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Activity Report 2013

Project-Team GALAAD

Geometry, algebra, algorithms

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME

Table of contents

Project-Team GALAAD

Keywords: Algorithmic Geometry, Computer Algebra, Real Numbers

Galaad is a joint project with Laboratoire J.A. Dieudonné U.M.R. C.N.R.S. n^O 6621, University of Nice Sophia-Antipolis.

Creation of the Project-Team: 2002 April 01, end of the Project-Team: 2013 December 31.

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2. Overall Objectives

2.1. Highlights of the Year

The paper [15] has been awarded the best paper price, 1st place, of the SIAM conference on Geometric and Physical Modeling 2013 (Denver, USA, Nov. 11-14).

2.2. Introduction

There is a shared vision that our day life environment will increasingly interact with a digital world, populated by captors, sensors, or devices used to simplify or improve some of our activities. Digital cameras, positioning systems, mobile phones, internet web interfaces are such typical examples which are nowadays completely standard tools. Interconnected with each other, these devices are producing, exchanging or processing digital data in order to interact with the physical world. Computing is becoming ubiquitous and this evolution raises new challenges to represent, analyze and transform this digital information.

From this perspective, geometry is playing an important role. There is a strong interaction between physical and digital worlds through geometric modeling and analysis. Understanding a physical phenomena can be done by analyzing numerical simulations on a digital representation of the geometry. Conversely developing digital geometry (as in Computer Aided Geometric Design – CAGD for short) is nowadays used to produce devices to overcome some physical difficulties (car, planes, ...). Obviously, geometry is not addressing directly problems related to storage or transmission of information, but it deals with structured and efficient representations of this information and methods to compute with these models.

Within this context, our research program aims at developing new and efficient methods for modeling geometry with algebraic representations. We don't see shapes just as set of points with simple neighbor information. In our investigations, we use richer algebraic models which provide structured and compact representation of the geometry, while being able to encode their important characteristic features.

The first challenge to be addressed is how to move from the digital world to an algebraic world. Our objective is to develop efficient methods which can transform digital data produced by cameras, laser scanners, observations or simulations into algebraic models involving few parameters. This is a way to structure the digital information and to further exploit its properties. This methodological investigations are connected with practical problems such as compression of data for exchange of geometric information, accurate description and simulation with manufactured objects, shape optimization in computer aided design, ...

A second challenge concerns operations and transformations on these algebraic representations. They require the development of dedicated techniques which fully exploit the algebraic characteristics of these representations. The theoretical foundations of our investigations are in algebraic geometry. This domain deals with the solutions of algebraic equations and its effective aspect concerns algorithms to compute and analyze them. It is an old, important and very active part of mathematics. Its combination with algorithmic developments for algebraic computation leads to new methods to treat effectively geometric problems. These investigations result in new contributions in commutative algebra, new algorithms in computer algebra, complexity analyses and/or software development for practical experimentation.

The third challenge is how to analyze and understand digital geometric data. In this approach, constructing algebraic representation and developing methods to compute with these models are the preliminary steps of our analysis process. The goal is to develop methods to extract some type of information we are searching from this data, such as topological descriptions, subdivisions in smooth components and adjacency relations, decomposition in irreducible components. The interplay between algebraic models and numerical computation is central in this activity. A main issue concerns the approximation of models and the certification of the computation.

3. Research Program

3.1. Introduction

Our scientific activity is structured according to three broad topics:

1. **Algebraic representations for geometric modeling**.
2. **Algebraic algorithms for geometric computing**,
3. **Symbolic-numeric methods for analysis**,

3.2. Algebraic representations for geometric modeling

Compact, efficient and structured descriptions of shapes are required in many scientific computations in engineering, such as "Isogeometric" Finite Elements methods, point cloud fitting problems or implicit surfaces defined by convolution. Our objective is to investigate new algebraic representations (or improve the existing ones) together with their analysis and implementations.

We are investigating representations, based on semi-algebraic models. Such non-linear models are able to capture efficiently complexes shapes, using few data. However, they required specific methods to solve the underlying non-linear problems, which we are investigating.

Effective algebraic geometry is a natural framework for handling shape representations. This framework not only provides tools for modeling but it also allows to exploit rich geometric properties.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

The use of piecewise algebraic representations also raises problems of approximation and reconstruction, on which we are working on. In this direction, we are studying B-spline function spaces with specified regularity associated to domain partitions.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. We are interested in exploiting these invariant properties, to develop compact and adapted representations of shapes.

3.3. Algebraic algorithms for geometric computing

This topic is directly related to polynomial system solving and effective algebraic geometry. It is our core expertise and many of our works are contributing to this area.

