Activity Report 2014

Team LEMON

Littoral, Environment: MOdels and Numerics

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Earth, Environmental and Energy Sciences

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Team LEMON

Keywords: Environment, Geophysics, Model Coupling, Multiscale Models, Numerical Methods

LEMON has been created officially on 01/01/2014 but in practice, we've only been working together since September, when the new Inria building in Montpellier was completed (with dedicated offices for the team members).
Fabien Campillo has joined the team on November 1st. For the sake of simplicity, all his activities in 2014 will hold in the MODEMIC activity report.

Creation of the Team: 2014 January 01.

1. Members

Research Scientists
Antoine Rousseau [Team leader, Inria, Researcher]
Fabien Campillo [Inria, Senior Researcher, from Nov 2014, HdR]

Faculty Members
Vincent Guinot [Univ. Montpellier II, Professor, HdR]
Fabien Marche [Univ. Montpellier II, Associate Professor, HdR]

PhD Student
Arnaud Duran [Univ. Montpellier II, PhD Candidate, until Sep 2014]

Administrative Assistant
Annie Aliaga [Inria, Team assistant]

Other
Alexis Pacholik [Inria, Master Intern, from Apr 2014 until Sep 2014]

2. Overall Objectives

2.1. Context

Coastal zones are the theatre for numerous interfaces. The main elements that come to mind are the sea/earth interface, saline/brackish/fresh water interfaces and sediment/biological world interfaces. These elements cause most of the phenomena met within coastal zones to be in fragile equilibrium or more often, in constant evolution. This is due to the evolving external pressures, such as anthropic activity or physical forces (tectonic features, tide, precipitations, storms, sea level rise, sediment transport, etc.). In order to illustrate the considerable importance of such a research project, let us underline the following figures:

- **60 % of the world population lives in a 100km wide coastal strip** (80% within 30km in Brittany),
- **current sea level rise** has occurred at a mean rate of 1.8 mm per year for the past century, and more recently at rates estimated near 2.8 ± 0.4 to 3.1 ± 0.7 mm per year (1993-2003). It is likely to rise in the future: IPCC recently anticipated a 1.5m sea level rise within the next century,

It results that **coastal management** requires the development of theoretical and applied models to facilitate the **decision process**. For example, a city that wants to develop a harbour needs to anticipate the time-evolution of urban floods. The construction of defense barriers to protect buildings and houses from natural hazards relies on the knowledge of potential submersion events, in a period where the impact of global climatic and anthropic changes on the coastal zone is expected to generate increased coastal risks (IPCC 2007 and 2013). One also needs to analyze *"what if"* scenarios for proposed changes in land use or land cover in coastal regions (such as French Mont Saint-Michel).
As a matter of fact, the software packages available for engineering applications are usually not satisfactory. More specifically, some modeling hypotheses (such as the hydrostatic approximation) should be weakened, and more appropriate numerical schemes should be implemented. What is proposed with LEMON is to **increase the quality of coastal engineering numerical tools**, thanks to better designed mathematical and numerical models.

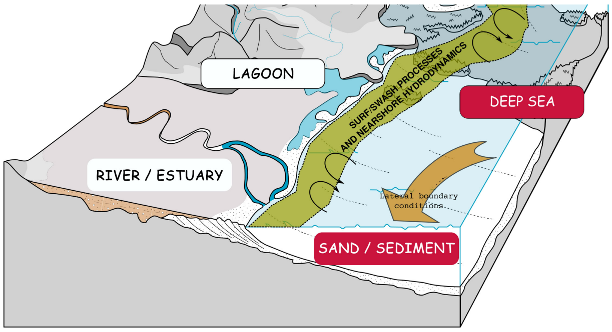

Figure 1. Examples of interacting nearshore processes. Courtesy F. Bouchette.

The mathematical modeling of the phenomena occurring within coastal zones and their interactions is currently a major scientific issue. If we want to model coastal zones, we have to consider the fact that they cover a very wide range of situations and that they are the result of several complex interacting phenomena (see Figure 1). More specifically, many time scales and space scales are involved and many physical and biological phenomena are in action. Moreover, within each zone, specific interactions between those phenomena make it an almost unique situation. Hence, we are far from having a database aggregating every possible situation. Modeling complex phenomena with the objective of building and improving management/decision tools requires the interaction of several models, each of them being dedicated to the simulation of a specific process. Such (mathematical and numerical) models usually exist but scarcely interact: therefore there is a need to understand how these bricks can be modified (forcing terms, boundary conditions) in order to be assembled. It will require a dialog with specialists of the application domain (geophysics, mechanical engineering, biology, hydrology, etc.) to help to develop new mathematical and numerical models for coastal engineering.
Developing more accurate and/or less CPU demanding models and coupling them together, LEMON will have a strong impact in the applications targeted and in coastal management.

