



IN PARTNERSHIP WITH:  
**CNRS**

**Université Nice - Sophia  
Antipolis**

Activity Report 2013

# **Project-Team NEUROMATHCOMP**

Mathematical and Computational  
Neuroscience

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

RESEARCH CENTER  
**Sophia Antipolis - Méditerranée**

THEME  
**Computational Neuroscience and  
Medicine**



## Table of contents

<b>1. Members</b>	<b>1</b>
<b>2. Overall Objectives</b>	<b>1</b>
2.1. Presentation	1
2.2. Highlights of the Year	2
<b>3. Research Program</b>	<b>2</b>
3.1. Neural networks dynamics	2
3.2. Mean-field approaches	3
3.3. Neural fields	3
3.4. Spike train statistics	3
3.5. Synaptic Plasticity	4
3.6. Visual neuroscience	4
3.7. Neuromorphic vision	5
<b>4. Software and Platforms</b>	<b>5</b>
4.1. Virtual Retina: A Large-Scale Simulator of Biological Retina	5
4.2. Event Neural Assembly Simulation	6
<b>5. New Results</b>	<b>6</b>
5.1. Neural Networks as dynamical systems	6
5.2. Mean field approaches	7
5.2.1. Asymptotic description of neural networks with correlated synaptic weights	7
5.2.2. Beyond dynamical mean-field theory of neural networks	7
5.3. Neural fields theory	7
5.3.1. Existence of localized solutions	7
5.3.2. A Center Manifold Result for Delayed Neural Fields Equations	7
5.3.3. Interplay between synaptic delays and propagation delays in neural fields equations	8
5.3.4. Stochastic neural field equations: A rigorous footing	8
5.4. Spike trains statistics	8
5.4.1. Decoding the retina with the first wave of spikes	8
5.4.2. Spike train statistics from empirical facts to theory: the case of the retina	9
5.4.3. Hearing the Maximum Entropy Potential of neuronal networks	9
5.4.4. Spatio-temporal spike trains analysis for large scale networks using maximum entropy principle and Monte-Carlo method	9
5.4.5. Spike train statistics and Gibbs distributions	10
5.4.6. A maximum likelihood estimator of neural network synaptic weights	10
5.5. Synaptic plasticity	10
5.5.1. Effects of Cellular Homeostatic Intrinsic Plasticity on Dynamical and Computational Properties of Biological Recurrent Neural Networks	10
5.5.2. Short-term synaptic plasticity in the deterministic Tsodyks-Markram model leads to unpredictable network dynamics	10
5.6. Visual Neuroscience	11
5.6.1. Bifurcation Study of a Neural Fields Competition Model with an Application to Perceptual Switching in Motion Integration	11
5.6.2. A Retinotopic Neural Fields Model of Perceptual Switching in 2D Motion Integration	11
<b>6. Partnerships and Cooperations</b>	<b>12</b>
6.1. National Initiatives	12
6.2. European Initiatives	12
6.2.1.1. BRAINSCALES	12
6.2.1.2. MATHEMACS	13
6.2.1.3. RENVISION	13
6.2.1.4. HBP	14

6.2.1.5.	NERVI	15
6.2.1.6.	FACETS-ITN	15
6.3.	International Initiatives	15
6.3.1.	Inria Associate Teams	15
6.3.2.	Inria International Partners	16
6.3.3.	Participation In other International Programs	16
6.4.	International Research Visitors	17
<b>7.</b>	<b>Dissemination</b> .....	<b>17</b>
7.1.	Scientific Animation	17
7.2.	Teaching - Supervision - Juries	17
7.2.1.	Teaching	17
7.2.2.	Supervision	18
7.2.3.	Juries	18
<b>8.</b>	<b>Bibliography</b> .....	<b>18</b>

# Project-Team NEUROMATHCOMP

**Keywords:** Computational Neurosciences, Neural Network, Network Dynamics, Stochastic Methods, Stochastic Modeling

*Creation of the Project-Team:* 2009 January 01.

## 1. Members

### Research Scientists

Olivier Faugeras [Team leader, Inria, Senior Researcher, HdR]  
Bruno Cessac [Inria, Senior Researcher, HdR]  
Pascal Chossat [CNRS, Senior Researcher, HdR]  
Pierre Kornprobst [Inria, Researcher, HdR]  
Romain Veltz [Inria, Researcher, from September 2013]

### Engineer

Andrew James Rankin [Inria, FP7 ERC NERVI project, until September 2013]

### PhD Students

Javier Baladron [Inria, FP7 FACETS-ITN project, until June 2013]  
Rodrigo Cofre Torres [Université de Nice Sophia-Antipolis]  
Diego Fasoli [Inria, FP7 FACETS-ITN project, until June 2013]  
Massimiliano Muratori [Université de Nice Sophia-Antipolis]  
Karthek Medathati [Inria, FP7 CE MATHEMACS project, from August 2013]  
Hassan Nasser [Inria, FP7 ERC NERVI project]  
Jamil Salhi [Inria, from April 2013]

### Post-Doctoral Fellows

Mathieu Galtier [CNRS, from December 2013]  
James Maclaurin [Inria, FP7 ERC NERVI project]  
Geoffrey Portelli [Inria, FP7 BRAINSCALES project]  
Wahiba Taouali [Inria, ANR KEOPS project, until February 2013]  
Li Yang [Inria, FP7 BRAINSCALES project, from August 2013]

### Administrative Assistant

Marie-Cécile Lafont [Inria]

### Others

Louis Capietto [Inria, from January 2013 until June 2013]  
Gaia Lombardi [Inria, from March 2013 until August 2013]  
Vincent Verdier [Université de Nice Sophia-Antipolis, until March 2013]

## 2. Overall Objectives

### 2.1. Presentation

NEUROMATHCOMP focuses on the exploration of the brain from the mathematical and computational perspectives.

We want to unveil the principles that govern the functioning of neurons and assemblies thereof and to use our results to bridge the gap between biological and computational vision.

Our work is quite mathematical but we make heavy use of computers for numerical experiments and simulations. We have close ties with several top groups in biological neuroscience. We are pursuing the idea that the "unreasonable effectiveness of mathematics" can be brought, as it has been in physics, to bear on neuroscience.

Computational neuroscience attempts to build models of neurons at a variety of levels, microscopic, i.e., the single neuron, the minicolumn containing of the order of one hundred or so neurons, mesoscopic, i.e., the macrocolumn containing of the order of  $10^4 - 10^5$  neurons, and macroscopic, i.e., a cortical area such as the primary visual area V1.

Modeling such assemblies of neurons and simulating their behavior involves putting together a mixture of the most recent results in neurophysiology with such advanced mathematical methods as dynamic systems theory, bifurcation theory, probability theory, stochastic calculus, theoretical physics and statistics, as well as the use of simulation tools.

We conduct research in the following main areas:

1. Neural networks dynamics
2. Mean-field approaches
3. Neural fields
4. Spike train statistics
5. Synaptic plasticity
6. Visual neuroscience
7. Neuromorphic vision

## 2.2. Highlights of the Year

Our PhD student H. Nasser has obtained an award and some financial support from Nice University to create a start up to develop a software (DataSpot) based on the methods developed in Enas.

# 3. Research Program

## 3.1. Neural networks dynamics

The study of neural networks is certainly motivated by the long term goal to understand how brain is working. But, beyond the comprehension of brain or even of simpler neural systems in less evolved animals, there is also the desire to exhibit general mechanisms or principles at work in the nervous system. One possible strategy is to propose mathematical models of neural activity, at different space and time scales, depending on the type of phenomena under consideration. However, beyond the mere proposal of new models, which can rapidly result in a plethora, there is also a need to understand some fundamental keys ruling the behaviour of neural networks, and, from this, to extract new ideas that can be tested in real experiments. Therefore, there is a need to make a thorough analysis of these models. An efficient approach, developed in our team, consists of analysing neural networks as dynamical systems. This allows to address several issues. A first, natural issue is to ask about the (generic) dynamics exhibited by the system when control parameters vary. This naturally leads to analyse the bifurcations occurring in the network and which phenomenological parameters control these bifurcations. Another issue concerns the interplay between neuron dynamics and synaptic network structure.

In this spirit, our team has been able to characterize the generic dynamics exhibited by models such as Integrate and Fire models [9], conductance-based Integrate and Fire models [2], [51], [17], models of epilepsy [70], effects of synaptic plasticity, homeostasis and intrinsic plasticity [21].

[Selected publications on this topic.](#)

## 3.2. Mean-field approaches

Modeling neural activity at scales integrating the effect of thousands of neurons is of central importance for several reasons. First, most imaging techniques are not able to measure individual neuron activity (“microscopic” scale), but are instead measuring mesoscopic effects resulting from the activity of several hundreds to several hundreds of thousands of neurons. Second, anatomical data recorded in the cortex reveal the existence of structures, such as the cortical columns, with a diameter of about  $50\mu\text{m}$  to  $1\text{mm}$ , containing of the order of one hundred to one hundred thousand neurons belonging to a few different species. The description of this collective dynamics requires models which are different from individual neurons models. In particular, when the number of neurons is large enough averaging effects appear, and the collective dynamics is well described by an effective mean-field, summarizing the effect of the interactions of a neuron with the other neurons, and depending on a few effective control parameters. This vision, inherited from statistical physics requires that the space scale be large enough to include a large number of microscopic components (here neurons) and small enough so that the region considered is homogeneous.

Our group is developing mathematical and numerical methods allowing on one hand to produce dynamic mean-field equations from the physiological characteristics of neural structure (neurons type, synapse type and anatomical connectivity between neurons populations), and on the other so simulate these equations. These methods use tools from advanced probability theory such as the theory of Large Deviations [7] and the study of interacting diffusions [1]. Our investigations have shown that the rigorous dynamics mean-field equations can have a quite more complex structure than the ones commonly used in the literature (e.g. [61]) as soon as realistic effects such as synaptic variability are taken into account. Our goal is to relate those theoretical results with experimental measurement, especially in the field of optical imaging. For this we are collaborating with the INT, Marseille.

