



IN PARTNERSHIP WITH:
CNRS

Université de Lorraine

Activity Report 2013

Project-Team VEGAS

Effective Geometric Algorithms for Surfaces and Visibility

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

RESEARCH CENTER
Nancy - Grand Est

THEME
**Algorithmics, Computer Algebra and
Cryptology**

Table of contents

1. Members	1
2. Overall Objectives	1
3. Application Domains	2
3.1. Computer graphics	2
3.2. Solid modeling	2
3.3. Fast prototyping	3
4. Software and Platforms	3
4.1. QI: Quadrics Intersection	3
4.2. Isotop: Topology and Geometry of Planar Algebraic Curves	3
4.3. CGAL: Computational Geometry Algorithms Library	4
4.4. Fast_polynomial: fast polynomial evaluation software	4
5. New Results	4
5.1. Classical and probabilistic computational geometry	4
5.1.1. Probabilistic complexity analysis of random geometric structures	5
5.1.2. Embedding geometric structures	5
5.1.3. Bounded-Curvature Shortest Paths	6
5.1.4. Approximating Geodesics in Meshes	6
5.1.5. On Point-sets that Support Planar Graphs	6
5.2. Non-linear computational geometry	6
5.2.1. Solving bivariate systems and topology of algebraic curves	6
5.2.2. Reflection through quadric mirror surfaces	7
5.2.3. Fast polynomial evaluation and composition	7
5.3. Combinatorics and combinatorial geometry	7
5.3.1. Simplifying inclusion-exclusion formulas	8
5.3.2. Helly numbers of acyclic families	8
5.3.3. Set systems and families of permutations with small traces	8
6. Partnerships and Cooperations	8
6.1. National Initiatives	8
6.1.1. ANR PRESAGE	8
6.1.2. ANR SingCAST	9
6.2. International Research Visitors	9
7. Dissemination	9
7.1. Scientific Animation	9
7.2. Teaching - Supervision - Juries	10
7.2.1. Teaching	10
7.2.2. Supervision	11
7.3. Popularization	11
8. Bibliography	11

Project-Team VEGAS

Keywords: Algorithmic Geometry, Computational Geometry, Computer Algebra

Creation of the Project-Team: 2005 August 01.

1. Members

Research Scientists

Sylvain Lazard [Team leader, Inria, Senior Researcher, HdR]
Xavier Goaoc [Inria, Researcher, until Aug. 2013, HdR]
Guillaume Moroz [Inria, Researcher]
Marc Pouget [Inria, Researcher]
Sylvain Petitjean [Inria, Senior Researcher, Head Inria Nancy Grand Est, HdR]

Faculty Member

Laurent Dupont [Univ. Lorraine, Associate Professor]

PhD Student

Yacine Bouzidi [Inria, CORDI-S since Oct. 2010. Univ. Lorraine since Oct. 2013]

Post-Doctoral Fellows

Eduardo Ferraz [Inria, since Oct. 2013]
Alfredo Hubard [Inria, since Oct. 2013]

Visiting Scientist

Heuna Kim [KAIST Korea, since Mar. 2013 until Apr. 2013]

Administrative Assistants

Aurélie Adam [Univ. Lorraine]
Helene Zganic [Inria]

Others

Oswald Hounkonnou [Inria, Master (M2) student, since Apr. 2013 until Aug. 2013]
Ioannis Psarros [Inria, Internship, since Apr. 2013 until Jul. 2013]
Judith Recknagel [Univ. Lorraine, Erasmus Internship, since May 2013 until Aug. 2013]

2. Overall Objectives

2.1. Overall Objectives

The main scientific objective of the **VEGAS** research team is to *contribute to the development of an effective geometric computing* dedicated to *non-trivial geometric objects*. Included among its main tasks are the study and development of new algorithms for the manipulation of geometric objects, the experimentation of algorithms, the production of high-quality software, and the application of such algorithms and implementations to research domains that deal with a large amount of geometric data, notably solid modeling and computer graphics.

Computational geometry has traditionally treated linear objects like line segments and polygons in the plane, and point sets and polytopes in three-dimensional space, occasionally (and more recently) venturing into the world of non-linear curves such as circles and ellipses. The methodological experience and the know-how accumulated over the last thirty years have been enormous.

For many applications, particularly in the fields of computer graphics and solid modeling, it is necessary to manipulate more general objects such as curves and surfaces given in either implicit or parametric form. Typically such objects are handled by approximating them by simple objects such as triangles. This approach is extremely important and it has been used in almost all of the usable software existing in industry today. It does, however, have some disadvantages. Using a tessellated form in place of its exact geometry may introduce spurious numerical errors (the famous gap between the wing and the body of the aircraft), not to mention that thousands if not hundreds of thousands of triangles could be needed to adequately represent the object. Moreover, the curved objects that we consider are not necessarily everyday three-dimensional objects, but also abstract mathematical objects that are not linear, that may live in high-dimensional space, and whose geometry we do not control. For example, the set of lines in 3D (at the core of visibility issues) that are tangent to three polyhedra span a piecewise ruled quadratic surface, and the lines tangent to a sphere correspond, in projective five-dimensional space, to the intersection of two quadratic hypersurfaces.

Effectiveness is a key word of our research project. By requiring our algorithms to be effective, we imply that the algorithms should be *robust*, *efficient*, and *versatile*. By robust we mean algorithms that do not crash on degenerate inputs and always output topologically consistent data. By efficient we mean algorithms that run reasonably quickly on realistic data where performance is ascertained both experimentally and theoretically. Finally, by versatile we mean algorithms that work for classes of objects that are general enough to cover realistic situations and that account for the *exact geometry* of the objects, in particular when they are curved.

3. Application Domains

3.1. Computer graphics

We are interested in the application of our work to virtual prototyping, which refers to the many steps required for the creation of a realistic virtual representation from a CAD/CAM model.

