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Activity Report 2013

Project-Team MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

RESEARCH CENTER
Paris - Rocquencourt

THEME
Stochastic approaches

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Project-Team MATHRISK

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Creation of the Team: 2012 January 01*, updated into Project-Team:* 2013 January 01.

1. Members

Research Scientists
Agnès Sulem [Team leader, Inria, Senior Researcher, HdR]
Bernard Lapeyre [Professor ENPC, HdR]
Benjamin Jourdain [ENPC-Cermics, Professor, HdR]
Aurélien Alfonsi [Professor ENPC-Cermics, HdR]

Faculty Members
Vlad Bally [Univ. Paris Est Marne-la-Vallée, Professor, HdR]
Damien Lamberton [Univ. Paris Est Marne-la-Vallée, Professor, until December 2013, HdR]

External Collaborators
Ahmed Kebaier [Univ. Paris XIII, Associate Professor]
Céline Labart [Univ. Savoie, Associate Professor]
Jérôme Lelong [ENSIMAG, Associate Professor]
Jean-Philippe Chancelier [ENPC]
Antonino Zanette [Univ. Udine]

PhD Student
Roxana-Larisa Dumitrescu [Univ. Paris Dauphine]

Post-Doctoral Fellow
Claudio Fontana [Inria, from Jan 2013 until Oct 2013]

Visiting Scientist
Xiao Wei [CIAS, Jul 2013]

Administrative Assistant
Martine Verneuille [AI, Inria]

2. Overall Objectives

2.1. Introduction

MathRisk is a joint Inria project-team with ENPC (CERMICS Laboratory) and the University Paris Est Marne-la-Vallée (UPEMLV, LAMA Laboratory), located in Rocquencourt and Marne-la-Vallée. https://www-rocq.inria.fr/mathfi/. This project is based on the former Mathfi project team.

Mathfi was founded in 2000, and was devoted to financial mathematics. The project was focused on advanced stochastic analysis and numerical techniques motivated by the development of increasingly complex financial products. Main applications concerned evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, and calibration of financial models. Special attention was paid to models with jumps, stochastic volatility models, asymmetry of information.

Crisis, deregulation, and impact on the research in finance.

The starting point of the development of modern finance theory is traditionally associated to the publication of the famous paper of Black and Scholes in 1973 [66]. Since then, in spite of sporadic crises, generally well overcome, financial markets have grown in a exponential manner. More and more complex exotic derivative products have appeared, on equities first, then on interest rates, and more recently on credit markets. The period between the end of the eighties and the crisis of 2008 can be qualified as the “golden age of financial mathematics”: finance became a quantitative industry, and financial mathematics programs flourished in top universities, involving seminal interplays between the worlds of finance and applied mathematics. During its 12 years existence, the Mathfi project team has extensively contributed to the development of modeling and computational methods for the pricing and hedging of increasingly complex financial products.

Since the crisis of 2008, there has been a critical reorientation of research priorities in quantitative finance with emphasis on risk. In 2008, the “subprime” crisis has questioned the very existence of some derivative products such as CDS (credit default swaps) or CDOs (collateralized debt obligations), which were accused to be responsible for the crisis. The nature of this crisis is profoundly different from the previous ones. It has negatively impacted the activity on the exotic products in general, - even on equity derivative markets-, and the interest in the modeling issues for these products. The perfect replication paradigm, at the origin of the success of the Black and Scholes model became unsound, in particular through the effects of the lack of liquidity. The interest of quantitative finance analysts and mathematicians shifted then to more realistic models taking into account the multidimensional feature and the incompleteness of the markets, but as such getting away from the “lost paradi(gm)” of perfect replication. These models are much more demanding numerically, and require the development of hedging risk measures, and decision procedures taking into account the illiquidity and various defaults.

Moreover, this crisis, and in particular the Lehman Brothers bankruptcy and its consequences, has underlined a systemic risk due to the strong interdependencies of financial institutions. The failure of one of them can cause a cascade of failures, thus affecting the global stability of the system. Better understanding of these interlinkage phenomena becomes crucial.

At the same time, independently from the subprime crisis, another phenomenon has appeared: deregulation in the organization of stock markets themselves. This has been encouraged by the Markets in Financial Instruments Directive (MIFID) which is effective since November, 1st 2007. This, together with the progress of the networks, and the fact that all the computers have now a high computation power, have induced arbitrage opportunities on the markets, by very short term trading, often performed by automatic trading. Using these high frequency trading possibilities, some speculating operators benefit from the large volatility of the markets. For example, the flash crash of May, 6 2010 has exhibited some perverse effects of these automatic speculating trading strategies. These phenomena are not well understood and the theme of high frequency trading needs to be explored.

To summarize, financial mathematics is facing the following new evolutions:

- the complete market modeling has become unsatisfactory to provide a realistic picture of the market and is replaced by incomplete and multidimensional models which lead to new modeling and numerical challenges.
- quantitative measures of risk coming from the markets, the hedging procedures, and the lack of liquidity are crucial for banks,
- uncontrolled systemic risks may cause planetary economic disasters, and require better understanding,
- deregulation of stock markets and its consequences lead to study high frequency trading.

The new project team MathRisk is designed to address these new issues, in particular dependence modeling, systemic risk, market microstructure modeling and risk measures. The research in modeling and numerical analysis remain active in this new context, motivated by new issues.

The MathRisk project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. https://www-rocq.inria.fr/mathfi/Premia/.

The MathRisk project is part of the Université Paris-Est **“Labex” BÉZOUT**.

2.2. Highlights of the Year

- AA. Sulem has been invited for a Plenary talk at IFIP TC 7 Conference on System Modelling and Optimization, Klagenfurt, Austria. September 2013 - http://ifip2013.uni-klu.ac.at/

- The paper of B. Jourdain with S. Méléard and W. Woyczynski "Lévy flights in evolutionary ecology", *Journal of Mathematical Biology*, has been honored by the prize La Recherche - Mathématiques 2013 - http://www.leprixlarecherche.com/palmares-2013

3. Research Program

3.1. Dependence modeling

Participants: Aurélien Alfonsi, Damien Lamberton, Bernard Lapeyre.

