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Activity Report 2013

Project-Team BIPOP

Modelling, Simulation, Control and Optimization of Non-Smooth Dynamical Systems

IN COLLABORATION WITH: Laboratoire Jean Kuntzmann (LJK)

RESEARCH CENTER
Grenoble - Rhône-Alpes

THEME
Optimization and control of dynamic systems

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Project-Team BIPOP

Keywords: modeling, Simulation, Nonsmooth Analysis, Optimization, System Analysis And Control

Creation of the Project-Team: 2004 April 01.

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2. Overall Objectives

2.1. Overall Objectives

Generally speaking, this project deals with nonregular systems, control, modelling and simulation, with emphasis on

- dynamic systems, mostly mechanical systems with unilateral constraints and Coulomb friction, but also electrical circuits with ideal diodes and transistors Mos [1], etc;
- numerical methods for nonsmooth optimization, and more generally the connection between continuous and combinatorial optimization.

3. Research Program

3.1. Dynamic non-regular systems

mechanical systems, impacts, unilateral constraints, complementarity, modeling, analysis, simulation, control, convex analysis

Dynamical systems (we limit ourselves to finite-dimensional ones) are said to be *non-regular* whenever some nonsmoothness of the state arises. This nonsmoothness may have various roots: for example some outer impulse, entailing so-called *differential equations with measure*. An important class of such systems can be described by the complementarity system

$$\begin{cases} \dot{x} = f(x, u, \lambda)\,, \\ 0 \leq y \perp \lambda \geq 0\,, \\ g(y, \lambda, x, u, t) = 0\,, \\ \text{re-initialization law of the state } x(\cdot)\ , \end{cases} \tag{1}$$

where $\perp$ denotes orthogonality; u is a control input. Now (1) can be viewed from different angles.

- Hybrid systems: it is in fact natural to consider that (1) corresponds to different models, depending whether $y_i = 0$ or $y_i > 0$ (y_i being a component of the vector y). In some cases, passing from one mode to the other implies a jump in the state x; then the continuous dynamics in (1) may contain distributions.
- Differential inclusions: $0 \leq y \perp \lambda \geq 0$ is equivalent to $-\lambda \in \mathrm{N}_K(y)$, where K is the nonnegative orthant and $\mathrm{N}_K(y)$ denotes the normal cone to K at y. Then it is not difficult to reformulate (1) as a differential inclusion.
- Dynamic variational inequalities: such a formalism reads as $\langle \dot{x}(t) + F(x(t), t), v - x(t)\rangle \geq 0$ for all $v \in K$ and $x(t) \in K$, where K is a nonempty closed convex set. When K is a polyhedron, then this can also be written as a complementarity system as in (1).

Thus, the 2nd and 3rd lines in (1) define the modes of the hybrid systems, as well as the conditions under which transitions occur from one mode to another. The 4th line defines how transitions are performed by the state x. There are several other formalisms which are quite related to complementarity. A tutorial-survey paper has been published [4], whose aim is to introduce the dynamics of complementarity systems and the main available results in the fields of mathematical analysis, analysis for control (controllability, observability, stability), and feedback control.

3.2. Nonsmooth optimization

optimization, numerical algorithm, convexity, Lagrangian relaxation, combinatorial optimization.

[1] metal-oxyde semiconductor

Here we are dealing with the minimization of a function f (say over the whole space R^n), whose derivatives are discontinuous. A typical situation is when f comes from dualization, if the primal problem is not strictly convex – for example a large-scale linear program – or even nonconvex – for example a combinatorial optimization problem. Also important is the case of spectral functions, where $f(x) = F(\lambda(A(x)))$, A being a symmetric matrix and λ its spectrum.

For these types of problems, we are mainly interested in developing efficient resolution algorithms. Our basic tool is bundling (Chap. XV of [10]) and we act along two directions:

- To explore application areas where nonsmooth optimization algorithms can be applied, possibly after some tayloring. A rich field of such application is combinatorial optimization, with all forms of relaxation [12], [11].
- To explore the possibility of designing more sophisticated algorithms. This implies an appropriate generalization of second derivatives when the first derivative does not exist, and we use advanced tools of nonsmooth analysis, for example [13].

4. Application Domains

4.1. Computational neuroscience

Modeling in neuroscience makes extensive use of nonlinear dynamical systems with a huge number of interconnected elements. Our current theoretical understanding of the properties of neural systems is mainly based on numerical simulations, from single cell models to neural networks. To handle correctly the discontinuous nature of integrate-and-fire networks, specific numerical schemes have to be developed. Our current works focus on event-driven, time-stepping and voltage-stepping strategies, to simulate accurately and efficiently neuronal networks. Our activity also includes a mathematical analysis of the dynamical properties of neural systems. One of our aims is to understand neural computation and to develop it as a new type of information science.

4.2. Electronic circuits

Whether they are integrated on a single substrate or as a set of components on a board, electronic circuits are very often a complex assembly of many basic components with non linear characteristics. The IC technologies now allow the integration of hundreds of millions of transistors switching at GHz frequencies on a die of $1\mathrm{cm}^2$. It is out of the question to simulate a complete IC with standard tools such as the SPICE simulator. We currently work on a dedicated plug-in able to simulate a whole circuit comprising various components, some modelled in a nonsmooth way.