When designing an automobile, detailed physical mockups of the interior are built to study the design and evaluate human factors and ergonomic issues. These hand-made prototypes are costly, time consuming, and difficult to modify. To shorten the design cycle and improve interactivity and reliability, realistic rendering and immersive virtual reality provide an effective alternative. A virtual prototype can replace a physical mockup for the analysis of such design aspects as visibility of instruments and mirrors, reachability and accessibility, and aesthetics and appeal.

Virtual prototyping encompasses most of our work on effective geometric computing. In particular, our work on 3D visibility should have fruitful applications in this domain. As already explained, meshing objects of the scene along the main discontinuities of the visibility function can have a dramatic impact on the realism of the simulations.

3.2. Solid modeling

Solid modeling, i.e., the computer representation and manipulation of 3D shapes, has historically developed somewhat in parallel to computational geometry. Both communities are concerned with geometric algorithms and deal with many of the same issues. But while the computational geometry community has been mathematically inclined and essentially concerned with linear objects, solid modeling has traditionally had closer ties to industry and has been more concerned with curved surfaces.

Clearly, there is considerable potential for interaction between the two fields. Standing somewhere in the middle, our project has a lot to offer. Among the geometric questions related to solid modeling that are of interest to us, let us mention: the description of geometric shapes, the representation of solids, the conversion between different representations, data structures for graphical rendering of models and robustness of geometric computations.

3.3. Fast prototyping

We work in collaboration with **CIRTES** on rapid prototyping. **CIRTES**, a company based in Saint-Dié-des-Vosges, has designed a technique called Stratoconception[®] where a prototype of a 3D computer model is constructed by first decomposing the model into layers and then manufacturing separately each layer, typically out of wood of standard thickness (e.g. 1 cm), with a three-axis CNC (Computer Numerical Controls) milling machine. The layers are then assembled together to form the object. The Stratoconception[®] technique is cheap and allows fast prototyping of large models.

When the model is complex, for example an art sculpture, some parts of the models may be inaccessible to the milling machine. These inaccessible regions are sanded out by hand in a post-processing phase. This phase is very consuming in time and resources. We work on minimizing the amount of work to be done in this last phase by improving the algorithmic techniques for decomposing the model into layers, that is, finding a direction of slicing and a position of the first layer.

4. Software and Platforms

4.1. QI: Quadrics Intersection

QI stands for “Quadrics Intersection”. QI is the first exact, robust, efficient and usable implementation of an algorithm for parameterizing the intersection of two arbitrary quadrics, given in implicit form, with integer coefficients. This implementation is based on the parameterization method described in [7], [10] and represents the first complete and robust solution to what is perhaps the most basic problem of solid modeling by implicit curved surfaces.

QI is written in C++ and builds upon the LiDIA computational number theory library [29] bundled with the GMP multi-precision integer arithmetic [28]. QI can routinely compute parameterizations of quadrics having coefficients with up to 50 digits in less than 100 milliseconds on an average PC; see [10] for detailed benchmarks.

Our implementation consists of roughly 18,000 lines of source code. QI has been registered at the Agence pour la Protection des Programmes (APP). It is distributed under the free for non-commercial use Inria license and will be distributed under the QPL license in the next release. The implementation can also be queried via a web interface [30].

Since its official first release in June 2004, QI has been downloaded six times a month on average and it has been included in the geometric library EXACUS developed at the Max-Planck-Institut für Informatik (Saarbrücken, Germany). QI is also used in a broad range of applications; for instance, it is used in photochemistry for studying the interactions between potential energy surfaces, in computer vision for computing the image of conics seen by a catadioptric camera with a paraboloidal mirror, and in mathematics for computing flows of hypersurfaces of revolution based on constant-volume average curvature.

4.2. Isotop: Topology and Geometry of Planar Algebraic Curves

ISOTOP is a Maple software for computing the topology of an algebraic plane curve, that is, for computing an arrangement of polylines isotopic to the input curve. This problem is a necessary key step for computing arrangements of algebraic curves and has also applications for curve plotting. This software has been developed since 2007 in collaboration with F. Rouillier from Inria Paris - Rocquencourt. It is based on the method described in [4] which incorporates several improvements over previous methods. In particular, our approach does not require generic position.

Isotop is registered at the APP (June 15th 2011) with reference IDDN.FR.001.240007.000.S.P.2011.000.10000. This version is competitive with other implementations (such as ALCIX and INSULATE developed at MPII Saarbrücken, Germany and TOP developed at Santander Univ., Spain). It performs similarly for small-degree curves and performs significantly better for higher degrees, in particular when the curves are not in generic position.

We are currently working on an improved version integrating our new bivariate polynomial solver.

4.3. CGAL: Computational Geometry Algorithms Library

Born as a European project, CGAL (<http://www.cgal.org>) has become the standard library for computational geometry. It offers easy access to efficient and reliable geometric algorithms in the form of a C++ library. CGAL is used in various areas needing geometric computation, such as: computer graphics, scientific visualization, computer aided design and modeling, geographic information systems, molecular biology, medical imaging, robotics and motion planning, mesh generation, numerical methods...

In computational geometry, many problems lead to standard, though difficult, algebraic questions such as computing the real roots of a system of equations, computing the sign of a polynomial at the roots of a system, or determining the dimension of a set of solutions. We want to make state-of-the-art algebraic software more accessible to the computational geometry community, in particular, through the computational geometric library CGAL. On this line, we contributed a model of the *Univariate Algebraic Kernel* concept for algebraic computations [32] (see Sections 8.2.2 and 8.4). This CGAL package improves, for instance, the efficiency of the computation of arrangements of polynomial functions in CGAL [34]. We are currently developing a model of the *Bivariate Algebraic Kernel* based a new bivariate polynomial solver.

