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Project-Team GECO

Geometric Control Design

RESEARCH CENTER
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THEME
Optimization and control of dynamic systems

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Project-Team GECO

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2. Overall Objectives

2.1. Overall Objectives

Motion planning is not only a crucial issue in control theory, but also a widespread task of all sort of human activities. The aim of the project-team is to study the various aspects preceding and framing *motion planning*: accessibility analysis (determining which configurations are attainable), criteria to make choice among possible trajectories, trajectory tracking (fixing a possibly unfeasible trajectory and following it as closely as required), performance analysis (determining the cost of a tracking strategy), design of implementable algorithms, robustness of a control strategy with respect to computationally motivated discretizations, etc. The viewpoint that we adopt comes from geometric control: our main interest is in qualitative and intrinsic properties and our focus is on trajectories (either individual ones or families of them).

The main application domain of GECO is *quantum control*. The importance of designing efficient transfers between different atomic or molecular levels in atomic and molecular physics is due to its applications to photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing.

A second application area concerns the control interpretation of phenomena appearing in *neurophysiology*. It studies the modeling of the mechanisms supervising some biomechanics actions or sensorial reactions such as image reconstruction by the primary visual cortex, eyes movement and body motion. All these problems can be seen as motion planning tasks accomplished by the brain.

As a third applicative domain we propose a system dynamics approach to *switched systems*. Switched systems are characterized by the interaction of continuous dynamics (physical system) and discrete/logical components. They provide a popular modeling framework for heterogeneous aspects issuing from automotive and transportation industry, energy management and factory automation.

2.2. Highlights of the Year

We edited two volumes on two different and challenging subjects, that is hybrid systems with constraints [12] and sub-Riemannian geometry [13].

3. Research Program

3.1. Geometric control theory

The main research topic of the project-team will be **geometric control**, with a special focus on **control design**. The application areas that we target are control of quantum mechanical systems, neurogeometry and switched systems.

Geometric control theory provides a viewpoint and several tools, issued in particular from differential geometry, to tackle typical questions arising in the control framework: controllability, observability, stabilization, optimal control... [29], [64] The geometric control approach is particularly well suited for systems involving nonlinear and nonholonomic phenomena. We recall that nonholonomicity refers to the property of a velocity constraint that is not equivalent to a state constraint.

The expression **control design** refers here to all phases of the construction of a control law, in a mainly open-loop perspective: modeling, controllability analysis, output tracking, motion planning, simultaneous control algorithms, tracking algorithms, performance comparisons for control and tracking algorithms, simulation and implementation.

We recall that

- **controllability** denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law ;
- **output tracking** refers to a control strategy aiming at keeping the value of some functions of the state arbitrarily close to a prescribed time-dependent profile. A typical example is **configuration tracking** for a mechanical system, in which the controls act as forces and one prescribes the position variables along the trajectory, while the evolution of the momenta is free. One can think for instance at the lateral movement of a car-like vehicle: even if such a movement is unfeasible, it can be tracked with arbitrary precision by applying a suitable control strategy;
- **motion planning** is the expression usually denoting the algorithmic strategy for selecting one control law steering the system from a given initial state to an attainable final one;
- **simultaneous control** concerns algorithms that aim at driving the system from two different initial conditions, with the same control law and over the same time interval, towards two given final states (one can think, for instance, at some control action on a fluid whose goal is to steer simultaneously two floating bodies.) Clearly, the study of which pairs (or n-uples) of states can be simultaneously connected thanks to an admissible control requires an additional controllability analysis with respect to the plain controllability mentioned above.

At the core of control design is then the notion of motion planning. Among the motion planning methods, a preeminent role is played by those based on the Lie algebra associated with the control system ([84], [71], [77]), those exploiting the possible flatness of the system ([58]) and those based on the continuation method ([96]). Optimal control is clearly another method for choosing a control law connecting two states, although it generally introduces new computational and theoretical difficulties.

Control systems with special structure, which are very important for applications are those for which the controls appear linearly. When the controls are not bounded, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a smoothly varying norm (representing a cost of the control), the resulting geometrical structure is called *sub-Riemannian*. Sub-Riemannian geometry thus appears as the underlying geometry of the nonholonomic control systems, playing the same role as Euclidean geometry for linear systems. As such, its study is fundamental for control design. Moreover its importance goes far beyond control theory and is an active field of research both in differential geometry ([83]), geometric measure theory ([59], [33]) and hypoelliptic operator theory ([45]).

