Activity Report 2013

Project-Team POEMS

Wave propagation: mathematical analysis and simulation

IN COLLABORATION WITH: Propagation des ondes : étude mathématique et simulation (POEMS)

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Project-Team POEMS

Keywords: Waves, Scientific Computation, Simulation, Numerical Methods, Inverse Problem

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2. Overall Objectives

2.1. Overall Objectives

The propagation of waves is one of the most common physical phenomena one can meet in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) and to the scale of the universe (electromagnetic waves, gravity waves), to the scale of the atom (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us.

The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in the various domains of physics and engineering science.

The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas justify together a research project in Scientific Computing entirely devoted to this theme.

The project POEMS is an UMR (Unité Mixte de Recherche) between CNRS, ENSTA and Inria (UMR 7231). The general activity of the project is oriented toward the conception, the analysis, the numerical approximation, and the control of mathematical models for the description of wave propagation in mechanics, physics, and engineering sciences.

Beyond the general objective of contributing to the progress of the scientific knowledge, four goals can be ascribed to the project:

- the development of an expertise relative to various types of waves (acoustic, elastic, electromagnetic, gravity waves, ...) and in particular for their numerical simulation,

- the treatment of complex problems whose simulation is close enough to real life situations and industrial applications,

- the development of original mathematical and numerical techniques,

- the development of computational codes, in particular in collaboration with external partners (scientists from other disciplines, industry, state companies...)

3. Research Program

3.1. Mathematical analysis and simulation of wave propagation

Our activity relies on the existence of mathematical models established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation (or a system of partial differential equations) of the hyperbolic type that are often (but not always) linear for most of the applications we are interested in. The prototype equation is the wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0,$$

which can be directly applied to acoustic waves but which also constitutes a simplified scalar model for other types of waves (This is why the development of new numerical methods often begins by their application to the wave equation). Of course, taking into account more realistic physics will enrich and complexify the basic models (presence of sources, boundary conditions, coupling of models, integro-differential or non linear terms,...)

It is classical to distinguish between two types of problems associated with these models: the time domain problems and the frequency domain (or time harmonic) problems. In the first case, the time is one of the variables of which the unknown solution depends and one has to face an evolution problem. In the second case (which rigorously makes sense only for linear problems), the dependence with respect to time is imposed a priori (via the source term for instance): the solution is supposed to be harmonic in time, proportional to $e^{i\omega t}$, where $\omega > 0$ denotes the pulsation (also commonly, but improperly, called the frequency). Therefore, the time dependence occurs only through this pulsation which is given a priori and plays the rôle of a parameter: the unknown is only a function of space variables. For instance, the wave equation leads to the Helmholtz wave equation (also called the reduced wave equation) :

$$-c^2 \Delta u - \omega^2 u = 0.$$

These two types of problems, although deduced from the same physical modelling, have very different mathematical properties and require the development of adapted numerical methods.

However, there is generally one common feature between the two problems: the existence of a dimension characteristic of the physical phenomenon: the wavelength. Intuitively, this dimension is the length along which the searched solution varies substantially. In the case of the propagation of a wave in an heterogeneous medium, it is necessary to speak of several wavelengths (the wavelength can vary from one medium to another). This quantity has a fundamental influence on the behaviour of the solution and its knowledge will have a great influence on the choice of a numerical method.

Nowadays, the numerical techniques for solving the basic academic and industrial problems are well mastered. A lot of companies have at their disposal computational codes whose limits (in particular in terms of accuracy or robustness) are well known. However, the resolution of complex wave propagation problems close to real applications still poses (essentially open) problems which constitute a real challenge for applied mathematicians. A large part of research in mathematics applied to wave propagation problems is oriented towards the following goals:

- the conception of new numerical methods, more and more accurate and high performing.
- the treatment of more and more complex problems (non local models, non linear models, coupled systems, periodic media).
- the study of specific phenomena or features such as guided waves, resonances,...
- the development of approximate models via asymptotic analysis with multiple scales (thin layers, boundary or interfaces, small homogeneities, homogenization, ...).
- imaging techniques and inverse problems related to wave propagation.

4. Application Domains

4.1. Introduction

We are concerned with all application domains where linear wave problems arise: acoustics and elastodynamics (including fluid-structure interactions), electromagnetism and optics, and gravity water waves. We give in the sequel some details on each domain, pointing out our main motivations and collaborations.

4.2. Acoustics

As the acoustic propagation in a fluid at rest can be described by a scalar equation, it is generally considered by applied mathematicians as a simple preliminary step for more complicated (vectorial) models. However, several difficult questions concerning coupling problems have occupied our attention recently. Aeroacoustics, or more precisely, acoustic propagation in a moving compressible fluid, is for our team a new and very challenging topic, which gives rise to a lot of open questions, from the modelling (Euler equations, Galbrun equations, Goldstein equation) to the numerical approximation of such models (which poses new difficulties). Our works in this area are partially supported by EADS and Airbus. The typical objective is to reduce the noise radiated by Airbus planes. Vibroacoustics, which concerns the interaction between sound propagation and vibrations of thin structures, also raises up a lot of relevant research subjects.

Both applications (aeroacoustics and vibroacoustics) led us in particular to develop an academic research between volumic methods and integral equations in time domain.

Finally, a particularly attractive application concerns the simulation of musical instruments, whose objectives are both a better understanding of the behavior of existing instruments and an aid for the manufacturing of new instruments. The modelling and simulation of the timpani, the guitar and the piano have been carried out in collaboration with A. Chaigne of ENSTA. This work will continue in the framework of the European Project BATWOMAN.

4.3. Electromagnetism

This is a particularly important domain, first because of the very important technological applications but also because the treatment of Maxwell's equations is much more technically involved from the mathematical point of view that the scalar wave equation. Applied mathematics for electromagnetism during the last ten years have mainly concerned stealth technology, electromagnetic compatibility, design of optoelectronic micro-components or smart materials. Stealth technology relies in particular on the conception and simulation of new absorbing materials (anisotropic, chiral, non-linear...). The simulation of antennas raises delicate questions related to the complexity of the geometry (in particular the presence of edges and corners). In optics, the development of the Mmcro and nano optics has made recently fantastic progress and the thematic of metamaterials (with negative index of refraction) opens new amazing applications. For all these reasons, we are developing an intense research in the following areas

- Highly accurate and hybrid numerical methods in collaboration with CEA (Gramat) and ONERA (Toulouse).
- Electromagmetic wave propagation in periodic media.
- Development of simplified approximate models by asymptotic anslysis for various applications : boundary layers, thin coatings, thin domains, thin wires and cables, ...
- Mathematical and numerical questions linked to the modeling of metamaterials.

4.4. Elastodynamics

Wave propagation in solids is with no doubt, among the three fundamental domains that are acoustics, electromagnetism and elastodynamics, the one that poses the most significant difficulties from mathematical and numerical points of view.

Our activity on this topic began with applications in geophysics, which unfortunately has been forced to slow down in the middle of the 90's due to the disengagement of French oil companies in matter of research. However it has seen a most welcomed rebound through new academic problems (in parficular surface waves, perfectly matched layers techniques, inverse problems in wave guides) and industrial contacts, more precisely with CEA-LIST with which we have developed a long term collaboration in the domain of non destructive testing by ultrasounds. The most recent problems we have been dealing with in this domain concern elastic wave propagation in plates, the modeling of piezoelectric devices or elastic wave propagation in highly heterogeneous media.

5. Software and Platforms

5.1. Software

5.1.1. Introduction

We are led to develop two types of softwares. The first one is prototype softwares : various softwares are developed in the framework of specific research contracts (and sometimes sold to the contractor) or during PhD theses. They may be also contributions to already existing softwares developed by other institutions such as CEA, ONERA or EDF. The second category is an advanced software which are intended to be developed, enriched and maintained over longer periods. Such software is devoted to help us for our own research and/or promote our research. We have chosen to present here our advanced software.

5.1.2. XLiFE++

Participants: Eric Lunéville, Nicolas Kielbasiewicz, Colin Chambeyron, Manh Ha Nguyen.

XLIFE++ is a new Finite Element library in C++ based on philosophy of the previous library MELINA in Fortran but with new capabilities (boundary elements and discontinuous Galerkin methods, more integrated tools – in particular mesh tools – and high performance computing skills, multithread and GPU computation. It is licensed under LGPL and developed in the context of the European project SIMPOSIUM (FP7/ICT, leader CEA/LIST, from september 2011 to august 2014). There are also academic partners : IRMAR, University of Rennes and LAMA, University of Marne-la-Vallée.

In 2012, as a reminder, all development tools were set up and all fundamental and major libraries were done. In 2013, developments have sped up. The Finite Elements, the Spectral Elements and the Boundary Elements computation cores have been implemented and are currently under testing. In addition to the implementation of direct and iterative solvers, an internal eigen solver is operational and coupled to external solver libraries (Arpack++, Umfpack, ...).

As far as inputs/outputs are concerned, XLIFE++ allows to export a solution to the visualization tool PARAVIEW and to read mesh files from GMSH, MELINA and PARAVIEW (vtk). Furthermore, mesh tools have been enriched and a C++ interface to the mesh tool GMSH is under development. XLIFE++ can now solve the Helmholtz equation with Neumann boundary conditions in any mesh. A first version of the library should be published soon.

6. New Results

6.1. Time domain wave propagation problems

6.1.1. Numerical methods in electromagnetism

Participant: Gary Cohen.

In the framework of contract GREAT, we implemented and compared two discontinuous Galerkin methods to solve Maxwell's equations for time dependent problems, the first using tetrahedral meshes (first used by Hesthaven), the second using hexahedral meshes with mass lumping. This comparison showed the undeniable superiority of the second method, 4-7 times faster (for orders from 2 to 4) for the same accuracy.

The ultimate goal of this program was the hybridization of those two types of meshes because the construction of purely hexahedral mesh for complex geometries is often difficult or almost impossible. A first approach was studied in the thesis Morgane Bergot, where the transition between the two grids was performed by the use of pyramids. The implementation of such elements is difficult and costly, we were interested in a transition mortars elements capable of hybridizing directly tetrahedra with flat faces and hexahedral with non-planar faces. This approach is promising and should lead to a rapid and efficient method. A theoretical study of the error and stability is conducted in collaboration with Eric Chung of CUHK (Chinese University of Hong Kong).

Moreover, always with E. Chung, we became interested in the construction of a discontinuous Galerkin method on hexahedral meshes offset for solving Maxwell's equations. This approach has two advantages : firstly, the shift naturally removes the spurious waves which appear with other approaches (which usually requires the introduction of a dissipative term to remove them). On the other hand, a phenomenon of super-convergence appears which should lead to a substantial time saving. A first study of the dispersion of this method led to a publication.

6.1.2. *Solving the Homogeneous Isotropic Linear Elastodynamics Equations Using Potentials and Finite Elements.*

Participants: Aliénor Burel, Patrick Joly.

The aim of this subject, investigated in collaboration with Marc Duruflé (Inria Bordeaux) and Sébastien Imperiale (Inria Saclay), is to use the classical theoretical decomposition of the elastodynamic displacement into two potentials referring to the pressure wave and the shear wave, and use it in a numerical framework. During the past two years, a method has been proposed for solving the Dirichlet problem (clamped boundary), successfully analysed and implemented, and for the free boundary conditions, we proposed an original method considering these boundary conditions as a perturbation of the Dirichlet conditions. This approach performs successfully in the time-harmonic regime but appears to give rise to severe instabilities in the time-dependent case after space and time discretization. Our investigations seem to prove that this instability is already present in the semi-discrete problem in space, but we are still looking for an explanation of this phenomenon.