4.4. Fast_polynomial: fast polynomial evaluation software

The library *fast_polynomial*¹ provides fast evaluation and composition of polynomials over several types of data. It is interfaced for the computer algebra system *Sage* and its algorithms are documented². This software is meant to be a first step toward a certified numerical software to compute the topology of algebraic curves and surfaces. It can also be useful as is and is submitted for integration in the computer algebra system *Sage*.

This software is focused on *fast online computation*, *multivariate evaluation*, *modularity*, and *efficiency*.

Fast online computation. The library is optimized for the evaluation of a polynomial on several point arguments given one after the other. The main motivation is numerical path tracking of algebraic curves, where a given polynomial criterion must be evaluated several thousands of times on different values arising along the path.

Multivariate evaluation. The library provides specialized fast evaluation of multivariate polynomials with several schemes, specialized for different types such as *mpz* big ints, *boost* intervals with hardware precision, *mpfi* intervals with any given precision, etc.

Modularity. The evaluation scheme can be easily changed and adapted to the user needs. Moreover, the code is designed to easily extend the library with specialization over new C++ objects.

Efficiency. The library uses several tools and methods to provide high efficiency. First, the code uses templates, such that after the compilation of a polynomial for a specific type, the evaluation performance is equivalent to low-level evaluation. Locality is also taken into account: the memory footprint is minimized, such that an evaluation using the classical Hörner scheme will use $O(1)$ temporary objects and divide and conquer schemes will use $O(\log n)$ temporary objects, where n is the degree of the polynomial. Finally, divide and conquer schemes can be evaluated in parallel, using a number of threads provided by the user.

5. New Results

5.1. Classical and probabilistic computational geometry

Participants: Xavier Goaoc, Guillaume Moroz, Sylvain Lazard, Marc Pouget.

¹http://trac.sagemath.org/sage_trac/ticket/13358

²<http://arxiv.org/abs/1307.5655>

5.1.1. Probabilistic complexity analysis of random geometric structures

Average-case analysis of data-structures or algorithms is commonly used in computational geometry when the more classical worst-case analysis is deemed overly pessimistic. Since these analyses are often intricate, the models of random geometric data that can be handled are often simplistic and far from "realistic inputs".

Complexity analysis of random geometric structures made simpler. In a joint work with Olivier Devillers and Marc Glisse (Inria Geometrica), we presented a new simple scheme for the analysis of geometric structures. While this scheme only produces results up to a polylog factor, it is much simpler to apply than the classical techniques and therefore succeeds in analyzing new input distributions related to smoothed complexity analysis. We illustrated our method on two classical structures: convex hulls and Delaunay triangulations. Specifically, we gave short and elementary proofs of the classical results that n points uniformly distributed in a ball in \mathbb{R}^d have a convex hull and a Delaunay triangulation of respective expected complexities $\tilde{\Theta}(n^{((d+1)/(d-1))})$ and $\tilde{\Theta}(n)$. We then prove that if we start with n points well-spread on a sphere, e.g. an (ϵ, κ) -sample of that sphere, and perturb that sample by moving each point randomly and uniformly within distance at most δ of its initial position, then the expected complexity of the convex hull of the resulting point set is $\tilde{\Theta}(\sqrt{n}^{(1-1/d)} \delta^{-(d-1)/(4d)})$. We presented these results in the *Symposium on Computational Geometry 2013* [20].

Monotonicity of the number of facets of random polytopes. We also proved a result on the size of the convex hull K_n of n points sampled uniformly in a convex set K . More precisely, let $u_n^{K,i}$ be the expected number of facets of dimension i of the convex hull. We proved that, in the plane, $u_n^{K,0}$ is an increasing sequence. In higher dimension, if K is a convex, smooth, compact body, then we showed that the sequence $u_n^{K,d-1}$ is asymptotically increasing. This result, published in the *Electronic Communications in Probability* [13], was obtained in collaboration with Olivier Devillers and Marc Glisse (Inria Geometrica) and Matthias Reitzner (Osnabruck Univ.).

Worst-case silhouette size of random polytopes. Finally, we studied from a probabilistic point of view the size of the silhouette of a polyhedron. While the silhouette size of a polyhedron with n vertices may be linear for some view points, several experimental and theoretical studies show a sublinear behavior for a wide range of constraints. The latest result on the subject proves a bound in $\Theta(\sqrt{n})$ on the size of the silhouette from a random view point of polyhedra of size n approximating non-convex surfaces in a reasonable way [9]. This result considers the polyhedron given and average the sizes of the silhouettes over all view points. This year, we addressed the problem of bounding the worst-case size of the silhouette where the average is taken over a set of polyhedra. Namely, we consider random polytopes defined as the convex hull of a Poisson point process on a sphere in \mathbb{R}^3 such that its average number of points is n . We show that the expectation over all such random polytopes of the maximum size of their silhouettes viewed from infinity is $\Theta(\sqrt{n})$. This work was done in collaboration with Marc Glisse (Inria Geometrica) and Julien Michel (Université de Poitiers) [24].

5.1.2. Embedding geometric structures

We continued working this year on the problem of embedding geometric objects on a grid of \mathbb{R}^3 . Essentially all industrial applications take, as input, models defined with a fixed-precision floating-point arithmetic, typically doubles. As a consequence, geometric objects constructed using exact arithmetic must be embedded on a fixed-precision grid before they can be used as input in other software. More precisely, the problem is, given a geometric object, to find a similar object representable with fixed-precision floating-point arithmetic, where similar means topologically equivalent, close according to some distance function, etc. We are working on the problem of rounding polyhedral subdivisions on a grid of \mathbb{R}^3 , where the only known method, due to Fortune in 1999, considers a grid whose refinement depends on the combinatorial complexity of the input, which does not solve the problem at hand. This project is joint work with Olivier Devillers (Inria Geometrica) and William Lenhart (Williams College, USA).

