IN PARTNERSHIP WITH:
Ecole Centrale Paris

Activity Report 2014

Project-Team REGULARITY

Probabilistic modelling of irregularity and application to uncertainties management

IN COLLABORATION WITH: Laboratoire de Mathématiques Appliquées aux Systèmes (MAS)

RESEARCH CENTER
Saclay - Île-de-France

THEME
Stochastic approaches

Table of contents

1. Members **1**
2. Overall Objectives **1**
3. Research Program **2**
3.1. Theoretical aspects: probabilistic modeling of irregularity 2
3.2. Tools for characterizing and measuring regularity 3
3.3. Stochastic models 6
4. Application Domains **9**
4.1. Uncertainties management 9
4.2. Risk modelling in finance 11
5. New Software and Platforms **11**
6. New Results **12**
6.1. Highlights of the Year 12
6.2. Modelling the exchange of cultural goods on the Internet 12
6.3. Financial risk analysis 12
6.4. Functional central limit theorem for multistable Lévy motions 13
6.5. Deviation inequalities for martingales with applications 13
6.6. Self-stabilizing Lévy motions 14
7. Bilateral Contracts and Grants with Industry **15**
8. Partnerships and Cooperations **15**
8.1. Regional Initiatives 15
8.2. International Initiatives 15
8.3. International Research Visitors 15
9. Dissemination **16**
9.1. Promoting Scientific Activities 16
9.1.1. Scientific events organisation 16
9.1.2. Journal 16
9.1.2.1. Member of the editorial board 16
9.1.2.2. Reviewer 16
9.2. Teaching - Supervision - Juries 16
9.2.1. Teaching 16
9.2.2. Supervision 16
9.2.3. Juries 16
9.3. Institutional commitment 16
10. Bibliography **16**

Project-Team REGULARITY

Keywords: Stochastic Modelling, Financial Mathematics, Signal And Image Processing, Stochastic Differential Equations

Creation of the Team: 2010 January 01*, updated into Project-Team:* 2012 January 01, end of the Project-Team: 2014 December 31.

1. Members

Research Scientist

Jacques Lévy Véhel [Team leader, Inria, Senior Researcher, HdR]

PhD Student

Benjamin Arras [Ecole Centrale de Paris, until Oct 2014]

Post-Doctoral Fellow

Xiequan Fan [Inria]

Administrative Assistant

Christine Aklouche [Inria]

2. Overall Objectives

2.1. Overall Objectives

Many phenomena of interest are analyzed and controlled through graphs or n-dimensional images. Often, these graphs have an *irregular aspect*, whether the studied phenomenon is of natural or artificial origin. In the first class, one may cite natural landscapes, most biological signals and images (EEG, ECG, MR images, ...), and temperature records. In the second class, prominent examples include financial logs and TCP traces.

Such irregular phenomena are usually not adequately described by purely deterministic models, and a probabilistic ingredient is often added. Stochastic processes allow to take into account, with a firm theoretical basis, the numerous microscopic fluctuations that shape the phenomenon.

In general, it is a wrong view to believe that irregularity appears as an epiphenomenon, that is conveniently dealt with by introducing randomness. In many situations, and in particular in some of the examples mentioned above, irregularity is a core ingredient that cannot be removed without destroying the phenomenon itself. In some cases, irregularity is even a necessary condition for proper functioning. A striking example is that of ECG: an ECG is inherently irregular, and, moreover, in a mathematically precise sense, an *increase* in its regularity is strongly correlated with a *degradation* of its condition.

In fact, in various situations, irregularity is a crucial feature that can be used to assess the behaviour of a given system. For instance, irregularity may the result of two or more sub-systems that act in a concurrent way to achieve some kind of equilibrium. Examples of this abound in nature (*e.g.* the sympathetic and parasympathetic systems in the regulation of the heart). For artifacts, such as financial logs and TCP traffic, irregularity is in a sense an unwanted feature, since it typically makes regulations more complex. It is again, however, a necessary one. For instance, efficiency in financial markets requires a constant flow of information among agents, which manifests itself through permanent fluctuations of the prices: irregularity just reflects the evolution of this information.

The aim of *Regularity* is a to develop a coherent set of methods allowing to model such “essentially irregular” phenomena in view of managing the uncertainties entailed by their irregularity.

Indeed, essential irregularity makes it more to difficult to study phenomena in terms of their description, modeling, prediction and control. It introduces *uncertainties* both in the measurements and the dynamics. It is, for instance, obviously easier to predict the short time behaviour of a smooth (*e.g.* C^1) process than of a nowhere differentiable one. Likewise, sampling rough functions yields less precise information than regular ones. As a consequence, when dealing with essentially irregular phenomena, uncertainties are fundamental in the sense that one cannot hope to remove them by a more careful analysis or a more adequate modeling. The study of such phenomena then requires to develop specific approaches allowing to manage in an efficient way these inherent uncertainties.

