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Team QUANTIC

QUANTum Information Circuits

RESEARCH CENTER
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THEME
Optimization and control of dynamic systems

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Team QUANTIC

Keywords: Quantum Physics, Automatic Control, Nonlinear Control, Network Dynamics

Creation of the Team: 2013 September 12.

1. Members

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2. Overall Objectives

2.1. Overall objectives

The research activities of QUANTIC team lie at the border between theoretical and experimental efforts in the emerging field of quantum systems engineering. Our research topics are in direct continuation of a historic research theme of Inria, classical automatic control, while opening completely new perspectives toward quantum control: by developing a new mathematical system theory for quantum circuits, we will realize the components of a future quantum information processing unit.

One of the unique features of our team concerns the large spectrum of our subjects going from the mathematical analysis of the physical systems (development of systematic mathematical methods for control and estimation of quantum systems), and the numerical analysis of the proposed solutions, to the experimental implementation of the quantum circuits based on these solutions. This is made possible by the constant and profound interaction between the applied mathematicians and the physicists in the group. Indeed, this close collaboration has already brought a significant acceleration in our research efforts. In a long run, this synergy should lead to a deeper understanding of the physical phenomena behind these emerging technologies and the development of new research directions within the field of quantum information processing.

Towards this ultimate task of practical quantum digital systems, the approach of the QUANTIC team is complementary to the one taken by teams with expertise in quantum algorithms. Indeed, we start from the specific controls that can be realistically applied on physical systems, to propose designs which combine them into *hardware shortcuts* implementing *robust* behaviors useful for quantum information processing. Whenever a significant new element of quantum engineering architecture is developed, the initial motivation is to provide an enabling technology with major impact for the groups working one abstraction layer higher: on quantum algorithms but also on e.g. secure communication and metrology applications.

3. Research Program

3.1. Hardware-efficient quantum information processing

The research activities of this section and those in next sections are done in collaboration with the permanent researchers of the future QUANTIC project-team, members of Laboratoire Pierre Aigrain, Benjamin Huard (CNRS) and François Mallet (UPMC), and of Centre Automatique et Systèmes, Pierre Rouchon (Mines Paristech). They have benefited from important scientific exchanges and collaborations with the teams of Serge Haroche, Jean-Michel Raimond and Michel Brune at Laboratoire Kastler Brossel (LKB) and Collège de France and those of Michel Devoret and Robert Schoelkopf at the department of Applied Physics of Yale University.

In this scientific program, we will explore various theoretical and experimental issues concerning protection and manipulation of quantum information. Indeed, the next, critical stage in the development of Quantum Information Processing (QIP) is most certainly the active quantum error correction (QEC). Through this stage one designs, possibly using many physical qubits, an encoded logical qubit which is protected against major decoherence channels and hence admits a significantly longer effective coherence time than a physical qubit. Reliable (fault-tolerant) computation with protected logical qubits usually comes at the expense of a significant overhead in the hardware (up to thousands of physical qubits per logical qubit). Each of the involved physical qubits still needs to satisfy the best achievable properties (coherence times, coupling strengths and tunability). More remarkably, one needs to avoid undesired interactions between various subsystems. This is going to be a major difficulty for qubits on a single chip.

The usual approach for the realization of QEC is to use many qubits to obtain a larger Hilbert space of the qubit register [72], [75]. By redundantly encoding quantum information in this Hilbert space of larger dimension one make the QEC tractable: different error channels lead to distinguishable error syndromes. There are two major drawbacks in using multi-qubit registers. The first, fundamental, drawback is that with each added physical qubit, several new decoherence channels are added. Because of the exponential increase of the Hilbert's space dimension versus the linear increase in the number of decay channels, using enough qubits, one is able to eventually protect quantum information against decoherence. However, multiplying the number of possible errors, this requires measuring more error syndromes. Note furthermore that, in general, some of these new decoherence channels can lead to correlated action on many qubits and this needs to be taken into account with extra care: in particular, such kind of non-local error channels are problematic for surface codes. The second, more practical, drawback is that it is still extremely challenging to build a register of more than on the order of 10 qubits where each of the qubits is required to satisfy near the best achieved properties: these properties include the coherence time, the coupling strengths and the tunability. Indeed, building such a register is not merely only a fabrication task but rather, one requirers to look for architectures such that, each individual qubit can be addressed and controlled independently from the others. One is also required to make sure that all the noise channels are well-controlled and uncorrelated for the QEC to be effective.

We have recently introduced a new paradigm for encoding and protecting quantum information in a quantum harmonic oscillator (e.g. a high-Q mode of a 3D superconducting cavity) instead of a multi-qubit register [4]. The infinite dimensional Hilbert space of such a system can be used to redundantly encode quantum information. The power of this idea lies in the fact that the dominant decoherence channel in a cavity is photon damping, and no more decay channels are added if we increase the number of photons we insert in the cavity. Hence, only a single error syndrome needs to be measured to identify if an error has occurred or not. Indeed, we are convinced that most early proposals on continuous variable QIP [49], [45] could be revisited taking into account the design flexibilities of Quantum Superconducting Circuits (QSC) and the new coupling regimes that are provided by these systems. In particular, we have illustrated that coupling a qubit to the cavity mode in the strong dispersive regime provides an important controllability over the Hilbert space of the cavity mode [51]. Through a recent experimental work [10], we benefit from this controllability to prepare superpositions of quasi-orthogonal coherent states, also known as Schrödinger cat states.