5.1.3. Bounded-Curvature Shortest Paths

We considered the problem of computing shortest paths having curvature at most one almost everywhere and visiting a sequence of n points in the plane in a given order. This problem is a sub-problem of the Dubins Traveling Salesman Problem and also arises naturally in path planning for point car-like robots in the presence of polygonal obstacles. We showed that when consecutive waypoints are distance at least four apart, this question reduces to a family of convex optimization problems over polyhedra in \mathbb{R}^n . This result, done in collaboration with Hyo-Sil Kim (KAIST) was published in the *SIAM Journal on Computing* [15].

5.1.4. Approximating Geodesics in Meshes

A standard way to approximate the distance between any two vertices p and q on a mesh is to compute, in the associated graph, a shortest path from p to q that goes through one of k sources, which are well-chosen vertices. Precomputing the distance between each of the k sources to all vertices of the graph yields an efficient computation of approximate distances between any two vertices. One standard method for choosing k sources, which has been used extensively and successfully for isometry-invariant surface processing, is the so-called *Farthest Point Sampling* (FPS), which starts with a random vertex as the first source, and iteratively selects the farthest vertex from the already selected sources.

We analyzed the stretch factor \mathcal{F}_{FPS} of approximate geodesics computed using FPS, which is the maximum, over all pairs of distinct vertices, of their approximated distance over their geodesic distance in the graph. We show that \mathcal{F}_{FPS} can be bounded in terms of the minimal value \mathcal{F}^* of the stretch factor obtained using an optimal placement of k sources as $\mathcal{F}_{FPS} \leq 2r_e^2\mathcal{F}^* + 2r_e^2 + 8r_e + 1$, where r_e is the ratio of the lengths of the longest and the shortest edges of the graph. This provides some evidence explaining why farthest point sampling has been used successfully for isometry-invariant shape processing. Furthermore, we showed that it is NP-complete to find k sources that minimize the stretch factor [25].

5.1.5. On Point-sets that Support Planar Graphs

A set of points is said universal if it supports a crossing-free drawing of any planar graph. For a planar graph with n vertices, if bends on edges of the drawing are permitted, universal point-sets of size n are known, but only if the bend-points are in arbitrary positions. If the locations of the bend-points must also be specified as part of the point-set, no result was known, and we prove that any planar graph with n vertices can be drawn on a universal set \mathcal{S} of $O(n^2/\log n)$ points with at most one bend per edge and with the vertices and the bend points in \mathcal{S} . If two bends per edge are allowed, we show that $O(n \log n)$ points are sufficient, and if three bends per edge are allowed, $\Theta(n)$ points are sufficient. When no bends on edges are permitted, no universal point-set of size $o(n^2)$ is known for the class of planar graphs. We show that a set of n points in balanced biconvex position supports the class of maximum-degree-3 series-parallel lattices. These results were published this year in the journal *Computational Geometry: Theory and Application* [14].

We also considered the setting in which graphs are drawn with curved edges. We proved that, surprisingly, there exists a universal set of n points in the plane for which every n -vertex planar graph admits a planar drawing in which the edges are drawn as a circular arc. This result was presented in the *Canadian Conference on Computational Geometry* [17].

5.2. Non-linear computational geometry

Participants: Guillaume Moroz, Sylvain Lazard, Marc Pouget, Yacine Bouzidi, Laurent Dupont.

5.2.1. Solving bivariate systems and topology of algebraic curves

In the context of our algorithm Isotop for computing the topology of algebraic curves [4], we work on the problem of solving a system of two bivariate polynomials. We focus on the problem of computing a Rational Univariate Representation (RUR) of the solutions, that is, roughly speaking, a univariate polynomial and two rational functions which map the roots of the polynomial to the two coordinates of the solutions of the system.

Separating linear forms. We first presented an algorithm for computing a separating linear form of a system of bivariate polynomials with integer coefficients, that is a linear combination of the variables that takes different values when evaluated at distinct (complex) solutions of the system. In other words, a separating linear form defines a shear of the coordinate system that sends the algebraic system in generic position, in the sense that no two distinct solutions are vertically aligned. The computation of such linear forms is at the core of most algorithms that solve algebraic systems by computing rational parameterizations of the solutions and, moreover, the computation of a separating linear form is the bottleneck of these algorithms, in terms of worst-case bit complexity. Given two bivariate polynomials of total degree at most d with integer coefficients of bitsize at most τ , our algorithm computes a separating linear form in $\tilde{O}_B(d^8 + d^7\tau)$ bit operations in the worst case, which decreases by a factor d^2 the best known complexity for this problem (\tilde{O}_B refers to the complexity where polylogarithmic factors are omitted and O_B refers to the bit complexity). This result was presented at the *International Symposium on Symbolic and Algebraic Computation* in 2013 [19] and submitted to a journal [23].

Solving bivariate systems & RURs. Given such a separating linear form, we also presented an algorithm for computing a RUR with worst-case bit complexity in $\tilde{O}_B(d^7 + d^6\tau)$ and a bound on the bitsize of its coefficients in $\tilde{O}(d^2 + d\tau)$. We showed in addition that isolating boxes of the solutions of the system can be computed from the RUR with $\tilde{O}_B(d^8 + d^7\tau)$ bit operations. Finally, we showed how a RUR can be used to evaluate the sign of a bivariate polynomial (of degree at most d and bitsize at most τ) at one real solution of the system in $\tilde{O}_B(d^8 + d^7\tau)$ bit operations and at all the $\Theta(d^2)$ real solutions in only $O(d)$ times that for one solution. These results were also presented at the *International Symposium on Symbolic and Algebraic Computation* in 2013 [18] and submitted to a journal [22].