4.3. Walking robots

As compared to rolling robots, the walking ones – for example hexapods – possess definite advantages whenever the ground is not flat or free: clearing obstacles is easier, holding on the ground is lighter, adaptivity is improved. However, if the working environment of the system is adapted to man, the biped technology must be preferred, to preserve good displacement abilities without modifying the environment. This explains the interest displayed by the international community in robotics toward humanoid systems, whose aim is to back man in some of his activities, professional or others. For example, a certain form of help at home to disabled persons could be done by biped robots, as they are able to move without any special adaptation of the environment.

4.4. Optimization

Optimization exists in virtually all economic sectors. Simulation tools can be used to optimize the simulated system. Another domain is parameter *identification* (Idopt or Estime teams), where the deviation between measurements and theoretical predictions must be minimized. Accordingly, giving an exhaustive list of applications is impossible. Some domains where Inria has been involved in the past, possibly through the former Promath and Numopt teams are: production management, geophysics, finance, molecular modeling, robotics, networks, astrophysics, crystallography, ...Our current applicative activity includes: the management of electrical production (deterministic or stochastic), the design and operation of telecommunication networks.

4.5. Computer graphics animation

Computer graphics animation is dedicated to the numerical modeling and simulation of physical phenomena featuring a high visual impact. Typically, deformable objects prone to strong deformation, large displacements, complex and nonlinear or even nonsmooth behavior, are of interest for this community. We are interested in two main mechanical phenomena: on the one hand, the behavior of slender (nonlinear) structures such as rods, plates and shells; on the other hand, the effect of frictional contact between rigid or deformable bodies. In both cases the goal is to design realistic, efficient, robust, and controllable computational models. Whereas the problem of collision detection has become a mature field those recent years, simulating the collision response (in particular frictional contacts) in a realistic, robust and efficient way, still remains an important challenge. Another related issue we began to study is the simulation of heterogeneous objects such as granular or fibrous materials, which requires the design of new high-scales models for dynamics and contacts; indeed, for such large systems, simulating each interacting particle/fiber individually would be too much time-consuming for typical graphics applications. We also pursue some study on the design of high-order models for slender structures such as rods, plates or shells. Our current activity includes the static inversion of mechanical objects, which is of great importance in the field of artistic design, for the making of movies and video games for example. Such problems typically involve geometric fitting and parameters identification issues, both resolved with the help of constrained optimization. Finally, we have just started to study certain discrepancies (inexistence of solution) due to the combination of incompatible models such as contacting rigid bodies subject to Coulomb friction.

5. Software and Platforms

5.1. Nonsmooth dynamics: Siconos

Participants: Vincent Acary, Maurice Brémond, Olivier Huber, Franck Pérignon.

In the framework of the European project Siconos, Bipop was the leader of the Work Package 2 (WP2), dedicated to the numerical methods and the software design for nonsmooth dynamical systems. The aim of this work is to provide a common platform for the simulation, modeling, analysis and control of abstract nonsmooth dynamical systems. Besides usual quality attributes for scientific computing software, we want to provide a common framework for various scientific fields, to be able to rely on the existing developments (numerical algorithms, description and modeling software), to support exchanges and comparisons of methods, to disseminate the know-how to other fields of research and industry, and to take into account the diversity of users (end-users, algorithm developers, framework builders) in building expert interfaces in Python and end-user front-end through Scilab.

After the requirement elicitation phase, the Siconos Software project has been divided into 5 work packages which are identified to software products:

1. SICONOS/NUMERICS This library contains a set of numerical algorithms, already well identified, to solve non smooth dynamical systems. This library is written in low-level languages (C,F77) in order to ensure numerical efficiency and the use of standard libraries (Blas, Lapack, ...)

2. SICONOS/KERNEL This module is an object-oriented structure (C++) for the modeling and the simulation of abstract dynamical systems. It provides the users with a set of classes to describe their nonsmooth dynamical system (dynamical systems, intercations, nonsmooth laws, ...) and to perform a numerical time integration and solving.
3. SICONOS/FRONT-END. This module is mainly an auto-generated wrapper in Python which provides a user-friendly interface to the Siconos libraries. A scilab interface is also provided in the Front-End module.
4. SICONOS/CONTROL This part is devoted to the implementation of control strategies of non smooth dynamical systems.
5. SICONOS/MECHANICS. This part is dedicated to the modeling and the simulation of multi-body systems with 3D contacts, impacts and Coulomb's friction. It uses the Siconos/Kernel as simulation engine but relies on a industrial CAD library (OpenCascade and pythonOCC) to deal with complex body geometries and to compute the contact locations and distances between B-Rep description and on Bullet for contact detection between meshes.

Further informations may be found at http://siconos.gforge.inria.fr/

5.2. Optimization

Participant: Claude Lemaréchal.

Essentially two possibilities exist to distribute our optimization software: library programs (say Modulopt codes), communicated either freely or not, depending on what they are used for, and on the other hand specific software, developed for a given application.

The following optimization codes have been developed in the framework of the former Promath project. They are generally available at http://www-rocq.inria.fr/~gilbert/modulopt/; M1QN3 is also distributed under GPL.

5.2.1. Code M1QN3

Optimization without constraints for problems with many variables ($n \geq 10^3$, has been used for $n = 10^6$). Technically, uses a limited-memory BFGS algorithm with Wolfe's line-search (see Chap. 4 of [3] for the terminology).