In this Scheme, the logical qubit is encoded in a four-component Schrödinger cat state. Continuous quantum non-demolition (QND) monitoring of a single physical observable, consisting of photon number parity, enables then the tractability of single photon jumps. We obtain therefore a first-order quantum error correcting code using only a single high-Q cavity mode (for the storage of quantum information), a single qubit (providing the non-linearity needed for controllability) and a single low-Q cavity mode (for reading out the error syndrome). As shown in Figure 1, this leads to a significant hardware economy for realization of a protected logical qubit. Our goal here is to push these ideas towards a reliable and hardware-efficient paradigm for universal quantum computation.

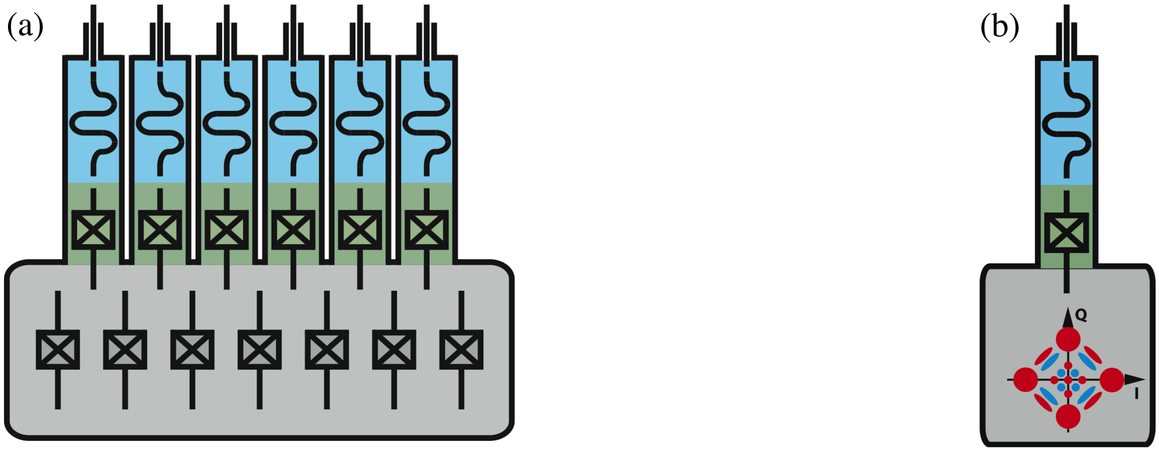

Figure 1. (a) A protected logical qubit consisting of a register of many qubits: here, we see a possible architecture for the Steane code [75] consisting of 7 qubits requiring the measurement of 6 error syndromes. In this sketch, 7 transmon qubits in a high-Q resonator and the measurement of the 6 error syndromes is ensured through 6 additional ancillary qubits with the possibility of individual readout of the ancillary qubits via independent low-Q resonators. (b) Minimal architecture for a protected logical qubit, adapted to circuit quantum electrodynamics experiments. Quantum information is encoded in a Schrödinger cat state of a single high-Q resonator mode and a single error syndrome is measured, using a single ancillary transmpn qubit and the associated readout low-Q resonator.

3.2. Reservoir (dissipation) engineering and autonomous stabilization of quantum systems

Being at the heart of any QEC protocol, the concept of feedback is central for the protection of the quantum information enabling many-qubit quantum computation or long-distance quantum communication. However, such a closed-loop control which requires a real-time and continuous measurement of the quantum system has been for long considered as counter-intuitive or even impossible. This thought was mainly caused by properties of quantum measurements: any measurement implies an instantaneous strong perturbation to the system's state. The concept of *quantum non-demolotion* (QND) measurement has played a crucial role in understanding and resolving this difficulty [30]. In the context of cavity quantum electro-dynamics (cavity QED) with Rydberg atoms [47], a first experiment on continuous QND measurements of the number of microwave photons was performed by the group at Laboratoire Kastler-Brossel (ENS) [46]. Later on, this ability of performing continuous measurements allowed the same group to realize the first continuous quantum feedback protocol stabilizing highly non-classical states of the microwave field in the cavity, the so-called photon number states [7] (this ground-breaking work was mentioned in the Nobel prize attributed to Serge

Haroche). The QUANTIC team contributed to the theoretical work behind this experiment [38], [21], [74], [23]. These contributions include the development and optimization of the quantum filters taking into account the quantum measurement back-action and various measurement noises and uncertainties, the development of a feedback law based on control Lyapunov techniques, and the compensation of the feedback delay.

In the context of circuit quantum electrodynamics (circuit QED) [37], recent advances in quantum-limited amplifiers [67], [77] have opened doors to high-fidelity non-demolition measurements and real-time feedback for superconducting qubits [2]. This ability to perform high-fidelity non-demolition measurements of a quantum signal has very recently led to quantum feedback experiments with quantum superconducting circuits [77], [66], [32]. Here again, the QUANTIC team has participated to one of the first experiments in the field where the control objective is to track a dynamical trajectory of a single qubit rather than stabilizing a stationary state (this experiment was performed by the members of the future QUANTIC project-team). Such quantum trajectory tracking could be further explored to achieve metrological goals such as the stabilization of the amplitude of a microwave drive [58].

