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Activity Report 2013

## **Project-Team FLUMINANCE**

Fluid Flow Analysis, Description and Control  
from Image Sequences

RESEARCH CENTER  
**Rennes - Bretagne-Atlantique**

THEME  
**Earth, Environmental and Energy  
Sciences**



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# Project-Team FLUMINANCE

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## 2. Overall Objectives

### 2.1. Overall Objectives

The research group that we have entitled FLUMINANCE from a contraction between the words “Fluid” and “Luminance” is dedicated to the extraction of information on fluid flows from image sequences and to the development of tools for the analysis and control of these flows. The objectives of the group are at the frontiers of several important domains. The group aims at providing in the one hand image sequence methods devoted to the analysis and description of fluid flows and in the other hand physically consistent models and operational tools to extract meaningful features characterizing or describing the observed flow and enabling decisions or actions. Such a twofold goal is of major interest for the inspection, the analysis and the monitoring of complex fluid flows, but also for control purpose of specific flows involved in industrial problems. To reach these goals we will mainly rely on data assimilation strategies and on motion measurement techniques. From a methodological point of view, the techniques involved for image analysis are either stochastic or variational. One of the main originality of the FLUMINANCE group is to combine cutting-edge researches on these methods with an ability to conduct proper intensive experimental validations on prototype flows mastered in laboratory. The scientific objectives decompose in three main themes:

- **Fluid flows characterization from images**

In this first axis, we aim at providing accurate measurements and consistent analysis of complex fluid flows through image analysis techniques. The application domain ranges from industrial processes and experimental fluid mechanics to environmental. This theme includes also the use of non-conventional imaging techniques such as Schlieren techniques, Shadowgraphs, holography. The objective will be here to go towards 3D dense velocity measurements.

- **Coupling dynamical model and image data**

We focus here on the study, through image data, of complex and partially known fluid flows involving complex boundary conditions, multi-phase fluids, fluids and structures interaction problems. Our credo is that image analysis can provide sufficiently fine observations on small and medium scales to construct models which, applied at medium and large scale, account accurately for a wider range of the dynamics scales. The image data and a sound modeling of the dynamical uncertainty at the observation scale should allow us to reconstruct the observed flow and to provide efficient real flows (experimental or natural) based dynamical modeling. Our final goal will be to go towards a 3D reconstruction of real flows, or to operate large motion scales simulations that fit real world flow data and incorporate an appropriate uncertainty modeling.

- **Control and optimization of turbulent flows**

We are interested on active control and more precisely on closed-loop control. The main idea is to extract reliable image features to act on the flow. This approach is well known in the robot control community, it is called visual servoing. More generally, it is a technique to control a dynamic system from image features. We plan to apply this approach on flows involved in various domains such as environment, transport, microfluidic, industrial chemistry, pharmacy, food industry, agriculture, etc.

## 3. Research Program

### 3.1. Estimation of fluid characteristic features from images

The measurement of fluid representative features such as vector fields, potential functions or vorticity maps, enables physicists to have better understanding of experimental or geophysical fluid flows. Such measurements date back to one century and more but became an intensive subject of research since the emergence of correlation techniques [35] to track fluid movements in pairs of images of a particles laden fluid or by the way of clouds photometric pattern identification in meteorological images. In computer vision, the estimation of the projection of the apparent motion of a 3D scene onto the image plane, referred to in the literature as optical-flow, is an intensive subject of researches since the 80's and the seminal work of B. Horn and B. Schunk [48]. Unlike to dense optical flow estimators, the former approach provides techniques that supply only sparse velocity fields. These methods have demonstrated to be robust and to provide accurate measurements for flows seeded with particles. These restrictions and their inherent discrete local nature limit too much their use and prevent any evolutions of these techniques towards the devising of methods supplying physically consistent results and small scale velocity measurements. It does not authorize also the use of scalar images exploited in numerous situations to visualize flows (image showing the diffusion of a scalar such as dye, pollutant, light index refraction, fluorocein,...). At the opposite, variational techniques enable in a well-established mathematical framework to estimate spatially continuous velocity fields, which should allow more properly to go towards the measurement of smaller motion scales. As these methods are defined through PDE's systems they allow quite naturally including as constraints kinematic properties or dynamic laws governing the observed fluid flows. Besides, within this framework it is also much easier to define characteristic features estimation procedures on the basis of physically grounded data model that describes the relation linking the observed luminance function and some state variables of the observed flow.

A substantial progress has been done in this direction with the design of dedicated dense estimation techniques to estimate dense fluid motion fields [4], [10], the setting up of tomographic techniques to carry out 3D velocity measurements [42], the inclusion of physical constraints to infer 3D motions in atmospheric satellite images [8] or the design of dynamically consistent velocity measurements to provide coherent motion fields from time resolved fluid flow image sequences [9]. These progresses have brought further accuracy and an improved spatial resolution for a variety of applications ranging from experimental fluid mechanics to geophysical sciences. For a detailed review of these approaches see [6].

We believe that such approaches must be first enlarged to the wide variety of imaging modalities enabling the observation of fluid flows. This covers for instance, the systematic study of motion estimation for the different channels of meteorological satellites, but also of other experimental imaging tools such as Shadowgraphs, Background oriented Schlieren, Schlieren [55], diffusive scalar images, fluid holography [56], or Laser Induced Fluorimetry. All these modalities offer the possibility to visualize time resolved sequences of the flow. The velocity measurement processes available to date for that kind of images suffer from a lack of physical relevancy to keep up with the increasing amount of fine and coherent information provided by the images. We think, and have begun to prove, that a significant step forward can be taken by providing new tools based on sound data models and adapted regularization functional, both built on physical grounds.

Additional difficulties arise when considering the necessity to go towards 3D measurements and 3D volumetric reconstruction of the observed flows (e.g., the tomographic PIV paradigm). First, unlike in the standard setup, the 2D images captured by the experimentalists only provide a partial information about the structure of the particles transported by the fluid. As a matter of fact, inverse problems have to be solved in order to recover this crucial information. Secondly, another issue stands in the increase of the underdetermination of the problem, that is the important decrease of the ratio between the number of observations and the total number of unknowns. In particular, this point asks for methodologies able to gather and exploit observations captured at different time instants. Finally, the dimensions of the problem (that is, the number of unknown) dramatically increase with the transition from the 2D to the 3D paradigm. This leads, as a by-product, to a significant amplification of the computational burden and requires the conception of efficient algorithms, exhibiting a reasonable scaling with the problem dimensions.

The first problem can be addressed by resorting to state-of-the-art methodologies pertaining to sparse representations. These techniques consist in identifying the solution of an inverse problem with the most “zero” components which, in the case of the tomographic PIV, turns out to be a physically relevant option. Hence, the design of sparse representation algorithms and the study of their conditions of success constitute an important research topic of the group. On the other hand, we believe that the dramatic increase of the under-determination appearing in the 3D setup can be tackled by combining tomographic reconstruction of several planar views of the flow with data assimilation techniques. These techniques enable to couple a dynamical model with incomplete observations of the flow. Each applicative situation under concern defines its proper required scale of measurement and a scale for the dynamical model. For instance, for control or monitoring purposes, very rapid techniques are needed whereas for analysis purpose the priority is to get accurate measurements of the smallest motion scales as possible. These two extreme cases imply the use of different models but also of different algorithmic techniques. Recursive techniques and large scale representation of the flow are relevant for the first case whereas batch techniques relying on the whole set of data available and models refined down to small scales have to be used for the latter case.

The question of the scale of the velocity measurement is also an open question that must be studied carefully. Actually, no scale considerations are taken into account in the estimation schemes. It is more or less abusively assumed that the measurements supplied have a subpixel accuracy, which is obviously erroneous due to implicit smoothness assumptions made either in correlation techniques or in variational estimation techniques. We are convinced that to go towards the measurement of the smaller scales of the flow it is necessary to introduce some turbulence or uncertainty subgrid modeling within the estimation scheme and also to devise alternative regularization schemes that fit well with phenomenological statistical descriptions of turbulence described by the velocity increments moments. As a by product such schemes should offer the possibility to have a direct characterization, from image sequences, of the flow turbulent regions in term of vortex tube,

area of pure straining, or vortex sheet. This philosophy should allow us to elaborate methods enabling the estimation of relevant characteristics of the turbulence like second-order structure functions, mean energy dissipation rate, turbulent viscosity coefficient, or dissipative scales.

We are planning to study these questions for a wide variety of application domains ranging from experimental fluid mechanics to geophysical sciences. We believe there are specific needs in different application domains that require clearly identified developments and modeling. Let us for instance mention meteorology and oceanography which both involve very specific dynamical modeling but also micro-fluidic applications or bio-fluid applications that are ruled by other types of dynamics.

### 3.2. Data assimilation and Tracking of characteristic fluid features

Real flows have an extent of complexity, even in carefully controlled experimental conditions, which prevents any set of sensors from providing enough information to describe them completely. Even with the highest levels of accuracy, space-time coverage and grid refinement, there will always remain at least a lack of resolution and some missing input about the actual boundary conditions. This is obviously true for the complex flows encountered in industrial and natural conditions, but remains also an obstacle even for standard academic flows thoroughly investigated in research conditions.