Our goal is to develop algebraic algorithms to efficiently perform geometric operations such as computing the intersection or self-intersection locus of algebraic surface patches, offsets, envelopes of surfaces, ...

The underlying representations behind the geometric models we consider are often of algebraic type. Computing with such models raises algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

Geometric objects are often described in a parametric form. For performing efficiently on these objects, it can also be interesting to manipulate implicit representations. We consider particular projections techniques based on new resultant constructions or syzygies, which allow to transform parametric representations into implicit ones. These problems can be reformulated in terms of linear algebra. We investigate methods which exploit this matrix representation based on resultant constructions.

They involve structured matrices such as Hankel, Toeplitz, Bezoutian matrices or their generalization in several variables. We investigate algorithms that exploit their properties and their implications in solving polynomial equations.

We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems.

We are also interested in subdivision methods, which are able to efficiently localise the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue while developing these methods is to analyse their practical and algorithmic behavior. Our aim is to obtain good complexity bounds and practical efficiency by exploiting the structure of the problem.

3.4. Symbolic numeric analysis

While treating practical problems, noisy data appear and incertitude has to be taken into account. The objective is to devise adapted techniques for analyzing the geometric properties of the algebraic models in this context.

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many context, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate the geometric objects while controlling the quality of approximation. We investigate subdivision techniques based on the algebraic formulation of our problems which allow us to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

The problem of decomposition and factorisation is also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. A typical problem in this direction is the problem of approximate factorization, which requires to analyze perturbations of the data, which enables us to break up the problem.

4. Application Domains

4.1. Shape modeling

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, etc. Optimizing these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship with the industrial contacts that we are developing in Computer Aided Geometric Design.

4.2. Shape processing

Many problems encountered in the application of computer sciences start from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where the geometry of distances plays a significant role, for example, in the reconstruction from NMR (Nuclear Magnetic Resonance) experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tend to be more and more easily used yield large set of data points from which one has to recover compact geometric model. We are working in collaboration with groups in agronomy on the problems of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems. Geometry is also highly involved in the numerical simulation of physical problems such as heat conduction, ship hull design, blades and turbines analysis, mechanical stress analysis. We apply our algebraic-geometric techniques in the isogeometric approach which uses the same (B-spline) formalism to represent both the geometry and the solutions of partial differential equations on this geometry.

5. Software and Platforms

5.1. Mathemagix, a free computer algebra environment

Participant: Bernard Mourrain.

http://www.mathemagix.org/

algebra, univariate polynomial, multivariate polynomial, matrices, series, fast algorithm, interpreter, compiler, hybrid software.

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments, based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on the embeding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

The project aims at building a bridge between symbolic computation and numerical analysis. It is structured by collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

- REALROOT: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms; exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterized, implicit with different type of coefficients; algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler AXEL.

Collaborators: Grégoire Lecerf, Joris van der Hoeven and Philippe Trébuchet.

5.2. Axel, a geometric modeler for algebraic objects

Participants: Nicolas Douillet, Anaïs Ducoffe [contact], Valentin Michelet, Bernard Mourrain, Hung Nguyen, Meriadeg Perrinel.

http://axel.inria.fr.

computational algebraic geometry, curve, implicit equation, intersection, parameterization, resolution, surface, singularity, topology

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The software contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterizations, B-splines, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, for detecting and computing self-intersection points of parameterized surfaces, for implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

The developments related to isogeometric analysis have been integrated as dedicated plugins. Optimization techniques and solvers for partial differential equations developed by R. Duvigneau (OPALE) have been connected.

The new version of the algebraic-geometric modelers based on the DTK platform is still developed in order to provide a better modularity and a better interface to existing computation facilities and geometric rendering interface. This software is intended to be multi-platform, and jobs are running nightly on the Continous Integration platform `ci.inria.fr` of Inria, performing builds and tests on Virtual Machines of different OS such as Fedora, Ubuntu, Windows.

AXEL is written in C++ and thanks to a wrapping system using SWIG, its data structures and algorithms can be integrated into C# programs, as well as Python and Java programs. This wrapper was used to integrate AXEL into the CAD software TopSolid, developed by Missler Company and written in C#. But it also enables AXEL to embed a Python interpreter.

Other functionalities were also added or improved: the scientific visualization was improved and it is now possible to create dynamic geometric model in AXEL.

The software is distributed as a source package, as well as binary packages for Linux, MacOSX and Windows. It is hosted at http://dtk.inria.fr/axel with some of its plugins developed on Inria's gforge server (http://gforge.inria.fr) The first version of the software has been downloaded more than 15000 times, since it is available. A new version, AXEL 2.3.1, was released at the end of this year.

