IN PARTNERSHIP WITH:
CNRS

Université de Bordeaux

Activity Report 2013

Project-Team REALOPT

Reformulations based algorithms for Combinatorial Optimization

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB)

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
Optimization, machine learning and statistical methods

Table of contents

Project-Team REALOPT

Keywords: Combinatorial Optimization, Mixed Integer Linear Programming, Operations Research

In collaboration with IMB and LABRI.

Creation of the Project-Team: 2009 January 01.

1. Members

Research Scientist
Ruslan Sadykov [Inria, Researcher]

Faculty Members
François Clautiaux [Univ. de Bordeaux, Professor, from Sep 2013]
Boris Detienne [Univ. Bordeaux, Associate Professor, from Sep 2013]
Arnaud Pêcher [Univ. de Bordeaux, Professor]
Pierre Pesneau [Univ. de Bordeaux, Associate Professor]
François Vanderbeck [Team leader, Univ. de Bordeaux, Professor]

Engineer
Romain Leguay [Inria, until Oct 2013]

PhD Students
Nastaran Rahmani [Inria]
Hugo Ribeiro Kramer [Visiting student on the SAMBA project]

Post-Doctoral Fellow
Jinil Han [Inria, Conseil Régional d'Aquitaine]

Visiting Scientist
Eduardo Uchoa Barboza [Visiting Professor on the SAMBA project, April]

Administrative Assistants
Sabine Delarboulas Cusin [Inria]
Sylvie Embolla [Inria]

Other
Silvia Ferretto [Internship, from March until June]

2. Overall Objectives

2.1. Overall Objectives

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decision set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space.

Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, extended reformulations (combinatorial relations can be formulated better with extra variables), constraint programming to actively reduce the solution domain through logical implications, Lagrangian and Bender's decomposition methods to produce powerful relaxations, multi-level programming to model a hierarchy of decision levels or recourse decision in the case of data adjustment, heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidates at all stage of the solution process, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique. The progress are measured in terms of the large scale of input data that can now be solved, the integration of many decision levels into planning models, and not least, the account taken for random data by way of modeling expectation (stochastic approaches) or worst-case bahavior (robust approaches).

Building on complementary expertise, our team's overall goals are threefold:

(i) Methodologies: To design tight formulations for specific problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. More broadly, to contribute to theoretical and methodological developments of exact approaches in combinatorial optimization, while extending the scope of applications.

(ii) Problem solving: To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, dynamic programming, robust and stochastic optimization, constraint programming, combinatorial algorithms and graph theory, by developing "efficient" algorithms for specific mathematical models. To tackle large-scale real-life applications, providing provably good approximate solutions by combining exact methods and heuristics.

(iii) Software platform: To provide prototypes of specific model solvers and generic software tools that build on our research developments, writing proof-of-concept code, while transferring our research findings to internal and external users.

2.2. Highlights of the Year

Our scientific contributions have been recognized by prestigious journals such as Mathematical Programming, EURO Journal on Computational Optimization, INFORMS Journal on Computing, European Journal of Operational Research, Transportation Science, European Journal of Combinatorics, and Combinatorica, as well as by our participations to prime scientific meetings. In particular, François Vanderbeck was invited as a keynote speaker at the European/American Operations Research Conference in Rome (his presentation was "Extended formulations, Column Generation, and stabilization: synergies in the benefit of large scale applications" which is a subject central to our team work this year); Arnaud Pêcher was invited speaker at the International Conference in Discrete Mathematics, India.

Our methodology of combining an extended formulation approach with Dantzig-Wolfe decomposition has proved able to handle the very large scale instances of railway fret transportation applications (as shown by the very competitive results obtained by Ruslan Sadykov) as well as power production planning at EDF (Jinil Han has managed to solve the Roadef Challenge Instances in a few minutes, while Boris Detienne develops a robust approach on that basis). The Samba associated-team project with Brasil has been at the core of our methodological research effort with a one-month-visit of Professor Uchoa, and a one-year-stay of his PhD student, Hugo Kramer. New industrial collaborative project have started: working on the dimensioning of a logistic fleet with Exeo-Solution (Pierre Pesneau is leading this project), on planning operations in wineries with Ertus Consulting, and on packing and cutting problems with Renault and Saint-Gobain (Francois Clautiaux is an expert on those).

The team is very fortunate to have made very successful appointments: a university professor, Francois Clautiaux, an assistant professor, Boris Detienne and an expert engineer, Issam Tahiri, to be appointed by the center of excellence - labex CPU. Let us also mention that the team is organizing the annual conference of the French Operation Research society (ROADEF) taking place in Bordeaux in February 2014.

3. Research Program

3.1. Introduction

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm* : (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a factional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM-CPLEX, FICO-Xpress-mp, or GUROBI). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify “efficiently” a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is robust optimization where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise in mathematical programming (polyhedral approaches, Dantzig-Wolfe decomposition, mixed integer programing, robust and stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems.

3.2. Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [76]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if X is a mixed integer program: $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$ where $P = \{x \in \mathbb{R}^{n+p} : Ax \leq b\}$ with matrix $(A, b) \in \mathbb{Q}^{m \times (n+p+1)}$, then $conv(X)$ is a polyhedron that can be described in terms of linear constraints, i.e. it writes as $conv(X) = \{x \in \mathbb{R}^{n+p} : C\,x \leq d\}$ for some matrix $(C, d) \in \mathbb{Q}^{m' \times (n+p+1)}$ although the dimension m' is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem $\min\{cx : x \in X\}$ and solving the *separation problem* over the associated polyhedron $conv(X)$: if $\tilde{x} \notin conv(X)$, find a linear inequality $\pi\,x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\tilde{x}$. Hence, for NP-hard problems, one can not hope to get a compact description of $conv(X)$ nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of $conv(X)$ and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities $C\,x \leq d$ can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

3.3. Decomposition and reformulation approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, the *cutting plane procedures* that we defined in the previous section are well-suited tools. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

3.4. Integration of Artificial Intelligence Techniques in Integer Programming

When one deals with combinatorial problems with a large number of integer variables, or tightly constrained problems, mixed integer programming (MIP) alone may not be able to find solutions in a reasonable amount of time. In this case, techniques from artificial intelligence can be used to improve these methods. In particular, we use primal heuristics and constraint programming.

Primal heuristics are useful to find feasible solutions in a small amount of time. We focus on heuristics that are either based on integer programming (rounding, diving, relaxation induced neighborhood search, feasibility pump), or that are used inside our exact methods (heuristics for separation or pricing subproblem, heuristic constraint propagation, ...).

Constraint Programming (CP) focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems Mixed Integer Programming (MIP), on the other hand, is known to be effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables. Many problems belong to the intersection of these two classes. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

3.5. Robust Optimization

Decision makers are usually facing several sources of uncertainty, such as the variability in time or estimation errors. A simplistic way to handle these uncertainties is to overestimate the unknown parameters. However, this results in over-conservatism and a significant waste in resource consumption. A better approach is to account for the uncertainty directly into the decision aid model by considering mixed integer programs that involve uncertain parameters. Stochastic optimization account for the expected realization of random data and optimize an expected value representing the average situation. Robust optimization on the other hand entails protecting against the worst-case behavior of unknown data. There is an analogy to game theory where one considers an oblivious adversary choosing the realization that harms the solution the most. A full worst case protection against uncertainty is too conservative and induces very high over-cost. Instead, the realization of random data are bound to belong to a restricted feasibility set, the so-called uncertainty set. Stochastic and robust optimization rely on very large scale programs where probabilistic scenarios are enumerated. There is hope of a tractable solution for realistic size problems, provided one develops very efficient ad-hoc algorithms. The techniques for dynamically handling variables and constraints (column-and-row generation and Bender's projection tools) that are at the core of our team methodological work are specially well-suited to this context.

3.6. Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

4. Application Domains

4.1. Introduction

Our group has tackled applications in logistics, transportation and routing [75], [74], [70], [72], in production planning [94] and inventory control [70], [72], in network design and traffic routing [51], [61], [68], [97], [49], [62], [81], [87], in cutting and placement problems [77], [78], [91], [92], [93], [95], and in scheduling [5], [82], [47].