3. Research Program

3.1. Theoretical aspects: probabilistic modeling of irregularity

The modeling of essentially irregular phenomena is an important challenge, with an emphasis on understanding the sources and functions of this irregularity. Probabilistic tools are well-adapted to this task, provided one can design stochastic models for which the regularity can be measured and controlled precisely. Two points deserve special attention:

- first, the study of regularity has to be *local*. Indeed, in most applications, one will want to act on a system based on local temporal or spatial information. For instance, detection of arrhythmias in ECG or of krachs in financial markets should be performed in "real time", or, even better, ahead of time. In this sense, regularity is a *local* indicator of the *local* health of a system.
- Second, although we have used the term "irregularity" in a generic and somewhat vague sense, it seems obvious that, in real-world phenomena, regularity comes in many colors, and a rigorous analysis should distinguish between them. As an example, at least two kinds of irregularities are present in financial logs: the local "roughness" of the records, and the local density and height of jumps. These correspond to two different concepts of regularity (in technical terms, Hölder exponents and local index of stability), and they both contribute a different manner to financial risk.

In view of the above, the *Regularity* team focuses on the design of methods that:

1. define and study precisely various relevant measures of local regularity,
2. allow to build stochastic models versatile enough to mimic the rapid variations of the different kinds of regularities observed in real phenomena,
3. allow to estimate as precisely and rapidly as possible these regularities, so as to alert systems in charge of control.

Our aim is to address the three items above through the design of mathematical tools in the field of probability (and, to a lesser extent, statistics), and to apply these tools to uncertainty management as described in the following section. We note here that we do not intend to address the problem of controlling the phenomena based on regularity, that would naturally constitute an item 4 in the list above. Indeed, while we strongly believe that generic tools may be designed to measure and model regularity, and that these tools may be used to analyze real-world applications, in particular in the field of uncertainty management, it is clear that, when it comes to control, application-specific tools are required, that we do not wish to address.

The research topics of the *Regularity* team can be roughly divided into two strongly interacting axes, corresponding to two complementary ways of studying regularity:

1. developments of tools allowing to characterize, measure and estimate various notions of local regularity, with a particular emphasis on the stochastic frame,
2. definition and fine analysis of stochastic models for which some aspects of local regularity may be prescribed.

These two aspects are detailed in sections 3.2 and 3.3 below.

3.2. Tools for characterizing and measuring regularity

Fractional Dimensions

Although the main focus of our team is on characterizing *local* regularity, on occasions, it is interesting to use a *global* index of regularity. Fractional dimensions provide such an index. In particular, the *regularization dimension*, that was defined in [31], is well adapted to the study stochastic processes, as its definition allows to build robust estimators in an easy way. Since its introduction, regularization dimension has been used by various teams worldwide in many different applications including the characterization of certain stochastic processes, statistical estimation, the study of mammographies or galactograms for breast carcinomas detection, ECG analysis for the study of ventricular arrhythmia, encephalitis diagnosis from EEG, human skin analysis, discrimination between the nature of radioactive contaminations, analysis of porous media textures, well-logs data analysis, agro-alimentary image analysis, road profile analysis, remote sensing, mechanical systems assessment, analysis of video games, ...(see http://regularity.saclay.inria.fr/theory/localregularity/biblioregdim for a list of works using the regularization dimension).

Hölder exponents

The simplest and most popular measures of local regularity are the pointwise and local Hölder exponents. For a stochastic process $\{X(t)\}_{t\in\mathbb{R}}$ whose trajectories are continuous and nowhere differentiable, these are defined, at a point t_0, as the random variables:

$$\alpha_X(t_0,\omega) = \sup\left\{\alpha : \limsup_{\rho\to 0}\ \sup_{t,u\in B(t_0,\rho)} \frac{|X_t - X_u|}{\rho^\alpha} < \infty\right\}, \tag{1}$$

and

$$\widetilde{\alpha}_X(t_0,\omega) = \sup\left\{\alpha : \limsup_{\rho\to 0}\ \sup_{t,u\in B(t_0,\rho)} \frac{|X_t - X_u|}{\|t-u\|^\alpha} < \infty\right\}. \tag{2}$$

Although these quantities are in general random, we will omit as is customary the dependency in ω and X and write $\alpha(t_0)$ and $\widetilde{\alpha}(t_0)$ instead of $\alpha_X(t_0,\omega)$ and $\widetilde{\alpha}_X(t_0,\omega)$.

The random functions $t \mapsto \alpha_X(t_0,\omega)$ and $t \mapsto \widetilde{\alpha}_X(t_0,\omega)$ are called respectively the pointwise and local Hölder functions of the process X.