This unavoidable deficiency of the experimental techniques is nevertheless more and more compensated by numerical simulations. The parallel advances in sensors, acquisition, treatment and computer efficiency allow the mixing of experimental and simulated data produced at compatible scales in space and time. The inclusion of dynamical models as constraints of the data analysis process brings a guaranty of coherency based on fundamental equations known to correctly represent the dynamics of the flow (e.g. Navier Stokes equations) [3], [5].

Conversely, the injection of experimental data into simulations ensures some fitting of the model with reality. When used with the correct level of expertise to calibrate the models at the relevant scales, regarding data validity and the targeted representation scale, this collaboration represents a powerful tool for the analysis and reconstruction of the flows. Automated back and forth sequencing between data integration and calculations have to be elaborated for the different types of flows with a correct adjustment of the observed and modeled scales. This appears more and more feasible when considering the sensitivity, the space resolution and above all the time resolution that the imaging sensors are reaching now.

That becomes particularly true, for instance, for satellite imaging, the foreseeable advances of which will soon give the right complement to the progresses in atmospheric and ocean modeling to dramatically improve the analysis and predictions of physical states and streams for weather and environment monitoring. In that domain, there is a particular interest in being able to combine image data, models and in-situ measurements, as high densities of data supplied by meteorological stations are available only for limited regions of the world, typically Europe and USA, while Africa, or the south hemisphere lack of refined and frequent *in situ* measurements. Moreover, we believe that such an approach can favor great advances in the analysis and prediction of complex flows interactions like those encountered in sea-atmosphere interactions, dispersion of polluting agents in seas and rivers, etc. In other domains we believe that image data and dynamical models coupling may bring interesting solutions for the analysis of complex phenomena which involve multi-phasic flows, interaction between fluid and structures, and the general case of flows with complex unknown border conditions.

The coupling approach can be extended outside the fluidics domain to complex dynamics that can be modeled either from physical laws or from learning strategies based on the observation of previous events [1]. This concerns for instance forest combustion, the analysis of the biosphere evolution, the observation and prediction of the melting of pack ice, the evolution of sea ice, the study of the consequences of human activity like deforestation, city growing, landscape and farming evolution, etc. All these phenomena are nowadays rapidly evolving due to global warming. The measurement of their evolution is a major societal interest for analysis purpose or risk monitoring and prevention.



To enable data and models coupling to achieve its potential, some difficulties have to be tackled. It is in particular important to outline the fact that the coupling of dynamical models and image data are far from being straightforward. The first difficulty is related to the space of the physical model. As a matter of fact, physical models describe generally the phenomenon evolution in a 3D Cartesian space whereas images provides generally only 2D tomographic views or projections of the 3D space on the 2D image plane. Furthermore, these views are sometimes incomplete because of partial occlusions and the relations between the model state variables and the image intensity function are otherwise often intricate and only partially known. Besides, the dynamical model and the image data may be related to spatio-temporal scale spaces of very different natures which increases the complexity of an eventual multiscale coupling. As a consequence of these difficulties, it is necessary generally to define simpler dynamical models in order to assimilate image data. This redefinition can be done for instance on an uncertainty analysis basis, through physical considerations or by the way of data based empirical specifications. Such modeling comes to define inexact evolution laws and leads to the handling of stochastic dynamical models. The necessity to make use and define sound approximate models, the dimension of the state variables of interest and the complex relations linking the state variables and the intensity function, together with the potential applications described earlier constitute very stimulating issues for the design of efficient data-model coupling techniques based on image sequences.

On top of the problems mentioned above, the models exploited in assimilation techniques often suffer from some uncertainties on the parameters which define them. Hence, a new emerging field of research focuses on the characterization of the set of achievable solutions as a function of these uncertainties. This sort of characterization indeed turns out to be crucial for the relevant analysis of any simulation outputs or the correct interpretation of operational forecasting schemes. In this context, the tools provided by the Bayesian theory play a crucial role since they encompass a variety of methodologies to model and process uncertainty. As a consequence, the Bayesian paradigm has already been present in many contributions of the Fluminance group in the last years and will remain a cornerstone of the new methodologies investigated by the team in the domain of uncertainty characterization.

This wide theme of research problems is a central topic in our research group. As a matter of fact, such a coupling may rely on adequate instantaneous motion descriptors extracted with the help of the techniques studied in the first research axis of the FLUMINANCE group. In the same time, this coupling is also essential with respect to visual flow control studies explored in the third theme. The coupling between a dynamics and data, designated in the literature as a Data Assimilation issue, can be either conducted with optimal control techniques [50], [51] or through stochastic filtering approaches [43], [46]. These two frameworks have their own advantages and deficiencies. We rely indifferently on both approaches.

### 3.3. Optimization and control of fluid flows with visual servoing

Fluid flow control is a recent and active research domain. A significant part of the work carried out so far in that field has been dedicated to the control of the transition from laminarity to turbulence. Delaying, accelerating or modifying this transition is of great economical interest for industrial applications. For instance, it has been shown that for an aircraft, a drag reduction can be obtained while enhancing the lift, leading consequently to limit fuel consumption. In contrast, in other application domains such as industrial chemistry, turbulence phenomena are encouraged to improve heat exchange, increase the mixing of chemical components and enhance chemical reactions. Similarly, in military and civilians applications where combustion is involved, the control of mixing by means of turbulence handling rouses a great interest, for example to limit infra-red signatures of fighter aircraft.

Flow control can be achieved in two different ways: passive or active control. Passive control provides a permanent action on a system. Most often it consists in optimizing shapes or in choosing suitable surfacing (see for example [39] where longitudinal riblets are used to reduce the drag caused by turbulence). The main problem with such an approach is that the control is, of course, inoperative when the system changes. Conversely, in active control the action is time varying and adapted to the current system's state. This approach requires an external energy to act on the system through actuators enabling a forcing on the flow through for instance blowing and suction actions [58], [45]. A closed-loop problem can be formulated as an optimal control

issue where a control law minimizing an objective cost function (minimization of the drag, minimization of the actuators power, etc.) must be applied to the actuators [36]. Most of the works of the literature indeed comes back to open-loop control approaches [53], [47], [52] or to forcing approaches [44] with control laws acting without any feedback information on the flow actual state. In order for these methods to be operative, the model used to derive the control law must describe as accurately as possible the flow and all the eventual perturbations of the surrounding environment, which is very unlikely in real situations. In addition, as such approaches rely on a perfect model, a high computational costs is usually required. This inescapable pitfall has motivated a strong interest on model reduction. Their key advantage being that they can be specified empirically from the data and represent quite accurately, with only few modes, complex flows' dynamics. This motivates an important research axis in the Fluminance group.

Another important part of the works conducted in Fluminance concerns the study of closed-loop approaches, for which the convergence of the system to a target state is ensured even in the presence of errors (related either to the flow model, the actuators, or the sensors) [41]. However, designing a closed loop control law requires the use of sensors that are both non-intrusive, accurate and adapted to the time and spacial scales of the phenomenon to monitor. Such sensors are unfortunately hardly available in the context of flow control. The only sensors currently used are wall sensors located in a limited set of measurement points [37], [40]. The difficulty is then to reconstruct the entire state of the controlled system from a model based only on the few measurements available on the walls [49]. Instead of relying on sparse measurements, we propose to use denser features estimated from images. With the capabilities of up-to-date imaging sensors, we can expect an improved reconstruction of the flow (both in space and time) enabling the design of efficient image based control laws. This formulation is referred to as visual servoing control scheme.

Visual servoing is a widely used technique for robot control. It consists in using data provided by a vision sensor for controlling the motions of a robot [38]. This technique, historically embedded in the larger domain of sensor-based control [54], can be properly used to control complex robotic systems or, as we showed it recently, flows [57].

Classically, to achieve a visual servoing task, a set of visual features,  $s$ , has to be selected from visual measurements,  $m$ , extracted from a current image. A control law is then designed so that these visual features reach a desired value,  $s^*$ , related to the target state of the system. The control principle consists in regulating to zero the error vector:  $e = s - s^*$ . To build the control law, the knowledge of the so-called *interaction matrix*  $L_s$  is usually required. This matrix links the time variation of  $s$  to the signal command  $u$ . However, computing this matrix in the context of flow control is far more complex than in the case of robot control as flows are associated to chaotic nonlinear systems living in infinite dimensional spaces. As such, it is possible to formalize the model through a Galerkin projection in terms of an ODE system for which classical control laws can be applied. It is also possible to express the system with finite difference approximations and to use discrete time control algorithms amenable to modern micro-controllers. Alternatively, one may develop control methods directly on the infinite dimensional system and then finally discretize the resulting process for implementation purpose. Each approach has its own advantages and drawbacks. For the first two, known control methods can be used at the expense of a great sensibility to space discretization. The last one is less sensitive to discretization errors but more difficult to set up. These practical issues and their related theoretical difficulties make this study a very interesting field of research.

