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Project-Team GECO

Geometric Control Design

RESEARCH CENTER
Saclay - Île-de-France

THEME
Optimization and control of dynamic systems

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Project-Team GECO

Keywords: Automatic Control, Nonlinear Control, Quantum Chemistry, System Analysis And Control, Tracking

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2. Overall Objectives

2.1. Overall Objectives

Motion planning is not only a crucial issue in control theory, but also a widespread task of all sort of human activities. The aim of the project-team is to study the various aspects preceding and framing *motion planning*: accessibility analysis (determining which configurations are attainable), criteria to make choice among possible trajectories, trajectory tracking (fixing a possibly unfeasible trajectory and following it as closely as required), performance analysis (determining the cost of a tracking strategy), design of implementable algorithms, robustness of a control strategy with respect to computationally motivated discretizations, etc. The viewpoint that we adopt comes from geometric control: our main interest is in qualitative and intrinsic properties and our focus is on trajectories (either individual ones or families of them).

The main application domain of GECO is *quantum control*. The importance of designing efficient transfers between different atomic or molecular levels in atomic and molecular physics is due to its applications to photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing.

A second application area concerns the control interpretation of phenomena appearing in *neurophysiology*. It studies the modeling of the mechanisms supervising some biomechanics actions or sensorial reactions such as image reconstruction by the primary visual cortex, eyes movement and body motion. All these problems can be seen as motion planning tasks accomplished by the brain.

As a third applicative domain we propose a system dynamics approach to *switched systems*. Switched systems are characterized by the interaction of continuous dynamics (physical system) and discrete/logical components. They provide a popular modeling framework for heterogeneous aspects issuing from automotive and transportation industry, energy management and factory automation.

3. Research Program

3.1. Geometric control theory

The main research topic of the project-team will be **geometric control**, with a special focus on **control design**. The application areas that we target are control of quantum mechanical systems, neurogeometry and switched systems.

Geometric control theory provides a viewpoint and several tools, issued in particular from differential geometry, to tackle typical questions arising in the control framework: controllability, observability, stabilization, optimal control... [32], [66] The geometric control approach is particularly well suited for systems involving nonlinear and nonholonomic phenomena. We recall that nonholonomicity refers to the property of a velocity constraint that is not equivalent to a state constraint.

The expression **control design** refers here to all phases of the construction of a control law, in a mainly open-loop perspective: modeling, controllability analysis, output tracking, motion planning, simultaneous control algorithms, tracking algorithms, performance comparisons for control and tracking algorithms, simulation and implementation.

We recall that

- **controllability** denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law ;
- **output tracking** refers to a control strategy aiming at keeping the value of some functions of the state arbitrarily close to a prescribed time-dependent profile. A typical example is **configuration tracking** for a mechanical system, in which the controls act as forces and one prescribes the position variables along the trajectory, while the evolution of the momenta is free. One can think for instance at the lateral movement of a car-like vehicle: even if such a movement is unfeasible, it can be tracked with arbitrary precision by applying a suitable control strategy;
- **motion planning** is the expression usually denoting the algorithmic strategy for selecting one control law steering the system from a given initial state to an attainable final one;
- **simultaneous control** concerns algorithms that aim at driving the system from two different initial conditions, with the same control law and over the same time interval, towards two given final states (one can think, for instance, at some control action on a fluid whose goal is to steer simultaneously two floating bodies.) Clearly, the study of which pairs (or n-uples) of states can be simultaneously connected thanks to an admissible control requires an additional controllability analysis with respect to the plain controllability mentioned above.

At the core of control design is then the notion of motion planning. Among the motion planning methods, a preeminent role is played by those based on the Lie algebra associated with the control system ([86], [73], [79]), those exploiting the possible flatness of the system ([60]) and those based on the continuation method ([98]). Optimal control is clearly another method for choosing a control law connecting two states, although it generally introduces new computational and theoretical difficulties.

Control systems with special structure, which are very important for applications are those for which the controls appear linearly. When the controls are not bounded, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a smoothly varying norm (representing a cost of the control), the resulting geometrical structure is called *sub-Riemannian*. Sub-Riemannian geometry thus appears as the underlying geometry of the nonholonomic control systems, playing the same role as Euclidean geometry for linear systems. As such, its study is fundamental for control design. Moreover its importance goes far beyond control theory and is an active field of research both in differential geometry ([85]), geometric measure theory ([61], [36]) and hypoelliptic operator theory ([48]).