6.1.3. *Limiting amplitude principle in a two-layered medium composed of a dielectric and a metamaterial*

Participants: Maxence Cassier, Christophe Hazard, Patrick Joly, Valentin Vinoles.

We are investigating this problem from both theoretical and numerical points of view. This is also the object of a collaboration with B. Gralak from the Institut Fresnel in Marseille.

This work is the time-domain counterpart of the research done at Poems about frequency domain analysis of metamaterials in electromagnetism, in the framework of the ANR Project METAMATH. One fundamental question is the link between the evolution / time harmonic problems via the limiting amplitude principle, in particular in the cases where the time harmonic problem fails to be well posed. This occurs, at certain frequencies, when one considers a transmission problem between a standard dielectric material and a dispersive material obeying for instance to the Drude model (other models as Lorentz materials or their generalization also give rise to the same results). Indeed, for well-chosen coefficients (which we refer as the critical case), there exist critical frequencies (only one frequency ω_c for the Drude model) for which the metamaterial behaves as a material whose equivalent electric permittivity and magnetic permeability are negative and precisely opposite to the ones of the dielectric medium : in such a situation, in the case of a plane interface, it is known that the time harmonic transmission problem is strongly ill-posed.

We have considered the evolution problem in a two-layered medium, when we consider a source term $f(x)\, e^{i\,\omega t}$ with frequency $\omega > 0$. In the non critical case, the limiting amplitude principle holds : for large times, the solution $u(x, t)$ of the evolution problem "converges" to a time harmonic solution of the form $u^\infty(x)\, e^{i\,\omega t}$. In the critical case, the limiting amplitude principle no longer holds. If $\omega \neq \omega_c$, the solution of the evolution problem behaves when $t \to +\infty$ to a "double frequency" solution of the form

$$u^\infty(x)\, e^{i\,\omega t} + u^\infty_{c,+}(x)\, e^{i\,\omega_c t} + u^\infty_{c,-}(x)\, e^{-i\,\omega_c t}.$$

If $\omega = \omega_c$, the solution blows up linearly at infinity :

$$u(x, t) \sim t\, u^\infty_c(x)\, e^{i\,\omega_c t} \quad (t \to +\infty)$$
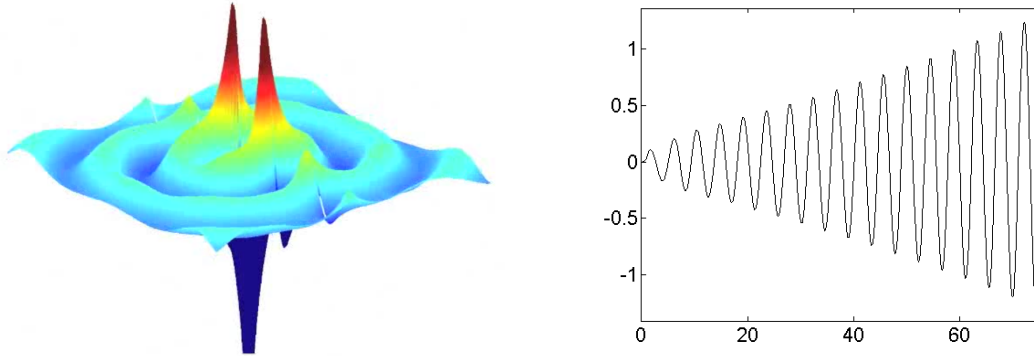
Figure 1. Left : the interface wave. Right : solution at one point as a function of time.

where the function $u_c^\infty(x)$ is "concentrated" near the interface : this can be interpreted as an "interface resonance" phenomenon. We have performed various numerical experiments (using in particular the stabilized PMLs evoked in section 6.3.2) that illustrate this resonance phenomenon (cf figure 1).

From the mathematical point of view, the method we have used consists in rewriting the original problem as an abstract Schrödinger equation

$$i\,\frac{du}{dt} + Au = F\,e^{i\,\omega t}$$

where A is a self-adjoint operator in an appropriate Hilbert space H. The key of the analysis is the spectral theory of the operator A. This permits a quasi-explicit representation of the solution via the (generalized) diagonalization of A. This is achieved by combining a partial Fourier transform along the interface with Sturm-Liouville type techniques in the orthogonal direction. In the critical case, the resonance phenomenon appears to be linked to the fact that A admits a (single) eigenvalue of infinite multiplicity.

6.1.4. Finite differences method for nonlinear acoustic waves with fractional derivatives

Participant: Jean-François Mercier.

This subject is developed in collaboration with Bruno Lombard from LMA.

We develop a numerical method to study the wave propagation in a 1-D guide with an array of Helmholtz resonators, considering large amplitude waves and viscous boundary layers. The model consists in two coupled equations: a nonlinear PDE for the velocity in the tube (Burgers like equation) and a linear ODE describing the pressure oscillations in the Helmholtz resonators. The dissipative and dispersive effects in the tube and in the necks of the resonators are modelled by fractional derivatives expressed as convolution products with singular kernels. Based on a diffusive representation, the convolution kernels of the fractional derivatives are replaced by a finite number of memory variables that satisfy local ordinary differential equations. The procedure to compute weights and nodes of the diffusive representation of fractional derivatives is optimized. Moreover an adequate coupling between the PDE and the ODE is introduced to be sure that the discrete energy is decreasing. A splitting strategy is then applied to the evolution equations to obtain a stable scheme under the optimal CFL condition: the propagative part is solved by a standard TVD scheme for hyperbolic equations, whereas the diffusive part is solved exactly. This approach is validated by comparisons with exact solutions. The properties of the full nonlinear solutions are investigated numerically. In particular, the existence of acoustic solitary waves, due to the competition between dispersion and nonlinear effects, is confirmed.

6.2. Time-harmonic diffraction problems

6.2.1. *Numerical computation of variational integral equation methods*
Participants: Marc Lenoir, Nicolas Salles.

The discretization of 3-D scattering problems by variational boundary element methods leads to the evaluation of such elementary integrals as

$$\int_{S \times T} G(x,y)\, v(x)\, w(y)\, dx\, dy \text{ and } \int_{S \times T} \frac{\partial}{\partial n_y} G(x,y)\, v(x)\, w(y)\, dx\, dy \tag{1}$$

where v and w are polynomial basis functions, G is the Green kernel and S and T two planar polygons from the discretization of the boundary. Due to the singularity of the kernel, the numerical evaluation of these integrals may lead to inaccurate results when S and T are close to each other. We split G and its gradient into a regular part which involves classical numerical techniques and a singular part subject to our method. This new method consists in integrating exactly integrals such as

$$I = \int_{S \times T} v(x) \frac{1}{\|x - y\|} w(y)\, dx\, dy \text{ and } J_\zeta = \int_{S \times T} \frac{x - y}{\|x - y\|^{1+\zeta}}\, dx\, dy,\ \zeta \in \{0, 2\}. \tag{2}$$

or numerically integrals such as:

$$\mathcal{L} = \int_{S \times T} v(x) \frac{e^{ik\|x - y\|}}{\|x - y\|} w(y)\, dx\, dy \tag{3}$$

where v and w are basis functions of order 0 or 1. The general approach relies on two steps.

Basic formulas : let $f(x, d)\ : \Omega \subset \mathbb{R}^n \times \mathbb{R} \longrightarrow \mathbb{R}$ a positively homogeneous function of degree q. By Euler's formula and Green's theorem we have the function $I(d)$ satisfies :

$$(q + n)\, I(d) = dI'(d) + \int_{\partial \Omega} (\overrightarrow{z} \,|\, \overrightarrow{\nu})\, f(z, d)\, d\gamma_z, \quad \text{with } I(d) = \int_\Omega f(z, d)\, dz \tag{4}$$

where $\overrightarrow{\nu}$ is the exterior normal to Ω. Provided $d^{-(q+n)} \int_\Omega f(z, d)\, dz \to 0$ as $d \to +\infty$ one obtains

$$I(d) = d^{q+n} \int_{\partial \Omega} (\overrightarrow{z} \,|\, \overrightarrow{\nu}) \int_d^{+\infty} \frac{f(z, t)}{t^{q+n+1}} dt\, d\gamma_z. \tag{5}$$

When $f(z, d)$ does not depend on d and $q + n \neq 0$ then

$$I = \frac{1}{q + n} \int_{\partial \Omega} (\overrightarrow{z} \,|\, \overrightarrow{\nu})\, f(z)\, d\gamma_z. \tag{6}$$

As long as the inner integral in (5) can be explicitly evaluated, both formulas reduce an n-dimensional integral to an $(n - 1)$ one. When Ω is an n-dimensional polyhedron (such as $S \times T$ with $n = 4$), $(\overrightarrow{z} \,|\, \overrightarrow{\nu})$ is constant on each $(n - 1)$-face of Ω, a simplification of crucial importance in the sequel.

The reduction process : we have obtained formulas for three types of geometrical configurations: S and T are (i) coplanar, (ii) in secant planes and (iii) in parallel planes. All these cases are treated using formulas (6) or (5) or both, depending on the relative positions of S and T. As an example, we present the simple but significant result for the self-influence coefficient ($S = T$). Let A_i be a vertex of the triangle, α_i the opposite side, B_i the orthogonal projection of A_i on α_i and $\gamma_i = \|A_i B_i\|$. After 3 successive reductions using formula (6), one obtains

$$I = \int_{S \times S} \frac{1}{\|x - y\|} dx dy = \frac{2\,|S|}{3} \sum_{i=1,3} \gamma_i R\left(A_i, \alpha_i\right), \tag{7}$$

where $R(A_i, \alpha_i)$ is given analytically by $((i, jk)$ being a circular permutation of $(1, 2, 3))$

$$R\left(A_i, \alpha_i\right) = \int_{\alpha_i} \frac{1}{\|A_i - y\|} dy = \arg\sinh \frac{\|B_i A_k\|}{\gamma_i} - \arg\sinh \frac{\|B_i A_j\|}{\gamma_i}. \tag{8}$$

Results for the 3-D Helmholtz equation with piecewise constant density have been obtained for all pairs of panels. Integral $\mathcal{L}$ (see formula (3)) can be reduced to a linear combination of 1-D or 2-D integrals when triangles have at least one common vertex; the resulting integrals have to be evaluated numerically but the final integrands are simple and regular on the domain of integration. For example, when $T = S$ and with piecewise constant basis functions, one has:

$$\mathcal{L} = \int_{S \times S} \frac{e^{ik\|x-y\|}}{\|x - y\|} \, dx \, dy = 4|S| \sum_{i=1}^{3} \gamma_i \int_{\alpha_i} f\left(\|A_i - y\|\right) ds_y \tag{9}$$

where $f(r) = i \dfrac{e^{ikr} - 1 - ikr + k^2 r^2/2}{k^3 r^4}$.

The extension to linear basis functions is in progress. Our method works also for 3-D Maxwell's equations with linear edge basis functions (for MFIE and EFIE). Despite some (possibly) lengthy calculations, the principle is rather straightforward and the method is quite flexible, leading to the reduction of 4-D integrals to a linear combination 1-D regular integrals which can be numerically or even explicitly evaluated. It is possible to use our method for Collocation technique, 2-D BEM and volume integral equations. A high degree of accuracy can be obtained, even in the case of nearly singular integrals. We will present the method and some results for 3-D Helmholtz equation.

6.2.2. *Integral equations for modelling eddy current non destructive testing experiments*
Participants: Marc Bonnet, Audrey Vigneron.

