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Project-Team ASPI

Applications of interacting particle systems to statistics

IN COLLABORATION WITH: Institut de recherche mathématique de Rennes (IRMAR)

RESEARCH CENTER
Rennes - Bretagne-Atlantique

THEME
Stochastic approaches

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Project-Team ASPI

Keywords: Monte Carlo Methods, Markovian Model, Rare Events, Particle Filtering, Tracking, Data Assimilation

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2. Overall Objectives

2.1. Overall Objectives

The scientific objectives of ASPI are the design, analysis and implementation of interacting Monte Carlo methods, also known as particle methods, with focus on

- statistical inference in hidden Markov models and particle filtering,
- risk evaluation and simulation of rare events,
- global optimization.

The whole problematic is multidisciplinary, not only because of the many scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have already contributed to establish the foundations of the field

> target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods, etc.

Intuitively speaking, interacting Monte Carlo methods are sequential simulation methods, in which particles

- *explore* the state space by mimicking the evolution of an underlying random process,
- *learn* their environment by evaluating a fitness function,
- and *interact* so that only the most successful particles (in view of the fitness function) are allowed to survive and to get offsprings at the next generation.

The effect of this mutation / selection mechanism is to automatically concentrate particles (i.e. the available computing power) in regions of interest of the state space. In the special case of particle filtering, which has numerous applications under the generic heading of positioning, navigation and tracking, in

> target tracking, computer vision, mobile robotics, wireless communications, ubiquitous computing and ambient intelligence, sensor networks, etc.,

each particle represents a possible hidden state, and is replicated or terminated at the next generation on the basis of its consistency with the current observation, as quantified by the likelihood function. With these genetic–type algorithms, it becomes easy to efficiently combine a prior model of displacement with or without constraints, sensor–based measurements, and a base of reference measurements, for example in the form of a digital map (digital elevation map, attenuation map, etc.). In the most general case, particle methods provide approximations of Feynman–Kac distributions, a pathwise generalization of Gibbs–Boltzmann distributions, by means of the weighted empirical probability distribution associated with an interacting particle system, with applications that go far beyond filtering, in

> simulation of rare events, global optimization, molecular simulation, etc.

The main applications currently considered are geolocalisation and tracking of mobile terminals, terrain–aided navigation, data fusion for indoor localisation, optimization of sensors location and activation, risk assessment in air traffic management, protection of digital documents.

3. Research Program

3.1. Interacting Monte Carlo methods and particle approximation of Feynman–Kac distributions

Monte Carlo methods are numerical methods that are widely used in situations where (i) a stochastic (usually Markovian) model is given for some underlying process, and (ii) some quantity of interest should be evaluated, that can be expressed in terms of the expected value of a functional of the process trajectory, which includes as an important special case the probability that a given event has occurred. Numerous examples can be found, e.g. in financial engineering (pricing of options and derivative securities) [40], in performance evaluation of communication networks (probability of buffer overflow), in statistics of hidden Markov models (state estimation, evaluation of contrast and score functions), etc. Very often in practice, no analytical expression is available for the quantity of interest, but it is possible to simulate trajectories of the underlying process. The idea behind Monte Carlo methods is to generate independent trajectories of this process or of an alternate instrumental process, and to build an approximation (estimator) of the quantity of interest in terms of the weighted empirical probability distribution associated with the resulting independent sample. By the law of large numbers, the above estimator converges as the size N of the sample goes to infinity, with rate $1/\sqrt{N}$ and the asymptotic variance can be estimated using an appropriate central limit theorem. To reduce the variance of the estimator, many variance reduction techniques have been proposed. Still, running independent Monte Carlo simulations can lead to very poor results, because trajectories are generated *blindly*, and only afterwards are the corresponding weights evaluated. Some of the weights can happen to be negligible, in which case the corresponding trajectories are not going to contribute to the estimator, i.e. computing power has been wasted.

A recent and major breakthrough, has been the introduction of interacting Monte Carlo methods, also known as sequential Monte Carlo (SMC) methods, in which a whole (possibly weighted) sample, called *system of particles*, is propagated in time, where the particles

- *explore* the state space under the effect of a *mutation* mechanism which mimics the evolution of the underlying process,
- and are *replicated* or *terminated*, under the effect of a *selection* mechanism which automatically concentrates the particles, i.e. the available computing power, into regions of interest of the state space.

In full generality, the underlying process is a discrete–time Markov chain, whose state space can be

> finite, continuous, hybrid (continuous / discrete), graphical, constrained, time varying, pathwise, etc.,

the only condition being that it can easily be *simulated*.

In the special case of particle filtering, originally developed within the tracking community, the algorithms yield a numerical approximation of the optimal Bayesian filter, i.e. of the conditional probability distribution of the hidden state given the past observations, as a (possibly weighted) empirical probability distribution of the system of particles. In its simplest version, introduced in several different scientific communities under the name of *bootstrap filter* [42], *Monte Carlo filter* [47] or *condensation* (conditional density propagation) algorithm [44], and which historically has been the first algorithm to include a redistribution step, the selection mechanism is governed by the likelihood function: at each time step, a particle is more likely to survive and to replicate at the next generation if it is consistent with the current observation. The algorithms also provide as a by–product a numerical approximation of the likelihood function, and of many other contrast functions for parameter estimation in hidden Markov models, such as the prediction error or the conditional least–squares criterion.