The pointwise Hölder exponent is a very versatile tool, in the sense that the set of pointwise Hölder functions of continuous functions is quite large (it coincides with the set of lower limits of sequences of continuous functions [6]). In this sense, the pointwise exponent is often a more precise tool (*i.e.* it varies in a more rapid way) than the local one, since local Hölder functions are always lower semi-continuous. This is why, in particular, it is the exponent that is used as a basis ingredient in multifractal analysis (see section 3.2). For certain classes of stochastic processes, and most notably Gaussian processes, it has the remarkable property that, at each point, it assumes an almost sure value [18]. SRP, mBm, and processes of this kind (see sections 3.3 and 3.3) rely on the sole use of the pointwise Hölder exponent for prescribing the regularity.

However, α_X obviously does not give a complete description of local regularity, even for continuous processes. It is for instance insensitive to "oscillations", contrarily to the local exponent. A simple example in the deterministic frame is provided by the function $x^\gamma \sin(x^{-\beta})$, where γ,β are positive real numbers. This so-called "chirp function" exhibits two kinds of irregularities: the first one, due to the term x^γ is measured by the pointwise Hölder exponent. Indeed, $\alpha(0)=\gamma$. The second one is due to the wild oscillations around 0, to which α is blind. In contrast, the local Hölder exponent at 0 is equal to $\frac{\gamma}{1+\beta}$, and is thus influenced by the oscillatory behaviour.

Another, related, drawback of the pointwise exponent is that it is not stable under integro-differentiation, which sometimes makes its use complicated in applications. Again, the local exponent provides here a useful complement to α, since $\widetilde{\alpha}$ is stable under integro-differentiation.

Both exponents have proved useful in various applications, ranging from image denoising and segmentation to TCP traffic characterization. Applications require precise estimation of these exponents.

Stochastic 2-microlocal analysis

Neither the pointwise nor the local exponents give a complete characterization of the local regularity, and, although their joint use somewhat improves the situation, it is far from yielding the complete picture.

A fuller description of local regularity is provided by the so-called *2-microlocal analysis*, introduced by J.M. Bony [46]. In this frame, regularity at each point is now specified by two indices, which makes the analysis and estimation tasks more difficult. More precisely, a function f is said to belong to the *2-microlocal space* $C_{x_0}^{s,s'}$, where $s+s'>0, s'<0$, if and only if its $m=[s+s']-$th order derivative exists around x_0, and if there exists $\delta>0$, a polynomial P with degree lower than $[s]-m$, and a constant C, such that

$$\left|\frac{\partial^m f(x)-P(x)}{|x-x_0|^{[s]-m}}-\frac{\partial^m f(y)-P(y)}{|y-x_0|^{[s]-m}}\right| \leq C|x-y|^{s+s'-m}(|x-y|+|x-x_0|)^{-s'-[s]+m}$$

for all x,y such that $0<|x-x_0|<\delta$, $0<|y-x_0|<\delta$. This characterization was obtained in [25], [32]. See [53], [54] for other characterizations and results. These spaces are stable through integro-differentiation, i.e. $f\in C_x^{s,s'}$ if and only if $f'\in C_x^{s-1,s'}$. Knowing to which space f belongs thus allows to predict the evolution of its regularity after derivation, a useful feature if one uses models based on some kind differential equations. A lot of work remains to be done in this area, in order to obtain more general characterizations, to develop robust estimation methods, and to extend the "2-microlocal formalism" : this is a tool allowing to detect which space a function belongs to, from the computation of the Legendre transform of an auxiliary function known as its *2-microlocal spectrum*. This spectrum provide a wealth of information on the local regularity.

In [18], we have laid some foundations for a stochastic version of 2-microlocal analysis. We believe this will provide a fine analysis of the local regularity of random processes in a direction different from the one detailed for instance in [55].We have defined random versions of the 2-microlocal spaces, and given almost sure conditions for continuous processes to belong to such spaces. More precise results have also been obtained for Gaussian processes. A preliminary investigation of the 2-microlocal behaviour of Wiener integrals has been performed.

Multifractal analysis of stochastic processes

A direct use of the local regularity is often fruitful in applications. This is for instance the case in RR analysis or terrain modeling. However, in some situations, it is interesting to supplement or replace it by a more global approach known as *multifractal analysis* (MA). The idea behind MA is to group together all points with same regularity (as measured by the pointwise Hölder exponent) and to measure the "size" of the sets thus obtained [28], [47], [50]. There are mainly two ways to do so, a geometrical and a statistical one.

In the geometrical approach, one defines the *Hausdorff multifractal spectrum* of a process or function X as the function: $\alpha\mapsto f_h(\alpha)=\dim\{t:\alpha_X(t)=\alpha\}$, where $\dim E$ denotes the Hausdorff dimension of the set E. This gives a fine measure-theoretic information, but is often difficult to compute theoretically, and almost impossible to estimate on numerical data.

