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Activity Report 2014

Project-Team MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

RESEARCH CENTER
Paris - Rocquencourt

THEME
Stochastic approaches

Table of contents

Project-Team MATHRISK

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Creation of the Team: 2012 January 01*, updated into Project-Team:* 2013 January 01.

1. Members

Research Scientist
Agnès Sulem [Team leader, Inria, Senior Researcher, HdR]

Faculty Members
Aurélien Alfonsi [ENPC, Professor, HdR]
Benjamin Jourdain [ENPC, Professor, HdR]
Bernard Lapeyre [ENPC, Professor, HdR]
Vlad Bally [Univ. Paris Est Marne la Vallée, Professor, HdR]
Damien Lamberton [Univ. Paris Est Marne la Vallée, Professor, HdR]
Romuald Elie [Univ. Paris Est Marne la Vallée, Professor, HdR]

PhD Students
Rui Chen [Fondation Sciences Mathématiques de Paris, from Oct 2014]
Roxana-Larisa Dumitrescu [Univ. Paris Dauphine]
Julien Reygner [Univ. Paris Est, until November 2014]

Post-Doctoral Fellow
Arnaud Lionnet [Inria, from December 2014]

Visiting Scientists
Oleg Kudryavtsev [Russian Customs Academy Rostoc on Don, July-August 2014]
Xiao Wei [CIAS, China, July-August 2014]

Administrative Assistant
Martine Verneuille [Inria]

Others
Ahmed Kebaier [Univ. Paris XIII, Associate Professor, Partner Researcher]
Céline Labart [Univ. de Savoie, Associate Professor,Partner Researcher]
Jérôme Lelong [ENSIMAG, Associate Professor, Partner Researcher]
Antonino Zanette [Univ. Udine, Italie, Associate Professor, Partner Researcher]
Jean-Philippe Chancelier [ENPC]

2. Overall Objectives

2.1. Overall Objectives

MathRisk is a joint Inria project-team with ENPC (CERMICS Laboratory) and the University Paris Est Marne-la-Vallée (UPEMLV, LAMA Laboratory), located in Rocquencourt and Marne-la-Vallée.
http://www.inria.fr/en/teams/mathrisk. Mathrisk is based on the former Mathfi project team. Mathfi was founded in 2000, and was devoted to financial mathematics. The project was focused on advanced stochastic analysis and numerical techniques motivated by the development of increasingly complex financial products. Main applications concerned evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, and calibration of financial models.

2.1.1. Crisis, deregulation, and impact on the research in finance

The starting point of the development of modern finance theory is traditionally associated to the publication of the famous paper of Black and Scholes in 1973 [54]. Since then, in spite of sporadic crises, generally well overcome, financial markets have grown in a exponential manner. More and more complex exotic derivative products have appeared, on equities first, then on interest rates, and more recently on credit markets. The period between the end of the eighties and the crisis of 2008 can be qualified as the "golden age of financial mathematics": finance became a quantitative industry, and financial mathematics programs flourished in top universities, involving seminal interplays between the worlds of finance and applied mathematics. During its 12 years existence, the Mathfi project team has extensively contributed to the development of modeling and computational methods for the pricing and hedging of increasingly complex financial products.

Since the crisis of 2008, there has been a critical reorientation of research priorities in quantitative finance with emphasis on risk. In 2008, the "subprime" crisis has questioned the very existence of some derivative products such as CDS (credit default swaps) or CDOs (collateralized debt obligations), which were accused to be responsible for the crisis. The nature of this crisis is profoundly different from the previous ones. It has negatively impacted the activity on the exotic products in general, - even on equity derivative markets-, and the interest in the modeling issues for these products. The perfect replication paradigm, at the origin of the success of the Black and Scholes model became unsound, in particular through the effects of the lack of liquidity. The interest of quantitative finance analysts and mathematicians shifted then to more realistic models taking into account the multidimensional feature and the incompleteness of the markets, but as such getting away from the "lost paradi(gm)" of perfect replication. These models are much more demanding numerically, and require the development of hedging risk measures, and decision procedures taking into account the illiquidity and various defaults.

Moreover, this crisis, and in particular the Lehman Brothers bankruptcy and its consequences, has underlined a systemic risk due to the strong interdependencies of financial institutions. The failure of one of them can cause a cascade of failures, thus affecting the global stability of the system. Better understanding of these interlinkage phenomena becomes crucial.

At the same time, independently from the subprime crisis, another phenomenon has appeared: deregulation in the organization of stock markets themselves. This has been encouraged by the Markets in Financial Instruments Directive (MIFID) which is effective since November, 1st 2007. This, together with the progress of the networks, and the fact that all the computers have now a high computation power, have induced arbitrage opportunities on the markets, by very short term trading, often performed by automatic trading. Using these high frequency trading possibilities, some speculating operators benefit from the large volatility of the markets. For example, the flash crash of May, 6 2010 has exhibited some perverse effects of these automatic speculating trading strategies. These phenomena are not well understood and the theme of high frequency trading needs to be explored.

To summarize, financial mathematics is facing the following new evolutions:

- the complete market modeling has become unsatisfactory to provide a realistic picture of the market and is replaced by incomplete and multidimensional models which lead to new modeling and numerical challenges.
- quantitative measures of risk coming from the markets, the hedging procedures, and the lack of liquidity are crucial for banks,
- uncontrolled systemic risks may cause planetary economic disasters, and require better understanding,
- deregulation of stock markets and its consequences lead to study high frequency trading.

The project team MathRisk is designed to address these new issues, in particular dependence modeling, systemic risk, market microstructure modeling and risk measures. The research in modeling and numerical analysis remain active in this new context, motivated by new issues.

The MathRisk project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. https://www.rocq.inria.fr/mathfi/Premia/index.html.

The MathRisk project is part of the Université Paris-Est **“Labex” BÉZOUT**.

3. Research Program

3.1. Dependence modeling

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton, Bernard Lapeyre.