While all this progress has led to a strong optimism about the possibility to perform active protection of quantum information against decoherence, the rather short dynamical time scales of these systems limit, to a great amount, the complexity of the feedback strategies that could be employed. Indeed, in such measurement-based feedback protocols, the time-consuming data acquisition and post-treatment of the output signal leads to an important latency in the feedback procedure.

The reservoir (dissipation) engineering [64] and the closely related coherent feedback [56] are considered as alternative approaches circumventing the necessity of a real-time data acquisition, signal processing and feedback calculations. In the context of quantum information, the decoherence, caused by the coupling of a system to uncontrolled external degrees of freedom, is generally considered as the main obstacle to synthesize quantum states and to observe quantum effects. Paradoxically, it is possible to intentionally engineer a particular coupling to a reservoir in the aim of maintaining the coherence of some particular quantum states. In a general viewpoint, these approaches could be understood in the following manner: by coupling the quantum system to be stabilized to a strongly dissipative ancillary quantum system, one evacuates the entropy of the main system through the dissipation of the ancillary one. By building the feedback loop into the Hamiltonian, this type of autonomous feedback obviates the need for a complicated external control loop to correct errors. On the experimental side, such autonomous feedback techniques have been used for qubit reset [1], single-qubit state stabilization [59], and the creation [25] and stabilization [50], [55][8] of states of multipartite quantum systems.

Such reservoir engineering techniques could be widely revisited exploring the flexibility in the Hamiltonian design for QSC. We have recently developed theoretical proposals leading to extremely efficient, and simple to implement, stabilization schemes for systems consisting of a single or two qubits [1] [53]. The experimental results based on these protocols have illustrated the efficiency of the approach [1], [8]. Through these experiments, we exploit the strong dispersive interaction [70] between superconducting qubits and a single low-Q cavity mode playing the role of a dissipative reservoir. Applying some continuous-wave (cw) microwave drives with well-chosen fixed frequencies, amplitudes, and phases, we engineer an effective interaction Hamiltonian which evacuates entropy from the qubits when an eventual perturbation occurs: by driving the qubits and cavity with continuous-wave drives, we induce an autonomous feedback loop which corrects the state of the qubits every time it decays out of the desired target state. The schemes are robust against small variations of the control parameters (drives amplitudes and phase) and require only some basic calibration. Finally, by avoiding resonant interactions between the qubits and the low-Q cavity mode, the qubits remain protected against the Purcell effect, which would reduce the coherence times.

3.3. System theory for quantum information processing

In parallel and in strong interactions with the above experimental goals, we develop systematic mathematical methods for dynamical analysis, control and estimation of composite and open quantum systems. These systems are built with several quantum subsystems whose irreversible dynamics results from measurements and/or decoherence. A special attention is given to spin/spring systems made with qubits and harmonic oscillators. These developments are done in the spirit of our recent contributions [68], [21], [73], [74], [23][6] [69] resulting from collaborations with the cavity quantum electrodynamics group of Laboratoire Kastler Brossel.

3.3.1. Stabilization by measurement-based feedback

The protection of quantum information via efficient QEC is a combination of (i) tailored dynamics of a quantum system in order to protect an informational qubit from certain decoherence channels, and (ii) controlled reaction to measurements that efficiently detect and correct the dominating disturbances that are not rejected by the tailored quantum dynamics.

In such feedback scheme, the system and its measurement are quantum objects whereas the controller and the control input are classical. The stabilizing control law is based on the past values of the measurement outcomes. During our work on the LKB photon box, we have developed, for single input systems subject to quantum non-demolition measurement, a systematic stabilization method [23]: it is based on a discrete-time formulation of the dynamics, on the construction of a strict control Lyapunov function and on an explicit compensation of the feedback-loop delay. Keeping the QND measurement assumptions, extensions of such stabilization schemes will be investigated in the following directions: finite set of values for the control input with application to the convergence analysis of the atomic feedback scheme experimentally tested in [79]; multi-input case where the construction by inversion of a Metzler matrix of the strict Lyapunov function is not straightforward; continuous-time systems governed by diffusive master equations; stabilization towards a set of density operators included in a target subspace; adaptive measurement by feedback to accelerate the convergence towards a stationary state as experimentally tested in [62]. Without the QND measurement assumptions, we will also address the stabilization of non-stationary states and trajectory tracking, with applications to systems similar to those considered in [2] [32].

3.3.2. Filtering, quantum state and parameter estimations

The performance of every feedback controller crucially depends on its online estimation of the current situation. This becomes even more important for quantum systems, where full state measurements are physically impossible. Therefore the ultimate performance of feedback correction depends on fast, efficient and optimally accurate state and parameter estimations.