3. Research Program

3.1. State of the Art

3.1.1. Shallow Water Models

Shallow Water (SW) wave dynamics and dissipation represent an important research field. This is because shallow water flows are the most common flows in geophysics. In shallow water regions, dispersive effects (non-hydrostatic pressure effects related to strong curvature in the flow streamlines) can become significant and affect wave transformations. The shoaling of the wave (the "steepening" that happens before the breaking) cannot be described with the usual Saint-Venant equations. To model such various evolutions, one has to use more sophisticated models (Boussinesq, Green-Naghdi...). Nowadays, the classical Saint-Venant equations can be solved numerically in an accurate way, allowing the generation of bores and the shoreline motion to be handled, using recent finite-volume or discontinuous-Galerkin schemes. In contrast, very few advanced works regarding the derivation and modern numerical solution of dispersive equations [28], [32], [60] are available in one dimensions, let alone in the multidimensional case. We can refer to [58], [35] for some linear dispersive equations, treated with finite-element methods, or to [32] for the first use of advanced high-order compact finite-volume methods for the Serre equations. Recent work undertaken during the ANR MathOCEAN [28] lead to some new 1D fully nonlinear and weakly dispersive models (Green-Naghdi like models) that allow to accurately handle the nonlinear waves transformations. High order accuracy numerical methods (based on a second-order splitting strategy) have been developed and implemented, raising a new and promising 1D numerical model. However, there is still a lack of new development regarding the multidimensional case.
In shallow water regions, depending on the complex balance between non-linear effects, dispersive effects and energy dissipation due to wave breaking, wave fronts can evolve into a large range of bore types, from purely breaking to purely undular bore. Boussinesq or Green-Naghdi models can handle these phenomena [26] . However, these models neglect the wave overturning and the associated dissipation, and the dispersive terms are not justified in the vicinity of the singularity. Previous numerical studies concerning bore dynamics using depth-averaged models have been devoted to either purely broken bores using NSW models [29], or undular bores using Boussinesq-type models [39]. Let us also mention [37] for tsunami modeling and [36], [48] for the dam-break problem. A model able to reproduce the various bore shapes, as well as the transition from one type of bore to another, is required. A first step has been made with the one-dimensional code [28], [56]. The SWASH project led by Zijlema at Delft [60] addresses the same issues.

3.1.2. Open boundary conditions and coupling algorithms

For every model set in a bounded domain, there is a need to consider boundary conditions. When the boundaries correspond to a modeling choice rather than to a physical reality, the corresponding boundary conditions should not create spurious oscillations or other unphysical behaviour at the artificial boundary. Such conditions are called **open boundary conditions** (OBC). They have been widely studied by applied mathematicians since the pionneering work of [38] on transparent boundary conditions. Deep studies of these operators have been performed in the case of linear equations, [43], [27], [53]. Unfortunately, in the case of geophysical fluid dynamics, this theory leads to nonlocal conditions (even in linear cases) that are not usable in numerical models. Most of current models (including high quality operational ones) modestly use a *no flux* condition (namely an homogeneous Neumann boundary condition) when a free boundary condition is required. But in many cases, Neumann homogeneous conditions are a very poor approximation of the exact transparent conditions. Hence the need to build higher order approximations of these conditions that remain numerically tractable.
Numerous physical processes are involved in coastal modeling, each of them depending on others (surface winds for coastal oceanography, sea currents for sandbars dynamics, etc.). Connecting two (or more) model solutions at their interface is a difficult task, that is often addressed in a simplified way from the mathematical viewpoint: this can be viewed as the one and only iteration of an iterative process. This results with a low quality coupled system, which could be improved either with additional iterations, and/or thanks to the improvement of interface boundary conditions and the use of OBC (see above). Promising results have been obtained in the framework of **ocean-atmosphere coupling** (in a simplified modeling context) in [49], where the use of advanced coupling techniques (based on domain decomposition algorithm) are introduced.

3.1.3. A need for upscaled shallow water models.

The mathematical modeling of **fluid-biology** coupled systems in lagoon ecosystems requires one or several water models. It is of course not necessary (and not numerically feasible) to use accurate non-hydrostatic turbulent models to force the biological processes over very long periods of time. There is a compromise to be reached between accurate (but untractable) fluid models such as the Navier-Stokes equations and simple (but imprecise) models such as [40].
In urbanized coastal zones, upscaling is also a key issue. This stems not only from the multi-scale aspects dealt with in the previous subsection, but also from modeling efficiency considerations.

The typical size of the relevant hydraulic feature in an urban area is between 0.1 m and 1.0 m, while the size of an urban area usually ranges from 10^3 m to 10^4 m. Refined flow computations (e.g. in simulating the impact of a tsunami) over entire coastal conurbations using a 2D horizontal model thus require 10^6 to 10^9 elements. From an engineering perspective, this makes both the CPU and man-supervised mesh design efforts unaffordable in the present state of technology.

Upscaling provides an answer to this problem by allowing macroscopic equations to be derived from the small-scale governing equations. The powerful, multiple scale expansion-based homogeneization technique [25], [24], [52] has been applied successfully to flow and transport upscaling in porous media, but its use is subordinated to the stringent assumptions of (i) the existence of a Representative Elementary Volume (REV), (ii) the scale separation principle, and (iii) the process is not purely hyperbolic at the microscopic scale, otherwise precluding the study of transient solutions [25]. Unfortunately, the REV has been shown recently not to exist in urban areas [42]. Besides, the scale separation principle is violated in the case of sharp transients (such as tsunami waves) impacting urban areas because the typical wavelength is of the same order of magnitude as the microscopic detail (the street/block size). Moreover, 2D shallow water equations are essentially hyperbolic, thus violating the third assumption.

These hurdles are overcome by averaging approaches. Single porosity-based, macroscopic shallow water models have been proposed [34], [41], [44] and applied successfully to urban flood modeling scale experiments [41], [50], [55]. They allow the CPU time to be divided by 10 to 100 compared to classical 2D shallow water models. Recent extensions of these models have been proposed in the form of integral porosity [54] and multiple porosity [42] shallow water models.