Collaboration with Gang Xu (Hangzhou Dianzi University, China), Julien Wintz (Dream), Elisa Berrini (MyCFD, Sophia), Angelos Mantzaflaris (GISMO library, Linz, Austria) and Laura Saini (Post-Doc GALAAD/Missler, TopSolid).

6. New Results

6.1. Algebraic representations for geometric modeling

6.1.1. Fitting ideals and multiple-points of surface parameterizations

Participant: Laurent Busé.

Parameterized algebraic surfaces are ubiquitous in geometric modeling and the determination of their singular loci is an important problem. Given a birational parameterization ϕ from $\mathbb{P}^2$ to $\mathbb{P}^3$ of a rational algebraic surface $\mathcal{S}$, the purpose of this work is to investigate the sets of points on $\mathcal{S}$ whose preimage consists in k or more points, counting multiplicity. In collaboration with Nicolas Botbol (University of Buenos Aires) and Marc Chardin (UMPC), we prove that they can be described in terms of Fitting ideals of some graded parts of the symmetric algebra associated to the parameterization ϕ. More precisely, we show that the drop of rank of a certain elimination matrix $M(\phi)$ at a given point $P \in \mathbb{P}^3$ is in relation with the fiber of the graph of ϕ over P. Thus, the Fitting ideals attached to $M(\phi)$ provide a filtration of the surface which is in correspondence with the degree and the dimension of the fibers of the graph of the parameterization ϕ. This property is linked with the double-point formulas that have been extensively studied in the field of intersection theory for finite maps.

This work has been accepted for presentation at the international conference MEGA 2013 and is submitted for publication [33].

6.1.2. Discriminant of a homogeneous and symmetric polynomial

Participant: Laurent Busé.

Polynomial algebra offers a standard approach to handle several problems in geometric modeling. A key tool is the discriminant of a well-constrained system of polynomial equations, which expresses the existence of a multiple root. In this work the factorization of a single homogeneous and symmetric polynomial is investigated. Indeed, in this setting the discriminant possesses a lot of symmetries and all of these symmetries produce an independent factor of the global discriminant. The two difficult points here are to prove that each of these factors are irreducible over a nice base ring and to determine its multiplicity in the factorization of the discriminant. This work, in collaboration with Anna Karasoulou (University of Athens) is still under progress.

6.1.3. On the cactus rank of cubic forms

In [14], we prove that the smallest degree of an apolar 0-dimensional scheme of a general cubic form in $n+1$ variables is at most $2n+2$, when $n \geq 8$, and therefore smaller than the rank of the form. For the general reducible cubic form the smallest degree of an apolar subscheme is $n+2$, while the rank is at least $2n$.

This is a work done by Alessandra Bernardi when she was post-doctorate for DECONSTRUCT IEF project, in collaboration with Kristian Ranestad (University of Oslo).

6.1.4. Grassmann secants and linear systems of tensors

For any irreducible non-degenerate variety $X \subset \mathbb{P}^r$, we relate in [11] the dimension of the s-th secant varieties of the Segre embedding of $\mathbb{P}^k \times X$ to the dimension of the (k,s)-Grassmann secant variety $GS_X(k,s)$ of X. We also give a criterion for the s-identifiability of X.

This is a work done by Alessandra Bernardi when she was post-doctorate for DECONSTRUCT IEF project, in collaboration with Edoardo Ballico (University of Trento), Maria Virgina Catalisano (DIPTEM, Genova), Luca Chiantini (University of Sienna).

6.1.5. Optimal analysis-aware parameterization of computational domain in 3D isogeometric analysis

Participants: André Galligo, Bernard Mourrain.

In isogeometric analysis framework, computational domain is exactly described using the same representation as that employed in the CAD process. For a CAD object, we can construct various computational domain with same shape but with different parameterization. One basic requirement is that the resulting parameterization should have no self-intersections. In [27], a linear and easy-to-check sufficient condition for injectivity of trivariate B-spline parameterization is proposed. By an example of 3D thermal conduction problem, we show that different parameterization of computational domain has different impact on the simulation result and efficiency in isogeometric analysis. For problems with exact solutions, we propose a shape optimization method to obtain optimal parameterization of computational domain. The proposed injective condition is used to check the injectivity of initial trivariate B-spline parameterization constructed by discrete Coons volume method, which is the generalization of discrete Coons patch method. Several examples and comparisons are presented to show the effectiveness of the proposed method. Compared with the initial parameterization during refinement, the optimal parameterization can achieve the same accuracy but with less degrees of freedom.