The volatility is a key concept in modern mathematical finance, and an indicator of the market stability. Risk management and associated instruments depend strongly on the volatility, and volatility modeling has thus become a crucial issue in the finance industry. Of particular importance is the assets *dependence* modeling. The calibration of models for a single asset can now be well managed by banks but modeling of dependence is the bottleneck to efficiently aggregate such models. A typical issue is how to go from the individual evolution of each stock belonging to an index to the joint modeling of these stocks. In this perspective, we want to model stochastic volatility in a *multidimensional* framework. To handle these questions mathematically, we have to deal with stochastic differential equations that are defined on matrices in order to model either the instantaneous covariance or the instantaneous correlation between the assets. From a numerical point of view, such models are very demanding since the main indexes include generally more than thirty assets. It is therefore necessary to develop efficient numerical methods for pricing options and calibrating such models to market data. As a first application, modeling the dependence between assets allows us to better handle derivatives products on a basket. It would give also a way to price and hedge consistenly single-asset and basket products. Besides, it can be a way to capture how the market estimates the dependence between assets. This could give some insights on how the market anticipates the systemic risk.

3.2. Liquidity risk

Participants: Aurélien Alfonsi, Anton Kolotaev, Marie-Claire Quenez, Agnès Sulem, Antonino Zanette.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the liquidity risk. Loosely speaking, liquidity risk is the risk that comes from the difficulty of selling (or buying) an asset. At the extreme, this may be the impossibility to sell an asset, which occured for “junk assets” during the subprime crisis. Hopefully, it is in general possible to sell assets, but this may have some cost. Let us be more precise. Usually, assets are quoted on a market with a Limit Order Book (LOB) that registers all the waiting limit buy and sell orders for this asset. The bid (resp. ask) price is the most expensive (resp. cheapest) waiting buy or sell order. If a trader wants to sell a single asset, he will sell it at the bid price. Instead, if he wants to sell a large quantity of assets, he will have to sell them at a lower price in order to match further waiting buy orders. This creates an extra cost, and raises important issues. From a short-term perspective (from few minutes to some days), this may be interesting to split the selling order and to focus on finding optimal selling strategies. This requires to model the market microstructure, i.e. how the market reacts in a short time-scale to execution orders. From a long-term perspective (typically, one month or more), one has to understand how this cost modifies portfolio managing strategies (especially delta-hedging or optimal investment strategies). At this time-scale, there is no need to model precisely the market microstructure, but one has to specify how the liquidity costs aggregate.

3.2.1. Long term liquidity risk.

On a long-term perspective, illiquidity can be approached via various ways: transactions costs [56], [57], [65], [71], [74], [89], [85], delay in the execution of the trading orders [90], [88], [67], trading constraints or restriction on the observation times (see e.g. [73] and references herein). As far as derivative products are concerned, one has to understand how delta-hedging strategies have to be modified. This has been considered for example by Cetin, Jarrow and Protter [87]. We plan to contribute on these various aspects of liquidity risk modeling and associated stochastic optimization problems. Let us mention here that the price impact generated by the trades of the investor is often neglected with a long-term perspective. This seems acceptable since the investor has time enough to trade slowly in order to eliminate its market impact. Instead, when the investor wants to make significant trades on a very short time horizon, it is crucial to take into account and to model how prices are modified by these trades. This question is addressed in the next paragraph on market microstructure.

3.2.2. Market microstructure.

The European directive MIFID has increased the competition between markets (NYSE-Euronext, Nasdaq, LSE and new competitors). As a consequence, the cost of posting buy or sell orders on markets has decreased, which has stimulated the growth of market makers. Market makers are posting simultaneously bid and ask orders on a same stock, and their profit comes from the bid-ask spread. Basically, their strategy is a "round-trip" (i.e. their position is unchanged between the beginning and the end of the day) that has generated a positive cash flow.

These new rules have also greatly stimulated research on market microstructure modeling. From a practitioner point of view, the main issue is to solve the so-called "optimal execution problem": given a deadline T, what is the optimal strategy to buy (or sell) a given amount of shares that achieves the minimal expected cost? For large amounts, it may be optimal to split the order into smaller ones. This is of course a crucial issue for brokers, but also market makers that are looking for the optimal round-trip.

Solving the optimal execution problem is not only an interesting mathematical challenge. It is also a mean to better understand market viability, high frequency arbitrage strategies and consequences of the competition between markets. For example when modeling the market microstructure, one would like to find conditions that allow or exclude round trips. Beyond this, even if round trips are excluded, it can happen that an optimal selling strategy is made with large intermediate buy trades, which is unlikely and may lead to market instability.

We are interested in finding synthetic market models in which we can describe and solve the optimal execution problem. A. Alfonsi and A. Schied (Mannheim University) [58] have already proposed a simple Limit Order Book model (LOB) in which an explicit solution can be found for the optimal execution problem. We are now interested in considering more sophisticated models that take into account realistic features of the market such as short memory or stochastic LOB. This is mid term objective. At a long term perspective one would like to bridge these models to the different agent behaviors, in order to understand the effect of the different quotation mechanisms (transaction costs for limit orders, tick size, etc.) on the market stability.

3.3. Contagion modeling and systemic risk

Participants: Benjamin Jourdain, Agnès Sulem.

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these. Ideally, one would like to be able to find capital requirements (such as the one proposed by the Basel committee) that ensure that the probability of multiple defaults is below some level.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular R. Cont et al. [59] and A. Minca [79]) or mean field interaction models (Garnier-Papanicolaou-Yang [72]). The recent approach in [59] seems very promising. It describes the financial network approach as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. Distress propagation in a financial system may be modeled as an epidemics on this graph. In the case of incomplete information on the structure of the interbank network, cascade dynamics may be reduced to the evolution of a multi-dimensional Markov chain that corresponds to a sequential discovery of exposures and determines at any time the size of contagion. Little has been done so far on the *control* of such systems in order to reduce the systemic risk and we aim to contribute to this domain.