A quantum filter takes into account imperfection and decoherence and provides the quantum state at time $t \geq 0$ from an initial value at $t = 0$ and the measurement outcomes between 0 and t. Quantum filtering goes back to the work of Belavkin [26] and is related to quantum trajectories [34], [36]. A modern and mathematical exposure of the diffusive models is given in [24]. In [80] a first convergence analysis of diffusive filters is proposed. Nevertheless the convergence characterization and estimation of convergence rate remain open and difficult problems. For discrete time filters, a general stability result based on fidelity is proven in [68], [73]. This stability result is extended to a large class of continuous-time filters in [22]. Further efforts are required to characterize asymptotic and exponential stability. Estimations of convergence rates are available only for quantum non-demolition measurements [27]. Parameter estimations based on measurement data of quantum trajectories can be formulated within such quantum filtering framework [40], [60].

We will continue to investigate stability and convergence of quantum filtering. We will also exploit our fidelity-based stability result to justify maximum likelihood estimation and to propose, for open quantum system, parameter estimation algorithms inspired of existing estimation algorithms for classical systems. We will also investigate a more specific quantum approach: it is noticed in [31] that post-selection statistics and "past quantum" state analysis [41] enhance sensitivity to parameters and could be interesting towards increasing the precision of an estimation.

3.3.3. Stabilization by interconnections

In such stabilization schemes, the controller is also a quantum object: it is coupled to the system of interest and is subject to decoherence and thus admits an irreversible evolution. These stabilization schemes are closely related to reservoir engineering and coherent feedback [64], [56]. The closed-loop system is then a composite system built with the original system and its controller. In fact, and given our particular recent expertise in this domain [6], [1], [8], this subsection is dedicated to further developing such stabilization techniques, both experimentally and theoretically.

The main analysis issues are to prove the closed-loop convergence and to estimate the convergence rates. Since these systems are governed by Lindblad differential equations (continuous-time case) or Kraus maps (discrete-time case), their stability is automatically guaranteed: such dynamics are contractions for a large set of metrics (see [63]). Convergence and asymptotic stability is less well understood. In particular most of the convergence results consider the case where the target steady-state is a density operator of maximum rank (see, e.g., [20][chapter 4, section 6]). When the goal steady-state is not full rank very few convergence results are available.

We will focus on this geometric situation where the goal steady-state is on the boundary of the cone of positive Hermitian operators of finite trace. A specific attention will be given to adapt standard tools (Lyapunov function, passivity, contraction and Lasalle's invariance principle) for infinite dimensional systems to spin/spring structures inspired of [6], [1], [8], [5] and their associated Fokker-Planck equations for the Wigner functions.

We will also explore the Heisenberg point of view in connection with recent results of the Inria project-team MAXPLUS (algorithms and applications of algebras of max-plus type) relative to Perron-Frobenius theory [44], [43]. We will start with [71] and [65] where, based on a theorem due to Birkhoff [28], dual Lindblad equations and dual Kraus maps governing the Heisenberg evolution of any operator are shown to be contractions on the cone of Hermitian operators equipped with Hilbert's projective metric. As the Heisenberg picture is characterized by convergence of all operators to a multiple of the identity, it might provide a mean to circumvent the rank issues. We hope that such contraction tools will be especially well adapted to analyzing quantum systems composed of multiple components, motivated by the facts that the same geometry describes the contraction of classical systems undergoing synchronizing interactions [76] and by our recent generalized extension of the latter synchronizing interactions to quantum systems [57].

Besides these analysis tasks, the major challenge in stabilization by interconnections is to provide systematic methods for the design, from typical building blocks, of control systems that stabilize a specific quantum goal (state, set of states, operation) when coupled to the target system. While constructions exist for so-called linear quantum systems [61], this does not cover the states that are more interesting for quantum applications. Various strategies have been proposed that concatenate iterative control steps for open-loop steering [78], [54] with experimental limitations. The characterization of Kraus maps to stabilize any types of states has also been established [29], but without considering experimental implementations. A viable stabilization by interaction has to combine the capabilities of these various approaches, and this is a missing piece that we want to address.

3.3.3.1. Perturbation methods

With this subsection we turn towards more fundamental developments that are necessary in order to address the complexity of quantum networks with efficient reduction techniques. This should yield both efficient mathematical methods, as well as insights towards unravelling dominant physical phenomena/mechanisms in multipartite quantum dynamical systems.

In the Schrödinger point of view, the dynamics of open quantum systems are governed by master equations, either deterministic or stochastic [47], [42]. Dynamical models of composite systems are based on tensor products of Hilbert spaces and operators attached to the constitutive subsystems. Generally, a hierarchy of different timescales is present. Perturbation techniques can be very useful to construct reliable models adapted to the timescale of interest.

To eliminate high frequency oscillations possibly induced by quasi-resonant classical drives, averaging techniques are used (rotating wave approximation). These techniques are well established for closed systems without any dissipation nor irreversible effect due to measurement or decoherence. We will consider in a first step the adaptation of these averaging techniques to deterministic Lindblad master equations governing the quantum state, i.e. the system density operator. Emphasis will be put on first order and higher order corrections based on non-commutative computations with the different operators appearing in the Lindblad equations. Higher order terms could be of some interest for the protected logical qubit of figure 1b. In future steps, we intend to explore the possibility to explicitly exploit averaging or singular perturbation properties in the design of coherent quantum feedback systems; this should be an open-systems counterpart of works like [52].