This is a joint work with Régis Duvigneau (Inria, EPI OPALE) and Xu Gang (College of computer - Hangzhou Dianzi University, China).

6.1.6. Constructing analysis-suitable parameterization of computational domain from CAD boundary by variational harmonic method

Participants: André Galligo, Bernard Mourrain.

In isogeometric analysis, parameterization of computational domain has great effects as mesh generation in finite element analysis. In the paper [26], based on the concept of harmonic mapping from the computational domain to parametric domain, a variational harmonic approach is proposed to construct analysis-suitable parameterization of computational domain from CAD boundary for 2D and 3D isogeometric applications. Different from the previous elliptic mesh generation method in finite element analysis, the proposed method focuses on isogeometric version, and converts the elliptic PDE into a nonlinear optimization problem, in which a regular term is integrated into the optimization formulation to achieve more uniform and orthogonal iso-parametric structure near convex (concave) parts of the boundary. Several examples are presented to show the efficiency of the proposed method in 2D and 3D isogeometric analysis.

This is a joint work with Régis Duvigneau (Inria, EPI OPALE) and Xu Gang (College of computer - Hangzhou Dianzi University, China).

6.1.7. Spline Spaces over Quadrangle Meshes with Complex Topologies

Participants: Meng Wu, André Galligo, Bernard Mourrain.

We study a new type of spline functions defined over a rectangular mesh equipped with an equivalence relation, in such a way that physical spaces with a complex topology can be represented as an homomorphic image of such meshes. We provide general definitions, a dimension formula for a subclass of these spline spaces, an explicit construction of their bases and also a process for local refinement. These developments, motivated by plane curvilinear mesh constructions are illustrated on several parametrization problems. Our main target in these constructions is to approximate isobaric lines of magnetic fields encountered in MHD (Magnetohydrodynamics) simulation for Tokamaks. Their particularity is that one of the isobaric curve has a node singularity.

This work is done in collaboration with Boniface Nkonga (Inria, EPI CASTOR and University of Nice).

6.1.8. Lagrangian Curves in Affine Symplectic 4-space

Participant: Evelyne Hubert.

Lagrangian curves in 4-space entertain intriguing relationships with second order deformation of plane curves under the special affine group and null curves in a 3-dimensional Lorentzian space form. In [39] we provide a natural affine symplectic frame for Lagrangian curves. It allows us to classify Lagrangrian curves with constant symplectic curvatures, to construct a class of Lagrangian tori and determine Lagrangian geodesics.

This is joint work with Emilio Musso, Dipartimento di Scienze Matematiche, Politecnico de Turino (Italy).

6.2. Algebraic algorithms for geometric computing

6.2.1. Implicit matrix representations of rational Bézier curves and surfaces

Participant: Laurent Busé.

In this work, we introduce and study a new implicit representation of rational Bézier curves and surfaces in the 3-dimensional space. Given such a curve or surface, this representation consists of a matrix whose entries depend on the space variables and whose rank drops exactly on this curve or surface. Our approach can be seen as an extension of the moving lines implicitization method introduced by Sederberg, from non-singular matrices to the more general context of singular matrices. First, we describe the construction of these new implicit matrix representations and their main geometric properties, in particular their ability to solve efficiently the inversion problem. Second, we show that these implicitization matrices adapt geometric problems, such as intersection problems, to the powerful tools of numerical linear algebra, in particular to one of the most important: the singular value decomposition. So, from the singular values of a given implicit matrix representation, we introduce a real evaluation function. We show that the variation of this function is qualitatively comparable to the Euclidean distance function. As an interesting consequence, we obtain a new determinantal formula for implicitizing a rational space curve or surface over the field of real numbers. Then, we show that implicit matrix representations can be used with numerical computations, in particular there is no need for symbolic computations to use them. We give some rigorous results explaining the numerical stability that we have observed in our experiments. We end the paper with a short illustration on ray tracing of parameterized surfaces.

This work has been accepted for presentation and publication at the SIAM conference on Geometric and Physical Modeling 2013 (Denver, USA, Nov. 11-14) [15]. It has been awarded the best paper price, 1st place.

6.2.2. Superfast solution of Toeplitz systems based on syzygy reduction

Participant: Bernard Mourrain.

In [22], we present a new superfast algorithm for solving Toeplitz systems. This algorithm is based on a relation between the solution of such problems and syzygies of polynomials or moving lines. We show an explicit connection between the generators of a Toeplitz matrix and the generators of the corresponding module of syzygies. We show that this module is generated by two elements and the solution of a Toeplitz system $Tu = g$ can be reinterpreted as the remainder of a vector depending on g, by these two generators. We obtain these generators and this remainder with computational complexity $O(n \log^2 n)$ for a Toeplitz matrix of size $n \times n$.