## 4. Application Domains

### 4.1. Introduction

By designing new approaches for the analysis of fluid-image sequences the FLUMINANCE group aims at contributing to several application domains of great interest for the community and in which the analysis of complex fluid flows plays a central role. The group focuses mainly on two broad application domains:

- Environmental sciences;
- Experimental fluid mechanics and industrial flows.

We detail hereafter these two application domains.

## 4.2. Environmental sciences

The first huge application domain concerns all the sciences that aim at observing the biosphere evolution such as meteorology, climatology or oceanography but also remote sensing study for the monitoring of meteorological events or human activities consequences. For all these domains image analysis is a practical and unique tool to *observe, detect, measure, characterize or analyze* the evolution of physical parameters over a large domain. The design of generic image processing techniques for all these domains might offer practical software tools to measure precisely the evolution of fluid flows for weather forecasting or climatology studies. It might also offer possibilities of close surveillance of human and natural activities in sensible areas such as forests, river edges, and valley in order to monitor pollution, floods or fire. The need in terms of local weather forecasting, risk prevention, or local climate change is becoming crucial for our tomorrow's life. At a more local scale, image sensors may also be of major utility to analyze precisely the effect of air curtains for safe packaging in agro-industrial.

## 4.3. Experimental fluid mechanics and industrial flows

In the domain of **experimental fluid mechanics**, the visualization of fluid flows plays a major role, especially for turbulence study since high frequency imaging has been made currently available. Together with analysis of turbulence at different scales, one of the major goals pursued at the moment by many scientists and engineers consists in studying the ability to manipulate a flow to induce a desired change. This is of huge technological importance to enhance or inhibit mixing in shear flows, improve energetic efficiency or control the physical effects of strain and stresses. This is for instance of particular interest for:

- military applications, for example to limit the infra-red signatures of fighter aircraft;
- aeronautics and transportation, to limit fuel consumption by controlling drag and lift effects of turbulence and boundary layer behavior;
- industrial applications, for example to monitor flowing, melting, mixing or swelling of processed materials, or preserve manufactured products from contamination by airborne pollutants, or in industrial chemistry to increase chemical reactions by acting on turbulence phenomena.

# 5. Software and Platforms

## 5.1. DenseMotion software - Estimation of 2D dense motion fields

**Participants:** Thomas Corpetti, Patrick Héas, Etienne Mémin.

This code allows the computation from two consecutive images of a dense motion field. The estimator is expressed as a global energy function minimization. The code enables the choice of different data models and different regularization functionals depending on the targeted application. Generic motion estimators for video sequences or fluid flows dedicated estimators can be set up. This software allows in addition the users to specify additional correlation based matching measurements. It enables also the inclusion of a temporal smoothing prior relying on a velocity vorticity formulation of the Navier-Stoke equation for Fluid motion analysis applications. The different variants of this code correspond to research studies that have been published in IEEE transaction on Pattern Analysis and machine Intelligence, Experiments in Fluids, IEEE transaction on Image Processing, IEEE transaction on Geo-Science and Remote Sensing. The binary of this code can be freely downloaded on the FLUID web site <http://fluid.irisa.fr>.

## 5.2. 2DLayeredMotion software - Estimation of 2D independent mesoscale layered atmospheric motion fields

**Participants:** Patrick Héas, Etienne Mémin.

This software enables to estimate a stack of 2D horizontal wind fields corresponding to a mesoscale dynamics of atmospheric pressure layers. This estimator is formulated as the minimization of a global energy function. It relies on a vertical decomposition of the atmosphere into pressure layers. This estimator uses pressure data and classification clouds maps and top of clouds pressure maps (or infra-red images). All these images are routinely supplied by the EUMETSAT consortium which handles the Meteosat and MSG satellite data distribution. The energy function relies on a data model built from the integration of the mass conservation on each layer. The estimator also includes a simplified and filtered shallow water dynamical model as temporal smoother and second-order div-curl spatial regularizer. The estimator may also incorporate correlation-based vector fields as additional observations. These correlation vectors are also routinely provided by the Eumetsat consortium. This code corresponds to research studies published in IEEE transaction on Geo-Science and Remote Sensing. It can be freely downloaded on the FLUID web site <http://fluid.irisa.fr>.

### 5.3. 3DLayeredMotion software - Estimation of 3D interconnected layered atmospheric motion fields

**Participants:** Patrick Héas, Etienne Mémin.

This software extends the previous 2D version. It allows (for the first time to our knowledge) the recovery of 3D wind fields from satellite image sequences. As with the previous techniques, the atmosphere is decomposed into a stack of pressure layers. The estimation relies also on pressure data and classification clouds maps and top of clouds pressure maps. In order to recover the 3D missing velocity information, physical knowledge on 3D mass exchanges between layers has been introduced in the data model. The corresponding data model appears to be a generalization of the previous data model constructed from a vertical integration of the continuity equation. This research study has been published in IEEE trans. on Geo-Science and Remote Sensing. The binary of this code can be freely downloaded on the FLUID web site <http://fluid.irisa.fr>.

### 5.4. Low-Order-Motion - Estimation of low order representation of fluid motion

**Participants:** Anne Cuzol, Etienne Mémin.

This code enables the estimation of a low order representation of a fluid motion field from two consecutive images. The fluid motion representation is obtained using a discretization of the vorticity and divergence maps through regularized Dirac measure. The irrotational and solenoidal components of the motion fields are expressed as linear combinations of basis functions obtained through the Biot-Savart law. The coefficient values and the basis function parameters are formalized as the minimizer of a functional relying on an intensity variation model obtained from an integrated version of the mass conservation principle of fluid mechanics. Different versions of this estimation are available. The code which includes a Matlab user interface can be downloaded on the FLUID web site <http://fluid.irisa.fr>. This program corresponds to a research study that has been published in the International Journal on computer Vision.

## 6. New Results

### 6.1. Fluid motion estimation

#### 6.1.1. Stochastic uncertainty models for motion estimation

**Participants:** Sébastien Beyou, Etienne Mémin, Emmanuel Saunier.

In this study we have proposed a stochastic formulation of the brightness consistency used principally in motion estimation problems. In this formalization the image luminance is modeled as a continuous function transported by a flow known only up to some uncertainties. Stochastic calculus then enables to built conservation principles which take into account the motion uncertainties. These uncertainties defined either from isotropic or anisotropic models can be estimated jointly to the motion estimates. Such a formulation besides providing estimates of the velocity field and of its associated uncertainties allows us to naturally define a linear multiresolution scale-space framework. The corresponding estimator, implemented within a local least squares approach, has shown to improve significantly the results of the corresponding deterministic estimator (Lucas and Kanade estimator). This fast local motion estimator provides results that are of the same order of accuracy than state-of-the-art dense fluid flow motion estimator for particle images. The uncertainties estimated supply a useful piece of information in the context of data assimilation. This ability has been exploited to define multiscale incremental data assimilation filtering schemes. This work has been recently published in *Numerical Mathematics: Theory, Methods and Applications* [14]. It is also described in Sébastien Beyou's PhD dissertation [11]. The development of an efficient GPU based version of this estimator recently started through the Inria ADT project FLUMILAB

### 6.1.2. 3D flows reconstruction from image data

**Participants:** Ioana Barbu, Kai Berger, Cédric Herzet, Etienne Mémin.

Our work focuses on the design of new tools for the estimation of 3D turbulent flow motion in the experimental setup of Tomo-PIV. This task includes both the study of physically-sound models on the observations and the fluid motion, and the design of low-complexity and accurate estimation algorithms. On the one hand, we investigate state-of-the-art methodologies such as "sparse representations" for the characterization of the observation and fluid motion models. On the other hand, we place the estimation problem into a probabilistic Bayesian framework and use state-of-the-art inference tools to effectively exploit the strong time-dependence on the fluid motion. In our previous work, we have focussed on the problem of reconstructing the particle positions from several two-dimensional images. Our approach was based on the exploitation of a particular family of sparse representation algorithms, leading to a good trade-off between performance and complexity. Moreover, we also tackled the problem of estimating the 3D velocity field of the fluid flow from two instances of reconstructed volumes of particles. Our approach was based on a generalization of the well-known Lucas-Kanade's motion estimator to 3D problems. A potential strength of the proposed approach is the possibility to consider a fully parallelized (and therefore very fast) hardware implementation. This year, we have focused on the design of new methodologies to jointly estimate the volume of particles and the velocity field from the received image data. Our approach is based on the minimization (with respect to both the position of the particles and the velocity field) of a cost function penalizing both the discrepancies with respect to a conservation equation and some prior estimates of particle positions. This work has led to one publication in an international conference (PIV13) [27] and one publication in a national conference (Fluvisu13) [31].

Since October 2013, with our new postdoctoral fellow Kai Berger, we have started a new direction of research targeting the volume reconstruction problem. In particular, we address the question of devising effective reconstruction procedures taking into account the limited computational budget available in practice. Our approach is based on the design of simple thresholding operators, allowing to reduce the dimension of the initial problem and amenable to fast parallel implementations.