3.2. Scientific Objectives

Our main challenge is: build and couple elementary models in coastal areas to improve their capacity to simulate complex dynamics. This challenge consists of three principal scientific objectives. First of all, each of the elementary models has to be consistently developed (regardless of boundary conditions and interactions with other processes). Then open boundary conditions (for the simulation of physical processes in bounded domains) and links between the models (interface conditions) have to be identified and formalized. Finally, models and boundary conditions (*i.e.* coupled systems) should be proposed, analyzed and implemented in a common platform.

3.2.1. Single process models and boundary conditions

The time-evolution of a water flow in a three-dimensional computational domain is classically modeled by Navier-Stokes equations for incompressible fluids. Depending on the physical description of the considered domain, these equations can be simplified or enriched. Consequently, there are **numerous water dynamics models** that are derived from the original Navier-Stokes equations, such as primitive equations, shallow water equations (see [33]), Boussinesq-type dispersive models [26]), etc. The aforementioned models have **very different mathematical natures**: hyperbolic *vs* parabolic, hydrostatic *vs* non-hydrostatic, inviscid *vs* viscous, etc. They all carry nonlinearities that make their mathematical study (existence, uniqueness and regularity of weak and/or strong solutions) highly challenging (not to speak about the $1M Clay competition for the 3D Navier Stokes equations, which may remain open for some time).

The objective is to focus on the mathematical and numerical modeling of models adapted to **nearshore dynamics**, accounting for complicated wave processes. There exists a large range of models, from the shallow water equations (eventually weakly dispersive) to some fully dispersive deeper models. All these models can be obtained from a suitable asymptotic analysis of the water wave equations (Zakharov formulation) and if the theoretical study of these equations has been recently investigated [47], there is still some serious numerical challenges. So we plan to focus on the derivation and implementation of robust and high order discretization methods for suitable two dimensional models, including enhanced fully nonlinear dispersive models and fully dispersive models, like the Matsuno-generalized approach proposed in [46]. Another objective is to study the shallow water dispersive models without any irrotational flow assumption. Such a study would be of great interest for the study of nearshore circulation (wave induced rip currents).

For obvious physical and/or computational reasons, our models are set in bounded domains. Two types of boundaries are considered: physical and mathematical. Physical boundaries are materialized by an existing interface (atmosphere/ocean, ocean/sand, shoreline, etc.) whereas mathematical boundaries appear with the truncation of the domain of interest. In the latter case, **open boundary conditions** are mandatory in order not to create spurious reflexions at the boundaries. Such boundary conditions being nonlocal and impossible to use in practice, we shall look for approximations. We shall obtain them thanks to the asymptotic analysis of the (pseudo-differential) boundary operators with respect to small parameters (viscosity, domain aspect ratio, Rossby number, etc.). Naturally, we **will seek the boundary conditions leading to the best compromise** between mathematical well-posedness and physical consistency. This will make extensive use of the mathematical theory of **absorbing operators** and their approximations [38].

3.2.2. Coupled systems

The Green-Naghdi equations provide a correct description of the waves up to the breaking point while the Saint-Venant equations are more suitable for the description of the surf zone (i.e. after the breaking). Therefore, the challenge here is first to **design a coupling strategy** between these two systems of equations, first in a simplified one-dimensional case, then to the two-dimensional case both on cartesian and unstructured grids. High order accuracy should be achieved through the use of flexible Discontinuous-Galerkin methods.
Additionally, we will couple our weakly dispersive shallow water models to other fully dispersive deeper water models. We plan to mathematically analyze the coupling between these models. In a first step, we have to understand well the mixed problem (initial and boundary conditions) for these systems. In a second step, these new mathematical development have to be embedded within a numerically efficient strong coupling approach. The deep water model should be fully dispersive (solved using spectral methods, for instance) and the shallow-water model will be, in a first approach, the Saint-Venant equations. Then, when the 2D extension of the currently developped Green-Naghdi numerical code will be available, the improved coupling with a weakly dispersive shallow water model should be considered.

In the context of Schwarz relaxation methods, usual techniques can be seen as the first iteration (not converged) of an iterative algorithm. Thanks to the work performed on efficient boundary conditions, we shall **improve the quality of current coupling algorithms**, allowing for qualitatively satisfying solutions **with a reduced computational cost** (small number of iterations).
We are also willing to explore the role of geophysical processes on some biological ones. For example, the design of optimal shellfish farms relies on confinement maps and plankton dynamics, which strongly depend on long-time averaged currents. Equations that model the time evolution of species in a coastal ecosystem are relatively simple from a modeling viewpoint: they mainly consist of ODEs, and possibly advection-diffusion equations. The issue we want to tackle is the choice of the fluid model that should be coupled to them, accounting for the important time scales discrepancy between biological (evolution) processes and coastal fluid dynamics. Discrimination criteria between refined models (such as turbulent Navier-Stokes) and cheap ones (see [40]) will be proposed.