This work in collaboration with E. Demaldent (CEA LIST) is concerned with developing boundary element solvers for modelling eddy current non destructive testing experiments, taking into account the probe, the probed part and the surrounding air. Attention is focused in implementing Galerkin-type formulations, overcoming ill-conditioning arising in configurations involving high contrasts, and fast solvers. Among several possible integral formulations based on either Maxwell's equations or the eddy-current model, a weighted coupled formulation using a loop-tree decomposition of the trial and test spaces was found to perform adequately over the whole range of values of physical parameters typical of eddy-current NDT experiments.

6.2.3. *Elastodynamic fast multipole method for semi-infinite domains.*
Participants: Marc Bonnet, Stéphanie Chaillat.

The use of the elastodynamic half-space Green's tensor in the FM-BEM is a very promising avenue for enhancing the computational performances of 3D BEM applied to analyses arising from e.g. soil-structure interaction or seismology. This work is concerned with a formulation and computation algorithm for the elastodynamic Green's tensor for the traction-free half-space allowing its use within a Fast Multipole Boundary Element Method (FM-BEM). Due to the implicit satisfaction of the traction-free boundary condition achieved by the Green's tensor, discretization of (parts of) the free surface is no longer required. Unlike the full-space fundamental solution, the elastodynamic half-space Green's tensor cannot be expressed in terms of usual kernels such as e^{ikr}/r or $1/r$. Its multipole expansion thus cannot be deduced from known expansions, and is formulated in this work using a spatial two-dimensional Fourier transform approach. The latter achieves the separation of variables which is required by the FMM. To address the critical need of an efficient quadrature for the 2D Fourier integral, whose singular and oscillatory character precludes using usual (e.g. Gaussian) rules, generalized Gaussian quadrature rules have been used instead. The latter were generated by tailoring for the present needs the methodology of Rokhlin's group. Extensive numerical tests have been conducted to demonstrate the accuracy and numerical efficiency of the proposed FMM. In particular, a complexity significantly lower than that of the non-multipole version was shown to be achieved. A full FM-BEM based on the proposed acceleration method for the half-space Green's tensor is currently under way. This treatment of the Green's tensor will be extended to other cases, e.g. layered semi-infinite media.

6.2.4. *Domain decomposition methods for time harmonic wave propagation*
Participants: Patrick Joly, Mathieu Lecouvez.

This work is motivated by a collaboration with the CEA-CESTA (B. Stupfel) through the PhD thesis of M. Lecouvez and is the object of a collaboration with F. Collino, co-advisor of the thesis with P. Joly.

We have considered first the case of the scalar Helmholtz equation for which we have developed a non overlapping iterative domain decomposition method based on the use of Robin type transmission conditions, in the spirit of previous works in the 90's by Collino, Desprès, and Joly.

The novelty of our approach consists in using new transmission conditions using some specific impedance operators in order to improve the convergence properties of the method (with respect to more standard Robin conditions). Provided that such operators have appropriate functional analytic properties, the theory shows that one achieves geometric convergence (in opposition the the slow algebraic convergence obtained with standard methods). These properties prevent the use of local impedance operator, a choice that was commonly done for the quest of optimized transmission conditions (following for instance the works of Gander, Japhet, Nataf). We propose a solution that uses nonlocal integral operators using appropriate Riesz potentials, the important feature of which being their singularity at the origin. To overcome the disadvantage of dealing with completely nonlocal operators, we suggest to work with truncated kernels, involving adequate smooth cut-off function. The results we have obtained are

- A complete theoretical justification of the exponential convergence of the algorithm in the 2D case for smooth enough interfaces. The extension to 3D is in progress : the case of a spherical interface is in particular completely understood.
- An heuristic analysis of the influence of the truncation procedure (several choices are possible) on the convergence of the method, together with a (semi-analytical) search for optimal values of the parameters involved in the method to improve the convergence rate.
- The implementation of the method in 2D and an intensive campaign of numerical validation of the method that appear to provide very good performance and seem to indicate that the method is quite robust with respect to increasing frequency (which remains to be proven). Let us however mention that, not so important but unexpected phenomena, due to space discretization, have been observed and remain to be explained. The implementation in 3D, in cooperation with M. Duruflé, is in progress.

The relevant application at CESTA being electromagnetism, the extension of the method to 3D Maxwell's equations, which proposes new non trivial difficulties, has been initiated.

As the development and the theoretical understanding of these new domain decomposition methods clearly exceed the content of one single thesis, we have proposed an ANR project on this topic, in collaboration with X. Claeys (Paris VI).

6.2.5. *Time harmonic aeroacoustics*

Participant: Jean-François Mercier.

This subject is treated in collaboration with Florence Millot (Cerfacs). We are still working on the numerical simulation of the acoustic radiation and scattering in presence of a mean flow. Up to now we have considered Galbrun's equation, but for 3D configurations it requires to introduce many unknowns. Therefore we focus now on the alternative model of Goldstein's equations. When the fluid flow and the source are potential, the acoustic perturbations are also potential and the velocity potential φ satisfy a simple scalar model. For a general flow, this model is slightly modified and is called Goldstein's equations. A new vectorial unknown ξ is introduced, satisfying a transport equation coupled to the velocity potential. φ satisfies the same modified Helmholtz's equation than in the potential flow case, in which ξ plays the role of a source term. The advantage of Goldstein's formulation compared to Galbrun's model is that the vectorial unknown vanishes in the areas where the flow is potential.

For a general flow ξ can be expressed versus φ as a convolution formula along the flow streamlines. The situation is much simpler for slow flows since the convolution formula can be simplified and the link between ξ and φ becomes explicit. Then Goldstein's equations can be solved by using continuous finite element (discontinuous elements must be used in the general case). We have proved theoretically that when replacing the general convolution formula by the "slow flow" approximation, the error on the velocity potential is small, bounded by the square of the flow velocity. This has been done for a simpler case, a shear flow, for which the streamlines are just parallel lines. Numerical tests have confirmed the square law for the error.

6.2.6. *Mathematical and numerical analysis of metamaterials*

Participants: Patrick Joly, Anne-Sophie Bonnet-Ben Dhia, Patrick Ciarlet, Sonia Fliss, Camille Carvalho, Valentin Vinoles, Christian Stohrer.

Metamaterials are artificial composite materials having the extraordinary electromagnetic property of negative permittivity and permeability at some frequencies . Both of sign-changing coefficients and high contrast homogenization raise new mathematical and numerical challenges. The ANR METAMATH is devoted to the study of those problems. We perform analysis both in time domain (see sections 6.1.3 and 6.3.2) and harmonic domain.

6.2.6.1. *Time-harmonic transmission problems involving metamaterials*

A special interest is devoted to the transmission of an electromagnetic wave between two media with opposite sign dielectric and/or magnetic constants. As a matter of fact, applied mathematicians have to address challenging issues, both from the theoretical and the discretization points of view. In particular, it can happen that the problem is not well-posed in the classical frameworks (H^1 for the scalar case, $H(curl)$ for the vector case). During 2013, we addressed the issues below.

The numerical analysis of the well-posed scalar eigenproblem discretized with a classical, H^1 conforming, finite element method, for arbitrarily shaped interfaces can be carried out with the help of T-coercivity. This work complements the paper Chesnel-Ciarlet, published in Numerische Mathematik, which handled simpler interface configurations (see also §6.2.6.2).

As a second topic, we investigated the case of a scattering problem with a 2D corner interface which can be ill-posed (in the classical H^1 framework). When this is the case, the part of the solution which does not belong to H^1 can be described as a wave which takes an infinite time to reach the corner: this "black-hole" phenomenon is observed in other situations (elastic wedges for example). We have proposed a numerical approach to approximate the solution which consists in adding some Perfectly Matched Layers in the neighbourhood of the corner. As an alternate choice, a T-coercivity approach is also being currently developed to solve the discrete problem.

Last, we studied the transmission problem in a purely 3D electromagnetic setting from a theoretical point of view. We proved that the Maxwell problem is well-posed if and only if the two associated scalar problems (with Dirichlet and Neumann boundary conditions) are well-posed. Numerical analysis of the discretized models (edge elements) is under way.

L. Chesnel left our project in March 2013 after he completed his PhD thesis on these topics. He is currently a post-doc fellow at Aalto University (Finland).

6.2.6.2. Modeling of plasmonic devices

Plasmonic surface waves occur at the interface between the vacuum (or a dielectric) and a metal, at optical frequencies, when the dielectric permittivity ε of the metal has a small imaginary part and a large negative real part. Neglecting the dissipation effects, we have to study electromagnetic problems with a sign-changing ε. An in-depth analysis has been done by Lucas Chesnel during his PhD. In the context of the PhD of Camille Carvalho, we extended the results obtained previously by Lucas Chesnel to more realistic configurations. First, we studied the diffraction of a transversely polarized plane wave by a cylindrical metallic inclusion, when the section of the inclusion presents edges (cf. §6.2.6.1). Then, we considered a related spectral problem in view of studying plasmonic guided waves. The spectral theory is far from obvious. In particular, we have to introduce a non-selfadjoint formulation which provides physical real eigenvalues and complex spurious ones. For both the diffraction problem and the spectral problem, a MATLAB code has been developed, where Perfectly Matched Layers are introduced at the corners to take into account the presence of black-hole waves seemingly absorbed by the corners. The convergence of the finite element discretization (including convergence of the eigenvalues) has been proved (see §6.2.6.1).

6.2.6.3. Study of metamaterials via numerical homogenization

Recently, we have started to study the numerical approximation of the full models, using the HMM (Heterogeneous Multiscale Method). Recall that the full model is obtained via periodization of a local model that includes slow and fast variations. With this HMM approach, computations are carried out on a global mesh, whereas the action of the test-functions is computed at a local level to take into account the fast variations. As a first step, we have begun by the application of HMM for the time-harmonic scalar problem. The case of uniformly bounded coefficients has been addressed. The more general case of non-uniformly bounded coefficients, also called the high-contrast case, is now under scrutiny. It is hoped that one can recover some extra-ordinary properties of the metamaterials with this latter case.

C. Stohrer arrived as a post-doc fellow this fall.

6.3. Absorbing boundary conditions and absorbing layers

6.3.1. *New transparent boundary conditions for time harmonic acoustic and elastic problems in anisotropic media*

Participants: Anne-Sophie Bonnet-Ben Dhia, Antoine Tonnoir, Sonia Fliss.

This topic is developed in collaboration with Vahan Baronian (CEA). Non destructive testing (NDT) is a common method to check the quality of structures and is widely used in industrial applications. Typically, in aircraft design, it is required to control structures like plates. Efficient and accurate numerical methods are required to simulate NDT experiments.

In our case, we want to study the diffraction of a time harmonic wave by a bounded defect in an infinite anisotropic elastic plate. The difficulty is to find a way to restrict the finite element computation to a small box containing the defect. Indeed classical methods such as the perfectly matched layers fail when the medium is anisotropic.

Up to now we considered the simpler case of an infinite dissipative 2D medium.

Our idea, inspired by the work of Sonia Fliss and Patrick Joly for periodic media, is to consider five domains recovering the whole plane:

- a square that surrounds the defect in which we have a finite element representation of the solution,
- and four half-spaces parallel to the four edges of the square, in which we can give an analytical representation of the solution thanks to the Fourier transform.

The different unknowns are coupled by well-chosen transmission relations which ensure the compatibility between the five representations.