The volatility is a key concept in modern mathematical finance, and an indicator of the market stability. Risk management and associated instruments depend strongly on the volatility, and volatility modeling has thus become a crucial issue in the finance industry. Of particular importance is the assets *dependence* modeling. The calibration of models for a single asset can now be well managed by banks but modeling of dependence is the bottleneck to efficiently aggregate such models. A typical issue is how to go from the individual evolution of each stock belonging to an index to the joint modeling of these stocks. In this perspective, we want to model stochastic volatility in a *multidimensional* framework. To handle these questions mathematically, we have to deal with stochastic differential equations that are defined on matrices in order to model either the instantaneous covariance or the instantaneous correlation between the assets. From a numerical point of view, such models are very demanding since the main indexes include generally more than thirty assets. It is therefore necessary to develop efficient numerical methods for pricing options and calibrating such models to market data. As a first application, modeling the dependence between assets allows us to better handle derivatives products on a basket. It would give also a way to price and hedge consistently single-asset and basket products. Besides, it can be a way to capture how the market estimates the dependence between assets. This could give some insights on how the market anticipates the systemic risk.

3.2. Liquidity risk

Participants: Aurélien Alfonsi, Agnès Sulem, Antonino Zanette.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the liquidity risk. Loosely speaking, liquidity risk is the risk that comes from the difficulty of selling (or buying) an asset. At the extreme, this may be the impossibility to sell an asset, which occurred for “junk assets” during the subprime crisis. Hopefully, it is in general possible to sell assets, but this may have some cost. Let us be more precise. Usually, assets are quoted on a market with a Limit Order Book (LOB) that registers all the waiting limit buy and sell orders for this asset. The bid (resp. ask) price is the most expensive (resp. cheapest) waiting buy or sell order. If a trader wants to sell a single asset, he will sell it at the bid price. Instead, if he wants to sell a large quantity of assets, he will have to sell them at a lower price in order to match further waiting buy orders. This creates an extra cost, and raises important issues. From a short-term perspective (from few minutes to some days), this may be interesting to split the selling order and to focus on finding optimal selling strategies. This requires to model the market microstructure, i.e. how the market reacts in a short time-scale to execution orders. From a long-term perspective (typically, one month or more), one has to understand how this cost modifies portfolio managing strategies (especially delta-hedging or optimal investment strategies). At this time-scale, there is no need to model precisely the market microstructure, but one has to specify how the liquidity costs aggregate.

3.2.1. Long term liquidity risk.

On a long-term perspective, illiquidity can be approached via various ways: transactions costs [41], [42], [53], [61], [67], [87], [83], delay in the execution of the trading orders [88], [86], [57], trading constraints or restriction on the observation times (see e.g. [63] and references herein). As far as derivative products are concerned, one has to understand how delta-hedging strategies have to be modified. This has been considered for example by Cetin, Jarrow and Protter [85]. We plan to contribute on these various aspects of liquidity risk modeling and associated stochastic optimization problems. Let us mention here that the price impact generated by the trades of the investor is often neglected with a long-term perspective. This seems acceptable since the investor has time enough to trade slowly in order to eliminate its market impact. Instead, when the investor wants to make significant trades on a very short time horizon, it is crucial to take into account and to model how prices are modified by these trades. This question is addressed in the next paragraph on market microstructure.

3.2.2. Market microstructure.

The European directive MIFID has increased the competition between markets (NYSE-Euronext, Nasdaq, LSE and new competitors). As a consequence, the cost of posting buy or sell orders on markets has decreased, which has stimulated the growth of market makers. Market makers are posting simultaneously bid and ask orders on a same stock, and their profit comes from the bid-ask spread. Basically, their strategy is a "round-trip" (i.e. their position is unchanged between the beginning and the end of the day) that has generated a positive cash flow.

These new rules have also greatly stimulated research on market microstructure modeling. From a practitioner point of view, the main issue is to solve the so-called "optimal execution problem": given a deadline T, what is the optimal strategy to buy (or sell) a given amount of shares that achieves the minimal expected cost? For large amounts, it may be optimal to split the order into smaller ones. This is of course a crucial issue for brokers, but also market makers that are looking for the optimal round-trip.

Solving the optimal execution problem is not only an interesting mathematical challenge. It is also a mean to better understand market viability, high frequency arbitrage strategies and consequences of the competition between markets. For example when modeling the market microstructure, one would like to find conditions that allow or exclude round trips. Beyond this, even if round trips are excluded, it can happen that an optimal selling strategy is made with large intermediate buy trades, which is unlikely and may lead to market instability.

We are interested in finding synthetic market models in which we can describe and solve the optimal execution problem. A. Alfonsi and A. Schied (Mannheim University) [45] have already proposed a simple Limit Order Book model (LOB) in which an explicit solution can be found for the optimal execution problem. We are now interested in considering more sophisticated models that take into account realistic features of the market such as short memory or stochastic LOB. This is mid term objective. At a long term perspective one would like to bridge these models to the different agent behaviors, in order to understand the effect of the different quotation mechanisms (transaction costs for limit orders, tick size, etc.) on the market stability.

3.3. Contagion modeling and systemic risk

Participants: Benjamin Jourdain, Agnès Sulem.

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these. Ideally, one would like to be able to find capital requirements (such as the one proposed by the Basel committee) that ensure that the probability of multiple defaults is below some level.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular R. Cont et al. [46] and A. Minca [72]) or mean field interaction models (Garnier-Papanicolaou-Yang [62]). The recent approach in [46] seems very promising. It describes the financial network approach as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. Distress propagation in a financial system may be modeled as an epidemics on this graph. In the case of incomplete information on the structure of the interbank network, cascade dynamics may be reduced to the evolution of a multi-dimensional Markov chain that corresponds to a sequential discovery of exposures and determines at any time the size of contagion. Little has been done so far on the *control* of such systems in order to reduce the systemic risk and we aim to contribute to this domain.

3.4. Stochastic analysis and numerical probability

3.4.1. Stochastic control

Participants: Vlad Bally, Jean-Philippe Chancelier, Marie-Claire Quenez, Agnès Sulem.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the default risk modeling, the interplay between information and performance, the model uncertainty and the associated robustness questions, and various nonlinearities. We address these questions by further developing the theory of stochastic control in a broad sense, including stochastic optimization, nonlinear expectations, Malliavin calculus, stochastic differential games and various aspects of optimal stopping.