This work is done in collaboration with Fabrice Rouillier (project-team Ouragan at Inria Paris-Rocquencourt).

5.2.2. Reflection through quadric mirror surfaces

We addressed the problem of finding the reflection point on a quadric mirror surfaces of a light ray emanating from a 3D point source P_1 and going through another 3D point P_2 , the camera center of projection. This is a classical problem known as Alhazen's problem dating from around 1000 A.D. and based on the work of Ptolemy around 150 A.D. [31], [33]. We proposed a new algorithm for this problem based on our algorithm for the computation of the intersection of quadrics [7], [30] and using a characterization the reflection point as the tangential intersection point between the mirror and an ellipsoid with foci P_1 and P_2 . The implementation is in progress. This work is done in collaboration with Nuno Gonçalves, University of Coimbra (Portugal).

5.2.3. Fast polynomial evaluation and composition

Evaluating a polynomial can be done with different evaluation schemes. The Hörner scheme for example allows to evaluate a polynomial of degree n in $O(n)$ arithmetic operations. When the cost of the arithmetic operations is constant, such as in floating point arithmetic, this leads to $O(n)$ binary operations. However, with integers, the size of the elements grows linearly after each multiplication and this may lead to $O(n^2)$ binary operations. This problem arises also with polynomial composition.

The best way to handle these cases is to use divide-and-conquer algorithms to keep a linear complexity in the degree up to logarithmic factors. State-of-the-art algorithms split at the highest pure power of 2 lower or equal to $\frac{n}{2}$. However when n is not a pure power of 2, this strategy might not be optimal.

We developed the library *fast_polynomial* to explore different divide-and-conquer schemes and observed notably that splitting at $\lfloor \frac{n}{2} \rfloor$ is more efficient in some cases. In particular, this evaluation scheme does not suffer the staircase effect observed in state-of-the-art evaluations. Experimentally, it is always faster than our own implementation of the classical divide-and-conquer scheme, and faster than the state of the art library *Flint 2* when the degree of the input polynomial is between 2^k and $2^k + 2^{k-1}$. These results are presented in the technical report [26].

5.3. Combinatorics and combinatorial geometry

Participant: Xavier Goaoc.

5.3.1. Simplifying inclusion-exclusion formulas

In a joint work with Jiří Matoušek, Pavel Paták, Zuzana Safernová, Martin Tancer (Charles University, Prague, Czech republic), we worked on computing simplified inclusion-exclusion formulas. Let $\mathcal{F} = \{F_1, F_2, \dots, F_n\}$ be a family of n sets on a ground set S , such as a family of balls in \mathbb{R}^d . For every finite measure μ on S , such that the sets of \mathcal{F} are measurable, the classical *inclusion-exclusion formula* asserts that $\mu(F_1 \cup F_2 \cup \dots \cup F_n) = \sum_{I: \emptyset \neq I \subseteq [n]} (-1)^{|I|+1} \mu(\bigcap_{i \in I} F_i)$; that is, the measure of the union is expressed using measures of various intersections. The number of terms in this formula is exponential in n , and a significant amount of research, originating in applied areas, has been devoted to constructing simpler formulas for particular families \mathcal{F} . We provide an upper bound valid for an arbitrary \mathcal{F} : we show that every system \mathcal{F} of n sets with m nonempty fields in the Venn diagram admits an inclusion-exclusion formula with $m^{O(\log^2 n)}$ terms and with ± 1 coefficients, and that such a formula can be computed in $m^{O(\log^2 n)}$ expected time. We also construct systems of n sets on n points for which every valid inclusion-exclusion formula has the sum of absolute values of the coefficients at least $\Omega(n^{3/2})$. This work was presented at the EUROCOMB conference [21] in September 2013.

5.3.2. Helly numbers of acyclic families

In a joint work with Éric Colin de Verdière (CNRS-ENS) and Grégory Ginot (IMJ-UPMC), we worked on applications of algebraic topology to combinatorial geometry, and more precisely on extending classical results on nerve complexes. The nerve complex of a family is an abstract simplicial complex that encode its intersection patterns. Nerves are widely used in computational geometry and topology, in particular in reconstruction problems where one aims at inferring the geometry of an object from a point sample while guaranteeing that the topology is correct. Indeed, the *nerve theorem* ensures that the nerve of a family of geometric objects has the same “topology” (formally: homotopy type) as the union of the objects whenever they form a “good cover”, that is, when any subset of the objects has an empty or contractible intersection. We relaxed this “good cover” condition to allow for families of non-connected sets. We defined an analogue of the nerve, called the *multinerve*, that is suitable for general acyclic families, and we proved that this combinatorial structure enjoys an analogue of the nerve theorem. Using multinerve, we could derive a new *topological Helly-type theorem* for acyclic families that generalizes previous results of Amenta, Kalai and Meshulam, and Matoušek. We finally used this new Helly-type theorem to (re)prove, in a unified way, bounds on transversal Helly numbers in *geometric transversal theory*. This article was submitted to the journal *Advances in mathematics* in 2012; it was accepted in 2013 and will appear in 2014 [16].

5.3.3. Set systems and families of permutations with small traces

In a joint work with Otfried Cheong (KAIST, South Korea) and Cyril Nicaud (Univ. Marne-La-Vallée), we studied two problems of the following flavor: how large can a family of combinatorial objects defined on a finite set be if its number of distinct “projections” on any small subset is bounded? We first consider set systems, where the “projections” is the standard notion of trace, and for which we generalized Sauer’s Lemma on the size of set systems with bounded VC-dimension. We then studied families of permutations, where the “projections” corresponds to the notion of containment used in the study of permutations with excluded patterns, and for which we delineated the main growth rates ensured by projection conditions. One of our motivations for considering these questions is the “geometric permutation problem” in geometric transversal theory, a question that has been open for two decades. This work was submitted to the *European Journal of Combinatorics* in 2012 and published in 2013 [12].

