



Activity Report 2014

Project-Team APICS

Analysis and Problems of Inverse type in
Control and Signal processing

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
**Optimization and control of dynamic
systems**

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Project-Team APICS

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2. Overall Objectives

2.1. Research Themes

The team develops constructive, function-theoretic approaches to inverse problems arising in modeling and design, in particular for electro-magnetic systems as well as in the analysis of certain classes of signals.

Data typically consist of measurements or desired behaviors. The general thread is to approximate them by families of solutions to the equations governing the underlying system. This leads us to consider various interpolation and approximation problems in classes of rational and meromorphic functions, harmonic gradients, or solutions to more general elliptic partial differential equations (PDE), in connection with inverse potential problems. A recurring difficulty is to control the singularities of the approximants.

The mathematical tools pertain to complex and harmonic analysis, approximation theory, potential theory, system theory, differential topology, optimization and computer algebra. Targeted applications include:

- identification and synthesis of analog microwave devices (filters, amplifiers),
- non-destructive control from field measurements in medical engineering (source recovery in magneto/electro-encephalography), paleomagnetism (determining the magnetization of rock samples), and nuclear engineering (plasma shaping in tokamaks).

In each case, the endeavor is to develop algorithms resulting in dedicated software.

3. Research Program

3.1. Introduction

Within the extensive field of inverse problems, much of the research by Apics deals with reconstructing solutions of classical elliptic PDEs from their boundary behavior. Perhaps the simplest example lies with harmonic identification of a stable linear dynamical system: the transfer-function f can be evaluated at a point $i\omega$ of the imaginary axis from the response to a periodic input at frequency ω . Since f is holomorphic in the right half-plane, it satisfies there the Cauchy-Riemann equation $\bar{\partial}f = 0$, and recovering f amounts to solve a Dirichlet problem which can be done in principle using, *e.g.* the Cauchy formula.

Practice is not nearly as simple, for f is only measured pointwise in the pass-band of the system which makes the problem ill-posed [72]. Moreover, the transfer function is usually sought in specific form, displaying the necessary physical parameters for control and design. For instance if f is rational of degree n , then $\bar{\partial}f = \sum_1^n a_j \delta_{z_j}$ where the z_j are its poles and δ_{z_j} is a Dirac unit mass at z_j . Thus, to find the domain of holomorphy (*i.e.* to locate the z_j) amounts to solve a (degenerate) free-boundary inverse problem, this time on the left half-plane. To address such questions, the team has developed a two-step approach as follows.

Step 1: To determine a complete model, that is, one which is defined at every frequency, in a sufficiently versatile function class (*e.g.* Hardy spaces). This ill-posed issue requires regularization, for instance constraints on the behavior at non-measured frequencies.

Step 2: To compute a reduced order model. This typically consists of rational approximation of the complete model obtained in step 1, or phase-shift thereof to account for delays. We emphasize that deriving a complete model in step 1 is crucial to achieve stability of the reduced model in step 2.

Step 1 relates to extremal problems and analytic operator theory, see Section 3.3.1. Step 2 involves optimization, and some Schur analysis to parametrize transfer matrices of given Mc-Millan degree when dealing with systems having several inputs and outputs, see Section 3.3.2.2. It also makes contact with the topology of rational functions, in particular to count critical points and to derive bounds, see Section 3.3.2. Step 2 raises further issues in approximation theory regarding the rate of convergence and the extent to which singularities of the approximant (*i.e.* its poles) tend to singularities of the approximated function; this is where logarithmic potential theory becomes instrumental, see Section 3.3.3.

Applying a realization procedure to the result of step 2 yields an identification procedure from incomplete frequency data which was first demonstrated in [78] to tune resonant microwave filters. Harmonic identification of nonlinear systems around a stable equilibrium can also be envisaged by combining the previous steps with exact linearization techniques from [36].

A similar path can be taken to approach design problems in the frequency domain, replacing the measured behavior by some desired behavior. However, describing achievable responses in terms of the design parameters is often cumbersome, and most constructive techniques rely on specific criteria adapted to the physics of the problem. This is especially true of filters, the design of which traditionally appeals to polynomial extremal problems [74], [59]. Apics contributed to this area the use of Zolotarev-like problems for multi-band synthesis, although we presently favor interpolation techniques in which parameters arise in a more transparent manner, see Section 3.2.2.

The previous example of harmonic identification quickly suggests a generalization of itself. Indeed, on identifying \mathbb{C} with \mathbb{R}^2 , holomorphic functions become conjugate-gradients of harmonic functions, so that harmonic identification is, after all, a special case of a classical issue: to recover a harmonic function on a domain from partial knowledge of the Dirichlet-Neumann data; when the portion of boundary where data are not available is itself unknown, we meet a free boundary problem. This framework for 2-D non-destructive control was first advocated in [64] and subsequently received considerable attention. It makes clear how to state similar problems in higher dimensions and for more general operators than the Laplacian, provided solutions are essentially determined by the trace of their gradient on part of the boundary which is the case for elliptic equations ¹ [25], [83]. Such questions are particular instances of the so-called inverse potential

problem, where a measure μ has to be recovered from the knowledge of the gradient of its potential (*i.e.*, the field) on part of a hypersurface (a curve in 2-D) encompassing the support of μ . For Laplace's operator, potentials are logarithmic in 2-D and Newtonian in higher dimensions. For elliptic operators with non constant coefficients, the potential depends on the form of fundamental solutions and is less manageable because it is no longer of convolution type. Nevertheless it is a useful concept bringing perspective on how problems could be raised and solved, using tools from harmonic analysis.

Inverse potential problems are severely indeterminate because infinitely many measures within an open set produce the same field outside this set; this phenomenon is called *balayage* [71]. In the two steps approach previously described, we implicitly removed this indeterminacy by requiring in step 1 that the measure be supported on the boundary (because we seek a function holomorphic throughout the right half space), and by requiring in step 2 that the measure be discrete in the left half-plane. The discreteness assumption also prevails in 3-D inverse source problems, see Section 4.2. Conditions that ensure uniqueness of the solution to the inverse potential problem are part of the so-called regularizing assumptions which are needed in each case to derive efficient algorithms.

To recap, the gist of our approach is to approximate boundary data by (boundary traces of) fields arising from potentials of measures with specific support. Note that it is different from standard approaches to inverse problems, where descent algorithms are applied to integration schemes of the direct problem; in such methods, it is the equation which gets approximated (in fact: discretized).

Along these lines, Apics advocates the use of steps 1 and 2 above, along with some singularity analysis, to approach issues of nondestructive control in 2-D and 3-D [43] [5], [2]. The team is currently engaged in two kinds of generalizations, to be described further in Section 3.2.1. The first deals with non-constant conductivities in 2-D, where Cauchy-Riemann equations characterizing holomorphic functions are replaced by conjugate Beltrami equations characterizing pseudo-holomorphic functions; next in line are 3-D situations that we begin to consider, see Sections 6.2 and 4.4. There, we seek applications to inverse free boundary problems such as plasma confinement in the vessel of a tokamak, or inverse conductivity problems like those arising in impedance tomography. The second generalization lies with inverse source problems for the Laplace equation in 3-D, where holomorphic functions are replaced by harmonic gradients; applications are to EEG/MEG and inverse magnetization problems in paleomagnetism, see Section 4.2.

The approximation-theoretic tools developed by Apics to handle issues mentioned so far are outlined in Section 3.3. In Section 3.2 to come, we describe in more detail which problems are considered and which applications are targeted.

3.2. Range of inverse problems

3.2.1. Elliptic partial differential equations (PDE)

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Christos Papageorgakis, Dmitry Ponomarev.

By standard properties of conjugate differentials, reconstructing Dirichlet-Neumann boundary conditions for a function harmonic in a plane domain, when these boundary conditions are known already on a subset E of the boundary, is equivalent to recover a holomorphic function in the domain from its boundary values on E . This is the problem raised on the half-plane in step 1 of Section 3.1. It makes good sense in holomorphic Hardy spaces where functions are entirely determined by their values on boundary subsets of positive linear measure, which is the framework for Problem (P) that we set up in Section 3.3.1. Such issues naturally arise in nondestructive testing of 2-D (or 3-D cylindrical) materials from partial electrical measurements on the boundary. For instance, the ratio between the tangential and the normal currents (the so-called Robin

¹There is a subtle difference here between dimension 2 and higher. Indeed, a function holomorphic on a plane domain is defined by its non-tangential limit on a boundary subset of positive linear measure, but there are non-constant harmonic functions in the 3-D ball, C^1 up to the boundary sphere, yet having vanishing gradient on a subset of positive measure of the sphere. Such a "bad" subset, however, cannot have interior points on the sphere.

coefficient) tells one about corrosion of the material. Thus, solving Problem (P) where ψ is chosen to be the response of some uncorroded piece with identical shape yields non destructive testing of a potentially corroded piece of material, part of which is inaccessible to measurements. This was an initial application of holomorphic extremal problems to non-destructive control [56], [60].

Another application by the team deals with non-constant conductivity over a doubly connected domain, the set E being now the outer boundary. Measuring Dirichlet-Neumann data on E , one wants to recover level lines of the solution to a conductivity equation, which is a so-called free boundary inverse problem. For this, given a closed curve inside the domain, we first quantify how constant the solution on this curve. To this effect, we state and solve an analog of Problem (P), where the constraint bears on the real part of the function on the curve (it should be close to a constant there), in a Hardy space of a conjugate Beltrami equation, of which the considered conductivity equation is the compatibility condition (just like the Laplace equation is the compatibility condition of the Cauchy-Riemann system). Subsequently, a descent algorithm on the curve leads one to improve the initial guess. For example, when the domain is regarded as separating the edge of a tokamak's vessel from the plasma (rotational symmetry makes this a 2-D situation), this method can be used to estimate the shape of a plasma subject to magnetic confinement. It was successfully applied, in collaboration with CEA (French nuclear agency) and the University of Nice (JAD Lab.), to data from *Tore Supra* [63]. The procedure is fast because no numerical integration of the underlying PDE is needed, as an explicit basis of solutions to the conjugate Beltrami equation in terms of Bessel functions was found in this case. Generalizing this approach in a more systematic manner to free boundary problems of Bernoulli type, using descent algorithms based on shape-gradient for such approximation-theoretic criteria, is an interesting prospect, still to be pursued.

The piece of work we just mentioned requires defining and studying Hardy spaces of the conjugate-Beltrami equation, which is an interesting topic by itself. For Sobolev-smooth coefficients of exponent greater than 2, this was done in references [4] and [14]. The case of the critical exponent 2 is treated in [34], which apparently provides the first example of well-posedness for the Dirichlet problem in the non-strictly elliptic case: the conductivity may be unbounded or zero on sets of zero capacity and, accordingly, solutions need not be locally bounded.

The 3-D version of step 1 in Section 3.1 is another subject investigated by Apics: to recover a harmonic function (up to a constant) in a ball or a half-space from partial knowledge of its gradient on the boundary. This prototypical inverse problem (*i.e.* inverse to the Cauchy problem for the Laplace equation) often recurs in electromagnetism. At present, Apics is involved with solving instances of this inverse problem arising in two fields, namely medical imaging *e.g.* for electroencephalography (EEG) or magneto-encephalography (MEG), and paleomagnetism (recovery of rocks magnetization) [2], [38], see Section 6.1. In this connection, we collaborate with two groups of partners: Athena Inria project-team, CHU La Timone, and BESA company on the one hand, Geosciences Lab. at MIT and Cerege CNRS Lab. on the other hand. The question is considerably more difficult than its 2-D counterpart, due mainly to the lack of multiplicative structure for harmonic gradients. Still, considerable progress has been made over the last years using methods of harmonic analysis and operator theory.