3.4. Stochastic analysis and numerical probability

3.4.1. Stochastic control

Participants: Vlad Bally, Jean-Philippe Chancelier, Marie-Claire Quenez, Agnès Sulem.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the default risk modeling,the interplay between information and performance, the model uncertainty and the associated robustness questions. We address these questions by further developing the theory of stochastic control in a broad sense, including stochastic optimization, nonlinear expectations, Malliavin calculus, stochastic differential games and various aspects of optimal stopping.

3.4.2. Simulation of stochastic differential equations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Vlad Bally, Damien Lamberton, Bernard Lapeyre, Jérôme Lelong, Céline Labart.

Effective numerical methods are crucial in the pricing and hedging of derivative securities. The need for more complex models leads to stochastic differential equations which cannot be solved explicitly, and the development of discretization techniques is essential in the treatment of these models. The project MathRisk addresses fundamental mathematical questions as well as numerical issues in the following (non exhaustive) list of topics: Multidimensional stochastic differential equations, High order discretization schemes, Singular stochastic differential equations, Backward stochastic differential equations.

3.4.3. Monte-Carlo simulations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Mohamed Sbai, Vlad Bally, Bernard Lapeyre, Ahmed Kebaier, Céline Labart, Jérôme Lelong, Sidi-Mohamed Ould-Aly, Abdelmounaim Abbas-Turki, Abdelkoddousse Ahida, Antonino Zanette, El Hadj Aly Dia.

Monte-Carlo methods is a very useful tool to evaluate prices especially for complex models or options. We carry on research on *adaptive variance reduction methods* and to use *Monte-Carlo methods for calibration* of advanced models.

This activity in the MathRisk team is strongly related to the development of the Premia software.

3.4.4. Optimal stopping

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton, Maxence Jeunesse, Ayech Bouselmi, Agnès Sulem, Marie-Claire Quenez.

The theory of American option pricing has been an incite for a number of research articles about optimal stopping. Our recent contributions in this field concern optimal stopping in models with jumps irregular obstacles, free boundary analysis, reflected BSDEs.

3.4.5. Malliavin calculus and applications in finance

Participants: Vlad Bally, Arturo Kohatsu-Higa, Agnès Sulem, Antonino Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [77]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [80], D. Bell [64] D. Ocone [82], B. Øksendal [91]) give expositions of the subject. See also V. Bally [61] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [81].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [78], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods.

We recall that the theory is based on an integration by parts formula of the form $E(f'(X))=E(f(X)Q)$. Here X is a random variable which is supposed to be “smooth” in a certain sense and non-degenerated. A basic example is to take $X=\sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequences of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y)=H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X))=E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y))\sim\frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1,...,X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X))=E(H(X-x)Q)\sim\frac{1}{M}\sum_{i=1}^{M}E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [70] and [69] and the papers by Bally et al., Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [76].

L. Caramellino, A. Zanette and V. Bally have been concerned with the computation of conditional expectations using Integration by Parts formulas and applications to the numerical computation of the price and the Greeks (sensitivities) of American or Bermudean options. The aim of this research was to extend a paper of Reigner and Lions who treated the problem in dimension one to higher dimension - which represent the real challenge in this field. Significant results have been obtained up to dimension 5 [63] and the corresponding algorithms have been implemented in the Premia software.

Moreover, there is an increasing interest in considering jump components in the financial models, especially motivated by calibration reasons. Algorithms based on the integration by parts formulas have been developed in order to compute Greeks for options with discontinuous payoff (e.g. digital options). Several papers and two theses (M. Messaoud and M. Bavouzet defended in 2006) have been published on this topic and the corresponding algorithms have been implemented in Premia. Malliavin Calculus for jump type diffusions - and more general for random variables with localy smooth law - represents a large field of research, also for applications to credit risk problems.

The Malliavin calculus is also used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P. Imkeller in [76]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [76]).

A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. These SDEs can be interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals, as introduced by Russo and Valois [84]. This methodology has been applied for utility maximization with insiders.

4. Application Domains

4.1. Application Domains

Risk management, Quantitative Finance, Computational Finance, Market Microstructure, Systemic Risk, Portfolio optimization, Risk modeling.

5. Software and Platforms

5.1. PREMIA

Participants: Antonino Zanette, Mathrisk Research Team, Agnès Sulem [correspondant].

Premia is a software designed for option pricing, hedging and financial model calibration. It is provided with it's C/C++ source code and an extensive scientific documentation. https://www-rocq.inria.fr/mathfi/Premia

The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is:

1. to be a powerful testing platform for comparing different numerical methods between each other;
2. to build a link between professional financial teams and academic researchers;
3. to provide a useful teaching support for Master and PhD students in mathematical finance.

- AMS: 91B28;65Cxx;65Fxx;65Lxx;65Pxx
- License: Licence Propriétaire (genuine license for the Consortium Premia)

- Type of human computer interaction: Console, interface in Nsp, Web interface
- OS/Middelware: Linux, Mac OS X, Windows
- APP: The development of Premia started in 1999 and 15 are released up to now and registered at the APP agency.
- Programming language: C/C++ librairie Gtk
- Documentation: the PNL library is interfaced via doxygen
- Size of the software: 280580 lines for the Src part only, that is 11 Mbyte of code, 130400 lines for PNL, 103 Mbyte of PDF files of documentation.
- interfaces : Nsp for Windows/Linux/Mac, Excel, binding Python, and a Web interface.
- Publications: [1], [68], [75], [83], [86], [55]

5.1.1. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. **Equity derivatives:**

   The following models are considered:

   Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

   For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods and Malliavin calculus based methods.

   Dynamic Hedging for Black-Scholes and jump models is available.

   Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. **Interest rate derivatives**

   The following models are considered:

   HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

   Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. **Credit derivatives: CDS, CDO**

   Reduced form models and copula models are considered.

   Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. **Hybrid products**

   PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. **Energy derivatives: swing options**

   Mean reverting and jump models are considered.

   Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

5.1.2. Premia design

Premia has managed to grow up over a period of more than a dozen years; this has been possible only because contributing an algorithm to Premia is subject to strict rules, which have become too stringent. To facilitate contributions, a standardized numerical library (PNL) has been developed by J. Lelong under the LGPL since 2009, which offers a wide variety of high level numerical methods for dealing with linear algebra, numerical integration, optimization, random number generators, Fourier and Laplace transforms, and much more. Everyone who wishes to contribute is encouraged to base its code on PNL and providing such a unified numerical library has considerably eased the development of new algorithms which have become over the releases more and more sophisticated. An effort will be made to continue and stabilize the development of PNL.

1. Development of the PNL. Here are the major 2013 contributions (by Jérôme Lelong):
    1. PNL relies on CMake for compiling.
    2. Add the sampling of new distributions: log-normal, inverse Gaussian, asymmetric double exponential distributions.
    3. Add the computation of eigenvalues and eigenvectors for complex matrices. Based on this new function, add the computation of the matrix logarithm for complex matrices.
    4. Add Newton's algorithm with Armijo line search.
    5. The top level PnlOjbect is modified to keep track of the number of references on an object to improve memory management in lists. This delicate change in the core of the library enabled us to speed codes based on lists by a great deal.
    6. Several other functions have also been added.
2. Premia
    1. The compilation of Premia is now based on CMake which is a cross-platform building tool. It allows us to maintain a single building chain and to automatically generate Makefiles or a Visual project. This technology change significantly improves our ability to generate Windows versions.
    2. Add support for PnlMatrix both in Premia VAR and in the Nsp toolbox.
    3. A model size change in the Nsp GUI automatically propagates to all parameters thanks to the addition a *Return* callback in the GUI.
    4. Some fixes in the core of Premia: several setters were broken.
    5. Refactor the credit toolbox to simplify the number of products.
    6. Scripts to generate new model templates have been significantly improved and reimplemented in Python.
    7. Improve the generic functions `Get`, `FGet`, `Show`, `PrintVar` and `FScanVar` to enable all the models to use them. This led us to remove a lot of code.

5.1.3. Algorithms implemented in Premia in 2013

Premia 15 was delivered to the consortium members in March 2013. It contains the following new algorithms:

- **Interest Rate, Inflation, FX**
    - Inflation products with stochastic volatility and stochastic interest rates. S. Singor, L. Grzelak C.W.Oosterlee D.D.B. van Bragt.
    - On cross-currency models with stochastic volatility and correlated interest rates. L. Grzelak C.W.Oosterlee. *Applied Math. Finance, to appear.*
    - Repricing the Cross Smile: An Analytic Joint Density. P.Austing. *preprint 2011*
- **Energy and Commodities**

   - Efficient Pricing of Commodity Options with Early-Exercise under the Ornstein–Uhlenbeck process. C.W.Oosterlee. B.Zhang. *preprint 2011*
   - A finite dimensional approximation for pricing moving average options. M. Bernhart P.Tankov X. Warin, *to appear in SIAM Journal on Financial Mathematics.*
   - Pricing and hedging spread options. R. Carmona V.Durrleman. *SIAM Rev. 45 (2003), no. 4, 627—685.*
   - Closed form spread option valuation. P.Bjerksund G. Stensland *Quantitative Finance, 2011.*
   - A Fourier transform method for spread option pricing. T. R. Hurd and Z. Zhou. *SIAM Journal on Financial Mathematics. 1, 142-–157, 2010.*
   - Multi-asset spread option pricing and hedging *Quantitative Finance, Vol.10, No3, 305-324, 2010.*
   - Unspanned Stochastic Volatility and the Pricing of Commodity Derivatives. A.B.Trolle E.Schwartz. *Rev. Financ. Stud. (2009) 22(11): 4423-4461*
   - Pricing Commodity Swaptions in Multifactor Models. K.Larsson. *The Journal of Derivatives Winter 2011, Vol. 19, No. 2,32-44.*
- **Equity Derivatives**
   - Importance sampling and Statistical Romberg Method. M.B. Alaya A.Kebaier K.Hajji
   - New approximations in local volatility models. E. Gobet, A.Suleiman
   - Componentwise splitting methods for pricing American options under stochastic volatility. Ikonen, S.; Toivanen,J. *Int. J. Theor. Appl. Finance 10 (2007), no. 2, 331–361.*
   - ADI finite difference schemes for option pricing in the Heston model with correlation. K.J. in 't Hout and S. Foulon. *Int. J. Numer. Anal. Mod. 7, 303–320 (2010).*
   - ADI schemes with Ikonen-Toivanen splitting for pricing American put options in the Heston model. T. Haentjens, K. in 't Hout and K. Volders. In: Numerical Analysis and Applied Mathematics, eds. T. E. Simos et. al., AIP Conf. Proc. 1281, 231-234 (2010).
   - Pricing of Timer Options. C. Bernard Z. Cui. *Journal of Computational Finance, to appear*
   - Efficient Simulation of the Double Heston Model. P.Gauthier D.Possamai.
   - Greedy methods method for basket options. T.Lelievre J.I.Acevedo
   - Pricing higher-dimensional American options using the stochastic grid method. C.W.Oosterlee S. Jain
   - Calibration in the Heston model. L. Abbas Turki

The software Premia 15 has been registered at the APP (Agence pour la Protection des Programmes) with the reference IDDN.FR.001.190010.012.S.C.2001.000.31000.

6. New Results

6.1. Credit risk

Participants: Aurélien Alfonsi, Céline Labart, Jérôme Lelong.

We have ended our study on stochastic local intensity model. We have shown by the mean of a particles system that this model is well defined and have obtained an efficient way to perform Monte-Carlo algorithms for this model.

6.2. Liquidity risk

Participants: Aurélien Alfonsi, A. Schied.

A. Alfonsi and A. Schied (Mannheim University) are working on price impact models that describe how the price is modified by large trades. The paper together with J. Acevedo on a time-dependent price impact is now accepted for publication. With A. Schied and F. Klöck [45], we have studied the cross price impact between different assets and identified conditions on the resilience of this impact that avoid manipulations strategies. With P. Blanc, we are working on the optimal execution problem when there are many large traders that modify the price.

6.3. Systemic Risk

Participants: Andreea Minca, Agnès Sulem.

We are working on the theory of the stochastic control of financial networks.