3.4.2. Optimal stopping

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton, Agnès Sulem, Marie-Claire Quenez.

The theory of American option pricing has been an incite for a number of research articles about optimal stopping. Our recent contributions in this field concern optimal stopping in models with jumps, irregular obstacles, free boundary analysis, reflected BSDEs.

3.4.3. Simulation of stochastic differential equations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Vlad Bally, Damien Lamberton, Bernard Lapeyre, Jérôme Lelong, Céline Labart.

Effective numerical methods are crucial in the pricing and hedging of derivative securities. The need for more complex models leads to stochastic differential equations which cannot be solved explicitly, and the development of discretization techniques is essential in the treatment of these models. The project MathRisk addresses fundamental mathematical questions as well as numerical issues in the following (non exhaustive) list of topics: Multidimensional stochastic differential equations, High order discretization schemes, Singular stochastic differential equations, Backward stochastic differential equations.

3.4.4. Monte-Carlo simulations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Vlad Bally, Bernard Lapeyre, Ahmed Kebaier, Céline Labart, Jérôme Lelong, Antonino Zanette.

Monte-Carlo methods is a very useful tool to evaluate prices especially for complex models or options. We carry on research on *adaptive variance reduction methods* and to use *Monte-Carlo methods for calibration* of advanced models.

This activity in the MathRisk team is strongly related to the development of the Premia software.

3.4.5. Malliavin calculus and applications in finance

Participants: Vlad Bally, Arturo Kohatsu-Higa, Agnès Sulem, Antonino Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [70]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [74], D. Bell [52] D. Ocone [76], B. Øksendal [89]) give expositions of the subject. See also V. Bally [47] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [75].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [71], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

We give below an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods.

We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be “smooth” in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequences of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M}E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [60] and [59] and the papers by Bally et al., Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [69].

L. Caramellino, A. Zanette and V. Bally have been concerned with the computation of conditional expectations using Integration by Parts formulas and applications to the numerical computation of the price and the Greeks (sensitivities) of American or Bermudean options. The aim of this research was to extend a paper of Reigner and Lions who treated the problem in dimension one to higher dimension - which represent the real challenge in this field. Significant results have been obtained up to dimension 5 [51] and the corresponding algorithms have been implemented in the Premia software.

Moreover, there is an increasing interest in considering jump components in the financial models, especially motivated by calibration reasons. Algorithms based on the integration by parts formulas have been developed in order to compute Greeks for options with discontinuous payoff (e.g. digital options). Several papers and two theses (M. Messaoud and M. Bavouzet defended in 2006) have been published on this topic and the corresponding algorithms have been implemented in Premia. Malliavin Calculus for jump type diffusions - and more general for random variables with locally smooth law - represents a large field of research, also for applications to credit risk problems.

The Malliavin calculus is also used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P. Imkeller in [69]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [69]).

A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. These SDEs can be interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals, as introduced by Russo and Valois [82]. This methodology has been applied for utility maximization with insiders.

4. Application Domains

4.1. Application Domains

Risk management, Quantitative finance, Computational Finance, Market Microstructure analysis, Systemic risk, Portfolio optimization, Risk modeling, Option pricing and hedging in incomplete markets, insurance.

5. New Software and Platforms

5.1. PREMIA

Participants: Antonino Zanette, Mathrisk Research Team, Agnès Sulem [correspondant].

5.1.1. Premia: general description

Premia is a software designed for option pricing, hedging and financial model calibration. It is provided with it's C/C++ source code and an extensive scientific documentation. https://www-rocq.inria.fr/mathfi/Premia

The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is:

1. to be a powerful testing platform for comparing different numerical methods between each other;
2. to build a link between professional financial teams and academic researchers;
3. to provide a useful teaching support for Master and PhD students in mathematical finance.

- AMS: 91B28;65Cxx;65Fxx;65Lxx;65Pxx
- License: Licence Propriétaire (genuine license for the Consortium Premia)
- Type of human computer interaction: Console, interface in Nsp, Web interface
- OS/Middelware: Linux, Mac OS X, Windows
- APP: The development of Premia started in 1999 and 15 are released up to now and registered at the APP agency. Premia 15 has been registered on 18/03/2013 and has the number IDDN.FR.001.190010.012.S.C.2001.000.31000
- Programming language: C/C++ librairie Gtk
- Documentation: the PNL library is interfaced via doxygen
- Size of the software: 280580 lines for the Src part only, that is 11 Mbyte of code, 130400 lines for PNL, 105 Mbyte of PDF files of documentation.
- interfaces : Nsp for Windows/Linux/Mac, Excel, binding Python, and a Web interface.
- Publications: [12], [58], [68], [79], [84], [40], [18].

5.1.2. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. **Equity derivatives**

   The following models are considered:

   Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

   For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods and Malliavin calculus based methods.

   Dynamic Hedging for Black-Scholes and jump models is available.

   Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. **Interest rate derivatives**

   The following models are considered:

   HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

   Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. **Credit derivatives: Credit default swaps (CDS), Collateralized debt obligations (CDO)**

   Reduced form models and copula models are considered.

   Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. **Hybrid products**

A PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. **Energy derivatives: swing options**

   Mean reverting and jump models are considered.

   Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

5.1.3. Premia design

Premia has managed to grow up over a period of fifteen years; this has been possible only because contributing an algorithm to Premia is subject to strict rules, which have become too stringent. To facilitate contributions, a standardized numerical library (PNL) has been developed by J. Lelong under the LGPL since 2009, which offers a wide variety of high level numerical methods for dealing with linear algebra, numerical integration, optimization, random number generators, Fourier and Laplace transforms, and much more. Everyone who wishes to contribute is encouraged to base its code on PNL and providing such a unified numerical library has considerably eased the development of new algorithms which have become over the releases more and more sophisticated. An effort is made to continue and stabilize the development of PNL. J. Ph Chancelier, B. Lapeyre and J. Lelong are using Premia and Nsp for Constructing a Risk Management Benchmark for Testing Parallel Architecture [18].

Development of the PNL in 2014 (J. Lelong) - Release 1.70. and 1.7.1 of the *PNL* library (http://pnl.gforge.inria.fr/).