Particle methods are currently being used in many scientific and engineering areas

> positioning, navigation, and tracking [43], [37], visual tracking [44], mobile robotics [38], [59], ubiquitous computing and ambient intelligence, sensor networks, risk evaluation and simulation of rare events [41], genetics, molecular simulation [39], etc.

Other examples of the many applications of particle filtering can be found in the contributed volume [23] and in the special issue of *IEEE Transactions on Signal Processing* devoted to *Monte Carlo Methods for Statistical Signal Processing* in February 2002, where the tutorial paper [25] can be found, and in the textbook [56] devoted to applications in target tracking. Applications of sequential Monte Carlo methods to other areas, beyond signal and image processing, e.g. to genetics, can be found in [55]. A recent overview can also be found in [29].

Particle methods are very easy to implement, since it is sufficient in principle to simulate independent trajectories of the underlying process. The whole problematic is multidisciplinary, not only because of the already mentioned diversity of the scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have contributed to establish the foundations of the field

> target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods.

These algorithms can be interpreted as numerical approximation schemes for Feynman–Kac distributions, a pathwise generalization of Gibbs–Boltzmann distributions, in terms of the weighted empirical probability distribution associated with a system of particles. This abstract point of view [35], [33], has proved to be extremely fruitful in providing a very general framework to the design and analysis of numerical approximation schemes, based on systems of branching and / or interacting particles, for nonlinear dynamical systems with values in the space of probability distributions, associated with Feynman–Kac distributions. Many asymptotic results have been proved as the number N of particles (sample size) goes to infinity, using techniques coming from applied probability (interacting particle systems, empirical processes [60]), see e.g. the survey article [35] or the textbooks [33], [32], and references therein

> convergence in $\mathbb{L}^p$, convergence as empirical processes indexed by classes of functions, uniform convergence in time, see also [52], [53], central limit theorem, see also [49], propagation of chaos, large deviations principle, etc.

The objective here is to systematically study the impact of the many algorithmic variants on the convergence results.

3.2. Statistics of HMM

Hidden Markov models (HMM) form a special case of partially observed stochastic dynamical systems, in which the state of a Markov process (in discrete or continuous time, with finite or continuous state space) should be estimated from noisy observations. The conditional probability distribution of the hidden state given past observations is a well–known example of a normalized (nonlinear) Feynman–Kac distribution, see 3.1. These models are very flexible, because of the introduction of latent variables (non observed) which allows to model complex time dependent structures, to take constraints into account, etc. In addition, the underlying Markovian structure makes it possible to use numerical algorithms (particle filtering, Markov chain Monte Carlo methods (MCMC), etc.) which are computationally intensive but whose complexity is rather small. Hidden Markov models are widely used in various applied areas, such as speech recognition, alignment of biological sequences, tracking in complex environment, modeling and control of networks, digital communications, etc.

Beyond the recursive estimation of a hidden state from noisy observations, the problem arises of statistical inference of HMM with general state space [30], including estimation of model parameters, early monitoring and diagnosis of small changes in model parameters, etc.

Large time asymptotics A fruitful approach is the asymptotic study, when the observation time increases to infinity, of an extended Markov chain, whose state includes (i) the hidden state, (ii) the observation, (iii) the prediction filter (i.e. the conditional probability distribution of the hidden state given observations at all previous time instants), and possibly (iv) the derivative of the prediction filter with respect to the parameter. Indeed, it is easy to express the log–likelihood function, the conditional least–squares criterion, and many other clasical contrast processes, as well as their derivatives with respect to the parameter, as additive functionals of the extended Markov chain.

The following general approach has been proposed

- first, prove an exponential stability property (i.e. an exponential forgetting property of the initial condition) of the prediction filter and its derivative, for a misspecified model,
- from this, deduce a geometric ergodicity property and the existence of a unique invariant probability distribution for the extended Markov chain, hence a law of large numbers and a central limit theorem for a large class of contrast processes and their derivatives, and a local asymptotic normality property,
- finally, obtain the consistency (i.e. the convergence to the set of minima of the associated contrast function), and the asymptotic normality of a large class of minimum contrast estimators.

This programme has been completed in the case of a finite state space [7], and has been generalized [36] under an uniform minoration assumption for the Markov transition kernel, which typically does only hold when the state space is compact. Clearly, the whole approach relies on the existence of an exponential stability property of the prediction filter, and the main challenge currently is to get rid of this uniform minoration assumption for the Markov transition kernel [34], [53], so as to be able to consider more interesting situations, where the state space is noncompact.

Small noise asymptotics Another asymptotic approach can also be used, where it is rather easy to obtain interesting explicit results, in terms close to the language of nonlinear deterministic control theory [48]. Taking the simple example where the hidden state is the solution to an ordinary differential equation, or a nonlinear state model, and where the observations are subject to additive Gaussian white noise, this approach consists in assuming that covariances matrices of the state noise and of the observation noise go simultaneously to zero. If it is reasonable in many applications to consider that noise covariances are small, this asymptotic approach is less natural than the large time asymptotics, where it is enough (provided a suitable ergodicity assumption holds) to accumulate observations and to see the expected limit laws (law of large numbers, central limit theorem, etc.). In opposition, the expressions obtained in the limit (Kullback–Leibler divergence, Fisher information matrix, asymptotic covariance matrix, etc.) take here a much more explicit form than in the large time asymptotics.