Other important classes of control systems are those modeling mechanical systems. The dynamics are naturally defined on the tangent or cotangent bundle of the configuration manifold, they have Lagrangian or Hamiltonian structure, and the controls act as forces. When the controls appear linearly, the resulting model can be seen somehow as a second-order sub-Riemannian structure (see [53]).

The control design topics presented above naturally extend to the case of distributed parameter control systems. The geometric approach to control systems governed by partial differential equations is a novel subject with great potential. It could complement purely analytical and numerical approaches, thanks to its more dynamical, qualitative and intrinsic point of view. An interesting example of this approach is the paper [33] about the controllability of Navier–Stokes equation by low forcing modes.

4. Application Domains

4.1. Quantum control

The issue of designing efficient transfers between different atomic or molecular levels is crucial in atomic and molecular physics, in particular because of its importance in those fields such as photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (NMR, control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing. This last application explicitly relies on the design of quantum gates, each of them being, in essence, an open loop control law devoted to a prescribed simultaneous control action. NMR is one of the most promising techniques for the implementation of a quantum computer.

Physically, the control action is realized by exciting the quantum system by means of one or several external fields, being them magnetic or electric fields. The resulting control problem has attracted increasing attention, especially among quantum physicists and chemists (see, for instance, [91], [96]). The rapid evolution of the domain is driven by a multitude of experiments getting more and more precise and complex (see the recent review [52]). Control strategies have been proposed and implemented, both on numerical simulations and on physical systems, but there is still a large gap to fill before getting a complete picture of the control properties of quantum systems. Control techniques should necessarily be innovative, in order to take into account the physical peculiarities of the model and the specific experimental constraints.

The area where the picture got clearer is given by finite dimensional linear closed models.

- **Finite dimensional** refers to the dimension of the space of wave functions, and, accordingly, to the finite number of energy levels.
- **Linear** means that the evolution of the system for a fixed (constant in time) value of the control is determined by a linear vector field.
- **Closed** refers to the fact that the systems are assumed to be totally disconnected from the environment, resulting in the conservation of the norm of the wave function.

The resulting model is well suited for describing spin systems and also arises naturally when infinite dimensional quantum systems of the type discussed below are replaced by their finite dimensional Galerkin approximations. Without seeking exhaustiveness, let us mention some of the issues that have been tackled for finite dimensional linear closed quantum systems:

- controllability [34],
- bounds on the controllability time [30],
- STIRAP processes [101],
- simultaneous control [74],
- optimal control ([70], [43], [54]),
- numerical simulations [80].

Several of these results use suitable transformations or approximations (for instance the so-called rotating wave) to reformulate the finite-dimensional Schrödinger equation as a sub-Riemannian system. Open systems have also been the object of an intensive research activity (see, for instance, [35], [71], [92], [49]).

In the case where the state space is infinite dimensional, some optimal control results are known (see, for instance, [39], [50], [67], [40]). The controllability issue is less understood than in the finite dimensional setting, but several advances should be mentioned. First of all, it is known that one cannot expect exact controllability on the whole Hilbert sphere [100]. Moreover, it has been shown that a relevant model, the quantum oscillator, is not even approximately controllable [93], [83]. These negative results have been more recently completed by positive ones. In [41], [42] Beauchard and Coron obtained the first positive controllability result for a quantum particle in a 1D potential well. The result is highly nontrivial and is based on Coron's return method (see [56]). Exact controllability is proven to hold among regular enough wave functions. In particular, exact controllability among eigenfunctions of the uncontrolled Schrödinger operator can be achieved. Other important approximate controllability results have then been proved using Lyapunov methods [82], [87], [68]. While [82] studies a controlled Schrödinger equation in $\mathbb{R}$ for which the uncontrolled Schrödinger operator has mixed spectrum, [87], [68] deal mainly with general discrete-spectrum Schrödinger operators.

In all the positive results recalled in the previous paragraph, the quantum system is steered by a single external field. Different techniques can be applied in the case of two or more external fields, leading to additional controllability results [59], [46].

The picture is even less clear for nonlinear models, such as Gross–Pitaevski and Hartree–Fock equations. The obstructions to exact controllability, similar to the ones mentioned in the linear case, have been discussed in [65]. Optimal control approaches have also been considered [38], [51]. A comprehensive controllability analysis of such models is probably a long way away.

4.2. Neurophysiology

At the interface between neurosciences, mathematics, automatics and humanoid robotics, an entire new approach to neurophysiology is emerging. It arouses a strong interest in the four communities and its development requires a joint effort and the sharing of complementary tools.