1. Sampling from new distributions: non central Chi squared, Poisson, Bernoulli.
2. When using quasi Monte Carlo sequences, sampling from any distribution resorts to using the inverse of the cumulative distribution function technique.
3. A CMake module is provided to automatically detect the library when used by third party codes.
4. Add a sparse matrix object with advanced functionalities provided by Blas & Lapack. This new object is handled by the MPI binding.
5. Complete refactoring of the Basis object to considerably speedup the evaluation functions. Multivariate polynomials are represented as tensor products of one variate polynomials. The matrix holding the tensor product now uses a sparse storage which avoids many operations leading to a zero value thus leading to an impressive reduction the computational time.
6. All random number generators are thread–safe.

5.1.4. Algorithms implemented in Premia in 2014

Premia 16 has been delivered to the Consortium Premia in March 2014. In this release we have developed the Haar Wavelets-based approach for quantifying credit portfolio losses, Monte Carlo simulations of Credit Value Adjustment (CVA) using Malliavin techniques, asymptotic and exact pricing options on variance and importance sampling, and multilevel methods for jump models.

It contains the following new algorithms:

5.1.4.1. Commodities, Forex (FX), Insurance, Credit Risk

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  Journal of Computational Finance. Volume 13 Number 3, Spring 2010.
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- Stochastic local intensity loss models with interacting particle system.
  A. Alfonsi, C. Labart, J. Lelong *Mathematical Finance, to appear*
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5.1.4.2. Equity Derivatives

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- A robust tree method for pricing American options with the Cox-Ingersoll-Ross interest rate model. E. Appolloni, L. Caramellino and A. Zanette
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  Andreoli, A., Caravenna, F, Dai Pra, P. Posta, G., *Advances in Applied Probability. (2012), 44(4), 1018-1051.*
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6. New Results

6.1. Highlights of the Year

B. Jourdain and A. Sulem : Guest editors of the special issue "Systemic Risk" of *Statistics and Risk Modeling*, 2014. [27]

The research project "Stochastic Control of Systemic Risk" has been awarded by the scientific council and Professional Fellows of Institut Europlace de Finance (EIF) and Labex Louis Bachelier (December 2014).

Roxana Dumitrescu, PhD student, received the price for collaborative actions during her PhD studies, delivered by Fondation des Sciences Mathématiques de Paris and CASDEN (November 2014).

Pierre Blanc, PhD student, has got the award of "Rising star of quantitative finance" for his talk on a price impact models with an exogeneous (Hawkes) flow of orders [29]. This prize was given by the Global Derivatives conference (Amsterdam, 12-16 May) to indicate the best work among PhD students.

6.2. Liquidity risk

Aurélien Alfonsi and his PhD student Pierre Blanc are working on the optimal execution problem when there are many large traders who modify the price. They consider an Obizhaeva and Wang model for the price impact, and they assume that the flow of market orders generated by the other traders is given by an exogenous process. They have shown that Price Manipulation Strategies (PMS) exist when the flow of order is a compound Poisson process. On the other hand, modeling this flow by a mutually exciting Hawkes process allows them with a particular parametrization to exclude these PMS. Besides, they are able to calculate explicitly the optimal execution strategy within the model [29]. They are now investigating how this model can fit market data.

6.3. Dependence modeling

With his PHD student J. Reygner, B. Jourdain has studied a mean-field version of rank-based models of equity markets, introduced by Fernholz in the framework of stochastic portfolio theory ([38]). When the number of companies grows to infinity, they obtain an asymptotic description of the market in terms of a stochastic differential equation nonlinear in the sense of McKean. The diffusion and drift coefficients depend on the cumulative distribution function of the current marginal law of the capitalizations. Using results on the longtime behavior of such SDEs derived in [66], they discuss the long-term capital distribution in this asymptotic model, as well as the performance of simple portfolio rules. In particular, they highlight the influence of the volatility structure of the model on the growth rates of portfolios.

Another approach to handle the question of stochastic modeling in a multidimensional framework consists in dealing with stochastic differential equations that are defined on matrices in order to model either the instantaneous covariance or the instantaneous correlation between the assets.

The research on the estimation of the parameters of a Wishart process has started this year together with the thesis of Clément Rey. A. Alfonsi, A. Kebaier and C. Rey are studying the Maximum Likelihood Estimator for the Wishart processes and in particular its convergence in the ergodic and the non ergodic case.

Correlation issues are crucial in the modeling of volatility. In his thesis, Ould Aly ([77]) proposes a revised version of Bergomi's model for the variance curve which proves to be very tractable for calibration and for the pricing of variance derivatives (see [23]). He also obtains results on the monotonicity of option prices with respect to the correlation between the stock price and the volatility in the Heston model (see [78]).

In [34], [15], L. Abbas-Turki and D. Lamberton study the monotonicity of option prices with respect to cross-asset correlations in a multidimensional Heston model.

Modeling the dependence is not only useful for the equity market. In credit risk, getting a model that describes the dynamic of the joint distribution of a basket of defaults is still a challenge. The Loss Intensity model proposed by Schönbucher allows to fit perfectly the marginal distributions of the number of defaults in a basket. Then, Stochastic Loss Intensity models extend this model and can also in principle fit the marginal distributions. However, these models appear as a non-linear differential equation with jumps. A Alfonsi, C. Labart and J. Lelong have shown that these models are well-defined by using a particles system ([44]). Besides, this particles system gives a very convenient way to run a Monte-Carlo algorithm and to compute expectations in this model. Interacting particle systems are studied by B. Jourdain and his PhD student Julien Reygner in [39], [21].

Application of optimal transport. A. Alfonsi and B. Jourdain study in [43] the Wasserstein distance between two probability measures in dimension n sharing the same copula C. The image of the probability measure dC by the vectors of pseudo-inverses of marginal distributions is a natural generalization of the coupling known to be optimal in dimension $n = 1$. In dimension $n > 1$, it turns out that for cost functions equal to the p-th power of the L^q norm, this coupling is optimal only when $p = q$ i.e. when the cost function may be decomposed as the sum of coordinate-wise costs.