The following results have been obtained using this approach

- the consistency of the maximum likelihood estimator (i.e. the convergence to the set M of global minima of the Kullback–Leibler divergence), has been obtained using large deviations techniques, with an analytical approach [45],

- if the abovementioned set M does not reduce to the true parameter value, i.e. if the model is not identifiable, it is still possible to describe precisely the asymptotic behavior of the estimators [46]: in the simple case where the state equation is a noise–free ordinary differential equation and using a Bayesian framework, it has been shown that (i) if the rank r of the Fisher information matrix I is constant in a neighborhood of the set M, then this set is a differentiable submanifold of codimension r, (ii) the posterior probability distribution of the parameter converges to a random probability distribution in the limit, supported by the manifold M, absolutely continuous w.r.t. the Lebesgue measure on M, with an explicit expression for the density, and (iii) the posterior probability distribution of the suitably normalized difference between the parameter and its projection on the manifold M, converges to a mixture of Gaussian probability distributions on the normal spaces to the manifold M, which generalized the usual asymptotic normality property,
- it has been shown [54] that (i) the parameter dependent probability distributions of the observations are locally asymptotically normal (LAN) [51], from which the asymptotic normality of the maximum likelihood estimator follows, with an explicit expression for the asymptotic covariance matrix, i.e. for the Fisher information matrix I, in terms of the Kalman filter associated with the linear tangent linear Gaussian model, and (ii) the score function (i.e. the derivative of the log–likelihood function w.r.t. the parameter), evaluated at the true value of the parameter and suitably normalized, converges to a Gaussian r.v. with zero mean and covariance matrix I.

3.3. Multilevel splitting for rare event simulation

See 4.2, and 5.3, 5.6, and 5.7.

The estimation of the small probability of a rare but critical event, is a crucial issue in industrial areas such as

> nuclear power plants, food industry, telecommunication networks, finance and insurance industry, air traffic management, etc.

In such complex systems, analytical methods cannot be used, and naive Monte Carlo methods are clearly unefficient to estimate accurately very small probabilities. Besides importance sampling, an alternate widespread technique consists in multilevel splitting [50], where trajectories going towards the critical set are given offsprings, thus increasing the number of trajectories that eventually reach the critical set. As shown in [5], the Feynman–Kac formalism of 3.1 is well suited for the design and analysis of splitting algorithms for rare event simulation.

Propagation of uncertainty Multilevel splitting can be used in static situations. Here, the objective is to learn the probability distribution of an output random variable $Y = F(X)$, where the function F is only defined pointwise for instance by a computer programme, and where the probability distribution of the input random variable X is known and easy to simulate from. More specifically, the objective could be to compute the probability of the output random variable exceeding a threshold, or more generally to evaluate the cumulative distribution function of the output random variable for different output values. This problem is characterized by the lack of an analytical expression for the function, the computational cost of a single pointwise evaluation of the function, which means that the number of calls to the function should be limited as much as possible, and finally the complexity and / or unavailability of the source code of the computer programme, which makes any modification very difficult or even impossible, for instance to change the model as in importance sampling methods.

The key issue is to learn as fast as possible regions of the input space which contribute most to the computation of the target quantity. The proposed splitting methods consists in (i) introducing a sequence of intermediate regions in the input space, implicitly defined by exceeding an increasing sequence of thresholds or levels, (ii) counting the fraction of samples that reach a level given that the previous level has been reached already, and (iii) improving the diversity of the selected samples, usually using an artificial Markovian dynamics. In this way, the algorithm learns

- the transition probability between successive levels, hence the probability of reaching each intermediate level,

- and the probability distribution of the input random variable, conditionned on the output variable reaching each intermediate level.

A further remark, is that this conditional probability distribution is precisely the optimal (zero variance) importance distribution needed to compute the probability of reaching the considered intermediate level.

Rare event simulation To be specific, consider a complex dynamical system modelled as a Markov process, whose state can possibly contain continuous components and finite components (mode, regime, etc.), and the objective is to compute the probability, hopefully very small, that a critical region of the state space is reached by the Markov process before a final time T, which can be deterministic and fixed, or random (for instance the time of return to a recurrent set, corresponding to a nominal behaviour).

The proposed splitting method consists in (i) introducing a decreasing sequence of intermediate, more and more critical, regions in the state space, (ii) counting the fraction of trajectories that reach an intermediate region before time T, given that the previous intermediate region has been reached before time T, and (iii) regenerating the population at each stage, through redistribution. In addition to the non–intrusive behaviour of the method, the splitting methods make it possible to learn the probability distribution of typical critical trajectories, which reach the critical region before final time T, an important feature that methods based on importance sampling usually miss. Many variants have been proposed, whether

- the branching rate (number of offsprings allocated to a successful trajectory) is fixed, which allows for depth–first exploration of the branching tree, but raises the issue of controlling the population size,
- the population size is fixed, which requires a breadth–first exploration of the branching tree, with random (multinomial) or deterministic allocation of offsprings, etc.

Just as in the static case, the algorithm learns

- the transition probability between successive levels, hence the probability of reaching each intermediate level,
- and the entrance probability distribution of the Markov process in each intermediate region.