The team is further concerned with 3-D generalizations and applications to non-destructive control of step 2 in Section 3.1. A typical problem is here to localize inhomogeneities or defaults such as cracks, sources or occlusions in a planar or 3-dimensional object, knowing thermal, electrical, or magnetic measurements on the boundary. These defaults can be expressed as a lack of harmonicity of the solution to the associated Dirichlet-Neumann problem, thereby posing an inverse potential problem in order to recover them. In 2-D, finding an optimal discretization of the potential in Sobolev norm amounts to solve a best rational approximation problem, and the question arises as to how the location of the singularities of the approximant (*i.e.* its poles) reflects the location of the singularities of the potential (*i.e.* the defaults we seek). This is a fairly deep issue in approximation theory, to which Apics contributed convergence results for certain classes of fields expressed as Cauchy integrals over extremal contours for the logarithmic potential [39], [53] [6]. Initial schemes to locate cracks or sources *via* rational approximation on planar domains were obtained this way [56], [43], [46]. It is remarkable that finite inverse source problems in 3-D balls, or more general algebraic surfaces, can be

approached using these 2-D techniques upon slicing the domain into planar sections [3], [9]. This bottom line generates a steady research activity within Apics, and again applications are sought to medical imaging and geosciences, see Sections 4.2, 4.3 and 6.1.

Conjectures can be raised on the behavior of optimal potential discretization in 3-D, but answering them is an ambitious program still in its infancy.

3.2.2. Systems, transfer and scattering

Participants: Laurent Baratchart, Matthias Caenepeel, Sylvain Chevillard, Sanda Lefteriu, Martine Olivi, Fabien Seyfert.

Through contacts with CNES (French space agency), members of the team became involved in identification and tuning of microwave electromagnetic filters used in space telecommunications, see Section 4.5. The initial problem was to recover, from band-limited frequency measurements, physical parameters of the device under examination. The latter consists of interconnected dual-mode resonant cavities with negligible loss, hence its scattering matrix is modeled by a 2×2 unitary-valued matrix function on the frequency line, say the imaginary axis to fix ideas. In the bandwidth around the resonant frequency, a modal approximation of the Helmholtz equation in the cavities shows that this matrix is approximately rational, of Mc-Millan degree twice the number of cavities.

This is where system theory comes into play, through the so-called *realization* process mapping a rational transfer function in the frequency domain to a state-space representation of the underlying system of linear differential equations in the time domain. Specifically, realizing the scattering matrix allows one to construct a virtual electrical network, equivalent to the filter, the parameters of which mediate in between the frequency response and the geometric characteristics of the cavities (*i.e.* the tuning parameters).

Hardy spaces provide a framework to transform this ill-posed issue into a series of regularized analytic and meromorphic approximation problems. More precisely, the procedure sketched in Section 3.1 goes as follows:

1. infer from the pointwise boundary data in the bandwidth a stable transfer function (*i.e.* one which is holomorphic in the right half-plane), that may be infinite dimensional (numerically: of high degree). This is done by solving a problem analogous to (P) in Section 3.3.1, while taking into account prior knowledge on the decay of the response outside the bandwidth, see [13] for details.
2. A stable rational approximation of appropriate degree to the model obtained in the previous step is performed. For this, a descent method on the compact manifold of inner matrices of given size and degree is used, based on an original parametrization of stable transfer functions developed within the team [13].
3. Realizations of this rational approximant are computed. To be useful, they must satisfy certain constraints imposed by the geometry of the device. These constraints typically come from the coupling topology of the equivalent electrical network used to model the filter. This network is composed of resonators, coupled according to some specific graph. This realization step can be recast, under appropriate compatibility conditions [8], as solving a zero-dimensional multivariate polynomial system. To tackle this problem in practice, we use Gröbner basis techniques and continuation methods which team up in the Dedale-HF software (see Section 5.4).

Let us mention that extensions of classical coupling matrix theory to frequency-dependent (reactive) couplings have lately been carried-out [1] for wide-band design applications, although further study is needed to make them computationally effective.

Subsequently Apics started to investigate issues pertaining to design rather than identification. Given the topology of the filter, a basic problem in this connection is to find the optimal response subject to specifications that bear on rejection, transmission and group delay of the scattering parameters. Generalizing the classical approach based on Chebyshev polynomials for single band filters, we recast the problem of multi-band response synthesis as a generalization of the classical Zolotarev min-max problem for rational functions [29] [11]. Thanks to quasi-convexity, the latter can be solved efficiently using iterative methods relying on linear programming. These were implemented in the software easy-FF (see Section 5.5). Currently, the team

is engaged in synthesis of more complex microwave devices like multiplexers and routers, which connect several filters through wave guides. Schur analysis plays an important role here, because scattering matrices of passive systems are of Schur type (*i.e.* contractive in the stability region). The theory originates with the work of I. Schur [77], who devised a recursive test to check for contractivity of a holomorphic function in the disk. The so-called Schur parameters of a function may be viewed as Taylor coefficients for the hyperbolic metric of the disk, and the fact that Schur functions are contractions for that metric lies at the root of Schur's test. Generalizations thereof turn out to be efficient to parametrize solutions to contractive interpolation problems [31]. Dwelling on this, Apics contributed differential parametrizations (atlases of charts) of lossless matrix functions [30][12], [10] which are fundamental to our rational approximation software RARL2 (see Section 5.1). Schur analysis is also instrumental to approach de-embedding issues, and provides one with considerable insight into the so-called matching problem. The latter consists in maximizing the power a multiport can pass to a given load, and for reasons of efficiency it is all-pervasive in microwave and electric network design, *e.g.* of antennas, multiplexers, wifi cards and more. It can be viewed as a rational approximation problem in the hyperbolic metric, and the team presently gets to grips with this hot topic using multipoint contractive interpolation in the framework of the (defense funded) ANR COCORAM, see Sections 6.3.1 and 8.2.1.

In recent years, our attention was driven by CNES and UPV (Bilbao) to questions about stability of high-frequency amplifiers, see Section 7.2. Contrary to previously discussed devices, these are *active* components. The response of an amplifier can be linearized around a set of primary current and voltages, and then admittances of the corresponding electrical network can be computed at various frequencies, using the so-called harmonic balance method. The initial goal is to check for stability of the linearized model, so as to ascertain existence of a well-defined working state. The network is composed of lumped electrical elements namely inductors, capacitors, negative *and* positive reactors, transmission lines, and controlled current sources. Our research so far focuses on describing the algebraic structure of admittance functions, so as to set up a function-theoretic framework where the two-steps approach outlined in Section 3.1 can be put to work. The main discovery so far is that the unstable part of each partial transfer function is rational, see Section 6.4.

3.3. Approximation

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Martine Olivi, Dmitry Ponomarev, Fabien Seyfert.

3.3.1. Best analytic approximation

In dimension 2, the prototypical problem to be solved in step 1 of Section 3.1 may be described as: given a domain $D \subset \mathbb{R}^2$, to recover a holomorphic function from its values on a subset K of the boundary of D . For the discussion it is convenient to normalize D , which can be done by conformal mapping. So, in the simply connected case, we fix D to be the unit disk with boundary unit circle T . We denote by H^p the Hardy space of exponent p , which is the closure of polynomials in $L^p(T)$ -norm if $1 \leq p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which makes it possible to speak of (traces of) analytic functions on the boundary.

To find an analytic function g in D matching some measured values f approximately on a sub-arc K of T , we formulate a constrained best approximation problem as follows.

(P) Let $1 \leq p \leq \infty$, K a sub-arc of T , $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behavior capturing *a priori* assumptions on the behavior of the model off K , while M is some admissible deviation thereof. The value of p reflects the type of stability which is sought and how much one wants to smooth out the data. The choice of L^p classes is suited to handle point-wise measurements.

To fix terminology, we refer to (P) as a *bounded extremal problem*. As shown in [42], [44], [50], the solution to this convex infinite-dimensional optimization problem can be obtained when $p \neq 1$ upon iterating with respect to a Lagrange parameter the solution to spectral equations for appropriate Hankel and Toeplitz operators. These spectral equations involve the solution to the special case $K = T$ of (P) , which is a standard extremal problem [66]:

(P_0) Let $1 \leq p \leq \infty$ and $\varphi \in L^p(T)$; find a function $g \in H^p$ such that $g - \varphi$ is of minimal norm in $L^p(T)$.

The case $p = 1$ is more or less open.

Various modifications of (P) can be set up in order to meet specific needs. For instance when dealing with lossless transfer functions (see Section 4.5), one may want to express the constraint on $T \setminus K$ in a point-wise manner: $|g - \psi| \leq M$ a.e. on $T \setminus K$, see [45]. In this form, the problem comes close to (but still is different from) H^∞ frequency optimization used in control [68], [76]. One can also impose bounds on the real or imaginary part of $g - \psi$ on $T \setminus K$, which is useful when considering Dirichlet-Neuman problems, see [70].

The analog of Problem (P) on an annulus, K being now the outer boundary, can be seen as a means to regularize a classical inverse problem occurring in nondestructive control, namely to recover a harmonic function on the inner boundary from Dirichlet-Neumann data on the outer boundary (see Sections 3.2.1, 4.2, 6.1.1, 6.2). It may serve as a tool to approach Bernoulli type problems, where we are given data on the outer boundary and we *seek the inner boundary*, knowing it is a level curve of the solution.. In this case, the Lagrange parameter indicates how to deform the inner contour in order to improve data fitting. Similar topics are discussed in Sections 3.2.1 and 6.2 for more general equations than the Laplacian, namely isotropic conductivity equations of the form $\operatorname{div}(\sigma \nabla u) = 0$ where σ is no longer constant. Then, the Hardy spaces in Problem (P) are those of a so-called conjugate Beltrami equation: $\bar{\partial} f = \nu \partial f$ [69], which are studied for $1 < p < \infty$ in [14], [4], [61] and [34]. Expansions of solutions needed to constructively handle such issues in the specific case of linear fractional conductivities (these occur in plasma shaping) have been expounded in [63].

Though originally considered in dimension 2, Problem (P) carries over naturally to higher dimensions where analytic functions get replaced by gradients of harmonic functions. Namely, given some open set $\Omega \subset \mathbb{R}^n$ and some \mathbb{R}^n -valued vector field V on an open subset O of the boundary of Ω , we seek a harmonic function in Ω whose gradient is close to V on O .

When Ω is a ball or a half-space, a substitute for holomorphic Hardy spaces is provided by the Stein-Weiss Hardy spaces of harmonic gradients [80]. Conformal maps are no longer available when $n > 2$, so that Ω can no longer be normalized. More general geometries than spheres and half-spaces have not been much studied so far.

On the ball, the analog of Problem (P) is

(P_1) Let $1 \leq p \leq \infty$ and $B \subset \mathbb{R}^n$ the unit ball. Fix O an open subset of the unit sphere $S \subset \mathbb{R}^n$. Let further $V \in L^p(O)$ and $W \in L^p(S \setminus O)$ be \mathbb{R}^n -valued vector fields. Given $M > 0$, find a harmonic gradient $G \in H^p(B)$ such that $\|G - W\|_{L^p(S \setminus O)} \leq M$ and $G - V$ is of minimal norm in $L^p(O)$ under this constraint.

When $p = 2$, Problem (P_1) was solved in [2] as well as its analog on a shell. The solution extends the one given in [42] for the 2-D case, using a generalization of Toeplitz operators. The case of the shell was motivated. An important ingredient is a refinement of the Hodge decomposition, that we call the *Hardy-Hodge decomposition*, allowing us to express a \mathbb{R}^n -valued vector field in $L^p(S)$, $1 < p < \infty$, as the sum of a vector field in $H^p(B)$, a vector field in $H^p(\mathbb{R}^n \setminus \bar{B})$, and a tangential divergence free vector field on S ; the space of such fields is denoted by $D(S)$. If $p = 1$ or $p = \infty$, L^p must be replaced by the real Hardy space or the space of functions with bounded mean oscillation. More generally this decomposition, which is valid on any sufficiently smooth surface (see Section 6.1), seems to play a fundamental role in inverse potential problems. In fact, it was first introduced formally on the plane to describe silent magnetizations supported in \mathbb{R}^2 (*i.e.* those generating no field in the upper half space) [38].