[Selected publications on this topic.](#)

## 3.3. Neural fields

Neural fields are a phenomenological way of describing the activity of population of neurons by delay integro-differential equations. This continuous approximation turns out to be very useful to model large brain areas such as those involved in visual perception. The mathematical properties of these equations and their solutions are still imperfectly known, in particular in the presence of delays, different time scales and of noise.

Our group is developing mathematical and numerical methods for analysing these equations. These methods are based upon techniques from mathematical functional analysis [6], bifurcation theory [10], equivariant bifurcation analysis, delay equations, and stochastic partial differential equations. We have been able to characterize the solutions of these neural fields equations and their bifurcations, apply and expand the theory to account for such perceptual phenomena as edge, texture [5], and motion perception. We have also developed a theory of the delayed neural fields equations, in particular in the case of constant delays and propagation delays that must be taken into account when attempting to model large size cortical areas [24]. This theory is based on center manifold and normal forms ideas. We are currently extending the theory to take into account various sources of noise using tools from the theory of stochastic partial differential equations.

[Selected publications on this topic.](#)

## 3.4. Spike train statistics

The neuronal activity is manifested by the emission of action potentials (“spikes”) constituting spike trains. Those spike trains are usually not exactly reproducible when repeating the same experiment, even with a very good control ensuring that experimental conditions have not changed. Therefore, researchers are seeking models for spike train statistics, assumed to be characterized by a canonical probabilities giving the statistics of spatio-temporal spike patterns. A current goal in experimental analysis of spike trains is to approximate this probability from data. Several approach exist either based on (i) generic principles (maximum likelihood, maximum entropy); (ii) phenomenological models (Linear-Non linear, Generalized Linear Model, mean-field); (iii) Analytical results on spike train statistics in Neural Network models.

Our group is working on those 3 aspects, on a fundamental and on a practical (numerical) level. On one hand, we have published analytical (and rigorous) results on statistics of spike trains in canonical neural network models (Integrate and Fire, conductance based with chemical and electric synapses) [3], [17], [50]. The main result is the characterization of spike train statistics by a Gibbs distribution whose potential can be explicitly computed using some approximations. Note that this result does not require an assumption of stationarity. We have also shown that the distributions considered in the cases (i), (ii), (iii) above are all Gibbs distributions [15]. On the other hand, we are proposing new algorithms for data processing [20]. We have developed a C++ software for spike train statistics based on Gibbs distributions analysis and freely available at <http://enas.gforge.inria.fr/v3/>. We are using this software in collaboration with several biologist groups involved in the analysis of retina spike trains (Centro de Neurociencia Valparaiso; Molecular Biology Lab, Princeton; Institut de la vision, Paris) [71], [29].

[Selected publications on this topic.](#)

### 3.5. Synaptic Plasticity

Neural networks show amazing abilities to evolve and adapt, and to store and process information. These capabilities are mainly conditioned by plasticity mechanisms, and especially synaptic plasticity, inducing a mutual coupling between network structure and neuron dynamics. Synaptic plasticity occurs at many levels of organization and time scales in the nervous system (Bienenstock, Cooper, and Munroe, 1982). It is of course involved in memory and learning mechanisms, but it also alters excitability of brain areas and regulates behavioral states (e.g. transition between sleep and wakeful activity). Therefore, understanding the effects of synaptic plasticity on neurons dynamics is a crucial challenge.

Our group is developing mathematical and numerical methods to analyse this mutual interaction. On one hand, we have shown that plasticity mechanisms, Hebbian-like or STDP, have strong effects on neuron dynamics complexity, such as dynamics complexity reduction, and spike statistics (convergence to a specific Gibbs distribution via a variational principle), resulting in a response-adaptation of the network to learned stimuli [65], [66], [4]. We are also studying the conjugated effects of synaptic and intrinsic plasticity in collaboration with H. Berry (Inria Beagle) and B. Delord, J. Naudé, ISIR team, Paris. On the other hand, we have pursued a geometric approach in which we show how a Hopfield network represented by a neural field with modifiable recurrent connections undergoing slow Hebbian learning can extract the underlying geometry of an input space [57]. We have also pursued an approach based on the ideas developed in the theory of slow-fast systems (in this case a set of neural fields equations) in the presence of noise and applied temporal averaging methods to recurrent networks of noisy neurons undergoing a slow and unsupervised modification of their connectivity matrix called learning [58].

[Selected publications on this topic.](#)

### 3.6. Visual neuroscience

Our group focuses on the visual system to understand how information is encoded and processed resulting in visual percepts. To do so, we propose functional models of the visual system using a variety of mathematical formalisms, depending on the scale at which models are built, such as spiking neural networks or neural fields. So far, our efforts have been focused on the study of retinal processing, edge and texture perception, motion integration at the level of V1 and MT cortical areas.

At the retina level, we are modeling its circuitry [12] and we are studying the statistics of the spike train output (see, e.g., the software ENAS <http://enas.gforge.inria.fr/v3/>). Real cell recordings are also analysed in collaboration with Institut de la Vision (Paris). For visual edges perception, we have used the theory of neural fields [11]. For visual textures perception, we have used a combination of neural fields theory and equivariant bifurcations theory [5]. At the level of V1-MT cortical areas, we have been investigating the temporal dynamics of motion integration for a wide range of visual stimuli [23], [67], [48], [8]. This work is done in collaboration with Institut de Neurosciences de la Timone (Marseille).

[Selected publications on this topic.](#)



### 3.7. Neuromorphic vision

From the simplest vision architectures in insects to the extremely complex cortical hierarchy in primates, it is fascinating to observe how biology has found efficient solutions to solve vision problems. Pioneers in computer vision had this dream to build machines that could match and perhaps outperform human vision. This goal has not been reached, at least not on the scale that was originally planned, but the field of computer vision has met many other challenges from an unexpected variety of applications and fostered entirely new scientific and technological areas such as computer graphics and medical image analysis. However, modelling and emulating with computers biological vision largely remains an open challenge while there are still many outstanding issues in computer vision.

Our group is working on neuromorphic vision by proposing bio-inspired methods following our progress in visual neuroscience. Our goal is to bridge the gap between biological and computer vision, by applying our visual neuroscience models to challenging problems from computer vision such as optical flow estimation [69], coding/decoding approaches [62], [63] or classification [53], [54].

*Selected publications on this topic.*

## 4. Software and Platforms

### 4.1. Virtual Retina: A Large-Scale Simulator of Biological Retina

**Participants:** Bruno Cessac, Maria-Jose Escobar [Universidad Técnica Federico Santa María, Valparaíso, Chile], Christobal Nettle [Universidad Técnica Federico Santa María, Valparaíso, Chile], Pierre Kornprobst, Adrien Wohrer [Group for Neural Theory - ENS, Paris, France].

Virtual Retina is a simulation software developed by Adrien Wohrer during his PhD [73], [72] that allows large-scale simulations of biologically-plausible retinas.

Virtual Retina has a variety of biological features implemented such as (i) spatio-temporal linear filter implementing the basic center/surround organization of retinal filtering, (ii) non-linear contrast gain control mechanism providing instantaneous adaptation to the local level of contrast; (iii) spike generation by one or several layers of ganglion cells paving the visual field.

Virtual Retina is under Inria CeCill C open-source licence, so that one can download it, install it and run it on one's own image sequences. Virtual Retina also offers a web service (v 2.1), so that users may test directly the main software on their own data, without any installation. This webservice was developed in collaboration with Nicolas Debeissat (engineer, 2002).

We are now interested in the analysis of the collective behavior of ganglion cells responses. To take this collective behavior into account, Virtual Retina needs to be extended since in its current version, ganglion cells are independent. The goal is to produce better retinal models from experimental recordings obtained with our collaborators at the Institut de la Vision (Olivier Marre and Serge Picaud), Evelyne Sernagor (New Castle University) and Luca Berdondini (IIT) using e.g. multi-electrode arrays. This will allow us to better understand the correlations between retina spikes trains and to improve the Virtual Retina model [72] in such a way that it could reproduce the retinal response at the population level. Another application is to the electric stimulation of a retina with implanted multi-electrode arrays in collaboration with the Institut de la Vision and the INT (Frédéric Chavane). Other evolutions of Virtual Retina are also investigated by external partners like the role/implementation of starburst amacrine cells involved in direction selectivity (collaboration with Universidad Técnica Federico Santa María, Valparaíso, Chile, and Centro de Neurociencia de Valparaíso) (see also e.g., [64]).

- IDDN number: IDDN.FR.001.210034.000.S.P.2007.000.31235
- Version: v 2.2.2 (September 2011)
- Link: <http://www-sop.inria.fr/neuromathcomp/public/software/virtualretina>

## 4.2. Event Neural Assembly Simulation

**Participants:** Bruno Cessac, Sélim Kraria [Inria DREAM], Gaia Lombardi, Hassan Nasser, Wahiba Tahouali.

Enas is a library providing numerical tools for the simulation of neural networks and the analysis of spike trains either coming from neural simulators or from biological experiments. The goal is to provide statistical methods allowing to estimate a spatio-temporal statistical model of spike train statistics (including thus pairwise spatio-temporal correlations, but also higher order correlations) from experimental rasters. More precisely, the algorithms are based on our theoretical results on spike trains statistical analysis via Gibbs distributions. We estimate a parametric Gibbs potential optimally characterizing the statistics of empirical spike trains (by minimisation of the Kullback-Leibler divergence between the empirical measure and the Gibbs measure). From this, classical statistical indicators such as firing rate, correlations, higher order moments and statistical entropy are obtained. Also, the form of the Gibbs potential provides essential informations on the underlying neural network and its structure. This method does not only allows us to estimate the spikes statistics but also to compare different models, thus answering such questions about the neural code as: are correlations (or time synchrony or a given set of spike patterns,...) significant with respect to rate coding?