### 6.1.3. Motion estimation techniques for turbulent fluid flows

**Participants:** Patrick Héas, Dominique Heitz, Cédric Herzet, Etienne Mémin.

In this study we have devised smoothing functional adapted to the multiscale structure of homogeneous turbulent flows. These regularization constraints ensue from a classical phenomenological description of turbulence. The smoothing is in practice achieved by imposing some scale-invariance principles between histograms of motion increments computed at different scales. Relying on a Bayesian formulation, an inference technique, based on likelihood maximization and marginalization of the motion variable, has been proposed to jointly estimate the fluid motion, the regularization parameters and a proper physical models. The performance of the proposed Bayesian estimator has been assessed on several image sequences depicting synthetic and

real turbulent fluid flows. The results obtained in the context of fully developed turbulence show that an improvement in terms of small-scale motion estimation can be achieved as compared to classical motion estimator. This work, performed within a collaboration with Pablo Mininni from the university of Buenos Aires, have been published in the IEEE Transactions on Pattern Analysis And Machine Learning [22].

#### 6.1.4. *Wavelet basis for multiscale motion estimation*

**Participants:** Patrick Héas, Cédric Herzet, Etienne Mémin.

In this study we focused on the implementation of a simple wavelet-based optical-flow motion estimator dedicated to the recovery of fluid motions. The wavelet representation of the unknown velocity field is considered. This scale-space representation, associated to a simple gradient-based optimization algorithm, sets up a natural multiscale/multigrid optimization framework for the optical flow estimation that can be combined to more traditional incremental multiresolution approaches. Moreover, a very simple closure mechanism, approximating locally the solution by high-order polynomials, is provided by truncating the wavelet basis at intermediate scales. This offers a very interesting alternative to traditional Particle Image Velocimetry techniques. As another alternative to this medium-scale estimator, we explored strategies to define estimation at finer scales. These strategies rely on the encoding of high-order smoothing functional on divergence free wavelet basis. This study has been published in the journal of Numerical Mathematics: Theory, Methods and Applications [19] and in the international Journal of Computer Vision [23]. This work has strongly benefited from a collaboration with Souleyman Kadri-Harouna (University of La Rochelle and who was formerly on a post-doctoral position in our team). The divergence free wavelets basis proposed in [24] constitutes the building blocks on which we have elaborated our wavelet based motion estimation solutions. We have otherwise pursue our collaboration with Chico university through the post-doc of Pierre Dérian on the GPU implementation of such motion estimator for Lidar data.

#### 6.1.5. *Sparse-representation algorithms*

**Participant:** Cédric Herzet.

The paradigm of sparse representations is a rather new concept which turns out to be central in many domains of signal processing. In particular, in the field of fluid motion estimation, sparse representation appears to be potentially useful at several levels: i) it provides a relevant model for the characterization of the velocity field in some scenarios; ii) it plays a crucial role in the recovery of volumes of particles in the 3D Tomo-PIV problem.

Unfortunately, the standard sparse representation problem is known to be NP hard. Therefore, heuristic procedures have to be devised to try and access to the solution of this problem. Among the popular methods available in the literature, one can mention orthogonal matching pursuit, orthogonal least squares and the family of procedures based on the minimization of  $\ell_p$  norms. In order to assess and improve the performance of these algorithms, theoretical works have been undertaken in order to understand under which conditions these procedures can succeed in recovering the "true" sparse vector.

This year, we have contributed to this research axis by deriving conditions of success for the algorithms mentioned above when some partial information is available about the position of the nonzero coefficients in the sparse vector. This paradigm is of interest in the Tomographic-PIV volume reconstruction problem: one can indeed expect volumes of particles at two successive instants to be pretty similar; any estimate of the position of the particles at one given instant can therefore serve as a prior estimate about their position at the next instant. The conditions of success of such procedure have been rigorously formalized in two publications in the IEEE Transactions on Information Theory [21], [26] and one publication in an international conference (SPARS13) [28].

## 6.2. Tracking, Data assimilation and model-data coupling

### 6.2.1. *Stochastic filtering for fluid motion tracking*

**Participants:** Sébastien Beyou, Anne Cuzol, Etienne Mémin.

Within the PhD thesis of Sébastien Beyou [11], we investigated the study of a recursive Bayesian filter for tracking velocity fields of fluid flows. We resort in this study to Monte-Carlo approximations based on the particle filtering paradigm. In particular, we investigated the use of the so-called ensemble Kalman filtering for fluid tracking problems. This kind of filters introduced for the analysis of geophysical fluids is based on the Kalman filter update equations. Nevertheless, unlike traditional Kalman filtering setting, the covariances of the estimation errors, required to compute the so-called Kalman gain, relies on an ensemble of forecasts. Such a process gives rise to a Monte-Carlo approximation for a family of non-linear stochastic filters enabling to handle state spaces of large dimension. The method we proposed can be seen as an extension of this technique that combines sequential importance sampling and the propagation law of an ensemble Kalman filter. This technique leads to an ensemble Kalman filter with an improved efficiency. Within this type of scheme, we have in particular investigated the introduction of a nonlinear direct image measurement operator. This modification of the filter provides very good results on 2D numerical and experimental flows even in the presence of strong noises. We assessed successfully its application to oceanic satellite images for the recovering of ocean streams. We have also studied the impact on the stochastic dynamics of auto-similar Gaussian noise mimicking statistical properties of turbulence and the introduction within an incremental ensemble analysis scheme of multiscale motion measurements. This work has been published in the Tellus A journal [15].

### 6.2.2. *Stochastic filtering technique for the tracking of closed curves*

**Participant:** Etienne Mémin.

We have studied a stochastic filtering technique for the tracking of closed curves along an image sequence. In that aim, we designed a continuous-time stochastic dynamics that allows us to infer inter-frame deformations. The curve is defined by an implicit level-set representation and the stochastic dynamics is expressed properly on the level-set function. It takes the form of a stochastic partial differential equation with a Brownian motion of low dimension. The evolution model we proposed combines local photometric information, deformations induced by the curve displacement and an uncertainty modeling of the dynamics. Specific choices of noise models and drift terms lead to an evolution law based on mean curvature as in classic level set methods, while other choices yield new evolution laws. The approach we propose is implemented through a particle filter, which includes color measurements characterizing the target and the background photometric probability densities respectively. The merit of this parameter free filter is demonstrated on various satellite image sequences depicting the evolution of complex geophysical flows. This work has been recently published in the Journal of Mathematical Imaging and Vision [13]. Let us note the method provides an empirical dynamical model learned recursively from a data flow. Its short time forecasting skills have been used in the context of weather-watch radar images within a fruitful collaboration with MeteoFrance.

### 6.2.3. *Sequential smoothing for fluid motion*

**Participants:** Anne Cuzol, Etienne Mémin.

In parallel to the construction of stochastic filtering techniques for fluid motions, we have proposed a new sequential smoothing method within a Monte-Carlo framework. This smoothing aims at reducing the temporal discontinuities induced by the sequential assimilation of discrete time data into continuous time dynamical models. The time step between observations can indeed be long in environmental applications for instance, and much longer than the time step used to discretize the model equations. While the filtering aims at estimating the state of the system at observations times in an optimal way, the objective of the smoothing is to improve the estimation of the hidden state between observation times. The method is based on a Monte-Carlo approximation of the filtering and smoothing distributions, and relies on a simulation technique of conditional diffusions. The proposed smoother can be applied to general non linear and multidimensional models. It has been applied to a turbulent flow in a high-dimensional context, in order to smooth the filtering results obtained from a particle filter with a proposal density built from an Ensemble Kalman procedure. This conditional simulation framework can also be used for filtering problem with low measurement noise. This has been explored through a collaboration with Jean-Louis Marchand (ENS Bretagne) in the context of vorticity tracking from image data.

#### 6.2.4. *Stochastic fluid flow dynamics under uncertainty*

**Participants:** Etienne Mémin, Valentin Resseguier.

In this research axis we aim at devising Eulerian expressions for the description of fluid flow evolution laws under uncertainties. Such an uncertainty is modeled through the introduction of a random term that allows taking into account large-scale approximations or truncation effects performed within the dynamics analytical constitution steps. This includes for instance the modeling of unresolved scales interaction in large eddies simulation (LES) or in Reynolds average numerical simulation (RANS), but also uncertainties attached to non-uniform grid discretization. This model is mainly based on a stochastic version of the Reynolds transport theorem. Within this framework various simple expressions of the drift component can be exhibited for different models of the random field carrying the uncertainties we have on the flow. We aim at using such a formalization within image-based data assimilation framework and to derive appropriate stochastic versions of geophysical flow dynamical modeling. This formalization has been published in the journal *Geophysical and Astrophysical Fluid Dynamics* [25]. Numerical simulation on divergence free wavelets basis of 3D viscous Taylor-Green vortex and Crow instability have been performed within a collaboration with Souleymane Kadri-Harouna. First promising results have been published in the TSFP8 conference [30]. Besides, we explore in the context of Valentin Resseguier's PhD the extension of such framework to oceanic models and to satellite image data assimilation. This PhD thesis takes place within a fruitful collaboration with Bertrand Chapron (CERSAT/IFREMER).