A family of extremely interesting problems concerns the understanding of the mechanisms supervising some sensorial reactions or biomechanics actions such as image reconstruction by the primary visual cortex, eyes movement and body motion.

In order to study these phenomena, a promising approach consists in identifying the motion planning problems undertaken by the brain, through the analysis of the strategies that it applies when challenged by external inputs. The role of control is that of a language allowing to read and model neurological phenomena. The control algorithms would shed new light on the brain's geometric perception (the so-called neurogeometry [89]) and on the functional organization of the motor pathways.

- A challenging problem is that of the understanding of the mechanisms which are responsible for the process of image reconstruction in the primary visual cortex V1.

  The visual cortex areas composing V1 are notable for their complex spatial organization and their functional diversity. Understanding and describing their architecture requires sophisticated modeling tools. At the same time, the structure of the natural and artificial images used in visual psychophysics can be fully disclosed only using rather deep geometric concepts. The word "geometry" refers here to the internal geometry of the functional architecture of visual cortex areas (not to the geometry of the Euclidean external space). Differential geometry and analysis both play a fundamental role in the description of the structural characteristics of visual perception.

  A model of human perception based on a simplified description of the visual cortex V1, involving geometric objects typical of control theory and sub-Riemannian geometry, has been first proposed by Petitot ([90]) and then modified by Citti and Sarti ([55]). The model is based on experimental observations, and in particular on the fundamental work by Hubel and Wiesel [64] who received the Nobel prize in 1981.

In this model, neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli arriving at a given point of the retina and oriented along a given direction. The retina is modeled by the real plane, while the directions at a given point are modeled by the projective line. The fiber bundle having as base the real plane and as fiber the projective line is called the *bundle of directions of the plane*.

From the neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them sensitive to stimuli arriving at a given point, oriented along any direction. In the same hypercolumn, relative to a point of the plane, we also find neurons that are sensitive to other stimuli properties, such as colors. Therefore, in this model the visual cortex treats an image not as a planar object, but as a set of points in the bundle of directions of the plane. The reconstruction is then realized by minimizing the energy necessary to activate orientation columns among those which are not activated directly by the image. This gives rise to a sub-Riemannian problem on the bundle of directions of the plane.

- Another class of challenging problems concern the functional organization of the motor pathways.

  The interest in establishing a model of the motor pathways, at the same time mathematically rigorous and biologically plausible, comes from the possible spillovers in robotics and neurophysiology. It could help to design better control strategies for robots and artificial limbs, yielding smoother and more progressive movements. Another underlying relevant societal goal (clearly beyond our domain of expertise) is to clarify the mechanisms of certain debilitating troubles such as cerebellar disease, chorea and Parkinson's disease.

  A key issue in order to establish a model of the motor pathways is to determine the criteria underlying the brain's choices. For instance, for the problem of human locomotion (see [37]), identifying such criteria would be crucial to understand the neural pathways implicated in the generation of locomotion trajectories.

  A nowadays widely accepted paradigm is that, among all possible movements, the accomplished ones satisfy suitable optimality criteria (see [99] for a review). One is then led to study an inverse optimal control problem: starting from a database of experimentally recorded movements, identify a cost function such that the corresponding optimal solutions are compatible with the observed behaviors.

  Different methods have been taken into account in the literature to tackle this kind of problems, for instance in the linear quadratic case [69] or for Markov processes [88]. However all these methods have been conceived for very specific systems and they are not suitable in the general case. Two approaches are possible to overcome this difficulty. The direct approach consists in choosing a cost function among a class of functions naturally adapted to the dynamics (such as energy functions) and to compare the solutions of the corresponding optimal control problem to the experimental data. In particular one needs to compute, numerically or analytically, the optimal trajectories and to choose suitable criteria (quantitative and qualitative) for the comparison with observed trajectories. The inverse approach consists in deriving the cost function from the qualitative analysis of the data.

4.3. Switched systems

Switched systems form a subclass of hybrid systems, which themselves constitute a key growth area in automation and communication technologies with a broad range of applications. Existing and emerging areas include automotive and transportation industry, energy management and factory automation. The notion of hybrid systems provides a framework adapted to the description of the heterogeneous aspects related to the interaction of continuous dynamics (physical system) and discrete/logical components.