This is a joint work with Houssam Khalil (Université Claude Bernard - Lyon I) and Michelle Schatzman (Institut Camille Jordan, Lyon).

6.2.3. Budan Tables of Real Univariate Polynomials

Participant: André Galligo.

The Budan table of f collects the signs of the iterated derivative of f. We revisit the classical Budan-Fourier theorem for a univariate real polynomial f and establish a new connexity property of its Budan table. In [18], we use this property to characterize the virtual roots of f, (introduced by Gonzales-Vega, Lombardi, Mahe in 1998); they are continuous functions of the coefficients of f. We also consider a property (P) of a polynomial f, which is generically satisfied, it eases the topological-combinatorial description and study of the Budan tables. A natural extension of the information collected by the virtual roots provides alternative representations of (P)-polynomials; while an attached tree structure allows a strati fication of the space of (P)-polynomials.

6.2.4. A polynomial approach for extracting the extrema of a spherical function and its application in diffusion MRI

Participant: Bernard Mourrain.

Antipodally symmetric spherical functions play a pivotal role in diffusion MRI (Magnetic Resonance Imaging) in representing sub-voxel-resolution microstructural information of the underlying tissue. This information is described by the geometry of the spherical function. In [20], we propose a method to automatically compute all the extrema of a spherical function. We then classify the extrema as maxima, minima and saddle-points to identify the maxima. We take advantage of the fact that a spherical function can be described equivalently in the spherical harmonic (SH) basis, in the symmetric tensor (ST) basis constrained to the sphere, and in the homogeneous polynomial (HP) basis constrained to the sphere. We extract the extrema of the spherical function by computing the stationary points of its constrained HP representation. Instead of using traditional optimization approaches, which are inherently local and require exhaustive search or re-initializations to locate multiple extrema, we use a novel polynomial system solver which analytically brackets all the extrema and refines them numerically, thus missing none and achieving high precision. To illustrate our approach we consider the Orientation Distribution Function (ODF). In diffusion MRI the ODF is a spherical function which represents a state-of-the-art reconstruction algorithm whose maxima are aligned with the dominant fiber bundles. It is, therefore, vital to correctly compute these maxima to detect the fiber bundle directions. To demonstrate the potential of the proposed polynomial approach we compute the extrema of the ODF to extract all its maxima. This polynomial approach is, however, not dependent on the ODF and the framework presented in this paper can be applied to any spherical function described in either the SH basis, ST basis or the HP basis.

This is a joint work with Aurobrata Ghosh (Inria, EPI ATHENA), Elias Tsigaridas (Inria, EPI POLSYS), Rachid Deriche (Inria, EPI ATHENA).

6.2.5. The geometry of sound-source localization using non-coplanar microphone arrays

The paper [29] addresses the task of sound-source localization from time delay estimates using arbitrarily shaped non-coplanar microphone arrays. We fully exploit the direct path propagation model and our contribution is threefold: we provide a necessary and sufficient condition for a set of time delays to correspond to a sound source position, a proof of the uniqueness of this position, and a localization mapping to retrieve it. The time delay estimation task is casted into a non-linear multivariate optimization problem constrained by necessary and sufficient conditions on time delays. Two global optimization techniques to estimate time delays and localize the sound source are investigated. We report an extensive set of experiments and comparisons with state-of-the-art methods on simulated and real data in the presence of noise and reverberations.

This is a joint work with Xavier Alameda-Pineda (Inria, EPI PERCEPTION) and Radu Horaud (Inria, EPI PERCEPTION).

6.2.6. Rational Invariants of a Group Action

Participant: Evelyne Hubert.

The article [28] is based on introductory lectures delivered at the Journées Nationales de Calcul Formel that took place at the Centre International de Recherche en Mathématiques (2013) in Marseille. We introduce basic notions on algebraic group actions and their invariants. Based on geometric considerations, we present algebraic constructions for a generating set of rational invariants. In particular the use of sections and quasi-sections contribute to increased efficiency and reduced output size. The notion of sections is refined compared to the cross-sections used in [9].

6.2.7. Rational Invariants of Finite Abelian groups

Participant: Evelyne Hubert.

In [36] we investigate the field of rational invariants of the linear action of a finite abelian group in the non modular case. By diagonalization, the group is accurately described by an integer matrix of exponents. We make use of linear algebra to compute a minimal generating set of invariants and the substitution to rewrite any invariant in terms of this generating set. We show that the generating set can be chosen to consist of polynomial invariants. As an application, we provide a symmetry reduction scheme for polynomial systems the solution set of which are invariant by the group action.