Just like solving problem (P) appeals to the solution of problem (P_0) , our ability to solve problem (P_1) will depend on the possibility to tackle the special case where $O = S$:

(P_2) Let $1 \leq p \leq \infty$ and $V \in L^p(S)$ be a \mathbb{R}^n -valued vector field. Find a harmonic gradient $G \in H^p(B)$ such that $\|G - V\|_{L^p(S)}$ is minimum.

Problem (P_2) is simple when $p = 2$ by virtue of the Hardy Hodge decomposition together with orthogonality of $H^2(B)$ and $H^2(\mathbb{R}^n \setminus \overline{B})$, which is the reason why we were able to solve (P_1) in this case. Other values of p cannot be treated as easily and are currently investigated by Apics, especially the case $p = \infty$ which is of particular interest and presents itself as a 3-D analog to the Nehari problem [75].

Companion to problem (P_2) is problem (P_3) below.

(P_3) Let $1 \leq p \leq \infty$ and $V \in L^p(S)$ be a \mathbb{R}^n -valued vector field. Find $G \in H^p(B)$ and $D \in D(S)$ such that $\|G + D - V\|_{L^p(S)}$ is minimum.

Note that (P_2) and (P_3) are identical in 2-D, since no non-constant tangential divergence-free vector field exists on T . It is no longer so in higher dimension, where both (P_2) and (P_3) arise in connection with source recovery in electro/magneto encephalography and paleomagnetism, see Sections 3.2.1 and 4.2.

3.3.2. Best meromorphic and rational approximation

The techniques set forth in this section are used to solve step 2 in Section 3.2 and instrumental to approach inverse boundary value problems for the Poisson equation $\Delta u = \mu$, where μ is some (unknown) distribution.

3.3.2.1. Scalar meromorphic and rational approximation

We put R_N for the set of rational functions with at most N poles in D . By definition, meromorphic functions in $L^p(T)$ are (traces of) functions in $H^p + R_N$.

A natural generalization of problem (P_0) is:

(P_N) Let $1 \leq p \leq \infty$, $N \geq 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Only for $p = \infty$ and f continuous is it known how to solve (P_N) in closed form. The unique solution is given by AAK theory (named after Adamjan, Arov and Krein), which connects the spectral decomposition of Hankel operators with best approximation [75].

The case where $p = 2$ is of special importance for it reduces to rational approximation. Indeed, if we write the Hardy decomposition $f = f^+ + f^-$ where $f^+ \in H^2$ and $f^- \in H^2(\mathbb{C} \setminus \overline{D})$, then $g_N = f^+ + r_N$ where r_N is a best approximant to f^- from R_N in $L^2(T)$. Moreover, r_N has no pole outside D , hence it is a *stable* rational approximant to f^- . However, in contrast to the case where $p = \infty$, this best approximant may *not* be unique.

The former Miaou project (predecessor of Apics) designed a dedicated steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; until now it seems to be the only procedure meeting this property. This gradient algorithm proceeds recursively with respect to N on a compactification of the parameter space [35]. Although it has proved to be effective in all applications carried out so far (see Sections 4.2, 4.5), it is still unknown whether the absolute minimum can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as is done by the RARL2 software, Section 5.1).

In order to establish global convergence results, Apics has undertaken a deeper study of the number and nature of critical points (local minima, saddle points...), in which tools from differential topology and operator theory team up with classical interpolation theory [47], [49]. Based on this work, uniqueness or asymptotic uniqueness of the approximant was proved for certain classes of functions like transfer functions of relaxation systems (*i.e.* Markov functions) [51] and more generally Cauchy integrals over hyperbolic geodesic arcs [54]. These are the only results of this kind. Research by Apics on this topic remained dormant for a while by reasons of opportunity, but revisiting the work [32] in higher dimension is still a worthy endeavor. Meanwhile, an analog to AAK theory was carried out for $2 \leq p < \infty$ in [50]. Although not as effective computationally, it was recently used to derive lower bounds [26]. When $1 \leq p < 2$, problem (P_N) is still quite open.

A common feature to the above-mentioned problems is that critical point equations yield non-Hermitian orthogonality relations for the denominator of the approximant. This stresses connections with interpolation, which is a standard way to build approximants, and in many respects best or near-best rational approximation may be regarded as a clever manner to pick interpolation points. This was exploited in [55], [52], and is used in an essential manner to assess the behavior of poles of best approximants to functions with branched singularities, which is of particular interest for inverse source problems (*cf.* Sections 5.6 and 6.1).

In higher dimensions, the analog of Problem (P_N) is best approximation of a vector field by gradients of discrete potentials generated by N point masses. This basic issue is by no means fully understood, and it is an exciting research prospect. It is connected with certain generalizations of Toeplitz or Hankel operators, and with constructive approaches to so-called weak factorizations for real Hardy functions [62].

Besides, certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus (*i.e.* a Schur function). In particular, Schur interpolation lately received renewed attention from the team, in connection with matching problems. There, interpolation data are subject to a well-known compatibility condition (positive definiteness of the so-called Pick matrix), and the main difficulty is to put interpolation points on the boundary of D while controlling both the degree and the extremal points of the interpolant. Results obtained by Apics in this direction generalize a variant of contractive interpolation with degree constraint studied in [67], see Section 6.3.1. We mention that contractive interpolation with nodes approaching the boundary has been a subsidiary research topic by the team in the past, which plays an interesting role in the spectral representation of certain non-stationary stochastic processes [40], [37]. The subject is intimately connected to orthogonal polynomials on the unit circle, and this line of investigation has recently evolved towards an asymptotic study of orthogonal polynomials on planar domains, which is an active area in approximation theory with application to quantum particle systems and Hele-Shaw flows. Section 6.5.1.

3.3.2.2. Matrix-valued rational approximation

Matrix-valued approximation is necessary to handle systems with several inputs and outputs but it generates additional difficulties as compared to scalar-valued approximation, both theoretically and algorithmically. In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the usual notion of degree for rational functions.

The basic problem that we consider now goes as follows: let $\mathcal{F} \in (H^2)^{m \times l}$ and n an integer; find a rational matrix of size $m \times l$ without poles in the unit disk and of McMillan degree at most n which is nearest possible to \mathcal{F} in $(H^2)^{m \times l}$. Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The scalar approximation algorithm derived in [35] and mentioned in Section 3.3.2.1 generalizes to the matrix-valued situation [65]. The first difficulty here is to parametrize inner matrices (*i.e.* matrix-valued functions analytic in the unit disk and unitary on the unit circle) of given McMillan degree n . Indeed, inner matrices play the role of denominators in fractional representations of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization). The set of inner matrices of given degree is a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (local parametrizations) and to handle changes of charts in the course of the algorithm. Such parametrization can be obtained using interpolation theory and Schur-type algorithms, the parameters of which are vectors or matrices ([30], [10], [12]). Some of these parametrizations are also interesting to compute realizations and achieve filter synthesis ([10] [12]). The rational approximation software “RARL2” developed by the team is described in Section 5.1.

Difficulties relative to multiple local minima of course arise in the matrix-valued case as well, and deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The case of rational functions of degree n or small perturbations thereof (the consistency problem) was solved in [48]. Matrix-valued Markov functions are the only known example beyond this one [33].

Let us stress that RARL2 seems the only algorithm handling rational approximation in the matrix case that demonstrably converges to a local minimum while meeting stability constraints on the approximant.

3.3.3. Behavior of poles of meromorphic approximants

Participant: Laurent Baratchart.

We refer here to the behavior of poles of best meromorphic approximants, in the L^p -sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. Normalizing the contour to be the unit circle T , we are back to Problem (P_N) in Section 3.3.2.1; invariance of the latter under conformal mapping was established in [5]. Research so far has focused on functions whose singular set inside the contour is zero or one-dimensional.

Generally speaking in approximation theory, assessing the behavior of poles of rational approximants is essential to obtain error rates as the degree goes large, and to tackle constructive issues like uniqueness. However, as explained in Section 3.2.1, Apics considers this issue foremost as a means to extract information on singularities of the solution to a Dirichlet-Neumann problem. The general theme is thus: *how do the singularities of the approximant reflect those of the approximated function?* This approach to inverse problem for the 2-D Laplacian turns out to be attractive when singularities are zero- or one-dimensional (see Section 4.2). It can be used as a computationally cheap initial condition for more precise but much heavier numerical optimizations which often do not even converge unless properly initialized. As regards crack detection or source recovery, this approach boils down to analyzing the behavior of best meromorphic approximants of a function with branch points. For piecewise analytic cracks, or in the case of sources, we were able to prove ([5], [6], [39]), that the poles of the approximants accumulate, when the degree goes large, to some extremal cut of minimum weighted logarithmic capacity connecting the singular points of the crack, or the sources [43]. Moreover, the asymptotic density of the poles turns out to be the Green equilibrium distribution on this cut in D , therefore it charges the singular points if one is able to approximate in sufficiently high degree (this is where the method could fail, because high-order approximation requires rather precise data).

The case of two-dimensional singularities is still an outstanding open problem.

It is remarkable that inverse source problems inside a sphere or an ellipsoid in 3-D can be approached with such 2-D techniques, as applied to planar sections (see Section 6.1). The technique is implemented in the software FindSources3D, see Section 5.6.

3.3.4. Miscellaneous

Participant: Sylvain Chevillard.

Sylvain Chevillard, joined team in November 2010. His coming resulted in Apics hosting a research activity in certified computing, centered on the software *Sollya* of which S. Chevillard is a co-author, see Section 5.7. On the one hand, *Sollya* is an Inria software which still requires some tuning to a growing community of users. On the other hand, approximation-theoretic methods at work in *Sollya* are potentially useful for certified solutions to constrained analytic problems described in Section 3.3.1. However, developing *Sollya* is not a long-term objective of Apics.

4. Application Domains

4.1. Introduction

Application domains are naturally linked to the problems described in Sections 3.2.1 and 3.2.2. By and large, they split into a systems-and-circuits part and an inverse-source-and-boundary-problems part, united under a common umbrella of function-theoretic techniques as described in Section 3.3.

4.2. Inverse source problems in EEG

Participants: Laurent Baratchart, Juliette Leblond.

This work is performed in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena Project-Team, and Jean-Paul Marmorat (Centre de mathématiques appliquées - CMA, École des Mines de Paris).

Solving overdetermined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to extrapolate incomplete data (see Section 3.2.1) is a necessary ingredient of the team’s approach to inverse source problems, in particular for applications to EEG. Indeed, the latter involves propagating the initial conditions through several layers of different conductivities, from the boundary shell down to the center of the domain where the singularities (*i.e.* the sources) lie. Once propagated to the innermost sphere, it turns out that traces of the boundary data on 2-D cross sections coincide with analytic functions with branched singularities in the slicing plane [3]. The singularities are related to the actual location of the sources, namely their moduli reach in turn a maximum when the plane contains one of the sources. Hence we are back to the 2-D framework of Section 3.3.3, and recovering these singularities can be performed *via* best rational approximation. The goal is to produce a fast and sufficiently accurate initial guess on the number and location of the sources in order to run heavier descent algorithms on the direct problem, which are more precise but computationally costly and often fail to converge if not properly initialized.

Numerical experiments give very good results on simulated data and we are now engaged in the process of handling real experimental data (see Sections 5.6 and 6.1), in collaboration with the Athena team at Inria Sophia Antipolis, neuroscience teams in partner-hospitals (la Timone, Marseille), and the BESA company (Munich).

4.3. Inverse magnetization problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Dmitry Ponomarev.

Generally speaking, inverse potential problems, similar to the one appearing in Section 4.2, occur naturally in connection with systems governed by Maxwell’s equation in the quasi-static approximation regime. In particular, they arise in magnetic reconstruction issues. A specific application is to geophysics, which led us to form the Inria Associate Team “IMPINGE” (Inverse Magnetization Problems IN GEosciences) together with MIT and Vanderbilt University. A recent collaboration with Cerege (CNRS, Aix-en-Provence), in the framework of the ANR-project MagLune, completes this picture, see Section 8.2.2.