The characterizing feature of switched systems is the collective aspect of the dynamics. A typical question is that of stability, in which one wants to determine whether a dynamical system whose evolution is influenced by a time-dependent signal is uniformly stable with respect to all signals in a fixed class ([76]).

The theory of finite-dimensional hybrid and switched systems has been the subject of intensive research in the last decade and a large number of diverse and challenging problems such as stabilizability, observability, optimal control and synchronization have been investigated (see for instance [97], [77]).

The question of stability, in particular, because of its relevance for applications, has spurred a rich literature. Important contributions concern the notion of common Lyapunov function: when there exists a Lyapunov function that decays along all possible modes of the system (that is, for every possible constant value of the signal), then the system is uniformly asymptotically stable. Conversely, if the system is stable uniformly with respect to all signals switching in an arbitrary way, then a common Lyapunov function exists [78]. In the *linear* finite-dimensional case, the existence of a common Lyapunov function is actually equivalent to the global uniform exponential stability of the system [84] and, provided that the admissible modes are finitely many, the Lyapunov function can be taken polyhedral or polynomial [44], [45], [57]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see [58] and references therein). Algebraic approaches to prove the stability of switched systems under arbitrary switching, not relying on Lyapunov techniques, have been proposed in [75], [31].

Other interesting issues concerning the stability of switched systems arise when, instead of considering arbitrary switching, one restricts the class of admissible signals, by imposing, for instance, a dwell time constraint [63].

Another rich area of research concerns discrete-time switched systems, where new intriguing phenomena appear, preventing the algebraic characterization of stability even for small dimensions of the state space [72]. It is known that, in this context, stability cannot be tested on periodic signals alone [47].

Finally, let us mention that little is known about infinite-dimensional switched system, with the exception of some results on uniform asymptotic stability ([81], [94], [95]) and some recent papers on optimal control ([62], [102]).

5. New Software and Platforms

5.1. IRHD

We develop a software for reconstruction of corrupted and damaged images, named IRHD (for Image Reconstruction via Hypoelliptic Diffusion). One of the main features of the algorithm on which the software is based is that it does not require any information about the location and character of the corrupted places. Another important advantage is that this method is massively parallelizable; this allows to work with sufficiently large images. Theoretical background of the presented method is based on the model of geometry of vision due to Petitot, Citti and Sarti. The main step is numerical solution of the equation of 3D hypoelliptic diffusion. IRHD is based on Fortran.

6. New Results

6.1. Highlights of the Year

We organized a thematic trimester on “Geometry, analysis and dynamics on sub-Riemannian manifolds” at the Institut Henri Poincaré (IHP), including 4 workshops, 4 research courses, 8 thematic days, several seminars. We also organized an associated school at CIRM with 4 introductory courses. The web pages of the events are:

http://www.cmap.polytechnique.fr/subriemannian/
http://www.cmap.polytechnique.fr/subriemannian/cirm/

6.2. New results: geometric control

Let us list some new results in sub-Riemannian geometry and hypoelliptic diffusion obtained by GECO's members.