#### 6.2.5. *Free surface flows reconstruction and tracking*

**Participants:** Dominique Heitz, Etienne Mémin, Cordelia Robinson, Yin Yang.

Characterizing a free-surface flow (space and time-dependent velocity and geometry) given observations/measures at successive times is an ubiquitous problem in fluid mechanic and in hydrology. Observations can consist of e.g. measurements of velocity, or like in this work of measurements of the geometry of the free-surface. Indeed, recently developed depth/range sensors allow to capture directly a rough 3D geometry of surfaces with high space and time resolution. We have investigated the performance of the Kinect and have shown that it is likely to capture temporal sequences of depth observations of wave-like surfaces with wavelengths and amplitudes sufficiently small to characterize medium/large scale flows. Several data assimilation methods have been experimented and compared to estimate both time dependent geometry and displacement field associated to a free-surface flow from a temporal sequence of Kinect data. This study has been conducted on synthetic and real-world data. Finally, we explored the application of such techniques to hydrological applications. These results have been recently submitted to *Journal of Computational Physics*.

#### 6.2.6. *Variational ensemble methods for data assimilation*

**Participants:** Dominique Heitz, Etienne Mémin, Cordelia Robinson, Yin Yang.

In this work, we aim at studying an ensemble based optimal control strategy for data assimilation. Such a formulation nicely combines the ingredients of ensemble Kalman filters and variational assimilation. In the same way as standard variational assimilation, it is formulated as the minimization of an objective function. However, similarly to ensemble filters, it introduces in its objective function an empirical ensemble-based background-error covariance and works in an off-line smoothing mode rather than sequentially like filtering approaches in a sequential filter. These techniques have the great advantage to avoid the introduction of tangent linear and adjoint models, which are necessary for standard incremental variational techniques. As the background error covariance matrix plays a key role in the variational process, our study particularly focuses on the generation of the analysis ensemble state with localization techniques. We compared the performances of both methods in different cases in which the system's component are fully observed or only partially. The comparisons have been leaded on the basis of a Shallow Water model.

#### 6.2.7. *Optimal control techniques for the coupling of large scale dynamical systems and image data*

**Participants:** Dominique Heitz, Etienne Mémin, Cordelia Robinson.



This work aims at investigating the use of optimal control techniques for the coupling of Large Eddies Simulation (LES) techniques and 2D image data. The objective is to reconstruct a 3D flow from a set of simultaneous time resolved 2D image sequences visualizing the flow on a set of 2D plans enlightened with laser sheets. This approach will be experimented on shear layer flows and on wake flows generated on the wind tunnel of Irstea Rennes. Within this study we wish also to explore techniques to enrich large-scale dynamical models by the introduction of uncertainty terms or through the definition of subgrid models from the image data. This research theme is related to the issue of turbulence characterization from image sequences. Instead of predefined turbulence models, we aim here at tuning from the data the value of coefficients involved in traditional LES subgrid models or in longer-term goal to learn empirical subgrid models directly from image data. An accurate modeling of this term is essential for Large Eddies Simulation as it models all the non resolved motion scales and their interactions with the large scales.

We have pursued the first investigations on a 4DVar assimilation technique, integrating PIV data and Direct Numerical Simulation (DNS), to reconstruct two-dimensional turbulent flows. The problem we are dealing with consists in recovering a flow obeying Navier-Stokes equations, given some noisy and possibly incomplete PIV measurements of the flow. By modifying the initial and inflow conditions of the system, the proposed method reconstructs the flow on the basis of a DNS model and noisy measurements. The technique has been evaluated in the wake of a circular cylinder. It denoises the measurements and increases the spatiotemporal resolution of PIV time series. These results have been recently published in the Journal of Computational Physics [20]. A paper has been also recently published on the denoising aspect in the (PIV13) international conference [29]. Along the same line of studies the 3D case is ongoing. The goal consists here to reconstruct a 3D flow from a set of simultaneous time resolved 2D images of planar sections of the 3D volume. This work is mainly conducted within the PhD of Cordelia Robinson. The development of the variational assimilation code has been initiated within a collaboration with A. Gronskis, S. Laizé (lecturer, Imperial College, UK) and Eric Lamballais (institut P' Poitiers). A High Reynolds number simulation of the wake behind a cylinder has been recently performed within this collaboration.

### 6.2.8. *Variational assimilation of images for large scale fluid flow dynamics with uncertainty*

**Participant:** Etienne Mémin.

In this work we explore the assimilation of a large scale representation of the flow dynamics with image data provided at a finer resolution. The velocity field at large scales is described as a regular smooth components whereas the complement component is a highly oscillating random velocity field defined on the image grid but living at all the scales. Following this route we have started to assess the performance of a variational assimilation technique with direct image data observation. Preliminary encouraging results obtained for a wavelet-based 2D Navier Stokes implementation and images of a passive scalar transported by the flow have been obtained. Large-scale simulation under uncertainty for the 3D viscous Taylor-Green vortex flow have been carried out and show promising results of the approach.

### 6.2.9. *Reduced-order models for flows representation from image data*

**Participants:** Cédric Herzet, Etienne Mémin, Véronique Souchaud.

One of the possibilities to neglect the influence of some degrees of freedom over the main characteristics of a flow consists in representing it as a sum of  $K$  orthonormal spatial basis functions weighted with temporal coefficients. To determine the basis function of this expansion, one of the usual approaches relies on the Karhunen-Loeve decomposition (referred to as proper orthogonal decomposition – POD – in the fluid mechanics domain). In practice, the spatial basis functions, also called modes, are the eigenvectors of an empirical auto-correlation matrix which is built from “snapshots” of the considered physical process.

In this axis of work we focus on the case where one does not have a direct access to snapshots of the considered physical process. Instead, the POD has to be built from the partial and noisy observation of the physical phenomenon of interest. Instances of such scenarios include situations where real instantaneous vector-field snapshots are estimated from a sequence of images. We have been working on several approaches dealing with such a new paradigm. A first approach consists in extending standard penalized motion-estimation algorithms

to the case where the sought velocity field is constrained to span a low-dimensional subspace. In particular, we have considered scenarios where the standard optical flow constraint (OFC) is no longer satisfied and one has therefore to resort to a Discrete Finite Difference (DFD) model. The non-linearity of the latter leads to several practical issues that we have addressed this year.

Within a collaboration with the University of Buenos Aires, we have also explored, a method that combines Proper Orthogonal Decomposition with a spectral technique to analyze and extract reduced order models of flows from time resolved data of velocity fields. This methodology, relying on the eigenfunctions of the Koopman operator, is specifically adapted to flows with quasi periodic orbits in the phase space. The technique is particularly suited to cases requiring a discretization with a high spatial and temporal resolution. The proposed analysis enables to decompose the flow dynamics into modes that oscillate at a single frequency. For each modes an energy content and a spatial structure can be put in correspondence. This approach has been assessed for a wake flow behind a cylinder at Reynolds number 3900 and has been recently published to the journal of Theoretical and Computational Fluid Dynamics [16]. The assessment of this method on oceanic model simulation data is on going.

## 6.3. Analysis and modeling of turbulent flows

### 6.3.1. Hot-wire anemometry at low velocities

**Participant:** Dominique Heitz.

A new dynamical calibration technique has been developed for hot-wire probes. The technique permits, in a short time range, the combined calibration of velocity, temperature and direction calibration of single and multiple hot-wire probes. The calibration and measurements uncertainties were modeled, simulated and controlled, in order to reduce their estimated values. Based on a market study the french patent application has been extended this year to a Patent Cooperation Treaty (PCT) application.

### 6.3.2. Numerical and experimental image and flow database

**Participant:** Dominique Heitz.

The goal was to design a database for the evaluation of the different techniques developed in the Fluminance group. The main challenge was to enlarge a database mainly based on two-dimensional flows, with three-dimensional turbulent flows. New synthetic image sequences based on homogeneous isotropic turbulence and on circular cylinder wake have been provided. These images have been completed with real image sequences based on wake and mixing layers flows. This new database provides different realistic conditions to analyse the performance of the methods: time steps between images, level of noise, Reynolds number, large-scale images. A Wake flow at high Reynolds number has been also simulated on one of the IDRIS super computer. This simulation, whose results analysis is on going, has been performed within a collaboration with Sylvain Laizet (Imperial College).

## 6.4. Visual servoing approach for fluid flow control

### 6.4.1. Minimization of the kinetic energy density in the 2D plane Poiseuille flow

**Participants:** Christophe Collewet, Xuan Quy Dao.

This works concerns the PhD thesis of Xuan-Quy Dao. This year we have focused on a way to ensure a strict decreasing of the kinetic energy density. In that purpose, we have first proposed an approach to increase the controlled degrees of freedom. Indeed, the classical way to model this flow leads to only two degrees of freedom. With so few degrees of freedom it is obviously impossible to reach high desired performances as the strict minimization of the kinetic energy density. This way to proceed leads to a better minimization of the kinetic energy density. We have also proposed on approach based on a local decoupling of the controlled degree of freedom of the system so that an exponential decoupled decrease of each components of the state vector is locally obtained. This work has been presented at the CFM conference (Congrès Français de Mécanique) [32].

