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Project-Team REGULARITY

Probabilistic modelling of irregularity and application to uncertainties management

IN COLLABORATION WITH: Laboratoire de Mathématiques Appliquées aux Systèmes (MAS)

RESEARCH CENTER
Saclay - Île-de-France

THEME
Stochastic approaches

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Project-Team REGULARITY

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2. Overall Objectives

2.1. Overall Objectives

Many phenomena of interest are analyzed and controlled through graphs or n-dimensional images. Often, these graphs have an *irregular aspect*, whether the studied phenomenon is of natural or artificial origin. In the first class, one may cite natural landscapes, most biological signals and images (EEG, ECG, MR images, ...), and temperature records. In the second class, prominent examples include financial logs and TCP traces.

Such irregular phenomena are usually not adequately described by purely deterministic models, and a probabilistic ingredient is often added. Stochastic processes allow to take into account, with a firm theoretical basis, the numerous microscopic fluctuations that shape the phenomenon.

In general, it is a wrong view to believe that irregularity appears as an epiphenomenon, that is conveniently dealt with by introducing randomness. In many situations, and in particular in some of the examples mentioned above, irregularity is a core ingredient that cannot be removed without destroying the phenomenon itself. In some cases, irregularity is even a necessary condition for proper functioning. A striking example is that of ECG: an ECG is inherently irregular, and, moreover, in a mathematically precise sense, an *increase* in its regularity is strongly correlated with a *degradation* of its condition.

In fact, in various situations, irregularity is a crucial feature that can be used to assess the behaviour of a given system. For instance, irregularity may the result of two or more sub-systems that act in a concurrent way to achieve some kind of equilibrium. Examples of this abound in nature (*e.g.* the sympathetic and parasympathetic systems in the regulation of the heart). For artifacts, such as financial logs and TCP traffic, irregularity is in a sense an unwanted feature, since it typically makes regulations more complex. It is again, however, a necessary one. For instance, efficiency in financial markets requires a constant flow of information among agents, which manifests itself through permanent fluctuations of the prices: irregularity just reflects the evolution of this information.

The aim of *Regularity* is a to develop a coherent set of methods allowing to model such "essentially irregular" phenomena in view of managing the uncertainties entailed by their irregularity.

Indeed, essential irregularity makes it more to difficult to study phenomena in terms of their description, modeling, prediction and control. It introduces *uncertainties* both in the measurements and the dynamics. It is, for instance, obviously easier to predict the short time behaviour of a smooth (*e.g.* C^1) process than of a nowhere differentiable one. Likewise, sampling rough functions yields less precise information than regular ones. As a consequence, when dealing with essentially irregular phenomena, uncertainties are fundamental in the sense that one cannot hope to remove them by a more careful analysis or a more adequate modeling. The study of such phenomena then requires to develop specific approaches allowing to manage in an efficient way these inherent uncertainties.

2.2. Highlights of the Year

J. Lévy Véhel was a finalist at the 2013 Humies competition in Amsterdam.

3. Research Program

3.1. Theoretical aspects: probabilistic modeling of irregularity

The modeling of essentially irregular phenomena is an important challenge, with an emphasis on understanding the sources and functions of this irregularity. Probabilistic tools are well-adapted to this task, provided one can design stochastic models for which the regularity can be measured and controlled precisely. Two points deserve special attention:

- first, the study of regularity has to be *local*. Indeed, in most applications, one will want to act on a system based on local temporal or spatial information. For instance, detection of arrhythmias in ECG or of krachs in financial markets should be performed in "real time", or, even better, ahead of time. In this sense, regularity is a *local* indicator of the *local* health of a system.
- Second, although we have used the term "irregularity" in a generic and somewhat vague sense, it seems obvious that, in real-world phenomena, regularity comes in many colors, and a rigorous analysis should distinguish between them. As an example, at least two kinds of irregularities are present in financial logs: the local "roughness" of the records, and the local density and height of jumps. These correspond to two different concepts of regularity (in technical terms, Hölder exponents and local index of stability), and they both contribute a different manner to financial risk.

In view of the above, the *Regularity* team focuses on the design of methods that:

1. define and study precisely various relevant measures of local regularity,
2. allow to build stochastic models versatile enough to mimic the rapid variations of the different kinds of regularities observed in real phenomena,
3. allow to estimate as precisely and rapidly as possible these regularities, so as to alert systems in charge of control.

Our aim is to address the three items above through the design of mathematical tools in the field of probability (and, to a lesser extent, statistics), and to apply these tools to uncertainty management as described in the following section. We note here that we do not intend to address the problem of controlling the phenomena based on regularity, that would naturally constitute an item 4 in the list above. Indeed, while we strongly believe that generic tools may be designed to measure and model regularity, and that these tools may be used to analyze real-world applications, in particular in the field of uncertainty management, it is clear that, when it comes to control, application-specific tools are required, that we do not wish to address.

The research topics of the *Regularity* team can be roughly divided into two strongly interacting axes, corresponding to two complementary ways of studying regularity:

1. developments of tools allowing to characterize, measure and estimate various notions of local regularity, with a particular emphasis on the stochastic frame,
2. definition and fine analysis of stochastic models for which some aspects of local regularity may be prescribed.

These two aspects are detailed in sections 3.2 and 3.3 below.

3.2. Tools for characterizing and measuring regularity

Fractional Dimensions

Although the main focus of our team is on characterizing *local* regularity, on occasions, it is interesting to use a *global* index of regularity. Fractional dimensions provide such an index. In particular, the *regularization dimension*, that was defined in [31], is well adapted to the study stochastic processes, as its definition allows to build robust estimators in an easy way. Since its introduction, regularization dimension has been used by various teams worldwide in many different applications including the characterization of certain stochastic processes, statistical estimation, the study of mammographies or galactograms for breast carcinomas detection, ECG analysis for the study of ventricular arrhythmia, encephalitis diagnosis from EEG, human skin analysis, discrimination between the nature of radioactive contaminations, analysis of porous media textures, well-logs data analysis, agro-alimentary image analysis, road profile analysis, remote sensing, mechanical systems assessment, analysis of video games, ...(see http://regularity.saclay.inria.fr/theory/localregularity/biblioregdim for a list of works using the regularization dimension).

Hölder exponents

The simplest and most popular measures of local regularity are the pointwise and local Hölder exponents. For a stochastic process $\{X(t)\}_{t\in\mathbb{R}}$ whose trajectories are continuous and nowhere differentiable, these are defined, at a point t_0, as the random variables:

$$\alpha_X(t_0,\omega) = \sup\left\{\alpha : \limsup_{\rho\to 0} \sup_{t,u\in B(t_0,\rho)} \frac{|X_t - X_u|}{\rho^\alpha} < \infty\right\}, \tag{1}$$

and

$$\widetilde{\alpha}_X(t_0,\omega) = \sup\left\{\alpha : \limsup_{\rho\to 0} \sup_{t,u\in B(t_0,\rho)} \frac{|X_t - X_u|}{\|t-u\|^\alpha} < \infty\right\}. \tag{2}$$

Although these quantities are in general random, we will omit as is customary the dependency in ω and X and write $\alpha(t_0)$ and $\widetilde{\alpha}(t_0)$ instead of $\alpha_X(t_0,\omega)$ and $\widetilde{\alpha}_X(t_0,\omega)$.

The random functions $t \mapsto \alpha_X(t_0,\omega)$ and $t \mapsto \widetilde{\alpha}_X(t_0,\omega)$ are called respectively the pointwise and local Hölder functions of the process X.

The pointwise Hölder exponent is a very versatile tool, in the sense that the set of pointwise Hölder functions of continuous functions is quite large (it coincides with the set of lower limits of sequences of continuous functions [6]). In this sense, the pointwise exponent is often a more precise tool (*i.e.* it varies in a more rapid way) than the local one, since local Hölder functions are always lower semi-continuous. This is why, in particular, it is the exponent that is used as a basis ingredient in multifractal analysis (see section 3.2). For certain classes of stochastic processes, and most notably Gaussian processes, it has the remarkable property that, at each point, it assumes an almost sure value [18]. SRP, mBm, and processes of this kind (see sections 3.3 and 3.3) rely on the sole use of the pointwise Hölder exponent for prescribing the regularity.