Coastal processes evolve at very different time scales: atmosphere (seconds/minutes), ocean (hours), sediment (months/years) and species evolution (years/decades). Their coupling can be seen as a *slow-fast* dynamical system, and a naïve way to couple them would be to pick the smallest time-step and run the two models together: but the computational cost would then be way too large. Consequently **homogenization techniques or other upscaling methods** should be used in order to account for these various time scales at an affordable computational cost. The research objectives are the following:

- So far, the proposed upscaled models have been validated against theoretical results obtained from refined 2D shallow water models and/or very limited data sets from scale model experiments. The various approaches proposed in the literature [30], [31], [34], [41], [42], [44], [50], [54], [55] have not been compared over the same data sets. Part of the research effort will focus on the extensive validation of the models on the basis of scale model experiments. Active cooperation will be sought with a number of national and international Academic partners involved in urban hydraulics (UCL Louvain-la-Neuve, IMFS Strasbourg, Irvine University California) with operational experimental facilities.
- Upscaling of source terms. Two types of source terms play a key role in shallow water models: geometry-induced source terms (arising from the irregular bathymetry) and friction/turbulence-induced energy loss terms. In all the upscaled shallow water models presented so far, only the large scale effects of topographical variations have been upscaled. In the case of wetting/drying phenomena and small depths (e.g. the *Camargue* tidal flats), however, it is forseen that subgrid-scale topographic variations may play a predominant role. Research on the integration of subgrid-scale topography into macrosocopic shallow water models is thus needed. Upscaling of friction/turbulence-induced head loss terms is also a subject for research, with a number of competing approaches available from the literature [41], [42], [54], [57].
- Upscaling of transport processes. The upscaling of surface pollutant transport processes in the urban environment has not been addressed so far in the literature. Free surface flows in urban areas are characterized by strongly variable (in both time and space) flow fields. Dead/swirling zones have been shown to play a predominant role in the upscaling of the flow equations [42], [54]. Their role is expected to be even stronger in the upscaling of contaminant transport. While numerical experiments indicate that the microscopic hydrodynamic time scales are small compared to the macroscopic time scales, theoretical considerations indicate that this may not be the case with scalar transport. Trapping phenomena at the microscopic scale are well-known to be upscaled in the form of fractional dynamics models in the long time limit [45], [51]. The difficulty in the present research is that upscaling is not sought only for the long time limit but also for all time scales. Fractional dynamics will thus probably not suffice to a proper upscaling of the transport equations at all time scales.

3.2.3. Numerical platform

As a long term objective, the team shall create a common architecture for existing codes, and also the future codes developed by the project members, to offer a simplified management of various evolutions and a single and well documented tool for our partners. It will aim to be self-contained including pre and post-processing tools (efficient meshing approaches, GMT and VTK libraries), but must of course also be opened to user's suggestions, and account for existing tools inside and outside Inria. This numerical platform will be dedicated to the simulation of all the phenomena of interest, including flow propagation, sediment evolution, model coupling on large scales, from deep water to the shoreline, including swell propagation, shoaling, breaking and run-up. This numerical platform clearly aims at becoming a reference software in the community. It should be used to **develop a specific test case** around Montpellier which embeds many processes and their mutual interactions: from the *Camargue* (where the Rhône river flows into the Mediterranean sea) to the *Étang de Thau* (a wide lagoon where shellfishes are plentiful), **all the processes studied in the project occur in a 100km wide region**, including of course the various hydrodynamics regimes (from the deep sea to the shoaling, surf and swash zones) and crucial morphodynamic issues (*e.g.* in the town of Sete).

4. Application Domains

4.1. Coastal Oceanography

Participants: Arnaud Duran, Fabien Marche, Antoine Rousseau.

Saint-Venant and Boussinesq equations have been widely applied until recently to model and simulate the propagation and transformations of waves in the nearshore area, over rapidly varying topography. However, the first equations do not include dispersive effects, and consequently have a domain of validity limited to the surf zone. The second set of equations overcome the limitations of the SV equations but relies on a "small amplitude assumption" and is therefore unable to model the whole range of waves transformations. This is the reason why they are usually called "weakly nonlinear Boussinesq equations". A better suited set of equations is known as the Green-Naghdi equations, but until recently, they have received far less attention, both from the theoretical and numerical point of view. In particular, there is no available numerical method of arbitrary ordre for 2d simulations on unstructured meshes. Additionally, the construction of rigorous positive preserving schemes is a paramount for the study of waves run-up.

4.2. Urban Floods

Participant: Vincent Guinot.

Floods have by identified by the National Accounting Authority (Cour des Comptes) to represent up to 1% of the GNP in terms of damage cost. For crisis management purposes, modeling urban floods at the scale of the conurbation is highly desirable. This however cannot be achieved in the current state of technology because of the meshing and computational cost (5569up to one billion cells being needed to mesh an entire urban area). This can be overcome by upscaling the shallow water equations so as to obtain large scale models that can operate three orders of magnitude faster than refined 2D models. Various upscaled versions of the upscaled 2D Shallow Water Equations have been proposed in the literature, some of which by members of the Lemon team. Further developments are being carried out, including the subgrid-scale description of topography variations and a better representation of energy dissipation terms. Laboratory experiments are also needed to discriminate between the various existing models.

4.3. River Hydraulics

Participants: Vincent Guinot, Antoine Rousseau.

Shallow Water (SW) models are widely used for the numerical modeling of river flows. Depending on the geometry of the domain, of the flow regime, and of required accuracy, either 1D or 2D SW models are implemented. It is thus necessary to couple 1D models with 2D models when both models are used to represent different portions of the same river. Moreover, when a river flows into the sea/ocean (e.g. the Rhône river in the Mediterranean), one may need to couple a 2D SW with a full 3D model (such as the Navier-Stokes equations) of the estuary. These issues have been widely addressed by the river-engineering community, but often with somehow crude approaches in terms of coupling algorithms. This may be improved thanks to more advanced boundary conditions, and with the use of Schwarz iterative methods for example.