In two related articles, we find the optimal strategy of a government who seeks to make equity infusions in a banking system prone to insolvency and to bank runs. The first article combines stochastic control and the random graph representation of the financial system developed in Andreea's thesis. The second article combines the network representation of a financial system and the solvency-based mechanism of contagion with another potent source of distress, which is funding illiquidity [31] and [60].

6.4. Estimation of the parameters of a Wishart process

Participants: Aurélien Alfonsi, Ahmed Kebaier, Clément Rey.

This research has started this year together with the thesis of Clément Rey. We are studying the Maximum Likelihood Estimator for the Wishart processes and in particular its convergence in the ergodic and the non ergodic case.

6.5. An Affine term structure model for interest rates that involve Wishart diffusions

Participants: Aurélien Alfonsi, E. Palidda.

Affine term structure models (Dai and Singleton, Duffie, ...) consider vector affine diffusions. Here, we would like to extend this model by including some Wishart dynamics, and to get a model that could better fit the market. We also develop some numerical pricing methods for this model to make its implementation possible.

6.6. Applications of optimal transport

Participants: Aurélien Alfonsi, Benjamin Jourdain, Arturo Kohatsu-Higa.

A. Alfonsi and B. Jourdain study the Wasserstein distance between two probability measures in dimension n sharing the same copula C. The image of the probability measure dC by the vectors of pseudo-inverses of marginal distributions is a natural generalization of the coupling known to be optimal in dimension $n = 1$. In dimension $n > 1$, it turns out that for cost functions equal to the p-th power of the L^q norm, this coupling is optimal only when $p = q$ i.e. when the cost function may be decomposed as the sum of coordinate-wise costs.

As another application of optimal transport, they are working with A. Kohatsu-Higa on the uniform in time estimation of the Wasserstein distance between the time-marginals of an elliptic diffusion and its Euler scheme. To generalize in higher dimension the result that they obtained previously in dimension one using the optimality of the explicit inverse transform, they compute the derivative of the Wasserstein distance with respect to the time variable thanks to the theory developped by Ambrosio Gigli and Savare. The abstract properties of the optimal coupling between the time marginals then enable them to estimate this time derivative.

6.7. Capital distribution and portfolio performance in the mean-field Atlas model

Participants: Benjamin Jourdain, J. Reygner.

B. Jourdain and J. Reygner study a mean-field version of rank-based models of equity markets, introduced by Fernholz in the framework of stochastic portfolio theory. They first obtain an asymptotic description of the market when the number of companies grows to infinity. They then discuss the long-term capital distribution in this asymptotic model, as well as the performance of simple portfolio rules. In particular, they highlight the influence of the volatility structure of the model on the growth rates of portfolios.

6.8. Public Private Partnerships

Participants: Gilles Edouard Espinosa, Caroline Hillairet, Benjamin Jourdain, Monique Pontier.

With Gilles Edouard Espinosa, Caroline Hillairet and Monique Pontier, Benjamin Jourdain is interested in the problem of outsourcing the debt for a big investment, according two situations: either the firm outsources both the investment (and the associated debt) and the exploitation to a private consortium, or the firm supports the debt and the investment but outsources the exploitation. They prove the existence of Stackelberg and Nash equilibria between the firm and the private consortium, in both situations. They compare the benefits of these contracts.They conclude with a study of what happens in case of incomplete information, in the sense that the risk aversion coefficient of each partner may be unknown by the other partner [51].

6.9. Backward Stochastic (Partial) Differential equations with jumps and stochastic control

Participants: Roxana-Larisa Dumitrescu, Marie-Claire Quenez, Agnès Sulem.

We have studied optimization problems for BSDEs with jumps, optimal stopping for dynamic risk measures induced by BSDEs with jumps and associated reflected BSDEs, and generalized Dynkin games associated to double barriers reflected BSDEs with jumps [32], [38], [42]. A. Sulem, with B. Øksendal and T. Zhang has also studied optimal stopping for Stochastic Partial Differential equations and associated reflected SPDEs [34], and optimal control of Forward-Backward SDEs [54].

6.10. Utility maximization and Arbitrage Theory

Participants: Claudio Fontana, Bernt Øksendal, Agnès Sulem.

B. Øksendal and A. Sulem have contributed to the issue of robust utility maximization in jump diffusion markets via a stochastic maximum approach and the links with robust duality [53].

In the period January - October 2013, the main subject of investigation of C. Fontana has been arbitrage theory, with a special emphasis on no-arbitrage conditions weaker than the classical notion of No Free Lunch with Vanishing Risk (NFLVR). In particular, in the context of financial market models based on diffusion processes (see [35]), we have provided a characterization of several no-arbitrage conditions as well as a generalization of the second fundamental theorem of asset pricing. In the context of jump-diffusion models under partial information (see [25]), we have studied the relation between market viability (in the sense of solvability of portfolio optimization problems) and the existence of a martingale measure given by the marginal utility of terminal wealth, without a-priori assuming no-arbitrage restrictions on the model. Finally, in the paper [41], we have provided a critical analysis of the paper Arbitrage, Approximate Arbitrage and the Fundamental Theorem of Asset Pricing (Wong & Heyde, 2010), where the authors aim at proposing an original and simple proof of the fundamental theorem of asset pricing in the context of incomplete diffusion-based models. We have shown that the method of Wong & Heyde (2010) can only work in the well-known case of complete markets, exhibiting an explicit counterexample.

6.11. Regularity of probability laws using an interpolation method

Participant: Vlad Bally.