To set up the context, recall that the Earth’s geomagnetic field is generated by convection of the liquid metallic core (geodynamo) and that rocks become magnetized by the ambient field as they are formed or after subsequent alteration. Their remanent magnetization provides records of past variations of the geodynamo, which is used to study important processes in Earth sciences like motion of tectonic plates and geomagnetic reversals. Rocks from Mars, the Moon, and asteroids also contain remanent magnetization which indicates the past presence of core dynamos. Magnetization in meteorites may even record fields produced by the young sun and the protoplanetary disk which may have played a key role in solar system formation.

For a long time, paleomagnetic techniques were only capable of analyzing bulk samples and compute their net magnetic moment. The development of SQUID microscopes has recently extended the spatial resolution to sub-millimeter scales, raising new physical and algorithmic challenges. This associate team aims at tackling them, experimenting with the SQUID microscope set up in the Paleomagnetism Laboratory of the department of Earth, Atmospheric and Planetary Sciences at MIT. Typically, pieces of rock are sanded down to a thin slab, and the magnetization has to be recovered from the field measured on a parallel plane at small distance above the slab.

Mathematically speaking, both inverse source problems for EEG from Section 4.2 and inverse magnetization problems described presently amount to recover the (3-D valued) quantity m (primary current density in case of the brain or magnetization in case of a thin slab of rock) from measurements of the vector potential:

$$\int_{\Omega} \frac{\operatorname{div} m(x') dx'}{|x-x'|}, \quad (1)$$

outside the volume Ω of the object. The difference is that the distribution m is located in a volume in the case of EEG, and on a plane in the case of rock magnetization. This results in quite different identifiability properties, see [38] and Section 6.1.2.

4.4. Free boundary problems

Participants: Laurent Baratchart, Juliette Leblond.

This work is conducted in part with Yannick Privat, CNRS, Lab. J.-L. Lions, Paris.

The team has engaged in the study of problems with variable conductivity σ , governed by a 2-D equation of the form $\operatorname{div}(\sigma \nabla u) = 0$. Such equations are in one-to-one correspondence with real parts of solutions to conjugate-Beltrami equations $\bar{\partial} f = \nu \partial \bar{f}$, so that complex analysis is a tool to study them, see [4], [14], [34]. This research was prompted by issues in plasma confinement for thermonuclear fusion in a tokamak, more precisely with the extrapolation of magnetic data on the boundary of the chamber from the outer boundary of the plasma, which is a level curve for the poloidal flux solving the original div-grad equation. Solving this inverse problem of Bernoulli type is of importance to determine the appropriate boundary conditions to be applied to the chamber in order to shape the plasma [58]. Investigations started in collaboration with CEA-IRFM (Cadarache) and the Laboratoire J.-A. Dieudonné at the Univ. of Nice-SA. Within the team, they now expand to cover Dirichlet-Neumann problems for larger classes of conductivities, *cf.* in particular [34] (see Section 6.2).

4.5. Identification and design of microwave devices

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

This is joint work with Stéphane Bila (XLIM, Limoges) and Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

One of the best training grounds for function-theoretic applications by the team is the identification and design of physical systems whose performance is assessed frequency-wise. This is the case of electromagnetic resonant systems which are of common use in telecommunications.

In space telecommunications (satellite transmissions), constraints specific to on-board technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study of the Helmholtz equation states that an essentially discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all cavities show the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since screws are conductors, they behave as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of an iris is opposite to that of a screw: no condition is imposed on a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 GHz.



Figure 1. Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that 16 quantities must be optimized. Quantities such as the diameter and length of the cavities, or the width of the 11 slits are fixed during the design phase.

Near the resonance frequency, a good approximation of Maxwell's equations is given by the solution of a second order differential equation. Thus, one obtains an electrical model of the filter as a sequence of electrically-coupled resonant circuits, each circuit being modeled by two resonators, one per mode, the resonance frequency of which represents the frequency of a mode, and whose resistance accounts for electric losses (current on the surface) of the cavities.

This way, the filter can be seen as a quadripole, with two ports, when plugged on a resistor at one end and fed with some potential at the other end. One is now interested in the power which is transmitted and reflected. This leads one to define a scattering matrix S , which may be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example on Figure 2), and the key step consists in expressing the components of the equivalent electrical circuit as functions of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the quality of design, in particular the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.* the underlying system may no longer have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the strategy for identification is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80MHz in the example).
- Solving bounded extremal problems for the transmission and the reflection (the modulus of the response being respectively close to 0 and 1 outside the interval measurement, cf. Section 3.3.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.

- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this example) via the Endymion or RARL2 software (cf. Section 3.3.2.2).
- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

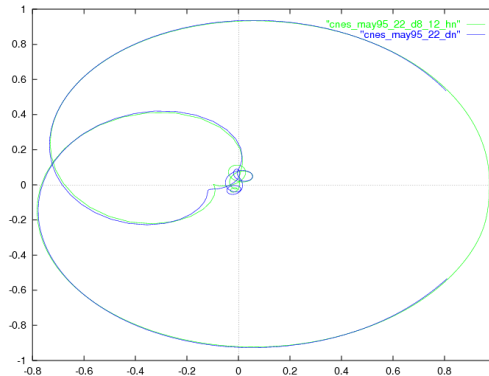


Figure 2. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22} .

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3} . This is illustrated by a reflection diagram (Figure 2). Non-physical couplings are less than 10^{-2} .

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, which are useful for the synthesis of repeating devices.

The team also investigates problems relative to the design of optimal responses for microwave devices. The resolution of a quasi-convex Zolotarev problems was proposed, in order to derive guaranteed optimal multi-band filter responses subject to modulus constraints [11]. This generalizes the classical single band design techniques based on Chebyshev polynomials and elliptic functions. The approach relies on the fact that the modulus of the scattering parameter $|S_{1,2}|$ admits a simple expression in terms of the filtering function $D = |S_{1,1}|/|S_{1,2}|$, namely

$$|S_{1,2}|^2 = \frac{1}{1 + D^2}.$$

The filtering function appears to be the ratio of two polynomials p_1/p_2 , the numerator of the reflection and transmission scattering factors, that can be chosen freely. The denominator q is obtained as the unique stable unitary polynomial solving the classical Feldtkeller spectral equation:

$$qq^* = p_1p_1^* + p_2p_2^*.$$

The relative simplicity of the derivation of a filter's response, under modulus constraints, owes much to the possibility of forgetting about Feldtkeller's equation and express all design constraints in terms of the filtering function. This no longer the case when considering the synthesis N -port devices for $N > 3$, like multiplexers, routers power dividers or when considering the synthesis of filters under matching conditions. The efficient derivation of multiplexers responses is among the team's recent investigation, where techniques based on constrained Nevanlinna-Pick interpolation problems are being considered (see Section 6.3.1).

Through contacts with CNES (Toulouse) and UPV (Bilbao), Apics got further involved three years ago with the design of amplifiers which, unlike filters, are active devices. A prominent issue here is stability. A twenty years back, it was not possible to simulate unstable responses, and only after building a device could one detect instability. The advent of so-called *harmonic balance* techniques, which compute steady state responses of linear elements in the frequency domain and look for a periodic state in the time domain of a network connecting these linear elements *via* static nonlinearities made it possible to compute the harmonic response of a (possibly nonlinear and unstable) device [82]. This has had tremendous impact on design, and there is a growing demand for software analyzers.

There are two types of stability involved. The first is stability of a fixed point around which the linearized transfer function accounts for small signal amplification. The second is stability of a limit cycle which is reached when the input signal is no longer small and truly nonlinear amplification is attained (*e.g.* because of saturation). Work by the team so far is concerned with the first type of stability, and emphasis is put on defining and extracting the "unstable part" of the response, see Section 6.4.

5. New Software and Platforms

5.1. RARL2

Participant: Martine Olivi [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see Section 3.3.2.2) <http://www-sop.inria.fr/apics/RARL2/rarl2.html>.

The software RARL2 computes, from a given matrix-valued function in $\overline{H}^{2^{m \times l}}$, a local best rational approximant in the L^2 norm, which is *stable and of prescribed McMillan degree* (see Section 3.3.2.2). It was initially developed in the context of linear (discrete-time) system theory and makes an heavy use of the classical concepts in this field. The matrix-valued function to be approximated can be viewed as the transfer function of a multivariable discrete-time stable system. RARL2 takes as input either:

- its internal realization,
- its first N Fourier coefficients,
- discretized (uniformly distributed) values on the circle. In this case, a least-square criterion is used instead of the L^2 norm.

It thus performs model reduction in case 1) and 2) and frequency data identification in case 3). In the case of band-limited frequency data, it could be necessary to infer the behavior of the system outside the bandwidth before performing rational approximation (see Section 3.2.2). An appropriate Möbius transformation allows to use the software for continuous-time systems as well.

The method is a steepest-descent algorithm. A parametrization of MIMO systems is used, which ensures that the stability constraint on the approximant is met. The implementation, in Matlab, is based on state-space representations.

The number of local minima can be large so that the choice of an initial point for the optimization may play a crucial role. In this connection, two methods can be used: 1) An initialization with a best Hankel approximant. 2) An iterative research strategy on the degree of the local minima, similar in principle to that of RARL2, increases the chance of obtaining the absolute minimum by generating, in a structured manner, several initial conditions.

RARL2 performs the rational approximation step in our applications to filter identification (see Section 4.5) as well as sources or cracks recovery (see Section 4.2). It was released to the universities of Delft, Maastricht, Cork, Brussels and Macao. The parametrization embodied in RARL2 was also used for a multi-objective control synthesis problem provided by ESTEC-ESA, The Netherlands. An extension of the software to the case of triple poles approximants is now available. It is used by FindSources3D (see Section 5.6).

5.2. RGC

Participant: Fabien Seyfert [corresponding participant].

Status: A stable version is maintained.

This software is developed in collaboration with Jean-Paul Marmorat (Centre de mathématiques appliquées (CMA), École des Mines de Paris).

The identification of filters modeled by an electrical circuit that was developed by the team (see Section 4.5) led us to compute the electrical parameters of the underlying filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called “the arrow form” [57] which is of particular interest since it is unique for a given transfer function and is easily computed. The computation of this realization is the first step of RGC. Subsequently, if the target realization is not in arrow form, one can nevertheless show that it can be deduced from the arrow-form by a complex-orthogonal change of basis. In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has many local and global minima. In fact, there is not even uniqueness of the filter realization for a given geometry. Moreover, it is often relevant to know all solutions of the problem, because the designer is not even sure, in many cases, which one is being handled. The assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software is not guaranteed to provide a solution. In contrast, the software Dedale-HF (see Section 5.4), which is the successor of RGC, is guaranteed to solve this constraint realization problem.

5.3. PRESTO-HF

Participant: Fabien Seyfert [corresponding participant].

Status: Currently under development. A stable version is maintained.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for microwave filters <http://www-sop.inria.fr/apics/Presto-HF>. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single shot:

- determination of delay components caused by the access devices (automatic reference plane adjustment),
- automatic determination of an analytic completion, bounded in modulus for each channel,
- rational approximation of fixed McMillan degree,
- determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies on RARL2 (see Section 5.1). Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Thales Alenia Space in Toulouse, Thales airborne systems and a license agreement has been recently negotiated with TAS-Espagna. XLIM (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements are currently being considered with the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada). A time-limited license has been bought by Flextronics for testing purposes.

5.4. Dedale-HF

Participant: Fabien Seyfert [corresponding participant].

Status: Currently under development. A stable version is maintained.

Dedale-HF is a software dedicated to solve exhaustively the coupling matrix synthesis problem in reasonable time for the filtering community. Given a coupling topology, the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of given filter characteristics. Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements (see Figure 3).

Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. corresponding to the user-specified filter characteristics. The reference files are computed off-line using Gröbner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such continuation techniques, combined with an efficient implementation of the integrator, drastically reduces the computational time.