5. New Software and Platforms

5.1. SW2D

Participant: Vincent Guinot.

Urban floods are usually simulated using two-dimensional shallow water models. A correct representation of the urban geometry and hydraulics would require that the average computational cell size be between 0.1 m and 1 m. The meshing and computation costs make the simulation of entire districts/conurbations impracticable in the current state of computer technology.

An alternative approach consists in upscaling the shallow water equations using averaging techniques. This leads to introducing storage and conveyance porosities, as well as additional source terms, in the mass and momentum balance equations. Various versions of porosity-based shallow water models have been proposed in the literature. The Shallow Water 2 Dimensions (SW2D) computational code embeds various finite volume discretizations of these models. Ituses fully unstructured meshes with arbitrary numbers of edges. The key features of the models and numerical techniques embedded in SW2D are

- specific momentum/energy dissipation models that are active only under transient conditions. Such models, that are not present in classical shallow water models, stem from the upscaling of the shallow water equations and prove essential in modeling the features of fast urban flow transients accurately
- modified HLLC solvers for an improved discretization of the momentum source terms stemming from porosity gradients
- higher-order reconstruction techniques that allow for faster and more stable calculations in the presence of wetting/drying fronts.

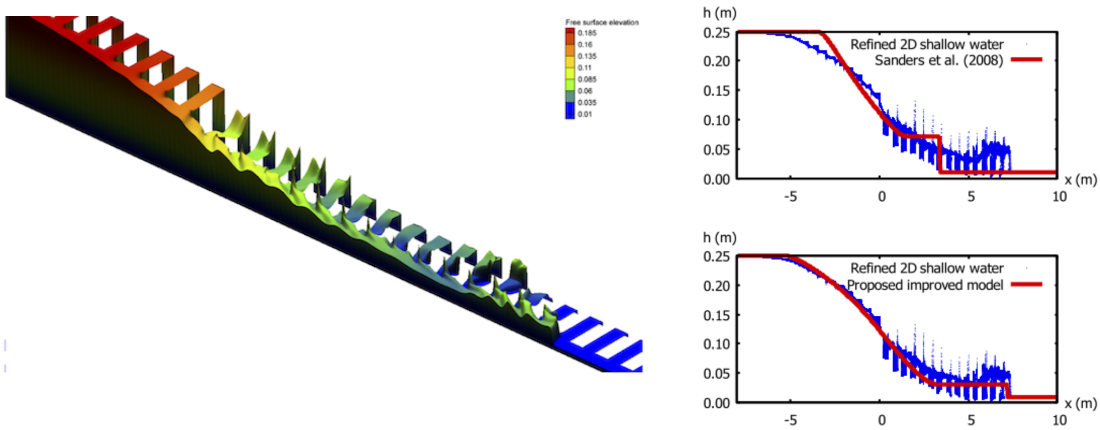

Figure 2. Propagation of a flood wave into a channel with lateral storage. Refined 2D simulation using the SW2D computational code

5.2. Stochastic Downscaling Method

Participant: Antoine Rousseau.

The computation of the wind at small scale and the estimation of its uncertainties is of particular importance for applications such as wind energy resource estimation. To this aim, starting in 2005, we have developed a new method based on the combination of an existing Numerical Weather Prediction model providing a coarse prediction, and a Lagrangian Stochastic Model for turbulent flows. This Stochastic Downscaling Method (SDM) requires a specific modeling of the turbulence closure, and involves various simulation techniques whose combination is totally original (such as Poisson solvers, optimal transportation mass algorithm, original Euler scheme for confined Langevin stochastic processes, and stochastic particle methods).

In 2013, the SDM code became the kernel of the wind farm modeling of the Fundacion Inria Chile with the Windpos project. In France, its development is going on through the collaborative Modéol project on the evaluation of wind potential.

This is a joint work with Mireille Bossy from the team TOSCA.

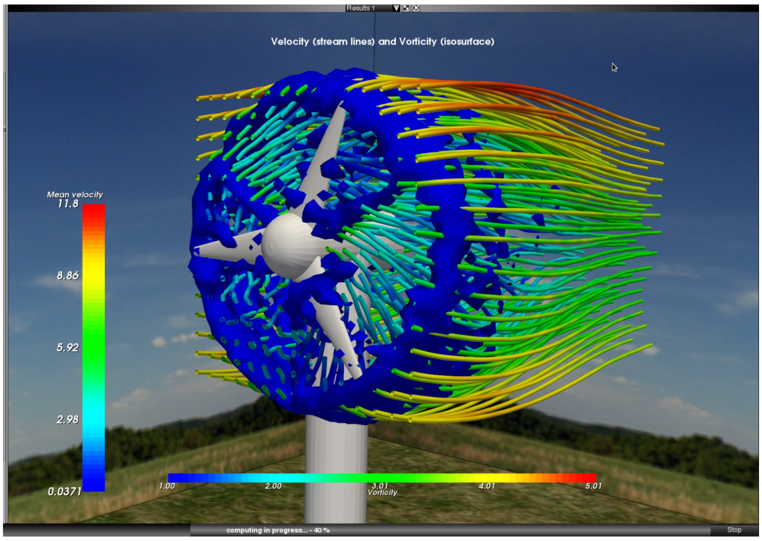

Figure 3. Velocity streamlines and vorticity around a wind mill (artistic view). WINDPOS Project.