This is joint work with George Labahn, University of Waterloo, Ontario (Canada).

6.2.8. Exact relaxation for polynomial optimization on semi-algebraic sets

Participants: Marta Abril Bucero, Bernard Mourrain.

In [31], we study the problem of computing by relaxation hierarchies the infimum of a real polynomial function f on a closed basic semialgebraic set S and the points where this infimum is reached, if they exist. We show that when the infimum is reached, a relaxation hierarchy constructed from the Karush-Kuhn-Tucker ideal is always exact and that the vanishing ideal of the KKT minimizer points is generated by the kernel of the associated moment matrix in that degree, even if this ideal is not zero-dimensional. We also show that this relaxation allows to detect when there is no KKT minimizer. We prove that the exactness of the relaxation depends only on the real points which satisfy these constraints. This exploits representations of positive polynomials as elements of the preordering modulo the KKT ideal, which only involves polynomials in the initial set of variables. The approach provides a uniform treatment of different optimization problems considered previously. Applications to global optimization, optimization on semialgebraic sets defined by regular sets of constraints, optimization on finite semialgebraic sets, real radical computation are given.

6.3. Symbolic-Numeric Analysis

6.3.1. Numerical Reconstruction of Convex Polytopes from Directional Moments

Participants: Mathieu Collowald, Evelyne Hubert.

In [35] we address the reconstruction of convex polytopes, in any dimension n, from the knowledge of a finite set of directional moments of the shape. Starting with the formula relating the projection of the vertices to the directional moments, we employ established numerical algorithms for generalized eigenvalues and interval interpolation to recover the coordinates of the vertices. We perform the reconstruction of a diamond cut using our novel method.

This is joint work with Annie Cuyt, Wen-Shin Lee and Oliver Salazar Celis from University of Antwerp (Belgium).

6.3.2. Bulbous Bow Shape Optimization

Participant: Bernard Mourrain.

The aim of the work [30] is to prove the usefulness of a bulbous bow for a fishing vessel, in terms of drag reduction, using an automated shape optimization procedure including hydrodynamic simulations. A bulbous bow is an appendage that is known to reduce the drag, thanks to its influence on the bow wave system. However, the definition of the geometrical parameters of the bulb, such as its length and thickness, is not intuitive, as both parameters are coupled with regards to their influence on the final drag. Therefore, we propose to use an automated shape optimization procedure, based on a high-fidelity flow solver, a surrogate model-based optimizer and a CAD-based geometrical model, to derive the characteristics of the bow geometry allowing to maximize the achievable drag reduction. The numerical tools are first presented, and then applied to the optimization of a bow shape for a real fishing vessel, in order to determine the optimal length and thickness of the bow for drag reduction purpose.

This is a joint work with Louis Blanchard and Régis Duvigneau (Inria, EPI OPALE), Elisa Berrini (MyCFD), Yann Roux (K-Epsilon) Eric Jean (Jean & Frasca Design).

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Grants with Industry

7.1.1. Algebraic-geometric methods for design and manufacturing

This collaboration between Inria and Missler in the context of Carnot program, aims at developing algebraic-geometric computational techniques for the control of machining tools. It focuses on the problem of pocket manufacturing and the computation of medial axis and of offsets of planar regions with piecewise algebraic boundaries. An integration of plugins related to AXEL platform into the CAGD modeler TOPSOLID developed by Missler is planned. Laura Saini is involved in this collaboration.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. GEOLMI

GEOLMI - Geometry and Algebra of Linear Matrix Inequalities with Systems Control Applications - is an ANR project working on topics related to the Geometry of determinantal varieties, positive polynomials, computational algebraic geometry, semidefinite programming and systems control applications.

The partners are LAAS-CNRS, Univ. de Toulouse (coordinator), LJK-CNRS, Univ. Joseph Fourier de Grenoble; Inria Sophia Antipolis Méditerranée; LIP6-CNRS Univ. Pierre et Marie Curie; Univ. de Pau et des Pays de l'Adour; IRMAR-CNRS, Univ. de Rennes.

More information available at http://homepages.laas.fr/henrion/geolmi.

8.1.2. ANEMOS

ANEMOS - Advanced Numeric for ELMs (Edge Localized Mode) : Modeling and Optimized Schemes - is an ANR project devoted to the numerical modelling study of such ELM control methods as Resonant Magnetic Perturbations (RMPs) and pellet ELM pacing both foreseen in ITER. The goals of the project are to improve understanding of the related physics and propose possible new strategies to improve effectiveness of ELM control techniques. The study of spline spaces for isogemetric finite element methods is proposed in this context.