Access to the database and integrator code is done via the web on <http://www-sop.inria.fr/apics/Dedale/WebPages>. The software is free of charge for academic research purposes: a registration is however needed in order to access full functionality. Up to now 90 users have registered world wide (mainly: Europe, U.S.A, Canada and China) and 4000 reference files have been downloaded.

A license for this software has been sold end of 2011 to TAS-Espagna, in order to tune filters with topologies having multiple solutions. For this, Dedale-HF teams up with Presto-HF.

5.5. easyFF

Participant: Fabien Seyfert.

Status: A stable version is maintained.

This software has been developed by Vincent Lunot (Taiwan Univ.) during his PhD. He still continues to maintain it.

EasyFF is a software dedicated to the computation of complex, in particular multi-band filtering functions. The software takes as input, specifications on the modulus of the scattering matrix (transmission and rejection), the filter's order and the number of transmission zeros. The output is an "optimal" filtering characteristic in the sense that it is the solution of an associated min-max Zolotarev problem. Computations are based on a Remez-type algorithm (if transmission zeros are fixed) or on linear programming techniques if transmission zeros are part of the optimization [11].

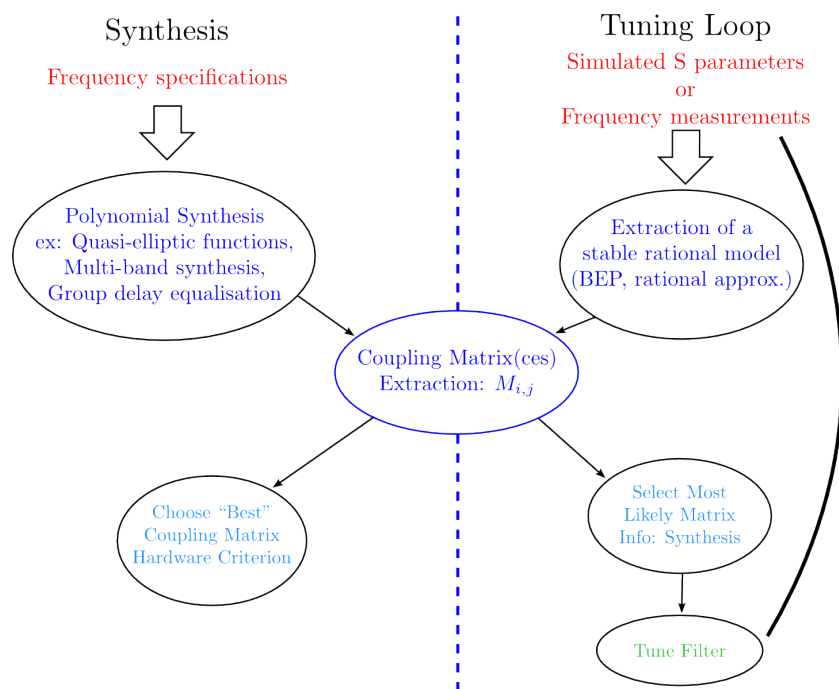


Figure 3. Overall scheme of the design and tuning process of a microwave filter.

5.6. FindSources3D

Participant: Juliette Leblond [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena Project-Team, and with Jean-Paul Marmorat (Centre de mathématiques appliquées - CMA, École des Mines de Paris).

FindSources3D ² is a software dedicated to source recovery for the inverse EEG problem, in 3-layer spherical settings, from point-wise data (see <http://www-sop.inria.fr/apics/FindSources3D/>). Through the algorithm described in [9] and Section 4.2, it makes use of the software RARL2 (Section 5.1) for the rational approximation step in plane sections.

A new release of FindSources3D is now available, which will be demonstrated and distributed, in particular to the medical team we maintain contact with (hosp. la Timone, Marseille). The preliminary step (“cortical mapping”) is now solved using expansion in spherical harmonics, along with a constrained approximation scheme.

Another release is being prepared, due to strong interest by the German company BESA GmbH ³, which develops EEG software for research and clinical applications. A deeper collaboration with this company started last year. Figure 4 shows good results on a two sources distribution recovered by FindSources3D from values of the potential at electrodes on a sphere (scalp) generated by BESA’s simulator. There, the localization error is satisfactory, see [28]. Altogether FindSources3D provides suitable initial guess to heavier dedicated recovery tools, including an estimate of the number of sources see Section 6.1.1.

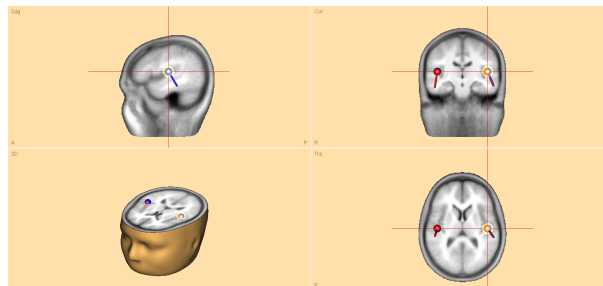


Figure 4. Recovered 2 sources by FindSources3D (courtesy of BESA).

5.7. Sollya

Participant: Sylvain Chevillard [corresponding participant].

Status: Currently under development. A stable version is maintained.

This software is developed in collaboration with Christoph Lauter (LIP6) and Mioara Joldes (LAAS).

Sollya is an interactive tool where the developers of mathematical floating-point libraries (libm) can experiment before actually developing code. The environment is safe with respect to floating-point errors, *i.e.* the user precisely knows when rounding errors or approximation errors happen, and rigorous bounds are always provided for these errors.

²CeCILL license, APP version 2.0 (2012): IDDN.FR.001.45009.001.S.A.2009.000.10000

³<http://www.besa.de/>

Among other features, it offers a fast Remez algorithm for computing polynomial approximations of real functions and also an algorithm for finding good polynomial approximants with floating-point coefficients to any real function. As well, it provides algorithms for the certification of numerical codes, such as Taylor Models, interval arithmetic or certified supremum norms.

It is available as a free software under the CeCILL-C license at <http://sollya.gforge.inria.fr/>.

6. New Results

6.1. Source recovery problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Christos Papageorgakis, Olga Permiakova, Dmitry Ponomarev.

The research in this section is partly joint work with Qian Tao (Univ. Macao).

It was proved in [38] that a vector field with $n + 1$ components on \mathbb{R}^n can be expressed uniquely as the sum of (the trace on \mathbb{R}^n of) a harmonic gradient in the upper half-space, of (the trace on \mathbb{R}^n of) a harmonic gradient in the lower half-space, and of a tangential divergence free vector field on \mathbb{R}^n . This decomposition, that we call the *Hardy-Hodge* decomposition, is valid not only for L^p vector fields as mentioned in Section 3.3.1, but in much more general distribution spaces like $W^{-\infty,p}$ which contains all distributions with compact support or $BMO^{-\infty}$ which contains all finite sums of derivatives of bounded functions. This year we extended the decomposition to smooth hypersurfaces, where divergence-free distributions may be defined as those annihilating tangential gradient vector fields. We also studied the case where the hypersurface is only Lipschitz smooth, and then we proved the decomposition in L^p provided that p is close enough to 2 (how close depends on the Lipschitz constant of the hypersurface).

The Hardy-Hodge decomposition was used in [38] to find the kernel of the planar magnetization operator, namely a potential of the form (1) with m supported in a plane generates the zero field above that plane if, and only if there is no harmonic gradient from below in the Hardy-Hodge decomposition of m . The above mentioned generalization is now to the effect that a magnetization supported on a bounded closed surface (e.g. a sphere) is silent in the unbounded component of the complement of that surface if, and only if there is no harmonic gradient from inside in its Hardy-Hodge decomposition. An article is being written on this topic.

We also considered the case where m is compactly supported in the bounded component of the complement of that surface. Then m is silent if and only if it is the sum of a divergence-free distribution and of finitely many derivatives of gradients of Sobolev functions having zero trace on the surface [41].

These results shed light on the indeterminacy of inverse source problems.

6.1.1. EEG

This work is conducted in collaboration with Maureen Clerc and Théo Papadopoulos from the Athena EPI, and with Jean-Paul Marmorat (Centre de mathématiques appliquées - CMA, École des Mines de Paris).

In 3-D, functional or clinically active regions in the cortex are often modeled by point-wise sources that must be localized from measurements of a potential on the scalp. Inside the cortex, identified to a ball after the cortical mapping step, the potential satisfies a Poisson equation whose right-hand side is a linear combination of gradients of Dirac masses (the sources in EEG). In the work [3] it was shown how best rational approximation on a family of circles, cut along parallel planes on the sphere, can be used to recover the sources when they are at most 2 of them. Later, results on the behavior of poles in best rational approximation of fixed degree to functions with branch points [6] helped justifying the technique for finitely many sources (see section 4.2).

The dedicated software FindSources3D (see section 5.6), developed, in collaboration with the team Athena and the CMA, dwells on these ideas. Functions to be approximated in 2-D slices turn out to have additional *multiple* poles at their branch points so that, in the rational approximation step, it is beneficial to consider approximants with multiple poles as well (for EEG data, one should consider *triple* poles). Though numerically observed in [9], there is no mathematical justification so far why these multiple poles are attracted more strongly than simple poles to the singularities of the approximated function. This intriguing property, however, definitely helps source recovery [28]. This year we used it to automatically estimate the “most plausible” number of sources (numerically: up to 3 at the moment). Such enhancements were prompted by a developing collaboration with the BESA company, which is interested in automatic detection of the number of sources (which was left to the user until recently).

Soon, magnetic data from MEG (magneto-encephalography) will become available together with EEG data; indeed, it is now possible to use simultaneously the corresponding measurement devices. We expect this to improve the accuracy of our algorithms.

In relation to other brain exploration modalities like electrical impedance tomography (EIT, see [16]), we also consider identifying electrical conductivity in the head. This is the topic of the PhD of C. Papageorgakis, co-advised with the Athena project-team and BESA GmbH. Specifically, in layered models, we are concerned with estimating conductivity of the skull (intermediate layer). Indeed, the skull consists of a hard bone part, the conductivity of which is more or less known, and spongy bone compartments whose conductivities may vary considerably with individuals.

A preliminary question in this connection is: can one uniquely recover a homogeneous skull conductivity from a single EEG recording when the sources and the conductivities of other layers are known? And if sources are not known, which additional information do we need? These are issues currently under investigation. To put them into perspective, recall the famous Caldèron problem of deducing a bounded (nonconstant) conductivity from the knowledge of all possible pairs consisting of a potential and its current flux at the boundary. In dimension 3, when the conductivity is not smooth (less than $3/2$ of a derivative), it is unknown whether the problem is even injective (*i.e.* if two conductivities can have the same pairs of boundary potential and flux). A weaker, discrete version of this problem is: if the conductivity takes on finitely many values and the geometry of the level sets is known, does a finite set of pairs of boundary potential and flux allow one to recover it? This is a significant question to be tackled for the purpose of source recovery in EEG with known geometry but unknown conductivities inside the head.

6.1.2. Inverse Magnetization problems

This work is carried out in the framework of the “équipe associée Inria” IMPINGE, comprising Eduardo Andrade Lima and Benjamin Weiss from the Earth Sciences department at MIT (Boston, USA) and Douglas Hardin and Edward Saff from the Mathematics department at Vanderbilt University (Nashville, USA),

Localizing magnetic sources from measurements of the magnetic field away from the support of the magnetization is the fundamental issue under investigation by IMPINGE. The goal is to determine magnetic properties of rock samples (*e.g.* meteorites or stalactites), from fine field measurements close to the sample that can nowadays be obtained using SQUIDS (superconducting coil devices). Currently, rock samples are cut into thin slabs and the magnetization distribution is considered to lie in a plane, which makes for a somewhat less indeterminate framework than EEG because “less” magnetizations can produce the same field (for the slab has no inner volume). Note however that EEG data consist of both potential and current values at the boundary, whereas in the present setting only values of the normal magnetic field are provided to us.