Other important classes of control systems are those modeling mechanical systems. The dynamics are naturally defined on the tangent or cotangent bundle of the configuration manifold, they have Lagrangian or Hamiltonian structure, and the controls act as forces. When the controls appear linearly, the resulting model can be seen somehow as a second-order sub-Riemannian structure (see [50]).

The control design topics presented above naturally extend to the case of distributed parameter control systems. The geometric approach to control systems governed by partial differential equations is a novel subject with great potential. It could complement purely analytical and numerical approaches, thanks to its more dynamical, qualitative and intrinsic point of view. An interesting example of this approach is the paper [30] about the controllability of Navier–Stokes equation by low forcing modes.

4. Application Domains

4.1. Quantum control

The issue of designing efficient transfers between different atomic or molecular levels is crucial in atomic and molecular physics, in particular because of its importance in those fields such as photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (NMR, control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing. This last application explicitly relies on the design of quantum gates, each of them being, in essence, an open loop control law devoted to a prescribed simultaneous control action. NMR is one of the most promising techniques for the implementation of a quantum computer.

Physically, the control action is realized by exciting the quantum system by means of one or several external fields, being them magnetic or electric fields. The resulting control problem has attracted increasing attention, especially among quantum physicists and chemists (see, for instance, [89], [94]). The rapid evolution of the domain is driven by a multitude of experiments getting more and more precise and complex (see the recent review [49]). Control strategies have been proposed and implemented, both on numerical simulations and on physical systems, but there is still a large gap to fill before getting a complete picture of the control properties of quantum systems. Control techniques should necessarily be innovative, in order to take into account the physical peculiarities of the model and the specific experimental constraints.

The area where the picture got clearer is given by finite dimensional linear closed models.

- **Finite dimensional** refers to the dimension of the space of wave functions, and, accordingly, to the finite number of energy levels.
- **Linear** means that the evolution of the system for a fixed (constant in time) value of the control is determined by a linear vector field.
- **Closed** refers to the fact that the systems are assumed to be totally disconnected from the environment, resulting in the conservation of the norm of the wave function.

The resulting model is well suited for describing spin systems and also arises naturally when infinite dimensional quantum systems of the type discussed below are replaced by their finite dimensional Galerkin approximations. Without seeking exhaustiveness, let us mention some of the issues that have been tackled for finite dimensional linear closed quantum systems:

- controllability [31],
- bounds on the controllability time [27],
- STIRAP processes [99],
- simultaneous control [72],
- optimal control ([68], [40], [51]),
- numerical simulations [78].

Several of these results use suitable transformations or approximations (for instance the so-called rotating wave) to reformulate the finite-dimensional Schrödinger equation as a sub-Riemannian system. Open systems have also been the object of an intensive research activity (see, for instance, [32], [69], [90], [46]).

In the case where the state space is infinite dimensional, some optimal control results are known (see, for instance, [36], [47], [65], [37]). The controllability issue is less understood than in the finite dimensional setting, but several advances should be mentioned. First of all, it is known that one cannot expect exact controllability on the whole Hilbert sphere [98]. Moreover, it has been shown that a relevant model, the quantum oscillator, is not even approximately controllable [91], [81]. These negative results have been more recently completed by positive ones. In [38], [39] Beauchard and Coron obtained the first positive controllability result for a quantum particle in a 1D potential well. The result is highly nontrivial and is based on Coron's return method (see [54]). Exact controllability is proven to hold among regular enough wave functions. In particular, exact controllability among eigenfunctions of the uncontrolled Schrödinger operator can be achieved. Other important approximate controllability results have then been proved using Lyapunov methods [80], [85], [66]. While [80] studies a controlled Schrödinger equation in $\mathbb{R}$ for which the uncontrolled Schrödinger operator has mixed spectrum, [85], [66] deal mainly with general discrete-spectrum Schrödinger operators.

In all the positive results recalled in the previous paragraph, the quantum system is steered by a single external field. Different techniques can be applied in the case of two or more external fields, leading to additional controllability results [57], [43].

The picture is even less clear for nonlinear models, such as Gross–Pitaevski and Hartree–Fock equations. The obstructions to exact controllability, similar to the ones mentioned in the linear case, have been discussed in [63]. Optimal control approaches have also been considered [35], [48]. A comprehensive controllability analysis of such models is probably a long way away.