4.2. Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form "at least two paths link each pair of terminals". We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [61]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today's optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronic system installation cost. We developed and compared several decomposition approaches [99], [98], [97] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route [96]. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [68], [69].

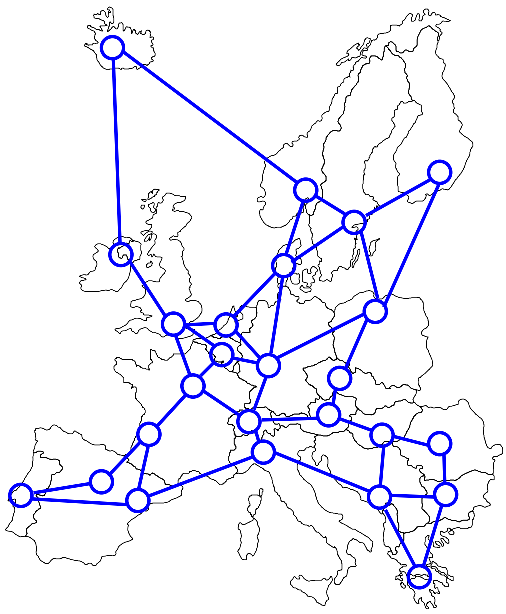

Figure 1. Design of a SDH/SONET european network where demands are multiplexed.

We studied several time dependent formulations for the unit demand vehicle routing problem [53], [52]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We also worked on the p-median problem, applying the matching theory to develop an efficient algorithm in Y-free graphs and to provide a simple polyhedral characterization of the problem and therefore a simple linear formulation [86] simplifying results from Baiou and Barahona.

We considered the multi-commodity transportation problem. Applications of this problem arise in, for example, rail freight service design, "less than truckload" trucking, where goods should be delivered between different locations in a transportation network using various kinds of vehicles of large capacity. A particularity here is that, to be profitable, transportation of goods should be consolidated. This means that goods are not delivered directly from the origin to the destination, but transferred from one vehicle to another in intermediate locations. We proposed an original Mixed Integer Programming formulation for this problem which is suitable for resolution by a Branch-and-Price algorithm and intelligent primal heuristics based on it.

For the problem of routing freight railcars, we proposed two algorithmes based on the column generation approach. These algorithmes have been testes on a set of real-life instances coming from a Russian freight real transportation company. Our algorithmes have been faster on these instances than the current solution approach being used by the company.

4.3. Packing and Covering Problems

Realopt team has a strong experience on exact methods for cutting and packing problems. These problems occur in logistics (loading trucks), industry (wood or steel cutting), computer science (parallel processor scheduling).

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [21]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs.

We also developped a branch-and-price algorithm for a variant of the bin-packing problem where the items are fragile. In [10] we studied empirically different branching schemes and different algorithms for solving the subproblems.

We studied a variant of the knapsack problem encountered in inventory routing problem [72]: we faced a multiple-class integer knapsack problem with setups [71] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory [65], [63], [67], [66]. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2d knapsack problem that uses our 2D packing feasibility check.

We are now organizing a european challenge on packing with society Renault. This challenge will be held in 2014 and will be about loading trucks under practical constraints.

4.4. Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a truncated branch-and-price algorithm: periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem; the global planning of customer visits is generated by solving a master program. Confronted with the issue of symmetry in time, we used a state-space relaxation idea. Our algorithm provides solutions with reasonable deviation from optimality for large scale problems (260 customer sites, 60 time periods, 10 vehicles) coming from industry [73]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [75].

Together with our partner company GAPSO from the associate team SAMBA, we worked on the equipment routing task scheduling problem [80] arising during port operations. In this problem, a set of tasks needs to be performed using equipments of different types with the objective to maximum the weighted sum of performed tasks.

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [60] and exact algorithms [47]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [46]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [82]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [84]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [85], [83].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [45], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [44] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [48] provide demonstrably stronger formulations for some problem classes than any previously proposed.

We have been developing robust optimization models and methods to deal with a number of applications like the above in which uncertainty is involved. In [55], [54], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [50]. We considered train timetabling problems and their re-optimization after a perturbation in the network [57], [56]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

Currently, we are conducting investigations on a real-world planning problem in the domain of energy production, in the context of a collaboration with EDF. The problem consists in scheduling maintenance periods of nuclear power plants as well as production levels of both nuclear and conventional power plants in order to meet a power demand, so as to minimize the total production cost. For this application, we used a Dantzig-Wolfe reformulation which allows us to solve realistic instances of the deterministic version of the problem. In practice, the input data comprises a number of uncertain parameters. We deal with a scenario-based stochastic demand with help of a Benders decomposition method. We are working on Multistage Robust Optimization approaches to take into account other uncertain parameters like the duration of each maintenance period, in a dynamic optimization framework. The main challenge adressed in this work is the joint management of different reformulations and solving techniques coming from the deterministic (Dantzig-Wolfe decomposition, due to the large scale nature of the problem), stochastic (Benders decomposition, due to the number of demand scenarios) and robust (reformulations based on duality and/or column and/or row generation due to maintenance extension scenarios) components of the problem.

5. Software and Platforms

5.1. BaPCod – a generic Branch-and-Price Code

Participants: Romain Leguay [Software Engineer], Pierre Pesneau, Ruslan Sadykov, François Vanderbeck [correspondant].

BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of a Dantzig-Wolfe reformulation technique. The reformulated problem is solved using a branch-and-price (column generation) algorithm. This software platform, made of C++ classes, offers a "*black-box*" implementation that does not require user input and is not application specific. The features are
(i) the automation of the Dantzig-Wolfe reformulation process (the user defines a mixed integer programming problem in a pseudo modeling language, defining variables and constraints, identifying subproblems. He can provide subproblem solvers if available, but he does not need to explicitly define the reformulation, the explicit form of the columns, their reduced cost, or the Lagrangian bounds.
(ii) a default column generation procedure with standard initialization and stabilization [1], [59] [89] [88] [27] and
(iii) a default branching scheme that is generic to all applications [7],
(iv) default primal heuristics specially developed for use in a decomposition framework [64], [79], [90].
The prototype software was/is used as background solver in our application studies and local PhD thesis. It also serves as the framework for our comparative study in a Inria assocaited team project and our transfert projects (the prototype enables us to be very responsive in our industrial contact).

See also the web page https://wiki.bordeaux.inria.fr/realopt/pmwiki.php/Project/BaPCod.

6. New Results

6.1. Extending the column generation paradigm

Building on our technical review [89] of methods for solving the Lagrangian Dual (with an analysis of the scope for hybridization) we have worked on methodologies that can be understood as an extension of the column generation approach in [22]. Working in an extended variable space allows one to develop tighter reformulations for mixed integer programs. To handle the size of the extended formulation, one can work with inner approximations defined and improved by generating dynamically variables and constraints. This so-called "column-and-row generation" procedure is revisited here in a unifying presentation that generalizes the column generation algorithm and extends to the case of working with an approximate extended formulation. A key benefit of this approach is that lifting pricing problem solutions in the space of the extended formulation permits their recombination into new subproblem solutions and results in faster convergence. The interest of the approach is evaluated numerically on machine scheduling, bin packing, generalized assignment, and multi-echelon lot-sizing problems. We compare a direct handling of the extended formulation, a standard column generation approach, and the "column-and-row generation" procedure. Within the Samba project we further showed that this stabilization offered by the recombination of solutions is complementary and adds up to stabilization techniques based on smoothing that were developed within Samba. These techniques have been applied in [26], [29].