### 6.4.2. Control behind a backward-facing step

**Participant:** Christophe Collewet.

This work is performed in the context of the PhD thesis of Nicolas Gautier from ESCPI in collaboration with J.L. Aider. The separated flow downstream a backward-facing step is studied using visual information for feedback. More precisely, flow velocity fields are computed from a real time optical flow algorithm. The control law we used is a simple PID controller. Even a better control law could be used, this study validates that visual servo control is an effective approach to control a flow.

### 6.4.3. Control of systems described by partial differential equations

**Participants:** Tudor-Bogdan Airimioaie, Christophe Collewet.

This work concerns principally the post-doctoral research of Tudor-Bogdan Airimioaie. It aims at controlling continuously evolving systems described by partial differential equations (PDEs). This is relevant in the context of the Fluminance team because fluid flows are infinite dimensional systems and can be rigorously described only through PDEs. In spite of this, practical approaches of flow control are based on low order numerical implementation relying on space and time discretization of the continuous system. This implies to setup strategies for model reduction that must be then in return properly understood with respect to the convergence of the control law. For finite dimensional implementations, one of the research directions pursued concerns the study on the benefit of increasing the controlled degrees of freedom (see the work of Xuan-Quy Dao). Another research direction, started recently, consists in improving control by using real-time estimation of a finite number of parameters related to the original infinite dimensional system. Indeed, this opens the possibility of improving performances by using more advanced robust linear parametric varying (LPV) control techniques existing in the literature. Two conference papers on these works have been submitted at the "7th AIAA Flow Control Conference".

## 7. Bilateral Contracts and Grants with Industry

### 7.1. Contrat CERSAT/IFREMER

This contract aims at studying image based data assimilation strategies for oceanic models incorporating random uncertainty terms. The goal targeted will consist in deriving appropriate stochastic version of oceanic model and on top of them to devise estimation procedures from noisy data to calibrate the associated subgrid models. This contract covers half of the funding of Valentin Resseguier PhD thesis.

## 8. Partnerships and Cooperations

### 8.1. Regional Initiatives

#### 8.1.1. Brittany concil ARED IMAGEO:

**Participants:** Cédric Herzet, Etienne Mémin, Véronique Souchaud.

*duration 36 months.* This project of the Brittany concil, which finances the PhD thesis of Véronique Souchaud, aims at studying methods for the estimation of reduced order modeling of fluid flows evolution laws from image sequences. The goal consists here at defining the estimation of a reduced basis describing the flow evolution as a motion estimation problem.

### 8.2. National Initiatives

#### 8.2.1. ANR-COSINUS PREVASSEMBLE: Ensemble methods for assimilation of observations and for prevision in Meteorology and Oceanography

**Participants:** Sébastien Beyou, Anne Cuzol, Etienne Mémin.

*duration 36 months.*

The purpose of this project is to further study ensemble methods -, and to develop their use for both assimilation of observations and forecast. Among the specific questions to be studied are the theory of Particle Filters and Ensemble Kalman Filters, the possibility of taking temporal correlation into account in ensemble assimilation, the precise assessment of what can and cannot be achieved in ensemble prediction, and the objective validation of ensemble methods.

The partners of this project are Laboratoire de Météorologie Dynamique/ENS (leader), Météo-France and three Inria groups (ALEA, ASPI, FLUMINANCE).

### **8.2.2. ANR SYSCOMM MSDAG: MultiScale Data Assimilation in Geophysics**

**Participants:** Patrick Héas, Dominique Heitz, Cédric Herzet, Etienne Mémin.

*duration 36 months.*

Changing scale is a well-known topic in physics (geophysics, fluid mechanics and turbulence, theoretical and statistical physics, mechanics, porous media, etc.). It has led to the creation of powerful sophisticated mathematical tools: renormalization, homogenization, etc. These ideas are also used in numerical analysis (the so-called multigrid approach) for solving efficiently partial differential equations. Data assimilation in Geophysics is a set of methods that allows to combine optimally numerical models in large spaces with large dataset of observations. At the confluence of these two topics, the goal of this project is to study how to embed the change of scales (a multiscale point of view) issue into the framework of geophysical data assimilation, which is a largely unexplored subject.

The partners of this 3 years project are the CEREAL/ CLIME Inria group (leader), the LSCE/CEA, the Inria groups MOISE and FLUMINANCE.

### **8.2.3. ANR SYSCOMM GeoFluids: Analyse et simulation d'écoulements fluides à partir de séquences d'images : application à l'étude d'écoulements géophysiques**

**Participants:** Dominique Heitz, Etienne Mémin.

*duration 48 months.*

The project Geo-FLUIDS focuses on the specification of tools to analyze geophysical fluid flows from image sequences. Geo-FLUIDS aims at providing image-based methods using physically consistent models to extract meaningful features describing the observed flow and to unveil the dynamical properties of this flow. The main targeted application domains concern Oceanography and Meteorology . The project consortium gathers the Inria research groups: FLUMINANCE (leader), CLIME and MOISE. The group of the "Laboratoire de Météorologie Dynamique" located at the ENS Paris, the IFREMER-CERSAT group located at Brest and the METEOFRANCE GMAP group in Toulouse.

### **8.2.4. ANR JCJC GERONIMO : Advanced GEophysical Reduced-Order Model construction from Image Observations**

**Participant:** Cédric Herzet.

*duration 48 months.* The GERONIMO project which starts in January 20014 aims at devising new efficient and effective techniques for the design of geophysical reduced-order models from image data. The project both arises from the crucial need of accurate low-order descriptions of highly-complex geophysical phenomena and the recent numerical revolution which has supplied the geophysical scientists with an unprecedented volume of image data. The project is placed in the intersection of several fields of expertise (Bayesian inference, matrix factorization, sparse representations, etc) which will be combined to handle the uncertainties associated to image measurements and to characterize the accurate reduced dynamical systems.

### **8.2.5. INSU-LEFE: Vers de nouvelles méthodes d'estimation de la sous-mésoéchelle océanique**

**Participants:** Patrick Héas, Cédric Herzet.

*duration 36 months.* This project tackles the problem of deriving a precise submesoscale characterization of ocean currents from satellite data. The targeted methodologies should in particular enable the exploitation of data of different nature (for example sea surface temperature or height) and/or resolutions. This 36-month project benefits from a strong collaboration with Guillaume Lapeyre (Laboratoire de Météorologie Dynamique, Ecole Normale Supérieure, Paris).

## 9. Dissemination

### 9.1. Scientific Animation

Christophe Collewet

- Technical program committees of ORASIS 2013 (journées francophones des jeunes chercheurs en vision par ordinateur)
- Reviewer for ICRA'13 (IEEE International conference on robotics and automation)

Dominique Heitz

- Member of IRSTEA "Comité directeur des Systèmes d'Information"
- Member of IRSTEA "Comité Technique Spécial"
- Responsible of the IRSTEA ACTA Team
- Reviewer for IEEE trans. Im. Proc, Exp in Fluid

Cédric Herzet

- Technical program committees of ICASSP 2013/2014 and SPARS 2013
- Project reviewer for the "Fond National de la Recherche Scientifique" (FNRS), Belgique
- Invited speaker at the CIMI Workshop, Toulouse, June 2013.
- Invited speaker at a local seminar at ParisTech, Paris, January 2013.
- Organizer of a monthly local seminar dedicated to sparse representations.

Etienne Mémin

- Invited speaker special session "Signaux & Images en Océanographie" GretsI de traitement du signal et des images, Assimilation d'images satellites océaniques: filtrage stochastique et définition de dynamiques adaptées Brest sep. 2013.
- invited speaker 51st AIAA Aerospace Sciences Meeting, jan. 2013. Fluid flow velocity measurements from image sequences, 51st AIAA Aerospace Sciences Meeting, jan. 2013.
- Invited speaker workshop "2D to 3D Ocean Dynamics from Space", Ifremer, Brest December 2013.
- invited speaker CIMI (Centre International de Mathématiques et d'Informatique - Trimestre EDP & Probabilités - Weather Forecast, jan. 2014
- Associate editor of the Journal of Computer Vision (IJCV)
- Associate editor of the journal of Image and Vision Computing (IVC)
- Reviewing for Tellus-A, IEEE Im. Proc., IEEE trans. Pat. Anal. Mach. Intel. , , m. Vis. Comp., Exp in Fluids, ICCV'13, Nonlinear Proc. in Geophysics.
- Responsible of the "Commission Développement Technologique" Inria-IRISA Rennes
- member of the "Commission Personnel" Inria-IRISA Rennes

## 9.2. Teaching - Supervision - Juries

### 9.2.1. Teaching

Licence : Dominique Heitz, Mécanique des fluides, 30h, niveau L2 INSA Rennes

Master : Dominique Heitz, Mécanique des fluides, 25h, niveau M1, Dep GMA INSA Rennes

Master : Cédric Herzet, Analyse de données, Mastere de Statistiques et Econométrie, 10h, niveau M1, Université de Rennes I

Master : Etienne Mémin, Analyse du mouvement, Mastere Informatique, 15h, niveau M2, Université de Rennes 1.

Master : Etienne Mémin, Vision par ordinateur , 15h, niveau M2, ESIR Université de Rennes 1.