4.2. Neurophysiology

At the interface between neurosciences, mathematics, automatics and humanoid robotics, an entire new approach to neurophysiology is emerging. It arouses a strong interest in the four communities and its development requires a joint effort and the sharing of complementary tools.

A family of extremely interesting problems concerns the understanding of the mechanisms supervising some sensorial reactions or biomechanics actions such as image reconstruction by the primary visual cortex, eyes movement and body motion.

In order to study these phenomena, a promising approach consists in identifying the motion planning problems undertaken by the brain, through the analysis of the strategies that it applies when challenged by external inputs. The role of control is that of a language allowing to read and model neurological phenomena. The control algorithms would shed new light on the brain's geometric perception (the so-called neurogeometry [87]) and on the functional organization of the motor pathways.

- A challenging problem is that of the understanding of the mechanisms which are responsible for the process of image reconstruction in the primary visual cortex V1.

The visual cortex areas composing V1 are notable for their complex spatial organization and their functional diversity. Understanding and describing their architecture requires sophisticated modeling tools. At the same time, the structure of the natural and artificial images used in visual psychophysics can be fully disclosed only using rather deep geometric concepts. The word “geometry" refers here to the internal geometry of the functional architecture of visual cortex areas (not to the geometry of the Euclidean external space). Differential geometry and analysis both play a fundamental role in the description of the structural characteristics of visual perception.

A model of human perception based on a simplified description of the visual cortex V1, involving geometric objects typical of control theory and sub-Riemannian geometry, has been first proposed by Petitot ([88]) and then modified by Citti and Sarti ([53]). The model is based on experimental observations, and in particular on the fundamental work by Hubel and Wiesel [62] who received the Nobel prize in 1981.

In this model, neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli arriving at a given point of the retina and oriented along a given direction. The retina is modeled by the real plane, while the directions at a given point are modeled by the projective line. The fiber bundle having as base the real plane and as fiber the projective line is called the *bundle of directions of the plane*.

From the neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them sensitive to stimuli arriving at a given point, oriented along any direction. In the same hypercolumn, relative to a point of the plane, we also find neurons that are sensitive to other stimuli properties, such as colors. Therefore, in this model the visual cortex treats an image not as a planar object, but as a set of points in the bundle of directions of the plane. The reconstruction is then realized by minimizing the energy necessary to activate orientation columns among those which are not activated directly by the image. This gives rise to a sub-Riemannian problem on the bundle of directions of the plane.

- Another class of challenging problems concern the functional organization of the motor pathways.

  The interest in establishing a model of the motor pathways, at the same time mathematically rigorous and biologically plausible, comes from the possible spillovers in robotics and neurophysiology. It could help to design better control strategies for robots and artificial limbs, yielding smoother and more progressive movements. Another underlying relevant societal goal (clearly beyond our domain of expertise) is to clarify the mechanisms of certain debilitating troubles such as cerebellar disease, chorea and Parkinson’s disease.

  A key issue in order to establish a model of the motor pathways is to determine the criteria underlying the brain’s choices. For instance, for the problem of human locomotion (see [34]), identifying such criteria would be crucial to understand the neural pathways implicated in the generation of locomotion trajectories.

  A nowadays widely accepted paradigm is that, among all possible movements, the accomplished ones satisfy suitable optimality criteria (see [97] for a review). One is then led to study an inverse optimal control problem: starting from a database of experimentally recorded movements, identify a cost function such that the corresponding optimal solutions are compatible with the observed behaviors.

  Different methods have been taken into account in the literature to tackle this kind of problems, for instance in the linear quadratic case [67] or for Markov processes [86]. However all these methods have been conceived for very specific systems and they are not suitable in the general case. Two approaches are possible to overcome this difficulty. The direct approach consists in choosing a cost function among a class of functions naturally adapted to the dynamics (such as energy functions) and to compare the solutions of the corresponding optimal control problem to the experimental data. In particular one needs to compute, numerically or analytically, the optimal trajectories and to choose suitable criteria (quantitative and qualitative) for the comparison with observed trajectories. The inverse approach consists in deriving the cost function from the qualitative analysis of the data.

4.3. Switched systems

Switched systems form a subclass of hybrid systems, which themselves constitute a key growth area in automation and communication technologies with a broad range of applications. Existing and emerging areas include automotive and transportation industry, energy management and factory automation. The notion of hybrid systems provides a framework adapted to the description of the heterogeneous aspects related to the interaction of continuous dynamics (physical system) and discrete/logical components.