The partners are IRFM, CEA, Cadarache; JAD, University of Nice - Sophia Antipolis; Inria, Bacchus; Maison de la Simulation CEA-CNRS-Inria-University of Orsay- University of Versailles St Quentin .

8.2. European Initiatives

8.2.1. FP7 Projects

8.2.1.1. TERRIFIC

Title: Towards Enhanced Integration of Design and Production in the Factory of the Future through Isogeometric Technologies

Type: COOPERATION (ICT)

Defi: PPP FoF: Digital factories: Manufactoring design and product lifecycle manage

Instrument: Specific Targeted Research Project (STREP)

Duration: September 2011 - August 2014

Coordinator: SINTEF, Oslo (Norway)

Others partners:

Alenia Aeronautica (Italy); Inria Méditerranée (France); Jozef Kepler universitet, Linz (Austria); JOTNE, Oslo (Norway); MAGNA, Steyr (Austria); Missler Software (France); Siemens AG (Germany); Technische Universität Kaiserslautern (Germany); University of Pavia (Italy).

See also: http://terrific-project.eu

Abstract: The project aims at significant improvement of the interoperability of computational tools for the design, analysis and optimization of functional products. An isogeometric approach is applied for selected manufacturing application areas (cars, trains, aircrafts) and for computer-aided machining. Computer Aided Design (CAD) and numerical simulation algorithms are vital technologies in modern product development, yet they are today far from being seamlessly integrated. Their interoperability is severely disturbed by inconsistencies in the mathematical approaches used. Efficient feedback from analysis to CAD and iterative refinement of the analysis model is a feature of isogeometric analysis, and would be an essential improvement for computer-based design optimization and virtual product development. Our vision is to provide and disseminate tangible evidence of the performance of the isogeometric approach in comparison to traditional ones in four important application areas as well as addressing interoperability and other issues that necessarily arise in a large-scale industrial introduction of isogeometry.

8.2.2. Collaborations in European Programs, except FP7

8.2.2.1. PHC TOURNESOL FL

Program: TOURNESOL

Project acronym: PHC TOURNESOL FL 2012 - 26409SH

Project title: Extracting multidimensional shapes

Duration: January 2012 - December 2013

Coordinator: E. Hubert (Inria), A. Cuyt (Universiteit Antwerpen)

Other partners: Inria Sophia-Antipolis (France); Universiteit Antwerpen (Belgium)

Abstract: We are working on the shape-from-moments problem : from measurement-like data, reconstructing a desired object. For many years, this problem has been solved and optimized in the 2D-case thanks to use of complex numbers. Thanks to a new formula, we want to stay in the real domain in order to generalize this problem to multidimensional shapes - in particular 3D-shapes. For more details about our project TOURNESOL: http://www-sop.inria.fr/teams/galaad/joomla/index.php/international-collaborations-147/173-tournesol.html. For more details about the program TOURNESOL: http://www.campusfrance.org/fr/tournesol-communaute-francaise.

8.3. International Initiatives

8.3.1. Participation In International Programs

We have a bilateral collaboration between Galaad and the University of Athens-DIT team ERGA, headed by Ioannis Emiris for the period August 2013-August 2014. It is supported by both Inria and the University of Athens.

Title: Algebraic algorithms in optimization

Abstract: In the past decade, algebraic approaches to optimization problems defined in terms of multivariate polynomials have been intensively explored and studied in several directions. One example is the work on semidefinite optimization and, more recently, convex algebraic geometry. This project aims to focus on algebraic approaches for optimization applications in the wide sense. We concentrate on specific tools, namely root counting techniques, the resultant, the discriminant and non-negative polynomials, on which the two teams have extensive collaboration and expertise. We examine applications in convex algebraic geometry as well as to a newer topic for the two teams, namely game theory. A common thread to these approaches is to exploit any (sparse) structure.

8.4. International Research Visitors

8.4.1. Visits of International Scientists

Wen-Shin Lee and Annie Cuyt (University of Antwerp, Belgium) visited from Monday June 3rd to Friday June 7th in the context of the TOURNESOL project.

8.4.2. Visits to International Teams

Evelyne Hubert was invited to La Trobe university for the whole month of January to carry on a collaboration with Peter van der Kamp on geometric curve flows and their integrability.

Evelyne Hubert and Bernard Mourrain were invited to the Institute of Mathematical Science at the National University of Singapore to participate to the 2 month long program *Inverse Moment Problems: the Crossroads of Analysis, Algebra, Discrete Geometry and Combinatorics*.

Evelyne Hubert and Bernard Mourrain visited Wen-Shin Lee and Annie Cuyt (University of Antwerp, Belgium) on November 20-21 in the context of the TOURNESOL project.