This work was motivated by previous papers of Nicolas Fournier, J. Printemps, E. Clément, A. Debusche and of myself, concerning the regularity of the law of the solutions of some equations with coefficients with little regularity - for example diffusion processes with Hölder coefficients (but also many other examples including jump type equations, Bolzmann equation or Stochastic PDE's). Since we do not have sufficient regularity the usual approach by Malliavin calculus fails in this framework. Then one may use an alternative idea which roughly speaking is the following: We approximate the law of the random variable X (the solution of the equation at hand) by a sequence $X(n)$ of random variables which are smooth and consequently we are able to establish integration by parts formulas for $X(n)$ and we are able to obtain the absolutely continuity of the law of $X(n)$ and to establish estimates for the density of the law of $X(n)$ and for its derivatives. Notice that the derivatives of the densities of $X(n)$ generally blow up - so we can not derive directly results concerning the density of the law of X. But, if the speed of convergence of $X(n)$ to X is stronger than the blow up, then we may obtain results concerning the density of the law of X. It turns out that this approach fits in the framework of interpolation spaces and that the criterion of regularity for the law of X amounts to the characterization of an interpolation space between a space of distributions and a space of smooth functions. Although the theory of interpolation spaces is very well developed and one already know to characterize the interpolation spaces for Sobolev spaces of positive and negative indices, we have not found in the (huge) literature a result which covers the problem we are concerned with. So, although our result may be viewed as an interpolation result, it is a new one. The above work is treated in the paper [62] (in collaboration with Lucia Caramellino). As an application we discussed in [48] the regularity of the law of a Wiener functional under a Hörmander type non degeneracy condition.

6.12. A stochastic parametric representation for the density of a Markov process

Participant: Vlad Bally.

Classical results in the PDE theory (due to A. Friedmann) assert that, under uniform ellipticity conditions, the law of a diffusion process has a continuous density (the approach of A. Friedmann is analytical and concerns PDE's instead of the corresponding diffusion process). The method developed by A. Friedmann becomes well known as the "parametric method". In collaboration with A. Kohatzu Higa [49] we gave a probabilistic approach which represents the probabilistic counterpart of the parametric method. We obtained a probabilistic representation for the density of the law of the solution of a SDE and more generaly, for a class of Markov processes including solutions of jump type SDE's. This representation may be considered as a perfect simulation scheme and so represents a starting point for Monte Carlo simulation. However the random variable which appears in the stochastic representation has infinite variance, so direct simulation gives unstable results (as some preliminary tests have proved). In order to obtain an efficient simulation scheme some more work on the reduction of variance has to be done.

6.13. Regularity of probability laws using an interpolation method

Participant: Vlad Bally.

The distance between two density functions and convergence in total variation. In collaboration with Lucia Caramellino we obtained estimates of the distance between the densities of the law of two random variables using an abstract variant of Malliavin calculus. We used these estimates in order to study the convergence in total variation of a sequence of random variables. This has been done in [47]. We are now working on more specific examples concerning the Central Limit Theorem. In the last years the convergence in entropy distance and in total variation distance for several variants of the CLT has been considered in papers of S. Bobkov, F. Gotze, G. Peccati, Y. Nourdin, D. Nualart and G. Polly. So this seems to be a very active research area. Moreover, in an working paper in collaboration with my Phd student R. Clement, we use the same methods in order to study the total variation distance between two Markov semigroups and in particular for approximation schemes. A special interest is devoted to higher order schemes - as for example the Victoire Nyomia scheme.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

PREMIA consortium: presently composed of Crédit Agricole CIB, and Natixis.

7.2. Bilateral Grants with Industry

Chair "Financial Risks" , Risk Foundation.
Partners: Ecole des Ponts ParisTech, Ecole Polytechnique, UPMC, Société Générale.

A. Alfonsi, B. Jourdain, B. Lapeyre.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

B. Jourdain is involved in the ANR Stab (2013/2016). Partners: Lyon1 and Paris-Dauphine.

8.1.2. Competitivity Clusters

Pôle Finance Innovation.

8.2. International Initiatives

8.2.1. Inria International Partners

8.2.1.1. Informal International Partners

- Center of Excellence program in Mathematics and Life Sciences at the Department of Mathematics, University of Oslo, Norway, (with B. Øksendal).
- Department of Mathematics, University of Manchester (with Tusheng Zhang, currently in charge of an EU-ITN program on BSDEs and Applications).
- Mannheim University (with Alexander Schied, Chair of Mathematics in Business and Economics, Department of Mathematics)
- Roma Tor Vergata University (Lucia Caramellino)
- Amsterdam University (Michel Velekoop)
- Delft University (Kees Oosterlee)
- Mexico University and CIMAT (Begonia Fernandez)
- Osaka and Ritsumeikan University (A. Kohatsu-Higa).
- Shandong University, China (Z. Chen)

8.3. International Research Visitors

8.3.1. Visits of International Scientists

- Alexander Schied, Mannheim University,
- Andreea Minca, Cornell University,
- Xin Guo, Berkeley University,
- Arturo Kohatsu Higa, Ritsumeikan University,

- Luis Ortiz Gracia, CWI - Centrum voor Wiskunde en Informatica, Amsterdam,
- Karel in 't Hout, University of Antwerpern,
- Lucia Caramellino, Tor Vergata University, Roma.

9. Dissemination

9.1. Scientific Animation

9.1.1. Collective responsabilities

- A. Alfonsi: Co-organizer of the working group seminar of MathRisk "Méthodes stochastiques et finance".
  In charge of the Master "Finance and Application" at the Ecole des Ponts.
- B. Jourdain:
  - Head of the doctoral school MSTIC, University Paris-Est,
  - Associate Editor of ESAIM:Proc,
  - Member of editorial board of American Journal of Algorithms and Computing.
- B. Jourdain and A. Sulem: Guest Editors of a special issue on systemic risk, to appear in *Statistics and Risk modeling*.
- C. Labart: reviewer for : Mathematical Reviews and Referee for Stochastic Processes and Their Applications, Mathematical Finance and Mathematics of computation, Journal of Computational Finance, SIAM Numerical Analysis and Annals of Applied Probability.
- D. Lamberton: associate editor of *Mathematical Finance* and co-editor of *ESAIM Probability and Statistics*.
  Research Vice President of Université Paris-Est Marne-la-Vallée.
- B. Lapeyre: Head of the doctoral department of Université Paris-Est.
- A. Sulem:

  - Editorship:
  - *Journal of Mathematical Analysis and Applications (JMAA)*
  - *International Journal of Stochastic Analysis (IJSA)*
  - *SIAM Journal on Financial Mathematics (SIFIN)*

  - Participation to selection committees:
  - Full Professor of Mathematics, Ecole Centrale de Paris, September 2013.
  - Researcher, Inria Sophia-Antipolis, 2013.