The characterizing feature of switched systems is the collective aspect of the dynamics. A typical question is that of stability, in which one wants to determine whether a dynamical system whose evolution is influenced by a time-dependent signal is uniformly stable with respect to all signals in a fixed class ([74]).

The theory of finite-dimensional hybrid and switched systems has been the subject of intensive research in the last decade and a large number of diverse and challenging problems such as stabilizability, observability, optimal control and synchronization have been investigated (see for instance [95], [75]).

The question of stability, in particular, because of its relevance for applications, has spurred a rich literature. Important contributions concern the notion of common Lyapunov function: when there exists a Lyapunov function that decays along all possible modes of the system (that is, for every possible constant value of the signal), then the system is uniformly asymptotically stable. Conversely, if the system is stable uniformly with respect to all signals switching in an arbitrary way, then a common Lyapunov function exists [76]. In the *linear* finite-dimensional case, the existence of a common Lyapunov function is actually equivalent to the global uniform exponential stability of the system [82] and, provided that the admissible modes are finitely many, the Lyapunov function can be taken polyhedral or polynomial [41], [42], [55]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see [56] and references therein). Algebraic approaches to prove the stability of switched systems under arbitrary switching, not relying on Lyapunov techniques, have been proposed in [73], [28].

Other interesting issues concerning the stability of switched systems arise when, instead of considering arbitrary switching, one restricts the class of admissible signals, by imposing, for instance, a dwell time constraint [61].

Another rich area of research concerns discrete-time switched systems, where new intriguing phenomena appear, preventing the algebraic characterization of stability even for small dimensions of the state space [70]. It is known that, in this context, stability cannot be tested on periodic signals alone [44].

Finally, let us mention that little is known about infinite-dimensional switched system, with the exception of some results on uniform asymptotic stability ([79], [92], [93]) and some recent papers on optimal control ([60], [100]).

5. Software and Platforms

5.1. IRHD

We develop a software for reconstruction of corrupted and damaged images, named IRHD (for Image Reconstruction via Hypoelliptic Diffusion). One of the main features of the algorithm on which the software is based is that it does not require any information about the location and character of the corrupted places. Another important advantage is that this method is massively parallelizable; this allows to work with sufficiently large images. Theoretical background of the presented method is based on the model of geometry of vision due to Petitot, Citti and Sarti. The main step is numerical solution of the equation of 3D hypoelliptic diffusion. IRHD is based on Fortran.

6. New Results

6.1. New results: geometric control

We start by presenting some results on motion planning and tracking algorithms.

- In [22] we study the complexity of the motion planning problem for control-affine systems. Such complexities are already defined and rather well-understood in the particular case of nonholonomic (or sub-Riemannian) systems. Our aim is to generalize these notions and results to systems with a drift. Accordingly, we present various definitions of complexity, as functions of the curve that is approximated, and of the precision of the approximation. Due to the lack of time-rescaling invariance of these systems, we consider geometric and parametrized curves separately. Then, we give some asymptotic estimates for these quantities.
- In [23] we study the problem of controlling an unmanned aerial vehicle (UAV) to provide a target supervision and to provide convoy protection to ground vehicles. We first present a control strategy based upon a Lyapunov–LaSalle stabilization method to provide supervision of a stationary target. The UAV is expected to join a pre-designed admissible circular trajectory around the target which is itself a fixed point in the space. Our strategy is presented for both HALE (High Altitude Long Endurance) and MALE (Medium Altitude Long Endurance) types UAVs. A UAV flying at a constant altitude (HALE type) is modeled as a Dubins vehicle (i.e. a planar vehicle with constrained turning radius and constant forward velocity). For a UAV that might change its altitude (MALE type), we use the general kinematic model of a rigid body evolving in $\mathbb{R}^3$. Both control strategies presented are smooth and unlike what is usually proposed in the literature these strategies asymptotically track a circular trajectory of exact minimum turning radius. We then consider the problem of adding to the tracking task an optimality criterion. In particular, we present the time-optimal control synthesis for tracking a circle by a Dubins vehicle. This optimal strategy, although much simpler than the point-to-point time-optimal strategy obtained by P. Souéres and J.-P. Laumond in the 1990s, is very rich. Finally, we propose control strategies to provide supervision of a moving target, that are based upon the previous ones.
- In [26] we prove the continuity and the Hölder equivalence w.r.t. an Euclidean distance of the value function associated with the L^1 cost of the control-affine system $\dot{q} = f_0(q) + \sum_{j=1}^{m} u_j f_j(q)$, satisfying the strong Hörmander condition. This is done by proving a result in the same spirit as the Ball-Box theorem for driftless (or sub-Riemannian) systems. The techniques used are based on a reduction of the control-affine system to a linear but time-dependent one, for which we are able to define a generalization of the nilpotent approximation and through which we derive estimates for the shape of the reachable sets. Finally, we also prove the continuity of the value function associated with the L^1 cost of time-dependent systems of the form $\dot{q} = \sum_{j=1}^{m} u_j f_j^t(q)$.

