchart

International Partner (Institution - Laboratory - Researcher):

MIT - Department of Earth, Atmospheric and Planetary Sciences (United States) - Benjamin Weiss

Duration: 2013 - 2015

See details at : http://www-sop.inria.fr/apics/IMPINGE/

The purpose of the associate team IMPINGE is to develop efficient algorithms to recover the magnetization distribution of rock slabs from measurements of the magnetic field above the slab using a SQUID microscope (developed at MIT). The US team also involves a group at Vanderbilt Univ.

8.3.2. Inria International Partners

8.3.2.1. Declared Inria International Partners

NSF CMG collaborative research grant DMS/0934630, "Imaging magnetization distributions in geological samples", with Vanderbilt University and the MIT (USA).

Cyprus NF grant "Orthogonal polynomials in the complex plane: distribution of zeros, strong asymptotics and shape reconstruction".

PHC Utique CMCU (led by Fédération Denis Poisson, Univ. Orléans), "Harmonic analysis and applications".

8.3.2.2. Informal International Partners

As mentioned in Sections 5.6 and 6.1.1, a cooperation with the German firm BESA [4] has started this year, which includes Athena Team (Inria Sophia-Antipolis-Méditerranée) and Centre de Mathématiques Appliquées of École des Mines de Paris. It is expected to be formalized soon, so as to include several developments of the software FindSources3D as well as a co-advised PhD.

8.4. International Research Visitors

8.4.1. Visits of International Scientists

- Douglas Hardin (Vanderbilt University, Nashville, USA, Jun 2013)
- Matteo Oldoni (Siae Microelettronica, Milano, Italy, Nov 2013)
- Vladimir Peller (Michigan University, East Lansing, from May until Jun 2013)
- Yannick Privat (CNRS, Univ. P. et M. Curie, Paris, Dec 2013).
- Tao Qian (University of Macau, Taipa, China, Jul 2013)
- Edward Saff (Vanderbilt University, Nashville, USA, from May until Jun 2013)
- Michael Stessin (New York state University at Albany, USA, Jun 2013)
- Nikos Stylianopoulos (Univ. of Cyprus).
- Ian Sloan (University of New South Wales, Sydney, Australia, Jun. 2013).
- Maxim Yattselev (Indiana University–Purdue University, Indianapolis, USA, Mar 2013)

8.4.1.1. Internships

- K. Bashtova, Master 2 Mathmods - UNSA (6 months), Inverse source problems for elecromagnetic fields, with physical applications.

8.5. List of international and industrial partners

- Collaboration under contract with Thales Alenia Space (Toulouse, Cannes, and Paris), CNES (Toulouse), XLIM (Limoges), University of Bilbao (Universidad del País Vasco / Euskal Herriko Unibertsitatea, Spain).
- Regular contacts with research groups at UST (Villeneuve d'Asq), Universities of Bordeaux-I (Talence), Orléans (MAPMO), Aix-Marseille (CMI-LATP), Nice Sophia Antipolis (Lab. JAD), Grenoble (IJF and LJK), Paris 6 (P. et M. Curie, Lab. JLL), Paris Diderot (LAREG-IGN), CWI (the Netherlands), MIT (Boston, USA), Vanderbilt University (Nashville USA), Steklov Institute (Moscow), Michigan State University (East-Lansing, USA), Texas A&M University (College Station USA), State University of New-York (Albany, USA), University of Oregon (Eugene, USA), Politecnico di Milano (Milan, Italy), University of Trieste (Italy), RMC (Kingston, Canada), University of Leeds (UK), of Maastricht (The Netherlands), of Cork (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien (Austria), TFH-Berlin (Germany), ENIT (Tunis), KTH (Stockholm), University of Cyprus (Nicosia, Cyprus), University of Macau (Macau, China), BESA company (Munich), SIAE Microelettronica (Milano).
- The project is involved in the GDR-project AFHP (CNRS), in the ANR (Astrid program) project COCORAM (with XLIM, Limoges, and DGA), in a EMS21-RTG NSF program (with MIT, Boston, and Vanderbilt University, Nashville, USA), in the Associate Inria Team IMPINGE (with MIT, Boston), and in a CSF program (with University of Cyprus).