The method has been validated successfully in the case of anisotropic acoustic media and the implementation for the case of elasticity is in progress. The mathematical properties of the formulation and the efficiency of the method strongly depend on the presence or not of overlaps between the finite element box and the four half-planes. The formulation with overlaps has good Fredholm properties but the well-posedness for all frequencies is proved only for the formulation without overlaps.

6.3.2. Perfectly Matched Layers in negative index metamaterials

Participants: Patrick Joly, Eliane Bécache, Valentin Vinoles.

The simulation of waves in unbounded domains requires methods to artificially truncate the computational domain. One of the most popular ones to do so is the Perfectly Matched Layers (PMLs) which are effective and stable for non dispersive isotropic media. For non dispersive anisotropic media, we established a necessary stability condition in 2004 : PMLs are unstable in presence of so called-backward waves.

We are interested here in dispersive media and more specifically in Negative Index Metamaterials (NIMs), also called left-handed media. Those media have negative permittivity and permeability at some frequencies due to microscopic resonating structures. Since the 1990s, NIMs are the subject of active researches due to their promising applications : superlens, cloaking, improved antenna, etc.

In a first step, we consider a simple model of NIMs : the Drude model. For this model, a plane wave analysis shows the simultaneous presence of both forward and backward waves and numerical simulations confirm the instability of standard PMLs (cf figure 2) that result from complex changes of variable leading to the following modification of the spatial derivatives

$$\partial_x \longrightarrow \left(1 + \frac{\sigma_x(x)}{i\omega}\right)^{-1} \partial_x \quad \text{and} \quad \partial_y \longrightarrow \left(1 + \frac{\sigma_y(y)}{i\omega}\right)^{-1} \partial_y$$

where $\sigma_x(x) > 0$ and $\sigma_y(y) > 0$ are the damping terms. Inspired by works of the physics community, we propose more general changes of variable

$$\partial_x \longrightarrow \left(1 + \frac{\sigma_x(x)}{i\omega\psi(\omega)}\right)^{-1} \partial_x \quad \text{and} \quad \partial_y \longrightarrow \left(1 + \frac{\sigma_y(y)}{i\omega\psi(\omega)}\right)^{-1} \partial_y$$

where $\psi(\omega)$ is a function to be chosen judiciously. We have generalised the previous necessary stability condition for those new PMLs, called Stabilized Perfectly Matched Layers, for dispersive media. This analysis allows us to understand the instabilities observed for standard PMLs in NIMs and to propose a choice of functions $\psi(\omega)$ which take into account the backward waves and stabilize the PMLs as confirmed by numerical simulations (cf figure 2).

6.3.3. Perfectly Matched Layers in plasmas

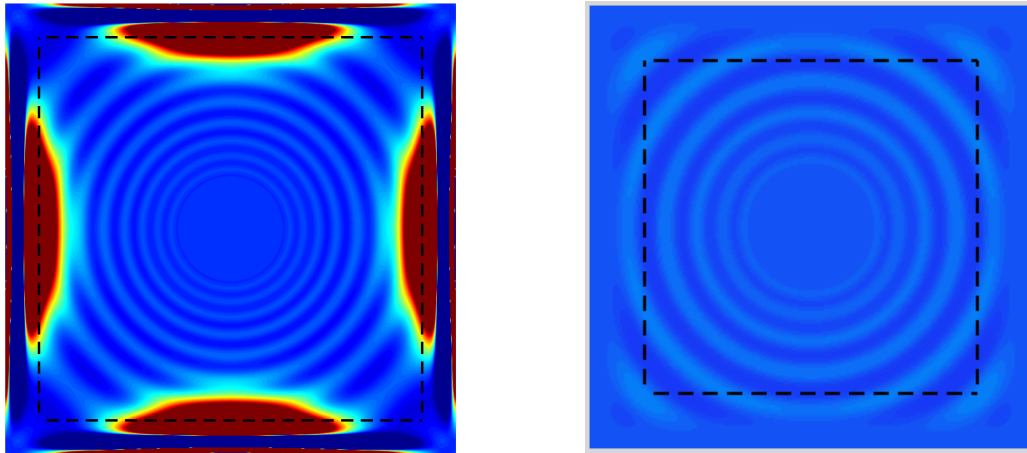**Participants:** Patrick Joly, Eliane Bécache, Valentin Vinoles.

Figure 2. Left : the standard PMLs are unstable. Right : the Stabilized PMLs are stable.

This work was done during the internship of Guillaume Chicaud in the framework of the ANR CHROME which concerns the study of electromagnetic wave propagation in plasmas. Our aim is to develop efficient and robust codes to simulate wave propagation in unbounded plasmas models. The simulation of waves in plasmas requires technics to bound the computational domain. As plasmas are dispersive media where backward waves may occur, the difficulties to construct stable PMLs are analogous to the ones encountered for Negative Index Metamaterials (cf 6.3.2). This work is a preliminary study of this topics, in a simplified model, the case of a 2D anisotropic uniaxial plasma. It consists first in analyzing the presence of backward waves with a plane wave analysis. The second step was to implement the equations using standard PMLs and to confirm the expected instabilities. Finally, we proposed stabilized PMLs (SPMLs), inspired by the work done in metamaterials (see section 6.3.2).

The continuation of this project will constitute the subject of the post-doc of Maryna Kachanovska.

6.4. Waveguides, resonances, and scattering theory

6.4.1. *An improved modal method in non uniform acoustic waveguides*
Participant: Jean-François Mercier.

This topic is developed in collaboration with Agnès Maurel (Langevin Institute ESPCI).
We develop modal methods to study the scattering of an acoustic wave in a non uniform waveguide. Usual modal approaches are efficient only when a rather large number of evanescent modes are taken into account. An improved representation has been proposed in which an additional transverse mode and an additional unknown modal component are introduced. This so called boundary mode helps to better satisfy the Neumann boundary conditions at the varying walls. A system of coupled ordinary differential equations is obtained and is found to remain coupled in the straight part of the waveguide which implies that the classical radiation condition cannot be applied directly at the inlet/outlet of the scattering region.

We revisit the coupled mode equations in order to derive an improved system, in which the additional mode can be identified as evanescent mode, and then adapted to define radiation conditions. This makes possible the implementation of efficient numerical multimodal methods (like the admittance matrix method) and also approximate solutions can be found using the Born approximation. The numerical tests have shown that our method is very efficient to reduce the number of degree of freedom: adding to the boundary mode, it is sufficient to take only the propagative modes to get very good results. This is in particular interesting at

low frequency when only the plane mode propagates. In the low frequency regime, the system can be solved analytically, using the Born approximation, leading to improved approximate equations compared to the usual Webster's approximation.

6.4.2. *Construction of non scattering perturbations in a waveguide*

Participants: Anne-Sophie Bonnet-Ben Dhia, Eric Lunéville.

This work is done in collaboration with Sergei Nazarov from Saint-Petersbourg University and during the internship of Yves Mbeutcha. We consider a two-dimensional homogeneous acoustic waveguide and we aim at designing deformations of the boundary which are invisible at a given frequency (or more generally at a finite number of given frequencies) in the sense that they are non scattering. To find such invisible perturbations, we take advantage of the fact that there are only a finite number of propagative modes at a given frequency in a waveguide. As a consequence, the invisibility is achieved by canceling a finite number of scattering coefficients, and an invisible deformation only produces an exponentially decreasing scattered field, not measurable in the far field.

The first step consists in studying the effect of a small deformation, of amplitude ε. The asymptotic analysis allows to derive the first order terms of the scattering coefficients, as integrals involving the function describing the deformation. This leads to express the deformation as a linear combination of some explicit (compactly supported) functions, so that invisibility is satisfied if and only if the coefficients of the linear combination are solution of a fixed point equation. The key point is that we can prove, using the results of the asymptotic analysis, that the function of this fixed point equation is a contraction for ε small enough. This proves the existence of invisible deformations of amplitude ε. Moreover, it provides a natural algorithm to compute the invisible deformation.

This has been tested numerically and the results are in perfect agreement with the theory. At low frequency, the good news is that ε can be taken quite large (the amplitude of the deformation may be half the size of the guide). But this deteriorates when the frequency increases.

6.4.3. *Localized modes in unbounded perturbed periodic media*

Participants: Patrick Joly, Sonia Fliss, Elizaveta Vasilevskaya.

This topic is investigated in collaboration with Bérangère Delourme (Univ. Paris XIII) and constitutes the subject of the E. Vasilevskaya's PhD thesis. We are interested in a 2D propagation medium which is a localized perturbation of a reference homogeneous periodic reference medium. This reference medium is a "thick graph", namely a thin structure (the thinness being characterized by the parameter $\delta > 0$) whose limit when δ tends to 0 is a periodic graph. This is for instance the case of the thick periodic ladder and the thick periodic rectangular grid of figure. The perturbation consists in changing only the geometry (and not the material properties) of the reference medium by modifying the thickness of one of the lines of the reference medium as illustrated by figure with the perturbed ladder and perturbed grid (see figure 3). The question we investigate is whether such a geometrical perturbation is able to produce localized eigenmodes (for the ladder) or guided modes (for the grid). We have investigated this question when the propagation model is the scalar Helmholtz equation with Neumann boundary conditions (in opposition to Dirichlet conditions that have been more studied in the literature - see the works by S. Nazarov for instance). This amounts to solving an eigenvalue problem for the Laplace operator in an unbounded domain : the associated self-adjoint operator has a continuous spectrum with a band gap structure and the eigenvalues are searched in the gaps.

With Neumann boundary conditions, we can use for the theoretical study an asymptotic analysis with respect to δ: indeed, it is well known (see in particular the works by Exner, Kuchment, Post) the limit model when δ tends to 0, is the Helmholtz equation on the graph : 1D Helmholtz equations on each branch completed by continuity and Kirchoff transmission conditions at each node. The geometrical perturbation of the original medium results into a perturbation of the Kirchoff conditions on the nodes of the modified line. The spectral analysis of the limit problem can be done completely by hand and the existence of eigenmodes for the thick medium is ensured, for δ small enough, by the existence of corresponding eigenmodes for a limit "1D operator"
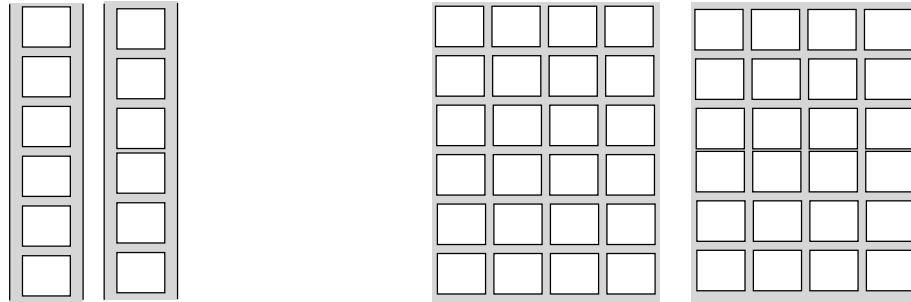
Figure 3. Left : periodic ladder (non perturbed/perturbed). Right : periodic "thick graph" (non perturbed/perturbed). The propagation domain is in grey.

whose spectrum appears to possess an infinity of band gaps in each of which eigenvalues can exist, due to the perturbation. Following this idea, we have been able to prove the existence of localized modes in the case of the ladder provided that the geometrical perturbation consists in diminishing the width of one rung. One can even prove that one can produce more and more localized modes, corresponding to larger and larger frequencies, when δ is smaller and smaller. On the contrary, we conjecture that there is no localized modes when we enlarge the rung. The extension of these results to the existence of guided modes in the case of the grid in progress.