Let us list some new results in sub-Riemannian geometry and hypoelliptic diffusion.

- In [1] we provide normal forms for 2D almost-Riemannian structures, which are generalized Riemannian structures on surfaces for which a local orthonormal frame is given by a Lie bracket generating pair of vector fields that can become collinear. Generically, there are three types of points: Riemannian points where the two vector fields are linearly independent, Grushin points where the two vector fields are collinear but their Lie bracket is not, and tangency points where the two vector fields and their Lie bracket are collinear and the missing direction is obtained with one more bracket. We consider the problem of finding normal forms and functional invariants at each type of point. We also require that functional invariants are "complete" in the sense that they permit to recognize locally isometric structures. The problem happens to be equivalent to the one of finding a smooth canonical parameterized curve passing through the point and being transversal to the distribution. For Riemannian points such that the gradient of the Gaussian curvature K is different from zero, we use the level set of K as support of the parameterized curve. For Riemannian points such that the gradient of the curvature vanishes (and under additional generic conditions), we use a curve which is found by looking for crests and valleys of the curvature. For Grushin points we use the set where the vector fields are parallel. Tangency points are the most complicated to deal with. The cut locus from the tangency point is not a good candidate as canonical parameterized curve since it is known to be non-smooth. Thus, we analyze the cut locus from the singular set and we prove that it is not smooth either. A good candidate appears to be a curve which is found by looking for crests and valleys of the Gaussian curvature. We prove that the support of such a curve is uniquely determined and has a canonical parametrization.

- The curvature discussed in [14] is a rather far going generalization of the Riemann sectional curvature. We define it for a wide class of optimal control problems: a unified framework including geometric structures such as Riemannian, sub-Riemannian, Finsler and sub-Finsler structures; a special attention is paid to the sub-Riemannian (or Carnot–Caratheodory) metric spaces. Our construction of the curvature is direct and naive, and it is similar to the original approach of Riemann. Surprisingly, it works in a very general setting and, in particular, for all sub-Riemannian spaces.
- In [15] we provide the small-time heat kernel asymptotics at the cut locus in three relevant cases: generic low-dimensional Riemannian manifolds, generic 3D contact sub-Riemannian manifolds (close to the starting point) and generic 4D quasi-contact sub-Riemannian manifolds (close to a generic starting point). As a byproduct, we show that, for generic low-dimensional Riemannian manifolds, the only singularities of the exponential map, as a Lagragian map, that can arise along a minimizing geodesic are A_3 and A_5 (in the classification of Arnol'd's school). We show that in the non-generic case, a cornucopia of asymptotics can occur, even for Riemannian surfaces.
- In [19] we study the evolution of the heat and of a free quantum particle (described by the Schroedinger equation) on two-dimensional manifolds endowed with the degenerate Riemannian metric $ds^2 = dx^2 + |x|^{-2\alpha} d\theta^2$, where $x \in \mathbb{R}$, $\theta \in \mathbb{T}$ and the parameter $\alpha \in \mathbb{R}$. For $\alpha \leq -1$ this metric describes cone-like manifolds (for $\alpha = -1$ it is a flat cone). For $\alpha = 0$ it is a cylinder. For $\alpha \geq 1$ it is a Grushin-like metric. We show that the Laplace–Beltrami operator Δ is essentially self-adjoint if and only if $\alpha \notin (-3, 1)$. In this case the only self-adjoint extension is the Friedrichs extension Δ_F, that does not allow communication through the singular set $\{x = 0\}$ both for the heat and for a quantum particle. For $\alpha \in (-3, -1]$ we show that for the Schroedinger equation only the average on θ of the wave function can cross the singular set, while the solutions of the only Markovian extension of the heat equation (which indeed is Δ_F) cannot. For $\alpha \in (-1, 1)$ we prove that there exists a canonical self-adjoint extension Δ_B, called bridging extension, which is Markovian and allows the complete communication through the singularity (both of the heat and of a quantum particle). Also, we study the stochastic completeness (i.e., conservation of the L^1 norm for the heat equation) of the Markovian extensions Δ_F and Δ_B, proving that Δ_F is stochastically complete at the singularity if and only if $\alpha \leq -1$, while Δ_B is always stochastically complete at the singularity.