9.1.2. Invitations and participations in conferences

- A. Alfonsi :
  - "Optimal execution and price manipulations", ANR Liquirisk Workshop, Paris Dauphine.
  - "Pathwise optimal transport bounds between a one-dimensional diffusion and its Euler scheme", (June at Mannheim, October at Evry).
- V. Bally
  - Visits to Ritsumeikan University (Japan), Collaboration with A. Kohatsu Higa, September 2013, and to Tor Vergata University, Roma (May-June and July 2013).
  - 29-31 August 2013. Conference for the 150 aniversary of the Mathematical Faculty of Bucharest. Romania

- R. Dumitrescu:

  Poster Presentation, SMAI Congress, Seignosse (Landes), 27-31 May 2013.
- C. Fontana:
  - 7th Bachelier Colloquium on Mathematical Finance and Stochastic Calculus, Métabief, 13-20 January 2013 (speaker);
  - visits Politecnico di Milano, collaboration with Emilio Barucci, 27-29 January, 27 March - 1 April, 13-14 May.
  - Seminar: Université Marne La Vallée, Laboratoire d'Analyse et de Mathématiques Appliquées, 8 February 2013;
  - Current Topics in Mathematical Finance, Wien, 18-19 April 2013 (participant);
  - Seminar: Politecnico di Milano, Department of Mathematics, 14 May 2013;
  - 30th International French Finance Association Conference, Lyon, 28-31 May 2013 (speaker & discutant);
  - 6th General AmaMeF and Banach Center Conference, Warsaw, 10-15 June 2013 (speaker);
  - Workshop on Stochastic Portfolio, Arbitrage Theory, Role of Information and Credit Risk, Beijing, 24-28 June 2013 (invited speaker);
  - Seminar: Ludwig-Maximilians-Universität Munich, 8 July 2013, and collaboration with Alessandro Gnoatto et Christa Cuchiero, 7-10 July ? 37th Meeting of the Italian Association for Mathematics Applied to Economics and Social Sciences (AMASES), 5-7 September 2013 (speaker);
  - Workshop on Mathematics in Finance, Zurich, 18 October 2013 (participant);
  - Seminar: ENSTA Paris, 28 October 2013.
- B. Jourdain:
  - colloquium Bezout, 31 January 2013, Discretization of stochastic differential equations.
  - BigMC Seminar, 21 February 2013 , Optimal sclaing of the transient phase of Metropolis-Hastings algorithms
  - Ergonum workshop , CIRM, 25-28 February 2013 : Optimal scaling of the transient phase of Metropolis-Hastings algorithms
  - Seminar of the chair "Financial risks", 22 April 2013 : American Put in an exponential Levy model with discrete dividends
  - German-Polish joint Conference on Probability and Mathematical Statistics, Torun, 6-9 juin 2013, Optimal scaling of the transient phase of Metropolis-Hastings algorithms
  - ANR Stab meeting, Paris Dauphine, 11-12 June 2013, Optimal transport bounds between a one-dimensional diffusion and its Euler scheme
  - LIASFMA Summer school : new developments in stochastic analysis : probability and PDE, interactions, Beijing, 8-12 july 2013 : Optimal scaling of the transient phase of Metropolis-Hastings algorithms
  - Scicade 2013, 19 September 2013, Optimal scaling of the transient phase of Metropolis-Hastings algorithms, in an invited session on MCMC algorithms
  - Seminar Stochastic Analysis, Oxford MAN Institute, 14 October 2013, Optimal transport bounds between a one-dimensional diffusion and its Euler scheme
  - Mathematics seminar of Marne-la-Vallée, 3 December 2013, Optimal scaling of the transient phase of Metropolis-Hastings algorithms
  - ANR Stab meeting, 16-17 December 2013, 4h lecture on "Discretization of stochastic differential equations : strong error, weak error and optimal transport"

- C. Labart:
  - Colloquium on Financial Mathematics and Numerical Probability, Beijing.
  - Local organizer of the international conference IMACS 2013, which took place in Annecy.
- J. Lelong:
  - Conference "Les Nouveaux Outils du Développement Durable", Paris, October 2013.
  - 9th IMACS Seminar on Monte Carlo, Annecy, July 2013.
  - Workshop on Financial Mathematics and Numerical Probability, Beijing, June 2013.
- A. Sulem:
  - Plenary talk at the IFIP TC 7 Conference on System Modelling and Optimization, Klagenfurt, Austria. September 2013
    http://ifip2013.uni-klu.ac.at/
  - Invited talk, "Stochastic Analysis and Mathematical Finance", September 2013, Bergen
  - "PDE and Mathematical Finance V", Stockholm, June 10-14, 2013.
    http://www.math.kth.se/pdefinance/2013/Welcome.html
  - Stochastics and Real World Models 2013, July 2013, Bielefeld, Germany.
    http://www.igk.math.uni-bielefeld.de/workshop2013
  - Nomura seminar, Mathematical Institute, University of Oxford , March 8 2013,
    http://www.maths.ox.ac.uk/events/seminars/upcoming/P10Y/4/1664
  - Stochastic Analysis and Applications Conference, Växjö, Sweden, June 2013,
  - Seminar of Probability, Rennes University, June 2013
  - Talk at *Horizon Maths 2013*, Fondation Sciences Mathématiques de Paris, Paris, December 16-17 2013.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

- A. Alfonsi:

- Modéliser, Programmer et Simuler, second year course at the Ecole des Ponts, France.
- Calibration, Volatilité Locale et Stochastique, third-year course at ENSTA (Master with Paris I).
- Traitement des données de marché : aspects statistiques et calibration, lecture for the Master at UPEMLV.
- Mesures de risque, Master course of UPEMLV and Paris VI.

- V. Bally:

- Mini course (6 hours) for Phd and Post Doc students, Ritzumeikan University, Japan, September 2013.
- "The Malliavin calculus and applications in finance", 30 hours, Master 2 Finance University Marne la Vallée,
- "Risk methods in acuarial science", 36 hours, Master IMIS, University Marne la Vallée.