[4] http://www.besa.de/

9. Dissemination

9.1. Scientific Animation

- F. Seyfert was invited to give a talk at the department "Optimization and System Theory" of KTH University (Stockholm, Sweden), on "Generalized Nevanlinna-Pick interpolation on the boundary"
- J. Leblond and D. Ponomarev gave communications at the *11th International Conference on Mathematical and Numerical Aspects of Wave Propagation (WAVES 2013)*, Tunis, June (http://www.lamsin.tn/waves13/). J. Leblond was invited to give a communication at the *Workshop on Control and Observation of Nonlinear Control Systems with Application to Medicine (CONCSAM)*, Honolulu, Hawaii, Sept. (http://math.hawaii.edu/control2013/).
- M. Olivi gave a talk at the SSSC 2013 conference in Grenoble (France) [25] and presented a poster at the ERNSI 2013 conference in Nancy (France).
- E. Pozzi gave communications at the *Spring School in Functional and Harmonic analysis and Operator theory*, Lens (may), at the *Workshop in Operator Theory, Harmonic and Complex Analysis*, Lille (may), and at the seminar of the Lab. J.-A. Dieudonné, Université Nice-Sophia Antipolis.
- L. Baratchart was an invited speaker at the workshop "Inverse Problems and Nonlinear Equations", May, Palaiseau. He was an invited speaker at the workshop "Frames and Bases in Banach Spaces of Holomorphic Functions", October, Bordeaux. He was an invited speaker at the conference in honor of A.A. Gonchar, November, Steklov Institute, Moscow. He was a speaker at CMFT 2013 (Shantou, China). He was an invited speaker at the seminar in Guangzhou University (China), the University of Bordeaux (LMB)and of Grenoble (Laboratoire J. Kuntzmann). He was a visitor at the University of Macao, MIT, and the University of Cyprus.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master, PhD: J. Leblond, Inverse source problems, 3h, Franco-German summer school for inverse problems and PDE, Univ. Brême, All.

9.2.2. Supervision

PhD: S. Chaabi, Analyse complexe et problèmes de Dirichlet dans le plan : équation de Weinstein et autres conductivités non-bornées, defended Dec.2d, 2013 (advisors: L. Baratchart, A. Borichev).

PhD in progress: D. Ponomarev, Inverse problems for planar conductivity and Schrödinger PDEs, since Nov. 2012 (advisors: J. Leblond, L. Baratchart).

PhD in progress: M. Caenepeel, A hierarchical framework for design oriented modeling, since Feb. 2013 (advisors: Y. Rolain, M. Olivi, F. Seyfert).

9.2.3. Juries

- L. Baratchart was a referee of the PhD. manuscript of J. Vayssettes (Univ. Poitiers).
- J. Leblond was a member of the hiring committee for a professor in applied mathematics, Univ. Lorraine, and of the PhD jury of A. Blandinières (Univ. Lyon), R. Tytgat (Aix-Marseille University), and A. Abdelmoula (Univ. Rennes, reviewer).
- M. Olivi was a member (reviewer) of the HdR jury of Sylvie Icart (Université de Nice-Sophia Antipolis, March).

9.3. Popularization

- J. Leblond is a member of the Committee MASTIC. She gave a communication within the “Café-in” of the Research Center (Sept.).

- M. Olivi is co-president with I. Castellani of the Committee MASTIC (Commission d'Animation et de Médiation Scientifique) https://project.inria.fr/mastic/. She is responsible for Scientific Mediation. She held a booth at the APMEP conference 2013 in Marseille (France).
- E. Pozzi was a member of the Committee MASTIC.
- S. Chevillard published a popularization blog post on the website "Mathémαtiques de la planète Terre 2013" (http://mpt2013.fr/) about the problem of inverse magnetization of rocks (cf. Section 4.3)

9.4. Community services

- L. Baratchart is a member of the Editorial Boards of *Constructive Methods and Function Theory* and *Complex Analysis and Operator Theory*. He is Inria's representative at the "conseil scientifique" of the Aix-Marseille University.
- S. Chevillard is representative at the "comité de centre" and at the "comité des projets" (Research Center Inria-Sophia). He was a member of the work-group "Books" whose assignment was to propose different scenarios regarding the future of the books currently stored at the library of the research center.
- J. Leblond is an elected member of the "Conseil Scientifique" of Inria. She is one of the two researchers in charge of the mission "Conseil et soutien aux chercheurs" within the Research Center. She is a member of the "Comité de Suivi National PRPS - QVT" (Prévention des Risques Psycho-Sociaux et la Qualité de Vie au Travail).
- M. Olivi is responsible for scientific mediation and co-president of the committee MASTIC.
- F. Seyfert is a member of CUMIR at Inria Sophia-Antipolis-Méditerrannée.

10. Bibliography

Major publications by the team in recent years

[1] S. Amari, F. Seyfert, M. Bekheit. *Theory of Coupled Resonator Microwave Bandpass Filters of Arbitrary Bandwidth*, in "Microwave Theory and Techniques, IEEE Transactions on", August 2010, vol. 58, n^{o} 8, pp. 2188 -2203

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Publications of the year

Doctoral Dissertations and Habilitation Theses

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[14] L. BARATCHART, D. HARDIN, E. A. LIMA, E. B. SAFF, B. WEISS. *Characterizing kernels of operators related to thin-plate magnetizations via generalizations of Hodge decompositions*, in "Inverse Problems", 2013, vol. 29, n^o^ 1 [*DOI :* 10.1088/0266-5611/29/1/015004], http://hal.inria.fr/hal-00919261

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