For the numerical computation of such localized modes, we have adapted the DtN approach discussed in the activity report of 2012. We gave in figure 4 an example of computed localized mode in the case of the ladder : this mode is geometrically confined at the neighbourhood of the modified rung.

6.5. Asymptotic methods and approximate models

6.5.1. *Homogenization and interfaces*
Participants: Sonia Fliss, Valentin Vinoles.

This topic is developed in collaboration with Xavier Claeys (LJLL, Paris VI).

The mathematical modelling of electromagnetic metamaterials and the homogenization theory are intimately related because metamaterials are precisely constructed by a periodic assembly of small resonating micro-structures involving dielectric materials presenting a high contrast with respect to a reference medium. In the framework of the ANR Metamath (see 6.2.6), we wish to look carefully at the treatment of boundaries and interfaces that are generally poorly taken into account by the first order homogenization.

This question is already relevant for standard homogenization (ie without high contrast). Indeed, the presence of a boundary induces a loss of accuracy due to the inadequateness of the standard homogenization approach to take into account boundary layer effects. Our objective is to construct approximate effective boundary conditions that would restore the desired accuracy.

We first considered a plane interface between a homogeneous and the periodic media in the standard case without high-contrast. We obtained high order transmission conditions between the homogeneous media and the periodic media. The technique we used involves matched asymptotic expansions combined with standard homogenization ansatz. Those conditions are non standard : they involve Laplace-Beltrami operators at the interface and requires to solve cell problems in infinite periodic waveguides. The derivation of the corresponding error estimates is in progress. The analysis is based on a original combination of Floquet-Bloch and a periodic version of Kondratiev technique.
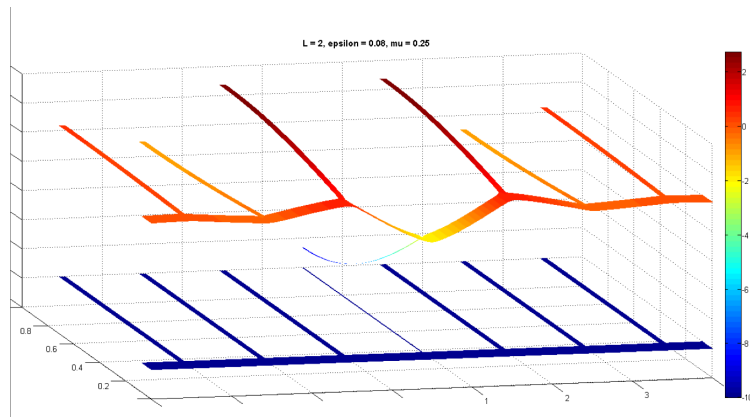
Figure 4. Localized mode in the perturbed ladder.

The next step will be to consider the same problem but with a high-contrast periodic media in collaboration with Guy Bouchitté, a french expert in high contrast homogenization.

6.5.2. *Effective boundary conditions for thin periodic coatings*
Participants: Mathieu Chamaillard, Patrick Joly.

This topic is the object of a collaboration with Houssem Haddar (CMAP École Polytechnique). We are interested in the construction of "equivalent" boundary condition for the diffraction of waves by an obstacle with smooth boundary Γ covered with a thin coating of width δ whose physical characteristics vary "periodically" along Γ with a period proportional to the small parameter δ. For a general boundary Γ, the notion of periodicity is ambiguous: we have chosen to define the coating as the image, or the deformation, by a smooth mapping ψ_Γ of a flat layer of width δ (the reference configuration) that preserves the normals, which appears consistent with a manufacturing process. The electromagnetic parameters in the coating are then defined as the images through ψ_Γ of periodic functions in the reference configuration.

We have first considered the case of the scalar wave equation when the homogeneous Neumann condition is applied on the boundary of the obstacle. Using an asymptotic analysis in δ, which combines homogenization and matched asymptotic expansions, we have been able to establish a second order boundary condition of the form

$$\partial_\nu u + (\delta B_\Gamma^1 + \delta^2 B_\Gamma^2)u = 0,$$

where B_Γ^1 and B_Γ^2 are second order tangential differential operators along Γ. The coefficients of these operators depend on both the geometrical characteristics of Γ (through the curvature tensor), the deformation mapping ψ_Γ and the material properties of the coating, through the resolution of particular unbounded cell problems in the flat reference configuration. When the coating is homogeneous, we have checked that one recovers the well known second order thin layer condition. We have moreover proven that this approximate condition provides in $\mathcal{O}(\delta^3)$.

6.5.3. *Thin Layers in Isotropic Elastodynamics*
Participants: Marc Bonnet, Aliénor Burel, Patrick Joly.

This research is concerned with the numerical modelling of non-destructive testing experiments using ultrasonic waves. Some materials, e.g. composite materials, involve thin layers of resin. The numerical modelling of such thin layers can be problematic as they result in very small spatial mesh sizes. To alleviate this difficulty, we develop an approach based on an asymptotic analysis with respect to the layer thickness ε, aiming to model the thin layer by approximate effective transmission conditions (ETCs), which remove the need to mesh the layer. So far, ETCs that are second-order accurate in ε have been formulated, justified, implemented and numerically validated, for 2-D and 3-D configurations involving planar interfaces of constant thickness. In particular, the continuous evolution problem is shown to be stable, and a time-stepping scheme that essentially preserves the stability requirement on the time step is proposed. Extension of this work to 2-D and 3-D configurations involving a curved layer is ongoing.

6.5.4. *Mathematical modelling of electromagnetic wave propagation in electric networks.*
Participants: Geoffrey Beck, Patrick Joly.

This topic is developed in collaboration with S. Imperiale (Inria Saclay) in the framework of the ANR project SODDA, in collaboration with CEA-LETI, about the non destructive testing of electric networks. This is the subject of the PhD thesis of G. Beck.

We investigate the question of the electromagnetic propagation in thin electric cables from a mathematical point of view via an asymptotic analysis with respect to the (small) transverse dimension of the cable: as it has been done in the past in mechanics for the beam theory from 3D elasticity, we use such an approach for deriving simplified effective 1D models from 3D Maxwell's equations.

During last year, we have achieved some progress in various directions:

- Single wire coaxial cables. This is the direct continuation of what has been done last year. Concerning the lowest order, the telegraphist's model, we have extended the error analysis, previously restricted to non lossy cylindrical cables to very general cases. Technically, this relies on time Laplace transform and new, parameter dependent, Poincaré-Friedrichs inequalities. From the numerical point of view, in collaboration with M. Duruflé, we have initiated a quantitative comparison between the full 3D model and our 1D model. Furthermore we have derived and studied a higher order generalized telegraphist's equation that include dispersive effects through nonlocal capacity and inductance operators. The corresponding mathematical analysis is in progress.

- Multiple wires cables. The objective here was to extend our approach to cables containing N conducting wires. Our results into a vectorial generalized telegraphist's model with $2N$ (2 for each wire) 1D unknowns, N electrical potentials and N currents. This model involves in particular a capacity matrix C, an inductance matrix L, a resistance matrix R and a conductance matrix G, whose properties have been deeply investigated, which allowed us to justify rigorously and extend some results from the electrical engineering literature. In the most general case, the effective models also involve time memory terms with matrix valued convolution kernels.

- Junction of cables. This is a new and essential step towards the modelling of networks. We have started the case of junctions of single wire cables via the method of matched asymptotic expansions in the spirit of the PhD thesis of A. Semin.

6.5.5. *Elastic wave propagation in strongly heterogeneous media*

Participants: Simon Marmorat, Patrick Joly.

This subject enters our long term collaboration with CEA-LIST on the development of numerical methods for time-domain non destructive testing experiments using ultra-sounds, and is realized in collaboration with Xavier Claeys (LJLL, Paris VI). We aim at developing an efficient numerical approach to simulate the propagation of waves in a medium made of many small heterogeneities, embedded in a smooth (or piecewise smooth) background medium, without any particular assumption (such as periodicity) on the spatial distribution of these heterogeneities. The figure 5 is a snapshot of a simulation inside such a medium, computed thanks to classical simulation tools: to reach satisfying accuracy, one has to use mesh refinement in the vicinity of the heterogeneities, which greatly increases the computational cost of the method.

By considering the medium with defects as a perturbation of the smooth one, we have derived an auxiliary model in the acoustic case, involving the defect-free wave operator and some volume Lagrange multipliers which account for the presence of the defects. These Lagrange multipliers are unknown functions defined on the defects and live in some infinite dimensional functional space. Exploiting the smallness of the defects, we have shown thanks to matched asymptotic analysis that the aforementioned functional space may be well described by a finite number N of profile functions: we propose an asymptotic model by looking for the Lagrange multipliers into the space spanned by these N profile functions, and we have shown that the error hence made is controlled by ε^N, ε being the characteristic size of the defects, assumed to be small.

On a computational point of view, the asymptotic model is much easier to solve than the original one since it can be discretized using a computation mesh that ignores the presence of the heterogeneities, the Lagrangian multipliers being computed by solving a linear system of size N. A resolution of this model has been implemented in the 1D and in the 2D case, and a rigorous error estimate has been established.

6.6. Imaging and inverse problems

6.6.1. *Sampling methods in waveguides*

Participants: Laurent Bourgeois, Sonia Fliss, Eric Lunéville, Anne-Claire Egloffe.
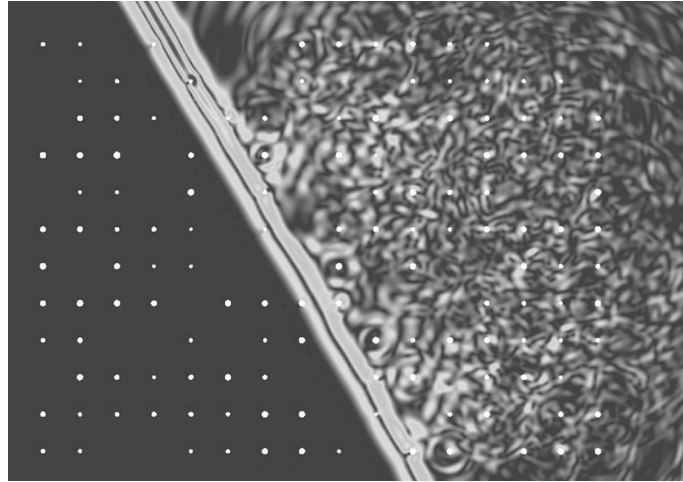
Figure 5. Snapshot of a simulation in the medium of interest, using high-order finite element method as well as local mesh refinement and local time stepping around the heterogeneities.

First, we have adapted the modal formulation of sampling methods (Linear Sampling Method and Factorization Method) to the case of a periodic waveguide in the acoustic case. This study is based on the analysis of the far field of scattering solutions in cylindrical waveguides, in particular for the fundamental solution, which enables us to obtain a far field formulation of sampling methods, and then a modal formulation of such methods. The aim of the inverse problem is to retrieve a defect (that is a loss of periodicity) from the scattered fields which correspond to the incident fields formed by the Floquet modes. Some convincing numerical experiments have shown the feasibility of the method. Secondly, going back to the homogeneous waveguide in the acoustic case, we have started a study of the sampling methods in a more realistic situation, that is the data (emission and reception) are measured on the boundary of the waveguide in the time domain. This was the subject of Anne-Claire Egloffe's post-doc. The aim is to use the modal formulation of the sampling methods at all frequencies and recompose the best possible image of the defect. Some first encouraging results have been obtained when the spectrum of the incident signal is centered at a rather low frequency (corresponding to 3 propagating guided modes).