6.4. Systemic risk

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular [46]), or mean field interaction models [62], [55]. Little has been done so far on the *control* of such systems and A. Sulem has started to contribute on these issues in the framework of random graph models in collaboration with A. Minca (Cornell University) and H. Amini (EPFL). In [22], [31], they consider a financial network described as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. Here, the distress propagation is modeled as an epidemics on this graph. They study the optimal intervention of a lender of last resort who seeks to make equity infusions in a banking system prone to insolvency and to bank runs, under complete and incomplete information of the failure cluster, in order to minimize the contagion effects.

R. Elie is studying risk systemic propagation and its links with mean field games.

6.5. Backward stochastic (partial) differential equations with jumps and stochastic control with nonlinear expectation

A. Sulem, M.C. Quenez and R. Dumitrescu have studied optimization problems for BSDEs with jumps [11], optimal stopping for dynamic risk measures induced by BSDEs with jumps and associated reflected BSDEs. [24], [80], [19]. They have also investigated optimal stopping with nonlinear expectation under ambiguity, and their links with nonlinear Hamilton Jacobi Bellman variational inequalities in the Markovian case. Moreover they have obtained dynamic programming principles for mixed optimal-stopping problems with nonlinear expectations. They have also explored the links between generalized Dynkin games and double barriers reflected BSDE with jumps [56]. Stochastic control of Itô-Lévy Processes with applications to finance are studied by A. Sulem and B. Øksendal in [25], [26]. We have also contributed to the theory of BSDEs and Forward-Backward SDEs which appear as the adjoint equations associated to stochastic maximum principles, and address various issues about the relation between information and performance in non Markovian stochastic control: In particular, in the context of jump-diffusion models under partial information, A. Sulem, C. Fontana and B. Øksendal study in [20] the relation between market viability (in the sense of solvability of portfolio optimization problems) and the existence of a martingale measure given by the marginal utility of terminal wealth, without a-priori assuming no-arbitrage restrictions on the model.

A. Sulem, with B. Øksendal and T. Zhang has studied optimal stopping for Stochastic Partial Differential equations and associated reflected SPDEs [91], and optimal control of Forward-Backward SDEs [90].

Stochastic maximum principles for singular mean-field games are obtained in [37] with applications to optimal irreversible investments under uncertainty.

R. Dumitrescu and C. Labart have proposed a numerical approximation for Doubly Reflected BSDEs with Jumps and RCLL obstacles [35].

R. Elie studies approximate hedging prices under various risk constraints. This is done in collaboration with P. Briand, Y. Hu, A. Matoussi, B. Bouchard, L. Moreau, J.F. Chassagneux, I. Kharroubi and R. Dumitrescu.

6.6. Option Pricing

Interest rates modeling. A. Alfonsi studies an affine term structure model for interest rates that involve Wishart diffusions (with E. Palidda) [28] . Affine term structure models (Dai and Singleton, Duffie, ...) consider vector affine diffusions. Here, we extend the Linear Gaussian Model (LGM) by including some Wishart dynamics, and to get a model that could better fit the market. We have obtained a price expansion around the LGM for Caplet and Swaption prices. Also, we present a second order discretization scheme that allow to calculate exotic prices with this model.

American Options. In joint work with Aych Bouselmi, D. Lamberton studied the asymptotic behavior of the exercise boundary near maturity for American put options in exponential Lévy models [34].

He is currently working with M. Pistorius on the approximation of American options by Canadian options, which originated from the work of Peter Carr.

Barrier Options. Numerical pricing of double barrier options is investigated by A. Zanette and coauthors in [16].

6.7. Discretization of stochastic differential equations

With his PhD student A. Al Gerbi and E. Clément, B. Jourdain is interested in the strong convergence properties of the Ninomiya-Victoir scheme which is known to exhibit order 2 of weak convergence. This study is aimed at analysing the use of this scheme either at each level or only at the finest level of a multilevel Monte Carlo estimator : indeed, the variance of a multilevel Monte Carlo estimator is related to the strong error between the two schemes used in the coarse and fine grids at each level. They prove strong convergence with order $1/2$ which is improved to order 1 when the vector fields corresponding to each Brownian coordinate in the SDE commute. They also check that the renormalized errors converge to affine SDEs with source terms involving the Lie brackets between these vector fields and, in the commuting case, their Lie brackets with the drift vector field. Last, they propose a modified Ninomiya-Victoir scheme, which, at the finest level of the multilevel Monte Carlo estimator, may be coupled with strong order 1 to a simpler scheme with weak order 1 recently proposed by Giles and Szpruch.

Using optimal transport tools, A. Alfonsi, B. Jourdain and A. Kohatsu-Higa have proved that the Wasserstein distance between the time marginals of an elliptic SDE and its Euler discretization with N steps is not larger than $\frac{C\sqrt{\log(N)}}{N}$. The logarithmic factor may be removed when the uniform time-grid is replaced by a grid still counting N points but refined near the origin of times [4] . To generalize in higher dimension the result that they obtained previously in dimension one using the optimality of the explicit inverse transform, they compute the derivative of the Wasserstein distance with respect to the time variable using the theory developed by Ambrosio Gigli and Savare. The abstract properties of the optimal coupling between the time marginals then enable them to estimate this time derivative [30].

6.8. Advanced Monte Carlo methods.

- **Adaptive variance reduction methods.** B. Jourdain and J. Lelong have pursued their work on adaptive Monte Carlo methods in several directions [17], [36].
- **Metropolis Hastings algorithm in large dimension.** With T. Lelièvre and B. Miasojedow, B. Jourdain considers the Random Walk Metropolis algorithm on $\mathbf{R}^n$ with Gaussian proposals, and when the target probability measure is the n-fold product of a one dimensional law. It is well-known that, in the limit n tends to infinity, starting at equilibrium and for an appropriate scaling of the variance and of the timescale as a function of the dimension n, a diffusive limit is obtained for each component of the Markov chain. They generalize this result when the initial distribution is not the target probability measure ([65]). The obtained diffusive limit is the solution to a stochastic differential equation nonlinear in the sense of McKean. In [64], they prove convergence to equilibrium for this equation. They also discuss practical counterparts in order to optimize the variance of the proposal distribution to accelerate convergence to equilibrium. The analysis confirms the interest of the constant acceptance rate strategy (with acceptance rate between 1/4 and 1/3).