Compared to existing software (**Pandora**; **Sigtool**; **Spyke Viewer**; **Orbital Spikes**) Enas offers new computational methods taking into account time constraints in neural networks (such as memory effects), based on theoretical methods rooted in statistical physics and applied mathematics. The algorithms used are based on linear programming, nonlinear parameter estimations, statistical methods. The C/C++ code has been organized as “bean java” to ease its use by programmers non specialized in advanced object programming. As a consequence the code is distributed in the form of an include source for the lightest and the most universal integration into users codes.

Event neural assembly simulation is developed under CeCILL C licence  
 APP logiciel Enas: IDDN.FR.OOI.360008.000.S.P.2009.000.10600.

It has benefited from the support of an ADT Inria from 2011 to 2013.

The software is freely downloadable at <http://enas.gforge.inria.fr/v3/download.html>.

Website: <http://enas.gforge.inria.fr/>

## 5. New Results

### 5.1. Neural Networks as dynamical systems

#### 5.1.1. Dynamics and spike trains statistics in conductance-based Integrate-and-Fire neural networks with chemical and electric synapses

**Participants:** Bruno Cessac, Rodrigo Cofré.

We investigate the effect of electric synapses (gap junctions) on collective neuronal dynamics and spike statistics in a conductance-based Integrate-and-Fire neural network, driven by a Brownian noise, where conductances depend upon spike history. We compute explicitly the time evolution operator and show that, given the spike-history of the network and the membrane potentials at a given time, the further dynamical evolution can be written in a closed form. We show that spike train statistics is described by a Gibbs distribution whose potential can be approximated with an explicit formula, when the noise is weak. This potential form encompasses existing models for spike trains statistics analysis such as maximum entropy models or Generalized Linear Models (GLM). We also discuss the different types of correlations: those induced by a shared stimulus and those induced by neurons interactions. This work has been presented in several conferences [40], [39], [39], [15] and published in Chaos, Solitons and Fractals [17].

## 5.2. Mean field approaches

### 5.2.1. *Asymptotic description of neural networks with correlated synaptic weights*

**Participants:** Olivier Faugeras, James Maclaurin.

We study the asymptotic law of a network of interacting neurons when the number of neurons becomes infinite. Given a completely connected network of neurons in which the synaptic weights are Gaussian correlated random variables, we describe the asymptotic law of the network when the number of neurons goes to infinity. We introduce the process-level empirical measure of the trajectories of the solutions to the equations of the finite network of neurons and the averaged law (with respect to the synaptic weights) of the trajectories of the solutions to the equations of the network of neurons. The main result of this work is that the image law through the empirical measure satisfies a large deviation principle with a good rate function which is shown to have a unique global minimum. Our analysis of the rate function allows us also to characterize the limit measure as the image of a stationary Gaussian measure defined on a transformed set of trajectories. This work is available on [ArXiv](#) and is under review for a Journal. A preliminary version has been presented at the CNS meeting [42].

### 5.2.2. *Beyond dynamical mean-field theory of neural networks*

**Participants:** Bruno Cessac, Massimiliano Muratori.

We consider a set of  $N$  firing rate neurons with discrete time dynamics and a leak term. The nonlinearity of the sigmoid is controlled by a parameter and each neuron has a firing threshold, Gaussian distributed (thresholds are uncorrelated). The network is fully connected with correlated Gaussian random synaptic weights, with mean zero and covariance matrix. When synaptic weights are uncorrelated the dynamic mean field theory allows us to draw the bifurcation diagram of the model in the thermodynamic limit ( $N$  tending to infinity): in particular there is sharp transition from fixed point to chaos characterized by the maximum Lyapunov exponent, which is known analytically in the thermodynamic limit. However, mean-field theory is exact only in the thermodynamic limit and when synaptic weights are uncorrelated. What are the deviations from mean-field theory observed when one departs from these hypotheses ? We have first studied the finite size dynamics. For finite  $N$  the maximal Lyapunov exponent has a plateau at 0 corresponding to a transition to chaos by quasi-periodicity where dynamics is at the edge of chaos. This plateau disappears in the thermodynamic limit. Thus, mean-field theory neglects an important finite-sized effect since neuronal dynamics at the edge of chaos has strong implications on learning performances of the network. We also studied the effect of a weak correlation on dynamics. Even when correlation is small, one detects an important deviation on the maximal Lyapunov exponent. This work has been presented at the CNS conference in Paris, 2013 [43].

## 5.3. Neural fields theory

### 5.3.1. *Existence of localized solutions*

**Participants:** Pascal Chossat, Grégory Faye, James Rankin.

We have started to tackle the problem of rigorously proving the existence of localized solutions to the neural fields equations. Existence of such solutions had been assumed or guessed from numerical simulations by other researchers. In a series of articles starting with [55] we have used ideas from the theory of ordinary differential equations (existence of homoclinic orbits) [56], [19], and the theory of partial differential equations (Swift-Hohenberg equation) [16] to show the existence of localized solutions for an extended variety of neural fields equations. This is important both theoretically and for neuroscience since these solutions are considered to characterize working (short-term) memory.

### 5.3.2. *A Center Manifold Result for Delayed Neural Fields Equations*

**Participants:** Olivier Faugeras, Romain Veltz.

We have developed a framework for the study of delayed neural fields equations and proved a center manifold theorem for these equations. Specific properties of delayed neural fields equations make it difficult to apply existing methods from the literature concerning center manifold results for functional differential equations. Our approach for the proof of the center manifold theorem uses the original combination of results from Vanderbauwhede et al. [1992] together with a theory of linear functional differential equations in a history space larger than the commonly used set of time-continuous functions. This work has appeared in the SIAM Journal on Mathematical Analysis [24].

### 5.3.3. *Interplay between synaptic delays and propagation delays in neural fields equations*

**Participant:** Romain Veltz.

Neural field equations describe the activity of neural populations at a mesoscopic level. Although the early derivation of these equations introduced space dependent delays coming from the finite speed of signal propagation along axons, there has been few studies concerning their role in shaping the nonlinear dynamics of neural activity. This is mainly due to the lack of analytical tractable models. On the other hand, constant delays have been introduced to model the synaptic transmission and the spike initiation dynamics. By incorporating the two kind of delays in the neural fields equations, we are able to find the Hopf bifurcation curves analytically which produce many Hopf-Hopf interactions. We use normal theory to study two different types of connectivity that reveals a surprisingly rich dynamical portrait. In particular, the shape of the connectivity strongly influences the spatiotemporal dynamics. This work has appeared in SIAM Journal on Applied Dynamical Systems [25].

### 5.3.4. *Stochastic neural field equations: A rigorous footing*

**Participants:** James Inglis, Olivier Faugeras.

We extend the theory of neural fields which has been developed in a deterministic framework by considering the influence spatio-temporal noise. The outstanding problem that we address here is the development of a theory that gives rigorous meaning to stochastic neural field equations, and conditions ensuring that they are well-posed. Previous investigations in the field of computational and mathematical neuroscience have been numerical for the most part. Such questions have been considered for a long time in the theory of stochastic partial differential equations, where at least two different approaches have been developed, each having its advantages and disadvantages. It turns out that both approaches have also been used in computational and mathematical neuroscience, but with much less emphasis on the underlying theory. We present a review of two existing theories and show how they can be used to put the theory of stochastic neural fields on a rigorous footing. We also provide general conditions on the parameters of the stochastic neural field equations under which we guarantee that these equations are well-posed. In so doing, we relate each approach to previous work in computational and mathematical neuroscience. We hope this will provide a reference that will pave the way for future studies (both theoretical and applied) of these equations, where basic questions of existence and uniqueness will no longer be a cause for concern. This work is available on [ArXiv](#) and is under review for a Journal.

## 5.4. Spike trains statistics

### 5.4.1. *Decoding the retina with the first wave of spikes*

**Participants:** John Barrett [Institute of Neuroscience, Medical School, Newcastle University, Newcastle UK], Pierre Kornprobst, Geoffrey Portelli, Evelyne Sernagor [Institute of Neuroscience, Medical School, Newcastle University, Newcastle UK].

Understanding how the retina encodes visual information remains an open question. Using MEAs on salamander retinas [60] showed that the relative latencies between some neuron pairs carry sufficient information to identify the phase of square-wave gratings (Using gratings of varying phase, spatial frequency, and contrast on mouse retinas, we extended this idea by systematically considering the relative order of all spike latencies, i.e. the shape of the first wave of spikes after stimulus onset. The discrimination task was to identify the phase among gratings of identical spatial frequency. We compared the performance (fraction correct predictions) of our approach under classical Bayesian and LDA decoders to spike count and response latency of each recorded neuron. Best results were obtained for the lowest spatial frequency. There, results showed that the spike count discrimination performance was higher than for latency under both the Bayesian ( $0,95\pm 0,02$  and  $0,75\pm 0,11$  respectively) and LDA ( $0,95\pm 0,01$  and  $0,62\pm 0,03$  respectively) decoders. The first wave of spikes decoder is ( $0,46\pm 0,06$ ) less efficient than the spike count. Nevertheless, it accounts for 50% of the overall performance. Interestingly, these results tend to confirm the rank order coding hypothesis [59] which we are currently investigating further.

This work has been presented in [45].

#### 5.4.2. *Spike train statistics from empirical facts to theory: the case of the retina*

**Participants:** Bruno Cessac, Adrian Palacios [CINV-Centro Interdisciplinario de Neurociencia de Valparaiso, Universidad de Valparaiso].

This work focuses on methods from statistical physics and probability theory allowing the analysis of spike trains in neural networks. Taking as an example the retina we present recent works attempting to understand how retina ganglion cells encode the information transmitted to the visual cortex via the optical nerve, by analyzing their spike train statistics. We compare the maximal entropy models used in the literature of retina spike train analysis to rigorous results establishing the exact form of spike train statistics in conductance-based Integrate-and-Fire neural networks. This work has been published in *Mathematical Problems in Computational Biology and Biomedicine*, F. Cazals and P. Kornprobst, Springer [29].