Contributions have been given to

- minimizing the asymptotic variance, obtained through a central limit theorem, with respect to the shape of the intermediate regions (selection of the importance function), to the thresholds (levels), to the population size, etc.
- controlling the probability of extinction (when not even one trajectory reaches the next intermediate level),
- designing and studying variants suited for hybrid state space (resampling per mode, marginalization, mode aggregation),

and in the static case, to

- minimizing the asymptotic variance, obtained through a central limit theorem, with respect to intermediate levels, to the Metropolis kernel introduced in the mutation step, etc.

A related issue is global optimization. Indeed, the difficult problem of finding the set M of global minima of a real–valued function V can be replaced by the apparently simpler problem of sampling a population from a probability distribution depending on a small parameter, and asymptotically supported by the set M as the small parameter goes to zero. The usual approach here is to use the cross–entropy method [57], [31], which relies on learning the optimal importance distribution within a prescribed parametric family. On the other hand, multilevel splitting methods could provide an alternate nonparametric approach to this problem.

3.4. Nearest neighbor estimates

This additional topic was not present in the initial list of objectives, and has emerged only recently.

In pattern recognition and statistical learning, also known as machine learning, nearest neighbor (NN) algorithms are amongst the simplest but also very powerful algorithms available. Basically, given a training set of data, i.e. an N–sample of i.i.d. object–feature pairs, with real–valued features, the question is how to generalize, that is how to guess the feature associated with any new object. To achieve this, one chooses some integer k smaller than N, and takes the mean–value of the k features associated with the k objects that are nearest to the new object, for some given metric.

In general, there is no way to guess exactly the value of the feature associated with the new object, and the minimal error that can be done is that of the Bayes estimator, which cannot be computed by lack of knowledge of the distribution of the object–feature pair, but the Bayes estimator can be useful to characterize the strength of the method. So the best that can be expected is that the NN estimator converges, say when the sample size N grows, to the Bayes estimator. This is what has been proved in great generality by Stone [58] for the mean square convergence, provided that the object is a finite–dimensional random variable, the feature is a square–integrable random variable, and the ratio k/N goes to 0. Nearest neighbor estimator is not the only local averaging estimator with this property, but it is arguably the simplest.

The asymptotic behavior when the sample size grows is well understood in finite dimension, but the situation is radically different in general infinite dimensional spaces, when the objects to be classified are functions, images, etc.

Nearest neighbor classification in infinite dimension In finite dimension, the k–nearest neighbor classifier is universally consistent, i.e. its probability of error converges to the Bayes risk as N goes to infinity, whatever the joint probability distribution of the pair, provided that the ratio k/N goes to zero. Unfortunately, this result is no longer valid in general metric spaces, and the objective is to find out reasonable sufficient conditions for the weak consistency to hold. Even in finite dimension, there are exotic distances such that the nearest neighbor does not even get closer (in the sense of the distance) to the point of interest, and the state space needs to be complete for the metric, which is the first condition. Some regularity on the regression function is required next. Clearly, continuity is too strong because it is not required in finite dimension, and a weaker form of regularity is assumed. The following consistency result has been obtained: if the metric space is separable and if some Besicovich condition holds, then the nearest neighbor classifier is weakly consistent. Note that the Besicovich condition is always fulfilled in finite dimensional vector spaces (this result is called the Besicovich theorem), and that a counterexample [3] can be given in an infinite dimensional space with a Gaussian measure (in this case, the nearest neighbor classifier is clearly nonconsistent). Finally, a simple example has been found which verifies the Besicovich condition with a noncontinuous regression function.

Rates of convergence of the functional k–nearest neighbor estimator Motivated by a broad range of potential applications, such as regression on curves, rates of convergence of the k–nearest neighbor estimator of the regression function, based on N independent copies of the object–feature pair, have been investigated when the object is in a suitable ball in some functional space. Using compact embedding theory, explicit and general finite sample bounds can be obtained for the expected squared difference between the k–nearest neighbor estimator and the Bayes regression function, in a very general setting. The results have also been particularized to classical function spaces such as Sobolev spaces, Besov spaces and reproducing kernel Hilbert spaces. The rates obtained are genuine nonparametric convergence rates, and up to our knowledge the first of their kind for k–nearest neighbor regression.

This emerging topic has produced several theoretical advances [1], [2] in collaboration with Gérard Biau (université Pierre et Marie Curie, ENS Paris and EPI CLASSIC, Inria Paris—Rocquencourt), and a possible target application domain has been identified in the statistical analysis of recommendation systems, that would be a source of interesting problems.

4. Application Domains

4.1. Localisation, navigation and tracking

See 5.11.

Among the many application domains of particle methods, or interacting Monte Carlo methods, ASPI has decided to focus on applications in localisation (or positioning), navigation and tracking [43], [37], which already covers a very broad spectrum of application domains. The objective here is to estimate the position (and also velocity, attitude, etc.) of a mobile object, from the combination of different sources of information, including

- a prior dynamical model of typical evolutions of the mobile, such as inertial estimates and prior model for inertial errors,
- measurements provided by sensors,
- and possibly a digital map providing some useful feature (terrain altitude, power attenuation, etc.) at each possible position.