However, α_X obviously does not give a complete description of local regularity, even for continuous processes. It is for instance insensitive to "oscillations", contrarily to the local exponent. A simple example in the deterministic frame is provided by the function $x^{\gamma} \sin(x^{-\beta})$, where γ, β are positive real numbers. This so-called "chirp function" exhibits two kinds of irregularities: the first one, due to the term x^{γ} is measured by the pointwise Hölder exponent. Indeed, $\alpha(0) = \gamma$. The second one is due to the wild oscillations around 0, to which α is blind. In contrast, the local Hölder exponent at 0 is equal to $\frac{\gamma}{1+\beta}$, and is thus influenced by the oscillatory behaviour.

Another, related, drawback of the pointwise exponent is that it is not stable under integro-differentiation, which sometimes makes its use complicated in applications. Again, the local exponent provides here a useful complement to α, since $\widetilde{\alpha}$ is stable under integro-differentiation.

Both exponents have proved useful in various applications, ranging from image denoising and segmentation to TCP traffic characterization. Applications require precise estimation of these exponents.

Stochastic 2-microlocal analysis

Neither the pointwise nor the local exponents give a complete characterization of the local regularity, and, although their joint use somewhat improves the situation, it is far from yielding the complete picture.

A fuller description of local regularity is provided by the so-called *2-microlocal analysis*, introduced by J.M. Bony [53]. In this frame, regularity at each point is now specified by two indices, which makes the analysis and estimation tasks more difficult. More precisely, a function f is said to belong to the *2-microlocal space* $C_{x_0}^{s,s'}$, where $s+s'>0, s'<0$, if and only if its $m=[s+s']-$th order derivative exists around x_0, and if there exists $\delta>0$, a polynomial P with degree lower than $[s]-m$, and a constant C, such that

$$\left|\frac{\partial^m f(x)-P(x)}{|x-x_0|^{[s]-m}}-\frac{\partial^m f(y)-P(y)}{|y-x_0|^{[s]-m}}\right| \leq C|x-y|^{s+s'-m}(|x-y|+|x-x_0|)^{-s'-[s]+m}$$

for all x, y such that $0<|x-x_0|<\delta$, $0<|y-x_0|<\delta$. This characterization was obtained in [25], [32]. See [64], [65] for other characterizations and results. These spaces are stable through integro-differentiation, i.e. $f \in C_x^{s,s'}$ if and only if $f' \in C_x^{s-1,s'}$. Knowing to which space f belongs thus allows to predict the evolution of its regularity after derivation, a useful feature if one uses models based on some kind differential equations. A lot of work remains to be done in this area, in order to obtain more general characterizations, to develop robust estimation methods, and to extend the "2-microlocal formalism" : this is a tool allowing to detect which space a function belongs to, from the computation of the Legendre transform of an auxiliary function known as its *2-microlocal spectrum*. This spectrum provide a wealth of information on the local regularity.

In [18], we have laid some foundations for a stochastic version of 2-microlocal analysis. We believe this will provide a fine analysis of the local regularity of random processes in a direction different from the one detailed for instance in [69].We have defined random versions of the 2-microlocal spaces, and given almost sure conditions for continuous processes to belong to such spaces. More precise results have also been obtained for Gaussian processes. A preliminary investigation of the 2-microlocal behaviour of Wiener integrals has been performed.

Multifractal analysis of stochastic processes

A direct use of the local regularity is often fruitful in applications. This is for instance the case in RR analysis or terrain modeling. However, in some situations, it is interesting to supplement or replace it by a more global approach known as *multifractal analysis* (MA). The idea behind MA is to group together all points with same regularity (as measured by the pointwise Hölder exponent) and to measure the "size" of the sets thus obtained [28], [54], [60]. There are mainly two ways to do so, a geometrical and a statistical one.

In the geometrical approach, one defines the *Hausdorff multifractal spectrum* of a process or function X as the function: $\alpha \mapsto f_h(\alpha) = \dim\{t : \alpha_X(t) = \alpha\}$, where $\dim E$ denotes the Hausdorff dimension of the set E. This gives a fine measure-theoretic information, but is often difficult to compute theoretically, and almost impossible to estimate on numerical data.

The statistical path to MA is based on the so-called *large deviation multifractal spectrum*:

$$f_g(\alpha) = \lim_{\varepsilon \to 0} \liminf_{n \to \infty} \frac{\log\ N_n^\varepsilon(\alpha)}{\log\ n},$$

where:

$$N_n^\varepsilon(\alpha) = \#\{k : \alpha - \varepsilon \le \alpha_n^k \le \alpha + \varepsilon\},$$

and α_n^k is the "coarse grained exponent" corresponding to the interval $I_n^k = \left[\frac{k}{n}, \frac{k+1}{n}\right]$, *i.e.*:

$$\alpha_n^k = \frac{\log |Y_n^k|}{-\log\ n}.$$

Here, Y_n^k is some quantity that measures the variation of X in the interval I_n^k, such as the increment, the oscillation or a wavelet coefficient.

The large deviation spectrum is typically easier to compute and to estimate than the Hausdorff one. In addition, it often gives more relevant information in applications.

Under very mild conditions (*e.g.* for instance, if the support of f_g is bounded, [27]) the concave envelope of f_g can be computed easily from an auxiliary function, called the *Legendre multifractal spectrum*. To do so, one basically interprets the spectrum f_g as a rate function in a large deviation principle (LDP): define, for $q \in \mathbb{R}$,

$$S_n(q) = \sum_{k=0}^{n-1} |Y_n^k|^q, \tag{3}$$

with the convention $0^q := 0$ for all $q \in \mathbb{R}$. Let:

$$\tau(q) = \liminf_{n \to \infty} \frac{\log S_n(q)}{-\log(n)}.$$

The Legendre multifractal spectrum of X is defined as the Legendre transform τ^* of τ:

$$f_l(\alpha) := \tau^*(\alpha) := \inf_{q \in \mathbb{R}} (q\alpha - \tau(q)).$$

To see the relation between f_g and f_l, define the sequence of random variables $Z_n := \log |Y_n^k|$ where the randomness is through a choice of k uniformly in $\{0, ..., n-1\}$. Consider the corresponding moment generating functions:

$$c_n(q) := -\frac{\log E_n[\exp(qZ_n)]}{\log(n)}$$

where E_n denotes expectation with respect to P_n, the uniform distribution on $\{0, ..., n-1\}$. A version of Gärtner-Ellis theorem ensures that if $\lim c_n(q)$ exists (in which case it equals $1 + \tau(q)$), and is differentiable, then $c^* = f_g - 1$. In this case, one says that the *weak multifractal formalism* holds, *i.e.* $f_g = f_l$. In favorable cases, this also coincides with f_h, a situation referred to as the *strong multifractal formalism*.

Multifractal spectra subsume a lot of information about the distribution of the regularity, that has proved useful in various situations. A most notable example is the strong correlation reported recently in several works between the narrowing of the multifractal spectrum of ECG and certain pathologies of the heart [61], [63]. Let us also mention the multifractality of TCP traffic, that has been both observed experimentally and proved on simplified models of TCP [2], [49].

Another colour in local regularity: jumps

As noted above, apart from Hölder exponents and their generalizations, at least another type of irregularity may sometimes be observed on certain real phenomena: discontinuities, which occur for instance on financial logs and certain biomedical signals. In this frame, it is of interest to supplement Hölder exponents and their extensions with (at least) an additional index that measures the local intensity and size of jumps. This is a topic we intend to pursue in full generality in the near future. So far, we have developed an approach in the particular frame of *multistable processes*. We refer to section 3.3 for more details.

3.3. Stochastic models

The second axis in the theoretical developments of the *Regularity* team aims at defining and studying stochastic processes for which various aspects of the local regularity may be prescribed.

Multifractional Brownian motion

One of the simplest stochastic process for which some kind of control over the Hölder exponents is possible is probably fractional Brownian motion (fBm). This process was defined by Kolmogorov and further studied by Mandelbrot and Van Ness, followed by many authors. The so-called "moving average" definition of fBm reads as follows:

$$Y_t = \int_{-\infty}^{0} \left[(t-u)^{H-\frac{1}{2}} - (-u)^{H-\frac{1}{2}}\right].\mathbb{W}(du) + \int_{0}^{t} (t-u)^{H-\frac{1}{2}}.\mathbb{W}(du),$$

where $\mathbb{W}$ denotes the real white noise. The parameter H ranges in $(0,1)$, and it governs the pointwise regularity: indeed, almost surely, at each point, both the local and pointwise Hölder exponents are equal to H.

Although varying H yields processes with different regularity, the fact that the exponents are constant along any single path is often a major drawback for the modeling of real world phenomena. For instance, fBm has often been used for the synthesis natural terrains. This is not satisfactory since it yields images lacking crucial features of real mountains, where some parts are smoother than others, due, for instance, to erosion.