- The article [14] presents simple controls that generate motion in the direction of high order Lie brackets. Whereas the naive use of piecewise constant controls requires the number of switchings to grow exponentially with the length of the bracket, we show that such motion is possible with sinusoidal controls whose sum of frequencies equals the length of the bracket. This work is closely related and motivated by the study of the complexity of sub-Riemannian geodesics for generic regular distributions, i.e., whose derived flag has maximal growth vector. Of particular interest is the approximation of curves transversal to the distribution by admissible curves. We also present a surprising example that shows that it is possible to simultaneously kill higher moments without increasing the number of self-intersections of the base curve.
- The curvature discussed in [18] is a rather far going generalization of the Riemann sectional curvature. We define it for a wide class of optimal control problems: a unified framework including geometric structures such as Riemannian, sub-Riemannian, Finsler and sub-Finsler structures; a special attention is paid to the sub-Riemannian (or Carnot-Caratheodory) metric spaces. Our construction of the curvature is direct and naive, and it is similar to the original approach by Riemann. Surprisingly, it works in a very general setting and, in particular, for all sub-Riemannian spaces.
- In [19] we prove sectional and Ricci-type comparison theorems for the existence of conjugate points along sub-Riemannian geodesics. In order to do that, we regard sub-Riemannian structures as a special kind of variational problems. In this setting, we identify a class of models, namely linear quadratic optimal control systems, that play the role of the constant curvature spaces. As an application, we prove a version of sub-Riemannian Bonnet–Myers theorem and we obtain some new results on conjugate points for 3D left-invariant sub-Riemannian structures.
- In the study of conjugate times in sub-Riemannian geometry, linear quadratic optimal control problems show up as model cases. In [1] we consider a dynamical system with a constant, quadratic Hamiltonian h, and we characterize the number of conjugate times in terms of the spectrum of the Hamiltonian vector field H. We prove the following dichotomy: the number of conjugate times is identically zero or grows to infinity. The latter case occurs if and only if H has at least one Jordan block of odd dimension corresponding to a purely imaginary eigenvalue. As a byproduct, we obtain bounds from below on the number of conjugate times contained in an interval in terms of the spectrum of H.
- A 3D almost-Riemannian manifold is a generalized Riemannian manifold defined locally by 3 vector fields that play the role of an orthonormal frame, but could become collinear on some set called the singular set. Under the Hormander condition, a 3D almost-Riemannian structure still has a metric space structure, whose topology is compatible with the original topology of the manifold. Almost-Riemannian manifolds were deeply studied in dimension 2. In [21] we start the study of the 3D case which appear to be reacher with respect to the 2D case, due to the presence of abnormal extremals which define a field of directions on the singular set. We study the type of singularities of the metric that could appear generically, we construct local normal forms and we study abnormal extremals. We then study the nilpotent approximation and the structure of the corresponding small spheres. We finally give some preliminary results about heat diffusion on such manifolds.
- In [22] we study spectral properties of the Laplace-Beltrami operator on two relevant almost-Riemannian manifolds, namely the Grushin structures on the cylinder and on the sphere. As for general almost-Riemannian structures (under certain technical hypothesis), the singular set acts as a barrier for the evolution of the heat and of a quantum particle, although geodesics can cross it. This is a consequence of the self-adjointness of the Laplace-Beltrami operator on each connected

component of the manifolds without the singular set. We get explicit descriptions of the spectrum, of the eigenfunctions and their properties. In particular in both cases we get a Weyl law with dominant term $E \log E$. We then study the effect of an Aharonov-Bohm non-apophantic magnetic potential that has a drastic effect on the spectral properties. Other generalized Riemannian structures including conic and anti-conic type manifolds are also studied. In this case, the Aharonov-Bohm magnetic potential may affect the self-adjointness of the Laplace-Beltrami operator.

- In [28] we investigate the number of geodesics between two points p and q on a contact sub-Riemannian manifold M. We show that the count of geodesics on M is controlled by the count on its nilpotent approximation at p (a contact Carnot group). For contact Carnot groups we give sharp bounds for a generic point q. Removing the genericity condition for q, geodesics might appear in families and we prove a similar statement for their topology. We study these families, and in particular we focus on the unexpected appearance of isometrically non-equivalent geodesics: families on which the action of isometries is not transitive. We apply the previous study to contact sub-Riemannian manifolds: we prove that for any given point $p \in M$ there is a sequence of points p_n such that $p_n \to p$ and that the number of geodesics between p and p_n grows unbounded (moreover these geodesics have the property of being contained in a small neighborhood of p).

New results on automatic control and motion planning for various type of applicative domains are the following.

- [8] is devoted to the problem of model-based prognostics for a Waste Water Treatment Plant (WWTP). Our aim is to predict degradation of certain parameters in the process, in order to anticipate malfunctions and to schedule maintenance. It turns out that a WWTP, together with the possible malfunction, has a specific structure: mostly, the malfunction appears in the model as an unknown input function. The process is observable whatever this unknown input is, and the unknown input can itself be identified through the observations. Due to this property, our method does not require any assumption of the type “slow dynamics degradation", as is usually assumed in ordinary prognostic methods. Our system being unknown-input observable, standard observer-based methods are enough to solve prognostic problems. Simulation results are shown for a typical WWTP.
- In [9] we study the problem of controlling an unmanned aerial vehicle (UAV) to provide a target supervision and/or to provide convoy protection to ground vehicles. We first present a control strategy based upon a Lyapunov-LaSalle stabilization method to provide supervision of a stationary target. The UAV is expected to join a predesigned admissible circular trajectory around the target which is itself a fixed point in the space. Our strategy is presented for both high altitude long endurance (HALE) and medium altitude long endurance (MALE) types of UAVs.
- In [12] we study how a particular spatial structure with a buffer impacts the number of equilibria and their stability in the chemostat model. We show that the occurrence of a buffer can allow a species to setup or on the opposite to go to extinction, depending on the characteristics of the buffer. For non-monotonic response function, we characterize the buffered configurations that make the chemostat dynamics globally asymptotically stable, while this is not possible with single, serial or parallel vessels of the same total volume and input flow. These results are illustrated with the Haldane kinetic function.
- In [15] and [25] we present new results on the path planning problem in the case study of the car with trailers. We formulate the problem in the framework of optimal nonholonomic interpolation and we use standard techniques of nonlinear optimal control theory for deriving hyperelliptic signals as controls for driving the system in an optimal way. The hyperelliptic curves contain as many loops as the number of nonzero Lie brackets generated by the system. We compare the hyperelliptic signals with the ordinary Lissajous-like signals that appear in the literature, we conclude that the former have better performance.
- In [27] we consider affine-control systems, i.e., systems in the form $\dot{q}(t) = f_0(q(t)) + \sum_{i=1}^{m} u_i(t) f_i(q(t))$. Here, the point q belongs to a smooth manifold M, the f_i's are smooth vector fields on M. This type of system appears in many applications for mechanical systems, quantum control, microswimmers, neuro-geometry of vision...