The statistical path to MA is based on the so-called *large deviation multifractal spectrum*:

$$f_g(\alpha)=\lim_{\varepsilon\to 0}\liminf_{n\to\infty}\frac{\log\ N_n^\varepsilon(\alpha)}{\log\ n},$$

where:

$$N_n^\varepsilon(\alpha)=\#\{k:\alpha-\varepsilon\leq\alpha_n^k\leq\alpha+\varepsilon\},$$

and α_n^k is the "coarse grained exponent" corresponding to the interval $I_n^k = \left[\frac{k}{n}, \frac{k+1}{n}\right]$, *i.e.*:

$$\alpha_n^k = \frac{\log |Y_n^k|}{-\log\ n}.$$

Here, Y_n^k is some quantity that measures the variation of X in the interval I_n^k, such as the increment, the oscillation or a wavelet coefficient.

The large deviation spectrum is typically easier to compute and to estimate than the Hausdorff one. In addition, it often gives more relevant information in applications.

Under very mild conditions (*e.g.* for instance, if the support of f_g is bounded, [27]) the concave envelope of f_g can be computed easily from an auxiliary function, called the *Legendre multifractal spectrum*. To do so, one basically interprets the spectrum f_g as a rate function in a large deviation principle (LDP): define, for $q \in \mathbb{R}$,

$$S_n(q) = \sum_{k=0}^{n-1} |Y_n^k|^q, \tag{3}$$

with the convention $0^q := 0$ for all $q \in \mathbb{R}$. Let:

$$\tau(q) = \liminf_{n\to\infty} \frac{\log S_n(q)}{-\log(n)}.$$

The Legendre multifractal spectrum of X is defined as the Legendre transform τ^* of τ:

$$f_l(\alpha) := \tau^*(\alpha) := \inf_{q\in\mathbb{R}} (q\alpha - \tau(q)).$$

To see the relation between f_g and f_l, define the sequence of random variables $Z_n := \log |Y_n^k|$ where the randomness is through a choice of k uniformly in $\{0, ..., n-1\}$. Consider the corresponding moment generating functions:

$$c_n(q) := -\frac{\log E_n[\exp(qZ_n)]}{\log(n)}$$

where E_n denotes expectation with respect to P_n, the uniform distribution on $\{0, ..., n-1\}$. A version of Gärtner-Ellis theorem ensures that if $\lim c_n(q)$ exists (in which case it equals $1 + \tau(q)$), and is differentiable, then $c^* = f_g - 1$. In this case, one says that the *weak multifractal formalism* holds, *i.e.* $f_g = f_l$. In favorable cases, this also coincides with f_h, a situation referred to as the *strong multifractal formalism*.

Multifractal spectra subsume a lot of information about the distribution of the regularity, that has proved useful in various situations. A most notable example is the strong correlation reported recently in several works between the narrowing of the multifractal spectrum of ECG and certain pathologies of the heart [51], [52]. Let us also mention the multifractality of TCP traffic, that has been both observed experimentally and proved on simplified models of TCP [2], [44].

Another colour in local regularity: jumps

As noted above, apart from Hölder exponents and their generalizations, at least another type of irregularity may sometimes be observed on certain real phenomena: discontinuities, which occur for instance on financial logs and certain biomedical signals. In this frame, it is of interest to supplement Hölder exponents and their extensions with (at least) an additional index that measures the local intensity and size of jumps. This is a topic we intend to pursue in full generality in the near future. So far, we have developed an approach in the particular frame of *multistable processes*. We refer to section 3.3 for more details.

3.3. Stochastic models

The second axis in the theoretical developments of the *Regularity* team aims at defining and studying stochastic processes for which various aspects of the local regularity may be prescribed.

Multifractional Brownian motion

One of the simplest stochastic process for which some kind of control over the Hölder exponents is possible is probably fractional Brownian motion (fBm). This process was defined by Kolmogorov and further studied by Mandelbrot and Van Ness, followed by many authors. The so-called "moving average" definition of fBm reads as follows:

$$Y_t = \int_{-\infty}^{0} \left[(t-u)^{H-\frac{1}{2}} - (-u)^{H-\frac{1}{2}} \right] .\mathbb{W}(du) + \int_{0}^{t} (t-u)^{H-\frac{1}{2}} .\mathbb{W}(du),$$

where $\mathbb{W}$ denotes the real white noise. The parameter H ranges in $(0,1)$, and it governs the pointwise regularity: indeed, almost surely, at each point, both the local and pointwise Hölder exponents are equal to H.