5.3. Action Dépollution

Participants: Antoine Rousseau, Alexis Pacholik.

Action Dépollution (see website in french) is a serious game made for learning how to purify fast and well a water reservoir, such as lakes. In the scope of the international initiative Mathematics of Planet Earth, this game shows an application of mathematics related to environmental education and sustainable development. The player can act as a researcher, that compares different strategies and looks for the best solution. The conception has been achieved in collaboration with the Inria project-team MODEMIC, and the realization with the help of the start-up Funkadelichik, sponsored by the french consortium Cap'Maths and Inria (Direction de la Communication).
This work is in connection with the INRA/Inria patent [19].

6. New Results

6.1. Highlights of the Year

Antoine ROUSSEAU and 5 co-authors released in 2014 the book *Brèves de Maths* [16]. This work (in french) selected more than 100 posts from the blog breves-de-maths.fr, in the framework of the international initiative "Mathematics of the Planet Earth". In this book (see cover 5), no complicated numbers, no weird equation, but short and clear sentences together with nice drawings to illustrate everyday life topics on our planet with the beauty of mathematics.

6.2. A Schwarz coupling method for dimensionally heterogeneous problem

Participant: Antoine Rousseau.

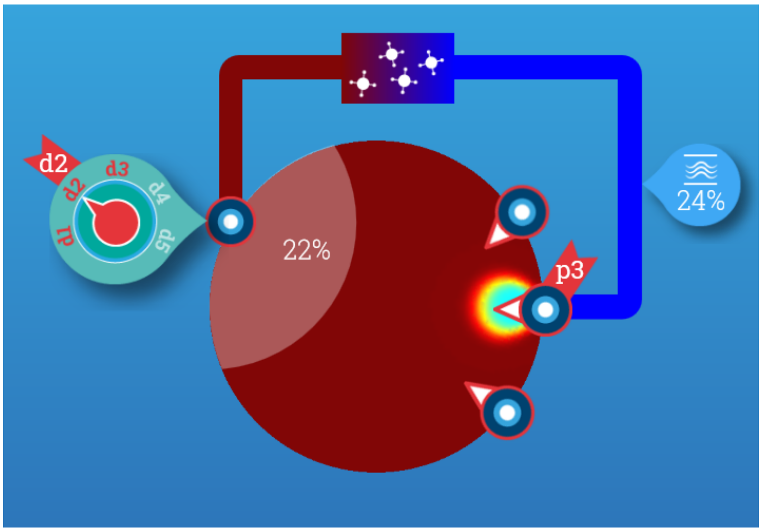

Figure 4. Player interface. Serious game Action Dépollution.

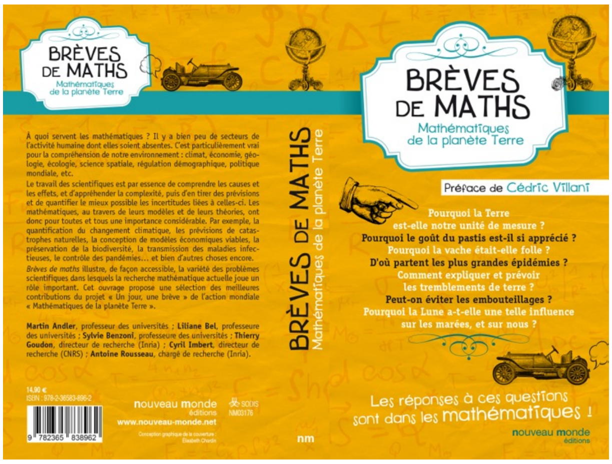

Figure 5. Brèves de Maths. Ed. Nouveau Monde, 2014

We study and analyze in [10] an efficient iterative coupling method for a dimensionally heterogeneous problem. We consider the case of 2-D Laplace equation with non symmetric boundary conditions with a corresponding 1-D Laplace equation. We first show how to obtain the 1-D model from the 2-D one by integration along one direction, by analogy with the link between shallow water equations and the Navier-Stokes system. Then, we focus on the design of an Schwarz-like iterative coupling method. We discuss the choice of boundary conditions at coupling interfaces. We prove the convergence of such algorithms and give some theoretical results related to the choice of the location of the coupling interface, and the control of the difference between a global 2-D reference solution and the 2-D coupled one. These theoretical results are illustrated numerically.

6.3. Bioremediation of water ressources

Participants: Antoine Rousseau, Alexis Pacholik.

Together with fellows from the MODEMIC team, we proposed a strategy for the bioremediation of water ressources such as lakes or lagoons. The originality is that the water treatment has to be done outside of the resource, in order not to dislocate its fragile ecological equilibrium.
The objective is to reach a targeted acceptable state for the resource within the minimal time. The patent [19] has been filed in connection with this work.

6.4. A well-balanced and positive preserving DG scheme for the SW equations

Participants: Arnaud Duran, Fabien Marche.

We consider in [5] the discontinuous Galerkin discretization of the nonlinear Shallow Water equations on unstructured triangulations. We propose an efficient combination of ingredients that leads to a simple high-order robust and well-balanced scheme, based on the alternative formulation of the equations known as the pre-balanced shallow water equations. We show that the preservation of the motionless steady states can be achieved, for an arbitrary order of polynomial expansion. Additionally, the preservation of the positivity of the water height is ensured using the recent method introduced in [59]. Some comparisons with a recent finite-volume MUSCL approach are also performed. The well-known tsunami test case shown in figures 6 and 7 has been computed here with high order DG scheme on unstructured triangulation.