We conclude the section by mentioning the book [17] that we edited, collecting some papers in honour of Andrei A. Agrachev for his 60th birthday. The book contains new results on sub-Riemannian geometry and more generally on the geometric theory of control.

6.3. New results: quantum control

New results have been obtained for the control of the bilinear Schrödinger equation.

- In [2] we present a sufficient condition for approximate controllability of the bilinear discrete-spectrum Schrödinger equation in the multi-input case. The controllability result extends to simultaneous controllability, approximate controllability in H^s, and tracking in modulus. The sufficient condition is more general than those present in the literature even in the single-input case and allows the spectrum of the uncontrolled operator to be very degenerate (e.g. to have multiple eigenvalues or equal gaps among different pairs of eigenvalues). We apply the general result to a rotating polar linear molecule, driven by three orthogonal external fields. A remarkable property of this model is the presence of infinitely many degeneracies and resonances in the spectrum.
- In [5] we consider the minimum time population transfer problem for a two level quantum system driven by two external fields with bounded amplitude. The controls are modeled as real functions and we do not use the Rotating Wave Approximation. After projection on the Bloch sphere, we treat the time-optimal control problem with techniques of optimal synthesis on 2D manifolds. Based on the Pontryagin Maximum Principle, we characterize a restricted set of candidate optimal trajectories. Properties on this set, crucial for complete optimal synthesis, are illustrated by numerical simulations. Furthermore, when the two controls have the same bound and this bound is small with respect to the difference of the two energy levels, we get a complete optimal synthesis up to a small neighborhood of the antipodal point of the initial condition.
- In [11] we investigate the controllability of quantum electrons trapped in a two-dimensional device, typically a metal oxide semiconductor (MOS) field-effect transistor. The problem is modeled by the Schrödinger equation in a bounded domain coupled to the Poisson equation for the electrical potential. The controller acts on the system through the boundary condition on the potential, on a part of the boundary modeling the gate. We prove that, generically with respect to the shape of the domain and boundary conditions on the gate, the device is controllable. We also consider control properties of a more realistic nonlinear version of the device, taking into account the self-consistent electrostatic Poisson potential.
- In [29] we prove the approximate controllability of a bilinear Schrödinger equation modelling a two trapped ions system. A new spectral decoupling technique is introduced, which allows to analyze the controllability of the infinite-dimensional system through finite-dimensional considerations.

6.4. New results: neurophysiology

- [3] presents a semidiscrete alternative to the theory of neurogeometry of vision, due to Citti, Petitot, and Sarti. We propose a new ingredient, namely, working on the group of translations and discrete rotations $SE(2,N)$. The theoretical side of our study relates the stochastic nature of the problem with the Moore group structure of $SE(2,N)$. Harmonic analysis over this group leads to very simple finite dimensional reductions. We then apply these ideas to the inpainting problem which is reduced to the integration of a completely parallelizable finite set of Mathieu-type diffusions (indexed by the dual of $SE(2,N)$ in place of the points of the Fourier plane, which is a drastic reduction). The integration of the the Mathieu equations can be performed by standard numerical methods for elliptic diffusions and leads to a very simple and efficient class of inpainting algorithms. We illustrate the performances of the method on a series of deeply corrupted images.
- In [4] and [7] we consider the problem of minimizing $\int_0^l \sqrt{\xi^2+K(s)^2}ds$ for a planar curve having fixed initial and final positions and directions. The total length l is free. Here s is the arclength parameter, $K(s)$ is the curvature of the curve and $\xi>0$ is a fixed constant. This problem comes

from a model of geometry of vision due to Petitot, Citti and Sarti. We study existence of local and global minimizers for this problem. In [7] we characterize sub-Riemannian geodesics and the range of the exponential map. In [4] we prove that if for a certain choice of boundary conditions there is no global minimizer, then there is neither a local minimizer nor a geodesic. We finally give properties of the set of boundary conditions for which there exists a solution to the problem.