To eliminate subsystems subject to fast convergence induced by decoherence, singular perturbation techniques can be used. They provide reduced models of smaller dimension via the adiabatic elimination of the rapidly converging subsystems. The derivation of the slow dynamics is far from being obvious (see, e.g., the computations of page 142 in [33] for the adiabatic elimination of low-Q cavity). Contrarily to the classical composite systems where we have to eliminate one component in a Cartesian product, we here have to eliminate one component in a tensor product. We will adapt geometric singular perturbations [39] and invariant manifold techniques [35] to such tensor product computations to derive reduced slow approximations of any order. Such adaptations will be very useful in the context of quantum Zeno dynamics to obtain approximations of the slow dynamics on the decoherence-free subspace corresponding to the slow attractive manifold.

Perturbation methods are also precious to analyze convergence rates. Deriving the spectrum attached to the Lindblad differential equation is not obvious. We will focus on the situation where the decoherence terms of the form $L\rho L^\dagger - (L^\dagger L\rho + \rho L^\dagger L)/2$ are small compared to the conservative terms $-i[H/\hbar, \rho]$. The difficulty to overcome here is the degeneracy of the unperturbed spectrum attached to the conservative evolution $\frac{d}{dt}\rho = -i[H/\hbar, \rho]$. The degree of degeneracy of the zero eigenvalue always exceeds the dimension of the Hilbert space. Adaptations of usual perturbation techniques [48] will be investigated. They will provide estimates of convergence rates for slightly open quantum systems. We expect that such estimates will help to understand the dependance on the experimental parameters of the convergence rates observed in [1], [8] [53].

As particular outcomes for the other subsections, we expect that these developments towards simpler dominant dynamics will guide the search for optimal control strategies, both in open-loop microwave networks and in autonomous stabilization schemes such as reservoir engineering. It will further help to efficiently compute explicit convergence rates and quantitative performances for all the intended experiments.

4. Application Domains

4.1. Quantum engineering

A new field of quantum systems engineering has emerged during the last few decades. This field englobes a wide range of applications including nano-electromechanical devices, nuclear magnetic resonance applications, quantum chemical synthesis, high resolution measurement devices and finally quantum information processing devices for implementing quantum computation and quantum communication. Recent theoretical and experimental achievements have shown that the quantum dynamics can be studied within the framework of estimation and control theory, but give rise to new models that have not been fully explored yet.

The QUANTIC team's activities are defined at the border between theoretical and experimental efforts of this emerging field with an emphasis on the applications in quantum information, computation and communication. The main objective of this interdisciplinary team is to develop quantum devices ensuring a robust processing of quantum information.

On the theory side, this is done by following a system theory approach: we develop estimation and control tools adapted to particular features of quantum systems. The most important features, requiring the development of new engineering methods, are related to the concept of measurement and feedback for composite quantum systems. The destructive and partial [1] nature of measurements for quantum systems lead to major difficulties in extending classical control theory tools. Indeed, design of appropriate measurement protocols and, in the sequel, the corresponding quantum filters estimating the state of the system from the partial measurement record, are themselves building blocks of the quantum system theory to be developed.

[1] Here the partiality means that no single quantum measurement is capable of providing the complete information on the state of the system.

On the experimental side, we develop new quantum information processing devices based on quantum superconducting circuits. Indeed, by realizing superconducting circuits at low temperatures and using microwave measurement techniques, the macroscopic and collective degrees of freedom such as the voltage and the current are forced to behave according to the laws of quantum mechanics. Our quantum devices are aimed to protect and process quantum information through these integrated circuits.

5. New Results

5.1. Highlights of the Year

- Experimental results in continuous measurement of error syndromes for a quantum error correction scheme developed by Mazyar Mirrahimi and his former PhD student Zaki Leghtas in close collaboration with the teams of Michel Devoret and Robert Schoelkopf (Department of Applied Physics of Yale University) have been published in Nature [13].
- Theoretical proposal on a new paradigm for universal quantum computation [12] has been chosen by the editors of the New Journal of Physics as an IOPselect paper for the novelty, significance and potential impact on future research.
- The EPOQ2 ANR Young Researcher project, led by Mazyar Mirrahimi, was highlighted in the 2013 annual report of Agence Nationale de la Recherche.

5.2. Dynamically protected cat-qubits: a new paradigm for universal quantum computation

Participant: Mazyar Mirrahimi.

In a close collaboration with the teams of Michel Devoret, Robert Schoelkopf and Liang Jiang (Department of Applied Physics, Yale university) and in particular a former member of our group, Zaki Leghtas, we have presented a new hardware-efficient paradigm for universal quantum computation. This paradigm is based on encoding, protecting and manipulating quantum information in a quantum harmonic oscillator. This proposal exploits multi-photon driven dissipative processes to encode quantum information in logical bases composed of Schrödinger cat states. More precisely, we consider two schemes. In a first scheme, a two-photon driven dissipative process is used to stabilize a logical qubit basis of two-component Schrödinger cat states. While such a scheme ensures a protection of the logical qubit against the photon dephasing errors, the prominent error channel of single-photon loss induces bit-flip type errors that cannot be corrected. Therefore, we have considered a second scheme based on a four-photon driven dissipative process which leads to the choice of four-component Schrödinger cat states as the logical qubit. Such a logical qubit can be protected against single-photon loss by continuous photon number parity measurements. Next, applying some specific Hamiltonians, we have provided a set of universal quantum gates on the encoded qubits of each of the two schemes. In particular, we have illustrated how these operations can be rendered fault-tolerant with respect to various decoherence channels of participating quantum systems. Finally, we have also proposed experimental schemes based on quantum superconducting circuits and inspired by methods used in Josephson parametric amplification, which should allow to achieve these driven dissipative processes along with the Hamiltonians ensuring the universal operations in an efficient manner.