6.6.2. *Space-time focusing on unknown scatterers*
Participants: Maxence Cassier, Patrick Joly, Christophe Hazard.

This topic concerns the studies about time-reversal in the context of Maxence Cassier's thesis. We are motivated by the following challenging question: in a propagative medium which contains several unknown scatterers, how can one generate a wave that focuses selectively on one scatterer not only in space, but also in time? In other words, we look for a wave that 'hits hard at the right spot'. Such focusing properties have been studied in the frequency domain in the context of the DORT method ("Decomposition of the Time Reversal Operator"). In short, an array of transducers first emits an incident wave which propagates in the medium. This wave interacts with the scatterers and the transducers measure the scattered field. The DORT method consists in doing a Singular Value Decomposition (SVD) of the scattering operator, that is, the operator which maps the input signals sent to the transducers to the measure of the scattered wave. It is now well understood that for small and distant enough scatterers, each singular vector associated with a non zero singular value generates a wave which focuses selectively on one scatterer. Can we take advantage of these spatial focusing properties in the frequency domain to find the input signals which generate a time-dependent wave which would be also focused in time? Since any frequency superposition of a family of singular vectors associated

with a given scatterer leads to a spatial focusing, the main question is to synchronize them by a proper choice of their phases. The method we propose is based on a particular SVD of the scattering operator related to its symmetry. The signals we obtain do not require the knowledge of the locations of the scatterers. We compare it with some "optimal" signals which require this knowledge. Our study is illustrated by a two dimensional acoustic model where both scatterers and transducers are assumed pointlike (see figure 6).
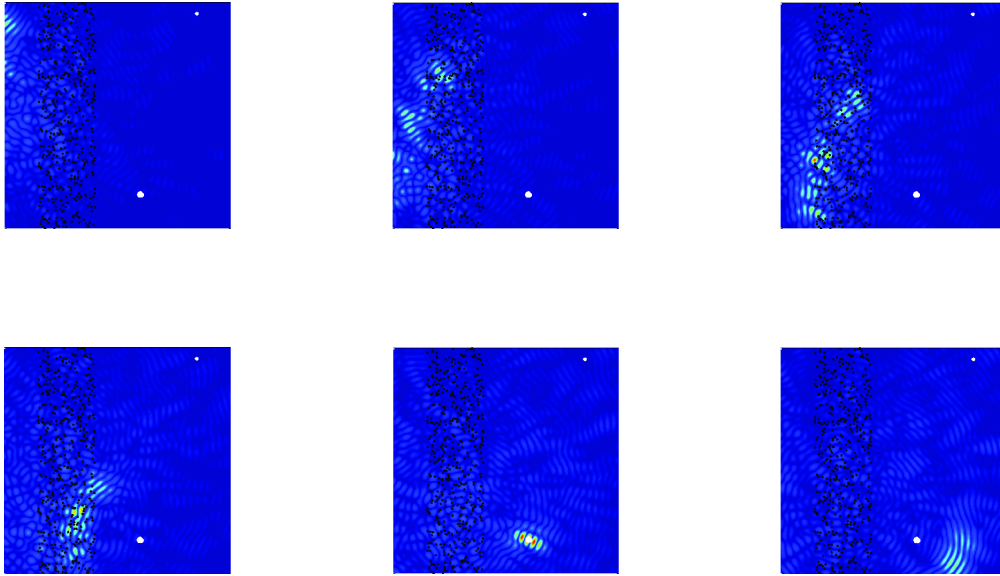

Figure 6. The case of a scattering reference medium perturbed by two obstacles (the white circles) : modulus of the field generated by 128 transducers (left edge of each figure) at different times.

6.6.3. *The exterior approach to retrieve obstacles*

Participant: Laurent Bourgeois.

This theme is a collaboration with Jérémi Dardé from IMT (Toulouse). The aim is to find a fixed Dirichlet obstacle in a bounded domain by using some redundant boundary conditions (Cauchy data) on the accessible part of the boundary, while the boundary conditions are unknown on the inaccessible part of the boundary. We wish to adapt the exterior approach developed for the Laplace equation and the Stokes system to the case of time evolution problems, in particular the heat equation. The exterior approach consists in defining a decreasing sequence of domains that converge in some sense to the obstacle. More precisely, such iterative approach is based on a combination of a quasi-reversibility method to update the solution of the ill-posed Cauchy problem outside the obstacle obtained at previous iteration and of a level set method to update the obstacle with the help of the solution obtained at previous iteration. We have already introduced two different mixed formulations of quasi-reversibility for the ill-posed heat equation with lateral Cauchy data in order to use standard Lagrange finite elements.

6.6.4. *Uniqueness and stability of inverse problems*

Participant: Laurent Bourgeois.

In collaboration with Laurent Baratchart and Juliette Leblond from APICS (Nice), we have proved uniqueness for the inverse Robin problem with a boundary coefficient in L^∞ in the 2D case, for the Laplace equation in the divergence form. The result is based on complex analysis. We have also established an abstract Lipschitz

stability result for inverse problems such that the set of parameters is a compact and convex subset of a finite dimensional space. In particular, such result can be applied to the previous inverse Robin problem.

6.6.5. *Interior transmission problem*
Participant: Anne-Sophie Bonnet-Ben Dhia.

This work is in collaboration with Lucas Chesnel (Aalto University, Finland). During this year, we investigated a two-dimensional interior transmission eigenvalue problem for an inclusion made of a composite material. This problem plays a central role in the theory of the corresponding inverse problem. We considered configurations where the difference between the parameters of the composite material and the ones of the background change sign on the boundary of the inclusion. In a first step, under some assumptions on the parameters, we extended the variational approach of the T-coercivity to prove that the transmission eigenvalues form at most a discrete set. In the process, we also provided localization results. Then, we study what happens when these assumptions are not satisfied. The main idea is that, due to very strong singularities that can occur at the boundary, the problem may lose Fredholmness in the natural H^1 framework. Using Kondratiev theory, we proposed a new functional framework where the Fredholm property is restored.

6.6.6. *Flaw identification using elastodynamic topological derivative or transmission eigenvalues*
Participants: Marc Bonnet, Rémi Cornaggia.

This work is in collaboration with C. Bellis (LMA, CNRS, Marseille), F. Cakoni (Univ. of Delaware, USA) and B. Guzina (Univ. of Minnesota, USA). The concept of topological derivative (TD) quantifies the perturbation induced to a given cost functional by the nucleation of an infinitesimal flaw in a reference defect-free body, and may serve as a flaw indicator function. In this work, the TD is derived for three-dimensional crack identification exploiting over-determined transient elastodynamic boundary data. This entails in particular the derivation of the relevant polarization tensor, here given for infinitesimal trial cracks in homogeneous or bi-material elastic bodies. Simple and efficient adjoint-state based formulations are used for computational efficiency, allowing to compute the TD field for arbitrarily shaped elastic solids. The latter is then used as an indicator function for the spatial location of the sought crack(s). The heuristic underpinning TD-based identification, which consists in deeming regions where the TD is most negative as the likeliest locations of actual flaws and on formulating higher-order topological expansions in the elastodynamic case, has (with C. Bellis and F. Cakoni) been given a partial justification within the limited framework of acoustic inverse scattering using far-field data. Current investigations (M. Bonnet, R. Cornaggia) include setting up and justifying the formulation of higher-order topological expansions for the elastostatic and elastodynamic cases.

Another ongoing research on a related topic addresses the use of transmission eigenvalues (TEs), i.e. values of the wave number for which the homogeneous interior transmission problem (ITP) related to the scattering scattering of time-harmonic elastic waves by aninhomogeneity D admits non-trivial solutions. This works (R. Cornaggia, in collaboration with C. Bellis, F. Cakoni, B. Guzina) aims on the one hand to understand better how to compute the TEs -if any- in the case where D's characteristics vary periodically. On the other hand it looks for how a previously obtained knowledge of the TE set could be the basis of an identification process. In a preliminary study considering 1-D elastic beams with periodically varying section over a length L, gradient elasticity was found to be a well-suited homogenization model to both compute the TEs and identify L, the periodic cell length and the damage parameter from available values of the TEs.

6.6.7. *Topological derivative in anisotropic elasticity*
Participant: Marc Bonnet.

This work is in collaboration with G. Delgado (PhD student, CMAP Ecole Polytechnique and EADS IW). Following up on previous work on the topological derivative (TD) of displacement-based cost functionals in anisotropic elasticity, a TD formula has been derived and justified for general cost functionals that involve strains (or displacement gradients) rather than displacements. The small-inclusion asymptotics of such cost functionals are quite different than in the previous case, due to the fact that the strain perturbation inside an

elastic inclusion has a finite, nonzero asymptotic value in the limit of a vanishingly small inclusion. Cost functionals of practical interest having this format include von Mises equivalent stress (often used in plasticity or failure criteria) and energy-norm error functionals for coefficient-identification inverse problems. This TD formulation has been tested on 2D and 3D numerical examples, some of them involving anisotropic elasticity and nonquadratic cost functionals.

6.6.8. *Energy functionals for elastic medium reconstruction using transient data*

Participant: Marc Bonnet.

This work is in collaboration with W. Aquino (Duke Univ., USA). Energy-based misfit cost functionals, known in mechanics as error in constitutive relation (ECR) functionals, are known since a long time to be well suited to (electrostatic, elastic,...) medium reconstruction. In this ongoing work, a transient elastodynamic version of this methodology is developed, with emphasis on its applicability to large time-domain finite element modeling of the forward problem. The formulation involves coupled transient forward and adjoint solutions, which greatly hinders large-scale computations. A computational approach combining an iterative treatment of the coupled problem and the adjoint to the discrete Newmark time-stepping scheme is found to perform well on cases where both the FE model and the identification problem are of large size (2D and 3D elastodynamic numerical experiments made so far involve up to half a million unknown for the discretized inverse problem), making the time-domain ECR functional a worthwhile tool for medium identification.

7. Bilateral Contracts and Grants with Industry

7.1. Contract POEMS-CEA-LIST-2

Participant: Anne-Sophie Bonnet-Ben Dhia.

Start : 09/01/2010, End : 07/31/2013. Administrator : ENSTA.
This contract is about the scattering of elastic waves by a stiffener in an anisotropic plate.

7.2. Contract POEMS-CEA-LIST-DIGITEO

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Antoine Tonnoir.

Start : 10/01/2011, End : 09/30/2014. Administrator : ENSTA.
This contract is about the scattering of elastic waves by a local defects in an anisotropic plate. It consists on the funding of Antoine Tonnoir's Phd.

7.3. Contract POEMS-DGA

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Patrick Joly.

Start : 09/01/2011, End : 12/31/2013. Administrator : ENSTA.
This contract is about the waveguide in photonic crystals : we want to develop new mathematical and numerical tools for the characterization, the study and the computation of the guided modes in photonic crystals.

7.4. Contract POEMS-CEA-LIST

Participants: Marc Bonnet, Audrey Vigneron.

Start : 01/01/2013, End : 12/31/2015. Administrator : ENSTA.
This contract is about the modelisation of Eddy current by integral equations.

7.5. Contract POEMS-SHELL

Participants: Stéphanie Chaillat, Patrick Ciarlet, Luca Desiderio.

Start : 10/01/2010, End : 09/31/2016. Administrator : CNRS.

This contract is about fast direct solvers to simulate seismic wave propagation in complex media.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

- ANR project *AEROSON: Simulation numérique du rayonnement sonore dans des géométries complexes en présence d'écoulements réalistes*
  Partners: EADS-IW, CERFACS, Laboratoire d'Acoutique de l'Université du Maine.
  Start : 10/01/2009, End : 02/28/2013. Administrator : CNRS. Coordinator : Jean-François Mercier.