6. Partnerships and Cooperations

6.1. National Initiatives

6.1.1. ANR PRESAGE

The white ANR grant PRESAGE brings together computational geometers (from the VEGAS and GEOMETRICA projects of Inria) and probabilistic geometers (from Universities of Rouen, Orléans and Poitiers) to tackle new probabilistic geometry problems arising from the design and analysis of geometric algorithms and data structures. We focus on properties of discrete structures induced by or underlying random continuous geometric objects.

This is a four year project, with a total budget of 400k€, that started on Dec. 31st, 2011. It is coordinated by Xavier Goaoc (VEGAS).

6.1.2. ANR SingCAST

The objective of the young-researcher ANR grant SingCAST is to intertwine further symbolic/numeric approaches to compute efficiently solution sets of polynomial systems with topological and geometrical guarantees in singular cases. We focus on two applications: the visualization of algebraic curves and surfaces and the mechanical design of robots.

After identifying classes of problems with restricted types of singularities, we plan to develop dedicated symbolic-numerical methods that take advantage of the structure of the associated polynomial systems that cannot be handled by purely symbolical or numerical methods. Thus we plan to extend the class of manipulators that can be analyzed, and the class of algebraic curves and surfaces that can be visualized with certification.

This is a 3.5 years project, with a total budget of 100k€, that will start on March 1st 2014, coordinated by Guillaume Moroz.

6.2. International Research Visitors

Nuno Gonçalves, University of Coimbra (Portugal), visited the VEGAS project for 1 week in January.

William J. Lenhart, Williams College (USA), visited the VEGAS project for 2 weeks in May..

6.2.1. Internships

Ioannis Psarros

Subject: Common tangents to ellipsoids in \mathbb{R}^3 .

Date: from Apr. 2013 until July 2013.

Institution: University of Athens, Greece.

Oswald Hounkounou

Subject: Study with computer algebra system of a conjecture relating the width of a convex polygon with the width of its inscribed triangles.

Date: from Apr. 2013 until Aug. 2013.

Institution: Telecom Nancy de l'université de Lorraine.

Judit Recknagel

Subject: Topology of planar singular curves resultant of two trivariate polynomials.

Date: from Apr. 2013 until Aug. 2013

Institution: Halle-Wittenberg university, Germany.

7. Dissemination

7.1. Scientific Animation

Program and Paper Committee:

- Sylvain Lazard: Program committee of the European Workshop on Computational Geometry (EuroCG'13).

Editorial responsibilities:

- Sylvain Petitjean: Editor of *Graphical Models* (Elsevier).

Workshop organizations:

- Sylvain Lazard co-organized with S. Whitesides (Victoria University) the 12th Inria - McGill - Victoria Workshop on Computational Geometry ³ at the Bellairs Research Institute of McGill University in Feb. (1 week workshop on invitation).
- Marc Pouget co-organized the *Journées Informatiques et Géométrie* ⁴ at Inria Nancy 14-15 Nov.
- Guillaume Moroz organized the session *Calcul formel et numérique* oh the *Rencontres Arithmétiques de l'Informatique Mathématique* ⁵ at the *Institut Henri Poincaré* Paris 18-20 Nov.

Other responsibilities:

- Sylvain Lazard: Head of the Inria Nancy-Grand Est *PhD and Post-doc hiring committee* (since 2009). Member of the *Bureau du Département Informatique de Formation Doctorale* of the *École Doctorale IAEM* (since 2009). “Chargé de formation par la recherche” for Inria Nancy-Grand Est.
- Laurent Dupont: *Member* of Commission Pédagogique Nationale Infocom/SRC (since 2011). *Member* of Commission Information Scientifique (Inria/Loria).
- Xavier Goaoc: Chair of the Inria *COST-GTRI committee* (2011– august 2013).
- Guillaume Moroz: *Vice delegate* of the *Commission des Utilisateurs des Moyens Informatiques pour la Recherche*.
- Sylvain Petitjean: Director of the Inria Nancy Grand-Est. Member of Inria’s *Executive committee*.
- Marc Pouget: Member of the CGAL Editorial Board (since 2008).

7.2. Teaching - Supervision - Juries

7.2.1. Teaching

Licence: Laurent Dupont, *Systèmes de Gestion de Bases de Données Avancé*, 40h, L3, Université de Lorraine (IUT Charlemagne).

Licence: Laurent Dupont, *Concepts et Outils Internet*, 40h, L1, Université de Lorraine (IUT Charlemagne).

Licence: Laurent Dupont, *Programmation Objet et Évènementielle*, 40h, L2, Université de Lorraine (IUT Charlemagne).

Licence: Laurent Dupont, *Rich Internet Applications*, 40h, L2, Université de Lorraine (IUT Charlemagne).

Licence: Laurent Dupont and Yacine Bouzidi, *Programmation de Sites Web Dynamiques*, 70h, L2, Université de Lorraine (IUT Charlemagne).

Licence: Laurent Dupont, *Algorithmique*, 80h, L1, Université de Lorraine (IUT Charlemagne)

Licence: Laurent Dupont *Programmation Objet*, 40h, L1, Université de Lorraine (IUT Charlemagne)

Master: Marc Pouget, *Introduction à la géométrie algorithmique*, 10.5h, M2, École Nationale Supérieure de Géologie, France.

Doctorat: Marc Pouget, *Postdoctoral Summer: Convex hulls and point location*, 15h, IMPA, Rio de Janeiro, Brazil.

Licence: Sylvain Lazard, *Algorithms and Complexity*, 25h, L3, Université de Lorraine.