6.2. New results: quantum control

New results have been obtained for the control of the bilinear Schrödinger equation.

- In [4] we show the approximate rotational controllability of a polar linear molecule by means of three nonresonant linear polarized laser fields. The result is based on a general approximate controllability result for the bilinear Schroedinger equation, with wavefunction varying in the unit sphere of an infinite-dimensional Hilbert space and with several control potentials, under the assumption that the internal Hamiltonian has discrete spectrum. A further general results, extending the above approach, are obtained in [16].
- In [5] we provide a short introduction to modern issues in the control of infinite dimensional closed quantum systems, driven by the bilinear Schroedinger equation. The first part is a quick presentation of some of the numerous recent developments in the fields. This short summary is intended to demonstrate the variety of tools and approaches used by various teams in the last decade. In a second part, we present four examples of bilinear closed quantum systems. These examples were extensively

studied and may be used as a convenient and efficient test bench for new conjectures. Finally, we list some open questions, both of theoretical and practical interest.

- In [6] we study the so-called spin-boson system, namely a spin-1/2 particle in interaction with a distinguished mode of a quantized bosonic field. We control the system via an external field acting on the bosonic part. Applying geometric control techniques to the Galerkin approximation and using perturbation theory to guarantee non-resonance of the spectrum of the drift operator, we prove approximate controllability of the system, for almost every value of the interaction parameter.
- In [9] and [25] we investigate the controllability of a quantum electron trapped in a two-dimensional device. The problem is modeled by the Schroedinger equation in a bounded domain coupled to the Poisson equation for the electrical potential. The controller acts on the system through the boundary condition on the potential, on a part of the boundary modeling the gate. We prove that, generically with respect to the shape and boundary conditions on the gate, the device is controllable. In [25] We also consider control properties of a more realistic nonlinear version of the device, taking into account the self-consistent electrostatic Poisson potential.
- In [18] we study the controllability of a closed control-affine quantum system driven by two or more external fields. We provide a sufficient condition for controllability in terms of existence of conical intersections between eigenvalues of the Hamiltonian in dependence of the controls seen as parameters. Such spectral condition is structurally stable in the case of three controls or in the case of two controls when the Hamiltonian is real. The spectral condition appears naturally in the adiabatic control framework and yields approximate controllability in the infinite-dimensional case. In the finite-dimensional case it implies that the system is Lie-bracket generating when lifted to the group of unitary transformations, and in particular that it is exactly controllable. Hence, Lie algebraic conditions are deduced from purely spectral properties. Another contribution of [18] is the proof that approximate and exact controllability are equivalent properties for general finite-dimensional quantum systems.

6.3. New results: neurophysiology

- In recent papers models of the human locomotion by means of an optimal control problem have been proposed. In this paradigm, the trajectories are assumed to be solutions of an optimal control problem whose cost has to be determined. The purpose of [3] is to analyze the class of optimal control problems defined in this way. We prove strong convergence result for their solutions on the one hand for perturbations of the initial and final points (stability), and on the other hand for perturbations of the cost (robustness).
- [8] analyses a class of optimal control problems on geometric paths of the euclidean space, that is, curves parametrized by arc length. In the first part we deal with existence and robustness issues for such problems and we define the associated inverse optimal control problem. In the second part we discuss the inverse optimal control problem in the special case of planar trajectories and under additional assumptions. More precisely we define a criterion to restrict the study to a convenient class of costs based on the analysis of experimentally recorded trajectories. This method applies in particular to the case of human locomotion trajectories.
- The article [17] presents an algorithm implementing the theory of neurogeometry of vision, described by Jean Petitot in his book. We propose a new ingredient, namely working on the group of translations and discrete rotations $\mathrm{SE}(2, N)$. We focus on the theoretical and numerical aspects of integration of an hypoelliptic diffusion equation on this group. Our main tool is the generalized Fourier transform. We provide a complete numerical algorithm, fully parallellizable.