#### 5.4.3. *Hearing the Maximum Entropy Potential of neuronal networks*

**Participants:** Bruno Cessac, Rodrigo Cofré.

We consider a spike-generating stationary Markov process whose transition probabilities are known. We show that there is a canonical potential whose Gibbs distribution, obtained from the Maximum Entropy Principle (MaxEnt), is the equilibrium distribution of this process. We provide a method to compute explicitly and exactly this potential as a linear combination of spatio-temporal interactions. The method is based on the Hammersley Clifford decomposition and on periodic orbits sampling. As an application, we establish an explicit correspondence between the parameters of the Ising model and the parameters of Markovian models like the Generalized-Linear Model. This work has been presented in several conferences [39], [27], and submitted to *Phys. Rev. Letters* [41], see also the research report [31].

#### 5.4.4. *Spatio-temporal spike trains analysis for large scale networks using maximum entropy principle and Monte-Carlo method*

**Participants:** Bruno Cessac, Olivier Marre [Institut de la Vision, Paris, France], Hassan Nasser.

Understanding the dynamics of neural networks is a major challenge in experimental neuroscience. For that purpose, a modelling of the recorded activity that reproduces the main statistics of the data is required. We present a review on recent results dealing with spike train statistics analysis using maximum entropy models (MaxEnt). Most of these studies have been focusing on modelling synchronous spike patterns, leaving aside the temporal dynamics of the neural activity. However, the maximum entropy principle can be generalized to the temporal case, leading to Markovian models where memory effects and time correlations in the dynamics are properly taken into account. We also present a new method based on Monte-Carlo sampling which is suited for the fitting of large-scale spatio-temporal MaxEnt models. The formalism and the tools presented will be essential to fit MaxEnt spatio-temporal models to large neural ensembles. This work has been presented in several conferences [39], [15], [44] and published in *Journal of Statistical Mechanics* [20].

#### 5.4.5. *Spike train statistics and Gibbs distributions*

**Participants:** Bruno Cessac, Rodrigo Cofré.

We introduce Gibbs distribution in a general setting, including non stationary dynamics, and present then three examples of such Gibbs distributions, in the context of neural networks spike train statistics: (i) Maximum entropy model with spatio-temporal constraints; (ii) Generalized Linear Models; (iii) Conductance based Integrate and Fire model with chemical synapses and gap junctions. This leads us to argue that Gibbs distributions might be canonical models for spike train statistics analysis. This work has published in *J. Physiol. Paris* [15].

#### 5.4.6. *A maximum likelihood estimator of neural network synaptic weights*

**Participants:** Bruno Cessac, Wahiba Taouali.

Given a conductance-based Integrate-and-Fire model where the spike statistics dependence on synaptic weights is known, can one reconstruct this network of synaptic weights from the observation of a raster plot generated by the network ? We have solved this inverse problem using an explicit expression of a maximum likelihood estimator based on the Newton-Raphson method. This estimator uses analytically computed gradients and Hessian of the likelihood function given by the product of conditional probabilities. The explicit form of these conditional probabilities can be found in [49]. Our results show that this method allows to estimate the set of connections weights knowing the input, the noise distribution and the leak function. This work has been presented in the CNS conference in Paris, 2013 [47].

### 5.5. Synaptic plasticity

#### 5.5.1. *Effects of Cellular Homeostatic Intrinsic Plasticity on Dynamical and Computational Properties of Biological Recurrent Neural Networks*

**Participants:** Hugues Berry, Bruno Cessac, Bruno Delord, Jérémie Naudé.

Homeostatic intrinsic plasticity (HIP) is a ubiquitous cellular mechanism regulating neuronal activity, cardinal for the proper functioning of nervous systems. In invertebrates, HIP is critical for orchestrating stereotyped activity patterns. The functional impact of HIP remains more obscure in vertebrate networks, where higher-order cognitive processes rely on complex neural dynamics. The hypothesis has emerged that HIP might control the complexity of activity dynamics in recurrent networks, with important computational consequences. However, conflicting results about the causal relationships between cellular HIP, network dynamics and computational performance have arisen from machine learning studies. In this work, we assess how cellular HIP effects translate into collective dynamics and computational properties in biological recurrent networks. We develop a realistic multi scale model including a generic HIP rule regulating the neuronal threshold with actual molecular signaling pathways kinetics, Dale's principle, sparse connectivity, synaptic balance and Hebbian synaptic plasticity (SP). Dynamic mean-field analysis and simulations unravel that HIP sets a working point at which inputs are transduced by large derivative ranges of the transfer function. This cellular mechanism insures increased network dynamics complexity, robust balance with SP at the edge of chaos, and improved input separability. Although critically dependent upon balanced excitatory and inhibitory drives, these effects display striking robustness to changes in network architecture, learning rates and input features. Thus, the mechanism we unveil might represent a ubiquitous cellular basis for complex dynamics in neural networks. Understanding this robustness is an important challenge to unravel principles underlying self-organization around criticality in biological recurrent neural networks. This work has been published in the *Journal of Neuroscience* [21].

#### 5.5.2. *Short-term synaptic plasticity in the deterministic Tsodyks-Markram model leads to unpredictable network dynamics*

**Participants:** Jesus Cortes, Mathieu Desroches, Serafim Rodrigues, Romain Veltz, Miguel Munoz, Terrence Sejnowski.

Short-term synaptic plasticity strongly affects the neural dynamics of cortical networks. The Tsodyks and Markram (TM) model for short-term synaptic plasticity accurately accounts for a wide range of physiological responses at different types of cortical synapses. We report a route to chaotic behavior via a Shilnikov homoclinic bifurcation that dynamically organizes some of the responses in the TM model. In particular, the presence of such a homoclinic bifurcation strongly affects the shape of the trajectories in the phase space and induces highly irregular transient dynamics; indeed, in the vicinity of the Shilnikov homoclinic bifurcation, the number of population spikes and their precise timing are unpredictable and highly sensitive to the initial conditions. Such an irregular deterministic dynamics has its counterpart in stochastic/network versions of the TM model: The existence of the Shilnikov homoclinic bifurcation generates complex and irregular spiking patterns and acting as a sort of springboard facilitates transitions between the down-state and unstable periodic orbits. The interplay between the (deterministic) homoclinic bifurcation and stochastic effects may give rise to some of the complex dynamics observed in neural systems.

This work has been published in the Proceedings of the National Academy of Sciences [52].

## 5.6. Visual Neuroscience

### 5.6.1. *Bifurcation Study of a Neural Fields Competition Model with an Application to Perceptual Switching in Motion Integration*

**Participants:** James Rankin, Andrew Meso [Institut de Neurosciences de la Timone, UMR 6193, CNRS, Marseille, France], Guillaume S. Masson [Institut de Neurosciences de la Timone, UMR 6193, CNRS, Marseille, France], Olivier Faugeras, Pierre Kornprobst.

In this work we have investigated the underlying mechanisms that gate multistable perception, by focusing on the presentation of 1:1 barber pole during long presentations, which is perceived to move in a direction that changes every few seconds. This phenomenon has been studied from the perspective of dynamical systems modeling and human psychophysics: From a modeling point of view, numerical tools from bifurcations analysis were applied to the study of a competition model posed as a feature-only neural field equation (with a continuous feature space) where adaptation and noise are implemented as mechanisms that can drive activity switches. Human psychophysics experiments were jointly done by INT (Institut de Neurosciences de la Timone, Marseille): Human observers were presented a moving grating stimulus over 15s while eye movements and reports of perceptual switches were recorded. Investigating the stimulus contrast, we found that the peak in switching rate observed experimentally occurs close to a bifurcation in the model that separates two mechanistic regimes. By identifying signatures of the switching predicted by the model with the behavioural data at different parts of the transition-contrast curve, we found for the first time, evidence for a dominance of driving mechanisms which shifts from noise dominated at low contrasts to adaptation dominated at higher contrasts.

This work has been published in [22], [23].

### 5.6.2. *A Retinotopic Neural Fields Model of Perceptual Switching in 2D Motion Integration*

**Participants:** Pierre Kornprobst, Guillaume S. Masson [Institut de Neurosciences de la Timone, UMR 6193, CNRS, Marseille, France], Kartheek Medathati, James Rankin.

In perceptual multistability a fixed but ambiguous stimulus can invoke multiple interpretations although only one can be held at a time. Visual motion stimuli are inherently ambiguous, for instance due to the aperture problem, which makes motion perception a complex inference task. The underlying cortical dynamics that select one percept out of multiple competing possibilities are not fully understood. Recent studies by [22] and [68] have tried to address this problem using the neural fields formalism. In [22], a switching behaviour for a classical psychophysics stimulus, the multistable barberpole, was successfully captured in a feature-only, one-layer model of MT with adaptation and noise. However, without a representation of space, only some very specific stimulus could be considered. The work reported in [68] provides a much more general framework for motion integration in a two layer-model, however, it fails to capture the switching behaviour as the mechanisms of adaptation and noise were not considered. Building on the strengths of both studies, we propose a model

that takes into account the spatial domain in a two-layer configuration whilst incorporating both adaptation and noise. Interactions between two layers processing local motion (V1 and MT) occurred through recurrent and lateral connections. The input stimuli are represented using direction of motion signals extracted using Reichardt detectors at corresponding 2D spatial locations. We use stimuli such as drifting bars and barberpoles to constrain the model to a suitable operating regime. In terms of computations, since the model is demanding, we implemented it using GPUs, extending the methods of [13]. Based on this implementation, we study dynamics of the model focusing on coherency in plaid motion (plaids and crossed barber pole).

This work has been presented in [28]

## 6. Partnerships and Cooperations

### 6.1. National Initiatives

#### 6.1.1. ANR

##### 6.1.1.1. KEOPS

See section “International Initiatives” below.