6.2. Interior point cutting plane strategy revisited for column generation

In [89], we identify what are the stabilization features that are built into variants of subgradient algorithms and polyhedral approaches. In [27], we further compare their theoretical performance and discuss their combination. Stabilization procedures for column generation can be viewed as cutting plane strategies in the dual. Exploiting the link between in-out separation strategies and dual price smoothing techniques for column generation, we derive a generic bound convergence property for algorithms using a smoothing feature. Such property adds to existing in-out asymptotic convergence results. In our study on In-Out Separation and Column Generation Stabilization by Dual Price Smoothing, we note that our convergence property adds to existing in-out asymptotic convergence results. Beyond theoretically convergence, we describe in [88] a proposal for effective finite convergence in practice and we develop a smoothing auto-regulating strategy that makes the need for parameter tuning obsolete. Practical speed-up convergence that are observed go from 20% to 500 %. These contributions turn stabilization by smoothing into a general purpose practical scheme that can be used into a generic column generation procedure. We conclude the paper by showing that the approach can be combined with an ascent method, leading to improved performances. Such combination might inspire novel cut separation strategies.

6.3. A MILP approach to minimize the number of late jobs with and without machine availability constraints

The study in [13] investigates scheduling problems that occur when the weighted number of late jobs that are subject to deterministic machine availability constraints have to be minimized. These problems can be modeled as a more general job selection problem. Cases with resumable, non-resumable, and semi-resumable jobs as well as cases without availability constraints are investigated. The proposed efficient mixed integer linear programming approach includes possible improvements to the model, notably specialized lifted knapsack cover cuts. The method proves to be competitive compared with existing dedicated methods: numerical experiments on randomly generated instances show that all 350-job instances of the test bed are closed for the well-known problem $1|r_i|\sum w_i U_i$. For all investigated problem types, 98.4% of 500-job instances can be solved to optimality within one hour.

6.4. Multidimensional dual-feasible functions

Dual-feasible functions have been used in the past to compute lower bounds and valid inequalities for different combinatorial optimization and integer programming problems. Until now, all the dual-feasible functions proposed in the literature were 1-dimensional functions. In [11] we extended the principles of dual-feasible functions to the m-dimensional case by introducing the concept of vector packing dual-feasible function. We explored the theoretical properties of these functions in depth, and we proposed general schemes for generating some instances of these functions. Additionally, we proposed and analyzed different new families of vector packing dual-feasible functions. All the proposed approaches were tested extensively using benchmark instances of the 2-dimensional vector packing problem. Our computational results showed that these functions can approximate very efficiently the best lower bounds for this problem.

6.5. New branch-and-price methods for variants of bin packing problems

We proposed branch-and-price methods for two variants of the well-known bin-packing problem.

The bin packing problem with conflicts consists in packing items in a minimum number of bins of limited capacity while avoiding joint assignments of items that are in con ict. The study in [21] demonstrates that a generic implementation of a Branch-and-Price algorithm using specific pricing oracle yields comparatively good performance for this problem. We use our black-box Branch-and-Price solver BaPCod, relying on its generic branching scheme and primal heuristics. We developed a dynamic programming algorithm for pricing when the conflict graph is an interval graph, and a depth-first-search branch-and-bound approach for pricing when the conflict graph has no special structure. The exact method was tested on instances from the literature where the conflict graph is an interval graph, as well as harder instances that we generated with an arbitrarily conflict graph and larger number of items per bin. Our computational experiment report sets new benchmark results for this problem, closing all open instances of the literature in one hour of CPU time.

In the bin-packing with fragile objects, we are given a set of objects, each characterized by a weight and a fragility, and a large number of uncapacitated bins. Our aim is to find the minimum number of bins needed to pack all objects, in such a way that in each bin the sum of the object weights is less than or equal to the smallest fragility of an object in the bin. The problem is known in the literature as the Bin Packing Problem with Fragile Objects, and appears in the telecommunication field, when one has to assign cellular calls to available channels by ensuring that the total noise in a channel does not exceed the noise acceptance limit of a call. In [10], we propose a branch-and-bound and several branch-and-price algorithms for the exact solution of the problem, and improve their performance by the use of lower bounds and tailored optimization techniques. In addition we also develop algorithms for the optimal solution of the related knapsack problem with fragile objects. We conduct an extensive computational evaluation on the benchmark set of instances, and show that the proposed algorithms perform very well.

6.6. Freight railcar routing

In some countries, the activities of managing railroads and managing a fleet of freight railcars are separated by a law. A state-owned company is in charge of the first activity. The control of freight railcars is separated between several independent companies. The main objective of such company is an effective management of its railcars. As these companies are commercial, the goal is to maximize the profit from the usage of their railcars. The profit of a company is mainly determined by the difference between the total gain it receives from satisfying requests for delivery of goods in railcars and the costs it pays to the state-owned company for exploiting the railroad network.

Consequently, the main optimization problem that every freight railcar management company faces can be formulated as follows. We need 1) to choose a set of transportation demands between stations in a railroad network, and 2) to fulfill these demands by appropriately routing the set of available railcars, while maximizing the total profit. We formulate this problem as a multi-commodity flow problem in a large space-time graph. Three approaches are proposed to solve the Linear Programming relaxation of this formulation: direct solution by an LP solver, a column generation approach based on the path reformulation, and a "column generation for extended formulations" approach [22]. In the latter, the multi-commodity flow formulation is solved iteratively by dynamic generation of arc flow variables. Three approaches have been tested on a set of real-life instances provided by one of the largest freight rail transportation companies in Russia. Instances with up to 10 millions of arc flow variables were solved within minutes of computational time [29], [39].

6.7. Reliable Service Allocation in Clouds with Memory and Capacity Constraints

In [25] we consider allocation problems that arise in the context of service allocation in Clouds. More specifically, on the one part we assume that each Physical Machine (denoted as PM) is offering resources (memory, CPU, disk, network). On the other part, we assume that each application in the IaaS Cloud comes as a set of services running as Virtual Machines (VMs) on top of the set of PMs. In turn, each service requires a given quantity of each resource on each machine where it runs (memory footprint, CPU, disk, network). Moreover, there exists a Service Level Agreement (SLA) between the Cloud provider and the client that can be expressed as follows: the client requires a minimal number of service instances which must be alive at the end of the day, with a given reliability (that can be converted into penalties paid by the provider). In this context, the goal for the Cloud provider is to find an allocation of VMs onto PMs so as to satisfy, at minimal cost, both capacity and reliability constraints for each service. In this paper, we propose a simple model for reliability constraints and we prove that it is possible to derive efficient heuristics.

6.8. On the Theta number of powers of cycle graphs

In [17] we give a closed formula for Lovász's theta number of the powers of cycle graphs C_k^d and of their complements, the circular complete graphs $K_{k/d}$. As a consequence, we establish that the circular chromatic number of a circular perfect graph is computable in polynomial time. We also derive an asymptotic estimate for the theta number of C_k^d.

6.9. Strong chromatic index of planar graphs with large girth

Let Δ be an integer. In [18], we prove that every planar graph with maximum degree Δ and girth at least $10\Delta + 46$ is strong $(2\Delta - 1)$-edge-colorable, that is best possible (in terms of number of colors) as soon as G contains two adjacent vertices of degree Δ. This improves the best previous result when $\Delta \geq 6$.

6.10. Computing clique and chromatic number of circular-perfect graphs in polynomial time

A main result of combinatorial optimization is that clique and chromatic number of a perfect graph are computable in polynomial time (Grötschel et al. in Combinatorica 1(2):169–197,1981). Perfect graphs have the key property that clique and chromatic number coincide for all induced subgraphs; in [19] we address the question whether the algorithmic results for perfect graphs can be extended to graph classes where the chromatic number of all members is bounded by the clique number plus one. We consider a well-studied superclass of perfect graphs satisfying this property, the circular-perfect graphs, and show that for such graphs both clique and chromatic number are computable in polynomial time as well. In addition, we discuss the polynomial time computability of further graph parameters for certain subclasses of circular-perfect graphs. All the results strongly rely upon Lovász's Theta function.