In some applications, another useful source of information is provided by

- a map of constrained admissible displacements, for instance in the form of an indoor building map,

which particle methods can easily handle (map-matching). This Bayesian dynamical estimation problem is also called filtering, and its numerical implementation using particle methods, known as particle filtering, has been introduced by the target tracking community [42], [56], which has already contributed to many of the most interesting algorithmic improvements and is still very active, and has found applications in

> target tracking, integrated navigation, points and / or objects tracking in video sequences, mobile robotics, wireless communications, ubiquitous computing and ambient intelligence, sensor networks, etc.

ASPI is contributing (or has contributed recently) to several applications of particle filtering in positioning, navigation and tracking, such as geolocalisation and tracking in a wireless network, terrain–aided navigation, and data fusion for indoor localisation.

4.2. Rare event simulation

See 3.3, and 5.3, 5.6, and 5.7.

Another application domain of particle methods, or interacting Monte Carlo methods, that ASPI has decided to focus on is the estimation of the small probability of a rare but critical event, in complex dynamical systems. This is a crucial issue in industrial areas such as

> nuclear power plants, food industry, telecommunication networks, finance and insurance industry, air traffic management, etc.

In such complex systems, analytical methods cannot be used, and naive Monte Carlo methods are clearly unefficient to estimate accurately very small probabilities. Besides importance sampling, an alternate widespread technique consists in multilevel splitting [50], where trajectories going towards the critical set are given offsprings, thus increasing the number of trajectories that eventually reach the critical set. This approach not only makes it possible to estimate the probability of the rare event, but also provides realizations of the random trajectory, given that it reaches the critical set, i.e. provides realizations of typical critical trajectories, an important feature that methods based on importance sampling usually miss.

ASPI is contributing (or has contributed recently) to several applications of multilevel splitting for rare event simulation, such as risk assessment in air traffic management, detection in sensor networks, and protection of digital documents.

5. New Results

5.1. Iterative isotone regression

Participant: Arnaud Guyader.

This is a collaboration with Nicolas Hengartner (Los Alamos), Nicolas Jégou (université de Rennes 2) and Eric Matzner–Løber (université de Rennes 2), and with Alexander B. Németh (Babeş Bolyai University) and Sándor Z. Németh (University of Birmingham).

We explore some theoretical aspects of a recent nonparametric method for estimating a univariate regression function of bounded variation. The method exploits the Jordan decomposition which states that a function of bounded variation can be decomposed as the sum of a non-decreasing function and a non-increasing function. This suggests combining the backfitting algorithm for estimating additive functions with isotonic regression for estimating monotone functions. The resulting iterative algorithm is called IIR (iterative isotonic regression). The main result in this work [22] states that the estimator is consistent if the number of iterations k_n grows appropriately with the sample size n. The proof requires two auxiliary results that are of interest in and by themselves: firstly, we generalize the well-known consistency property of isotonic regression to the framework of a non-monotone regression function, and secondly, we relate the backfitting algorithm to the von Neumann algorithm in convex analysis. We also analyse how the algorithm can be stopped in practice using a data-splitting procedure.

With the geometrical interpretation linking this iterative method with the von Neumann algorithm, and making a connection with the general property of isotonicity of projection onto convex cones, we derive in [14] another equivalent algorithm and go further in the analysis.

5.2. Mutual nearest neighbors

Participant: Arnaud Guyader.

This is a collaboration with Nicolas Hengartner (Los Alamos).

Motivated by promising experimental results, this work [13] investigates the theoretical properties of a recently proposed nonparametric estimator, called the MNR (mutual nearest neighbors) rule, which estimates the regression function $m(x) = E[Y|X = x]$ as follows: first identify the k nearest neighbors of x in the sample, then keep only those for which x is itself one of the k nearest neighbors, and finally take the average over the corresponding response variables. We prove that this estimator is consistent and that its rate of convergence is optimal. Since the estimate with the optimal rate of convergence depends on the unknown distribution of the observations, we also have adaptation results by data-splitting.

5.3. Adaptive multilevel splitting

Participants: Frédéric Cérou, Arnaud Guyader, Florent Malrieu.

This is a collaboration with Pierre Del Moral (EPI ALEA, Inria Bordeaux—Sud Ouest).

We show that an adaptive version of multilevel splitting for rare events is strongly consistent. We also show that the estimates satisfy a CLT (central limit theorem), with the same asymptotic variance as the non-adaptive algorithm with the optimal choice of the parameters. It is a strong and general result, that generalizes some of our previous results, and the proof is quite technical and involved.

5.4. Total variation estimates for the TCP process

Participant: Florent Malrieu.

This is a collaboration with Jean-Baptiste Bardet (université de Rouen), Alejandra Christen (University of Chile), Arnaud Guillin (université de Clermont–Ferrand), and Pierre–André Zitt (université de Paris–Est Marne–la–Vallée).

The TCP window size process appears in the modeling of the famous Transmission Control Protocol used for data transmission over the Internet. This continuous time Markov process takes its values in $[0, \infty)$, is ergodic and irreversible. The sample paths are piecewise linear deterministic and the whole randomness of the dynamics comes from the jump mechanism. The aim of [27] is to provide quantitative estimates for the exponential convergence to equilibrium, in terms of the total variation and Wasserstein distances, using coupling methods. The technique could be applied to a large class of Markov processes as well.

5.5. On the stability of planar randomly switched systems

Participant: Florent Malrieu.

This is a collaboration with Michel Benaïm (université de Neuchâtel), Stéphane Le Borgne (IRMAR) and Pierre–André Zitt (université de Paris–Est Marne–la–Vallée).