Although varying H yields processes with different regularity, the fact that the exponents are constant along any single path is often a major drawback for the modeling of real world phenomena. For instance, fBm has often been used for the synthesis natural terrains. This is not satisfactory since it yields images lacking crucial features of real mountains, where some parts are smoother than others, due, for instance, to erosion.

It is possible to generalize fBm to obtain a Gaussian process for which the pointwise Hölder exponent may be tuned at each point: the *multifractional Brownian motion (mBm)* is such an extension, obtained by substituting the constant parameter $H \in (0,1)$ with a *regularity function* $H : \mathbb{R}_+ \to (0,1)$.

mBm was introduced independently by two groups of authors: on the one hand, Peltier and Levy-Vehel [29] defined the mBm $\{X_t;\ t \in \mathbb{R}_+\}$ from the moving average definition of the fractional Brownian motion, and set:

$$X_t = \int_{-\infty}^{0} \left[(t-u)^{H(t)-\frac{1}{2}} - (-u)^{H(t)-\frac{1}{2}} \right] .\mathbb{W}(du) + \int_{0}^{t} (t-u)^{H(t)-\frac{1}{2}} .\mathbb{W}(du),$$

On the other hand, Benassi, Jaffard and Roux [45] defined the mBm from the harmonizable representation of the fBm, *i.e.*:

$$X_t = \int_{\mathbb{R}} \frac{e^{it\xi} - 1}{|\xi|^{H(t)+\frac{1}{2}}} .\widehat{\mathbb{W}}(d\xi),$$

where $\widehat{\mathbb{W}}$ denotes the complex white noise.

The Hölder exponents of the mBm are prescribed almost surely: the pointwise Hölder exponent is $\alpha_X(t) = H(t) \wedge \alpha_H(t)$ a.s., and the local Hölder exponent is $\widetilde{\alpha}_X(t) = H(t) \wedge \widetilde{\alpha}_H(t)$ a.s. Consequently, the regularity of the sample paths of the mBm are determined by the function H or by its regularity. The multifractional Brownian motion is our prime example of a stochastic process with prescribed local regularity.

The fact that the local regularity of mBm may be tuned *via* a functional parameter has made it a useful model in various areas such as finance, biomedicine, geophysics, image analysis, ... A large number of studies have been devoted worldwide to its mathematical properties, including in particular its local time. In addition, there is now a rather strong body of work dealing the estimation of its functional parameter, *i.e.* its local regularity. See http://regularity.saclay.inria.fr/theory/stochasticmodels/bibliombm for a partial list of works, applied or theoretical, that deal with mBm.

Self-regulating processes

We have recently introduced another class of stochastic models, inspired by mBm, but where the local regularity, instead of being tuned "exogenously", is a function of the amplitude. In other words, at each point t, the Hölder exponent of the process X verifies almost surely $\alpha_X(t) = g(X(t))$, where g is a fixed deterministic function verifying certain conditions. A process satisfying such an equation is generically termed a *self-regulating process* (SRP). The particular process obtained by adapting adequately mBm is called the self-regulating multifractional process [3]. Another instance is given by modifying the Lévy construction of Brownian motion [4]. The motivation for introducing self-regulating processes is based on the following general fact: in nature, the local regularity of a phenomenon is often related to its amplitude. An intuitive example is provided by natural terrains: in young mountains, regions at higher altitudes are typically more irregular than regions at lower altitudes. We have verified this fact experimentally on several digital elevation models [8]. Other natural phenomena displaying a relation between amplitude and exponent include temperatures records and RR intervals extracted from ECG [9].

To build the SRMP, one starts from a field of fractional Brownian motions $B(t,H)$, where (t,H) span $[0,1] \times [a,b]$ and $0 < a < b < 1$. For each fixed H, $B(t,H)$ is a fractional Brownian motion with exponent H. Denote:

$$\overline{\underline{X}}_{\alpha'}^{\beta'} \quad = \quad \alpha' + (\beta' - \alpha')\,\frac{X - \min_K(X)}{\max_K(X) - \min_K(X)}$$

the affine rescaling between α' and β' of an arbitrary continuous random field over a compact set K. One considers the following (stochastic) operator, defined almost surely:

$$\begin{array}{rcl} \Lambda_{\alpha',\beta'} \;:\; \mathcal{C}\left([0,1],[\alpha,\beta]\right) & \rightarrow & \mathcal{C}\left([0,1],[\alpha,\beta]\right) \\ Z(.) & \mapsto & \overline{\underline{B(.,g\left(Z(.)\right))}}_{\alpha'}^{\beta'} \end{array}$$

where $\alpha \le \alpha' < \beta' \le \beta$, α and β are two real numbers, and α', β' are random variables adequately chosen. One may show that this operator is contractive with respect to the sup-norm. Its unique fixed point is the SRMP. Additional arguments allow to prove that, indeed, the Hölder exponent at each point is almost surely $g(t)$.