This proposal was published in New Journal of Physics [12] and has also been chosen by the editor as an IOPselect paper for the novelty, significance and potential impact on future research.

5.3. Tracking photon jumps with repeated quantum non-demolition parity measurements

Participant: Mazyar Mirrahimi.

Quantum error correction (QEC) is required for a practical quantum computer because of the fragile nature of quantum information. In quantum error correction, information is redundantly stored in a large quantum state space and one or more observables must be monitored to reveal the occurrence of an error, without disturbing the information encoded in an unknown quantum state. Such observables, typically multi-quantum-bit parities, must correspond to a special symmetry property inherent in the encoding scheme. Measurements of these observables, or error syndromes, must also be performed in a quantum non-demolition way (projecting without further perturbing the state) and more quickly than errors occur. Previously, quantum non-demolition measurements of quantum jumps between states of well-defined energy have been performed in systems such as trapped ions, electrons, cavity quantum electrodynamics, nitrogen?vacancy centres and superconducting quantum bits. So far, however, no fast and repeated monitoring of an error syndrome had been achieved. Mazyar Mirrahimi has participated to an experiment performed by the group of Robert Schoelkopf (Department of Applied Physics, Yale University) where the quantum jumps of a possible error syndrome, namely the photon number parity of a microwave cavity, were tracked by mapping this property onto an ancilla quantum bit, whose only role is to facilitate quantum state manipulation and measurement. This quantity is just the error syndrome required in a QEC scheme proposed by Mazyar Mirrahimi and his former PhD student, Zaki Leghtas, and in a close collaboration with the teams of Michel Devoret and Robert Schoelkopf. This scheme should lead to a hardware-efficient protected quantum memory using Schrödinger cat states (quantum superpositions of different coherent states of light) in a harmonic oscillator [4]. We demonstrated the projective nature of this measurement onto a region of state space with well-defined parity by observing the collapse of a coherent state onto even or odd cat states. The measurement is fast compared with the cavity lifetime, has a high single-shot fidelity and has a 99.8 per cent probability per single measurement of leaving the parity unchanged. In combination with the deterministic encoding of quantum information in cat states realized earlier [10], the quantum non-demolition parity tracking that we have demonstrated represents an important step towards implementing an active system that extends the lifetime of a quantum bit. This result was published in Nature [9].

5.4. Dissipation-induced continuous quantum error correction for superconducting circuits

Participants: Joachim Cohen, Mazyar Mirrahimi.

Quantum error correction (QEC) is a crucial step towards long coherence times required for efficient quantum information processing (QIP). One major challenge in this direction concerns the fast real-time analysis of error syndrome measurements and the associated feedback control. Recent proposals on autonomous QEC (AQEC) have opened new perspectives to overcome this difficulty. As a sequel to our recent contributions to autonomous stabilization of maximally entangled states of superconducting qubits [53],[8], we have designed an AQEC scheme based on quantum reservoir engineering adapted to superconducting qubits. We have focused on a three-qubit bit-flip code, where three transmon qubits are dispersively coupled to a few low-Q resonator modes. By applying only continuous-wave drives of fixed but well-chosen frequencies and amplitudes, we engineer an effective interaction Hamiltonian to evacuate the entropy created by eventual bit-flip errors. We have provided a full analytical and numerical study of the protocol, while introducing the main limitations on the achievable error correction rates. This result was published in Physical Review A [11].

5.5. Continuous generation and stabilization of mesoscopic field superposition states in a quantum circuit

Participants: Ananda Roy, Mazyar Mirrahimi.

While dissipation is widely considered as being harmful for quantum coherence, it can, when properly engineered, lead to the stabilization of non-trivial pure quantum states. In a close collaboration with the teams of Michel Devoret and Douglas Stone (Department of Applied Physics, Yale University), and in the framework of a 6 months visit by Ananda Roy (PhD student at Yale), we proposed a scheme for continuous generation and stabilization of Schrödinger cat states in a cavity using dissipation engineering [15]. The scheme consists in first generating non-classical photon states with definite parity by means of a two-photon drive and dissipation, and then stabilizing these transient states against single-photon decay. The single-photon stabilization is autonomous, and is implemented through a second engineered bath, which exploits the photon number dependent frequency-splitting due to Kerr interactions in the strongly dispersive regime of circuit QED. Starting with the Hamiltonian of the baths plus cavity, we derived an effective model of only the cavity photon states along with analytic expressions for relevant physical quantities, such as the stabilization rate. The deterministic generation of such cat states is one of the key ingredients in performing universal quantum computation.

5.6. Extending robustness and randomization from consensus to symmetrization algorithms

Participant: Alain Sarlette.