- ANR project *PROCOMEDIA: Propagation d'ondes en milieux complexes*
  Partners: ESPCI, Laboratoire d'Acoutique de l'Université du Maine, Departamento de Fisica de la Universidad de Chile.
  Start : 04/01/2011, End : 03/30/2014. Administrator : CNRS. Coordinator for POEMS : Jean-François Mercier.

- ANR project *METAMATH: modélisation mathématique et numérique pour la propagation des ondes en présence de métamatériaux.* Partners: EPI DEFI (Inria Saclay), IMATH-Université de Toulon, DMIA-ISAE.
  Start : 12/01/2011, End : 11/30/2015. Administrator : Inria. Coordinator : Sonia Fliss.

- ANR project *CHROME: Chauffage , réflectométrie et Ondes pour les plasmas magnétiques)* Partners: Université Pierre et Marie Curie (Paris 6), Université de Lorraine
  Start : 10/01/2012, End : 10/01/2015 Administrator : Inria Coordinator for POEMS: Eliane Bécache

- ANR project *SODDA: Diagnostic de défauts non francs dans les réseaux de câbles* Partners: CEA LIST, ESYCOM, LGEP (Supelec)
  Start : 10/01/2012, End : 10/01/2015 Administrator : Inria Coordinator for Poems: Patrick Joly

- ANR project *RAFFINE: Robustesse, Automatisation et Fiabilité des Formulations INtégrales en propagation d'ondes : Estimateurs a posteriori et adaptivité* Partners: CERFACS, EADS, IMACS, ONERA, Thales
  Start : January 2013. End : december 2016. Administrator : Inria. Coordinator: Marc Bonnet.

- ANR project *ARAMIS: Analyse de méthodes asymptotiques robustes pour la simulation numérique en mécaniques* Partners: Université de Pau, Université technologique de Compiègne
  Start : january 2013. End : december 2016. Administrator : Université de Pau. Participants for POEMS: Marc Bonnet

8.1.2. Competitivity Clusters

- GDR Ultrasons: this GDR, which regroups more than regroup 15 academic and industrial research laboratories in Acoustics and Applied Mathematics working on nondestructive testing. It has been renoveled this year with the participation of Great Britain.

8.2. European Initiatives

8.2.1. FP7 Project : SIMPOSIUM

Title: Simulation Platform for Non Destructive Evaluation of Structures and Materials

Type: COOPERATION (ICT)

Defi: PPP FoF: Digital factories: Manufactoring design and product lifecycle manage

Instrument: Integrated Project (IP)

Duration: September 2011 - August 2014

Coordinator: CEA (Pierre Calmont) (France)

See also: http://cordis.europa.eu/fetch?CALLER=PROJICT&ACTION=D&CAT=PROJ&RCN=99653

8.3. International Research Visitors

8.3.1. Visits of International Scientists

- *Sergei Nazarov*, Professor at the University of Saint-Petersbourg.

9. Dissemination

9.1. Scientific Animation

9.1.1. Membership

- A. S. Bonnet-Ben Dhia is the Head of the Electromagnetism Group at CERFACS (Toulouse)
- A. S. Bonnet-Ben Dhia is in charge of the relations between l'ENSTA and the Master "Dynamique des Structures et des Systèmes Couplés (Responsible : Etienne Balmes)".
- A. S. Bonnet-Ben Dhia chairs the scientific council of the CNRS Institure for Engineering and Systems Sciences (INSIS)
- A. S. Bonnet-Ben Dhia is associate editor of SINUM (SIAM Journal of Numerical Analysis).
- M. Bonnet is associate editor of European Journal of Mechanics A/Solids
- M. Bonnet is associate editor of Engineering Analyses with Boundary Elements
- M. Bonnet is on the editorial board of *Inverse Problems*.
- M. Bonnet is on the editorial board of *Computational Mechanics*.
- P. Ciarlet is an editor of DEA (Differential Equations and Applications)
- P. Ciarlet is an editor of CAMWA (Computers & Mathematics with Applications)
- P. Ciarlet is an editor of ESAIM:M2AN (Mathematical Modeling and Numerical Analysis)
- G. Cohen is a scientific expert of ONERA.
- P. Joly is a member of the scientific committee of CEA-DAM.
- P. Joly is a member of the Scientific Committee of the Seminar in Applied Mathematics of College de France (P. L. Lions).
- P. Joly is an editor of the journal Mathematical Modeling and Numerical Analysis.
- P. Joly is a member of the Book Series Scientific Computing of Springer Verlag.
- M. Lenoir is a member of the Commission de Spécialistes of CNAM.
- M. Lenoir is in charge of Master of Modelling and Simulation at INSTN.
- E. Lunéville is the Head of UMA (Unité de Mathématiques Appliquées) at ENSTA.
- The Project organizes the monthly Seminar Poems (Coordinators: S. Chaillat, S. Marmorat)

9.1.2. Organisation of conferences

- POEMS has co-organized the international conference WAVES 2013 (Tunis june 2013).
- Patrick Joly was co-organizer of the Oberwolfach seminar (Computational Acoustics and Electromagnetism) in February 2013.
- Sonia Fliss organized the CEA-EDF-Inria summer school (Waves in periodic media : mathematical and numerical aspects), ENSTA-Paristech in april 2013.

- Anne-Sophie Bonnet-Ben Dhia co-organized a worshop (Mathematical methods for spectral problems : applications to waveguides, periodic media and metamaterials), Helsinki in Mars 2013.
- Anne-Sophie Bonnet-Ben Dhia organized a worshop on waveguides, ENSTA-Paristech in october 2013.
- Stéphanie Chaillat organized a worshop (Error Estimates and Adaptive Mesh Refinement Strategies for Boundary Elements Methods), ENSTA-Paristech in may 2013.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Éliane Bécache

- *Compléments sur la méthode des éléments finis*, ENSTA Paristech, (2nd year).

Marc Bonnet:

- *Problèmes inverses*, Master TACS (ENS Cachan) and DSMSC (Centrale Paris)
- *Méthodes intégrales*, Master TACS (ENS Cachan)
- *Equations intégrales et multipôles rapides*, Ecole doctorale MODES (Univ. Paris Est, Marne la Vallée)
- *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA Paristech (1st year)

Anne-Sophie Bonnet-Ben Dhia

- *Outils élémentaires d'analyse pour les EDP*, ENSTA Paristech (1st year).
- *Propagation dans les guides d'ondes*, ENSTA Paristech (3rd year).
- *Théorie spectrale des opérateurs autoadjoints et applications aux guides optiques*, ENSTA Paristech (2nd year).
- *Propagation des ondes*, Ecole Centrale de Paris (M2).

Laurent Bourgeois

- *Outils élémentaires pour l'analyse des EDP*, ENSTA Paristech (1st year)
- *Fonction de la variable complexe*, ENSTA Paristech (2nd year)
- *Inverse problems: mathematical analysis and numerical algorithms*, Université Pierre et Marie Curie (Master 2)

Aliénor Burel

- *Calculus, L1 PCST*, Université Paris-Sud XI (1st year)
- *Systèmes Linéaires, L3 MAP, Matlab*, Université Paris-Sud XI (3rd year)
- *Probabilités*, IUT d'informatique, Université Paris-Sud XI (2nd year)
- *Analyse*, IUT d'informatique, Université Paris-Sud XI (1st year)

Camille Carvalho

- *Optimisation Quadratique*, ENSTA Paristech (1st year)
- *Fonction d'une variable complexe*, ENSTA Paristech (2nd year)
- *Tutorat pour les élèves en difficulté en mathématiques appliquées*, ENSTA Paristech (1st year)

Maxence Cassier

- *Système dynamique: Stabilité et Commande*, ENSTA Paristech (1st year)
- *Introduction à MATLAB*, ENSTA Paristech (1st year)
- *Fonctions d'une variable complexe*, ENSTA-Paristech, (2nd year)

- *Tutorat pour élèves en difficulté en mathématiques appliquées*, ENSTA Paristech (1st year)

Stéphanie Chaillat

- *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA Paristech (1st year)
- *Fonctions d'une variable complexe*, ENSTA,ENSTA Paristech (2nd year)

Patrick Ciarlet

- *The finite element method*, ENSTA Paristech (2nd year)
- *The finite element method – extensions*, ENSTA Paristech (2nd year)
- *Theory and algorithms for distributed computing*, ENSTA Paristech (3rd year), and Master "Modeling and Simulation" (M2)
- *Maxwell's equations and their discretization*, ENSTA Paristech (3rd year), and Master "Modeling and Simulation" (M2)

Rémi Cornaggia:

- *Mécanique* , Polytech' Paris Sud, Orsay (PeiP, 1st year)
- *Physique des solides* , Université Paris 11, Orsay (L3 "Physique et mécanique")
- *Méthodes numériques pour la physique* , Université Paris 11, Orsay (M1 "Mécanique physique")

Sonia Fliss

- *Méthode des éléments finis*, ENSTA Paristech (2nd year)
- *Programmation scientifique et simulation numérique*, ENSTA Paristech (2nd year)
- *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA Paristech (1st year).
- *Local perturbations of periodic media*, International Summer School in Mathematics on the topic "Periodic structures in Applied Mathematics", Georg-August Universität, Gottingen.

Christophe Hazard

- *Outils élémentaires d'analyse pour les EDP*, ENSTA Paristech (1st year)
- *Théorie spectrale des opérateurs autoadjoints et applications aux guides optiques*, ENSTA Paristech (2nd year).

Patrick Joly

- *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA Paristech (1st year).
- *Outils élémentaires d'analyse pour les EDP*, ENSTA Paristech (1st year).

Nicolas Kielbasiewicz

- *Programmation scientifique et simulation numérique*, ENSTA-Paristech (2nd year)
- *Parallélisme et calcul réparti*, ENSTA-Paristech (Master 2)

Marc Lenoir

- *Fonctions d'une variable complexe*, ENSTA Paristech (2nd year)
- *Equations intégrales* ENSTA Paristech (Master 2)

Simon Marmorat

- *Introduction à la discrétisation des équations aux dérivées partielles et à leur discrétisation*, ENSTA Paristech (1st year)
- *La méthode des éléments finis*, ENSTA Paristech (2nd year)

Jean-François Mercier

- *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA Paristech (1st year)
- *Fonctions d'une variable complexe*, ENSTA-Paristech, ENSTA Paristech (2nd year)
- *Théorie spectrale des opérateurs autoadjoints et application aux guides optiques*, ENSTA Paristech (Master 2)

Nicolas Salles

- *Analyse et séries de Fourier*, Université Paris-Sud XI (L2)
- *Systèmes Linéaires (Matlab)*, Université Paris-Sud XI (L3)
- *Calcul scientifique*, Université Paris-Sud XI (L3)
- *Géométrie et Analyse de Fourier*, Université Paris-Sud XI (L2)

Elizaveta Vasilevskaya

- *Algèbre linéaire*, Institut Galigée (L1)
- *Mathématiques générales*, Univ. Paris XIII (L1)
- *Mathématiques générales*, Univ. Paris XIII (L2)

Valentin Vinoles

- *La méthode des éléments finis*, ENSTA Paristech (2nd year)
- *Introduction à MATLAB*, ENSTA-Paristech, ENSTA Paristech (1st year)