5.2.2. Code M2QN1

Optimization with simple bound-constraints for (small) problems: D is a parallelotope in R^n. Uses BFGS with Wolfe's line-search and active-set strategy.

5.2.3. Code N1CV2

Minimization without constraints of a convex nonsmooth function by a proximal bundle method (Chap. XV of [10], Chap. 9 of [3]).

5.2.4. Modulopt

In addition to codes such as above, the Modulopt library contains application problems, synthetic or from the real world. It is a field for experimentation, functioning both ways: to assess a new algorithm on a set of test-problems, or to select among several codes one best suited to a given problem.

5.3. Simulation of fibrous materials subject to frictional contact

5.3.1. MECHE: Modeling Entangling within Contacting hair fibErs

Participants: Florence Bertails-Descoubes, Gilles Daviet, Alexandre Derouet-Jourdan, Romain Casati, Laurence Boissieux.

The software MECHE was essentially developed during the MECHE ADT (2009-2011, research engineer: Gilles Daviet), for simulating the dynamics of assemblies of thin rods (such as hair), subject to contact and friction. Currently, this software is extensively used by two PhD students (A. Derouet-Jourdan and R. Casati) and continues to be enriched with new rod models and inversion modules. This software combines a panel of well-accepted models for rods (ranging from reduced coordinates to maximal coordinates models, and including models recently developed by some members of the group) with classical as well as innovative schemes for solving the problem of frictional contact (incorporating the most recent results of the group, as well as the new contact solver we published in [8]). The aim of this software is twofold: first, to compare and analyze the performance of nonsmooth schemes for the frictional contact problem, in terms of realism (capture of dry friction, typically), robustness, and computational efficiency. A first study of this kind was conducted in 2010-2011 onto the different rod models that were available in the software. New studies are planned for evaluating further rod models. Second, we believe such a software will help us understand the behavior of a fibrous material (such as hair) through virtual experiments, thanks to which we hope to identify and understand some important emergent phenomena. A careful validation study against experiments started to be conducted in 2011 in collaboration with physicists from L'Oréal. Once this discrete elements model will be fully validated, our ultimate goal would be to build a continuous macroscopic model for the hair medium relying on nonsmooth laws. The core of this software was transferred to L'Oréal in 2011, and to AGT Digital in early 2013, by Gilles Daviet and Florence Bertails-Descoubes. It was also used for generating a number of simulations supporting at least 4 of our research publications.

5.3.2. Cloc: super-space clothoids

Participants: Romain Casati, Florence Bertails-Descoubes.

This software implements the super-space clothoid model published this year in [25]. This model consists of a new dynamic rod primitive relying upon high-order elements with a linear curvature (clothoidal arcs). The source code of this software is distributed from our webpages from December 2013, based on a dual licensing policy: a free GPLv.3 license, mainly dedicated to academics; and a commercial license, mainly dedicated to industry.

5.3.3. APPROCHE: APPROximate Curves with HElices

Participants: Alexandre Derouet-Jourdan, Florence Bertails-Descoubes.

APPROCHE is a software that implements the 3d floating tangents algorithm published. The algorithm takes as input a set of curves, either represented as splines or sequences of points, and fits each curve to a C^1-smooth piecewise helix. This software has been transferred to L'Oréal in December 2013 and some source code will be made freely available to academics under the GPLv.3 licence.

6. New Results

6.1. Multiple impacts modelling

Participants: Bernard Brogliato, Ngoc-Son Nguyen.

The work consists of studying two systems: the rocking block and tapered chains of balls, using the Darboux-Keller model of multiple impacts previously developed. The objectives are threefold: 1) show that the model predicts well the motion by careful comparisons with experimental data found in the literature, 2) study the system's dynamics and extract critical kinetic angles that allow the engineer to predict the system's gross motion, 3) develop numerical code inside the SICONOS platform that incorporates the model of multiple impact. The influence of the kinetic angles in the rocking block motion with friction is analysed as well, numerically. Extensive experimental works have been conducted by our colleague C. Liu at PKU on a disc-ball system. Results are in [30], [24] [64], and in the monograph [15]. Another work is dedicated to analysing the influence of bilateral holonomic constraints on the well-posedness of the complementarity problem obtained from the (frictionless) unilateral constraints. Gauss' principle extension to this case is also analysed [22].

6.2. Discrete-time sliding mode control

Participants: Vincent Acary, Bernard Brogliato, Olivier Huber, Bin Wang.

This topic concerns the study of time-discretized sliding-mode controllers. Inspired by the discretization of nonsmooth mechanical systems, we propose implicit discretizations of discontinuous, set-valued controllers. This is shown to result in preservation of essential properties like simplicity of the parameters tuning, suppression of numerical chattering, reachability of the sliding surface after a finite number of steps, and disturbance attenuation by a factor h or h^2 [36]. This work is part of the ANR project CHASLIM. Within the framework of CHASLIM we have performed many experimental validations on the electropneumatic setup of IRCCyN (Nantes), which nicely confirm our theoretical and numerical predictions: the implicit implementation of sliding mode control, drastically improves the input and output chattering behaviours. In particular the high frequency bang-bang controllers which are observed with explicit discretizations, are completely suppressed.