An example of a two dimensional SRMP with function $g(x) = 1 - x^2$ is displayed on figure 1.

We believe that SRP open a whole new and very promising area of research.

Multistable processes

Non-continuous phenomena are commonly encountered in real-world applications, *e.g.* financial records or EEG traces. For such processes, the information brought by the Hölder exponent must be supplemented by some measure of the density and size of jumps. Stochastic processes with jumps, and in particular Lévy processes, are currently an active area of research.

The simplest class of non-continuous Lévy processes is maybe the one of stable processes [56]. These are mainly characterized by a parameter $\alpha \in (0,2]$, the *stability index* ($\alpha = 2$ corresponds to the Gaussian case, that we do not consider here). This index measures in some precise sense the intensity of jumps. Paths of stable processes with α close to 2 tend to display "small jumps", while, when α is near 0, their aspect is governed by large ones.

In line with our quest for the characterization and modeling of various notions of local regularity, we have defined *multistable processes*. These are processes which are "locally" stable, but where the stability index α is now a function of time. This allows to model phenomena which, at times, are "almost continuous", and at others display large discontinuities. Such a behaviour is for instance obvious on almost any sufficiently long financial record.

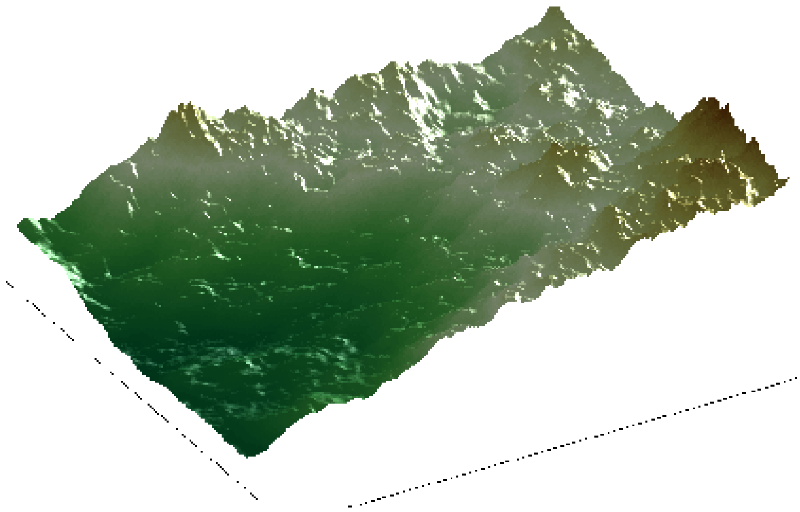

Figure 1. Self-regulating miltifractional process with $g(x) = 1 - x^2$

More formally, a multistable process is a process which is, at each time u, tangent to a stable process [49]. Recall that a process Y is said to be tangent at u to the process Y'_u if:

$$\lim_{r \to 0} \frac{Y(u + rt) - Y(u)}{r^h} = Y'_u(t), \tag{4}$$

where the limit is understood either in finite dimensional distributions or in the stronger sense of distributions. Note Y'_u may and in general will vary with u.

One approach to defining multistable processes is similar to the one developed for constructing mBm [29]: we consider fields of stochastic processes $X(t, u)$, where t is time and u is an independent parameter that controls the variation of α. We then consider a “diagonal” process $Y(t) = X(t, t)$, which will be, under certain conditions, “tangent” at each point t to a process $t \mapsto X(t, u)$.

A particular class of multistable processes, termed “linear multistable multifractional motions” (lmmm) takes the following form [11], [10]. Let $(E, \mathcal{E}, m)$ be a σ-finite measure space, and Π be a Poisson process on $E \times \mathbb{R}$ with mean measure $m \times \mathcal{L}$ ($\mathcal{L}$ denotes the Lebesgue measure). An lmmm is defined as:

$$Y(t) = a(t) \sum_{(\mathsf{X},\mathsf{Y}) \in \Pi} \mathsf{Y}^{<-1/\alpha(t)>} \left(|t - \mathsf{X}|^{h(t) - 1/\alpha(t)} - |\mathsf{X}|^{h(t) - 1/\alpha(t)} \right) \quad (t \in \mathbb{R}). \tag{5}$$

where $x^{<y>} := \text{sign}(x)|x|^y$, $a : \mathbb{R} \to \mathbb{R}^+$ is a C^1 function and $\alpha : \mathbb{R} \to (0, 2)$ and $h : \mathbb{R} \to (0, 1)$ are C^2 functions.