6.4. New results: switched systems

- In [2] we study the control system $\dot{x} = Ax + \alpha(t)bu$ where the pair (A, b) is controllable, $x \in \mathbb{R}^2$, $u \in \mathbb{R}$ is a scalar control and the unknown signal $\alpha : \mathbb{R}_+ \to [0, 1]$ is (T, μ)-persistently exciting (PE), i.e., there exists $T \geq \mu > 0$ such that, for all $t \in \mathbb{R}_+$, $\int_t^{t+T} \alpha(s)ds \geq \mu$. We are interested in the stabilization problem of this system by a linear state feedback $u = -Kx$. In [2], we positively answer a question asked in [52] and prove the following: Assume that the class of (T, μ)-PE signals is restricted to those which are M-Lipschitzian, where $M > 0$ is a positive constant. Then, given any $C > 0$, there exists a linear state feedback $u = -Kx$ where K only depends on (A, b) and T, μ, M so that, for every M-Lipschitzian (T, μ)-PE signal, the rate of exponential decay of the time-varying system $\dot{x} = (A - \alpha(t)bK)x$ is greater than C.

  In [20] we consider a family of linear control systems $\dot{x} = Ax + \alpha Bu$ where α belongs to a given class of persistently exciting signals. We seek maximal α-uniform stabilisation and destabilisation by means of linear feedbacks $u = Kx$. We extend previous results obtained for bidimensional single-input linear control systems to the general case as follows: if the pair (A, B) verifies a certain Lie bracket generating condition, then the maximal rate of convergence of (A, B) is equal to the maximal rate of divergence of $(-A, -B)$. We also provide more precise results in the general single-input case, where the above result is obtained under the sole assumption of controllability of the pair (A, B).

  The paper [24] considers the stabilization to the origin of a persistently excited linear system by means of a linear state feedback, where we suppose that the feedback law is not applied instantaneously, but after a certain positive delay (not necessarily constant). The main result is that, under certain spectral hypotheses on the linear system, stabilization by means of a linear delayed feedback is indeed possible, generalizing a previous result already known for non-delayed feedback laws.

  Several problems and results related with persistent excitation and stabilization are discussed in the survey [11]. These problems and results deal with both finite- and infinite-dimensional systems.

- In [7] we consider several time-discretization algorithms for singularly perturbed switched systems. The algorithms correspond to different sampling times and the discretization procedure respects the splitting of each mode in fast and slow dynamics. We study whether such algorithms preserve the asymptotic or quadratic stability of the original continuous-time singularly perturbed switched system.

- In [10] we consider affine switched systems as perturbations of linear ones, the equilibria playing the role of perturbation parameters. We study the stability properties of an affine switched system under arbitrary switching, assuming that the corresponding linear system is uniformly exponentially stable. It turns out that the affine system admits a minimal invariant set Ω, whose properties we investigate. In the two-dimensional bi-switched case when both subsystems have non-real eigenvalues we are able to characterize Ω completely and to prove that all trajectories of the system converge to Ω. We also explore the behavior of minimal-time trajectories in Ω by constructing optimal syntheses.

- In [21] we give a collection of converse Lyapunov–Krasovskii theorems for uncertain retarded differential equations. We show that the existence of a weakly degenerate Lyapunov–Krasovskii functional is a necessary and sufficient condition for the global exponential stability of the linear retarded functional differential equations. This is carried out using the switched system transformation approach.

7. Partnerships and Cooperations

7.1. Regional Initiatives

- **Digitéo project CONGEO.** CONGEO (2009–2013) is financed by Digitéo in the framework of the DIM *Logiciels et systèmes complexes*. It focuses on the neurophysiology applications. U. Boscain, Y. Chitour (leader), F. Jean and P. Mason are part of the project.

- **Digitéo project 2012-061D SSyCoDyC.** SSyCoDyC (2013–2014) is financed by Digitéo in the framework of the DIM *Hybrid Systems and Sensing Systems*. It focuses on the application of techniques of hybrid systems to the analysis of retarded equations with time-varying delays. SSyCoDyC finances the post-doc fellowship of Ihab Haidar and is coordinated by Paolo Mason and Mario Sigalotti.

7.2. National Initiatives

- **ANR project GCM.** The project ANR GCM (*programme blanc*, 2009–13) involves the great majority of GECO's members (permanent and external). It focuses on various theoretical aspects of geometric control and on quantum control. It is coordinated by J.-P. Gauthier.