6.3. Dissipativity preserving methods

Participants: Vincent Acary, Bernard Brogliato.

This work concerns the analysis of so-called theta-methods applied to linear complementarity systems that are dissipative (in the sense of Willems). Necessary and sufficient conditions for dissipativity preservation after the time-discretization are derived (preservation of the storage function, the supply rate and the dissipation function). The possible state jumps are also analyzed [57]. It is shown that excepted when the system is state lossless and theta = 0.5, the conditions for dissipativity preservation are very stringent. In this article we also provide (for the first time, to the best of our knowledge) a rigorous definition of numerical dissipation, which remained until now a vague notion in numerical analysis.

6.4. Lur'e set-valued dynamical systems

Participants: Bernard Brogliato, Aneel Tanwani, Christophe Prieur.

Lur'e systems are quite popular in Automatic Control since the fifties. Set-valued Lur'e systems possess a static feedback nonlinearity that is a multivalued function. This study consists in the mathematical analysis (existence and uniqueness of solutions) and the stability analysis (Lyapunov stability, invariance principle) of classes of set-valued Lur'e systems, with applications in complementarity dynamical systems, relay systems, mechanical systems with dry friction, electrical circuits, etc. Our works in this field started in [51]. The results in [53] extend those in [52] with an accurate characterization of the maximal monotonicity of the central operator of these systems, which consists of a projection-like operator. Concrete and verifiable criteria are provided for the above classes (complementarity, relay systems). Results on state observers for classes of Lur'e systems (namely: Moreau's sweeping process of first and second order, and with prox-regular sets) are proposed in [47], [39]. Therein the convexity is replaced by the far more general notion of prox-regularity, which destroys the monotonicity.

6.5. Analysis of Limit Cycles in Piecewise Linear Systems

Participants: Vincent Acary, Bernard Brogliato, Valentina Sessa.

Autonomous piecewise linear systems in the Lur'e form may exhibit periodic steady-state oscillations. For many practical systems belonging to this class the period and the shape of the oscillation is difficult to be predicted a priori. In this work the complementarity approach is used to tackle the issue. The complementarity formalism is used to represent the closed-loop system and a phase condition acting as an anchor equation for the periodic solution. By discretizing the dynamics a mixed complementarity problem is formulated. The corresponding solution provides an accurate prediction of the steady-state oscillation and its period. Numerical results show the effectiveness of the proposed technique for the computation of stable and sliding periodic solutions. The analysis of the steady-state solution of a Colpitts oscillator is considered as an illustration. This work has been presented at CDC 2013 in [37].

6.6. Simulation and stability of piecewise linear gene networks

Participants: Vincent Acary, Arnaud Tonnelier, Bernard Brogliato.

This work has been done in collaboration with the IBIS project team, it is reported in [45], [19]. Gene regulatory networks control the response of living cells to changes in their environment. A class of piecewise-linear (PWL) models, which capture the switch-like interactions between genes by means of step functions, has been found useful for describing the dynamics of gene regulatory networks. The step functions lead to discontinuities in the right-hand side of the differential equations. This has motivated extensions of the PWL models based on differential inclusions and Filippov solutions, whose analysis requires sophisticated numerical tools. We present a method for the numerical analysis of one proposed extension, called Aizerman-Pyatnitskii (AP)-extension, by reformulating the PWL models as a mixed complementarity system (MCS). This allows the application of powerful methods developed for this class of nonsmooth dynamical systems, in particular those implemented in the Siconos platform. We also show that under a set of reasonable biological assumptions, putting constraints on the right-hand side of the PWL models, AP-extensions and classical Filippov (F)-extensions are equivalent. This means that the proposed numerical method is valid for a range of different solution concepts. We illustrate the practical interest of our approach through the numerical analysis of three well-known networks developed in the field of synthetic biology.

In addition, we have investigated oscillatory regimes in repressilator-type models with piecewise linear dynamics [48]. We derived exact analytical conditions for oscillations and showed that the relative location between the dissociation constants of the Hill functions and the ratio of kinetic parameters determines the possibility of oscillatory activities. We also computed analytically the probability of oscillations. Results suggest that a switch-like coupling behaviour, a time-scale separation and a repressilator-type architecture with an even number of elements facilitate the emergence of sustained oscillations in biological systems.

6.7. Numerical analysis and simulation of mechanical systems with constraints

6.7.1. Event-capturing schemes for nonsmooth mechanical systems

Participant: Vincent Acary.

To perform the numerical time integration of nonsmooth mechanical systems, the family of event-capturing time–stepping schemes are the most robust and efficient tools. Nevertheless, they suffer from several drawbacks : a) a low–order accuracy (at best at order one), b) a drift phenomena when the unilateral constraints are treated at the velocity level and c) a poor "energetic" behavior in terms of stabilizing the high–frequency dynamics. We proposed self-adapting schemes by applying time–discontinuous Galerkin methods to the measure differential equation in [31]. In order to satisfy in discrete time, the impact law and the constraints at the position and the velocity level, an adaptation of the well-known Gear–Gupta–Leimkuhler approach has been developed in [18]. Finally, the energetic behavior of the standard Moreau–Jean scheme has been addressed in [26] by developing a Newmark–type scheme for nonsmooth dynamics.