6.9. Numerical Probability

6.9.1. Regularity of probability laws using an interpolation method

This work was motivated by previous studies by N. Fournier, J. Printemps, E. Clément, A. Debusche and V. Bally, on the regularity of the law of the solutions of stochastic differential equations with low regularity coefficients - such as diffusion processes with Hölder coefficients or many other examples including jump type equations, Boltzmann equation or Stochastic PDE's. Since we do not have sufficient regularity, the usual approach by Malliavin calculus fails in this framework. We use the following alternative idea: We approximate the law of the random variable X (the solution of the equation at hand) by a sequence $X(n)$ of random variables which are smooth. Consequently we are able to establish integration by parts formulas for $X(n)$, to obtain the absolutely continuity of the law of $X(n)$, and to establish estimates for the density of the law of $X(n)$ and its derivatives. Note that the derivatives of the densities of $X(n)$ generally blow up - so we can not derive directly results concerning the density of the law of X. But, if the speed of convergence of $X(n)$ to X is faster than the blow up, then we may obtain results concerning the density of the law of X. It turns out that this approach fits in the framework of interpolation spaces and that the criterion of regularity for the law of X amounts to the characterization of an interpolation space between a space of distributions and a space of smooth functions. Although the theory of interpolation spaces is very well developed and one already knows how to characterize the interpolation spaces for Sobolev spaces of positive and negative indices, we have not found in the (huge) literature a result which covers the problem we are concerned with. So, although our result may be viewed as an interpolation result, it is a new one. The above work is treated in the paper [48] by V. Bally and Lucia Caramellino. As an application we discussed in [50] the regularity of the law of a Wiener functional under a Hörmander type non degeneracy condition.

6.9.2. A stochastic parametrix representation for the density of a Markov process.

Classical results of PDE theory (due to A. Friedmann) assert that, under uniform ellipticity conditions, the law of a diffusion process has a continuous density (the approach of A. Friedmann is analytical and concerns PDE's instead of the corresponding diffusion process). The method developed by A. Friedmann is known as the "parametrix method". V. Bally In collaboration with A. Kohatzu Higa gave a probabilistic approach which represents the probabilistic counterpart of the parametrix method [33]. They obtained a probabilistic representation for the density of the law of the solution of a SDE and more generally, for a class of Markov processes including solutions of jump type SDE's. This representation may be considered as a perfect simulation scheme and so represents a starting point for Monte Carlo simulation. However the random variable which appears in the stochastic representation has infinite variance, so direct simulation gives unstable results (as some preliminary tests have proved). In order to obtain an efficient simulation scheme some more work on the reduction of variance has to be done - and this does not seem trivial.

6.9.3. The distance between two density functions and convergence in total variation.

V. Bally and L. Caramellino have obtained estimates of the distance between the densities of the law of two random variables using an abstract variant of Malliavin calculus. They used these estimates in order to study the convergence in total variation of a sequence of random variables. This has been done in [49]. They are now working on more specific examples concerning the Central Limit Theorem [32]. In the last years the convergence in entropy distance and in total variation distance for several variants of the CLT has been considered in papers by S. Bobkov, F. Götze, G. Peccati, Y. Nourdin, D. Nualart and G. Poly. This is a very active research. Moreover, in an working paper in collaboration with his Phd student R. Clement, V. Bally uses similar methods in order to study the total variation distance between two Markov semigroups and for approximation schemes purposes. A special interest is devoted to higher order schemes such as the Victoir Nyomia scheme.

6.9.4. An invariance principle for stochastic series (U- Statistics).

Vlad Bally and Lucia Caramellino are working on invariance principles for stochastic series of polynomial type. In the case of polynomials of degree one we must have the classical Central Limit Theorem (for random variables which are not identically distributed). For polynomials of higher order we are in the framework of the so called U statistics which have been introduced by Hoffdings in t 1948 and which play an important role in modern statistics. Our contribution in this topic concerns convergence in total variation distance for this type of objects. We use abstract Malliavin calculus and more generally, the methods mentioned in the above paragraph.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

- Consortium PREMIA, Natixis - Inria
- Consortium PREMIA, Crédit Agricole CIB - Inria

7.2. Bilateral Grants with Industry

- Chair X-ENPC-UPMC-Société Générale "Financial Risks" of the Risk fondation : A. Alfonsi, B. Jourdain, B. Lapeyre

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

B. Jourdain is involved in the ANR Stab (2013/2016). Partners: Lyon1 and Paris-Dauphine.

8.1.2. Competitivity Clusters

Pôle Finance Innovation.

8.2. International Initiatives

8.2.1. Inria International Partners

8.2.1.1. Informal International Partners

- Center of Excellence program in Mathematics and Life Sciences at the Department of Mathematics, University of Oslo, Norway, (B. Øksendal).
- Department of Mathematics, University of Manchester (Tusheng Zhang, currently in charge of an EU-ITN program on BSDEs and Applications).
- Mannheim University (Alexander Schied, Chair of Mathematics in Business and Economics, Department of Mathematics)
- Roma Tor Vergata University (Lucia Caramellino)
- Ritsumeikan University (A. Kohatsu-Higa).

8.3. International Research Visitors

8.3.1. Visits of International Scientists

- Arturo Kohatsu-Higa, Ritsumeikan University, 3 months

- Lucia Caramellino, Tor Vergata University, Roma, 2 weeks

- Oleg Kudryavtsev, Rostov University , 2 months

- Xiao Wei, Beijing university, 2 months

8.3.2. Visits to International Teams

8.3.2.1. Research stays abroad

- V. Bally, Ritsumeikan University, Japan, one month
- A. Sulem:

  - "Adjunct Professorship", Center of Mathematics for Applications (CMA), University of Oslo, Norway, 1st Semester 2014.

  - Participation to the "Stochastics in Environmental and Financial Economics" program, Centre of Advanced Studies of the Norwegian Academy of Sciences and Letters, Oslo, Last term 2014.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific events organisation

- R. Dumitrescu: Co-organizer of the young researchers in Mathematics Seminar of Université Paris Dauphine.