6.11. Computing the clique number of a-perfect graphs in polynomial time

A main result of combinatorial optimization is that clique and chromatic number of a perfect graph are computable in polynomial time (Grötschel, Lovasz and Schrijver 1981). This result relies on polyhedral characterizations of perfect graphs involving the stable set polytope of the graph, a linear relaxation defined by clique constraints, and a semi-definite relaxation, the Theta-body of the graph. A natural question is whether the algorithmic results for perfect graphs can be extended to graph classes with similar polyhedral properties. In [20] we consider a superclass of perfect graphs, the a-perfect graphs, whose stable set polytope is given by constraints associated with generalized cliques. We show that for such graphs the clique number can be computed in polynomial time as well. The result strongly relies upon Fulkersons's antiblocking theory for polyhedra and Lovasz's Theta function.

7. Bilateral Contracts and Grants with Industry

7.1. Contract with EDF on maintenance planning

We are currently working on a project aiming to plan the energy production and the maintenance breaks for a set of nuclear power plants generating electricity. We consider the large-scale power plant maintenance scheduling and production planning problem submitted by EDF to the 2010 Euro/Roadef Challenge. Two types of power plants are used to satisfy a customer demand over a specific time horizon. Type 1 plants can operate continuously while Type 2 plants have to be shut down regularly for refuelling and maintenance, and cannot produce during outage periods. The decision to be made consists of the dates of outages, the amount of refuel for Type 2 plants, and production level for both types of plants. The objective is to minimize the average cost of refuelling and production on various demand scenarios. In this work, we propose a novel column generation approach based on extended formulation which enables to solve within a few minutes a deterministic instance of the problem on a three years horizon, which is within the time frame of the operational tools currently used by EDF. Moreover, the approach can easily account for various demand scenarios. Our approach is tested on real life instances within a rolling horizon framework.
This project is carried in collaboration between EDF R&D (OSIRIS lab) Inria team Dolphin and Realopt.

7.2. Collaboration with ERTUS on sanitary treatment planning

In planning winary operations (most importantly sanitary treatments on the wine tree) under wheather forcast uncertainty, one searches for solutions that remain feasible and "cheap" in case of perturbation in the data. We consider the planning and scheduling of the operations that arise over a one-year horizon. More precisely, the operations to be sheduled include tasks related to soil care, or grape tree care: cutting, line building, thinning out leaves, ..., and chemical treatments. The latter are a main focus of our study since one of the principal goals of better planning is to reduce the amount of chemical treatments by selecting the appropriate products and schemes, but also by spacing out treatements while guarantying a desease free vineyard with some confidence. Each of the scheduled tasks requires its own resource, so the planning also triggers equipement and raw products selection decisions. The objective is to minimize both equipment and product costs augmented by an evaluation of the hazard of chemical product use. The planning should be "robust" to seasonal variations on the proper time frame for scheduling tasks.

7.3. Collaboration with Exeo-Solutions on dimensionning a vehicle fleet for waste collection

Through the internships of Damien Trut and Youcef Magnouche in Exeo, and the current work of Pierre Pesneau, we study the optimization of partitionning a urban area into zones that shall be assigned to vehicles for waste collection. The goal is to minimize the distance traversed by the vehicles in each zone. This can be modeled as a clustering problem with side constraints: zones assigned to a same cluster must be contiguous and satisfy capacity and time constraints.

7.4. Collaboration with B-Travel on a yield management problem

Through the PhD thesis of Martin Bué (in collaboration with inria team Dolphin), we are now working with society B-Travel on pricing and yield management. The goal is to find the best prices and incentives in the context of professional travel. The techniques used are based on network-flow formulations and mathematical programming.

7.5. Collaboration with Vekia on an employee-scheduling problem

Through the PhD thesis of Matthieu Gérard (in collaboration with inria team Dolphin), we are now investigating a very rich version of employee-scheduling problem. We have designed an efficient algorithm for computing the best shift for each employee, based on dynamic programming. This method is used in a greedy algorithm to find solutions in a faster manner, and in a branch-and-price method to prove the optimality of the solution.

8. Partnerships and Cooperations

8.1. Regional Initiatives

Region Aquitaine is supporting a post-doc in our team. Jinil Han has been recruited to constribute to our team effort to develop efficient decomposition based approach to real-life combinatorial optimization problems. Jinil's research aims at enhancing performance of such approach and prepare the way to high performance computing through parallelisation. Jinil's mission extends to problem solving that serves both as a motivation and an proof-of-concept. Jinil has contributed so far to warm-starting the methods and to convergence accelaration through stabilization techniques [59]. Jinil has pushed the column generation for extented formlation method to the limit on the EDF application [58].

8.2. National Initiatives

Pierre Pesneau has got a grant from the OR research group from **CNRS** to finance mission between Bordeaux and Paris within the context of a collaboration with University Paris 6 (P. Fouilhoux) and University Paris 13 (S. Borne, R. Grappe, M. Lacroix). This collaboration aims to study polyhedral properties and algorithmic aspects to the problem of connected graph partitioning.

8.3. International Initiatives

8.3.1. Inria Associate Teams: SAMBA

Title: “Synergies for Ameliorations and Mastering of Branch-and-Price Algorithms”

International Partner (Institution - Laboratory - Researcher):

> Pontificia Universidade Catolica do Rio de Janeiro (Brazil) - ATD-Lab - Marcus Poggi, and Universidade Federal Fluminense (UFF), Brazil - Eduardo Uchoa.

Duration: 2011 - 2013

See also: https://realopt.bordeaux.inria.fr/?page_id=573

The so-called Dantzig-Wolfe decomposition approach has not yet made its way into general purpose solvers for Mixed Integer Programming (MIP). Despite its proved efficiency, the use of the method is currently restricted to specific applications and requires ad-hoc algorithms developed by experts. Our project is to develop general purpose algorithms to make this method generic. We shall focus in particular on (i) preprocessing procedures, (ii) warm-starting, (iii) stabilization (to improve convergence), (iv) strategies for combining cut and column generation, and (v) primal heuristics. The project builds on the accumulated experience of both the Brazilian and the French teams that have done pioneering work in tackling complex applications and deriving generic solution strategies using this decomposition approach. The new algorithms are implemented and tested in the software platform BaPCod. Hence, the collaborative research on methodological developments should lead to, as a bi-product, a Version 2 of BaPCod as a state-of-the-art Branch-and-Price-and-Cut Solver. This prototype should (i) serve as proof-of-concept code for the research planned in thisproject and beyond, (ii) enable us to achieve new benchmark results on key problems,(iii) provide incentive for the use of the method by non experts, (iv) leverage technology transfer to industry.

8.3.2. Participation in other International Programs

- Collaboration with University of Minho through FCT Project MST4IRTO: New models and solution techniques for integrated and real-time optimization in the supply chain.

8.4. International Research Visitors

8.4.1. Visits of International Scientists

- Eduardo Uchoa, Professor at Universidade Federal Fluminense (UFF), has visited the University Bordeaux for one month in April 2013.
- Hugo Kramer, PhD student at Universidade Federal Fluminense (UFF), is visiting the University Bordeaux for one year in 2013-2014.

8.4.1.1. Internships

- Silvia Ferretto, from the University of Padova (It) has done her Master intership with us from March until June.

8.4.2. Visits to International Teams

- Ruslan Sadykov visited the Universidade Federal Fluminense (UFF) for two weeks in March 2013.
- Francois Vanderbeck visited PUC-Rio and UFF for two weeks in March 2013.
- Francois Vanderbeck visited Marcos Goycoolea (Prof.), Operations Research and Complex Systems Group School of Business, Universidad Adolfo Ibañez, Chile, for 10 days in November 2013.

9. Dissemination

9.1. Scientific Animation

Until June 2013, Pierre Pesneau was a member of the organizing comittee of the Polyhedral and Combinatorial Optimization working group of the CNRS Operations Research group. In this scope, he organized in April, 26th seminars arround the field of “Semi-Definite Programming”.