It is possible to generalize fBm to obtain a Gaussian process for which the pointwise Hölder exponent may be tuned at each point: the *multifractional Brownian motion (mBm)* is such an extension, obtained by substituting the constant parameter $H \in (0,1)$ with a *regularity function* $H : \mathbb{R}_+ \to (0,1)$.

mBm was introduced independently by two groups of authors: on the one hand, Peltier and Levy-Vehel [29] defined the mBm $\{X_t;\ t \in \mathbb{R}_+\}$ from the moving average definition of the fractional Brownian motion, and set:

$$X_t = \int_{-\infty}^{0} \left[(t-u)^{H(t)-\frac{1}{2}} - (-u)^{H(t)-\frac{1}{2}}\right].\mathbb{W}(du) + \int_{0}^{t} (t-u)^{H(t)-\frac{1}{2}}.\mathbb{W}(du),$$

On the other hand, Benassi, Jaffard and Roux [51] defined the mBm from the harmonizable representation of the fBm, *i.e.*:

$$X_t = \int_{\mathbb{R}} \frac{e^{it\xi}-1}{|\xi|^{H(t)+\frac{1}{2}}}.\widehat{\mathbb{W}}(d\xi),$$

where $\widehat{\mathbb{W}}$ denotes the complex white noise.

The Hölder exponents of the mBm are prescribed almost surely: the pointwise Hölder exponent is $\alpha_X(t) = H(t) \wedge \alpha_H(t)$ a.s., and the local Hölder exponent is $\widetilde{\alpha}_X(t) = H(t) \wedge \widetilde{\alpha}_H(t)$ a.s. Consequently, the regularity of the sample paths of the mBm are determined by the function H or by its regularity. The multifractional Brownian motion is our prime example of a stochastic process with prescribed local regularity.

The fact that the local regularity of mBm may be tuned *via* a functional parameter has made it a useful model in various areas such as finance, biomedicine, geophysics, image analysis, ... A large number of studies have been devoted worldwide to its mathematical properties, including in particular its local time. In addition, there is now a rather strong body of work dealing the estimation of its functional parameter, *i.e.* its local regularity. See http://regularity.saclay.inria.fr/theory/stochasticmodels/bibliombm for a partial list of works, applied or theoretical, that deal with mBm.

Self-regulating processes

We have recently introduced another class of stochastic models, inspired by mBm, but where the local regularity, instead of being tuned "exogenously", is a function of the amplitude. In other words, at each point t, the Hölder exponent of the process X verifies almost surely $\alpha_X(t) = g(X(t))$, where g is a fixed deterministic function verifying certain conditions. A process satisfying such an equation is generically termed a *self-regulating process* (SRP). The particular process obtained by adapting adequately mBm is called the self-regulating multifractional process [3]. Another instance is given by modifying the Lévy construction of Brownian motion [4]. The motivation for introducing self-regulating processes is based on the following general fact: in nature, the local regularity of a phenomenon is often related to its amplitude. An intuitive example is provided by natural terrains: in young mountains, regions at higher altitudes are typically more irregular than regions at lower altitudes. We have verified this fact experimentally on several digital elevation models [8]. Other natural phenomena displaying a relation between amplitude and exponent include temperatures records and RR intervals extracted from ECG [9].

To build the SRMP, one starts from a field of fractional Brownian motions $B(t, H)$, where (t, H) span $[0, 1] \times [a, b]$ and $0 < a < b < 1$. For each fixed H, $B(t, H)$ is a fractional Brownian motion with exponent H. Denote:

$$\underline{\overline{X}}_{\alpha'}^{\beta'} = \alpha' + (\beta' - \alpha') \frac{X - \min_K(X)}{\max_K(X) - \min_K(X)}$$

the affine rescaling between α' and β' of an arbitrary continuous random field over a compact set K. One considers the following (stochastic) operator, defined almost surely:

$$\begin{array}{rcl} \Lambda_{\alpha',\beta'} \ : \mathcal{C}\left([0,1], [\alpha,\beta]\right) & \to & \mathcal{C}\left([0,1], [\alpha,\beta]\right) \\ Z(.) & \mapsto & \underline{\overline{B(., g(Z(.))}}_{\alpha'}^{\beta'} \end{array}$$

where $\alpha \le \alpha' < \beta' \le \beta$, α and β are two real numbers, and α', β' are random variables adequately chosen. One may show that this operator is contractive with respect to the sup-norm. Its unique fixed point is the SRMP. Additional arguments allow to prove that, indeed, the Hölder exponent at each point is almost surely $g(t)$.

An example of a two dimensional SRMP with function $g(x) = 1 - x^2$ is displayed on figure 1.

We believe that SRP open a whole new and very promising area of research.

Multistable processes

Non-continuous phenomena are commonly encountered in real-world applications, *e.g.* financial records or EEG traces. For such processes, the information brought by the Hölder exponent must be supplemented by some measure of the density and size of jumps. Stochastic processes with jumps, and in particular Lévy processes, are currently an active area of research.

The simplest class of non-continuous Lévy processes is maybe the one of stable processes [71]. These are mainly characterized by a parameter $\alpha \in (0, 2]$, the *stability index* ($\alpha = 2$ corresponds to the Gaussian case, that we do not consider here). This index measures in some precise sense the intensity of jumps. Paths of stable processes with α close to 2 tend to display "small jumps", while, when α is near 0, their aspect is governed by large ones.

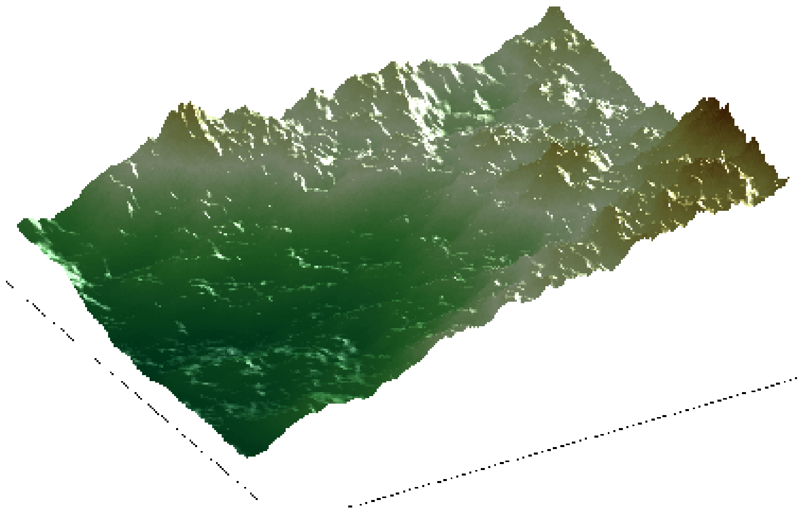

Figure 1. Self-regulating miltifractional process with $g(x) = 1 - x^2$

In line with our quest for the characterization and modeling of various notions of local regularity, we have defined *multistable processes*. These are processes which are "locally" stable, but where the stability index α is now a function of time. This allows to model phenomena which, at times, are "almost continuous", and at others display large discontinuities. Such a behaviour is for instance obvious on almost any sufficiently long financial record.

More formally, a multistable process is a process which is, at each time u, tangent to a stable process [59]. Recall that a process Y is said to be tangent at u to the process Y'_u if:

$$\lim_{r \to 0} \frac{Y(u + rt) - Y(u)}{r^h} = Y'_u(t), \tag{4}$$

where the limit is understood either in finite dimensional distributions or in the stronger sense of distributions. Note Y'_u may and in general will vary with u.

One approach to defining multistable processes is similar to the one developed for constructing mBm [29]: we consider fields of stochastic processes $X(t, u)$, where t is time and u is an independent parameter that controls the variation of α. We then consider a "diagonal" process $Y(t) = X(t, t)$, which will be, under certain conditions, "tangent" at each point t to a process $t \mapsto X(t, u)$.

A particular class of multistable processes, termed "linear multistable multifractional motions" (lmmm) takes the following form [11], [10]. Let $(E, \mathcal{E}, m)$ be a σ-finite measure space, and Π be a Poisson process on $E \times \mathbb{R}$ with mean measure $m \times \mathcal{L}$ ($\mathcal{L}$ denotes the Lebesgue measure). An lmmm is defined as:

$$Y(t) = a(t) \sum_{(\mathsf{X},\mathsf{Y}) \in \Pi} \mathsf{Y}^{<-1/\alpha(t)>} \left(|t - \mathsf{X}|^{h(t) - 1/\alpha(t)} - |\mathsf{X}|^{h(t) - 1/\alpha(t)} \right) \quad (t \in \mathbb{R}). \tag{5}$$

where $x^{<y>} := \text{sign}(x)|x|^y$, $a : \mathbb{R} \to \mathbb{R}^+$ is a C^1 function and $\alpha : \mathbb{R} \to (0, 2)$ and $h : \mathbb{R} \to (0, 1)$ are C^2 functions.