6.7.2. Numerical time–integration methods for event-detecting schemes.

Participants: Vincent Acary, Bernard Brogliato, Mounia Haddouni.

The CIFRE thesis of M. Haddouni concerns the numerical simulation of mechanical systems subject to holonomic bilateral constraints, unilateral constraints and impacts. This work is performed in collaboration with ANSYS and the main goal is to improve the numerical time–integration in the framework of event-detecting schemes. Between nonsmooth events, time integration amounts to numerically solving a differential algebraic equations (DAE) of index 3. We have compared dedicated solvers (Explicit RK schemes, Half-explicit schemes, generalizes α-schemes) that solve reduced index formulations of these systems. Since the drift of the constraints is crucial for the robustness of the simulation through the evaluation of the index sets of active contacts, we have proposed some recommendations on the use of the solvers of dedicated to index-2 DAE. This work has been presented in [35], [40].

6.7.3. Multibody systems woth contact, friction and clearances

Participants: Vincent Acary, Bernard Brogliato, Narendra Akadkhar.

The PhD thesis of N. Akadkhar under contract with Schneider Electric concerns the numerical simulation of mechanical systems with unilateral constraints and friction, where the presence of clearances in imperfect joints plays a crucial role. A first work deals with four-bar planar mechanisms with clearances at the joints, which induce unilateral constraints and impacts, rendering the dynamics nonsmooth. The objective is to determine sets of parameters (clearance value, restitution coefficients, friction coefficients) such that the system's trajectories stay in a neighborhood of the ideal mechanism (*i.e.* without clearance) trajectories. The analysis is based on numerical simulations obtained with the projected Moreau-Jean time-stepping scheme. These results have been submitted to the ENOC 2014 conference. It is planned to extend these simulations to frictional cases and to mechanisms of circuit breakers.

6.8. Mechanical rods

6.8.1. High-order models of mechanical rods

Participants: Florence Bertails-Descoubes, Romain Casati.

Reduced-coordinate models for rods such as the articulated rigid body model or the super-helix model [50] are able to capture the bending and twisting deformations of thin elastic rods while strictly and robustly avoiding stretching deformations. In this work we are exploring new reduced-coordinate models based on a higher-order geometry. Typically, elements are defined by a polynomial curvature function of the arc length, of degree $d \geq 1$. The main difficulty compared to the super-helix model (where $d = 0$) is that the kinematics has no longer a closed form. Last year, in R. Casati's PhD's thesis, we extended this result to the full 3D case. The key idea was to integrate the rod's kinematics using power series expansion, and to design an accurate and efficient computational algorithm adapted to floating point arithmetics. Our method nicely propagates to the computation of the full dynamic of a linked chain of 3d clothoid. This year we thoroughly compared our methods against other rod models from the literature, in terms of both accuracy and computational efficiency. Our results demonstrate that our model is competitive compared to former models, and yields a better trade-off in the case of highly curly rods. All these results were published and presented this year at SIGGRAPH [25]. The source code is also freely distributed under a GPLv.3 license (see Section 5.3).

6.8.2. Inverse modeling of mechanical rods subject to frictional contact

Participants: Florence Bertails-Descoubes, Alexandre Derouet-Jourdan, Gilles Daviet.

Controlling the input shape of slender structures such as rods is desirable in many design applications (such as hairstyling, reverse engineering, etc.), but solving the corresponding inverse problem is not straightforward. In [54], [55] we noted that reduced-coordinates models such as the super-helix are well-suited for static inversion in presence of gravity.

We are facing two main difficulties: 1/ the geometrical fitting of a piecewise helix to an arbitrary input curve and 2/ the inversion a super-helix subject to gravity *and* contacting forces.

6.8.2.1. Geometrical fitting: from an arbitrary smooth curve to a C^1 piecewise helix

Participants: Florence Bertails-Descoubes, Alexandre Derouet-Jourdan.

In A. Derouet-Jourdan's PhD's thesis (co-supervised by Joëlle Thollot, EPI Maverick), we solved this problem by extending to 3d the floating tangents algorithm introduced in 2d in [54]. In this new method, only tangents are strictly interpolated while points are displaced in an optimal way so as to lie in a feasible configuration, *i.e.*, a configuration that is compatible with the interpolation by a helix. Our approach relies upon the co-helicity condition found by Ghosh [56], which was however only partially proved in [56]. To ensure the existence of the helix and prove its uniqueness in the general case, we complete the proof which serves as the basis for our reconstruction algorithm.

Our method proves to be efficient and robust as it can successfully handle large and complex datasets from real curve aquisitions, such as the capture of hair fibers or the magnetic field of a star. We also compared our method against a standard nonlinear least-squares methods. Unlike the optimization approach which often fail to converge in the case of frizzy input curves, our method remains extremely fast regardless the complexity of the input curves. The set of these results was published this year at Computer-Aided Geometric Design [28]. This work has been transferred to L'Oréal in December 2013. Some source code is also freely released for academics under the GPLv.3 license (see Section 5.3).

6.8.2.2. Inverse modeling of a super-helix assembly subject to frictional contact

Participants: Florence Bertails-Descoubes, Alexandre Derouet-Jourdan, Gilles Daviet.