Licence: Yacine Bouzidi, *Certification informatique et internet*, 54h, L1, Université de Lorraine.

³Workshop on Computational Geometry

⁴<http://jig2013.sciencesconf.org/>

⁵<http://raim2013.lip6.fr/>

Licence: Yacine Bouzidi, *Langage orienté objet, Java*, 24h, L3, Université de Lorraine.

Licence: Yacine Bouzidi, *Langage d'interrogation des bases de données*, 20h, L3, Université de Lorraine.

7.2.2. Supervision

PhD in progress : Yacine Bouzidi, Résolution de systèmes bivariés et topologie de courbes planes, Oct. 2010, Sylvain Lazard et Marc Pouget.

7.3. Popularization

Guillaume Moroz: Member of the organizing committee of the *Olympiades académiques de mathématiques*.

8. Bibliography

Major publications by the team in recent years

- [1] C. BORCEA, X. GOAOC, S. LAZARD, S. PETITJEAN. *Common Tangents to Spheres in \mathbb{R}^3* , in "Discrete & Computational Geometry", 2006, vol. 35, n^o 2, pp. 287-300 [DOI : 10.1007/s00454-005-1230-Y], <http://hal.inria.fr/inria-00100261/en>
- [2] H. BRÖNNIMANN, O. DEVILLERS, V. DUJMOVIC, H. EVERETT, M. GLISSE, X. GOAOC, S. LAZARD, H.-S. NA, S. WHITESIDES. *Lines and free line segments Tangent to Arbitrary Three-dimensional Convex Polyhedra*, in "SIAM Journal on Computing", 2007, vol. 37, n^o 2, pp. 522-551 [DOI : 10.1137/S0097539705447116], <http://hal.inria.fr/inria-00103916/en>
- [3] H. BRÖNNIMANN, H. EVERETT, S. LAZARD, F. SOTTILE, S. WHITESIDES. *Transversals to line segments in three-dimensional space*, in "Discrete & Computational Geometry", 2005, vol. 34, n^o 3, pp. 381 - 390 [DOI : 10.1007/s00454-005-1183-1], <http://hal.inria.fr/inria-00000384/en>
- [4] J. CHENG, S. LAZARD, L. PEÑARANDA, M. POUGET, F. ROULLIER, E. P. TSIGARIDAS. *On the topology of real algebraic plane curves*, in "Mathematics in Computer Science", 2010, <http://hal.inria.fr/inria-00517175/en>
- [5] O. CHEONG, X. GOAOC, A. HOLMSEN, S. PETITJEAN. *Hadwiger and Helly-type theorems for disjoint unit spheres*, in "Discrete & Computational Geometry", 2008, vol. 39, n^o 1-3, pp. 194-212, <http://hal.inria.fr/inria-00103856/en>
- [6] O. DEVILLERS, V. DUJMOVIC, H. EVERETT, X. GOAOC, S. LAZARD, H.-S. NA, S. PETITJEAN. *The expected number of 3D visibility events is linear*, in "SIAM Journal on Computing", 2003, vol. 32, n^o 6, pp. 1586-1620 [DOI : 10.1137/S0097539702419662], <http://hal.inria.fr/inria-00099810/en>
- [7] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: I. The Generic Algorithm; II. A Classification of Pencils; III. Parameterizing Singular Intersections*, in "Journal of Symbolic Computation", 2008, vol. 43, pp. 168–191, 192–215, 216–232, <http://hal.inria.fr/inria-00186090/en/>, <http://hal.inria.fr/inria-00186091/en/>

- [8] H. EVERETT, D. LAZARD, S. LAZARD, M. SAFEY EL DIN. *The Voronoi diagram of three lines*, in "Journal of Discrete and Computational Geometry", 2009, vol. 42, n^o 1, pp. 94-130 [DOI : 10.1007/s00454-009-9173-3], <http://www.springerlink.com/content/f5601q6324664k2p/?p=6d7bb74bf9df40b0b7756b3a5153809f&pi=5>, <http://hal.inria.fr/inria-00431518/en>
- [9] M. GLISSE, S. LAZARD. *An Upper Bound on the Average Size of Silhouettes*, in "Discrete & Computational Geometry", 2008, vol. 40, n^o 2, pp. 241-257 [DOI : 10.1007/s00454-008-9089-3], <http://hal.inria.fr/inria-00336571/en>
- [10] S. LAZARD, L. PEÑARANDA, S. PETITJEAN. *Intersecting Quadrics: An Efficient and Exact Implementation*, in "Computational Geometry", 2006, vol. 35, n^o 1-2, pp. 74–99, <http://hal.inria.fr/inria-00000380/en>
- [11] S. PETITJEAN. *Invariant-based characterization of the relative position of two projective conics*, in "Non-Linear Computational Geometry", I. Z. EMIRIS, F. SOTTILE, T. THEOBALD (editors), Springer, 2008, <http://hal.inria.fr/inria-00335968/en>