The team members are regular referees for the best journals of the field. Besides, we are called on the scientific comittee of international and national conferences:

- Program comittee member of INOC 2013, International Network Optimization Conference, May 20-22, 2013
- Program comittee member for CPAIOR 2014, Eleventh International Conference on Integration of Artificial Intelligence (AI) and Operations Research (OR) techniques in Constraint Programming
- Program comittee for ISCO 2014, 3rd International Symposium on Combinatorial Optimization
- Program comittee for ICGT 2014, 9th International colloquium on graph theory and combinatorics, June 30-July 4, 2014.
- Program comittee for ROADEF 2014, 15ème congrès annuel de la Société française de recherche opérationnelle et d'aide à la décision (ROADEF)

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Licence : Pierre Pesneau, Modèles et Méthodes d'Optimisation, 30h, L2, Université de Bordeaux, France

Licence : Pierre Pesneau, Système et Programmation, 59h, L2, Université de Bordeaux, France

Licence : Arnaud Pêcher, Programmation objet et impérative, 200h, DUT, Université de Bordeaux, France

Licence : Arnaud Pêcher, Théorie des graphes, 16h, DUT, Université de Bordeaux, France

Master : François Clautiaux, Programmation linéaire 1, 15h TD, M1, Université de Bordeaux, France

Master : François Clautiaux, Introduction à l'optimisation en nombres entiers, 15h TD, M1, Université de Bordeaux, France

Master : François Clautiaux, Programmation en C++, 60h TD, M1, Université de Bordeaux, France

Master : François Clautiaux, Recherche opérationnelle, 16h TD, M1, Institut Polytechnique de Bordeaux, France

Master : Boris Detienne, Gestion des opérations et planification de production, 30h TD, M2, Université de Bordeaux, France

Master : Boris Detienne, Programmation Linéaire 1, 14h TD, M1, Université de Bordeaux, France

Master : Boris Detienne, Programmation Linéaire 2, 14h TD, M1, Université de Bordeaux, France

Master : Boris Detienne, Introduction à l'optimisation en nombres entiers, 14h TD, M1, Université de Bordeaux, France

Master : Boris Detienne, Modèles de flot, 14h TD, M1, Université de Bordeaux, France

Master : Boris Detienne, Projet informatique, 9h TD, M2, Université de Bordeaux, France

Master : Pierre Pesneau, Combinatoire et Routage, 14h, M1, Université de Bordeaux et ENSEIRB IPB, France

Master : Ruslan Sadykov, Introduction à la Programmation par Contraintes, 30 HETD, M2, Université de Bordeaux, France

Master : Ruslan Sadykov, Modélisation, Optimisation, Complexité, et Algorithmes, 50 HETD, M2, CNAM Aquitaine, France

Master : François Vanderbeck, Recherche opérationnelle, 15h cours, M1, Institut Polytechnique de Bordeaux, France

Master : François Vanderbeck, Flot et routage, 15h cours, M1, Université de Bordeaux, France

Master : François Vanderbeck, Programmation linéaire 2, 15h cours, M1, Université de Bordeaux, France

Master : François Vanderbeck, Programmation entière, 50h cours/TD, M2, Université de Bordeaux, France

9.2.2. Supervision

PhD in progress : Nastaran Rahmani, Planning and Routing via Decomposition Approaches, Ruslan Sadykov & François Vanderbeck.

PhD in progress : Nadia Dahmani, Problèmes de packing multi-critères, 01/09/2010, François Clautiaux, El-Ghazali Talbi

PhD in progress : Matthieu Gérard, Résolution de problèmes d'optimisation dans le commerce de détail, 01/09/2012, François Clautiaux.

PhD in progress : Martin Bué, Gestion du revenu dans le cadre du voyage professionnel, 01/09/2012, François Clautiaux, Luce Brotcorne

PhD in progress : Nicolas Dupin, Optimisation robuste des arrêts de centrales nucléaires pour maintenance, 01/09/2012, François Clautiaux, François Vanderbeck

PhD : Sagnik Sen, Oriented graph colourings, 01/01/2011 - 03/02/2014, Arnaud Pêcher

9.2.3. Juries

- François Clautiaux : Evaluation (rapporteur) of the PhD thesis of Louise Brac de la Perrière (Université de Technologie de Compiègne)
- François Clautiaux : Evaluation (rapporteur) of the PhD thesis of Marat Mesyagutov (University of Dresden)
- Boris Detienne: Evaluation (examinateur) of the PhD thesis of Xavier Libeaut (University of Angers)
- Arnaud Pêcher : Evaluation (examinateur) of the PhD thesis of Pierre Aboulker (University of Paris 7)
- Arnaud Pêcher : Evaluation (examinateur) of the PhD thesis of Alberto Passuelo (University of Bordeaux)
- François Vanderbeck : Evaluation (rapporteur) of the PhD thesis of Stephen J Maher (University of New South Wales, Australia)
- François Vanderbeck : Evaluation (rapporteur) of the PhD thesis of Amal Benhamiche (University Paris Dauphine)
- François Vanderbeck : Evaluation (examinateur) of the PhD thesis of Pierre-Louis Poirion (ENSTA-Paristech UMA –CEDRIC)

10. Bibliography

Major publications by the team in recent years

[1] O. BRIANT, C. LEMARÉCHAL, P. MEURDESOIF, S. MICHEL, N. PERROT, F. VANDERBECK. *Comparison of Bundle and Classical Column Generation*, in "Mathematical Programming", 2008, vol. 113, n^{o} 2, pp. 299–344

[2] S. COULONGES, A. PÊCHER, A. WAGLER. *Characterizing and bounding the imperfection ratio for some classes of graphs*, in "Mathematical Programming, Series A", 2009, vol. 118, n^{o} 1, pp. 37-46, http://hal.archives-ouvertes.fr/hal-00308150/en/

[3] L. GOUVEIA, P. PESNEAU. *On extended formulations for the precedence constrained asymmetric traveling salesman problem*, in "Networks", 2006, vol. 48, n^o 6, pp. 77–89

[4] A. PÊCHER, A. WAGLER. *Almost all webs are not rank-perfect*, in "Mathematical Programming", 2006, vol. 105, n^o 2-3, pp. 311–328

[5] R. SADYKOV, L. A. WOLSEY. *Integer Programming and Constraint Programming in Solving a Multimachine Assignment Scheduling Problem with Deadlines and Release Dates*, in "INFORMS Journal on Computing", 1 2006, vol. 18, n^o 2, pp. 209–217

[6] F. VANDERBECK. *On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm*, in "Operations Research", 2000, vol. 48, n^o 1, pp. 111–128

[7] F. VANDERBECK. *Branching in Branch-and-Price: a Generic Scheme*, in "Mathematical Programming, Series A", 2011, vol. 130, pp. 249-294 [*DOI :* 10.1007/S10107-009-0334-1], http://hal.inria.fr/inria-00311274

[8] F. VANDERBECK. *Computational Study of a Column Generation algorithm for Bin Packing and Cutting Stock problems*, in "Mathematical Programming, Ser. A", 1999, vol. 86, pp. 565–594

[9] F. VANDERBECK, L. WOLSEY. *Reformulation and Decomposition of Integer Programs*, in "50 Years of Integer Programming 1958-2008", Springer, 2010, CORE DP 2009/16 CORE DP 2009/16 [*DOI :* 10.1007/978-3-540-68279-0_13], http://hal.inria.fr/inria-00392254

Publications of the year

Articles in International Peer-Reviewed Journals

[10] M. ALBA MARTÍNEZ, F. CLAUTIAUX, M. DELL'AMICO, M. IORI. *Exact algorithms for the bin packing problem with fragile objects*, in "Discrete Optimization", August 2013, vol. 10, n^o 3, pp. 210-223 [*DOI :* 10.1016/J.DISOPT.2013.06.001], http://hal.inria.fr/hal-00909480

[11] C. ALVES, J. M. VALÉRIO DE CARVALHO, F. CLAUTIAUX, J. RIETZ. *Multidimensional dual-feasible functions and fast lower bounds for the vector packing problem*, in "European Journal of Operational Research", 2014, vol. 233, n^o 1, pp. 43-63, http://hal.inria.fr/hal-00909508

[12] F. CLAUTIAUX, A. JOUGLET, A. MOUKRIM. *A new graph-theoretical model for k-dimensional guillotine-cutting problems*, in "INFORMS Journal on Computing", 2013, vol. 25, n^o 1, pp. 72-86 [*DOI :* 10.1287/IJOC.1110.0478], http://hal.inria.fr/hal-00943849