### 6.2. European Initiatives

#### 6.2.1. FP7 Projects

##### 6.2.1.1. BRAINSCALES

Title: BrainScaleS: Brain-inspired multiscale computation in neuromorphic hybrid systems

Type: COOPERATION (ICT)

Defi: Brain-inspired multiscale computation in neuromorphic hybrid systems

Instrument: Integrated Project (IP)

Objectif: FET proactive 8: Brain Inspired ICT

Duration: January 2011 - December 2014

Coordinator: Universitaet Ruprecht- Karls Heidelberg (Germany)

Other Partners: Nederlandse Akademie van Wetenschappen, Amsterdam; Universitet For Miljo Og Biovitenskap, Aas; Universitat Pompeu Fabra, Barcelona; University of Cambridge; Debreceni Egyetem, Debrecen; Technische Universität Dresden; CNRS-UNIC, Gif-sur- Yvette; CNRS-INCM, Marseille; CNRS-ISM, Marseille; TUG, Graz; Ruprecht-Karls-Universität Heidelberg; Forschungszentrum Jülich GmbH, Jülich; EPFL LCN, Lausanne; EPFL- BBP, Lausanne; The University Of Manchester, Manchester; KTH, Stockholm; Universität Zürich.

See also <http://brainscales.kip.uni-heidelberg.de/>

Inria contact: Olivier Faugeras

Abstract: The BrainScaleS project aims at understanding function and interaction of multiple spatial and temporal scales in brain information processing. The fundamentally new approach of BrainScaleS lies in the in-vivo biological experimentation and computational analysis. Spatial scales range from individual neurons over larger neuron populations to entire functional brain areas. Temporal scales range from milliseconds relevant for event based plasticity mechanisms to hours or days relevant for learning and development. In the project generic theoretical principles will be extracted to enable an artificial synthesis of cortical-like cognitive skills. Both, numerical simulations on petaflop supercomputers and a fundamentally different non-von Neumann hardware architecture will be employed for this purpose. Neurobiological data from the early perceptual visual and somatosensory systems will be combined with data from specifically targeted higher cortical areas. Functional databases as well as novel project-specific experimental tools and protocols will be developed and



used. New theoretical concepts and methods will be developed for understanding the computational role of the complex multi-scale dynamics of neural systems in-vivo. Innovative in-vivo experiments will be carried out to guide this analytical understanding. Multiscale architectures will be synthesized into a non-von Neumann computing device realised in custom designed electronic hardware. The proposed Hybrid Multiscale Computing Facility (HMF) combines microscopic neuromorphic physical model circuits with numerically calculated mesoscopic and macroscopic functional units and a virtual environment providing sensory, decision-making and motor interfaces. The project also plans to employ petaflop supercomputing to obtain new insights into the specific properties of the different hardware architectures. A set of demonstration experiments will link multiscale analysis of biological systems with functionally and architecturally equivalent synthetic systems and offer the possibility for quantitative statements on the validity of theories bridging multiple scales. The demonstration experiments will also explore non-von Neumann computing outside the realm of brain-science. BrainScaleS will establish close links with the EU Brain-i-Nets and the Blue Brain project at the EPFL Lausanne. The consortium consists of a core group of 10 partners with 13 individual groups. Together with other projects and groups the BrainScaleS consortium plans to make important contributions to the preparation of a FET flagship project. This project will address the understanding and exploitation of information processing in the human brain as one of the major intellectual challenges of humanity with vast potential applications.

This project started on January 1st, 2011 and is funded for four years.

#### 6.2.1.2. MATHEMACS

Title: Mathematics of Multilevel Anticipatory Complex Systems

Type: Collaborative project (generic) (FP7-ICT)

Defi: develop a mathematical theory of complex multilevel systems and their dynamics.

Instrument: Integrated Project (IP)

Duration: October 2012 - September 2015

Coordinator: Fatihcan Atay, Max Planck Institute for Mathematics in the Sciences, Leipzig (Germany)

Other Partners: Max Planck Institute for Mathematics in the Sciences (Leipzig, Germany), Universität Bielefeld (Germany), Chalmers University of Technology (Gothenburg, Sweden), Ca'Foscari University of Venice (Italy), Università Politecnica delle Marche (Ancona, Italy).

See also: <http://www.mathemacsc.eu/description.html>

Inria contact: Olivier Faugeras

Abstract: The MATHEMACS project aims to develop a mathematical theory of complex multi-level systems and their dynamics. This is done through a general formulation based on the mathematical tools of information and dynamical systems theories. To ensure that the theoretical framework is at the same time practically applicable, three key application areas are represented within the project, namely neurobiology, human communication, and economics. These areas not only provide some of the best-known epitomes of complex multi-level systems, but also constitute a challenging test bed for validating the generality of the theory since they span a vast range of spatial and temporal scales. Furthermore, they have an important common aspect; namely, their complexity and self-organizational character is partly due to the anticipatory and predictive actions of their constituent units. The MATHEMACS project contends that the concepts of anticipation and prediction are particularly relevant for multi-level systems since they often involve different levels. Thus, as a further unique feature, the project includes the mathematical representation and modeling of anticipation in its agenda for understanding complex multi-level systems.

This project started on October 1st, 2012 and is funded for four years.

#### 6.2.1.3. RENVISION

Type: COOPERATION, FP7 FET (Future Emerging technology) proactive program: Neuro-Bio-Inspired Systems Call 9 Objective 9.11

Defi: Retina-inspired ENcoding for advanced VISION tasks (RENVISION)

Instrument: Specific Targeted Research Project

Duration: March 2013 - February 2016

Coordinator: Vittorio Murino, PAVIS, IIT (Italy)

Partner: PAVIS, IIT (Italy), NBT, IIT (Italy), NAPH, IIT (Italy), The Institute of Neuroscience, Newcastle University (UK), Institute for Adaptive and Neural Computation, The University of Edinburgh (UK), Neuromathcomp project-team, Inria (France)

Inria contact: Pierre Kornprobst

Abstract: The retina is a sophisticated distributed processing unit of the central nervous system encoding visual stimuli in a highly parallel, adaptive and computationally efficient way. Recent studies show that rather than being a simple spatiotemporal filter that encodes visual information, the retina performs sophisticated non-linear computations extracting specific spatio-temporal stimulus features in a highly selective manner (e.g. motion selectivity). Understanding the neurobiological principles beyond retinal functionality is essential to develop successful artificial computer vision architectures.

RENVISION's goal is, therefore, twofold:

- To achieve a comprehensive understanding of how the retina encodes visual information through the different cellular layers;
- To use such insights to develop a retina-inspired computational approach to high-level computer vision tasks.

To this aim, exploiting the recent advances in high-resolution light microscopy 3D imaging and high-density multielectrode array technologies, RENVISION will be in an unprecedented position to investigate pan-retinal signal processing at high spatio-temporal resolution, integrating these two technologies in a novel experimental setup. This will allow for simultaneous recording from the entire population of ganglion cells and functional imaging of inner retinal layers at near-cellular resolution, combined with 3D structural imaging of the whole inner retina. The combined analysis of these complex datasets will require the development of novel multimodal analysis methods.

Resting on these neuroscientific and computational grounds, RENVISION will generate new knowledge on retinal processing. It will provide advanced pattern recognition and machine learning technologies to ICTs by shedding a new light on how the output of retinal processing (natural or modelled) allows solving complex vision tasks such as automated scene categorization and human action recognition.

#### 6.2.1.4. HBP

Type: COOPERATION, FET Flagship' project

Defi: Understanding the brain

Instrument: FET Flagship' project

Duration: October 2013 - March 2016

Coordinator: EPFL (Switzerland)

Partner: see <http://www.humanbrainproject.eu>.

Inria contact: Olivier Faugeras

Abstract: The Human Brain Project (HBP) is supported by the European Union as a 'FET Flagship' project and the 86 institutions involved will receive one billion euro in funding over ten years. HBP should lay the technical foundations for a new model of ICT-based brain research, driving integration between data and knowledge from different disciplines, and catalysing a community effort to achieve a new understanding of the brain, new treatments for brain disease and new brain-like computing technologies. <http://www.humanbrainproject.eu>

#### 6.2.1.5. NERVI

Program: ERC IDEAS

Project acronym: NerVi

Project title: From single neurons to visual perception

Coordinator: Olivier Faugeras

Duration: January 2009 - December 2013

Abstract: The project is to develop a formal model of information representation and processing in the part of the neocortex that is mostly concerned with visual information. This model will open new horizons in a well-principled way in the fields of artificial and biological vision as well as in computational neuroscience. Specifically the goal is to develop a universally accepted formal framework for describing complex, distributed and hierarchical processes capable of processing seamlessly a continuous flow of images. This framework features notably computational units operating at several spatiotemporal scales on stochastic data arising from natural images. Mean-field theory and stochastic calculus are used to harness the fundamental stochastic nature of the data, functional analysis and bifurcation theory to map the complexity of the behaviours of these assemblies of units. In the absence of such foundations, the development of an understanding of visual information processing in man and machines could be greatly hindered. Although the proposal addresses fundamental problems, its goal is to serve as the basis for ground-breaking future computational development for managing visual data and as a theoretical framework for a scientific understanding of biological vision.

#### 6.2.1.6. FACETS-ITN

Title: FACETS-ITN

Instrument: Initial Training Network (ITN)

Duration: September 2009 - August 2013

Coordinator: Universität Heidelberg- Ruprecht-Karls (Germany)

Inria contact: O. Faugeras

See also <http://facets.kip.uni-heidelberg.de/ITN/index.html>

This 'Marie-Curie Initial Training Network' (funded by the EU) involves 15 groups at European Research Universities, Research Centers and Industrial Partners in 6 countries. Website: <http://facets.kip.uni-heidelberg.de/ITN/index.html>

## 6.3. International Initiatives

### 6.3.1. Inria Associate Teams

#### 6.3.1.1. CORTINA

Title: Retina neural network coding

Inria principal investigator: Bruno CESSAC

International Partner (Institution - Laboratory - Researcher):

Technical University Federico Santa Maria, Valparaíso (Chile) - Electronics Engineering  
Department - Bruno CESSAC

Duration: 2011 - 2013

See also: <http://cortex.loria.fr/Projects/Cortina>

Much progress has been made in the last decades in understanding the basic organization and function of the nervous system in general. Contributions to this end have come from various domains including computational neuroscience and numerical science of the information in general. The goal of this associate team is to combine our complementary expertise, from experimental biology and mathematical models (U de Valparaiso and U Federico Santa-Maria) to computational neuroscience (CORTEX and NEUROMATHCOMP), in order to develop numerical tools for the study and characterization of neural coding and related sensory-motor loops. Recording and modeling spike trains from the retina neural network, an accessible part of the brain, is a difficult task that our partnership can address, what constitute an excellent and unique opportunity to work together sharing our experience and to focus in developing computational tools for methodological innovations. To understand how the neural spike coding from natural image sequences works we are addressing the following issues: How visual signals are coded at earlier steps in the case of natural vision? What are their functions? What are the computational coding principles explaining (in artificial or biological system) the statistical properties of natural images? We wish to advance our actual knowledge in natural and artificial visual signals processing and apply it to the field of education; to foster better capacities for learning and memory; sensory prosthesis design, to will help unpaired sensory persons to sense the world and physical rehabilitation, among others. In the context of the cooperation between the Inria and Chile, we propose to develop new neural decoding algorithms that are transverse to several field and applications.