- A. Alfonsi: Co-organizer of the working group seminar of MathRisk “Méthodes stochastiques et finance”. http://cermics.enpc.fr/~alfonsi/GTMSF.html

9.1.2. Administrative and scientific responsabilites

- A. Alfonsi: In charge of the Master “Finance and Application” at the Ecole des Ponts.
- B. Jourdain: Head of the doctoral school MSTIC, University Paris-Est
- D. Lamberton: Vice-president for research at Université Paris-Est Marne-la-Vallée
- A. Sulem, Scientific Coordinator for the evaluation of the Inria theme *Stochastic Approaches*, Paris, March 2014.

9.1.3. Scientific events selection

- A. Alfonsi:
  - "Pathwise optimal transport bounds between a one-dimensional diffusion and its Euler scheme", Oberwolfach and Paris 13 seminar in May.
  - "Stochastic Local Intensity Loss Models with Interacting Particle Systems", Bachelier conference in June, Bruxelles.
  - "Dynamic optimal execution in a mixed-market-impact Hawkes price model", SIAM-SMAI Conference on Financial Mathematics: Advanced Modeling and Numerical Methods (June), Workshop "Stochastic analysis for risk modeling", CIRM (September) and Finance and Stochastics seminar, Imperial College.
  - Invitation Imperial College, Antoine Jacquier (19- 20 November)
- V. Bally

  - Statistics, jump processes and Malliavin calculus: recent applications, Barcelona June 2014

  - SPA - 2014, 37th conference on Stochastic Processes and their applications, Buenos Aires, July 2014.

  - invited lecture "Convergence in total variation distance using an interpolation method", Colloquium of the Mathematics department, Ritsumeikan University, Japan, December.

- R. Dumitrescu

  - Inria Junior Seminar, 13 February
  - Séminaire Jeunes Chercheurs, Université Paris Dauphine, 7 March
  - Colloque Mathématiques en Mouvement, Fondation des Sciences Mathématiques de Paris, 28 May
  - Conference Cycle Thématique en Mathématiques Financières, Paris, 20 June
  - Premier Séminaire Bachelier Paris-Londres, Paris, 25-26 September
- R. Elie

  Colloquium Bachelier, Metabief, January 2014

  Seminar Ecole Polytechnique, February 2014

  Seminar Marne la Vallée, March 2014

  Seminar ETH, Zurich, July 2014

  Seminar University of Santa Barbara (UCSB), September 2014

  Seminar University of Southern California, Los Angeles, September 2014

  Seminar of Risk analysis, Stanford University, October 2014

  Seminar on stochastics, Humboldt & TU, Berlin, December 2014

  - 2 weeks visit ETH Zurich, July 2014

  - 2 weeks visit, University of Santa Barbara (UCSB), September 2014
- B. Jourdain:

  Bachelier seminar, IHP, Paris, 4 April 2014 : Capital distribution and portfolio in the mean-field Atlas model

  Paris 13 university seminar, 21 May 2014: Estimation of the Wasserstein distance between the marginals of a diffusion and its Euler scheme

  Workshop Computational methods for statistical mechanics, ICMS Edimbourg 2-6 June 2014 : Optimal scaling of the transient phase of Metropolis Hastings algorithms

  Special session on Stochastic processes and spectral theory for partial differential equations and boundary value problems, AIMS 2014 Madrid, 7-11 July 2014 : A trajectorial interpretation of the dissipations of entropy and Fisher information for stochastic differential equations

  Workshop Advances in stochastic analysis for risk modeling, CIRM 8-12 September 2014 : Long-time behavior of the mean-field Atlas model

  London-Paris Bachelier workshop 25-26 september 2014 Paris : Long-time behavior of the mean-field Atlas model

  Seminar ENSTA-CMAP, 6 October 2014 : Capital distribution and portfolio performance in the mean-field Atlas model

  Seminar of the chair “Financial risks”, 21 november 2014 : Estimation of the Wasserstein distance between the marginals of a diffusion and its Euler scheme
- D. Lamberton

  Invited to give some lectures in the framework of the workshop in Quantitative Finance at the University of Bologna in May 2014. Title: "A short course on American options".
- A. Sulem

  - Plenary speaker, Mathematics of Systemic Risk, Pacific Institute for the Mathematical Sciences, University of British Columbia, Vancouver, Canada, July 27-31 2014, Title: "Control of interbank contagion under partial information"

- Plenary speaker at Stochastics in Environmental and Financial Economics, Centre of Advanced Studies CAS of the Norwegian Academy of Sciences and Letters, Oslo, September 2014

- Workshop "Stochastic analysis for risk modeling", CIRM, Lumigny, 8-12 September 2014. http://www.cirm.univ-mrs.fr/, Title: “Dynamic Programming Principle for Combined Optimal Stopping and Stochastic Control with f-conditional Expectation”

- Bachelier seminar, IHP, Paris, November 7 2014. Title: " Optimal control of interbank contagion under partial information", https://sites.google.com/site/seminairebachelierparis/

- Seminar on stochastic methods and Finance, Inria Paris, October 6th, "Control of interbank contagion under partial information", http://cermics.enpc.fr/~alfonsi/GTMSF.html

9.1.4. Journal

9.1.4.1. Member of the editorial board

- R. Elie

  Associate editor of *SIAM Journal on Financial Mathematics (SIFIN)* (since November 2014)

- D. Lamberton

  Associate editor of:

  - Mathematical Finance,

  - ESAIM Probability & Statistics

- A. Sulem

  Associate editor of

  – 2011- Present: *Journal of Mathematical Analysis and Applications (JMAA)*

  – 2009- Present: *International Journal of Stochastic Analysis (IJSA)*

  – 2008- Present: *SIAM Journal on Financial Mathematics (SIFIN)*

9.1.4.2. Reviewer

The members of the team reviewed numerous papers for various journals.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

License

A. Alfonsi, “Modéliser, Programmer et Simuler”, second year course at the Ecole des Ponts.

R. Dumitrescu, Applied courses (Travaux Dirigés) in Linear Algebra, 35h, L2, Université Paris Dauphine

R. Elie, Algebra (UPEMLV, L2), Probability (UPEMLV, L3)

B. Jourdain, "Probability theory and statistics", first year, ENPC, France

B. Jourdain, Introduction to probability theory", first year, Ecole Polytechnique, France

Master

- A. Alfonsi , “Calibration, Volatilité Locale et Stochastique”, third-year course at ENSTA (Master with Paris I).