In A. Derouet-Jourdan's PhD's thesis (co-supervised by Joëlle Thollot, EPI Maverick), we bring a first solution to the challenging problem consisting in identifying the intrinsic geometry of a fiber assembly under gravity and (unknown) frictional external and mutual contacts, from a single configuration geometry (a set of geometric curves). Taking an arbitrary fiber assembly geometry (such as hair) as input together with corresponding interacting meshes (such as the body mesh), we interpret the fiber assembly shape as a static equilibrium configuration of a fiber assembly simulator, in the presence of gravity as well as fiber-mesh and fiber-fiber frictional contacts. Assuming fibers parameters are homogeneous and lie in a plausible range of physical values, we show that this large, underdetermined inverse problem can be formulated as a well-posed constrained optimization problem (second-order cone quadratic program), which can be solved robustly and efficiently by leveraging the frictional contact solver of our direct simulator for fiber assemblies [8]. Our method was successfully applied to the animation of various hair geometries, ranging from synthetic hairstyles manually designed by an artist to the most recent human hair data reconstructed from capture. These results were published this year at SIGGRAPH Asia [27].

6.9. Threshold in neural models

Participant: Arnaud Tonnelier.

We studied the threshold for spike initiation in two-dimensionnal neural models. A threshold criterion that depends on both membrane voltage and recovery (or adaptation) variable is proposed. Our approach provides a simple and unified framework that can account for adapting threshold, threshold variability, dynamic threshold, inhibition-induced spike and postinhibitory facilitation. Implications on neural modeling and on neural dynamics are discussed.

6.10. Nonsmooth modes in chains of impact oscillators

Participants: Vincent Acary, Guillaume James, Franck Pérignon.

Chains of impact oscillators arise for example as finite-element models of thin oscillating mechanical structures (a string under tension or a clamped beam) contacting rigid obstacles. Nonlinear periodic waves are observed in experiments on such systems, but relatively little is known from a theoretical point of view on their existence and stability. In 2008, Gendelman and Manevitch have analyzed the existence and stability of nonlinear localized modes (breathers) for discrete linear chains with a single node undergoing rigid impacts. In this work, we introduce a numerical method allowing to compute branches of time-periodic solutions when an arbitrary number of nodes undergo rigid impacts without energy dissipation. For this purpose, we reformulate the search of periodic solutions as a boundary value problem incorporating unilateral constraints. We illustrate this numerical approach by computing different families of breathers and nonlinear normal modes. Our method is much more effective than a numerical continuation of periodic solutions based on compliant models, which requires to integrate stiff differential equations and lead to costly numerical continuation. These results have been submitted to the ENOC 2014 conference.

6.11. Traveling waves in spatially discrete excitable media

Participants: José Eduardo Morales, Arnaud Tonnelier, Guillaume James.

The propagation of traveling waves in excitable media is a widespread phenomenon, with applications ranging from forest fires to electrical signals propagating along nerve fibers. The case of spatially discrete excitable models is notoriously difficult to analyze. In particular, for the discrete FitzHugh-Nagumo reaction-diffusion system, the existence of pulses for a general class of bistable nonlinearities has been proved only recently (Hupkes and Sandstede, 2010). The existence of pulses under more general types of interactions (e.g. elastic instead of diffusive) remains an open question, as well as traveling wave propagation in higher-dimensional systems. These problems will be tackled in the PhD thesis of J.-E. Morales (advisors A. Tonnelier and G. James), which started on November 2013. J.-E. Morales has started to analyze pulse propagation in the excitable Burridge-Knopoff model, which finds applications in the context of nonlinear friction. This model includes elastic interactions between particles, and an additional difficulty linked with nonsmoothness of the (multivalued) Coulomb friction law.

6.12. Nonlinear waves in granular chains

Participants: Guillaume James, Bernard Brogliato, Ngoc-Son Nguyen.

Granular chains made of aligned beads interacting by contact (e.g. Newton's cradle) are widely studied in the context of impact dynamics and acoustic metamaterials. When a large number of beads are present, their dynamics can be described by infinite-dimensional differential equations, which possess a limited smoothness when unilateral Hertzian contact interactions are considered. In this context, we have developed and analyzed new reduced-order models describing nonlinear wave propagation in such systems. In the work [49] (collaboration with D.Pelinovsky, McMaster Univ.), we analyze small amplitude slowly modulated compression waves in the limit when the exponent of the Hertz force is close to unity. From a multiple scale analysis, we derive a new type of Korteweg-de Vries equation with logarithmic nonlinearity allowing to approximate wave profiles, in particular solitary wave solutions.

In addition the LZB model introduced in [14] has been extensively used to numerically investigate wave phenomena in chains of aligned balls (tapered, monodisperse, anti-tapered, stepped chains). Thorough comparisons with experimental results reported in the Granular Matter literature have been made. The results are reported in the monograph [15].

6.13. Robotics

6.13.1. Lexicographic Least-Squares solver

Participants: Pierre-Brice Wieber, Dimitar Dimitrov.