In fact, lmmm are somewhat more general than said above: indeed, the couple (h, α) allows to prescribe at each point, under certain conditions, both the pointwise Hölder exponent and the local intensity of jumps. In this sense, they generalize both the mBm and the linear multifractional stable motion [72]. From a broader perspective, such multistable multifractional processes are expected to provide relevant models for TCP traces, financial logs, EEG and other phenomena displaying time-varying regularity both in terms of Hölder exponents and discontinuity structure.

Figure 2 displays a graph of an lmmm with linearly increasing α and linearly decreasing H. One sees that the path has large jumps at the beginning, and almost no jumps at the end. Conversely, it is smooth (between jumps) at the beginning, but becomes jaggier and jaggier as time evolves.

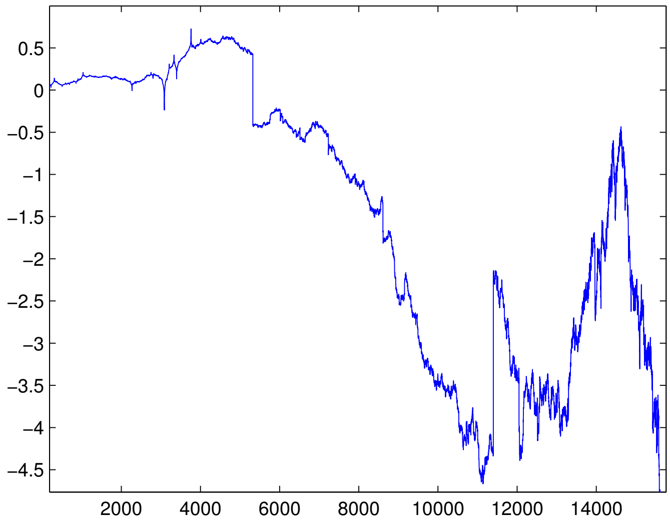

Figure 2. Linear multistable multifractional motion with linearly increasing α and linearly decreasing H

Multiparameter processes

In order to use stochastic processes to represent the variability of multidimensional phenomena, it is necessary to define extensions for indices in $\mathbb{R}^N$ $(N \geq 2)$ (see [66] for an introduction to the theory of multiparameter processes). Two different kinds of extensions of multifractional Brownian motion have already been considered: an isotropic extension using the Euclidean norm of $\mathbb{R}^N$ and a tensor product of one-dimensional processes on each axis. We refer to [15] for a comprehensive survey.

These works have highlighted the difficulty of giving satisfactory definitions for increment stationarity, Hölder continuity and covariance structure which are not closely dependent on the structure of $\mathbb{R}^N$. For example, the Euclidean structure can be unadapted to represent natural phenomena.

A promising improvement in the definition of multiparameter extensions is the concept of *set-indexed processes*. A set-indexed process is a process whose indices are no longer “times” or “locations” but may be some compact connected subsets of a metric measure space. In the simplest case, this framework is a generalization of the classical multiparameter processes [62]: usual multiparameter processes are set-indexed processes where the indexing subsets are simply the rectangles $[0, t]$, with $t \in \mathbb{R}_+^N$.

Set-indexed processes allow for greater flexibility, and should in particular be useful for the modeling of censored data. This situation occurs frequently in biology and medicine, since, for instance, data may not be constantly monitored. Censored data also appear in natural terrain modeling when data are acquired from sensors in presence of hidden areas. In these contexts, set-indexed models should constitute a relevant frame.

A set-indexed extension of fBm is the first step toward the modeling of irregular phenomena within this more general frame. In [20], the so-called *set-indexed fractional Brownian motion (sifBm)* was defined as the mean-zero Gaussian process $\{\mathbf{B}^H_U;\ U \in \mathcal{A}\}$ such that

$$\forall U, V \in \mathcal{A};\quad E[\mathbf{B}^H_U\ \mathbf{B}^H_V] = \frac{1}{2}\left[m(U)^{2H} + m(V)^{2H} - m(U \bigtriangleup V)^{2H}\right]$$

where $\mathcal{A}$ is a collection of connected compact subsets of a measure metric space and $0 < H \leq \frac{1}{2}$.

This process appears to be the only set-indexed process whose projection on increasing paths is a one-parameter fractional Brownian motion [19]. The construction also provides a way to define fBm's extensions on non-euclidean spaces, *e.g.* indices can belong to the unit hyper-sphere of $\mathbb{R}^N$. The study of fractal properties needs specific definitions for increment stationarity and self-similarity of set-indexed processes [22]. We have proved that the sifBm is the only Gaussian set-indexed process satisfying these two (extended) properties.

In the specific case of the indexing collection $\mathcal{A} = \{[0,t], t \in \mathbb{R}^N_+\} \cup \{\varnothing\}$, the sifBm can be seen as a multiparameter extension of fBm which is called *multiparameter fractional Brownian motion (MpfBm)*. This process differs from the Lévy fractional Brownian motion and the fractional Brownian sheet, which are also multiparameter extensions of fBm (but do not derive from set-indexed processes). The local behaviour of the sample paths of the MpfBm has been studied in [14]. The self-similarity index H is proved to be the almost sure value of the local Hölder exponent at any point, and the Hausdorff dimension of the graph is determined in function of H.

The increment stationarity property for set-indexed processes, previously defined in the study of the sifBm, allows to consider set-indexed processes whose increments are independent and stationary. This generalizes the definition of Bass-Pyke and Adler-Feigin for Lévy processes indexed by subsets of $\mathbb{R}^N$, to a more general indexing collection. We have obtained a Lévy-Khintchine representation for these set-indexed Lévy processes and we also characterized this class of Markov processes.

4. Application Domains

4.1. Uncertainties management

Our theoretical works are motivated by and find natural applications to real-world problems in a general frame generally referred to as uncertainty management, that we describe now.

Since a few decades, modeling has gained an increasing part in complex systems design in various fields of industry such as automobile, aeronautics, energy, etc. Industrial design involves several levels of modeling: from behavioural models in preliminary design to finite-elements models aiming at representing sharply physical phenomena. Nowadays, the fundamental challenge of numerical simulation is in designing physical systems while saving the experimentation steps.

As an example, at the early stage of conception in aeronautics, numerical simulation aims at exploring the design parameters space and setting the global variables such that target performances are satisfied. This iterative procedure needs fast multiphysical models. These simplified models are usually calibrated using high-fidelity models or experiments. At each of these levels, modeling requires control of uncertainties due to simplifications of models, numerical errors, data imprecisions, variability of surrounding conditions, etc.

One dilemma in the design by numerical simulation is that many crucial choices are made very early, and thus when uncertainties are maximum, and that these choices have a fundamental impact on the final performances.

Classically, coping with this variability is achieved through *model registration* by experimenting and adding fixed *margins* to the model response. In view of technical and economical performance, it appears judicious to replace these fixed margins by a rigorous analysis and control of risk. This may be achieved through a probabilistic approach to uncertainties, that provides decision criteria adapted to the management of unpredictability inherent to design issues.

From the particular case of aircraft design emerge several general aspects of management of uncertainties in simulation. Probabilistic decision criteria, that translate decision making into mathematical/probabilistic terms, require the following three steps to be considered [58]:

1. build a probabilistic description of the fluctuations of the model's parameters (*Quantification* of uncertainty sources),
2. deduce the implication of these distribution laws on the model's response (*Propagation* of uncertainties),
3. and determine the specific influence of each uncertainty source on the model's response variability (*Sensitivity Analysis*).

The previous analysis now constitutes the framework of a general study of uncertainties. It is used in industrial contexts where uncertainties can be represented by *random variables* (unknown temperature of an external surface, physical quantities of a given material, ... at a given *fixed time*). However, in order for the numerical models to describe with high fidelity a phenomenon, the relevant uncertainties must generally depend on time or space variables. Consequently, one has to tackle the following issues:

- *How to capture the distribution law of time (or space) dependent parameters, without directly accessible data?* The distribution of probability of the continuous time (or space) uncertainty sources must describe the links between variations at neighbor times (or points). The local and global regularity are important parameters of these laws, since it describes how the fluctuations at some time (or point) induce fluctuations at close times (or points). The continuous equations representing the studied phenomena should help *to propose models for the law of the random fields*. Let us notice that interactions between various levels of modeling might also be used to derive distributions of probability at the lowest one.
- The navigation between the various natures of models needs a kind of *metric* which could *mathematically describe the notion of granularity or fineness* of the models. Of course, the local regularity will not be totally absent of this mathematical definition.
- All the various levels of conception, preliminary design or high-fidelity modelling, require *registrations by experimentation* to reduce model errors. This *calibration* issue has been present in this frame since a long time, especially in a deterministic optimization context. The random modeling of uncertainty requires the definition of a systematic approach. The difficulty in this specific context is: statistical estimation with few data and estimation of a function with continuous variables using only discrete setting of values.