[13] B. DETIENNE. *A Mixed Integer Linear Programming approach to minimize the number of late jobs with and without machine availability constraints*, in "European Journal of Operational Research", October 2013 [*DOI :* 10.1016/J.EJOR.2013.10.052], http://hal.inria.fr/hal-00880908

[14] M. T. GODINHO, L. GOUVEIA, P. PESNEAU. *Natural and Extended formulations for the Time-Dependent Traveling Salesman Problem*, in "Discrete Applied Mathematics", 2014, vol. 164, pp. 138-153 [*DOI :* 10.1016/J.DAM.2011.11.019], http://hal.inria.fr/hal-00648451

[15] J. HAN, K. LEE, C. LEE, S. PARK. *Exact Algorithms for a Bandwidth Packing Problem with Queueing Delay Guarantees*, in "INFORMS Journal on Computing", August 2013, vol. 25, n^{o} 3, pp. 585-596 [*DOI :* 10.1287/IJOC.1120.0523], http://hal.inria.fr/hal-00857862

[16] J. HAN, C. LEE, S. PARK. *A Robust Scenario Approach for the Vehicle Routing Problem with Uncertain Travel Times*, in "Transportation Science", August 2013 [*DOI :* 10.1287/TRSC.2013.0476], http://hal.inria.fr/hal-00857871

[17] A. PÊCHER, C. BACHOC, A. THIERY. *On the Theta number of powers of cycle graphs*, in "Combinatorica", 2014, to appear, http://hal.inria.fr/hal-00920927

[18] A. PÊCHER, G. J. CHANG, M. MONTASSIER, A. RASPAUD. *Strong chromatic index of planar graphs with large girth*, in "Discussiones Mathematicae Graph Theory", 2014, to appear, http://hal.inria.fr/hal-00920932

[19] A. PÊCHER, A. WAGLER. *Computing clique and chromatic number of circular-perfect graphs in polynomial time*, in "Mathematical Programming A", 2013, vol. 141, pp. 121-133, http://hal.inria.fr/hal-00760621

[20] A. PÊCHER, A. WAGLER. *Computing the clique number of a-perfect graphs in polynomial time*, in "European Journal of Combinatorics", 2014, vol. 35, pp. 449-458, http://hal.inria.fr/hal-00920846

[21] R. SADYKOV, F. VANDERBECK. *Bin Packing with conflicts: a generic branch-and-price algorithm*, in "INFORMS Journal on Computing", 2013, vol. 25, n^{o} 2, pp. 244-255 [*DOI :* 10.1287/IJOC.1120.0499], http://hal.inria.fr/inria-00539869

[22] R. SADYKOV, F. VANDERBECK. *Column Generation for Extended Formulations*, in "EURO Journal on Computational Optimization", May 2013, vol. 1, n^{o} 1-2, pp. 81-115 [*DOI :* 10.1007/S13675-013-0009-9], http://hal.inria.fr/hal-00661758

Invited Conferences

[23] A. PÊCHER. *Interval graphs for orthogonal packing problems*, in "International Conference in Discrete Mathematics", Dharwad, India, June 2013, http://hal.inria.fr/hal-00920835

[24] F. VANDERBECK. *Extended formulations, Column Generation, and stabilization: synergies in the benefit of large scale applications*, in "EURO INFORMS 26", Rome, Italy, July 2013, http://hal.inria.fr/hal-00845318

International Conferences with Proceedings

[25] O. BEAUMONT, L. EYRAUD-DUBOIS, P. PESNEAU, P. RENAUD-GOUD. *Reliable Service Allocation in Clouds with Memory and Capacity Constraints*, in "Resilience 2013, in conjunction with EuroPar 2013", Aachen, Germany, September 2013, http://hal.inria.fr/hal-00850125

[26] J. HAN, R. SADYKOV, F. VANDERBECK. *Parallel Machine Scheduling with Generalized Precedence Relations*, in "Multidisciplinary International Scheduling Conference: Theory & Applications", Gent, Belgium, August 2013, http://hal.inria.fr/hal-00857856

[27] A. PESSOA, R. SADYKOV, E. UCHOA, F. VANDERBECK. *In-Out Separation and Column Generation Stabilization by Dual Price Smoothing*, in "12th International Symposium on Experimental Algorithms", Rome, Italy, V. BONIFACI, C. DEMETRESCU, A. MARCHETTI-SPACCAMELA (editors), June 2013, Sringer,

June 2013, vol. LNCS 7933, pp. 354-365 [*DOI :* 10.1007/978-3-642-38527-8], http://hal.inria.fr/hal-00750412

[28] A. PÊCHER, M. MONTASSIER, A. RASPAUD. *Strong chromatic index of planar graphs with large girth*, in "Eurocomb'13-European Conference on Combinatorics, Graph Theory and Applications", Pisa, Italy, September 2013, to appear, http://hal.inria.fr/hal-00920842

[29] R. SADYKOV, A. LAZAREV, V. SHIRYAEV, A. STRATONNIKOV. *Solving a Freight Railcar Flow Problem Arising in Russia*, in "ATMOS - 13th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems - 2013", Sophia Antipolis, France, Dagstuhl Open Access Series in Informatics, 2013 [*DOI :* 10.4230/OASIcs.ATMOS.2013.55], http://hal.inria.fr/hal-00857914

Conferences without Proceedings

[30] S. BORNE, P. FOUILHOUX, R. GRAPPE, M. LACROIX, P. PESNEAU. *Circuit and bond polytopes in series-parallel graphs*, in "JPOC8 - 8èmes Journées Polyèdres et Optimisation Combinatoire", Clermont-Ferrand, France, 2013, http://hal.inria.fr/hal-00909472

[31] S. BORNE, P. FOUILHOUX, R. GRAPPE, M. LACROIX, P. PESNEAU. *Branch-and-Cut algorithm for the connected-cut problem*, in "ROADEF - 15ème congrès annuel de la Société française de recherche opérationnelle et d'aide à la décision", Bordeaux, France, February 2014, http://hal.inria.fr/hal-00946285

[32] M. BUÉ, F. CLAUTIAUX, L. BROTCORNE. *Conception d'un système de rangement à deux niveaux*, in "ROADEF - 15ème congrès annuel de la Société française de recherche opérationnelle et d'aide à la décision", Bordeaux, France, February 2014, http://hal.inria.fr/hal-00946388

[33] M. GERARD, F. CLAUTIAUX. *planification hebdomadaire sur plusieurs sites d'un personnel hétérogène et poly-compétent sous contraintes multiples*, in "ROADEF - 15ème congrès annuel de la Société française de recherche opérationnelle et d'aide à la décision", Bordeaux, France, February 2014, http://hal.inria.fr/hal-00946320

[34] J. HAN, P. BENDOTTI, B. DETIENNE, G. PETROU, M. PORCHERON, R. SADYKOV, F. VANDERBECK. *Extended Formulation for Maintenance Planning at Power Plants*, in "ROADEF - 15ème congrès annuel de la Société française de recherche opérationnelle et d'aide à la décision", Bordeaux, France, February 2014, http://hal.inria.fr/hal-00946294

[35] H. H. KRAMER, M. FAMPA, V. KÖHLER, E. UCHOA, F. VANDERBECK. *Staged Column Generation Approach for the Software Clustering Problem*, in "ROADEF - 15ème congrès annuel de la Société française de recherche opérationnelle et d'aide à la décision", Bordeaux, France, February 2014, http://hal.inria.fr/hal-00946286

[36] A. LINHARES, R. SADYKOV, F. VANDERBECK, L. DI PUGLIA PUGLIESE, F. GUERRIERO. *Dynamic programming algorithms for the (Elementary) Resource Constrained Shortest Path Problem*, in "26th European Conference on Operational Research", Rome, Italy, 2013, http://hal.inria.fr/hal-00857928

[37] A. PARREAU, F. CLAUTIAUX. *Coloration bornée avec multiplicité*, in "ROADEF - 15ème congrès annuel de la Société française de recherche opérationnelle et d'aide à la décision", Bordeaux, France, February 2014, http://hal.inria.fr/hal-00946329