9.2.2. Supervision

PhD : Nicolas Salles, "Stabilisation du calcul des singularités dans les méthodes d'équations intégrales variationnelles", September 2013, Marc Lenoir and Eliane Bécache

PhD in progress : Aliénor Burel, "Méthodes numériques pour les ondes élastiques en présence d'interfaces minces et de milieux mous", October 2010, Patrick Joly and Marc Bonnet

PhD in progress : Maxence Cassier, "Focalisation par retournement temporel sur des obstacles diffractants.", October 2010, Christophe Hazard and Patrick Joly

PhD in progress : Matthieu Chamaillard, "Conditions aux limites effectives pour des revêtements minces périodiques", October 2011, Patrick Joly

PhD in progress : Simon Marmorat, "Etude d'un modèle asymptotique et de son couplage avec une approche par éléments finis pour simuler la propagation d'ondes ultrasonores dans un milieu complexe perturbé par de petites inclusions", Mars 2012, Patrick Joly

PhD in progress : Antoine Tonnoir, "Simulation numérique de la diffraction d'ondes ultrasonores par un défaut localisé dans une plaque élastique anisotrope", October 2011, Anne-Sophie Bonnet-Ben Dhia and Sonia Fliss

PhD in progress : Audrey Vigneron, "Formulations intégrales pour la simulation du contrôle non destructif par courants de Foucault", November 2011, Marc Bonnet

PhD in progress : Rémi Cornaggia, "Asymptotique petit-défaut de fonctions-coût et son application en identification: justifications théorique et expérimentale, extensions", October 2012, Marc Bonnet

PhD in progress : Geoffrey Beck, "Modélisation de la propagation d'ondes électromagnétiques dans des câbles co-axiaux", October 2012, Patrick Joly

PhD in progress : Elizaveta Vasilevskaia, "Modes localisés dans les guides d'onde quantiques", November 2012, Patrick Joly

PhD in progress : Camille Carvalho, "Étude théorique et numérique de guides d'ondes plasmoniques", October 2012, Anne-Sophie Bonnet-Ben Dhia and Patrick Ciarlet

PhD in progress : Valentin Vinoles, "Analyse asymptotique des équations de Maxwell en présence de métamatériaux", October 2012, Sonia Fliss and Patrick Joly

PhD in progress : Mathieu Lecouvez, "Méthodes de décomposition de domaine optimisées pour la propagation d'ondes en régime harmonique", March 2012, Patrick Joly

PhD in progress : Luca Desiderio, "Efficient visco-eleastic wave propagation in 3D for high contrast media", October 2013, Stéphanie Chaillat and Patrick Ciarlet

9.3. Popularization

Marc Bonnet:

- Opening workshop of UCL Centre for Inverse Problems (London, UK, March 2013),
- Conference honoring Andreas Kirsch for his 60th birthday (Bad Herrenalb, Germany, April 2013),
- International Conference on Novel Directions in Inverse Scattering (Newark DE, USA, July 2013),
- Singular Days (Rennes, France, August 2013)

Laurent Bourgeois

- *On sampling methods to identify defects in a periodic waveguide from far field data*, Inverse Problems: Scattering, Tomography and Parameter Identification, Conference honoring Andreas Kirsch for his 60th birthday, Bad Herrenalb - Germany, 8-11/4/2013
- *On Lipschitz stability for a class of inverse problems*, Inverse Problems and Nonlinear equations, Ecole Polytechnique, Palaiseau, 22-24/05/2013
- *On the far field of scattering solutions in a periodic waveguide. Part II : The inverse problem*, Waves conference, Gammarth, Tunisie, 3-7/6/2013

Aliénor Burel

- *Using potentials in elastodynamics : a challenge for FEM*, WONAPDE, Concepciòn (Chile), January 14-18th
- *Utilisation des potentiels en élastodynamique : un challenge pour les méthodes éléments finis ?*, GTN Orsay, Université Paris-Sud, February, 19th
- *Effective Transmission Conditions for Thin-Layer Transmission Problems in Elastodynamics*, WAVES '13, Gammarth (Tunisia), June 3-7th
- *Utilisation des potentiels en élastodynamique : un challenge pour les méthodes éléments finis ?*, Poems Seminar, Palaiseau, June 27th

Camille Carvalho

- *Plasmonic cavity modes with sign-changing permittivity*, WAVES, Tunis, June
- *Plasmonic cavity modes: black-hole phenomena captured by Perfectly Matched Layers*, PIERS, Stockholm, August

Maxence Cassier

- *Space-time focusing on unknown obstacles*, *International conference in applied mathematics, Heraklion, Greece, September 2013.*
- *Space-time focusing for acoustic waves* (poster session), *International conference on novel directions in inverse scattering, honoring David Colton, Newark, United states, July 2013.*
- *Selective focusing on unknown scatterers*, *Maxence Cassier, Christophe Hazard and Patrick Joly, Waves conference, Tunis, Tunisia, June 2013.*
- *Selective focusing for time-dependent waves*, *Maxence Cassier, Christophe Hazard and Patrick Joly, Workshop: Inverse problems and imaging, institut Henri Poincaré, Paris, France, February 2013.*

- *Selective focusing for time-dependent waves*, Maxence Cassier, Christophe Hazard and Patrick Joly, Workshop: Computational electromagnetism and acoustics, Oberwolfach, Germany, January 2013.

Stéphanie Chaillat

- *Fast multipole accelerated boundary integral equation method for 3-D elastodynamic problems in a half-space*, Séminaire EDP LJK, Grenoble, France, November 2013.

- *Fast multipole accelerated boundary integral equation method for 3-D elastodynamic problems in a half-space*, Séminaire Equipe ENI MSSMAT, Chatenay-Malabry, France, November 2013.

- *A fast and adaptive algorithm for the inverse medium problem based on Singular Value Decomposition*. 3rd European Conference on Computational Optimization, Chemnitz, Germany, July 2013.

- *An Adapted Fast Multipole Accelerated Boundary Element Method for 3D Elastodynamics*, SIAM in the Geosciences, Padua, Italy, June 2013.

- *Fast Multipole Accelerated Boundary Element Method for problems in an elastic Half-Space*, WAVES 2013, Tunis, Tunisia, June 2013.

- *Fast multipole accelerated boundary integral equation method for 3-D elastodynamic problems in a half-space*, Séminaire ISTERRE, Grenoble, France, February 2013.

- *Fast multipole accelerated boundary integral equation method for 3-D elastodynamic problems in a half-space*, Séminaire du LMA, Marseille, France, January 2013.

- *Comparison of two Fast Multipole Accelerated BEMs for 3D elastodynamic problems in semi-infinite media*, IABEM 2013, Santiago, Chile, January 2013.

Patrick Ciarlet

- *Strong convergence for Gauss' law with edge elements*, Mafelap'13, Uxbridge (G.-B.), 10-14/06/2013

- *Numerical approximation of transmission problems with sign changing coefficients*, Journées Singulières Augmentées, Rennes, 26-30/08/2013

Sonia Fliss

- *DtN approach for the exact computation of guided modes in a photonic crystal waveguides*, Séminaire EDP de Metz, Metz, January 18th.

- *Sufficient conditions for existence of guided modes in photonic crystal waveguide or how useful can be a numerical method*, Mathematical Methods for spectral problems, University of Helsinki, March 5th-7th

- *Transparent boundary conditions in periodic media*, HF 2013, Nancy, March 19th-21th

- *Scattering in locally perturbed periodic waveguides : forward and inverse problems*, Applied Analysis for the Material Sciences with a special hommage to Michael Vogelius, CIRM Marseille, May 27th-31st

- *On the far field of scattering solutions in a periodic waveguide. Part 1: the forward problem*, Waves 2013, Tunis, June 3rd-7th

Patrick Joly

- *Numerical simulation of a grand piano*, Conference WONAPDE, Concepcion, Chile, January 2013

- *A rigorous approach to the propagation of electromagnetic waves in co-axial cables*, Workshop EMSCA,Weierstrass Institute, Berlin, Germany, May 2013

- *Simulation numérique d'un piano de concert*, Journées EDPs, Contrôle et Musique, Université Pierre et Marie Curie, Paris, May 2013

- *A rigorous approach to the propagation of electromagnetic waves in co-axial cables* , Conference WAVES2013, Gammarth, Tunisia, June 2013
- *Quasi-localtransmission conditions and iterative domain decomposition methods for time harmonic wave propagation*, International Conference on Novel Directions in Inverse Scattering, Newark (Delaware), USA, Juillet 2013
- *Riesz potentials and quasi-local transmission condition for iterative non overlapping domain decomposition methods for the Helmholtz equation*, Conference JSA 2013, Rennes, France, August 2013
- *Perfectly Matched Layers for time domain wave propagation : overview and recent progress*, Conference CEDYA 2013, Castellon, Spain, September 2013
- *Conditions de transmission quasi-locales et méthodes de décomposition de domaine pour la propagation d'ondes en régime harmonique*, CMAP Seminar, Ecole Polytechnique, Palaiseau, France, December 2013

Simon Marmorat

- *An improved linear sampling method in the time domain*, WONAPDE 2013, Concepción, Chile, January.
- *Time domain computation of the scattering of waves by small heterogeneities*, Waves 2013, Tunis, Tunisia, June.
- *An asymptotic model for the scattering of waves by small heterogeneities*, Inria Junior Seminar, Rocquencourt, France, November.
- *Méthodes d'échantillonnage pour la diffraction inverse en fréquence et en temps*, POems Seminar, ENSTA, France, December.

Jean-François Mercier

- *Aeroacoustics in a waveguide with a shear flow*, Anne-Sophie Bonnet-BenDhia, Jean-François Mercier et Florence Millot, 11th International Conference on Mathematical and Numerical Aspects of Wave Propagation (WAVES'13), Tunisia (june 2013)
- *Numerical modeling of nonlinear acoustic waves with fractional derivatives*, B. Lombard et J.-F. Mercier, 11th International Conference on Mathematical and Numerical Aspects of Wave Propagation (WAVES'13), Tunisia (june 2013)

Nicolas Salles

- *Explicit evaluation of integrals arising in Galerkin BEM*, XIV International Conference on Boundary Element & Meshless Techniques, Palaiseau (France), July 2013

Antoine Tonnoir

- *New transparent boundary conditions for time harmonic acoustic diffraction in anisotropic media*, Waves International conference on Mathematical and numerical aspects of waves, Tunis, June 2013

Elizaveta Vasilevskaya

- *Localized modes in perturbed ladder-like periodic waveguides*, Workshop on waveguides, ENSTA ParisTech, Palaiseau, France, October 2013

10. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

[1] N. SALLES. , *Calcul des singularités dans les méthodes d'équations intégrales variationnelles*, Université Paris Sud - Paris XI, September 2013, http://hal.inria.fr/tel-00877482

Articles in International Peer-Reviewed Journals

[2] B. BANERJEE, T. WALSH, W. AQUINO, M. BONNET. *Large Scale Parameter Estimation Problems in Frequency-Domain Elastodynamics Using an Error in Constitutive Equation Functional*, in "Computer Methods in Applied Mechanics and Engineering", 2013, vol. 253, pp. 60-72 [*DOI :* 10.1016/J.CMA.2012.08.023], http://hal.inria.fr/hal-00732291

[3] C. BELLIS, M. BONNET. *Qualitative identification of cracks using 3D transient elastodynamic topological derivative: formulation and FE implementation*, in "Computer Methods in Applied Mechanics and Engineering", 2013, vol. 253, pp. 89-105 [*DOI :* 10.1016/J.CMA.2012.10.006], http://hal.inria.fr/hal-00741515

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