Moreover, a multi-physical context must be added to these questions. The complex system design is most often located at the interface between several disciplines. In that case, modeling relies on a coupling between several models for the various phenomena and design becomes a *multidisciplinary optimization* problem. In this uncertainty context, the real challenge turns robust optimization to manage technical and economical risks (risk for non-satisfaction of technical specifications, cost control).

We participate in the uncertainties community through several collaborative research projects. As explained above, we focus on essentially irregular phenomena, for which irregularity is a relevant quantity to capture the variability (e.g. certain biomedical signals, terrain modeling, financial data, etc.). These will be modeled through stochastic processes with prescribed regularity.

4.2. Biomedical Applications

ECG analysis and modelling

ECG and signals derived from them are an important source of information in the detection of various pathologies, including *e.g.* congestive heart failure, arrhythmia and sleep apnea. The fact that the irregularity of ECG bears some information on the condition of the heart is well documented (see *e.g.* the web resource http://www.physionet.org). The regularity parameters that have been studied so far are mainly the box and regularization dimensions, the local Hölder exponent and the multifractal spectrum [61], [63]. These have been found to correlate well with certain pathologies in some situations. From a general point of view, we participate in this research area in two ways.

- First, we use refined regularity characterizations, such as the regularization dimension, 2-microlocal analysis and advanced multifractal spectra for a more precise analysis of ECG data. This requires in particular to test current estimation procedures and to develop new ones.
- Second, we build stochastic processes that mimic in a faithful way some features of the dynamics of ECG. For instance, the local regularity of RR intervals, estimated in a parametric way based on a modelling by an mBm, displays correlations with the amplitude of the signal, a feature that seems to have remained unobserved so far [3]. In other words, RR intervals behave as SRP. We believe that modeling in a simplified way some aspects of the interplay between the sympathetic and parasympathetic systems might lead to an SRP, and to explain both this self-regulating property and the reasons behind the observed multifractality of records. This will open the way to understanding how these properties evolve under abnormal behaviour.

Pharmacodynamics and patient drug compliance

Poor adherence to treatment is a worldwide problem that threatens efficacy of therapy, particularly in the case of chronic diseases. Compliance to pharmacotherapy can range from 5% to 90%. This fact renders clinical tested therapies less effective in ambulatory settings. Increasing the effectiveness of adherence interventions has been placed by the World Health Organization at the top list of the most urgent needs for the health system. A large number of studies have appeared on this new topic in recent years [75], [74]. In collaboration with the pharmacy faculty of Montréal university, we consider the problem of compliance within the context of multiple dosing. Analysis of multiple dosing drug concentrations, with common deterministic models, is usually based on patient full compliance assumption, *i.e.*, drugs are administered at a fixed dosage. However, the drug concentration-time curve is often influenced by the random drug input generated by patient poor adherence behaviour, inducing erratic therapeutic outcomes. Following work already started in Montréal [67], [68], we consider stochastic processes induced by taking into account the random drug intake induced by various compliance patterns. Such studies have been made possible by technological progress, such as the "medication event monitoring system", which allows to obtain data describing the behaviour of patients.

We use different approaches to study this problem: statistical methods where enough data are available, model-based ones in presence of qualitative description of the patient behaviour. In this latter case, piecewise deterministic Markov processes (PDP) seem a promising path. PDP are non-diffusion processes whose evolution follows a deterministic trajectory governed by a flow between random time instants, where it undergoes a jump according to some probability measure [55]. There is a well-developed theory for PDP, which studies stochastic properties such as extended generator, Dynkin formula, long time behaviour. It is easy to cast a simplified model of non-compliance in terms of PDP. This has allowed us already to obtain certain properties of interest of the random concentration of drug [37]. In the simplest case of a Poisson distribution, we have obtained rather precise results that also point to a surprising connection with infinite Bernouilli convolutions [37], [13], [12]. Statistical aspects remain to be investigated in the general case.

5. Software and Platforms

5.1. FracLab

Participants: Paul Balança, Jacques Lévy Véhel [correspondant].

FracLab was developed for two main purposes:

1. propose a general platform allowing research teams to avoid the need to re-code basic and advanced techniques in the processing of signals based on (local) regularity.
2. provide state of the art algorithms allowing both to disseminate new methods in this area and to compare results on a common basis.

FracLab is a general purpose signal and image processing toolbox based on fractal, multifractal and local regularity methods. FracLab can be approached from two different perspectives:

- (multi-) fractal and local regularity analysis: A large number of procedures allow to compute various quantities associated with 1D or 2D signals, such as dimensions, Hölder and 2-microlocal exponents or multifractal spectra.
- Signal/Image processing: Alternatively, one can use FracLab directly to perform many basic tasks in signal processing, including estimation, detection, denoising, modeling, segmentation, classification, and synthesis.

A graphical interface makes FracLab easy to use and intuitive. In addition, various wavelet-related tools are available in FracLab.

FracLab is a free software. It mainly consists of routines developed in MatLab or C-code interfaced with MatLab. It runs under Linux, MacOS and Windows environments. In addition, a "stand-alone" version (*i.e.* which does not require MatLab to run) is available.

Fraclab has been downloaded several thousands of times in the last years by users all around the world. A few dozens laboratories seem to use it regularly, with more than four hundreds registered users. Our ambition is to make it the standard in fractal softwares for signal and image processing applications. We have signs that this is starting to become the case. To date, its use has been acknowledged in roughly three hundreds research papers in various areas such as astrophysics, chemical engineering, financial modeling, fluid dynamics, internet and road traffic analysis, image and signal processing, geophysics, biomedical applications, computer science, as well as in mathematical studies in analysis and statistics (see http://fraclab.saclay.inria.fr/ for a partial list with papers). In addition, we have opened the development of FracLab so that other teams worldwide may contribute. Additions have been made by groups in Australia, England, France, the USA, and Serbia.

Last year, we produced a major release of FracLab (version 2.1). This year, we corrected a number of bugs.

6. New Results

6.1. Stochastic integration with respect to the Rosenblatt process.

Participant: Benjamin Arras.

From a theoretical perspective to more concrete applications, fractional Brownian motion (fbm) is a fruitful and rich mathematical object. From its stochastic analysis, initiated during the nineties, several theories of stochastic integration have emerged so far. Indeed, fbm is, in general, not a semimartingale neither a Markov process. These theories rely on different properties of the stochastic integrator process and are then of different natures. Despite the quite large number of these strategies, we can group them into two fundamentally distinct categories: the pathwise and the probabilistic approaches. The probabilistic one requires highly evolved stochastic analysis tools. Indeed, the Malliavin calculus as well as Hida's distribution theory have been used in order to define stochastic integration with respect to fractional Brownian motion ([56], [52]) and more general Gaussian processes ([47]). Moreover, fbm belongs to an important class of stochastic processes, namely, the Hermite processes. This class appears in non-central limit theorems for processes defined as integrals or partial sums of non-linear functionals of stationary Gaussian sequences with long-range dependence (see [57]). They admit the following representation for all $d \geq 1$:

$$\forall t > 0 \quad Y_t^{H,d} = c(H_0) \int_{\mathbb{R}} ... \int_{\mathbb{R}} \left(\int_0^t \prod_{j=1}^{d} (s - x_j)_+^{H_0 - 1} ds \right) dB_{x_1} ... dB_{x_d}$$

where $c(H_0)$ is a normalizing constant such that $\mathbb{E}[|Y_1^{H,d}|^2] = 1$ and $H_0 = \frac{1}{2} + \frac{H-1}{d}$ with $H \in (\frac{1}{2}, 1)$. For $d = 1$, one recovers fractional Brownian motion. These processes share many properties with fbm. Namely, they are H-self-similar processes with stationary increments. They possess the same covariance structure, exhibit long range-dependence and their sample paths are almost-surely δ-Hölder continuous, for every $\delta < H$. For $d = 2$, the process is called the Rosenblatt process. This process has received lots of interest in the past and more recent years. Stochastic calculus with respect to the Rosenblatt process has been developped in [73] from both, the pathwise type calculus and Malliavin calculus points of view. Even if these two approaches are successful in order to define a stochastic integral with respect to the Rosenblatt process, the Malliavin calculus one fails to give an Itô's formula for the Rosenblatt process in the divergence sense. In [42], by means of white noise distribution theory, we obtain the following result:
Theorem: Let $(a, b) \in \mathbb{R}_+^*$ such that $a \le b < \infty$. Let F be an entire analytic function of the complex variable verifying:

$$\exists N \in \mathbb{N}, \exists C > 0, \forall z \in \mathbb{C} \quad |F(z)| \le C(1 + |z|)^N \exp(\frac{1}{\sqrt{2}b^H} |\Im(z)|)$$