In fact, lmmm are somewhat more general than said above: indeed, the couple (h, α) allows to prescribe at each point, under certain conditions, both the pointwise Hölder exponent and the local intensity of jumps. In this sense, they generalize both the mBm and the linear multifractional stable motion [57]. From a broader perspective, such multistable multifractional processes are expected to provide relevant models for TCP traces, financial logs, EEG and other phenomena displaying time-varying regularity both in terms of Hölder exponents and discontinuity structure.

Figure 2 displays a graph of an lmmm with linearly increasing α and linearly decreasing H. One sees that the path has large jumps at the beginning, and almost no jumps at the end. Conversely, it is smooth (between jumps) at the beginning, but becomes jaggier and jaggier as time evolves.

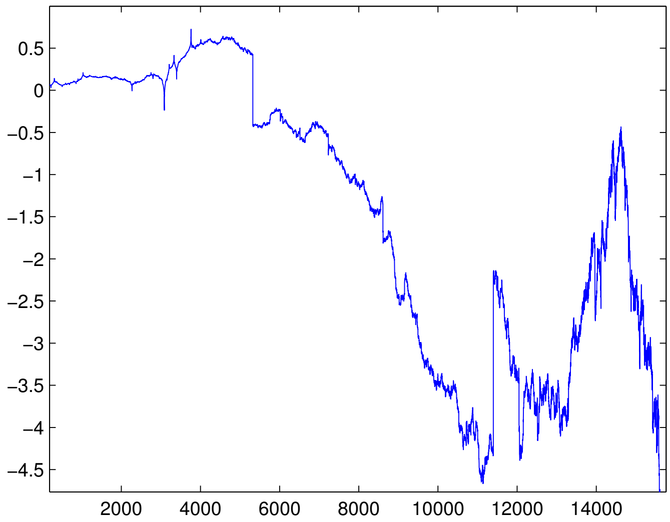

Figure 2. Linear multistable multifractional motion with linearly increasing α and linearly decreasing H

4. Application Domains

4.1. Uncertainties management

Our theoretical works are motivated by and find natural applications to real-world problems in a general frame generally referred to as uncertainty management, that we describe now.

Since a few decades, modeling has gained an increasing part in complex systems design in various fields of industry such as automobile, aeronautics, energy, etc. Industrial design involves several levels of modeling: from behavioural models in preliminary design to finite-elements models aiming at representing sharply physical phenomena. Nowadays, the fundamental challenge of numerical simulation is in designing physical systems while saving the experimentation steps.

As an example, at the early stage of conception in aeronautics, numerical simulation aims at exploring the design parameters space and setting the global variables such that target performances are satisfied. This iterative procedure needs fast multiphysical models. These simplified models are usually calibrated using high-fidelity models or experiments. At each of these levels, modeling requires control of uncertainties due to simplifications of models, numerical errors, data imprecisions, variability of surrounding conditions, etc.

One dilemma in the design by numerical simulation is that many crucial choices are made very early, and thus when uncertainties are maximum, and that these choices have a fundamental impact on the final performances.

Classically, coping with this variability is achieved through *model registration* by experimenting and adding fixed *margins* to the model response. In view of technical and economical performance, it appears judicious to replace these fixed margins by a rigorous analysis and control of risk. This may be achieved through a probabilistic approach to uncertainties, that provides decision criteria adapted to the management of unpredictability inherent to design issues.

From the particular case of aircraft design emerge several general aspects of management of uncertainties in simulation. Probabilistic decision criteria, that translate decision making into mathematical/probabilistic terms, require the following three steps to be considered [48]:

1. build a probabilistic description of the fluctuations of the model's parameters (*Quantification* of uncertainty sources),
2. deduce the implication of these distribution laws on the model's response (*Propagation* of uncertainties),
3. and determine the specific influence of each uncertainty source on the model's response variability (*Sensitivity Analysis*).

The previous analysis now constitutes the framework of a general study of uncertainties. It is used in industrial contexts where uncertainties can be represented by *random variables* (unknown temperature of an external surface, physical quantities of a given material, ... at a given *fixed time*). However, in order for the numerical models to describe with high fidelity a phenomenon, the relevant uncertainties must generally depend on time or space variables. Consequently, one has to tackle the following issues:

- *How to capture the distribution law of time (or space) dependent parameters, without directly accessible data?* The distribution of probability of the continuous time (or space) uncertainty sources must describe the links between variations at neighbor times (or points). The local and global regularity are important parameters of these laws, since it describes how the fluctuations at some time (or point) induce fluctuations at close times (or points). The continuous equations representing the studied phenomena should help *to propose models for the law of the random fields*. Let us notice that interactions between various levels of modeling might also be used to derive distributions of probability at the lowest one.
- The navigation between the various natures of models needs a kind of *metric* which could *mathematically describe the notion of granularity or fineness* of the models. Of course, the local regularity will not be totally absent of this mathematical definition.
- All the various levels of conception, preliminary design or high-fidelity modelling, require *registrations by experimentation* to reduce model errors. This *calibration* issue has been present in this frame since a long time, especially in a deterministic optimization context. The random modeling of uncertainty requires the definition of a systematic approach. The difficulty in this specific context is: statistical estimation with few data and estimation of a function with continuous variables using only discrete setting of values.