Figure 5 presents a schematic view of the experimental setup: the sample lie on a horizontal plane at height 0 and its support is included in a rectangle. The vertical component B_z of the field produced by the sample is measured on points of a horizontal $N \times N$ rectangular grid at height h .

We set up last year a heuristic procedure to recover regularly spaced dipolar magnetizations, *i.e.* magnetizations composed of dipoles placed at the points of a regular rectangular $n \times n$ grid. The latter seems general enough a model class to approximate magnetizations commonly encountered in samples. However, for reasons of computational complexity, n is significantly smaller than N which limits the power of the model. Each

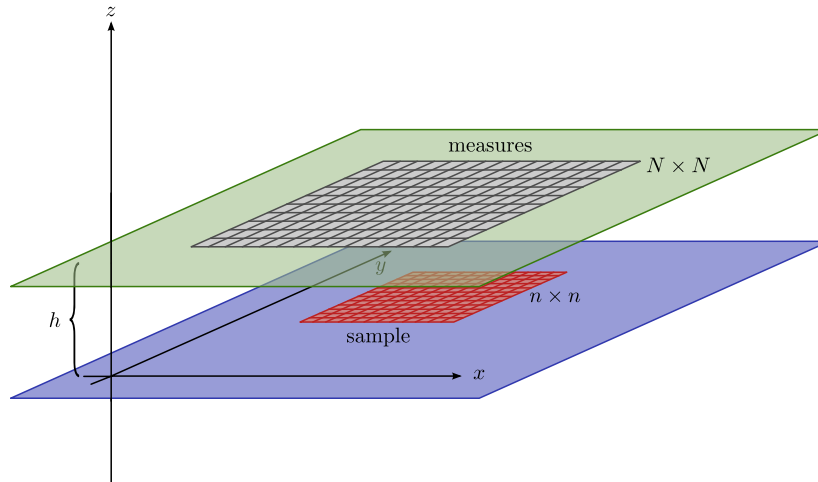


Figure 5. Schematic view of the experimental setup

dipole of the $n \times n$ grid is determined by the 3 components of its moment, thus the magnetization can be represented by a real $3n^2$ -vector. If we denote by A the matrix of the operator that maps such a vector X to the vector b of measurements (which belongs to \mathbb{R}^{N^2}), we want to find X such that AX is close to b . For computational simplicity, we use a Euclidean criterion $\|AX - b\|_2$, which reduces the problem to a singular value decomposition of A . The inverse problem being ill-posed, A is poorly conditioned and we must resort to a regularization technique. The one we developed initially has been based on iteratively cropping the support of b , using a threshold on the intensity of the dipoles at each step, so as to reduce the number of active components in b . Preliminary experiments were performed last year on synthetic data and also on a real example (Lunar spherule).

This year, we performed more systematic experiments on real data (namely Allende chondrules and Hawaiian basalt) provided by the SQUID scanning microscope at MIT lab. Cropping the support of b using thresholding has proved efficient to improve ill-conditioning for samples with localized support embedded in the slab (e.g., chondrules). On the other hand, when the support of the sample is spread out (e.g., Hawaiian basalt), the reduction of active components of b was insignificant. We used this inversion procedure to estimate the net moment. The importance of the latter has been emphasized by the geophysicists at MIT for at least two reasons: firstly it yields important geological information on the sample in particular to estimate the magnitude of the ambient magnetic field at the time the rock was formed. Secondly, it may to some extent be measured independently, using a magnetometer, thereby allowing one to cross-validate the approach. A third, computational reason is that knowledge of the net moment should pave the way to a numerically stable reconstruction of an equivalent unidirectional magnetization. The support of the latter would provide us with valuable information to test for unidirectionality of the true magnetization, which is an important question to physicists.

When the support can be significantly shrunk while keeping the residue small (i.e., explaining the data satisfactorily), estimates of the net moment based on the dipolar model obtained by inversion seem to be good. They apparently supersede the measurements by magnetometers as well as by dipole fitting procedures set up at MIT. It is interesting to notice that the magnetization obtained by our inversion procedure, either before or after shrinking the support, often does not resemble the true magnetization, even when it yields correct moment and field. This can be seen on synthetic examples and may be surmised on real data, thereby

confirming that recovering the net moment and recovering the magnetization are rather different problems, the latter being considerably more ill-posed than the former.

One specific difficulty with chondrule-type examples has been to account for their thickness: they are indeed small spheres and their 3-D character cannot be completely ignored. In order to use the inversion procedure set up in the plane, we investigated the following question. Assume that the sample has some thickness, but small enough that the magnetization at a point $P = (x, y, z)$ inside the sample depends only on x and y (possibly weighted by some function that depends only on z), *i.e.* that it is of the form $m(x, y)\phi(z)$. If we consider a (truly) planar magnetization with the same distribution $m(x, y)$ but on a plane lying at some nonzero height ε , how to choose ε so as to produce a field at height h which is closest to the field produced by the thick magnetization? This has been the object of the internship of Olga Permiakova who used local expansion of the dipole-to-field map (see her report ⁴). An article is being written on this subject.

The case where the magnetization is flat but spread out on the sample is more difficult. First of all, the computational effort becomes significant and led us to use the cluster at Inria Sophia Antipolis. We succeeded in obtaining full inversions for the Hawaiian basalt. The residue (approximation error) is moderate but not impressively small, which indicates that we reach the limit of modeling magnetizations by a regular grid of dipoles. However the computation of the moment compares favorably with estimates previously obtained by a different technique at MIT lab. Still, using a cluster and two days of evaluation to obtain a coarse estimate of the net moment of a sample is rather inefficient and calls for new investigations.

We also experimented an alternative regularization procedure, based on L^2 minimization under L^1 penalty as solved by the SALSA algorithm. Such methods are quite popular today for sparse recovery. However, the computational load, as well as the quality of the results, do not differ significantly from those obtained previously.

We now develop new methods in order to estimate the net moment of the magnetization, based on improvements of previously used Fourier techniques, and recently we reformulated the problem with the help of Kelvin transforms. It has been realized that the success of net moment recovery hinges on the ability to extrapolate the measurements. In particular, we managed to considerably improve previous estimates by means of data extension based on dipolar field asymptotics.

In the course of inverting the field map, we singled out magnetizations which are numerically (almost) silent from above though not from below. This illustrates how ill-posed (unstable) the problem, as theory predicts that no compactly supported magnetization can be exactly silent from above without being also exactly silent from below. Although such magnetizations seem to have small moment and therefore do not endanger the possibility of recovering the net moment, their existence is certainly an obstacle to inversion of the field map without extra measurements or hypotheses (*e.g.*, measuring from below or assuming unidirectionality).

In the course of the doctoral work by D. Ponomarev, the study of the 2D spectral decomposition of the truncated Poisson operator has been undertaken. It is a simplified version of the relation between the magnetization and the magnetic potential. We considered several formulations in terms of singular integral equations and matrix Riemann-Hilbert problems, and focused on finding closed form solutions for various approximations of the Poisson operator in terms of a the ratio between the distance h to the measurement plane and the sample support size.

Lately, Apics became a partner of the ANR project MagLune, dealing with Lunar magnetism, a in collaboration with the Geophysics and Planetology Department of Cerege, CNRS, Aix-en-Provence, see section 8.2.2. The research is just starting, and will focus on computing net moments of lunar rock samples collected by NASA.

6.2. Boundary value problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Dmitry Ponomarev.

⁴http://www-sop.inria.fr/apics/IMPINGE/Documents/Report_Permiakova_Olga.pdf

Collaboration with Laurent Bourgeois (ENSTA ParisTech, Lab. Poems), Elodie Pozzi (Univ. Bordeaux, IMB), Emmanuel Russ (Univ. Grenoble, IJF).

Generalized Hardy classes

As we mentioned in section 4.4, 2-D diffusion equations of the form $\operatorname{div}(\sigma \nabla u) = 0$ with real non-negative valued conductivity σ can be viewed as compatibility conditions for the so-called conjugate Beltrami equation: $\bar{\partial} f = \nu \partial \bar{f}$ with $\nu = (1 - \sigma)/(1 + \sigma)$ [4]. Thus, the conjugate Beltrami equation is a means to replace the initial second order diffusion equation by a first order system of two real equations, merged into a single complex one. Hardy spaces under study here are those of this conjugate Beltrami equation: they are comprised of solutions to that equation in the considered domain whose L^p means over curves tending to the boundary of the domain remain bounded. They will for example replace holomorphic Hardy spaces in problem (P) when dealing with non-constant (isotropic) conductivity. Their traces merely lie in L^p ($1 < p < \infty$), which is suitable for identification from point-wise measurements, and turn out to be dense on strict subsets of the boundary. This allows one to state Cauchy problems as bounded extremal issues in L^p classes of generalized analytic functions, in a manner which is reminiscent of what we discussed for analytic functions in section 3.3.1.

The study of such Hardy spaces for Lipschitz σ was reduced in [4] to that of spaces of pseudo-holomorphic functions with bounded coefficients, which were apparently first considered on the disk by S. Klimentov. Solutions factorize as $e^s F$, where F is a holomorphic Hardy function while s is in the Sobolev space $W^{1,r}$ for all $r < \infty$ (Bers factorization), and the analog to the M. Riesz theorem holds which amounts to solvability of the Dirichlet problem with L^p boundary data. The case of finitely connected domains was carried out in [14].

This year, we addressed in [25] the uniqueness issue for the classical Robin inverse problem on a Lipschitz-smooth domain $\Omega \subset \mathbb{R}^n$, with L^∞ Robin coefficient, L^2 Neumann data and isotropic conductivity of class $W^{1,r}(\Omega)$, $r > n$. The Robin inverse problem consists in recovering the ratio of the normal derivative and the solution (the so-called Robin coefficient) on a subset of the boundary, knowing them on the complementary subset. We showed that uniqueness of the Robin coefficient on a subset of the boundary, given Cauchy data on the complementary subset, does hold when $n = 2$ whenever the boundary subsets are of positive Lebesgue measure. We also showed that this no longer holds in higher dimension, and we gave counterexamples when $n = 3$. The subsets in these counterexamples look very bad, and it is natural to ask whether uniqueness prevails if they have interior points. This raises an interesting open issue on harmonic gradients, namely: can a nonzero harmonic function vanish together with its normal derivative on a subset of the boundary of positive measure, and still the Robin coefficient is bounded in a neighborhood of that set? This question is worth investigating

Best constrained analytic approximation

Several questions about the behavior of solutions to the bounded extremal problem (P) in section 3.3.1, and of some generalizations thereof, are still under study by Apics.. We considered additional interpolation constraints on the disk in problem (P), and derived new stability estimates for the solution [24]. An article is being written on the subject. Ongoing work is geared towards applications of [24]. New insight leads us to relate these results to overdetermined boundary value problems for 2D Laplace equations on irregular boundaries. This has applications in set-ups where measurements are obtained from oddly distributed sensors. Treating some of the measurements as pointwise interpolation constraints seems a reasonable strategy in comparison with interpolation of the data along a geometrically complicated boundary. Such interpolation constraints arise naturally in inverse boundary problems like plasma shaping, when some of the measurements are performed inside the chamber of the tokamak, see section 4.4.

6.3. Matching problems and their applications - De-embedding of filters in multiplexers

Participants: Laurent Baratchart, Martine Olivi, Sanda Lefteriu, David Martinez Martinez, Fabien Seyfert.

This work has been done in collaboration with Stéphane Bila (Xlim, Limoges, France), Hussein Ezzedin (Xlim, Limoges, France), Damien Pacaud (Thales Alenia Space, Toulouse, France), Giuseppe Macchiarella (Politecnico di Milano, Milan, Italy), and Matteo Oldoni (Siae Microelettronica, Milan, Italy).