[38] M. POGGI DE ARAGÃO, F. CASTILLA, R. SADYKOV, F. VANDERBECK. *The Vehicle Routing Problem with Time Windows, Limited Number of Vehicles and Exact Operation Synchronization (m-VRPTWEOS)*, in "26th European Conference on Operational Research", Rome, Italy, 2013, http://hal.inria.fr/hal-00857924

[39] R. SADYKOV, A. LAZAREV, V. SHIRYAEV, A. STRATONNIKOV. *Freight railcar routing problem in Russia*, in "26th European Conference on Operational Research", Rome, Italy, 2013, http://hal.inria.fr/hal-00857920

[40] F. VANDERBECK, A. PESSOA, R. SADYKOV, E. UCHOA. *Stabilization techniques for Column Generation: towards automated schemes*, in "EURO INFORMS 26", Rome, Italy, July 2013, http://hal.inria.fr/hal-00845858

Scientific Books (or Scientific Book chapters)

[41] P. BELOTTI, S. CAFIERI, J. LEE, L. LIBERTI, A. J. MILLER. *On the composition of convex envelopes for quadrilinear terms*, in "Optimization, Simulation, and Control", Springer Optimization and Its Applications, Springer Verlag, 2013, pp. 1-16 [*DOI :* 10.1007/978-1-4614-5131-0_1], http://hal.inria.fr/hal-00769671

[42] A. PÊCHER, A. WAGLER. *Beyond Perfection: Computational Results for Superclasses*, in "Facets of Combinatorial Optimization - Festschrift for Martin Grötschel", M. JÜNGER, G. REINELT (editors), Springer, September 2013, pp. 133-161, http://hal.inria.fr/hal-00920940

Other Publications

[43] D. C. PORUMBEL, F. CLAUTIAUX. , *Convergent Dual Bounds Using an Aggregation of Set-Covering Constraints for Capacitated Problems*, 2013, http://hal.inria.fr/hal-00747375

References in notes

[44] K. AKARTUNALI, A. J. MILLER. , *A Computational Analysis of Lower Bounds for Big Bucket Production Planning Problems*, 2009, http://hal.archives-ouvertes.fr/hal-00387105/en/

[45] K. AKARTUNALI, A. J. MILLER. *A heuristic approach for big bucket multi-level production planning problems*, in "European Journal of Operational Research", 2009, pp. 396-411, http://hal.archives-ouvertes.fr/hal-00387052/en/

[46] P. BAPTISTE, R. SADYKOV. *On Scheduling a Single Machine to Minimize a Piecewise Linear Objective Function : A Compact MIP Formulation*, in "Naval Research Logistics / Naval Research Logistics An International Journal", 2009, vol. 56, n^{o} 6, pp. 487–502, http://hal.inria.fr/inria-00387012/en/

[47] P. BAPTISTE, R. SADYKOV. *Time Indexed Formulations for Scheduling Chains on a Single Machine: An Application to Airborne Radars*, in "European Journal of Operational Research", 2009, http://hal.inria.fr/inria-00339639/en/

[48] M. CONSTANTINO, A. J. MILLER, M. VAN VYVE. *Mixing MIR Inequalities with Two Divisible Coefficients*, in "Mathematical Programming, Series A", 2009, pp. 1–1, http://hal.archives-ouvertes.fr/hal-00387098/en/

[49] G. DAHL, D. HUYGENS, A. R. MAHJOUB, P. PESNEAU. *On the k-edge disjoint 2-hop-constrained paths polytope*, in "Operations Research Letters", 2005, vol. 34, pp. 577–582

[50] B. DENTON, A. J. MILLER, H. BALASUBRAMANIAN, T. HUSCHKA. *Optimal Allocation of Surgery Blocks to Operating Rooms Under Uncertainty*, in "Operations Research", 2009, pp. 1–1, http://hal.archives-ouvertes.fr/hal-00386469/en/

[51] B. FORTZ, A. R. MAHJOUB, S. T. MCCORMICK, P. PESNEAU. *Two-edge connected subgraphs with bounded rings: polyhedral results and branch-and-cut*, in "Mathematical Programming", 2006, vol. 105, n^o 1, pp. 85–111

[52] M. T. GODINHO, L. GOUVEIA, T. L. MAGNANTI, P. PESNEAU, J. PIRES. , *On a Time-Dependent Model for the Unit Demand Vehicle Routing Problem*, Centro de Investigacao Operacional da Universidade de Lisboa, 2007, n^o 11-2007

[53] M. T. GODINHO, L. GOUVEIA, T. L. MAGNANTI, P. PESNEAU, J. PIRES. *On Time-Dependent Model for Unit Demand Vehicle Routing Problems*, in "International Conference on Network Optimization, INOC", International Network Optimization Conference (INOC), 2007

[54] Y. GUAN, S. AHMED, A. J. MILLER, G. NEMHAUSER. *On formulations of the stochastic uncapacitated lot-sizing problem*, in "Operations Research Letters", 2006, vol. 34, pp. 241-250

[55] Y. GUAN, S. AHMED, G. NEMHAUSER, A. J. MILLER. *A branch-and-cut algorithm for the stochastic uncapacitated lot-sizing problem*, in "Mathematical Programming", 2006, vol. 105, pp. 55-84

[56] L. GÉLY, G. DESSAGNE, P. PESNEAU, F. VANDERBECK. *A multi scalable model based on a connexity graph representation*, in "11th International Conference on Computer Design and Operation in the Railway and Other Transit Systems COMPRAIL'08", Toledo, Spain, September 2008

[57] L. GÉLY. *Real-time train scheduling at SNCF*, in "1st Workshop on Robust Planning and Rescheduling in Railways", ARRIVAL meeting on Robust planning and Rescheduling in Railways, April 2007

[58] J. HAN, P. BENDOTTI, B. DETIENNE, G. PETROU, M. PORCHERON, R. SADYKOV, F. VANDERBECK. *Extended Formulation for Maintenance Planning at Power Plants*, in "ROADEF 2014. Conférence Française de Recherche Opérationnelle et d'Aide à la Décision", 2014

[59] J. HAN, P. PESNEAU, A. PESSOA, E. UCHOA, F. VANDERBECK. *Column Generation Stabilization using Dual Smoothing: Theory & Practice*, in "Column Generation 2012", Bromont, Canada, June 2012, http://hal.inria.fr/hal-00750573

[60] Y. HENDEL, R. SADYKOV. *Timing problem for scheduling an airborne radar*, in "Proceedings of the 11th International Workshop on Project Management and Scheduling", Istanbul, Turkey, April 2008, pp. 132-135

[61] D. HUYGENS, M. LABBÉ, A. R. MAHJOUB, P. PESNEAU. *The two-edge connected hop-constrained network design problem: Valid inequalities and branch-and-cut*, in "Networks", 2007, vol. 49, n^o 1, pp. 116-133

[62] D. HUYGENS, A. R. MAHJOUB, P. PESNEAU. *Two edge-disjoint hop-constrained paths and polyhedra*, in "SIAM Journal of Discrete Mathematics", 2004, vol. 18, n^o 2, pp. 287–312

[63] C. JONCOUR. , *Problèmes de placement 2D et application à l ordonnancement : modélisation par la théorie des graphes et approches de programmation mathématique*, University Bordeaux I, December 2010

[64] C. JONCOUR, S. MICHEL, R. SADYKOV, D. SVERDLOV, F. VANDERBECK. *Column Generation based Primal Heuristics*, in "Submitted to ISCO 2010", 2010

[65] C. JONCOUR, A. PÊCHER, P. PESNEAU, F. VANDERBECK. *Mathematical programming formulations for the orthogonal 2d knapsack problem*, in "Livre des résumé du 9ème Congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision", February 2008, pp. 255–256, http://hal.archives-ouvertes.fr/hal-00307152/en/