The paper [28] illustrates some surprising instability properties that may occur when stable ODE's are switched using Markov dependent coefficients. Consider the random process (X_t) solution of $dX_t/dt = A(I_t)X_t$ where (I_t) is a Markov process on $\{0,1\}$ and A_0 and A_1 are real Hurwitz matrices on $\mathbb{R}^2$. Assuming that there exists $\lambda \in (0,1)$ such that $(1-\lambda)A_0 + \lambda A_1$ has a positive eigenvalue, we establish that the norm of X_t may converge to 0 or infinity, depending on the the jump rate of the process I. An application to product of random matrices is studied. This work can be viewed as a probabilistic counterpart of the paper [26] by Baldé, Boscain and Mason.

5.6. Marginalization in rare event simulation for switching diffusions

Participant: François Le Gland.

This is a collaboration with Anindya Goswami (IISER, Poone).

Switching diffusions are continuous–time Markov processes with a hybrid continuous / finite state space. A rare but critical event (such as a scalar function of the continuous component of the state exceeding a given threshold) can occur for several reasons:

- the process can remain in *nominal* mode, where the critical event is very unlikely to occur,
- or the process can switch in some *degraded* mode, where the critical event is much more likely to occur, but the switching itself is very unlikely to occur.

Not only is it important to accurately estimate the (very small) probability that the critical event occurs before some fixed final time, but it is also important to have an accurate account on the reason why it occured, or in other words to estimate the probability of the different modes. A classical implementation of the multilevel splitting would not be efficient. Indeed, as soon as (even a few) samples paths switch to a *degraded* mode, these sample paths will dominate and it will not be possible to estimate the contribution of samples paths in the *nominal* mode. Moreover, sampling the finite component of the state is not efficient to accurately estimate the (very small) probability of rare but critical modes. A more efficient implementation is based on marginalization, i.e. in sampling jointly the continuous component and the probability distribution of the finite component given the past continuous component [18]. The latter is a probability vector, known as the Wonham filter, that satisfies a deterministic equation.

5.7. Combining importance sampling and multilevel splitting for rare event simulation

Participants: François Le Gland, Damien–Barthélémy Jacquemart.

This is a collaboration with Jérôme Morio (ONERA, Palaiseau).

The problem is to accurately estimate the (very small) probability that a rare but critical event (such as a scalar function of the state exceeding a given threshold) occurs before some fixed final time. Multilevel splitting is a very efficient solution, in which sample paths are propagated and are replicated when some intermediate events occur. Events that are defined in terms of the state variable only (such as a scalar function of the state exceeding an intermediate threshold) are not a good design. A more efficient but more complicated design would be to let the intermediate events depend also on time. An alternative design is to keep intermediate events simple, defined in terms of the state variable only, and to make sure that samples that exceed the threshold early are replicated more than samples that exceed the same threshold later [19].

5.8. Sequential data assimilation: ensemble Kalman filter vs. particle filter

Participants: François Le Gland, Valérie Monbet.

The contribution has been to prove (by induction) the asymptotic normality of the estimation error, i.e. to prove a central limit theorem for the ensemble Kalman filter. Explicit expression of the asymptotic variance has been obtained for linear Gaussian systems (where the exact solution is known, and where EnKF is unbiased). This expression has been compared with explicit expressions of the asymptotic variance for two popular particle filters: the bootstrap particle filter and the so–called optimal particle filter, that uses the next observation in the importance distribution.

5.9. Non–homogeneous Markov–switching models

Participant: Valérie Monbet.

This is a collaboration with Pierre Ailliot (université de Bretagne occidentale, Brest).

We have developped various hidden non–homogeneous Markov–switching models for description and simulation of univariate and multivariate time series. Considered application are in weather variables modelling but also in economy. The main originality of the proposed models is that the hidden Markov chain is not homogeneous, its evolution depending on the past wind conditions or other covariates. It is shown that it permits to reproduce complex non–linearities.

5.10. Dynamical partitioning of directional ocean wave spectra

Participant: Valérie Monbet.

This is a collaboration with Pierre Ailliot (université de Bretagne occidentale, Brest) and Christophe Maisondieu (IFREMER, Brest).

Directional wave spectra generally exhibit several peaks due to the coexistence of wind sea generated by local wind conditions and swells originating from distant weather systems. The paper [24] proposes a new algorithm for partitioning such spectra and retrieving the various systems which compose a complex sea-state. It is based on a sequential Monte Carlo algorithm which allows to follow the time evolution of the various systems. The proposed methodology is validated on both synthetic and real spectra and the results are compared with a method commonly used in the literature.

5.11. Track–before–detect

Participants: François Le Gland, Alexandre Lepoutre.

This is a collaboration with Olivier Rabaste (ONERA, Palaiseau).

The problem considered in [20] is tracking one or several targets in a track–before–detect (TBD) context using particle filters. These filters require the computation of the likelihood of the complex measurement given the target states. This likelihood depends on the complex amplitudes of the targets. When the complex amplitude fluctuates over time, time coherence of the target cannot be taken into account. However, for the single target case, spatial coherence of this amplitude can be taken into account to improve the filter performance, by marginalizing the likelihood of the complex measurement over the amplitude parameter. The marginalization depends on the fluctuation law considered. We show that for the Swerling 1 model the likelihood of the complex measurement can be obtained analytically in the multi-target case. For the Swerling 0 model no closed form can be obtained in the general multi–target setting. Therefore we resort to some approximations to solve the problem. Finally, we demonstrate with Monte Carlo simulations the gain of this method both in detection and in estimation compared to the classic method that works with the square modulus of the complex signal.