6.5. New results: switched systems

- In [6] we consider a family of linear control systems $\dot{x} = Ax + \alpha Bu$ on $\mathbb{R}^d$, where α belongs to a given class of persistently exciting signals. We seek maximal α-uniform stabilization and destabilization by means of linear feedbacks $u = Kx$. We extend previous results obtained for bidimensional single-input linear control systems to the general case as follows: if there exists at least one K such that the Lie algebra generated by A and BK is equal to the set of all $d \times d$ matrices, then the maximal rate of convergence of (A, B) is equal to the maximal rate of divergence of (A, B). We also provide more precise results in the general single-input case, where the above result is obtained under the simpler assumption of controllability of the pair (A, B).
- The paper [10] considers the stabilization to the origin of a persistently excited linear system by means of a linear state feedback, where we suppose that the feedback law is not applied instantaneously, but after a certain positive delay (not necessarily constant). The main result is that, under certain spectral hypotheses on the linear system, stabilization by means of a linear delayed feedback is indeed possible, generalizing a previous result already known for non-delayed feedback laws.
- In [16] and [26] we give a collection of converse Lyapunov–Krasovskii theorems for uncertain retarded differential equations. We show that the existence of a weakly degenerate Lyapunov–Krasovskii functional is a necessary and sufficient condition for the global exponential stability of the linear retarded functional differential equations. This is carried out using the switched system transformation approach.
- Consider a continuous-time linear switched system on $\mathbb{R}^n$ associated with a compact convex set of matrices. When it is irreducible and its largest Lyapunov exponent is zero there always exists a Barabanov norm associated with the system. In [23] we deal with two types of issues: (a) properties of Barabanov norms such as uniqueness up to homogeneity and strict convexity; (b) asymptotic behaviour of the extremal solutions of the linear switched system. Regarding Issue (a), we provide partial answers and propose four related open problems. As for Issue (b), we establish, when $n = 3$, a Poincaré-Bendixson theorem under a regularity assumption on the set of matrices. We then revisit a noteworthy result of N.E. Barabanov describing the asymptotic behaviour of linear switched system on $\mathbb{R}^3$ associated with a pair of Hurwitz matrices $\{A, A + bc^T\}$. After pointing out a fatal gap in Barabanov's proof we partially recover his result by alternative arguments.
- In [24] we address the exponential stability of a system of transport equations with intermittent damping on a network of $N \geq 2$ circles intersecting at a single point O. The N equations are coupled through a linear mixing of their values at O, described by a matrix M. The activity of the intermittent damping is determined by persistently exciting signals, all belonging to a fixed class. The main result is that, under suitable hypotheses on M and on the rationality of the ratios between the lengths of the circles, such a system is exponentially stable, uniformly with respect to the persistently exciting signals. The proof relies on an explicit formula for the solutions of this system, which allows one to track down the effects of the intermittent damping.

7. Partnerships and Cooperations

7.1. Regional Initiatives

- Project *Stabilité des systèmes à excitation persistante*, Program MathIng, Labex LMH, 2013-2016. This project is about different stability properties for systems whose damping is intermittently activated. The coordinator is Mario Sigalotti. The other members are Yacine Chitour and Guilherme Mazanti.
- **Digitéo project 2012-061D SSyCoDyC.** SSyCoDyC (2013–2014) is financed by Digitéo in the framework of the DIM *Hybrid Systems and Sensing Systems*. It focuses on the application of techniques of hybrid systems to the analysis of retarded equations with time-varying delays. SSyCoDyC has financed the post-doc fellowship of Ihab Haidar and was coordinated by Paolo Mason and Mario Sigalotti.
- iCODE is the Institute for Control and Decision of the Idex Paris Saclay. It was launched in March 2014 for two years until June 2016. iCODE's aims are fostering research, spin-offs creation, training and diffusion of Control and Decision in Paris-Saclay. To those aims, iCODE has received a budget of 980Keuros, supported by *investissements d'avenir*. The scientific topics addressed by iCODE are organized in four research initiatives:
  - Control & Neuroscience
  - Large-scale systems & Smart grids
  - Behavioral Economics
  - White research initiative.

  iCODE is coordinated by Yacine Chitour (L2S-Univ. Paris Sud), associated member and collaborator of GECO. Mario Sigalotti is member of the Steering Committee.