We have been working on Multi-Objective Least-Squares problems with inequality constraints for the last few years, focusing especially on the Lexicographic case. A previous collaboration with LAAS-CNRS and CEA-LIST led to the development of a software, SOTH , based on Complete Orthogonal Decompositions, which has become a *de facto* reference in robotics when controlling robots (mobile, manipulator or humanoid) through constraints. The focus this year in the Bipop team has been to accelerate computations by reworking the inner matrix decomposition by combining QR and LU decompositions. The resulting solver, called LexLS, is approximately 5 times faster than the previous SOTH solver on most problems. But the main result has been to show both in theory and practice that it is faster to solve a Lexicographic problem than a Weighted problem, on the contrary to popular beliefs both in robotics and optimization theory. That leads to a reversal of popular approaches that prefer to solve weighted problems (thought to be faster to solve) as approximations to lexicographic problems (thought to be slower to solve).

6.13.2. Mobile manipulation by humanoid robots

Participants: Pierre-Brice Wieber, Dimitar Dimitrov, Alexander Sherikov, Jory Lafaye.

The realization of mobile manipulation by humanoid robots requires the handling of two simultaneous problems: taking care of the dynamic balance of the robot, what is usually done with Model Predictive Control (MPC) schemes, and redundant motion and force control of the whole body of the robot, what is usually done with a Quadratic Program, or a more advanced Lexicographic Least-Squares problem (see above). These two problems are usually solved in sequence: an MPC scheme first computes the necessary motion of the feet and Center of Mass (CoM) of the robot, then motion and force redundancy of the whole body of the robot is resolved. We have observed that this sequence corresponds to a lexicographic order between two objectives, feet and CoM motion first, the rest of the body after, which limits the possibility to tackle scenarios where we would like the motion of the CoM of the robot to be driven by the motion of the rest of the body of the robot, for example to catch an object with the hand. We have proposed therefore to reorganize the order between these different objectives, building on the LexLS solver presented above.

6.13.3. Reactive trajectory generation

Participants: Pierre-Brice Wieber, Dimitar Dimitrov, Saed Al Homsy, Matthieu Guilbert.

The goal of the ongoing collaboration with Adept Technologies is to generate near time optimal trajectories in the presence of moving obstacles in real time. Results are not public yet due to industrial constraints.

6.14. Optimization

6.14.1. Semidefinite programming and combinatorial optimization

Participant: Jérôme Malick.

We have worked with Frederic Roupin (Prof. at Paris XIII) and Nathan Krislock (Assistant Prof. at North Illinois University, USA) on the use of semidefinite programming to solve combinatorial optimization problems to optimality.

We proposed a new family of semidefinite bounds for 0-1 quadratic problems with linear or quadratic constraints [61]. We have embedded the new bounds within branch-and-bound algorithms to solve 2 standard combinatorial optimization problems to optimality.

- *Max-cut.* We developed [60] an improved bounding procedure obtained by reducing two key parameters (the target level of accuracy and the stopping tolerance of the inner Quasi-Newton engine) to zero, and iteratively adding triangle inequality cuts. We also precisely analyzed its theoretical convergence properties. We show that our method outperform the state-of-the-art solver ([62]) on the large test-problems.
- *Heaviest k-subgraph problems*. Adapting the techniques we developped for the max-cut problem, we have proposed in [59] an big improvement of the first algorithm (up to 10 times faster). For the first time, we were able to solve exactly k-cluster instances of size 160. In practice, our method works particularly fine on the most difficult instances (with a large number of vertices, small density and small k).

We have also been working on a generic online semidefinite-based solver for binary quadratic problems using the generality of [61]. Finally, a first web interface for our solvers and our data sets are available online at http://lipn.univ-paris13.fr/BiqCrunch/.

6.14.2. On computing marginal prices in electricity production

Participants: Jérôme Malick, Sofia Zaourar.

Unit-commitment optimization problems in electricity production are large-scale, nonconvex and heterogeneous, but they are decomposable by Lagrangian duality. Realistic modeling of technical production constraints makes the dual objective function computed inexactly though. An inexact version of the bundle method has been dedicated to tackle this difficulty [58]. We have worked on two projects related to solving dual unit-commitment problem by inexact bundle methods.

- *Stabilization.* We observed that the computed optimal dual variables show a noisy and unstable behaviour, that could prevent their use as price indicator. We have proposed a simple and controllable way to stabilize the dual optimal solutions, by penalizing the total variation of the prices [63]. Our illustrations on the daily electricity production optimization of EDF show a strinking stabilization at a negligible cost.
- *Acceleration.* We have worked with Welington Oliveira (IMPA, Brazil) on the acceleration of inexact bundle methods by taking advantage of cheap-to-get inexact information on the objective function which comes without any tighness guarantee though. We came up with a new family of bundle methods incorporating this coarse inexact information, to get better iterates. We have studied the convergence of these method and we have conducted numerical experimentation on unit-commitment problems and on two-stage linear problems show a subtantial gain in the overall computing time. This research is about to be released in a preprint in HAL

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

- Schneider Electric: thèse de Narendra Akadkhar.
- Ansys France: thèse de Mounia Haddouni.
- Aldebaran: thèse de Jory Lafaye.
- Adept Technology: thèse de Saed al Homsi.

7.1.1. L'OREAL - contrat d'étude 2012-2013

Participant: Florence Bertails-Descoubes.

Contrat d'étude with L'Oréal, from in December 2012 until April 2013. The topic was the automatic generation of the geometry of a hair wisp given some statistical properties such as density or curliness distribution.

7.1.2. AGT Digital - contrat de collaboration de recherche et de transfert 2013

Participants: Florence Bertails-Descoubes, Gilles Daviet.