The problem considered in [21] is detecting and tracking a single radar target with amplitude fluctuation Swerling 1 and 3 in a track–before–detect context with particle filter. Those fluctuations are difficult to take into account as they are uncoherent from measurement to measurement. Thus, conventionnal filters work on square modulus of the complexe signal to remove the unknown phase of complex amplitude and the marginalized over the law of the modulus but they lose the spatial coherence of the amplitude in the measurement. We show in this paper that complex measurements can be marginalized directly while taking into account the spatial coherence of the complex amplitude. Finally, we show the benefit of this method both in detection and in estimation via Monte Carlo simulations.

6. Bilateral Contracts and Grants with Industry

6.1. Bilateral contracts with industry

6.1.1. DUCATI: Optimization of sensors location and activation — contract with DGA / Techniques navales

Participant: François Le Gland.

See 3.3 and 4.2

Inria contract ALLOC 7326 — April 2013 to December 2016.

This is a collaboration with Christian Musso (ONERA, Palaiseau) and with Sébastien Paris (LSIS, université du Sud Toulon Var), related with the supervision of the PhD thesis of Yannick Kenne.

The objective of this project is to optimize the position and activation times of a few sensors deployed by one or several platforms over a search zone, so as to maximize the probability of detecting a moving target. The difficulty here is that the target can detect an activated sensor before it is detected itself, and it can then modify its own trajectory to escape from the sensor. This makes the optimization problem a spatio–temporal problem. The activity in the beginning of this project has been to study different ways to merge two different solutions to the optimization problem : a fast, though suboptimal, solution developped by ONERA in which sensors are deployed where and when the probability of presence of a target is high enough, and the optimal population–based solution developped by LSIS and Inria in a previous contract (Inria contract ALLOC 4233) with DGA / Techniques navales.

7. Partnerships and Cooperations

7.1. National initiatives

7.1.1. PDMP Inférence, Évolution, Contrôle et Ergodicité (PIECE) — ANR Jeunes Chercheuses et Jeunes Chercheurs

Participant: Florent Malrieu.

January 2013 to December 2016.

Piecewise deterministic markov processes (PDMP) are non-diffusive stochastic processes which naturally appear in many areas of applications as communication networks, neuron activities, biological populations or reliability of complex systems. Their mathematical study has been intensively carried out in the past two decades but many challenging problems remain completely open. This project aims at federating a group of experts with different backgrounds (probability, statistics, analysis, partial derivative equations, modelling) in order to pool everyone's knowledge and create new tools to study PDMPs. The main lines of the project relate to estimation, simulation and asymptotic behaviors (long time, large populations, multi-scale problems) in the various contexts of application.

7.2. International initiatives

7.2.1. Inria international partners

Arnaud Guyader collaborates with the group of Nicolas Hengartner at Los Alamos National Laboratories, on the development of fast algorithms to simulate rare events, and on iterative bias reduction techniques in nonparametric estimation. This collaboration has a long record of bilateral visits, and a succesful co–direction of a PhD thesis.

7.3. International research visitors

7.3.1. Visits to international teams

Arnaud Guyader has been invited by Nicolas Hengartner to visit LANL (Los Alamos National Laboratories) in July 2013.

François Le Gland has been invited by Arunabha Bagchi to visit the department of applied mathematics of the University of Twente in Enschede, in October 2013.

8. Dissemination

8.1. Scientific animation

François Le Gland is a member of the organizing committee of the *46èmes Journées de Statistique*, to be held in Rennes in June 2014.

Florent Malrieu has coordinated the spring semester of the Labex Henri Lebesgue on perspectives in analysis and probability, from April to September 2013, and he has co–organized the workshop on piecewise deterministic Markov processes in May 2013. He is also the coordinator of the ANR project PIECE (programme Jeunes Chercheuses et Jeunes Chercheurs), see 7.1.1. He is the coordinator, with Tony Lelièvre (CERMICS, ENPC, Marne–la–Vallée and EPI MICMAC, Inria Paris—Rocquencourt), of the SMAI meeting series *EDP / Probabilités*. He his a member of the scientific and organizing committee of the conference in honour of the 60th birthday of Dominique Bakry, to be held in Toulouse in December 2014.

Valérie Monbet has co–organized with Pierre Ailliot (université de Bretagne occidentale, Brest) two workshops *Space–Time Data Analysis in Oceanography and Meteorology I* held in Berder in May 2013 and *Space–Time Data Analysis in Oceanography and Meteorology II* held in Landéda in November 2013.

François Le Gland is a member of the “conseil d’UFR” of the department of mathematics of université de Rennes 1.

Valérie Monbet is a member of the “comité de direction” and of the “conseil” of IRMAR (institut de recherche mathématiques de Rennes, UMR 6625). She is also a member of the “conseil scientifique” of the department of mathematics of université de Rennes 1.

8.2. Teaching, supervision, thesis committees

8.2.1. Teaching

Arnaud Guyader is a member of the committee of “oraux blancs d’agrégation de mathématiques” for ENS Cachan at Ker Lann.