### 9.2.2. Supervision

PhD & HdR :

PhD: Sébastien Béyou, Estimation de la vitesse des courants marins à partir de séquences d'images satellitaires. Université de Rennes I, 12/07/2011, Etienne Mémin [11].

PhD in progress : Ioana Barbu, Estimation volumique de mouvement fluides /'a partir de séquence d'images, 01/11/2010, Cédric Herzet et Etienne Mémin

PhD in progress : Qui Dao, Commande des écoulements fluides par asservissement visuel, 01/10/2010, Christophe Collewet

PhD in progress : Valentin Resseguier, Image based assimilation of geophysical flows models under uncertainty, Bertrand Chapron (IFREMER) and Etienne Mémin

PhD in progress : Cordelia Robinson, Assimilation de données images dans un modèle LES : application à la reconstruction d'écoulements turbulents tridimensionnels , 01/11/2011, Dominique Heitz et Etienne Mémin

PhD in progress : Yin Yang , Assimilation d'images par techniques variationnelle ensembliste , 01/11/2011, Etienne Mémin

### 9.2.3. Juries

Etienne Mémin

- Rapporteur de la thèse de Vincent Chabot, Université J. Fourier, Grenoble, 15 Novembre 2013.
- Président du Jury de thèse de Karim Driffi, Université de Bretagne Sud, Juillet 2013.
- Examineur Jury HDR d'Arthur Vidard, Université J. Fourier, decembre 2012.

## 9.3. Popularization

Etienne Mémin

- E. Mémin. Ou vont les nuages ?, Un jour, une brève, Mathématiques de la planète terre, (Brève)
- Invited paper in the journal "Revue française de photogrammetry et de Télédétection", Outils méthodologiques d'analyse d'images MSG : estimation du mouvement, suivi de masses nuageuses et détection de fronts, with T. Corpetti, V. Dubreuil, E. Mémin, O. Planchon, C. Thomas.
- Invited paper in the journal de la Société Française de Statistique, Image data assimilation with filtering methods, with Anne Cuzol.

# 10. Bibliography

## Major publications by the team in recent years

- [1] E. ARNAUD, E. MÉMIN. *Partial linear Gaussian model for tracking in image sequences using sequential Monte Carlo methods*, in "International Journal of Computer Vision", 2007, vol. 74, n° 1, pp. 75-102

- [2] C. COLLEWET, E. MARCHAND. *Modeling complex luminance variations for target tracking*, in "IEEE Int. Conf. on Computer Vision and Pattern Recognition, CVPR'08", Anchorage, Alaska, June 2008, pp. 1–7
- [3] T. CORPETTI, P. HÉAS, E. MÉMIN, N. PAPADAKIS. *Pressure image assimilation for atmospheric motion estimation*, in "Tellus Series A: Dynamic Meteorology and Oceanography", 2009, vol. 61, n<sup>o</sup> 1, pp. 160–178, [http://www.irisa.fr/fluminance/publi/papers/2008\\_Tellus\\_Corpetti.pdf](http://www.irisa.fr/fluminance/publi/papers/2008_Tellus_Corpetti.pdf)
- [4] T. CORPETTI, D. HEITZ, G. ARROYO, E. MÉMIN, A. SANTA-CRUZ. *Fluid experimental flow estimation based on an optical-flow scheme*, in "Experiments in fluids", 2006, vol. 40, pp. 80–97
- [5] A. CUZOL, E. MÉMIN. *A stochastic filter technique for fluid flows velocity fields tracking*, in "IEEE Trans. Pattern Analysis and Machine Intelligence", 2009, vol. 31, n<sup>o</sup> 7, pp. 1278–1293
- [6] D. HEITZ, E. MÉMIN, C. SCHNOERR. *Variational Fluid Flow Measurements from Image Sequences: Synopsis and Perspectives*, in "Experiments in fluids", 2010, vol. 48, n<sup>o</sup> 3, pp. 369–393
- [7] C. HERZET, K. WORADIT, H. WYMEERSCH, L. VANDENDORPE. *Code-Aided Maximum-Likelihood Ambiguity Resolution Through Free-Energy Minimization*, in "IEEE Trans. Signal Processing", 2010, vol. 58, n<sup>o</sup> 12, pp. 6238–6250
- [8] P. HÉAS, E. MÉMIN. *3D motion estimation of atmospheric layers from image sequences*, in "IEEE Trans. on Geoscience and Remote Sensing", 2008, vol. 46, n<sup>o</sup> 8, pp. 2385–2396
- [9] N. PAPADAKIS, E. MÉMIN. *A variational technique for time consistent tracking of curves and motion*, in "Journal of Mathematical Imaging and Vision", 2008, vol. 31, n<sup>o</sup> 1, pp. 81–103, <http://www.irisa.fr/fluminance/publi/papers/Papadakis-Memin-JMIV07.pdf>
- [10] J. YUAN, C. SCHNOERR, E. MÉMIN. *Discrete orthogonal decomposition and variational fluid flow estimation*, in "Journal of Mathematical Imaging and Vision", 2007, vol. 28, n<sup>o</sup> 1, pp. 67–80, <http://www.irisa.fr/fluminance/publi/papers/Yuan-et-al-JMIV06.pdf>

## Publications of the year

### Doctoral Dissertations and Habilitation Theses

- [11] S. BEYOU. , *Estimation de la vitesse des courants marins à partir de séquences d'images satellitaires*, Université Rennes 1, July 2013, <http://hal.inria.fr/tel-00870722>
- [12] K. SODJAVI. , *Etude expérimentale de la turbulence dans une couche de mélange anisotherme*, Université Rennes 1, March 2013, <http://hal.inria.fr/tel-00921507>

### Articles in International Peer-Reviewed Journals

- [13] C. AVENEL, E. MÉMIN, P. PÉREZ. *Stochastic level set dynamics to track closed curves through image data*, in "Journal of Mathematical Imaging and Vision", August 2013 [DOI : 10.1007/s10851-013-0464-1], <http://hal.inria.fr/hal-00854420>

- [14] S. BEYOU, T. CORPETTI, S. GORTHI, E. MÉMIN. *Fluid flow estimation with multiscale ensemble filters based on motion measurements under location uncertainty*, in "Numerical Mathematics: Theory , Methods and Applications", January 2013, vol. 6, n<sup>o</sup> 1, pp. 21-46, <http://hal.inria.fr/hal-00736457>
- [15] S. BEYOU, A. CUZOL, S. S. GORTHI, E. MÉMIN. *Weighted Ensemble Transform Kalman Filter for Image Assimilation*, in "Tellus A", January 2013, vol. 65, pp. 1-33 [DOI : 10.3402/TELLUSA.v65i0.18803], <http://hal.inria.fr/hal-00793804>
- [16] A. CAMMILLERI, F. GUÉNIAT, J. CARLIER, L. PASTUR, E. MÉMIN, F. LUSSEYRAN, G. ARTANA. *POD-Spectral Decomposition for Fluid Flow Analysis and Model Reduction*, in "Theoretical and Computational Fluid Dynamics", February 2013, vol. 27, n<sup>o</sup> 125 [DOI : 10.1007/s00162-013-0293-2], <http://hal.inria.fr/hal-00793380>
- [17] T. CHACON REBELLO, R. LEWANDOWSKI. *A Variational Finite Element Model for Large-Eddy Simulations of Turbulent Flows*, in "Chinese Annals of Mathematics. Series B", 2013, vol. 34, n<sup>o</sup> 5, pp. 667-682 [DOI : 10.1007/s11401-013-0796-6], <http://hal.inria.fr/hal-00833231>
- [18] A. A. DUNCA, R. LEWANDOWSKI. *Error estimates in approximate deconvolution models*, in "Communications in Mathematical Sciences", December 2013, 11 p. , <http://hal.inria.fr/hal-00675968>
- [19] P. DÉRIAN, P. HÉAS, C. HERZET, E. MÉMIN. *Wavelets and Optical Flow Motion Estimation*, in "Numerical Mathematics: Theory, Methods and Applications", 2013, vol. 6, pp. 116-137, <http://hal.inria.fr/hal-00737566>
- [20] A. GRONSKIS, D. HEITZ, E. MÉMIN. *Inflow and initial conditions for direct numerical simulation based on adjoint data assimilation*, in "Journal of Computational Physics", 2013, vol. 242, 480 p. [DOI : 10.1016/J.JCP.2013.01.051], <http://hal.inria.fr/hal-00819311>
- [21] C. HERZET, C. SOUSSEN, J. IDIER, R. GRIBONVAL. *Exact recovery conditions for sparse representations with partial support information*, in "IEEE Transactions on Information Theory", November 2013, vol. 59, n<sup>o</sup> 11, pp. 7509-7524 [DOI : 10.1109/TIT.2013.2278179], <http://hal.inria.fr/hal-00907646>
- [22] P. HÉAS, C. HERZET, E. MÉMIN, D. HEITZ, P. D. MININNI. *Bayesian estimation of turbulent motion*, in "IEEE Transactions on Pattern Analysis and Machine Intelligence", June 2013, vol. 35, n<sup>o</sup> 6, pp. 1343-1356 [DOI : 10.1109/TPAMI.2012.232], <http://hal.inria.fr/hal-00745814>
- [23] S. KADRI HAROUNA, P. DÉRIAN, P. HÉAS, E. MÉMIN. *Divergence-free Wavelets and High Order Regularization*, in "International Journal of Computer Vision", May 2013, vol. 103, n<sup>o</sup> 1, pp. 80-99 [DOI : 10.1007/s11263-012-0595-7], <http://hal.inria.fr/hal-00646104>
- [24] S. KADRI HAROUNA, V. PERRIER. *Effective construction of divergence-free wavelets on the square*, in "Journal of Computational and Applied Mathematics", March 2013, vol. 240, pp. 74-86 [DOI : 10.1016/J.CAM.2012.07.029], <http://hal.inria.fr/hal-00727038>
- [25] E. MÉMIN. *Fluid flow dynamics under location uncertainty*, in "Geophysical and Astrophysical Fluid Dynamics", December 2013, <http://hal.inria.fr/hal-00852874>
- [26] C. SOUSSEN, R. GRIBONVAL, J. IDIER, C. HERZET. *Joint k-step analysis of Orthogonal Matching Pursuit and Orthogonal Least Squares*, in "IEEE Transactions on Information Theory", May 2013, vol. 59, n<sup>o</sup> 5, pp. 3158-3174 [DOI : 10.1109/TIT.2013.2238606], <http://hal.inria.fr/hal-00637003>