7.2. European Initiatives

7.2.1. FP7 Projects

Program: ERC Starting Grant

Project acronym: GeCoMethods

Project title: Geometric Control Methods for the Heat and Schroedinger Equations

Duration: 1/5/2010 - 1/5/2015

Coordinator: Ugo Boscain

Abstract: The aim of this project is to study certain PDEs for which geometric control techniques open new horizons. More precisely we plan to exploit the relation between the sub-Riemannian distance and the properties of the kernel of the corresponding hypoelliptic heat equation and to study controllability properties of the Schroedinger equation.

All subjects studied in this project are applications-driven: the problem of controllability of the Schroedinger equation has direct applications in Laser spectroscopy and in Nuclear Magnetic Resonance; the problem of nonisotropic diffusion has applications in cognitive neuroscience (in particular for models of human vision).

Participants. Main collaborator: Mario Sigalotti. Other members of the team: Andrei Agrachev, Riccardo Adami, Thomas Chambrion, Grégoire Charlot, Yacine Chitour, Jean-Paul Gauthier, Frédéric Jean.

7.3. International Initiatives

7.3.1. Inria International Partners

7.3.1.1. Informal International Partners

SISSA (Scuola Internazionale Superiore di Studi Avanzati), Trieste, Italy.

Sector of Functional Analysis and Applications, Geometric Control group. Coordinator: Andrei A. Agrachev.

We collaborate with the Geometric Control group at SISSA mainly on subjects related with sub-Riemannian geometry. Thanks partly to our collaboration, SISSA has established an official research partnership with École Polytechnique.

7.3.2. Participation In other International Programs

- Laboratoire Euro Maghrébin de Mathématiques et de leurs Interactions (LEM2I) http://www.lem2i.cnrs.fr/
- GDRE Control of Partial Differential Equations (CONEDP) http://www.ceremade.dauphine.fr/~glass/GDRE/

8. Dissemination

8.1. Promoting Scientific Activities

8.1.1. Scientific events organisation

- Davide Barilari, Ugo Boscain and Mario Sigalotti were organizers of the IHP trimester “Geometry, Analysis and Dynamics on Sub-Riemannian Manifolds”, Fall 2014, Institut Henri Poincaré, Paris.
- Ugo Boscain and Mario Sigalotti were organizers of the CIRM School (Marseille) “Sub-Riemannian manifolds: from geodesics to hypoelliptic diffusion”, September 2014.

8.1.1.1. Member of editorial boards

- Ugo Boscain is Associate Editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing Editor of Journal of Dynamical and Control Systems
- Mario Sigalotti is Associate Editor of Journal of Dynamical and Control Systems
- Ugo Boscain is Associate Editor of ESAIM Control, Optimisation and Calculus of Variations
- Ugo Boscain is Associate Editor of Mathematical Control and Related Fields
- Ugo Boscain is Associate editor of Analysis and Geometry in Metric Spaces

8.2. Teaching - Supervision - Juries

8.2.1. Supervision

- PhD: Moussa Gaye, “Some problems of geometric analysis in almost-Riemannian geometry and of stability of switching systems", supervisors: Ugo Boscain, Yacine Chitour, Paolo Mason, defended in November 2014.
- PhD in progress: Guiherme Mazanti, “Stabilité et taux de convergence pour les systèmes à excitation persistante", started in 1/9/2013, supervisors: Yacine Chitour, Mario Sigalotti.

8.2.2. Juries

- Ugo Boscain was referee for the PhD thesis of Sylvain Arguillere, Paris 6, July 2014.
- Ugo Boscain was member of the commission for the PhD defense of Laurent Sifre, Ecole Polytechnique, October 2014.
- Mario Sigalotti was member of the commission for the PhD defense of Dolly Tatiana Manrique Espindola, Université de Lorraine, December 2014.
- Ugo Boscain was member of the commission for the HDR of Gregoire Charlot, Universite de Grenoble, September 2014.
- Mario Sigalotti was member of the commission for a MCF position at INPT ENSEEIHT, Toulouse.

- Ugo Boscain was member of the jury for positions of CR at INSMI.

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