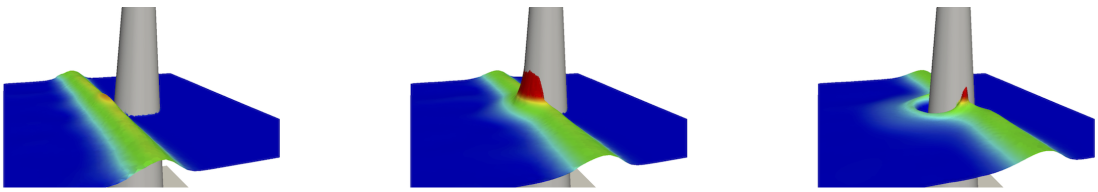

Figure 6. Tsunami wave over a conical island - Lateral view of the free surface at times t=5,6 and 7s.

6.5. A well-balanced and positive preserving DG scheme for the GN equations

Participants: Arnaud Duran, Fabien Marche.

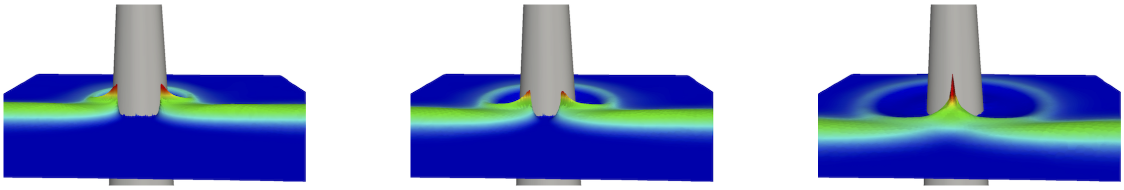

Figure 7. Tsunami wave over a conical island - Rear view of the free surface at times t=8,9 and 10s.

We introduce in [4] a discontinuous-Galerkin Finite-Element method to approximate the solutions of a new family of 1d Green-Naghdi models. These new models are shown to be more computationally efficient, while being asymptotically equivalent to the initial formulation with regard to the shallowness parameter. Using the free surface instead of the water height as a conservative variable, the models are recasted under a pre-balanced formulation and discretized using a nodal expansion basis. Independently from the polynomial degree in the approximation space, the preservation of the motionless steady-states is automatically ensured, and the water height positivity is enforced. A simple numerical procedure devoted to stabilize the computations in the vicinity of broken waves is also described. The validity of the resulting model is assessed through extensive numerical validations.

6.6. A new class of fully nonlinear and weakly dispersive Green-Naghdi models

Participant: Fabien Marche.

We introduce in [8] a new class of two-dimensional fully nonlinear and weakly dispersive Green-Naghdi equations over varying topography. These new Green-Naghdi systems share the same order of precision as the standard one but have a mathematical structure which makes them much more suitable for the numerical resolution, in particular in the demanding case of two dimensional surfaces. For these new models, we develop a high order, well balanced, and robust numerical code relying on a hybrid finite volume and finite difference splitting approach. The hyperbolic part of the equations is handled with a high-order finite volume scheme allowing for breaking waves and dry areas. The dispersive part is treated with a finite difference approach. Higher order accuracy in space and time is achieved through WENO reconstruction methods and through an SSP-RK time stepping. Particular effort is made to ensure positivity of the water depth.

6.7. Upscaling transfer properties in heterogeneous porous media

Participant: Vincent Guinot.

In [9] the passive solute transport was studied in a periodic, artificial porous medium. A Laplace analysis of the breakthrough curves indicates that the widely used, classical Advection-Dispersion (AD) model cannot reproduce the contaminant transport features accurately. Neither can fractional dynamics-based, anomalous dispersion models. The models failing to reproduce the features of contaminant transport is shown to be due to the Fick-like, gradient-based operator used to represent dispersion, that induces infinite signal propagation speed, even when fractional models are used. The Laplace analysis shows that advection processes are predominant at all time and space scales. The size of the Representative Elementary Volume is shown to be 20 to 30 periods.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

7.1.1. *Free surface hydraulics*

The finite volume-based, SW2D computational code (see Software section) is used by the Cereg Ingénierie company on a regular basis to carry out flood risk assessment studies. The code is constantly being developed on a work-for-hire basis depending on the company needs. The developments mostly concern pre- and post-processing functionalities, as well as specific hydraulic modules.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. *ANR*

Fabien MARCHE is member of the ANR project BonD (PI Sylvie Benzoni), 2013-2017.

Fabien MARCHE is member of the ANR project ACHYLLES (PI Rodolphe Turpault), 2014-2017

8.2. International Initiatives

8.2.1. *Inria International Labs*

Antoine ROUSSEAU visited Inria Chile in April, 2014 (2 weeks, see Associate Teams below) in order to prepare an application for a research center on marine energies in Chile.
This application is coordinated by DCNS Energies Marines and also involves Inria Chile and PUC University (Santiago).

8.2.2. *Inria Associate Teams*

Antoine ROUSSEAU collaborates with the ANESTOC partners (TOSCA at Inria Sophia and Rolando Rebolledo at PUC, Santiago, Chile) on the stochastic analysis of renewable energies. Together with Mireille Bossy (TOSCA), AR supervises the research of two engineers in Chile: Jacques Morice and Cristián Paris.