- A. Alfonsi , "Traitement des données de marché : aspects statistiques et calibration”, lecture for the Master at UPEMLV.

- A. Alfonsi , “Mesures de risque”, Master course of UPEMLV and Paris VI.

- V. Bally, "The Malliavin calculus and applications in finance", 30h, Master 2 Finance, Université Marne la Vallée

- V. Bally, " Interest Rates", 20h, Master 2 Finance, Université Marne la Vallée

- V. Bally, " Risk methodes in actuarial science", 36h, Master IMIS, Université Marne la Vallée

- R. Dumitrescu, Applied courses (Travaux Dirigés) Asset pricing by absence of arbitrage opportunities (Master 2 MASEF) , 35h, Université Paris Dauphine

- R. Elie : Imperfect markets modeling M2 Master MASEF (Paris-Dauphine), Stochastic calculus (UPEMLV, ENSAE); Introduction to mathematical finance (UPEMLV), Statistics for big data and applications (ENPC)

- B. Jourdain, B. Lapeyre , "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, University Paris- Est Marne-la-Vallée ;

- J.-F. Delmas, B.Jourdain, "Jump processes with applications to energy markets", 3rd year ENPC and Master Recherche Mathématiques et Application, University Paris- Est Marne-la-Vallée

- B. Jourdain, Stochastic numerical methods", 3rd year, Ecole Polytechnique, France

- B. Jourdain, projects in finance and numerical methods, 3rd year, Ecole Polytechnique, France

- A. Sulem, "Finite difference for PDEs in Finance", Master 2 MASEF, Université Paris IX-Dauphine, Département Mathématiques et Informatique de la Décision et des Organisations (MIDO), 18h.

- A. Sulem , Master of Mathematics, University of Luxemburg, 22 h lectures and responsible of the module "Numerical Methods in Finance".

9.2.2. Supervision

PhD

- J. Mint Moustapha : "Modelling and simulation of vehicle traffic : statistical analysis of insertion models and probabilistic simulation of a kinetic model, Université Paris Est, November 13 2014, Adviser: B. Jourdain [73].

- J. Reygner : "Longtime behaviour of particle systems : applications in physics, finance and PDEs" , Université Pierre et Marie Curie, November 24 2014, advisers: B. Jourdain and L. Zambotti [81]

PhD in progress :

- Anis Al Gerbi, "Discretization of stochastic differential equations and systemic risk modeling", Paris-Est Cermics, Adviser: B. Jourdain

- Pierre Blanc, "Price impact on marker orders and limit order books (from Nov. 2012), Ecole des Ponts, Adviser : A. Alfonsi

- Rui Chen, "Stochastic Control of mean field systems and applications to systemic risk, from September 2014, Université Paris-Dauphine, Adviser A. Sulem

- Roxana Dumitrescu, "Stochastic control with nonlinear expectation, stochastic targets and applications to risk optimization", from September 2012, Université Paris-Dauphine, Adviser A. Sulem and R. Elie.

- Paulo Pigato, "Lower bounds for the density of the solution of SDE's under the weak Hörmander condition, and applications in finance", Advisers: V. Bally and A. Dai Pra, University of Padova.

- Victor Rabiet : "On a class of jump type stochastic equations", Université Paris-Est Marne la Vallée, Advisers: V. Bally (75 %) and E. Locherbach

- Clément Rey (from Oct. 2012), " High order discretization schemes for singular diffusions", Ecole des Ponts, Advisers : A. Alfonsi and Vlad Bally.

- Benjamin Schannes, 2014, "Statistical learning and actuarial applications", Adviser: R. Elie.

9.2.3. Juries

R. Elie: Report on the PhD thesis of Christoph Mainberger, "Essays on Supersolutions of BSDEs and Equilibrium Pricing in Generalized Capital Asset Pricing Models", Humboldt, Berlin
R. Elie: Report on the PhD thesis of Xuzhe Zhao, Multi-modes switching problems via BSDEs, Université du Mans
R. Elie: PhD thesis of Adrian Iuga, "Analyse et modélisation du processus de formation de prix à travers les échelles. Market Impact", Université Paris-Est
R. Elie: HDR: Idris Kharroubi, Représentations et approximations probabilistes en contrôle stochastique et finance mathématique, Université Paris-Dauphine

B. Jourdain: Report on the Habilitation thesis “Contributions à l’étude du comportement en temps long de processus stochastiques”, of Fabien Panloup, Institut de Mathématiques de Toulouse, Université Paul Sabatier, December 4th 2014
B. Jourdain: Report on the Habilitation thesis "Contributions aux méthodes numériques pour le filtrage et l’optimisation stochastique" of Nadia Oudjane, University Paris 7, Defense on 22 January 2015.

A. Sulem : Committee for the recrutment of an Assistant Professor ("Maitre de conférences") in Financial Mathematics and numerical probability, Laboratoire de probabilités (LPMA), Université Paris-Diderot, May 2014.

9.3. Popularization

- A. Sulem, "Nouvelles directions de recherche en Mathématiques financières, Inria Alumni, jam session on Risk, November 25th, Inria, Paris.

10. Bibliography

Major publications by the team in recent years

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[2] A. AHDIDA, A. ALFONSI. *Exact and high order discretization schemes for Wishart processes and their affine extensions*, in "Annals of Applied Probability", 2013, vol. 23, n^o 3, pp. 1025-1073 [*DOI :* 10.1214/12-AAP863], http://hal.inria.fr/hal-00491371

[3] A. ALFONSI. *High order discretization schemes for the CIR process: Application to affine term structure and Heston models*, in "Stochastic Processes and their Applications", 2010, vol. 79, pp. 209-237, http://www.ams.org/journals/mcom/2010-79-269/S0025-5718-09-02252-2/home.html

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