Publications of the year

Articles in International Peer-Reviewed Journals

- [12] O. CHEONG, X. GOAOC, C. NICAUD. *Set Systems and Families of Permutations with Small Traces*, in "European Journal of Combinatorics", 2013, vol. 34, pp. 229-239, <http://hal.inria.fr/hal-00752064>
- [13] O. DEVILLERS, M. GLISSE, X. GOAOC, G. MOROZ, M. REITZNER. *The monotonicity of f-vectors of random polytopes*, in "Electronic Communications in Probability", 2013, vol. 18, n^o 23 [DOI : 10.1214/ECP.v18-2469], <http://hal.inria.fr/hal-00805690>
- [14] V. DUJMOVIC, W. EVANS, S. LAZARD, W. LENHART, G. LIOTTA, D. RAPPAPORT, S. WISMATH. *On Point-sets that Support Planar Graphs*, in "Computational Geometry : Theory and Applications", 2013, vol. 43, n^o 1, pp. 29–50 [DOI : 10.1016/J.COMGEO.2012.03.003], <http://hal.inria.fr/hal-00684510>
- [15] X. GOAOC, H.-S. KIM, S. LAZARD. *Bounded-Curvature Shortest Paths through a Sequence of Points using Convex Optimization*, in "SIAM Journal on Computing", 2013, vol. 42, n^o 2, pp. 662-684 [DOI : 10.1137/100816079], <http://hal.inria.fr/hal-00927100>
- [16] É. C. DE VERDIÈRE, G. GINOT, X. GOAOC. *Helly numbers of acyclic families*, in "Advances in Mathematics", 2014, vol. 253, pp. 163-193, Minor changes [DOI : 10.1016/J.AIM.2013.11.004], <http://hal.inria.fr/hal-00646166>

International Conferences with Proceedings

- [17] P. ANGELINI, D. EPPSTEIN, F. FRATI, M. KAUFMANN, S. LAZARD, T. MCHEDLIDZE, M. TEILLAUD, A. WOLFF. *Universal Point Sets for Planar Graph Drawings with Circular Arcs*, in "Canadian Conference on Computational Geometry", Waterloo, Canada, July 2013, pp. 117-122, <http://hal.inria.fr/hal-00846953>
- [18] Y. BOUZIDI, S. LAZARD, M. POUGET, F. ROUILLIER. *Rational Univariate Representations of Bivariate Systems and Applications*, in "ISSAC - 38th International Symposium on Symbolic and Algebraic Computation", Boston, United States, 2013, pp. 109-116, <http://hal.inria.fr/hal-00809430>

- [19] Y. BOUZIDI, S. LAZARD, M. POUGET, F. ROUILLIER. *Separating Linear Forms for Bivariate Systems*, in "ISSAC - 38th International Symposium on Symbolic and Algebraic Computation", Boston, United States, 2013, pp. 117-124, <http://hal.inria.fr/hal-00809425>
- [20] O. DEVILLERS, M. GLISSE, X. GOAOC. *Complexity Analysis of Random Geometric Structures Made Simpler*, in "30th Annual Symposium on Computational Geometry", Rio, Brazil, 2013, pp. 167-175 [DOI : 10.1145/2462356.2462362], <http://hal.inria.fr/hal-00833774>

Conferences without Proceedings

- [21] X. GOAOC, J. MATOUŠEK, P. PATÁK, Z. SAFERNOVÁ, M. TANCER. *Simplifying inclusion-exclusion formulas*, in "European Conference on Combinatorics, Graph Theory and Applications", Pisa, Italy, 2013, 14 p. , <http://hal.inria.fr/hal-00764182>

Research Reports

- [22] Y. BOUZIDI, S. LAZARD, M. POUGET, F. ROUILLIER. , *Rational Univariate Representations of Bivariate Systems and Applications*, Inria, March 2013, n^o RR-8262, 26 p. , <http://hal.inria.fr/hal-00802698>
- [23] Y. BOUZIDI, S. LAZARD, M. POUGET, F. ROUILLIER. , *Separating linear forms for bivariate systems*, Inria, March 2013, n^o RR-8261, 20 p. , <http://hal.inria.fr/hal-00802693>
- [24] M. GLISSE, S. LAZARD, J. MICHEL, M. POUGET. , *Silhouette of a random polytope*, Inria, July 2013, n^o RR-8327, 13 p. , <http://hal.inria.fr/hal-00841374>
- [25] P. KAMOUSHI, S. LAZARD, A. MAHESHWARI, S. WUHRER. , *Analysis of Farthest Point Sampling for Approximating Geodesics in a Graph*, November 2013, 13 p. , <http://hal.inria.fr/hal-00927643>
- [26] G. MOROZ. , *Fast polynomial evaluation and composition*, April 2013, <http://hal.inria.fr/hal-00846961>

Other Publications

- [27] J. RECKNAGEL. , *Topology of planar singular curves resultant of two trivariate polynomials*, Institute for Computer Science, Martin-Luther-University, Halle-Wittenberg, August 2013, <http://hal.inria.fr/hal-00927768>

References in notes

- [28] , *GMP: the GNU MP Bignum Library*, The Free Software Foundation, <http://gmplib.org/>
- [29] , *LiDIA: a C++ Library for Computational Number Theory*, Darmstadt University of Technology, <http://www.informatik.tu-darmstadt.de/TI/LiDIA>
- [30] , *QI: a C++ package for parameterizing intersections of quadrics*, 2005, LORIA, Inria Lorraine, VEGAS project, <http://www.loria.fr/equipes/vegas/qi>
- [31] M. BAKER. *Alhazen's problem*, in "American J. Mathematics", 1881, vol. 4, n^o 1, pp. 327-331
- [32] E. BERBERICH, M. HEMMER, S. LAZARD, L. PEÑARANDA, M. TEILLAUD. *Algebraic kernel*, in "CGAL User and Reference Manual, 3.6 edition", CGAL EDITORIAL BOARD (editor), CGAL Editorial board, 2010, <http://hal.inria.fr/inria-00537545/en>

- [33] G. GLAESER. *Reflections on spheres and cylinders of revolution*, in "J. Geometry and Graphics", 1999, vol. 3, n^o 2, pp. 121-139
- [34] S. LAZARD, L. PEÑARANDA, E. P. TSIGARIDAS. *Univariate Algebraic Kernel and Application to Arrangements*, in "International Symposium on Experimental Algorithms – SEA Experimental Algorithms, 8th International Symposium, SEA 2009", Allemagne Dortmund, Springer, 2009, vol. 5526/2009, pp. 209-220, <http://hal.inria.fr/inria-00431559/en/>