[66] C. JONCOUR, A. PÊCHER. *Consecutive ones matrices for multi-dimensional orthogonal packing problems*, in "Journal of Mathematical Modelling and Algorithms", 2012, vol. 11, n^o 1, pp. 23-44 [*DOI :* 10.1007/s10852-011-9167-z], http://hal.inria.fr/hal-00652574

[67] C. JONCOUR, A. PÊCHER, P. VALICOV. *MPQ-trees for the orthogonal packing problem*, in "Journal of Mathematical Modelling and Algorithms", March 2012, vol. 11, n^o 1, pp. 3-22 [*DOI :* 10.1007/s10852-011-9159-z], http://hal.archives-ouvertes.fr/hal-00611528

[68] P. MEURDESOIF, P. PESNEAU, F. VANDERBECK. *Meter installation for monitoring network traffic*, in "International Conference on Network Optimization, INOC", International Network Optimization Conference (INOC), 2007

[69] P. MEURDESOIF, P. PESNEAU, F. VANDERBECK. *A Branch-and-Cut algorithm to optimize sensor installation in a network*, in "Graph and Optimization Meeting GOM2008", France Saint-Maximin, 2008

[70] S. MICHEL. , *Optimisation des tournées de véhicules combinées à la gestion de stock*, Université Bordeaux 1, Dec 2006

[71] S. MICHEL, N. PERROT, F. VANDERBECK. *Knapsack Problems with Setups*, in "European Journal of Operational Research", 2009, vol. 196, pp. 909-918, http://hal.inria.fr/inria-00232782/en/

[72] S. MICHEL, F. VANDERBECK. , *A Column Generation based Tactical Planning Method for Inventory Routing*, Inria, 2008, http://hal.inria.fr/inria-00169311/en/

[73] S. MICHEL, F. VANDERBECK. *A Column Generation based Tactical Planning Method for Inventory Routing*, in "Operations Research", 2012, vol. 60, n^o 2, pp. 382-397, http://hal.inria.fr/inria-00169311

[74] M. MOURGAYA, F. VANDERBECK. *The Periodic Vehicle Routing Problem: classification and heuristic*, in "RAIRO-OR", April-June 2006, vol. 40, n^o 2, pp. 169–194

[75] M. MOURGAYA, F. VANDERBECK. *Column generation based heuristic for tactical planning in multi period vehicle routing*, in "European Journal of Operational Research", 2007, vol. 183, n^o 3, pp. 1028-1041

[76] M. PADBERG, G. RINALDI. *A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems*, in "SIAM Review", 1991, vol. 33, n^o 1, pp. 60–100

[77] N. PERROT. , *Advanced IP column generation strategies for the cutting stock stock problem and its variants*, University Bordeaux 1, 2005

[78] N. PERROT, F. VANDERBECK. *Industrial Cutting Stock Problem*, in "Collection Sciences, Technologie Informatique", PRESSES UNIVERSITAIRES DE VALENCIENNES (editor), ROADEF 2006, 2006

[79] P. PESNEAU, R. SADYKOV, F. VANDERBECK. *Feasibility Pump Heuristics for Column Generation Approaches*, in "11th International Symposium on Experimental Algorithms", Bordeaux, France, LNCS series, Springer, June 2012, http://hal.inria.fr/hal-00686255

[80] M. POGGI, D. PECIN, M. REIS, C. FERREIRA, K. NEVES, R. SADYKOV, F. VANDERBECK. *Equipment/Operator task scheduling with BAPCOD*, in "Column Generation 2012", Bromont, Canada, June 2012, http://hal.inria.fr/hal-00750575

[81] A. PÊCHER, A. WAGLER. *Almost all webs are not rank-perfect*, in "Mathematical Programming", 2006, vol. 105, pp. 311–328

[82] R. SADYKOV. *A branch-and-check algorithm for minimizing the sum of the weights of the late jobs on a single machine with release dates*, in "European Journal of Operations Research", 2008, vol. 189, n^{o} 3, pp. 1284–1304, http://dx.doi.org/10.1016/j.ejor.2006.06.078

[83] R. SADYKOV. , *A polynomial algorithm for a simple scheduling problem at cross docking terminals*, Inria, 2009, RR-7054, http://hal.inria.fr/inria-00412519/en/

[84] R. SADYKOV. *On scheduling malleable jobs to minimise the total weighted completion time*, in "13th IFAC Symposium on Information Control Problems in Manufacturing", Russie Moscow, 2009, http://hal.inria.fr/inria-00339646/en/

[85] R. SADYKOV. *Scheduling incoming and outgoing trucks at cross docking terminals to minimize the storage cost*, in "Annals of Operations Research", 2012, vol. 201, n^{o} 1, pp. 423-440 [*DOI :* 10.1007/s10479-012-1232-0], http://hal.inria.fr/inria-00539849

[86] G. STAUFFER. *The p-median Polytope of Y-free Graphs: An Application of the Matching Theory*, in "Operations Research Letters", 2008, http://hal.inria.fr/inria-00442282

[87] A. SUTTER, F. VANDERBECK, L. A. WOLSEY. *Optimal Placement of Add/Drop Multiplexers: Heuristic and Exact Algorithms*, in "Operations Research", 1998, vol. 46, n^{o} 5, pp. 719–728

[88] F. VANDERBECK, J. HAN, P. PESNEAU, A. PESSOA, R. SADYKOV, E. UCHOA. *Column Generation Stabilization using Dual Smoothing: Theory and Practice*, in "Informs Annual Meeting: Informatics Rising", Phoenix, États-Unis, October 2012, http://hal.inria.fr/hal-00750576

[89] F. VANDERBECK, J. HAN, P. PESNEAU, A. PESSOA, R. SADYKOV, E. UCHOA. *Unifying procedures for Cut-Column Generation and Stabilization*, in "Workshop on Integer Programming", Valparaiso, Chili, March 2012, http://hal.inria.fr/hal-00750579

[90] F. VANDERBECK, C. JONCOUR, S. MICHEL, P. PESNEAU, A. PESSOA, M. POGGI, R. SADYKOV, E. UCHOA. *Primal heuristics for branch-and-price*, in "21th International Symposium on Mathematical Programming (ISMP 2012)", Berlin, Allemagne, August 2012, http://hal.inria.fr/hal-00750578

[91] F. VANDERBECK. *Exact algorithm for minimising the number of setups in the one-dimensional cutting stock problem*, in "Operations Research", 2000, vol. 48, n^{o} 5, pp. 915–926

[92] F. VANDERBECK. *A nested decomposition approach to a 3-stage 2-dimensional cutting stock problem*, in "Management Science", 2001, vol. 47, n^{o} 2, pp. 864–879

[93] F. VANDERBECK. *Extending Dantzig's Bound to the Bounded Multiple-class Knapsack Problem*, in "Mathematical Programming, Ser. A", 2002, vol. 94, n^{o} 1, pp. 125–136

[94] F. VANDERBECK. *Lot-sizing with start-up times*, in "Management Science", 1998, vol. 44, n^{o} 10, pp. 1409–1425

[95] F. VANDERBECK. *Computational study of a column generation algorithm for bin packing and cutting stock problems*, in "Mathematical Programming, Ser. A", 1999, vol. 86, pp. 565–594

[96] B. VIGNAC, B. JAUMARD, F. VANDERBECK. , *Hierarchical Heuristic for the GRWA Problem in WDM Networks with Delay Constraints*, Inria, 2009, 18 p. , http://hal.inria.fr/inria-00415513/en/

[97] B. VIGNAC, F. VANDERBECK, B. JAUMARD. , *Nested Decomposition Approach to an Optical Network Design Problem*, Inria, 2009, 18 p. , http://hal.inria.fr/inria-00415500/en/

[98] B. VIGNAC, F. VANDERBECK, B. JAUMARD. , *Reformulation and Decomposition Approaches for Traffic Routing in Optical Networks*, Inria, 2009, 36 p. , http://www.math.u-bordeaux.fr/~fv/papers/grwaWP.pdf, http://hal.inria.fr/inria-00392256/en/

[99] B. VIGNAC. , *Résolution d'un problème de groupage dans le réseaux optiques maillés*, Université de Montréal, January 2010