Antoine ROUSSEAU collaborates with the DYNECOS2 partners (MODEMIC at Inria Sophia and Hector Ramirez at CMM, Santiago, Chile) on the bioremediation of natural resources.

In the framework for these two collaborations, AR visited Inria Chile in April, 2014 (2 weeks). See the TOSCA (resp. MODEMIC) project team activity report for more information on the ANESTOC (resp. DYNECOS) associate team.

8.2.3. *Inria International Partners*

8.2.3.1. Informal International Partners

Vincent GUINOT collaborates with B.F. Sanders (Irvine University, Californie, USA)

Vincent GUINOT collaborates with S. Soares-Frazao (Unité de Génie Civil, Université catholique de Louvain, Belgium)

Fabien MARCHE and Antoine ROUSSEAU collaborate with R. Cienfuegos (PUC University, Santiago, Chile)

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. *Scientific events selection*

9.1.1.1. Member of the conference program committee

Fabien MARCHE is member of the scientific committee of Advances in Numerical modeling of Hydrodynamics, 2015.

9.1.2. Journal

9.1.2.1. Member of the editorial board

Vincent GUINOT : Journal of Hydroinformatics.

Antoine ROUSSEAU : Discrete and Continuous Dynamical Systems, Series S.

9.1.2.2. Reviewer

Fabien MARCHE : Advances in Applied Mathematics and Mechanics, International Journal for Numerical Methods in Fluids, Journal of Applied and Computational Mathematics, Journal of Computational Physics, Journal of Scientific Computing and SIAM Journal on Scientific Computing.

Vincent GUINOT : Journal of Hydrology and Journal of Hydroinformatics.

Antoine ROUSSEAU : Applied Numerical Mathematics, International Journal for Numerical Methods in Fluids.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

V. Guinot, Mécanique des fluides, 72h ETD, L3, Polytech'Montpellier, France

V. Guinot, Hydraulique à surface libre, 60h ETD, L3, Polytech'Montpellier, France

V. Guinot, Méthodes Mathématiques pour l'Ingénieur, 18h ETD, M1, Polytech'Montpellier, France

V. Guinot, Hydraulique des Réseaux, 30h ETD, M1, Polytech'Montpellier, France

V. Guinot, Mécanique des Fluides, Master SPAE, 36h ETD, M1, UMontpellier, France

V. Guinot, Transitoires hydrauliques, 54 h ETD, M1, Polytech'Montpellier, France

V. Guinot, tutorat de stages ingénieur, 15h ETD, M1, Polytech'Montpellier, France

V. Guinot, Modélisation hydraulique à surface libre 2D, 6h ETD, M2, Polytech'Montpellier, France

V. Guinot, Projet Industriel de Fin d'Etudes (PIFE), 30h ETD, M2, Polytech'Montpellier, France

V. Guinot, Tutorat de Stage de fin d'études ingénieur, 18h ETD, M2, Polytech'Montpellier, France

F. Marche, Biomaths, 72h TD., L1, Université Montpellier 2, France

F. Marche, Analyse numérique des EDP, 24H CM, 12H TD, 15H TP., M1, Université Montpellier 2, France

F. Marche, Calcul scientifique avancé, 26H CM, M2R, Université Montpellier 2, France

A. Rousseau, Weather forecast and geophysical fluids, 10h, M2R, Labex CIMI, Toulouse, France

9.2.2. Supervision

PhD : Arnaud Duran, *Modeling, analysis and simulations of shallow water flows*, Université Montpellier II, December 2014, Fabien Marche.

PhD in progress : Mehdi Pierre Daou, *Développement d'une méthodologie de couplage multi-modèles avec changements de dimension. Validation sur un cas-test réaliste en dynamique littorale*, May 2013, Eric Blayo (EPI MOISE) and Antoine Rousseau

9.2.3. Juries

Vincent GUINOT : HDR of C. Delenne (UM2), Section CNU 60, december 2014.

Antoine ROUSSEAU : CR2 competition in Inria Lille, spring 2014.

Antoine ROUSSEAU : Inria Research Position competition, campaign #1, spring 2014.

Antoine ROUSSEAU : Inria Research Position competition, campaign #2, summer 2014.

9.3. Popularization

Antoine ROUSSEAU is co-author of the book [16] (and blog) *Brèves de Maths* (Ed. Nouveau Monde, Nov. 2014)

Antoine ROUSSEAU gave several conferences for highschool students and their teachers in France, on the topics of mathematical modeling for environmental sciences:

Fête de la Science, Oct. 2014, Genopolys Montpellier (with Twitter reports on the national account @EnDirectDuLabo)

Fête de la Science, Oct. 2014, Centre International de Valbonne

Initiation à la recherche, Oct. 2014, Lycée Saint-Sernin, Toulouse

Antoine ROUSSEAU gave a public lecture for Inria staff *Café In'* in Inria Sophia, Dec. 2014

Antoine ROUSSEAU was invited to the *Société Informatique de France* annual workshop SIF 2014 to participate to a round table on scientific outreach

Antoine ROUSSEAU is member of the national Inria network for scientific outreach *Médiation scientifique*

Arnaud Duran gave a public lecture for high school students at Collège Jean Bène (Pézénas, France), Feb 2014.

10. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

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Invited Conferences

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