6.3.1. Matching problems and their applications

Filter synthesis is usually performed under the hypothesis that both ports of the filter are loaded on a constant resistive load (usually 50 Ohm). In complex systems, filters are however cascaded with other devices, and end up being loaded, at least at one port, on a non purely resistive frequency varying load. This is for example the case when synthesizing a multiplexer: each filter is here loaded at one of its ports on a common junction. Thus, the load is by construction non constant with the frequency, and not purely resistive either. Likewise, in an emitter-receiver, the antenna is followed by a filter. Whereas the antenna can usually be regarded as a resistive load at some frequencies, this is far from being true on the whole working band. A mismatch between the antenna and the filter, however, causes irremediable power losses, both in emission and transmission. Our goal is therefore to develop a filter synthesis method that allows to match varying loads on specific frequency bands.

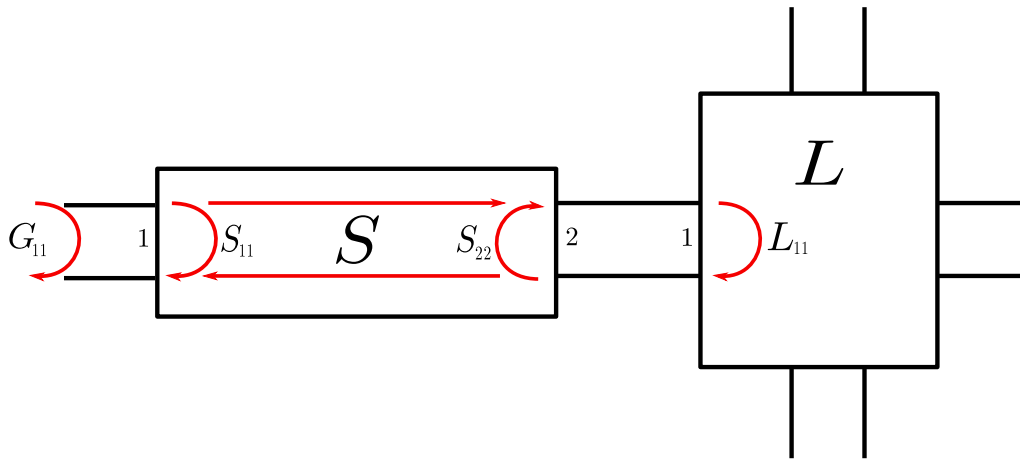


Figure 6. Filter plugged on a system with reflexion coefficient L_{11}

Figure 6 shows a filter with scattering matrix S , plugged at its right port on a frequency varying load with reflexion parameter $L_{1,1}$. If the filter is lossless, simple algebraic manipulations show that on the frequency axis the reflexion parameter satisfies:

$$|G_{1,1}| = \left| \frac{S_{2,2} - \overline{L_{1,1}}}{1 - S_{2,2}L_{1,1}} \right|.$$

The matching problem of minimizing $|G_{1,1}|$ amounts therefore to minimize the pseudo-hyperbolic distance between the filter's reflexion parameter $S_{2,2}$ and the load's reflexion $L_{1,1}$, on a given frequency band. For a broad class of filters, namely those that can be modeled by a circuit of n coupled resonators, the scattering matrix S is a rational function of McMillan degree n in the frequency. The matching problem appears therefore as a rational approximation problem in hyperbolic metric. When n is fixed, the latter is non-convex and led us to seek methods to derive good initial guesses for classical descent algorithms. To this effect, if $S_{2,2} = p/q$ we considered the following interpolation problem: given n frequency points $w_1 \cdots w_n$ and a transmission polynomial r , to find a unitary polynomial p of degree n such that:

$$j = 1..n, \quad \frac{p}{q}(w_j) = \overline{L_{1,1}(w_j)}$$

where q is the unique monic Hurwitz polynomial of degree n satisfying the Feldtkeller equation

$$qq^* = pp^* + rr^*,$$

which accounts for the losslessness of the filter. This problem can be seen as an extended Nevanlinna-Pick interpolation problem, that was considered in [67] when the interpolation points w_j lie in the *open* left half-plane. The method in the last reference does not extend to imaginary interpolation point and we used rather different, differential-topological techniques to prove that this problem has a unique solution, which can be computed by continuation. In the setting of multiplexer synthesis, where this result must be applied recursively to each filter, we showed the existence of a fixed point for the tuning procedure, based on Brouwer's fixed point theorem. These results were presented at the MTNS [18], at the plenary of session of Ernsi workshop 2014, and they lie at the heart of the ANR Cocoram on co-integration of filters and antennas (8.2.1). Implementation of the continuation algorithm has been done under contract with CNES and yields encouraging results. Generalizations of the interpolation problem where the monic condition is relaxed are under study in the framework of co-integration of filters and antennas.

6.3.2. De-embedding of multiplexers

This work is pursued in collaboration with Thales Alenia Space, Siae Microelettronica, Xlim and under contract with CNES-Toulouse (see section 7.1).

Let S be the scattering parameters of a multiplexer composed of a N -port junction with response T and $N - 1$ filters with responses F_1, \dots, F_{N-1} , as plotted on Figure 7. The de-embedding problem is to recover the F_k and it can be stated under various hypotheses. Last year we studied this problem when S and T are known [79] but no special structure for the F_k is assumed. It was shown that for generic T and for $N > 3$, the de-embedding problem has a unique solution. In practice, however, the junction's response is far from being generic, as it is usually obtained *via* assembly of T-junctions. This makes the problem extremely sensitive to measurement noise. It was also noticed that in practical applications, scattering measurements of the junction are hardly available.

It is therefore natural to consider the following de-embedding problem. Given S , and under the assumption that

- the F_k are rational of known McMillan degree,
- the coupling geometry of their circuital realization is known,

what can be said about the filter's response? Note that the above assumptions do not bear on the junction. Nevertheless, we showed that the filter's responses are identifiable up to a constant matrix chained at their nearest port to the junction [73]. It was proved also that the uncertainty induced by the chain matrix bears only on the resonant frequency of the last cavity of each filter, as well as on their output coupling. Most of the filters' parameters can therefore be recovered in principle. The approach is constructive and relies on rational approximation to certain scattering parameters, as well as on some extraction procedure similar to Darlington's synthesis. Software development is under way and experimental studies have started on data provided to us by Thales Alenia Space and by Siae Microelettronica. A mid-term objective is to extend Presto-HF (see Section 5.3) so as to handle de-embedding problems for multiplexers and more generally for multi-ports.

6.4. Stability of amplifiers

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

This work is performed under contract with CNES-Toulouse and the University of Bilbao. The goal is to help designing amplifiers, in particular to detect instability at an early stage of the design.

Currently, electrical engineers from the University of Bilbao, under contract with CNES (the French Space Agency), use heuristics to detect instability before an amplifying circuit is physically built. Our goal is to set up a rigorously founded algorithm, based on properties of transfer functions of such amplifiers, which belong to particular classes of analytic functions.

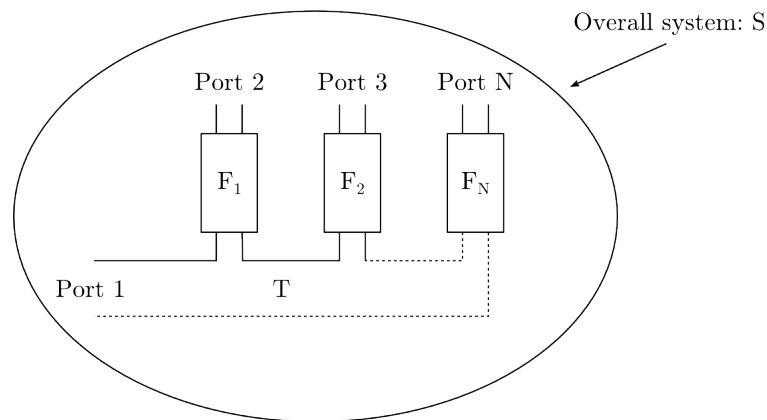


Figure 7. Multiplexer made of a junction T and filtering devices $F_1, F_2 \dots F_N$

In non-degenerate cases, non-linear electrical components can be replaced by their first order approximation when studying stability in the small signal regime. Using this approximation, diodes appear as negative resistors and transistors as current sources controlled by the voltage at certain nodes of the circuit.

Over the last three years, we studied several features of transfer functions of amplifying electronic circuits:

- We characterized the class of transfer functions which can be realized with ideal components linearized active components, together with standard passive components (resistors, inductors, capacitors and transmission lines). It is exactly the field of rational functions in the complex variable and in the hyperbolic cosines and identity-times-hyperbolic sines of polynomials of degree 2 with real negative roots.
- We introduced a realistic notion of stability, by terming stable a circuit whose transfer function belongs to H^∞ , as long a sufficiently high resistor is added in parallel to that circuit.
- We constructed unstable circuits having no pole in the right half-plane, which came as a surprise to our partners.
- In order to circumvent these pathological examples, we introduced a realistic hypothesis that there are small inductive and capacitive effects to active components. Our main result is that a realistic circuit without poles on the imaginary axis is unstable if and only if it has poles in the right half-plane. Moreover, there can only be finitely many of them.

This year, we were led to modify our definition of stability, taking a hint from scattering theory. We say that a transfer function Z is stable whenever $(R - Z)/(R + Z)$ belongs to H^∞ with uniformly bounded H^∞ -norm for all R large enough. Equivalently, this means that the circuit can amplify signals but not require an unbounded amount of energy from the primary power circuit. This new definition is really about energy, hence is more natural. Also, it allows us a unified characterization in the corner case where instabilities are located on the imaginary axis. We obtained this way a nice characterization: Z is stable if and only if it has no pole in the open right half plane, while each pole it may have on the imaginary axis is simple and has a residue with strictly positive real part. We published a research report [23] and an article is being written to report on our results.

6.5. Approximation

Participant: Laurent Baratchart.

6.5.1. Orthogonal Polynomials

This is joint work with Nikos Stylianopoulos (Univ. of Cyprus).

We study the asymptotic behavior of weighted orthogonal polynomials on a bounded simply connected plane domain Ω . The n -th orthogonal polynomial P_n has degree n , positive leading coefficient, and satisfies

$$\int_{\Omega} P_n \overline{P_k} w \, dm = \delta_{n,k}$$

where w is an integrable positive weight and $\delta_{n,k}$ is the Kronecker symbol. When Ω is smooth while w is Hölder-continuous and non-vanishing, it is known that

$$P_n(z) = \left(\frac{n+1}{\pi} \right)^{1/2} \Phi^n \frac{\Phi'}{S_w(z)} \{1 + o(1)\},$$

locally uniformly outside the convex hull of $\overline{\Omega}$, where Φ is the conformal map from the complement of Ω onto the complement of the unit disk and S_w is the so-called Szegő function of the trace of w on the boundary $\partial\Omega$ [81]. If we compare it with classical exterior Szegő asymptotics, the formula asserts that P_n behaves asymptotically like the n -th orthogonal polynomial with respect to a weight supported on $\partial\Omega$ (the trace of w), up to a factor $\sqrt{(n+1)/\pi}$.

When Ω is the unit disk, we proved this result under unprecedented weak assumptions on w , namely $w(re^{i\theta})$ should converge in $L^p(T)$ as $r \rightarrow 1$ for some $p > 1$ and its \log^- should be bounded in the real Hardy space H^1 . An article is being written on these findings and the case of a smooth domain Ω , more general than a disk, is under examination.

6.5.2. Meromorphic approximation

This is joint work with Maxim Yattselev (Purdue Univ. at Indianapolis, USA).

We proved in [6] that the normalized counting measure of poles of best H^2 approximants of degree n to a function analytically continuable, except over finitely many branchpoints lying outside the unit disk, converges to the Green equilibrium distribution of the compact set of minimal Green capacity outside of which the function is single valued (the normalized counting measure is the probability measure with equal mass at each pole). This result warrants source recovery techniques used in section 6.1.1. Here we consider the corresponding problem for best uniform meromorphic approximants on the unit circle (so-called AAK approximants after Adamjan, Arov and Krein), in the case where the function may have poles and essential singularities. This year, we established a similar result when the function has finitely many essential singularities. The general case is still pending.