In the framework of a collaboration with Francesco Ticozzi (University of Padova) on common points between quantum and classical network dynamics, we developed a general "symmetrization" framework which covers robust ways to generate dynamics in several algorithmic and control contexts [18]. The starting point was the question of generalizing so-called "consensus" algorithms to networks composed of quantum units. In order to define state information exchange without requiring state communication (an impossible feat given the quantum no-cloning theorem), an operational viewpoint on consensus had been proposed by Alain Sarlette and co-authors in the previous year. In this new result, the scope of this operational viewpoint is considerably extended by considering it as a "symmetrization" procedure with respect to some discrete group, completely abstracting away the actual action space. It is shown that this abstraction covers existing procedures ranging from network synchronization to random state generation (not in networks) and averaging-based open-loop control procedures. The interest of viewing those procedures under the common "symmetrization" framework proposed is twofold: convergence proofs follow from a general result that we have established; and robustness to randomized actions and (specific) parameter uncertainties is shown to carry over from the "consensus" literature. It is further anticipated that the approach might be a guideline for new algorithmic designs in the future.

5.7. Accelerating consensus by spectral clustering and polynomial filters

Participant: Alain Sarlette.

The previous work of Alain Sarlette about quantum consensus and symmetrization has been further explored towards quantum-induced accelerations of algorithms, thermalization processes and random walks. This work is still at a preliminary stage. It has been noticed that some non-quantum acceleration possibilities were not fully explored and this has led to two publications that establish preliminary clarifications for our main goal. In [17], a standing conjecture has been proved which claims that if only the spectral gap of a graph is known (i.e. a bound on its lowest and largest eigenvalues), then by adding m local memories to each node no faster convergence can be obtained than by adding $m = 2$ local memories. The conjecture is proved with an analogy to root locus techniques, and a network-centric (e.g. information-theory-based) argument for this fact is currently missing, but at least the fact has been established. This allows for direct comparisons with "quantum random walk" accelerations, which obtain the same speed as $m = 2$ but with a different tweak, that is based among others on more knowledge of the network structure. In this spirit, we have clarified in [16] how classical consensus with time-varying filters can benefit from knowledge of extra bounds on the graph eigenvalue locations (without knowing them exactly, which is the case considered in the existing literature). This work also observes how the speed-up trades off with robustness to network modifications.

5.8. Integral control on Lie groups

Participant: Alain Sarlette.

A big challenge for the long-term control of interacting networks is their robustness to systematic biases. Integral control is a standard way to counter them when a target output can be measured. This method has been originally proposed, and extensively studied, for linear systems. However when the system (output) evolves on a nonlinear state space, the standard "integration" technique cannot be straightforwardly applied. Especially for global motions on spaces like the circle, sphere or (real or complex) rotation groups, the output integration viewpoint becomes problematic. We have hence proposed a new viewpoint on integral control, based on integrating the intended input [19]. For linear state spaces, it is equivalent to the standard definition. For nonlinear state spaces, this viewpoint can be transposed verbatim modulo introduction of a transport map on the tangent bundle, which is almost always present for control design purposes. In particular for systems on Lie groups, which are ubiquitous in robotics and in quantum physics, a full analysis of fully actuated systems has been proposed. The more challenging extension to underactuated systems is underway.

6. Partnerships and Cooperations

6.1. National Initiatives

6.1.1. Towards QUANTIC project-team / PSL structuring project TOCOSUQI*

In the framework of the creation of the QUANTIC project-team, we have continued our going collaboration with the non-Inria members of this future project-team (not yet official members of QUANTIC). Indeed, we have a close collaboration with the experimental physicists Benjamin Huard and Françlis Mallet at ENS and applied mathematician Pierre Rouchon at Mines Paristech. These collaborations include all the subjects introduced in the above research program. In the framework of these collaborations, we have also benefited from a 2-year PSL* funding from september 2013 to August 2015. The funding was, in particular, used for the 6 months visit of Ananda Roy, PhD student at Yale university. The PSL* project TOCOSUQI (Tools of the control of superconducting quantum circuits) aims at developing new system theory tools for preparing, manipulating and protecting non-classical states of a microwave field in the framework of quantum Josephson circuits and circuit quantum electrodynamics, and applying them directly in the experiments.

6.1.2. ANR project EPOQ2

This young researchers ANR project, entitled “Estimation problems for quantum and quantum-like systems” and led by Mazyar Mirrahimi, was run between October 2009 and June 2014. This project had contributed to the development of a system theory approach in quantum engineering, with applications, in particular, within the field of quantum information processing. After important and fruitful collaborations with the physicists at Laboratoire Kastler-Brossel, ENS, our activities turned towards the feedback control of quantum systems taking into account the destructive character of quantum measurements. This later on led to new collaborations with the Physicists at Yale university which will be detailed in the sequel. EPOQ2 was highlighted in the 2013 annual report of Agence Nationale de la Recherche.

6.1.3. ANR project GEARED

This three-year collaborative ANR project, entitled “Reservoir engineering quantum entanglement in the microwave domain” and coordinated by Mazyar Mirrahimi, started on October 2014. The participants of the project are Daniel Esteve and Fabien Portier (Quantronics group, CEA Saclay), François Mallet and Benjamin Huard (Laboratoire Pierre Aigrain, ENS), Nicolas Roch and Olivier Buisson (Institut Neel, Grenoble) and Mazyar Mirrahimi (Inria). This project deals with robust generation of entanglement as a key resource for quantum information processing (quantum simulation, computation and communication). The entangled states are difficult to generate and sustain as interaction with a noisy environment leads to rapid loss of their unique quantum properties. Through Geared we intend to investigate different complementary approaches to master the entanglement of microwave photons coupled to quantum superconducting circuits.