- R. Dumitrescu:

- Evaluation d'actifs financiers par absence d'opportunité d'arbitrage (3rd year ENSAE, M2 MASEF Dauphine and M2 Modélisation aléatoire Paris Diderot).
- Calcul stochastique appliqué à la finance (2nd year ENSAE).
- Algèbre linéaire (L2, Paris Dauphine).

- B. Jourdain:

- Course "Probability theory and statistics", 1st year ENPC.
- Course "Introduction to probability theory", 1st year, Ecole Polytechnique.
- Course "Stochastic numerical methods", 3rd year, Ecole Polytechnique.
- Projects in finance and numerical methods, 3rd year, Ecole Polytechnique.

- B. Jourdain, B. Lapeyre:

- Course "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée

- J.-F. Delmas, B. Jourdain:

- Course "Jump processes with applications to energy markets", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée

- D. Lamberton: Master course “Calcul stochastique et applications en finance", Université Paris-Est Marne-la-Vallée.

- B. Lapeyre:

- Monte-Carlo methods in finance, 3rd year ENPC and Master Recherche Mathématiques et Application, University Paris-Est Marne-la-Vallée
- "Finance : mathematical and numerical aspects", 2nd year ENPC, professor, 15 hours/year

- J. Lelong:

- Oct. 2012 – May 2013 : supervisor of 2 third year students from Ensimag to work on an adaptive Monte Carlo method for jump diffusion models. See the research report “Importance sampling for jump processes and applications to finance” I am co-responsible for the *Financial Engineering* major at Ensimag.
- Lectures on “Parallel programming in financial mathematics” at Ensimag (third year course)
- Lectures on “Monte-Carlo methods in financial engineering” at Ensimag (third year course)
- Lectures on “Numerical methods for pricing American options” at Ensimag (third year course)
- Lectures on “Martingales and stochastic approximation” at Ensimag (third year course)
- Lectures on “Interest rate and Foreign exchange derivatives” at Ensimag (third year course)
- Lectures on “Numerical programming in C++” at Ensimag (second year course)
- Supervision of third year students for a long term project on “Pricing structured products”
- Supervision of second year students for long term projects on numerical finance.

- A. Sulem:

- Master of Mathematics, "Numerical methods in Finance", Luxembourg University, 20 hours
- Master course, Université Paris IX-Dauphine, Department "Mathématiques et Informatique de la Décision et des Organisations" (MIDO), Master MASEF, 21 hours “Numerical Methods for PDEs in Finance"

9.2.2. Supervision

PhD theses defended in 2013:

- José Infante Acevedo, Méthodes et modèles numériques appliquées aux risques du marché et à l’évaluation financière, Université Paris-Est, December 9 2013, Adviser: A. Alfonsi and T. Lelièvre.

-Ayech Bouselmi, "Processus de Lévy et options américaines", University of Marne la Vallée, December 11 2013, Adviser : D. Lamberton.

- Maxence Jeunesse, "Etude de deux problèmes de contrôle stochastique : Put Americain avec dividendes discrets et principe de programmation dynamique avec contraintes en probabilités", Université Paris-Est, 29 january 2013, Adviser: B. Jourdain.

PhD in progress:

- Clément Rey: ENPC and UPMLV. Weak error analysis of discretization schemes for some financial processes. Advisers: A. Alfonsi and V. Bally (from Oct. 2012).

- Pierre Blanc: Modeling the price impact of limit and market orders. Adviser: A. Alfonsi. (from Nov. 2012).

- Anis Al Gerbi : Discretization of stochastic differential equations and systemic risk modeling. Adviser B. Jourdain (from Sept. 2013).

- J. Reygner, Longtime behaviour of particle systems : applications in physics, finance and PDEs, Advisers: B. Jourdain and L. Zambotti (from Sept. 2011).

- A. Kritoglou, Stochastic modelling for ferromagnetic materials, Since sept. 2013 : Advisers: J. Lelong and S. Labbé (from Sept. 2013).

- R. Dumitrescu, Reflected BSDEs with jumps, 2nd year : Advisers: A. Sulem and R. Elie (from Sept. 2012).

- Victor Rabiet, On a class of jump type stochastic equations, Adviser : V. Bally (from Sept. 2009).

- Paolo Pigato, Lawer bounds for the density of the solution of SDE's under the weak Hörmander condition, and applications in finance, Advisers: V. Bally and A. Dai Pra.

- Ernesto Palidda (Since September 2010), Crédit Agricole, Operation Research group, Adviser: B. Lapeyre (from Sept. 2010).

- Marouen Iben Taarit (from September 2013), Cifre industrial contract Natixis/ENPC, Adviser: B. Lapeyre.

- Jyda Mint Moustapha (Since November 2012), IFSTTAR, Adisers: D. Daucher and B. Jourdain.

9.2.3. Participation to PhD committees

- A. Alfonsi
  - Florian Klöck, "Regularity of Market Impact Models, Time-Dependent Impact, Dark Pools and Multivariate Transient Impact", University of Mannheim, June 18 2013.
  - Timothée Papin,"Pricing of corporate loan: Credit risk and Liquidity cost", Université Paris-Dauphine, September 25 2013
- A. Sulem:
  - Myriana Grigorova, *Quelques liens entre la théorie de l'intégration non-additive et les domaines de la finance et de l'assurance*, (President of the committee), Laboratoire de Probabilités et Modèles Aléatoires (LPMA), Université Paris VII, October 18 2013
  - Fabien Guilbaud, *Contrôle optimal dans des carnets d'ordres limites*, (President of the committee), LPMA, Université Paris VII, 1 February 2013

10. Bibliography

Major publications by the team in recent years

[1] , *Numerical Methods implemented in the Premia Software*, 2009, Bankers, Markets, Investors, Introduction by Agnès Sulem and A. Zanette

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[13] B. ØKSENDAL, A. SULEM, T. ZHANG. *Optimal control of stochastic delay equations and time-advanced backward stochastic differential equations*, in "Advances in Applied Probability", 2011, vol. 43, pp. 572 - 596

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Publications of the year

Doctoral Dissertations and Habilitation Theses

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