François Le Gland gives a course on Kalman filtering and hidden Markov models, at université de Rennes 1, within the SISEA (signal, image, systèmes embarqués, automatique, école doctorale MATISSE) track of the master in electronical engineering and telecommunications, a 3rd year course on Bayesian filtering and particle approximation, at ENSTA (école nationale supérieure de techniques avancées), Paris, within the systems and control module, a 3rd year course on linear and nonlinear filtering, at ENSAI (école nationale de la statistique et de l’analyse de l’information), Ker Lann, within the statistical engineering track, and a 3rd year course on hidden Markov models, at Télécom Bretagne, Brest.

Florent Malrieu has given a doctoral course on piecewise deterministic Markov processes (PDMP) proposed as a complementary scientific training to PhD students of école doctorale MATISSE. He has also contributed to a pedagogical article in the *Revue de Mathématiques Spéciales*.

Valérie Monbet gives several courses on data analysis, on time series, and on mathematical statistics, all at université de Rennes 1 within the master on statistics and econometrics. She is also the director of the master on statistics and econometry at université de Rennes 1.

8.2.2. *Supervision*

François Le Gland has been supervising one PhD student

- Paul Bui Quang, title: *Particle approximation and the Laplace method for Bayesian filtering*, université de Rennes 1, defense in July 2013, funding: ONERA grant, co–direction: Christian Musso (ONERA, Palaiseau).

and he is currently supervising two PhD students

- Alexandre Lepoutre, provisional title: *Detection issues in track–before–detect*, université de Rennes 1, started in October 2010, expected defense in 2014, funding: ONERA grant, co–direction: Olivier Rabaste (ONERA, Palaiseau).
- Damien–Barthélémy Jacquemart, provisional title: *Rare event methods for the estimation of collision risk*, université de Rennes 1, started in October 2011, expected defense in 2014, funding: DGA / ONERA grant, co–direction: Jérôme Morio (ONERA, Palaiseau).

Florent Malrieu is currently supervising one PhD student

- Florent Bouguet, provisional title: *Coupling methods for PDMP*, université de Rennes 1, started in October 2013, co–direction : Jean–Christophe Breton (université de Rennes 1),

Valérie Monbet is currently supervising one PhD student

- Julie Bessac, provisional title: *Space time modelling of wind fields*, université de Rennes 1, started in October 2011, co–direction : Pierre Ailliot (université de Bretagne Occidentale),

and she is a member of the PhD thesis advisory committee of

- Jérôme Weiss, provisional title: *Modelling of extreme storm surge series*, funding : CIFRE grant with EDF R&D, direction : Michel Benoît (Laboratoire d'Hydraulique Saint-Venant).

8.2.3. *Thesis committees*

François Le Gland has been a member of the committees for the PhD thesis of Mathieu Chouchane (université de la Méditerrannée, advisors: Mustapha Ouladsine and Sébastien Paris) and for the habilitation thesis of Jérôme Morio (université de Rennes 1) and he as been a reviewer for the PhD thesis of Mélanie Bocquel (University of Twente, advisors: Arunabha Bagchi and Hans Driessen).

Florent Malrieu has been a member of the committees for the PhD theses of Bertrand Cloez (université Paris–Est Marne–la–Vallée, advisor: Djalil Chafaï) and Alexandre Genadot (université Pierre et Marie Curie, advisor: Michèle Thieullen) and he as been a reviewer for the PhD thesis of David Godinho Pereira (université Paris–Est Créteil, advisor: Nicolas Fournier).

Valérie Monbet has been a member of the committees for the PhD theses of Paul Bui Quang (université de Rennes 1, advisors: François Le Gland and Christian Musso) and Sébastien Béyou (université de Rennes 1, advisor: Étienne Mémin).

8.3. Participation in workshops, seminars, lectures, etc.

In addition to presentations with a publication in the proceedings, which are listed at the end of the document in the bibliography, members of ASPI have also given the following presentations.

Arnaud Guyader has been invited to give a talk on simulation and estimation of rare events and extreme quantiles, at the ESSEC working group on *Risk*, in October 2013 and at the université Pierre et Marie Curie working group on *Extreme Value Theory*, in December 2013. He has also given a talk on estimation of mutual nearest neighbors, at the *45èmes Journées de Statistique*, held in Toulouse in May 2013.

François Le Gland has been invited to give a talk on the ensemble Kalman filter, at the department of applied mathematics of the University of Twente, in October 2013.

Valérie Monbet has given a talk on dynamical partitioning of directional ocean wave spectra, at the workshop on *Space–Time Data Analysis in Oceanography and Meteorology I*, held in Berder in May 2013 and a talk on stochastic weather generators with non–homogeneous hidden Markov switching, at the workshop on *Space–Time Data Analysis in Oceanography and Meteorology II*, held in Landéda in November 2013. She has given a talk on stochastic weather generators, at the MODNAT workshop on *Modelling of Natural Events*, held at ONERA Palaiseau in October 2013, and at the *5ème École Interdisciplinaire de Rennes sur les Systèmes Complexes — Stochasticité dans les Systèmes Complexes : Désordre, Hasard, Incertitudes*, held in October 2013, and a talk on stochastic weather generators and switching auto–regressive models, and application to temperature series, at the PEPER workshop on *Extreme Value Theory and Risk Assessment in Climate Sciences*, held in Aussois in December 2013.

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