Then, we have in $(S)^*$:

$$F(X_b^H) - F(X_a^H) = \int_a^b F^{(1)}(X_t^H) \diamond \dot{X}_t^H dt + \sum_{k=2}^{\infty} \left(H\kappa_k(X_1^H) \int_a^b \frac{t^{Hk-1}}{(k-1)!} F^{(k)}(X_t^H) dt + 2^k \int_a^b F^{(k)}(X_t^H) \diamond \dot{X}_t^{H,k} dt \right),$$

where $\{X_t^H\} = \{Y_t^{H,2}\}$, $\{\dot{X}_t^H\}$ is the Rosenblatt noise, $\{\kappa_k(X_1^H); k \ge 2\}$ the non-zero cumulants of the Rosenblatt distribution, $\diamond$ the Wick product and $\{\{X_t^{H,k}\} : k \ge 2\}$ a sequence of processes defined by:

$$\forall t \ge 0 \quad X_t^{H,k} = \int_{\mathbb{R}} \int_{\mathbb{R}} \underbrace{(...((f_t^H \otimes_1 f_t^H) \otimes_1 f_t^H)... \otimes_1 f_t^H)}_{k-1 \times \otimes_1}(x_1, x_2) dB_{x_1} dB_{x_2}$$

with $f_t^H(x_1, x_2) = c(H) \int_0^t \prod_{j=1}^2 (s - x_j)_+^{\frac{H}{2}-1} ds$ and $\otimes_1$ is the contraction of order 1.
Moreover, in the same setting, we obtain the following "isometry" result for the Rosenblatt noise integral of sufficiently "good" integrand processes:
Theorem: Let $\{\phi_t; t \in I\}$ be a stochastic process such that for all $t \in I$ (I an interval), $\phi_t \in (L^2)$ and such that the Rosenblatt noise integral of $\{\phi_t\}$ exists in $(S)^*$. Moreover, let us assume that:

$$\sum_{m=0}^{+\infty} (m+2)! \int_I \int_I |t-s|^{2(H-1)} < f_m(.,t); f_m(.,s) >_{L^2(\mathbb{R}^m)} dt ds < +\infty,$$

where $\phi_t = \sum_{m=0}^{+\infty} I_m(f_m(.,t))$. Thus, we have:

$$\begin{aligned}
\mathbb{E}[(\int_I \phi_t \diamond \dot{X}_t^H dt)^2] &= H(2H-1) \int_I \int_I |t-s|^{2(H-1)} \mathbb{E}[\phi_t \phi_s] ds dt \\
&+ 4\sqrt{\frac{H(2H-1)}{2}} \int_I \int_I |t-s|^{H-1} \mathbb{E}[D_{\sqrt{d(H)}\delta_s \circ I_+^{\frac{H}{2}}}(\phi_t) D_{\sqrt{d(H)}\delta_t \circ I_+^{\frac{H}{2}}}(\phi_s)] ds dt \\
&+ \int_I \int_I \mathbb{E}[(D_{\sqrt{d(H)}\delta_s \circ I_+^{\frac{H}{2}}})^2(\phi_t)(D_{\sqrt{d(H)}\delta_t \circ I_+^{\frac{H}{2}}})^2(\phi_s)] ds dt,
\end{aligned}$$

where $D_{\sqrt{d(H)}\delta_s \circ I_+^{\frac{H}{2}}}$ is the derivative operator in the direction $\sqrt{d(H)}\delta_s \circ I_+^{\frac{H}{2}}$.
Finally, in the last section of [42], we compare our approach to the one of [73]. More specifically, we prove that the stochastic integral with respect to the Rosenblatt process built using Malliavin calculus corresponds with the Rosenblatt noise integral when both of them exist.

Proposition: Let $\{\phi_t; t \in [0;T]\}$ be a stochastic process such that $\phi \in L^2(\Omega; \mathcal{H}) \cap L^2([0,T]; \mathbb{D}^{2,2})$ and $\mathbb{E}[\int_0^T \int_0^T ||D_{s_1,s_2}\phi||^2_{\mathcal{H}}]ds_1 ds_2 < \infty$ where

$$\mathcal{H} = \{f : [0;T] \to \mathbb{R}; \int_0^T \int_0^T f(s)f(t)|t-s|^{2H-2} ds dt < \infty\}.$$

Then, $\{\phi_t\}$ is Skorohod integrable and $(S)^*$-integrable with respect to the Rosenblatt process, $\{Z_t^H\}_{t\in[0;T]}$, and we have:

$$\int_0^T \phi_t \delta Z_t^H = \int_0^T \phi_t \diamond \dot{Z}_t^H dt$$

6.2. Sample path properties of multifractional Brownian motion

Participants: Paul Balança, Erick Herbin [supervision].

In [50], we have investigated the geometry of the sample paths of multifractional Brownian motion. Several representations of mBm exist, including the classic integral form:

$$\forall t \in \mathbf{R}; \quad X_t = \frac{1}{\Gamma\left(H(t)+\frac{1}{2}\right)} \int_{\mathbf{R}} \left[(t-u)_+^{H(t)-1/2} - (-u)_+^{H(t)-1/2}\right] \mathrm{d}W_u,$$

where $H : \mathbf{R} \mapsto (0,1)$ is a continuous function. Interestingly, we observe that geometric properties obtained in the probabilistic literature usually rely on a key assumption on the behaviour of the Hurst function:

$$H \text{ is a } \beta\text{-Hölder continuous function such that } \forall t \in \mathbf{R},\ H(t) < \beta. \qquad (\mathcal{H}_0) \tag{6}$$

Under the previous hypothesis, the local regularity of the mBm at t corresponds to the geometry of a fractional Brownian motion of parameter $H(t)$. Nevertheless, it has been shown in [15] that when this assumption does not hold, the sample path properties are not as simple and straightforward. More precisely, the latter has proved that the Hölder exponents satisfy at every $t \in \mathbf{R}$:

$$\alpha_{X,t} = H(t) \wedge \alpha_{H,t} \quad \text{and} \quad \widetilde{\alpha}_{X,t} = H(t) \wedge \widetilde{\alpha}_{H,t} \quad \text{a.s.} \tag{7}$$

This result has been recently improved in [48], observing that the pointwise exponent can even be random under some assumptions on H.

Therefore, the main goal of this work was to obtain a more complete characterization of the geometry of the general mBm. We have first focused on the Hölder regularity of the sample paths, using for this purpose a deterministic representation of the fractional Brownian field:

$$B^{\pm}(t,H) = \frac{\pm 1}{\Gamma\left(H-\frac{1}{2}\right)} \int_{\mathbf{R}} B_u \left[(t-u)_{\pm}^{H-3/2} - (-u)_{\pm}^{H-3/2}\right] \mathrm{d}u, \tag{8}$$

where $H \geq 1/2$ and B is a continuous Brownian motion. Hence, observing that the mBm almost surely corresponds to the fractional integration of a Brownian motion, we have been able to use the 2-microlocal formalism and its interesting connections with fractional operators. As a consequence, we have proved that the pointwise exponent of the mBm almost surely satisfies:

$$\forall t \in \mathbf{R}; \quad \alpha_{X,t} = H(t) \wedge m_{t,H(t)} \alpha_{H,t}, \tag{9}$$

where $m_{t,H(t)}$ is defined as the multiplicity of the fractional Brownian field at (t, H), i.e.

$$m_{t,H} = \inf \left\{ k \in \mathbf{N} \setminus \{0\} : \partial_H^k B(t,H) \neq 0 \right\}.$$

We have also been able to obtain some uniform lower bounds on the 2-microlocal frontier, which are optimal under some mild assumptions on the Hurst function.