9. Dissemination

9.1. Scientific Animation

Laurent Busé

- is a member of the Jury of the french "agrégation externe de mathématiques".

André Galligo

- is part of the Advisory Board of the MEGA conference series and took part this year to its program committee.

Evelyne Hubert

- is an associate editor of the Journal of Symbolic Computation (http://www.journals.elsevier.com/journal-of-symbolic-computation),
- is part of the Advisory Board of the MEGA conference series and, as such, took part to the program committee of this year conference,
- edited, with Elizabeth Mansfield (University of Kent, UK) and Gloria Mari Beffa (University of Wisconsin, USA) a special issue of the journal *Foundations of Computational Mathematics* (Volume 13, Issue 4, August 2013) in honor of Peter Olver (University of Minnesota, USA). Cf. http://link.springer.com/journal/10208,
- is part of the local Inria Committee for Invited Professors and Postodoctoral Fellows,
- was elected to the new hiring committee at the mathematics department of the University of Nice (CPRH 25-26-60),
- co-organized the mini-symposia *Computational Aspects of Moving Frame* that was part of the *SIAM Conference on Applied Algebraic Geometry* that took place in Fort Collins, Colorado, August 1-4.

Bernard Mourrain

- is an associate editor of the Journal of Symbolic Computation (http://www.journals.elsevier.com/journal-of-symbolic-computation),
- is member of the Advisory Board of the MEGA conference series and chair of the program committee of MEGA'13 in Frankfurt, June 3-7,
- is co-editing, with Alicia Dickenstein and Jan Draisma, a special issue of the Journal of Symbolic Computation, after the conference MEGA'13,
- organized the mini-symposia *Algebraic Geometry, Moment Problems and Applications* that was part of the *SIAM Conference on Applied Algebraic Geometry* that took place in Fort Collins, Colorado, August 1-4,
- is chair of the local Inria Committee for Courses and Conferences.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Licence: Mathieu Collowald, Statistics (Exercices), 20h ETD, L1 MASS, University of Nice-Sophia Antipolis, France

Licence: Mathieu Collowald, Algebra (Exercices), 44h ETD, L1 I, University of Nice-Sophia Antipolis, France

Master: Laurent Busé, Curves and Surfaces, 60h ETD, MAM4, Ecole Polytechnique of the University of Nice-Sophia Antipolis, France

Master: Laurent Busé, Elimination theory for geometric modeling, 40h ETD, M2 of mathematics, University of Nice-Sophia Antipolis, France

Doctoral: Evelyne Hubert, Invariants of Group Actions, courses at the french workshop *Journées Nationales de Calcul Formel* (May 2013), 3h, CIRM Luminy, France.

9.2.2. Supervision

9.2.2.1. Ph.D. thesis in progress

Marta Abril Bucero, Moment matrices, real algebraic geometry and polynomial optimization. Advisor: Bernard Mourrain.

Mathieu Collowald, Integral representation of shapes for feature conservation or extraction. Advisor: Evelyne Hubert.

9.2.2.2. Internship

Matthieu Dien, *Décomposition de Tenseur Symétrique : Algorithme et Implémentation* 3.5 months, Master 1. Advisor: Bernard Mourrain.

Anna Karasoulou, *Discriminant of multivariate homogeneous symmetric polynomials*, 6 months, doctoral. Advisor: Laurent Busé.

9.2.3. Juries

Laurent Busé was a member of the PhD committee of Amir Bagheri, "Cohomologie locale, les idéaux gradués et leurs puissances", 25th September, University Pierre et Marie Curie, Paris, France.

Bernard Mourrain was member of the Ph.D. committee of Tatjana A. Kalinka, “Changing representation of curves and surfaces: exact and approximate methods”, University of Athenes, 15th March; of the Ph.D. committee of Remy Imbach, “Résolution de contraintes géométriques en guidant une méthode homotopique par la géométrie”, University of Strasbourg, 8th October; of the HDR committee of G. Lecerf, “Algorithmique des polynômes à plusieurs variables : opérations élémentaires, factorisation, élimination”, University of Orsay, 26th November.

Evelyne Hubert was a member of the PhD committee of Christine Jost, *Topics in Computational Algebraic Geometry and Deformation Quantization*, Stockholm University, March 11th, 2013; and Cédric Zanni, *Skeleton based Implicit Modeling and Applications*, Université de Grenoble, December 6th 2013. She also took part to the mid-term PhD evaluation of Jules Svartz, *Systèmes polynômiaux présentant des symétries : Algorithmes, Complexité, Applications*, Université de Paris 6, June 20th, 2013.

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