### **6.3.2. Inria International Partners**

#### *6.3.2.1. Declared Inria International Partners*

Paul Bressloff, Professor of applied mathematics at the University of Utah (USA) specialising in mathematical neuroscience, has been selected for an Inria International Chair. He will be visiting the Sophia-Antipolis Méditerranée research center two months every year for five years, starting in 2014.

### **6.3.3. Participation In other International Programs**

#### *6.3.3.1. ANR KEOPS*

Type: Algorithms for modeling the visual system: From natural vision to numerical applications.

Principal Investigator: Thierry Viéville (Mnemosyne)

International partner:

- Institution: University of Valparaiso (Chile)
- Laboratory: Centro Interdisciplinario de Neurociencia de Valparaiso
- Researcher: Adrian PALACIOS

International partner:

- Institution: UTFSM Valparaiso (Chile)
- Laboratory: Direccion General de Investigacion y Postgrado de Valparaiso
- Researcher: Maria-Jose ESCOBAR

Duration: 2011 - 2013

See also: <http://cortex.loria.fr/Research/Keops>

Abstract: KEOpS attempts to study and model the non-standard behavior of retinal (ganglion cells) sensors observed in natural scenarios. KEOpS also attempts to incorporate the resulting models into real engineering applications as new dynamical early-visual modules. The retina, an accessible part of the brain, is a unique model for studying the neural coding principles for natural scenarios. A recent study proposes that some visual functions (e.g. movement, orientation, anticipatory temporal prediction, contrast), thought to be the exclusive duty of higher brain centers, are actually carried at the retina level. The anatomical and physiological segregation of visual scenes into spatial, temporal and chromatic channels begins at the retina through the action of local neural networks.

However, how the precise articulation of this neural network contributes to local solutions and global perception necessary to resolve natural task remains in general a mystery. KEOpS thus attempts to study the complexity of retinal ganglion cells (the output to the brain) behaviors observed in natural scenarios and to apply this result to artificial visual systems. We revisit both the retinal neural coding information sent to the brain, and at the same time, the development of new engineering applications inspired by the understanding of such neural encoding mechanisms. We develop an innovative formalism that takes the real (natural) complexity of retinal responses into account. We also develop new dynamical early-visual modules necessary to solve visual problems task.

## 6.4. International Research Visitors

### 6.4.1. Visits of International Scientists

- Alexander Cerquera, Universidad Antonio Nariño, Facultad de Ingeniería Electrónica y Biomédica, Colombia, 25-29/03/2013.
- Antonio Galves, Statistics Department. Instituto de Matemática e Estatística · Universidade de São Paulo. 25-27/06/2013.
- Eva Loecherbach, Maths department, Cergy University. 25-27/06/2013.

#### 6.4.1.1. Internships

- Gaia Lombardi, M2 Internship, March-August 2013.

## 7. Dissemination

### 7.1. Scientific Animation

Bruno Cessac is a reviewer for the CONYCIT (Chile) and COFECUB (Brasil) program and for the journals *Physica D*, *Nonlinearity*, *Chaos*, *Journal of Statistical Physics*, *IEEE Transaction in Neural Networks*, *Journal of Mathematical Biology*, *Journal of Computational Neuroscience*. He is in charge of internships organisation in the Master of Computational Biology, Nice.

Olivier Faugeras is co-Editor in Chief of the *Journal of Mathematical Science* (JMN). He is a member of the French Academy of Sciences and of the French Academy of Technology. He is a member of panel 1 for the ERC Frontier Research Grants.

Pierre Kornprobst was a member of the program committee of the 6th Pacific-Rim Symposium on Image and Video Technology. He co-edited with Frédéric Cazals the book entitled "*Modeling in Computational Biology and Medicine: A Multidisciplinary Endeavor*" [30], which illustrates the program taught in the Master of Science in Computational Biology (Website: <http://cbb.unice.fr>) that they launched in 2009.

### 7.2. Teaching - Supervision - Juries

#### 7.2.1. Teaching

Licence 1 : Massimiliano Muratori, TP de physique, 45 h, L1, Ecole d'ingénieurs Polytech.

Licence 2 : Rodrigo Cofre, Traitement du signal, 50h, L2, Université Nice Sophia Antipolis, France.

Licence 2 : Massimiliano Muratori, TD d'électromagnétisme , 18h, L2, Ecole d'ingénieurs Polytech.

Licence 3 : Hassan Nasser, Electronique numérique, 36h, L3, Université Nice Sophia Antipolis, France

License 3 : Hassan Nasser, Microprocesseurs, 28h, L3, Université Nice Sophia Antipolis, France

Master 2: Bruno Cessac, *Neuronal dynamics*, 36 hours, Master of Computational Biology and Biomedicine, Université Nice Sophia Antipolis, France.

Master 2: Olivier Faugeras, Etienne Tanré (TOSCA EPI) and Romain Veltz, teach the module *Mathematical Methods for Neuroscience*, M2, ENS Paris, France. The teaching load is mostly distributed between Etienne Tanré and Romain Veltz, 24h each.

### 7.2.2. Supervision

PhD: Javier Baladron, «Parallel implementations of mean field and neural field equations», Université Nice Sophia Antipolis, 2013, supervised by Olivier Faugeras.

PhD: Diego Fasoli, «Mean-field theory of realistic spiking neurons», Université Nice Sophia Antipolis, 2013, supervised by Olivier Faugeras.

PhD in progress: Hassan Nasser, «Reproducing and anticipating retinal responses», defence planned in March 2014, supervised by Bruno Cessac.

PhD in progress: Rodrigo Cofre-Torres, «Statistics of spike trains and neuronal structures», defense planned in 2014, supervised by Bruno Cessac.

PhD interrupted: Massimiliano Muraatori, « Mean field equations for neural networks and synaptic correlations», supervised by Bruno Cessac. The student has decided to stop his PhD after one year to teach in college.

PhD in progress: Kartheek Medathati, « Perception du mouvement et attention: Des neurosciences à la vision artificielle», defence planned in 2016, co-supervised by Pierre Kornprobst and Guillaume Masson (Institut de Neurosciences de la Timone, UMR 6193, CNRS, Marseille, France).

### 7.2.3. Juries

Bruno Cessac. Jury member of Diégo Fasoli's PhD thesis "Mean-field theory of realistic spiking neurons". Nice, 25-09-2013.

Bruno Cessac. Reviewer of Christophe Magnani's PhD Thesis, "Analyse Sinusoïdale Quadratique de la Fonction Neuronale", Paris Descartes, 10-12-13.

Olivier Faugeras, Chair of the Jury of Mikhail Bogdanov PhD Thesis, Nice 9-12-2013.

Olivier Faugeras, Reviewer for the HDR manuscript of Christophe Pouzat, Paris-Descartes, 10-2013.

## 8. Bibliography

### Major publications by the team in recent years

- [1] J. BALADRON PEZOA, D. FASOLI, O. FAUGERAS, J. TOUBOUL. *Mean-field description and propagation of chaos in networks of Hodgkin-Huxley and FitzHugh-Nagumo neurons*, in "The Journal of Mathematical Neuroscience", 2012, vol. 2, n° 1, <http://www.mathematical-neuroscience.com/content/2/1/10>
- [2] B. CESSAC. *A discrete time neural network model with spiking neurons. Rigorous results on the spontaneous dynamics*, in "J. Math. Biol.", 2008, vol. 56, n° 3, pp. 311-345, <http://lanl.arxiv.org/abs/0706.0077>
- [3] B. CESSAC. *A discrete time neural network model with spiking neurons II. Dynamics with noise.*, in "Journal of Mathematical Biology", 2011, vol. 62, n° 6, pp. 863-900 [DOI : 10.1007/s00285-010-0358-4], <http://lanl.arxiv.org/pdf/1002.3275>
- [4] B. CESSAC, H. ROSTRO-GONZALEZ, J.-C. VASQUEZ, T. VIÉVILLE. *How Gibbs distribution may naturally arise from synaptic adaptation mechanisms: a model based argumentation*, in "J. Stat. Phys.", 2009, vol. 136, n° 3, pp. 565-602 [DOI : 10.1007/s10955-009-9786-1], <http://lanl.arxiv.org/abs/0812.3899>