Contrat de collaboration de recherche et de transfert with AGT Digital, from January 2013 until August 2013. AGT-Digital is a French start-up localized in Paris and specialized in the production of virtual hair models for the entertainment industry as well as for virtual hairstyling applications. The goal of this project was to transfer our work on the simulation of fiber assemblies suject to frictional contact [8] as well as to develop new features in line with the production pipeline under the Maya software. Gilles Daviet was hired on this project during 6 months as an Inria engineer to perform these software developments.

7.1.3. L'OREAL - contrat de collaboration de recherche et de transfert 2013-2014

Participants: Florence Bertails-Descoubes, Alexandre Derouet-Jourdan.

Contrat de collaboration de recherche et de transfert with L'Oréal, from October 2013 until April 2014. The goal was to transfer software corresponding to our recent work on the inversion of isolated fibers under gravity [54],[28] (especially the APPROCHE source code) while ensuring compatibility between different software.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

- CHASLIM Chattering Free Sliding Mode Control: ANR BLAN 2011 BS03 007 01 (octobre 2011–octobre 2015), coordinator B. Brogliato.

8.2. International Research Visitors

- Valentina Sessa from the University of Benevento, Italy, DIS, a six–month internship as a PhD student under the supervision of V. Acary and B. Brogliato,.

8.2.1. Visits of International Scientists

Visit of Prof. Yury Starovetsky from Technion, Israel, four weeks in 2013.

9. Dissemination

9.1. Scientific Animation

- B. Brogliato member of IPC of the IFAC Nonlinear Systems Conference NOLCOS 2013, Toulouse, September 2013. Reviewer for IEEE Trans. Automatic Control, SIAM J. Control Optimization, Multibody System Dynamics, European J. of Mechanics A/Solids, Automatica, IMA J. Applied Mathematics, etc.
- F. Bertails-Descoubes was a member of the international technical Program Committee of ACM SIGGRAPH 2013, and a member of the international technical Program Committee of Eurographics 2014. She has been a reviewer in 2013 for ACM Transactions on Graphics and Computer Aided Geometric Design.
- V. Acary is co-animator (with R. Leine ETH Zurich) of the European network for nonsooth dynamics. Member of the ENOC (EUROMECH Nonlinear Oscillations Conference) comittee. Reviewer in 2013 for Math reviews, Computer Methods in Applied Mechanics and Engineering, International Journal for Numerical Methods in Engineering, ASME Journal of Computational and Nonlinear Dynamics, Applied numerical Mathematics, American Control Conference, 52st IEEE Conference on Decision and Control (CDC 2013)

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master: Bernard Brogliato, Nonsmooth Dynamical Systems, 15 h. TD, M2, université de Limoges, France

Master: F. Bertails-Descoubes, Optimisation numérique, 30h equiv. TD, niveau M1, ENSIMAG, Grenoble INP

Master: J. Malick, Optimisation numérique, 60h equiv. TD, niveau M1, ENSIMAG, Grenoble INP

Master: Vincent Acary, Simulation of Nonsmooth Dynamical Systems, 9 h. TD, M2, université de Limoges, France

9.2.2. Supervision

PhD : Alexandre Derouet-Jourdan, *Inversion statique de fibres : de la géométrie de courbes 3d à l'équilibre d'une assemblée de tiges mécaniques en contact frottant*, Grenoble University, 7 novembre 2013, Florence Bertails-Descoubes et Joëlle Thollot

PhD in progress : Mounia Haddouni, 01 mai 2012, Vincent Acary et Bernard Brogliato

PhD in progress : Olivier Huber, 01 octobre 2011, Vincent Acary et Bernard Brogliato

PhD in progress : Narendra Akahdkar, 01 décembre 2012, Vincent Acary et Bernard Brogliato

PhD in progress : Sofia Zaourar, 01 octobre 2011, Jérôme Malick et Bernard Brogliato

PhD in progress : Romain Casati, 01 octobre 2011, Florence Bertails-Descoubes et Bernard Brogliato

PhD in progress : Alejandro Blumentals, 01 octobre 2013, Florence Bertails-Descoubes et Bernard Brogliato

PhD in progress : Gilles Daviet, 01 octobre 2013, Florence Bertails-Descoubes, Bruno Raffin (LIG) et Pierre Saramito (LJK)

PhD in progress : Federico Pierucci, 01 octobre 2012, Jérôme Malick et Zaid Harchaoui et Anatoli Ioudilski

PhD in progress : Saed al Homsi, 01 octobre 2012, Pierre-Brice Wieber et Bernard Brogliato

PhD in progress : Jory Lafaye, 01 octobre 2012, Pierre-Brice Wieber et Bernard Brogliato

PhD in progress : Jose Eduardo Morales Morales, *Travelling pulses in spatially discrete excitable media*, 15 november, 2013, Arnaud Tonnelier and Guillaume James

9.2.3. Juries

- B. Brogliato, jury HdR de J. Bastien, Université Lyon 1, rapporteur
- B. Brogliato, jury de thèse de A. Thorin, LMSX, rapporteur
- B. Brogliato, jury HdR de A. Girard, Grenoble Université, président
- V. Acary, jury de thèse de H. Rammal, Université de Limoges, examinateur

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