The second direction of our study has concerned the fractal dimension of the graph of the mBm. Interestingly, and on the contrary to fBm, we have to distinguish the Box and Hausdorff dimensions in our result. The first happens to be the easiest one to study and is closely related to the geometry of H itself. Therefore, with probability one,

$$\forall t \in \mathbf{R} \setminus \{0\}; \quad \dim_{\mathrm{B},t} \mathrm{Gr}(X) = (2 - H(t)) \vee \dim_{\mathrm{B},t} \mathrm{Gr}(H), \tag{10}$$

where $\dim_{\mathrm{B},t}$ denotes the localized Box dimension at t.

To study the Hausdorff dimension the graph, we need a slightly different approach which makes use of parabolic Hausdorff dimension. We first define for all $t \in \mathbf{R}$ a *parabolic metric* ϱ_H on $\mathbf{R}^2$, with $H > 0$: $\varrho_H\left((u,x);(v,y)\right) := \max\left(|u-v|^H, |x-y|\right)$. For any set $A \subset \mathbf{R}^2$, we denote by $\dim_{\mathcal{H}}(A;\varrho_H)$ the *parabolic Hausdorff dimension* of A. It is defined similarly to the classic Hausdorff dimension using covering balls relatively to the metric ϱ_H, i.e. it corresponds to the infimum of $s \geq 0$ for which

$$\lim_{\delta \to 0} \inf \left\{ \sum_{i=0}^{\infty} \text{diam } (O_i;\varrho_H)^s : (O_i)_{i \in N} \text{ is a } \delta\text{-cover of } A \right\} < \infty$$

Studying the local Hausdorff dimension of the graph of the mBm, we have proved that with probability one

$$\forall t \in \mathbf{R} \setminus \{0\}; \quad \dim_{\mathrm{H},t} \mathrm{Gr}(X) = 1 + H(t)\left(\dim_{\mathrm{H},t}\left(\mathrm{Gr}(H);\varrho_{H(t)}\right) - 1\right). \tag{11}$$

Even though this result might seem counter-intuitive, it can be checked that it induced the classic equality $\dim_{\mathrm{H},t} \mathrm{Gr}(X) = 2 - H(t)$ when the mBm satisfies the assumption $\mathcal{H}_0$. Interestingly, we observe that a similar expression has also emerged recently in the study [70] of the Hausdorff dimension of a fractional Brownian motion with variable drift. Finally, we also note this result can also been extended to images of fractal sets by the multifractional Brownian motion.

6.3. Large Deviations Inequalities

Participant: Xiequan Fan.

Let $(\xi_i)_{i=1,\ldots,n}$ be a sequence of independent and centered random variables satisfying Bernstein's condition, for a constant $\varepsilon > 0$,

$$|\mathbb{E}\xi_i^k| \leq \frac{1}{2} k! \varepsilon^{k-2} \mathbb{E}\xi_i^2, \quad \text{for all } k \geq 2 \text{ and all } i = 1, \ldots, n. \tag{12}$$

Denote by

$$S_n = \sum_{i=1}^n \xi_i \quad \text{and} \quad \sigma^2 = \sum_{i=1}^n \mathbb{E}\xi_i^2. \tag{13}$$

The well-known Bernstein inequality (1946) states that, for all $x > 0$,

$$\mathbb{P}(S_n > x\sigma) \ \leq \ \inf_{\lambda \geq 0} \mathbb{E} e^{\lambda (S_n - x\sigma)}. \tag{14}$$

In the i.i.d. case, Cramér (1938) has established a large deviation expansion under the condition $\mathbb{E}e^{|\xi_1|} < \infty$. For all $0 \leq x = o\left(\sqrt{n}\right)$, one has

$$\frac{\mathbb{P}(S_n > x\sigma)}{1 - \Phi(x)} = e^{\frac{x^3}{\sqrt{n}} \lambda\left(\frac{x}{\sqrt{n}}\right)} \left[1 + O\left(\frac{1+x}{\sqrt{n}}\right)\right], \quad n \to \infty, \tag{15}$$

where $\lambda(\cdot) = c_1 + c_2 \frac{x}{\sqrt{n}} + ...$ is the Cramér series and the values $c_1, c_2, ...$ depend on the distribution of ξ_1.

Bahadur-Rao (1960) proved the following sharp large deviations similar to (15). Assume Cramér's condition. Then, for given $y > 0$, there is a constant c_y depending on the distribution of ξ_1 and y such that

$$\mathbb{P}\left(\frac{S_n}{n} > y\right) = \frac{\inf_{\lambda \geq 0} \mathbb{E} e^{\lambda (S_n - yn)}}{\sigma_y t_y \sqrt{2\pi n}} \left[1 + O\left(\frac{c_y}{n}\right)\right], \quad n \to \infty, \tag{16}$$

where t_y, σ_y and c_y depend on the distribution of ξ_1 and y.

We present an improvement on Bernstein's inequality. In particular, we establish a sharp large deviation expansion similar to the classical results of Cramér and Bahadur-Rao. The following theorem is our main result.

Theorem 0.1 Assume Bernstein's condition. Then, for all $0 \leq x < \frac{1}{12}\frac{\sigma}{\varepsilon}$,

$$\mathbb{P}(S_n > x\sigma) = \inf_{\lambda \geq 0} \mathbb{E} e^{\lambda (S_n - x\sigma)} F\left(x, \frac{\varepsilon}{\sigma}\right), \tag{17}$$

where $\sqrt{2\pi} M(x)$ is the Mills ratio, the function

$$F\left(x, \frac{\varepsilon}{\sigma}\right) = M(x) + 28\,\theta R\left(4x\varepsilon/\sigma\right) \frac{\varepsilon}{\sigma} \tag{18}$$

with

$$R(t) = \frac{\left(1 - t + 6t^2\right)^3}{\left(1 - 3t\right)^{3/2} \left(1 - t\right)^7}, \qquad 0 \leq t < \frac{1}{3}, \tag{19}$$

and $|\theta| \leq 1$. In particular, in the i.i.d. case, for all $0 \leq x = o(\sqrt{n}), n \to \infty$,

$$\left| \mathbb{P}(S_n > x\sigma) - M(x) \inf_{\lambda \geq 0} \mathbb{E} e^{\lambda (S_n - x\sigma)} \right| = O\left(\frac{1}{\sqrt{n}} \inf_{\lambda \geq 0} \mathbb{E} e^{\lambda (S_n - x\sigma)}\right) \tag{20}$$

and thus

$$\frac{\mathbb{P}(S_n > x\sigma)}{M(x) \inf_{\lambda \geq 0} \mathbb{E} e^{\lambda (S_n - x\sigma)}} = 1 + o\left(1\right). \tag{21}$$

6.4. A fractional Brownian field indexed by L^2 and a varying Hurst parameter

Participant: Alexandre Richard.

Using structures of Abstract Wiener Spaces and their reproducing kernel Hilbert spaces, we define a fractional Brownian field indexed by a product space $(0,1/2] \times L^2(T,m)$, where the first coordinate corresponds to the Hurst parameter of fractional Brownian motion. This field encompasses a large class of existing fractional Brownian processes, such as Lévy fractional Brownian motion and multiparameter fractional Brownian motion, and provides a setup for new ones. We prove that it has good incremental variance in both coordinates and derive certain continuity and Hölder regularity properties. Then, we apply these general results to multiparameter and set-indexed processes, which proves the existence of processes with prescribed local Hölder regularity on general indexing collections.

The family of fBm can be considered for the different Hurst parameters as a single Gaussian process indexed by $(h,t) \in (0,1) \times \mathbb{R}_+$, which is the position we adopt. Besides, the "time" indexing is replaced by any separable L^2 space. We prove that there exists a Gaussian process indexed by $(0,1/2] \times L^2(T,m)$, with the additional constraint that the variance of its increments is as well behaved as it was on $(0,1) \times \mathbb{R}_+$, that is, for any compact of L^2, there is a constant $C > 0$ such that for any f in this compact, and any $h, h' \in (0,1/2)$,

$$\mathbb{E}\left(B_f^h - B_f^{h'}\right)^2 \leq C\ (h-h')^2. \tag{22}$$

When looking at the L^2–fBf with a fixed h, we have the following covariance: for each $h \in (0,1/2]$,

$$k_h : (f,g) \in L^2 \times L^2 \mapsto \frac{1}{2}\left(m(f^2)^{2h} + m(g^2)^{2h} - m(|f-g|^2)^{2h}\right)\ . \tag{23}$$

An important subclass of these processes is formed by processes restricted to indicator functions of subsets of T. In particular, multiparameter when $(T,m) = (\mathbb{R}_+^d, \text{Leb.})$, and more largely set-indexed processes [62],[20] naturally appear and thus motivate generalization *b)*, besides the inherent interest of studying processes over an abstract space.