- [5] P. CHOSSAT, O. FAUGERAS. *Hyperbolic planforms in relation to visual edges and textures perception*, in "Plos Comput Biol", December 2009, vol. 5, n<sup>o</sup> 12, <http://dx.doi.org/doi:10.1371/journal.pcbi.1000625>
- [6] O. FAUGERAS, F. GRIMBERT, J.-J. SLOTINE. *Abolute stability and complete synchronization in a class of neural fields models*, in "SIAM Journal of Applied Mathematics", September 2008, vol. 61, n<sup>o</sup> 1, pp. 205–250
- [7] O. FAUGERAS, J. TOUBOUL, B. CESSAC. *A constructive mean field analysis of multi population neural networks with random synaptic weights and stochastic inputs*, in "Frontiers in Computational Neuroscience", 2009, vol. 3, n<sup>o</sup> 1 [DOI : 10.3389/NEURO.10.001.2010], <http://arxiv.org/abs/0808.1113>
- [8] E. TLAPELE, G. S. MASSON, P. KORNPORST. *Modelling the dynamics of motion integration with a new luminance-gated diffusion mechanism*, in "Vision Research", August 2010, vol. 50, n<sup>o</sup> 17, pp. 1676–1692, <http://dx.doi.org/10.1016/j.visres.2010.05.022>
- [9] J. TOUBOUL, O. FAUGERAS. *A Markovian event-based framework for stochastic spiking neural networks*, in "Journal of Computational Neuroscience", April 2011, vol. 30, <http://www.springerlink.com/content/81736mn03j2221m7/fulltext.pdf>
- [10] R. VELTZ, O. FAUGERAS. *Local/Global Analysis of the Stationary Solutions of Some Neural Field Equations*, in "SIAM Journal on Applied Dynamical Systems", August 2010, vol. 9, n<sup>o</sup> 3, pp. 954–998 [DOI : 10.1137/090773611], <http://arxiv.org/abs/0910.2247>
- [11] R. VELTZ. , *Nonlinear analysis methods in neural field models*, Université Paris Est, 2011, <ftp://ftp-sop.inria.fr/neuromathcomp/publications/phds/veltz-11.pdf>
- [12] A. WOHRER, P. KORNPORST. *Virtual Retina : A biological retina model and simulator, with contrast gain control*, in "Journal of Computational Neuroscience", 2009, vol. 26, n<sup>o</sup> 2, 219 p. , DOI 10.1007/s10827-008-0108-4

## Publications of the year

### Doctoral Dissertations and Habilitation Theses

- [13] J. BALADRON PEZOA. , *Exploring the neural codes using parallel hardware*, Université Nice Sophia Antipolis, June 2013, <http://hal.inria.fr/tel-00847333>
- [14] D. FASOLI. , *Traiter le cerveau avec les neurosciences : théorie de champ-moyen, effets de taille finie et capacité de codage des réseaux de neurones stochastiques*, Université Nice Sophia Antipolis, September 2013, <http://hal.inria.fr/tel-00850289>

### Articles in International Peer-Reviewed Journals

- [15] B. CESSAC, R. COFRE. *Spike train statistics and Gibbs distributions*, in "Journal of Physiology - Paris", 2013, vol. 107, n<sup>o</sup> 5, pp. 360-368, <http://hal.inria.fr/hal-00850155>
- [16] P. CHOSSAT, G. FAYE. *Pattern Formation for the Swift-Hohenberg Equation on the Hyperbolic Plane*, in "Journal of Dynamics and Differential Equations", 2013, pp. 1–47 [DOI : 10.1007/s10884-013-9308-3], <http://hal.inria.fr/hal-00845612>

- [17] R. COFRÉ, B. CESSAC. *Dynamics and spike trains statistics in conductance-based Integrate-and-Fire neural networks with chemical and electric synapses*, in "Chaos, Solitons and Fractals", 2013, vol. 50, pp. 13-31 [DOI : 10.1016/J.CHAOS.2012.12.006], <http://hal.inria.fr/hal-00846091>
- [18] G. FAYE, P. CHOSSAT. *A spatialized model of textures perception using structure tensor formalism*, in "Networks and Heterogeneous Media", 2013, <http://hal.inria.fr/hal-00807371>
- [19] G. FAYE, J. RANKIN, P. CHOSSAT. *Localized states in an unbounded neural field equation with smooth firing rate function: a multi-parameter analysis*, in "Journal of Mathematical Biology", 2013, vol. 66, n<sup>o</sup> 6, pp. 1303-1338 [DOI : 10.1007/s00285-012-0532-Y], <http://hal.inria.fr/hal-00807366>
- [20] H. NASSER, O. MARRE, B. CESSAC. *Spatio-temporal spike train analysis for large scale networks using the maximum entropy principle and Monte Carlo method*, in "Journal of Statistical Mechanics: Theory and Experiment", 2013, vol. 2013, n<sup>o</sup> 03, 41 p. [DOI : 10.1088/1742-5468/2013/03/P03006], <http://hal.inria.fr/hal-00846160>
- [21] J. NAUDÉ, B. CESSAC, H. BERRY, B. DELORD. *Effects of Cellular Homeostatic Intrinsic Plasticity on Dynamical and Computational Properties of Biological Recurrent Neural Networks*, in "Journal of Neuroscience", 2013, vol. 33, n<sup>o</sup> 38, pp. 15032-15043 [DOI : 10.1523/JNEUROSCI.0870-13.2013], <http://hal.inria.fr/hal-00844218>
- [22] J. RANKIN, A. MESO, G. MASSON, O. FAUGERAS, P. KORNPBST. *Bifurcation Study of a Neural Fields Competition Model with an Application to Perceptual Switching in Motion Integration*, in "Journal of Computational Neuroscience", 2013, <http://hal.inria.fr/hal-00920528>
- [23] J. RANKIN, E. TLAPALE, R. VELTZ, O. FAUGERAS, P. KORNPBST. *Bifurcation analysis applied to a model of motion integration with a multistable stimulus*, in "Journal of Computational Neuroscience", 2013, vol. 34, n<sup>o</sup> 1, pp. 103-124, 10.1007/s10827-012-0409-5 [DOI : 10.1007/s10827-012-0409-5], <http://hal.inria.fr/hal-00845593>
- [24] R. VELTZ, O. FAUGERAS. *A Center Manifold Result for Delayed Neural Fields Equations*, in "SIAM Journal on Mathematical Analysis", 2013, vol. 45, n<sup>o</sup> 3, pp. 1527-1562 [DOI : 10.1137/110856162], <http://hal.inria.fr/hal-00850382>
- [25] R. VELTZ. *Interplay Between Synaptic Delays and Propagation Delays in Neural Field Equations*, in "SIAM Journal on Applied Dynamical Systems", 2013, vol. 12, n<sup>o</sup> 3, pp. 1566-1612 [DOI : 10.1137/120889253], <http://hal.inria.fr/hal-00850391>

### Invited Conferences

- [26] B. CESSAC. *Spike trains analysis, Gibbs distributions and (Variable Length ?) Markov chains*, in "VLMC days, 5-6 March 2013", Dijon, France, 2013, <http://hal.inria.fr/hal-00845619>

### International Conferences with Proceedings

- [27] B. CESSAC, R. COFRE. *Can we hear the shape of a Maximum Entropy potential from a spike train?*, in "Bernstein Conference", Tübingen, Germany, 2013, <http://hal.inria.fr/hal-00851436>



- [28] N. V. K. MEDATHATI, J. RANKIN, P. KORNPBOST, G. S. MASSON. *A retinotopic neural fields model of perceptual switching in 2D motion integration*, in "Bernstein Conference", Tübingen, Germany, 2013, <http://hal.inria.fr/hal-00850097>

### Scientific Books (or Scientific Book chapters)

- [29] B. CESSAC, A. PALACIOS. *Spike Train Statistics from Empirical Facts to Theory: The Case of the Retina*, in "Modeling in Computational Biology and Biomedicine: A Multidisciplinary Endeavor", F. CAZALS, P. KORNPBOST (editors), Springer, 2013, <http://hal.inria.fr/hal-00640507>

### Books or Proceedings Editing

- [30] F. CAZALS, P. KORNPBOST (editors). , *Modeling in Computational Biology and Medicine: A Multidisciplinary Endeavor*, Springer, 2013, 315 p. [DOI : 10.1007/978-3-642-31208-3], <http://hal.inria.fr/hal-00845616>

### Research Reports

- [31] B. CESSAC, R. COFRE. , *Estimating maximum entropy distributions from periodic orbits in spike trains*, Inria, July 2013, n<sup>o</sup> RR-8329, <http://hal.inria.fr/hal-00842776>
- [32] O. FAUGERAS, J. INGLIS. , *Stochastic neural field equations: A rigorous footing*, November 2013, 40 p. , <http://hal.inria.fr/hal-00907555>
- [33] O. FAUGERAS, J. MACLAURIN. , *A large deviation principle for networks of rate neurons with correlated synaptic weights*, February 2013, 71 p. , <http://hal.inria.fr/hal-00785627>
- [34] H. NASSER, B. CESSAC. , *Parameters estimation for spatio-temporal maximum entropy distributions: application to neural spike trains*, January 2014, <http://hal.inria.fr/hal-00927080>
- [35] J. RANKIN, A. MESO, G. MASSON, O. FAUGERAS, P. KORNPBOST. , *Bifurcation Study of a Neural Fields Competition Model with an Application to Perceptual Switching in Motion Integration*, Inria, February 2013, n<sup>o</sup> RR-8220, 37 p. , <http://hal.inria.fr/hal-00783525>
- [36] E. TEFTEF, M.-J. ESCOBAR, A. ASTUDILLO, C. CARVAJAL, B. CESSAC, A. PALACIOS, T. VIÉVILLE, F. ALEXANDRE. , *Modeling non-standard retinal in/out function using computer vision variational methods*, Inria, January 2013, n<sup>o</sup> RR-8217, 28 p. , <http://hal.inria.fr/hal-00783091>
- [37] R. VELTZ. , *Interplay between synaptic delays and propagation delays in neural fields equations*, Inria, January 2013, n<sup>o</sup> RR-8020, 53 p. , <http://hal.inria.fr/hal-00780444>

### Other Publications

- [38] P. BELTRAME, P. CHOSSAT. , *Onset of intermittent octahedral patterns in spherical Bénard convection*, February 2014, <http://hal.inria.fr/hal-00945597>
- [39] B. CESSAC, R. COFRE. *Space-time correlations in spike trains and the neural code*, in "Mathematics and neuroscience a dialogue", Utrecht, Netherlands, September 2013, <http://hal.inria.fr/hal-00861405>