6.2. European Initiatives

6.2.1. Collaborations with Major European Organizations

Partner 1: University of Padova

Alain Sarlette has been pursued a fruitful collaboration with the group of Francesco Ticozzi on "dynamical systems aspects of quantum systems": besides concluding their work on "symmetrization and quantum consensus", mainly initiated before A.S. joined Inria, a novel line of work in the direction of quantum thermalization and quantum random walks has been explored. Further joint work for the future is planned about among others generalized Markovian feedback and weak reservoir engineering.

Partner 2: Ghent University.

A. Sarlette is establishing a collaboration with applied mathematicians interested in quantum control at UGent (Dirk Aeyels and Lode Wylleman) in the framework of thesis co-supervisions. One PhD student is co-supervised with Dirk Aeyels in the framework of Belgian Inter-University Attraction Poles "Dynamical Systems, Control and Optimization" network 2013-2017. A second PhD student is also co-supervised with Dirk Aeyels in the framework of Chinese Scholarship Council and Flanders Research Fund grant "Developing control mechanisms to counter biases and drifts in coordination", 2013-2015. Finally, benefiting from a UGent starting grant on "Coordination control algorithms inspired from nonlinear PDEs and lattices", 2013-2017, Alain Sarlette also supervises a third PhD student at Ghent University.

6.3. International Initiatives

6.3.1. Inria International Partners

6.3.1.1. Declared Inria International Partners

The collaborations with the teams of Michel H. Devoret, Robert J. Schoelkopf, Liang Jiang and Steven M. Girvin, enforced through a two year sabbatical visit of Mazyar Mirrahimi at Yale university, have led to a set of contributions ranging from the theoretical analysis and performance optimization of ongoing experiments on weak quantum measurements [2] and preparation of non-classical field states through single photon Kerr effect [3] to the design of new experiments on single qubit cooling [1] and stabilization of maximally entangled states of superconducting qubits [8] by reservoir engineering techniques. Through these collaborations, Mazyar Mirrahimi and his former PhD student, Zaki Leghtas, currently a postdoc with Michel H. Devoret's group, have introduced a new direction for hardware-efficient universal quantum computation [4], [5]. These theoretical proposals have already led to groundbreaking experiments [10], [9].

6.4. International Research Visitors

6.4.1. Visits of International Scientists

Ananda Roy, Yale university, Department of Applied Physics, PhD student from the groups of A. Douglas Stone and Michel H. Devoret, has visited us for sixth month from February through July 2014.

6.4.2. Visits to International Teams

6.4.2.1. Research stays abroad

Mazyar Mirrahimi spent four months in the Quantronics Laboratory of Michel H. Devoret and in the Rob Schoelkopf Lab at Yale University.

7. Dissemination

7.1. Promoting Scientific Activities

7.1.1. Scientific events selection

7.1.1.1. member of the conference program committee

M. Mirrahimi was a member of the international conference program committee for the PRACQSYS (The Principles and Applications of Control in Quantum Systems) 2014, held at Isaac Newton Institute, Cambridge University, UK.

7.1.2. Journal

7.1.2.1. member of the editorial board

M. Mirrahimi is an associate editor of System and Control Letters.

7.1.2.2. reviewer

A. Sarlette has been a reviewer for several automatic control and dynamical systems journals or conferences.

M. Mirrahimi has been a reviewer for several automatic control and physics journals or conferences.

7.2. Teaching - Supervision - Juries

7.2.1. Teaching

M. Mirrahimi and P. Rouchon gave an introductif 3-hours mini-course on quantum systems at MTNS (Mathematical Theory of Networks and Systems) 2014, Groningen.

M. Mirrahimi gave a 8-hours mini-course on the “Stability and stabilization of partial differential equations” at the Institute for Research in Fundamental Sciences, Tehran, Iran.

7.2.2. Supervision

A. Sarlette is co-supervising 3 PhD students with his former institution UGent. One of them is working on (quantum) network algorithms accelerations and intends to address other quantum control questions in parallel, possibly joining Inria next year.

PhD in progress: Joachim Cohen, “Towards fault-tolerant quantum computation adapted to circuit QED experiments”, Sept 2013-August 2016, Advisor: Mazyar Mirrahimi.

Visiting PhD student: Ananda Roy, “Towards a robust source of Schrödinger cat states”, Feb 2014-July 2014, Advisor: Mazyar Mirrahimi.

7.3. Popularization

The high profile publications in Nature and Science are popularizing the new field of quantum engineering.

A. Sarlette has been giving talks on "reservoir engineering" to the (non-quantum) control community during several seminars.

M. Mirrahimi was an invited plenary speaker at CANUM (Congrès National d’Analyse Numérique) 2014, where he gave a public presentation on quantum feedback and quantum reservoir engineering.

8. Bibliography

Major publications by the team in recent years

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Publications of the year

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