7.3. European Initiatives

7.3.1. FP7 Projects

Program: ERC Starting Grant

Project acronym: GeCoMethods

Project title: Geometric Control Methods for the Heat and Schroedinger Equations

Duration: 1/5/2010 - 1/5/2015

Coordinator: Ugo Boscain

Abstract: The aim of this project is to study certain PDEs for which geometric control techniques open new horizons. More precisely we plan to exploit the relation between the sub-Riemannian distance and the properties of the kernel of the corresponding hypoelliptic heat equation and to study controllability properties of the Schroedinger equation.

All subjects studied in this project are applications-driven: the problem of controllability of the Schroedinger equation has direct applications in Laser spectroscopy and in Nuclear Magnetic Resonance; the problem of nonisotropic diffusion has applications in cognitive neuroscience (in particular for models of human vision).

Participants. Main collaborator: Mario Sigalotti. Other members of the team: Andrei Agrachev, Riccardo Adami, Thomas Chambrion, Grégoire Charlot, Yacine Chitour, Jean-Paul Gauthier, Frédéric Jean.

7.4. International Initiatives

7.4.1. Inria International Partners

7.4.1.1. Informal International Partners

SISSA (Scuola Internazionale Superiore di Studi Avanzati), Trieste, Italy.

Sector of Functional Analysis and Applications, Geometric Control group. Coordinator: Andrei A. Agrachev.

We collaborate with the Geometric Control group at SISSA mainly on subjects related with sub-Riemannian geometry. Thanks partly to our collaboration, SISSA has established an official research partnership with École Polytechnique.

7.4.2. Participation In other International Programs

- Laboratoire Euro Maghrébin de Mathématiques et de leurs Interactions (LEM2I)
  http://www.lem2i.cnrs.fr/
- GDRE Control of Partial Differential Equations (CONEDP)
  http://www.ceremade.dauphine.fr/~glass/GDRE/

8. Dissemination

8.1. Scientific Animation

8.1.1. Conference organization

- Y. Chitour, P. Mason and M. Sigalotti organized a double session named “Stability of switched systems: theoretical and computational aspects” for the 52nd IEEE Conference on Decision and Control, Firenze, Italy, December 10-13, 2013.
- Y. Chitour, P. Mason and M. Sigalotti organized a session named “Control issues for switched systems” for the 52nd IEEE Conference on Decision and Control, Firenze, Italy, December 10-13, 2013.

8.1.2. Editorial activity

- U. Boscain is Associate Editor of Journal of Dynamical and Control Systems, ESAIM Control, Optimisation and Calculus of Variations, Mathematical Control and Related Fields. He is also referee for Journal of Differential equations, AIMS Book series: Applied mathematics, SIAM J. Control Optim., Automatica, Rendiconti dei Lincei, Matematica ed Applicazioni, Physica A...and for the conferences ACC, CDC, MTNS...
- M. Sigalotti is Associate Editor of Journal of Dynamical and Control Systems. He is also referee for IEEE TAC, SIAM J. Control Optim., Automatica, MathSciNet, Journal of Functional Analysis...and for the conferences CDC, ACC, IFAC...

8.2. Teaching - Supervision - Juries

8.2.1. Supervision

PhD: Dario Prandi, “Geometric control and PDEs", supervisors: Ugo Boscain, Mario Sigalotti, defended in september 2013.

PhD in progress: Moussa Gaye, “Some problems of geometric analysis in almost-Riemannian geometry and of stability of switching systems", 1/9/2011, supervisors: Ugo Boscain, Paolo Mason.

PhD in progress: Guiherme Mazanti, “Stabilité et taux de convergence pour les systèmes à excitation persistante", 1/9/2013, supervisors: Yacine Chitour, Mario Sigalotti.

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International Conferences with Proceedings

[4] U. Boscain, M. Caponigro, M. Sigalotti. *Controllability of the bilinear Schrödinger equation with several controls and application to a 3D molecule*, in "2012 IEEE 51st Annual Conference on Decision and Control (CDC)", Maui, HI, United States, February 2013, pp. 3038-3043 [*DOI :* 10.1109/CDC.2012.6426289], http://hal.inria.fr/hal-00691706

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Scientific Books (or Scientific Book chapters)

[11] Y. Chitour, G. Mazanti, M. Sigalotti. *Stabilization of persistently excited linear systems*, in "Hybrid Systems with Constraints", Wiley-ISTE, 2013, pp. 85-120, http://hal.inria.fr/hal-00923619

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