To define this field, we used fractional operators on the Wiener space W introduced in [56], and first expressed the fractional Brownian field (indexed by $(0,1/2] \times \mathbb{R}_+$) as a white noise integral over W:

$$\left\{ \int_W \langle \mathcal{K}_h R_h(\cdot,t), w\rangle \ \mathrm{d}\mathbb{B}_w,\ \ (h,t) \in (0,1/2] \times \mathbb{R}_+ \right\}\ ,$$

The advantage of this approach is to allow the transfer of techniques of calculus on the Wiener space to any other linearly isometric space with the same structure (those spaces are called Abstract Wiener Spaces). Using the separability and reproducing kernel property of the Cameron-Martin spaces built from the kernels $k_h, h \in (0,1/2]$, we prove the existence of a Brownian field $\{\mathbf{B}_{h,f},\ \ h \in (0,1/2], f \in L^2(T,m)\}$ over some probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Some Hilbert space analysis then provides the desired bound (22). Then, we used this to derive a sufficient condition for almost sure continuity of the fractional Brownian field, in terms of metric entropy.

For fixed h, we proved that the h-fractional Brownian motion has the strong local nondeterminism property, which allowed to compute a sharp estimate of its small deviations, that is, for a compact K of L^2:

$$\exp\left(-C\ \ N(K,d_h,\varepsilon)\right) \leq \mathbb{P}\left(\sup_{f\in K}|\mathbf{B}_f^h| \leq \varepsilon\right) \leq \exp\left(-C^{-1}\ \ N(K,d_h,\varepsilon)\right)\ ,$$

where $N(K,d_h,\varepsilon)$ is the metric entropy of K, i.e., the minimal number of balls necessary to cover K with d_h-balls (the metric induced by the h-fBm) of radius at most ε.

Finally, we looked at the Hölder regularity of the fBf, when the L^2 indexing collection is restricted to the indicator functions of the rectangles of $\mathbb{R}^d$ (multiparameter processes) or to some indexing collection (in the sense of [62]). This restriction permits to use local Hölder regularity exponents, in the flavour of what was done in [24]. When a regular path $\mathbf{h} : L^2 \to (0, 1/2]$ is specified, this defines a multifractional Brownian field as $\mathbf{B}_f^{\mathbf{h}} = \mathbf{B}_{\mathbf{h}(f),f}$, whose Hölder regularity at each point is proved to equal $\mathbf{h}(f)$ almost surely.

6.5. Self-stabilizing processes

Participants: Xiequan Fan, Jacques Lévy Véhel.

In collaboration with K. Falconer, University of St Andrews.

Self-stabilizing processes are càdlàg processes whose local intensiy of jumpd depend on amplitude. We have investigated two paths to define such processes. The first one is based on a modification of the celebrated Lévy construction of Brownian motion.

The second one starts from a stochastic differentiel equation, and allows one to build Markov processes, a useful feature in applications such as financial modelling [40], [41].

6.6. Multifractal spectra of multistable Lévy motion

Participant: Jacques Lévy Véhel.

In collaboration with R. Le Guével, University of Rennes.

As a follow-up to the work in [34] we have computed the Hausdorf, large deviation, and Legendre multifractal spectra of multistable Lévy motion. It turns out that the shape of the Hausdorf multifractal spectrum is much more complex than could be expected considering the corresponding spectrum of plain Lévy motion. Also, the large deviation spectrum reveals more information on the fine structure of the process than the Hausdorf one, a situation reminiscent of what has already been observed for the model we have developped previously for TCP traffic [2],[39].

6.7. Self-regulating processes for the modelling of geophysical signals

Participant: Jacques Lévy Véhel.

In collaboration with A. Echelard and A. Philippe, University of Nantes.

We have shown that various geophysical signals, and in particular temperature records, can be modelled with self-regulating processes as introduced in [4]. For this purpose, we have used an estimator of the self-regulating function proposed in [44]. Such a modelling allows one to gain further insight on the fine structure of the evolution of temperatures.

6.8. Regularity-preserving signal denoising

Participant: Jacques Lévy Véhel.

In collaboration with A. Echelard.

We have proposed a new wavelet-based method for signal denoising, that allows one to recover the local Hölder regularity of the original signal under weak assumptions [43]. The algotihm is a modification of the well-known wavelet thresholding procedure, where "small" coefficients are not put to zero, but modified in a way governed by the behaviour of large scale coefficients. This will have applications in the frame of our Tandem project on the analysis of radar images.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

The Tandem Project is a consortium involving several industrial companies (e.g. Bull Amesys) and some research laboratories (e.g. CMAP). The aim is to detect landmines from 3D radar images.

8. Partnerships and Cooperations

8.1. National Initiatives

Erick Herbin is member of the CNRS Research Groups:

- GDR Mascot Num, devoted to stochastic analysis methods for codes and numerical treatment;
- GDR Math-Entreprise, devoted to mathematical modeling of industrial issues.

8.2. International Initiatives

8.2.1. Inria International Partners

8.2.1.1. Informal International Partners

- Regularity collaborates with Bar Ilan university on theoretical developments around set-indexed fractional Brownian motion and set-indexed Lévy processes. The PhD thesis of Alexandre Richard is co-supervised by Erick Herbin and Ely Merzbach.
- Regularity collaborates with Michigan State University (Prof. Yimin Xiao) on the study of fine regularity of multiparameter fractional Brownian motion.
- Regularity collaborates with St Andrews University (Prof. Kenneth Falconer) on the study of multistable processes.
- Regularity collaborates with Acadia University (Prof. Franklin Mendivil) on the study of fractal strings, certain fractals sets, and the study of the regularization dimension.
- Regularity collaborates with Milan University (Prof. Davide La Torre) on the study of certain economic growth models.

8.3. International Research Visitors

8.3.1. Visits of International Scientists

Ely Merzbach (Bar-Ilan University) visited the team for one month.

9. Dissemination

9.1. Scientific Animation

Benjamin Arras attented the *Journées de Probabilités* at Orléans from 17th june 2013 to 21rst june 2013. During a talk, he presented his research regarding wavelets expansion and Hölderian regularity of stochastic processes belonging to Wiener chaoses.
Benjamin Arras attented classes during the "Ecole de probabilités de Saint-Flour" at Saint-Flour from 10th july 2013 to 21rst july 2013.
Paul Balança attented the *Journées de Probabilités* at Orléans from 17th june 2013 to 21rst june 2013: Presentation on 2-microlocal analysis and Lévy processes. Paul Balança attented the *7th international conference on Lévy processes* at Wroclaw: Poster on 2-microlocal analysis and Lévy processes.
Alexandre Richard attented the *Journées de Probabilités* at Orléans from 17th june 2013 to 21rst june 2013: Presentation on fractional Brownian fields in abstract Wiener spaces.
Alexandre Richard attented classes during the "Ecole de probabilités de Saint-Flour" at Saint-Flour from 10th july 2013 to 21rst july 2013.
Xiequan Fan is a reviewer for Mathematical Reviews (AMS).
Jacques Lévy Véhel is associate editor of the journal *Fractals*.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

- Licence: Erick Herbin, Probability course at Ecole Centrale Paris (20h).
- Master: Erick Herbin, Advanced Probability course at Ecole Centrale Paris (30h).
- Master: Erick Herbin and Jacques Lévy Véhel, Brownian Motion and Stochastic Calculus course at Ecole Centrale Paris (30h).
- Master: Jacques Lévy Véhel teaches a course on Wavelets and Fractals at Ecole Centrale Nantes (8h).
- Licence: Benjamin Arras, Analysis, Probability and PDE, 3x10 hours, L3, Ecole Centrale Paris.
- Licence: Paul Balança, Analysis, Probability, 2x10 hours, L3, Ecole Centrale Paris.
- Master: Benjamin Arras, Brownian motion and Stochastic Calculus, 20 hours, M2, Ecole Centrale Paris.
- Master: Paul Balança, Advanced Probability, 18 hours, M1, Ecole Centrale Paris.

9.2.2. Supervision

PhD in progress : Benjamin Arras, Self-similar processes in higher order chaoses, started in September 2011, supervised by J. Lévy Véhel.

PhD in progress : Paul Balança, Stochastic 2-microlocal analysis of SDEs, started in October 2010, supervised by Erick Herbin.

PhD in progress : Alexandre Richard, Regularity of set-indexed processes and construction of a set-indexed process with varying local regularity , started in October 2010, supervised by Erick Herbin and E. Merzbach.

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