7. Bilateral Contracts and Grants with Industry

7.1. Contract CNES-Inria-XLIM

This contract (reference Inria: 7066, CNES: 127 197/00) involving CNES, XLIM and Inria, focuses on the development of synthesis algorithms for N -ports microwave devices. The objective is to derive analytical procedures for the design of multiplexers and routers, as opposed to "black box optimization" which is usually employed in this field (for $N \geq 3$). Emphasis at the moment bears on so-called "star-topologies".

7.2. Contract CNES-Inria-UPV/EHU

This contract (reference CNES: RS14/TG-0001-019) involving CNES, University of Bilbao (UPV/EHU) and Inria aims at setting up a methodology for testing the stability of amplifying devices. The work at Inria is concerned with the design of frequency optimization techniques to identify the unstable part of the linearized response and analyze the linear periodic components.

7.3. Contract BESA GmbH-Inria

This is a research agreement between Inria (Apics and Athena teams) and the German company BESA ⁵, which deals with head conductivity estimation and co-advising of the doctoral work of C. Papageorgakis, see Section 6.1.1. BESA is funding half of the corresponding research grant, the other half is supported by Region PACA (BDO), see Section 8.1.1.

8. Partnerships and Cooperations

8.1. Regional Initiatives

8.1.1. Contract Provence Alpes Côte d'Azur (PACA) Region - Inria, BDO

Contract (no. 2014-05764) funding the research grant of C. Papageorgakis, see Sections 6.1.1, 7.3.

8.2. National Initiatives

8.2.1. ANR

The ANR (Astrid) project COCORAM (Co-design et co-intégration de réseaux d'antennes actives multi-bandes pour systèmes de radionavigation par satellite) started January 2014. We are associated with three other teams from XLIM (Limoges University), respectively specialized in filters, antennas and amplifiers design. The core idea of the project is to work on the co-integration of various microwave devices in the context of GPS satellite systems in particular it provides us with an opportunity to work on matching problems (see Section 6.3.1).

8.2.2. ANR MagLune

The ANR project MagLune (Magnétisme de la Lune) has been approved by July 2014. It involves the Cerege (Centre de Recherche et d'Enseignement de Géosciences de l'Environnement, joint laboratory between Université Aix-Marseille, CNRS and IRD), the IGP (Institut de Physique du Globe de Paris) and ISTerre (Institut des Sciences de la Terre). Associated with Cerege are Inria (Apics team) and Irphe (Institut de Recherche sur les Phénomènes Hors Équilibre, joint laboratory between Université Aix-Marseille, CNRS and École Centrale de Marseille). The goal of this project (led by geologists) is to understand the past magnetic activity of the Moon, especially to answer the question whether it had a dynamo in the past and which mechanisms were at work to generate it. Apics will participate in the project by providing mathematical tools and algorithms to recover the remanent magnetization of rock samples from the moon on the basis of measurements of the magnetic field it generates. The techniques described in Section 6.1 are instrumental for this purpose.

8.3. European Initiatives

8.3.1. Collaborations with Major European Organizations

Apics is part of the European Research Network on System Identification (ERNSI) since 1992.

System identification deals with the derivation, estimation and validation of mathematical models of dynamical phenomena from experimental data.

8.4. International Initiatives

8.4.1. Inria Associate Teams

8.4.1.1. IMPINGE

⁵<http://www.besa.de/>

Title: Inverse Magnetization Problems IN GEosciences.

Inria principal investigator: Laurent Baratchart

International Partner (Institution - Laboratory - Researcher):

MIT - Department of Earth, Atmospheric and Planetary Sciences (United States) - Benjamin Weiss

Duration: 2013 - 2015

See details at : <http://www-sop.inria.fr/apics/IMPINGE/>

The purpose of the associate team IMPINGE is to develop efficient algorithms to recover the magnetization distribution of rock slabs from measurements of the magnetic field above the slab using a SQUID microscope (developed at MIT). The US team also involves a group at Vanderbilt Univ.

8.4.2. Inria International Partners

8.4.2.1. Declared Inria International Partners

MIT-France seed funding is a competitive collaborative research program ran by the Massachusetts Institute of Technology (Cambridge, Ma, USA). Together with E. Lima and . Weiss from the Earth and Planetary Sciences dept. at MIT, Apics obtained two-years support from the above-mentioned program to run a project entitled: “Development of Ultra-high Sensitivity Magnetometry for Analyzing Ancient Rock Magnetism”

Cyprus NF grant was obtained by N. Stylianopoulos (Univ. Cyprus) to conduct joint research with L. Baratchart, E.B. Saff (Vanderbilt Univ.) and V. Totik (Univ. Szeged, Hungary). The title of the grant is: “Orthogonal polynomials in the complex plane: distribution of zeros, strong asymptotics and shape reconstruction”.

8.5. International Research Visitors

8.5.1. Visits of International Scientists

- Doug Hardin (Vanderbilt Univ., Nashville, USA, Aug 2014)
- Benjamin Lanfer (BESA, Munich, Germany, Oct 2014)
- Eduardo A. Lima (MIT, Cambridge, USA, Mar 2014)
- Moncef Mahjoub (ENIT LAMSIN, Tunis, Tunisia, Jun 2014)
- Michael Northington (Vanderbilt Univ., Nashville, USA, Aug 2014)
- Yves Rolain (Vrije Universiteit Brussel, Belgium, June 2014)
- Maxim Yattselev (Indiana University–Purdue University, Indianapolis, USA, May 2014)

8.5.1.1. Internships

- Olga Permiakova, Master 2 Computational Biology - UNSA (5 months), Inverse source problem for electromagnetic fields, with physical applications.

8.6. List of international and industrial partners

- Collaboration under contract with Thales Alenia Space (Toulouse, Cannes, and Paris), CNES (Toulouse), XLIM (Limoges), University of Bilbao (Universidad del País Vasco / Euskal Herriko Unibertsitatea, Spain), BESA company (Munich), Flextronics.

- Regular contacts with research groups at UST (Villeneuve d'Asq), Universities of Bordeaux-I (Talence), Orléans (MAPMO), Aix-Marseille (CMI-LATP), Nice Sophia Antipolis (Lab. JAD), Grenoble (IJF and LJK), Paris 6 (P. et M. Curie, Lab. JLL), Inria Saclay (Lab. Poems), Cerege-CNRS (Aix-en-Provence), CWI (the Netherlands), MIT (Boston, USA), Vanderbilt University (Nashville USA), Steklov Institute (Moscow), Michigan State University (East-Lansing, USA), Texas A&M University (College Station USA), University of Urana-Champaign at Indianapolis (Indianapolis, USA), Politecnico di Milano (Milan, Italy), University of Trieste (Italy), RMC (Kingston, Canada), University of Leeds (UK), of Maastricht (The Netherlands), of Cork (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien (Austria), TFH-Berlin (Germany), ENIT (Tunis), KTH (Stockholm), University of Cyprus (Nicosia, Cyprus), University of Macau (Macau, China), SIAE Microelettronica (Milano).
- The project is involved in the GDR-project AFHP (CNRS), in the ANR (Astrid program) project COCORAM (with XLIM, Limoges, and DGA), in the ANR (Défis de tous les savoirs program) project MagLune (with Cerege, IPGP, ISTerre, Irphe), in a MIT-France collaborative seed funding, in the Associate Inria Team IMPINGE (with MIT, Boston), and in a CSF program (with University of Cyprus).

9. Dissemination

9.1. Promoting Scientific Activities

- L. Baratchart was a plenary speaker at Constructive Functions 2014 (June 2014) in Nashville, USA (TN). He was an invited speaker at the Complex Analysis Meeting of the Russian Academy of Sciences (April 2014) in Saint Petersburg, Russia, at the International Conference on Orthogonal Polynomials, Integrable Systems and their Applications (October 2014) in Shanghai, China, and at the conference Foundations of Constructive Mathematics (December 2014) in Montevideo. He was a visitor at Vanderbilt university, at MIT, at the University of Macao and at the University of Cyprus. He was a speaker at the seminar of Université de Bordeaux.
- M. Caenepeel gave a talk at the 33th Benelux Meeting on Systems and Control (The Netherlands) at the 18th IEEE Workshop on Signal and Power Integrity in Ghent (Belgium) and he presented a poster at the ERNSI meeting in Ostende (Belgium).
- S. Chevillard gave a talk at PICOOF 2014 (May 2014) in Hammamet, Tunisia, at Constructive Functions 2014 (June 2014) in Nashville, USA (TN). He was an invited speaker at “Journée scientifique SMAI-SIGMA 2014” (November 2014) in Paris.
- J. Leblond organized an invited session at PICOOF 2014 ⁶ (May 2014). A poster about joint work on source estimation in EEG was presented at OHBM 2014 ⁷ [28].
- S. Lefteriu was an invited speaker at the Max Planck Institute and presented a poster at the meeting of the working group GT Identification.
- M. Olivi gave a talk at the MTNS 2014 conference in Groningen (The Netherlands) [18].
- D. Ponomarev gave a talk at the 10th AIMS Conference on Dynamical Systems, Differential Equations and Applications (July 2014) , in Madrid, Spain, at the seminar of the team Analyse, Géométrie, Topologie (AGT), Institut de Mathématiques de Marseille, Aix-Marseille Université (May 2014), and at the seminar of the team Defi, Inria Saclay - Ecole Polytechnique (Nov. 2014).
- F. Seyfert gave a talk at the MTNS 2014 in Groningen, at the IMS 2014 in Tampa and was invited to give a plenary lecture at the Ernsi meeting in Ostende.

⁶<http://www.lamsin.tn/picof14/>

⁷<http://www.humanbrainmapping.org/i4a/pages/index.cfm?pageID=3565>

9.1.1. Scientific events selection

9.1.1.1. member of the conference program committee

L. Baratchart was a member of the program committee of MTNS (Mathematical Theory of Networks and Systems) 2014, Groningen, The Netherlands.

9.1.2. Journal

9.1.2.1. member of the editorial board

L. Baratchart is a member of the Editorial Boards of *Constructive Methods and Function Theory* and *Complex Analysis and Operator Theory*.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Colles: S. Chevillard is giving “Colles” at Centre International de Valbonne (CIV) (2 hours per week).

9.2.2. Supervision

PhD in progress: D. Ponomarev, Inverse problems for planar conductivity and Schrödinger PDEs, since Nov. 2012 (advisors: J. Leblond, L. Baratchart).

PhD in progress: M. Caenepel, The development of models for the design of RF/microwave filters, since Feb. 2013 (advisors: Y. Rolain, M. Olivi, F. Seyfert).

PhD in progress: C. Papageorgakis, Conductivity model estimation, since Oct 2014 (advisors: J. Leblond, M. Clerc, B. Lanfer).

9.2.3. Juries

- M. Olivi was a referee of the PhD manuscript of P. Vuillemin (Univ. Toulouse) and of the PhD manuscript of F. Cheng (Univ. Lorraine).
- J. Leblond was a member of the PhD defense committee of L. Jassionnesse (Univ. Dijon, Nov 2014).
- F. Seyfert was a member of the PhD defense committee of Le Ha Vy Nguyen (Univ. Paris Sud, Inria project DISCO)

9.3. Popularization

- L. Baratchart was a speaker at “Café in” (Oct. 2014, Inria Sophia-Antipolis-Méditerranée).
- J. Leblond is a member of the Committee MASTIC. She was an invited speaker at the seminar associated with the lecture by G. Berry at the Collège de France (Jan. 2014).
- M. Olivi is president of the Committee MASTIC (Commission d’Animation et de Médiation Scientifique) <https://project.inria.fr/mastic/>. She is responsible for Scientific Mediation.

9.4. Community services

- S. Chevillard is representative at the “comité de centre” and at the “comité des projets” (Research Center Inria-Sophia).
- J. Leblond is an elected member of the “Conseil Scientifique” and of the “Commission Administrative Paritaire” of Inria. She is one of the two researchers in charge of the mission “Conseil et soutien aux chercheurs” within the Research Center.
- M. Olivi is responsible for scientific mediation and co-president of the committee MASTIC.

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