- [40] R. COFRE, B. CESSAC. *Dynamics and spike trains statistics in conductance-based Integrate-and-Fire neural networks with chemical and electric synapses*, in "Twenty Second Annual Computational Neuroscience Meeting : CNS 2013", Paris, France, July 2013, 58 p. [DOI : 10.1186/1471-2202-14-S1-P58], <http://hal.inria.fr/hal-00842297>
- [41] R. COFRE, B. CESSAC. , *Hearing the Maximum Entropy Potential of neuronal networks*, January 2014, <http://hal.inria.fr/hal-00861397>
- [42] O. FAUGERAS, J. MACLAURIN. *A large deviation principle for networks of rate neurons with correlated synaptic weights*, in "Twenty Second Annual Computational Neuroscience Meeting : CNS 2013", Paris, France, July 2013, 252 p. [DOI : 10.1186/1471-2202-14-S1-P252], <http://hal.inria.fr/hal-00842310>
- [43] M. MURATORI, B. CESSAC. *Beyond dynamical mean-field theory of neural networks*, in "Twenty Second Annual Computational Neuroscience Meeting : CNS 2013", Paris, France, July 2013, 60 p. [DOI : 10.1186/1471-2202-14-S1-P60], <http://hal.inria.fr/hal-00842309>
- [44] H. NASSER, S. KRARIA, B. CESSAC. *EnaS: a new software for neural population analysis in large scale spiking networks*, in "Twenty Second Annual Computational Neuroscience Meeting : CNS 2013", Paris, France, July 2013, 57 p. [DOI : 10.1186/1471-2202-14-S1-P57], <http://hal.inria.fr/hal-00842303>
- [45] G. PORTELLI, J. M. BARRETT, E. SERNAGOR, P. KORNPBOST. *Decoding the retina with the first wave of spikes*, in "European Retina Meeting - 2013", Alicante, Spain, October 2013, <http://hal.inria.fr/hal-00920543>
- [46] J. RANKIN, D. AVITABILE, J. BALADRON PEZOA, G. FAYE, D. J. B. LLOYD. , *Continuation of localised coherent structures in nonlocal neural field equations*, 2013, 21 p. , submitted for peer review, <http://hal.inria.fr/hal-00850408>
- [47] W. TAOUALI, B. CESSAC. *A maximum likelihood estimator of neural network synaptic weights*, in "Twenty Second Annual Computational Neuroscience Meeting : CNS 2013", Paris, France, July 2013, 59 p. [DOI : 10.1186/1471-2202-14-S1-P59], <http://hal.inria.fr/hal-00842312>

## References in notes

- [48] J. BOUECKE, E. TLAPOLE, P. KORNPBOST, H. NEUMANN. *Neural Mechanisms of Motion Detection, Integration, and Segregation: From Biology to Artificial Image Processing Systems*, in "EURASIP Journal on Advances in Signal Processing", 2011, vol. 2011, special issue on Biologically inspired signal processing: Analysis, algorithms, and applications [DOI : 10.1155/2011/781561], <http://hal.inria.fr/hal-00784429>
- [49] B. CESSAC. *A view of Neural Networks as dynamical systems*, in "International Journal of Bifurcations and Chaos", 2010, vol. 20, n<sup>o</sup> 6, pp. 1585-1629 [DOI : 10.1142/S0218127410026721], <http://lanl.arxiv.org/abs/0901.2203>
- [50] B. CESSAC. *Statistics of spike trains in conductance-based neural networks: Rigorous results*, in "The Journal of Mathematical Neuroscience", 2011, vol. 1, n<sup>o</sup> 8, pp. 1-42 [DOI : 10.1186/2190-8567-1-8], <http://www.mathematical-neuroscience.com/content/1/1/8>
- [51] B. CESSAC, T. VIÉVILLE. *On Dynamics of Integrate-and-Fire Neural Networks with Adaptive Conductances*, in "Frontiers in neuroscience", July 2008, vol. 2, n<sup>o</sup> 2, <http://hal.inria.fr/inria-00338369>

- [52] J. M. CORTES, M. DESROCHES, S. RODRIGUES, R. VELTZ, M. A. MUÑOZ, T. J. SEJNOWSKI. *Short-term synaptic plasticity in the deterministic Tsodyks-Markram model leads to unpredictable network dynamics*, in "Proceedings of the National Academy of Sciences", 2013, vol. 110, n<sup>o</sup> 41, pp. 16610–16615
- [53] M.-J. ESCOBAR, P. KORNPBOST. *Action recognition via bio-inspired features: The richness of center-surround interaction*, in "Computer Vision and Image Understanding", 2012, vol. 116, n<sup>o</sup> 5, 593–605 p. , <http://hal.inria.fr/hal-00849935>
- [54] M.-J. ESCOBAR, G. S. MASSON, T. VIÉVILLE, P. KORNPBOST. *Action Recognition Using a Bio-Inspired Feedforward Spiking Network*, in "International Journal of Computer Vision", 2009, vol. 82, n<sup>o</sup> 3, 284 p. , <ftp://ftp-sop.inria.fr/neuromathcomp/publications/2009/escobar-masson-et-al:09.pdf>
- [55] G. FAYE. *Reduction method for studying localized solutions of neural field equations on the Poincaré disk*, in "Comptes Rendus de l'Académie des Sciences, Mathématique", February 2012, vol. 350, n<sup>o</sup> 3-4, pp. 161–166, <http://www.sciencedirect.com/science/article/pii/S1631073X12000337>
- [56] G. FAYE, J. RANKIN, D. J. B. LLOYD. *Localized radial bumps of a neural field equation on the Euclidean plane and the Poincaré disk*, in "Nonlinearity", April 2012
- [57] M. GALTIER, O. FAUGERAS, P. BRESSLOFF. *Hebbian Learning of Recurrent Connections: A Geometrical Perspective*, in "Neural Computation", September 2012, vol. 24, n<sup>o</sup> 9, pp. 2346–2383
- [58] M. GALTIER, G. WAINRIB. *Multiscale analysis of slow-fast neuronal learning models with noise*, in "Journal of Mathematical Neuroscience", 2012, vol. 2, n<sup>o</sup> 13, <http://www.mathematical-neuroscience.com/content/2/1/13/abstract>
- [59] J. GAUTRAIS, S. THORPE. *Rate Coding vs Temporal Order Coding : a theoretical approach*, in "Biosystems", 1998, vol. 48, pp. 57–65
- [60] T. GOLLISCH, M. MEISTER. *Rapid Neural Coding in the Retina with Relative Spike Latencies*, in "Science", 2008, vol. 319, pp. 1108–1111, DOI: 10.1126/science.1149639
- [61] B. H. JANSEN, V. G. RIT. *Electroencephalogram and visual evoked potential generation in a mathematical model of coupled cortical columns*, in "Biological Cybernetics", 1995, vol. 73, pp. 357–366
- [62] K. MASMOUDI, M. ANTONINI, P. KORNPBOST. *Frames for Exact Inversion of the Rank Order Coder*, in "IEEE Transactions on Neural Networks and Learning Systems", 2012, vol. 23, n<sup>o</sup> 2, pp. 353–359, <http://dx.doi.org/10.1109/TNNLS.2011.2179557>
- [63] K. MASMOUDI, M. ANTONINI, P. KORNPBOST. *Streaming an image through the eye: The retina seen as a dithered scalable image coder*, in "Signal Processing-Image Communication", 2012, <http://dx.doi.org/10.1016/j.image.2012.07.005>
- [64] T. MASQUELIER. *Relative spike time coding and STDP-based orientation selectivity in the early visual system in natural continuous and saccadic vision: a computational model*, in "Journal of Computational Neuroscience", 2011, <http://dx.doi.org/10.1007/s10827-011-0361-9>

- [65] B. SIRI, H. BERRY, B. CESSAC, B. DELORD, M. QUOY. *Effects of Hebbian learning on the dynamics and structure of random networks with inhibitory and excitatory neurons.*, in "Journal of Physiology-Paris", 2007
- [66] B. SIRI, H. BERRY, B. CESSAC, B. DELORD, M. QUOY. *A Mathematical Analysis of the Effects of Hebbian Learning Rules on the Dynamics and Structure of Discrete-Time Random Recurrent Neural Networks*, in "Neural Computation", December 2008, vol. 20, n<sup>o</sup> 12, 12 p.
- [67] E. TLAPALE, P. KORNPORST, G. S. MASSON, O. FAUGERAS. *A Neural Field Model for Motion Estimation*, in "Mathematical Image Processing", S. VERLAG (editor), Springer Proceedings in Mathematics, 2011, vol. 5, pp. 159–180, <http://dx.doi.org/10.1007/978-3-642-19604-1>
- [68] E. TLAPALE, G. S. MASSON, P. KORNPORST. *Modelling the dynamics of motion integration with a new luminance-gated diffusion mechanism*, in "Vision Research", August 2010, vol. 50, n<sup>o</sup> 17, pp. 1676–1692, <http://dx.doi.org/10.1016/j.visres.2010.05.022>
- [69] E. TLAPALE. , *Modelling the dynamics of contextual motion integration in the primate*, Université Nice Sophia Antipolis, January 2011, <ftp://ftp-sop.inria.fr/neuromathcomp/publications/phds/tlapale-11.pdf>
- [70] J. TOUBOUL, F. WENDLING, P. CHAUVEL, O. FAUGERAS. *Neural Mass Activity, Bifurcations, and Epilepsy*, in "Neural Computation", December 2011, vol. 23, n<sup>o</sup> 12, pp. 3232–3286
- [71] J.-C. VASQUEZ, A. PALACIOS, O. MARRE, M. J. BERRY, B. CESSAC. *Gibbs distribution analysis of temporal correlations structure in retina ganglion cells*, in "J. Physiol. Paris", May 2012, vol. 106, n<sup>o</sup> 3-4, pp. 120-127, <http://arxiv.org/abs/1112.2464>
- [72] A. WOHRER, P. KORNPORST. *Virtual Retina : A biological retina model and simulator, with contrast gain control*, in "Journal of Computational Neuroscience", 2009, vol. 26, n<sup>o</sup> 2, 219 p. , DOI 10.1007/s10827-008-0108-4
- [73] A. WOHRER. , *Model and large-scale simulator of a biological retina with contrast gain control*, University of Nice Sophia-Antipolis, 2008