### International Conferences with Proceedings

- [27] I. BARBU, C. HERZET, E. MÉMIN. *Joint Estimation of Volume and Velocity in TomoPIV*, in "10th international symposium on particle image velocimetry - PIV13", Delft, Netherlands, July 2013, 45 p. , <http://hal.inria.fr/hal-00880712>
- [28] M. DESOUBEAUX, C. HERZET, W. PUECH, G. LE GUELVOUT. *Enhanced Blind Decoding of Tardos Codes with New Map-Based Functions*, in "IEEE 15th International Workshop on Multimedia Signal Processing (MMSP)", Pula, Italy, October 2013, <http://hal.inria.fr/hal-00907670>
- [29] A. GRONSKIS, C. ROBINSON, D. HEITZ, E. MÉMIN. *A 4DVar PIV-data assimilation for flow spatio-temporal reconstruction*, in "International Symposium on Particle Image Velocimetry", Delft, Netherlands, July 2013, <http://hal.inria.fr/hal-00914037>
- [30] S. KADRI HAROUNA, E. MÉMIN. *Navier-Stokes simulation with fluid particles location uncertainty*, in "Int. Symposium on Turbulence and Shear Flow Phenomena (TSFP-8)", Poitiers, France, August 2013, <http://hal.inria.fr/hal-00914033>

### National Conferences with Proceedings

- [31] I. BARBU, C. HERZET, E. MÉMIN. *Estimation 3D jointe des volumes et des vitesses pour la tomographie PIV*, in "15ème Congrès Français de Visualisation et de Traitement d'Images en Mécanique des Fluides", Orléans, France, November 2013, 68 p. , <http://hal.inria.fr/hal-00880719>
- [32] X.-Q. DAO, C. COLLEWET. *Minimisation de l'énergie cinétique transitoire dans l'écoulement 3D plan de Poiseuille commandé*, in "Congrès Français de Mécanique", Bordeaux, France, August 2013, <http://hal.inria.fr/hal-00838247>

### Conferences without Proceedings

- [33] C. SOUSSEN, C. HERZET, J. IDIER, R. GRIBONVAL. *Unrecoverable subsets by OMP and Basis Pursuit*, in "Signal Processing with Adaptive Sparse Structured Representations, SPARS 2013", Lausanne, Switzerland, July 2013, <http://hal.inria.fr/hal-00811641>

### Research Reports

- [34] I. D. LANDAU, T.-B. AIRIMITOAI, M. ALMA. , *Adaptive Feedforward Compensation Algorithms for Active Vibration Control with Mechanical Coupling and Local Feedback - a unified approach*, January 2014, <http://hal.inria.fr/hal-00922912>

### References in notes

- [35] R. ADRIAN. *Particle imaging techniques for experimental fluid mechanics*, in "Annal Rev. Fluid Mech.", 1991, vol. 23, pp. 261-304
- [36] T. BEWLEY. *Flow control: new challenges for a new Renaissance*, in "Progress in Aerospace Sciences", 2001, vol. 37, pp. 21-58
- [37] K. BREUER. *Sensors, Actuators and Algorithms for practical implementations of turbulence boundary layer control*, in "3rd Japan Symposium on Turbulence and Flow Control", Tokyo, Japan, March 2002

- 
- [38] F. CHAUMETTE, S. HUTCHINSON. *Visual servoing and visual tracking*, in "Handbook of Robotics", B. SICILIANO, O. KHATIB (editors), Springer, 2008, chap. 24, pp. 563–583
- [39] H. CHOI, P. MOIN, J. KIM. *Direct numerical simulation of turbulent flow over riblets*, in "Journal of Fluid Mechanics", 1993, vol. 255, pp. 503–539
- [40] S. S. COLLIS, R. D. JOSLIN, A. SEIFERT, V. THEOFILIS. *Issues in active flow control: theory, control, simulation, and experiment*, in "Progress in Aerospace Sciences", 2004, vol. 4, pp. 237–289
- [41] E. DOEBELIN. , *Control System Principles and Design*, John Wiley and Sons New York, 1985
- [42] G. E. ELSINGA, F. SCARANO, B. WIENEKE, B. W. VAN OUDHEUSDEN. *Tomographic particle image velocimetry*, in "Exp. in Fluids", 2006, vol. 41, pp. 933–947
- [43] G. EVENSEN. *Sequential data assimilation with a non linear quasi-geostrophic model using Monte Carlo methods to forecast error statistics*, in "J. Geophys. Res.", 1994, vol. 99 (C5), n<sup>o</sup> 10, pp. 143–162
- [44] J. FAVIER. , *Contrôle d'écoulements : approche expérimentale et modélisation de dimension réduite*, Institut National Polytechnique de Toulouse, 2007
- [45] J. FAVIER, A. KOURTA, G. LEPLAT. *Control of flow separation on a wing profile using PIV measurements and POD analysis*, in "IUTAM Symposium on Flow Control and MEMS", London, UK, September 19–22, 2006
- [46] N. GORDON, D. SALMOND, A. SMITH. *Novel approach to non-linear/non-Gaussian Bayesian state estimation*, in "IEEE Processing-F", April 1993, vol. 140, n<sup>o</sup> 2
- [47] A. GUÉGAN, P. SCHMID, P. HUERRE. *Optimal energy growth and optimal control in swept Hiemenz flow*, in "J. Fluid Mech.", 2006, vol. 566, pp. 11–45
- [48] B. HORN, B. SCHUNCK. *Determining Optical Flow*, in "Artificial Intelligence", August 1981, vol. 17, n<sup>o</sup> 1–3, pp. 185–203
- [49] J. HÖPFNER, M. CHEVALIER, T. BEWLEY, D. S. HENNINGSON. *State estimation in wall-bounded flow systems. Part 1. Perturbed laminar flows*, in "Journal of Fluid Mechanics", 2005, vol. 534, pp. 263–294
- [50] F.-X. LE DIMET, O. TALAGRAND. *Variational algorithms for analysis and assimilation of meteorological observations: theoretical aspects*, in "Tellus", 1986, n<sup>o</sup> 38A, pp. 97–110
- [51] J. LIONS. , *Optimal Control of Systems Governed by Partial Differential Equations*, Springer-Verlag, 1971
- [52] L. MATHELIN, O. LE MAÎTRE. *Robust control of uncertain cylinder wake flows based on robust reduced order models*, in "Computer and Fluids", 2009, vol. 38, pp. 1168–1182
- [53] B. PROTAS, J. WESFREID. *Drag force in the open-loop control of the cylinder wake in the laminar regime*, in "Physics of Fluids", February 2002, vol. 14, n<sup>o</sup> 2, pp. 810–826

- 
- [54] C. SAMSON, M. LE BORGNE, B. ESPIAU. , *Robot Control: the Task Function Approach*, Clarendon Press, Oxford, United Kingdom, 1991
- [55] G. SETTLES. , *Schlieren and shadowgraph techniques*, Springer Verlag series on experimental fluid mechanics, 2001
- [56] A. SVIZHER, J. COHEN. *Holographic particle image velocimetry system for measurements of hairpin vortices in air channel flow*, in "Exp. in Fluids", 2006, vol. 40, n<sup>o</sup> 5, pp. 708-722
- [57] R. TATSAMBON FOMENA, C. COLLEWET. *Fluid Flow Control: a Vision-Based Approach*, in "International Journal of Flow Control", 2011, vol. 3, n<sup>o</sup> 23, pp. 133–169, <http://hal.inria.fr/hal-00701080>
- [58] I. WYGNANSKI. *Boundary layer flow control by periodic addition of momentum*, in "4th AIAA Shear Flow Control Conference", USA, June 29-July 2, 1997