Moreover, a multi-physical context must be added to these questions. The complex system design is most often located at the interface between several disciplines. In that case, modeling relies on a coupling between several models for the various phenomena and design becomes a *multidisciplinary optimization* problem. In this uncertainty context, the real challenge turns robust optimization to manage technical and economical risks (risk for non-satisfaction of technical specifications, cost control).

We participate in the uncertainties community through several collaborative research projects. As explained above, we focus on essentially irregular phenomena, for which irregularity is a relevant quantity to capture the variability (e.g. certain biomedical signals, terrain modeling, financial data, etc.). These will be modeled through stochastic processes with prescribed regularity.

4.2. Risk modelling in finance

- A striking feature of many financial logs is that they are both irregular in the Hölder sense and display jumps. Furthermore, the local roughness as well as the size of jumps typically vary in time. This hints that multifractional multistable processes may provide well-adapted models. As a first step, we shall investigate the simple case of multistable Lévy motions and concentrate on understanding how a time-varying α function translates in terms of risk, in particular for VaR computation. This will require both a deeper understanding of the stochastic properties of these processes and a fine analysis of the microstructure of financial logs.
- In another direction, we will study whether multifractional Brownian motion (mBm) and SRP provide useful models in the frame of financial modeling. Fractional Brownian motion-based option pricing and portfolio selection has attracted a lot of interest in recent years. This process is certainly a more adequate model than pure Brownian motion, as many studies have shown. However, it is also clear that it suffers various limitations. One of the most obvious is that the local regularity of financial logs is not constant, as is apparent on any sufficiently long sample. The most direct way of generalizing fractional Brownian motion to account for this fact is to consider mBm, as we have done in [35], using the theory of stochastic calculus with respect to mBm that we have recently developed in [39], [38]. Another possibility is to use SRP. This requires to extend both the theoretical results (mainly those related to stochastic calculus) and their applications (pricing, portfolio selection) beyond the case of fractional Brownian motion. A disadvantage of mBm is that, in order to price for instance, one has to know the regularity function ahead of time, which usually requires additional assumptions, or to build a model for its evolution. This problem is not present for the SRP: no further information is required once the function relating the amplitude and the regularity has been identified. On the other hand, stochastic integration with respect to SRP (which is neither a Gaussian process nor a semi-martingale) does not seem to be within reach at present, since little is known indeed about this process. This nevertheless constitutes one of our long term goals.

5. New Software and Platforms

5.1. FracLab

Participant: Jacques Lévy Véhel [correspondant].

FracLab was developed for two main purposes:

1. propose a general platform allowing research teams to avoid the need to re-code basic and advanced techniques in the processing of signals based on (local) regularity.
2. provide state of the art algorithms allowing both to disseminate new methods in this area and to compare results on a common basis.

FracLab is a general purpose signal and image processing toolbox based on fractal, multifractal and local regularity methods. FracLab can be approached from two different perspectives:

- (multi-) fractal and local regularity analysis: A large number of procedures allow to compute various quantities associated with 1D or 2D signals, such as dimensions, Hölder and 2-microlocal exponents or multifractal spectra.
- Signal/Image processing: Alternatively, one can use FracLab directly to perform many basic tasks in signal processing, including estimation, detection, denoising, modeling, segmentation, classification, and synthesis.

A graphical interface makes FracLab easy to use and intuitive. In addition, various wavelet-related tools are available in FracLab.

FracLab is a free software. It mainly consists of routines developed in MatLab or C-code interfaced with MatLab. It runs under Linux, MacOS and Windows environments. In addition, a “stand-alone” version (*i.e.* which does not require MatLab to run) is available.

Fraclab has been downloaded several thousands of times in the last years by users all around the world. A few dozens laboratories seem to use it regularly, with more than four hundreds registered users. Our ambition is to make it the standard in fractal softwares for signal and image processing applications. We have signs that this is starting to become the case. To date, its use has been acknowledged in roughly three hundreds and fifty research papers in various areas such as astrophysics, chemical engineering, financial modeling, fluid dynamics, internet and road traffic analysis, image and signal processing, geophysics, biomedical applications, computer science, as well as in mathematical studies in analysis and statistics (see http://fraclab.saclay.inria.fr/ for a partial list with papers). In addition, we have opened the development of FracLab so that other teams worldwide may contribute. Additions have been made by groups in Australia, England, France, the USA, and Serbia.

6. New Results

6.1. Highlights of the Year

The article "Christiane’s Hair" by Jacques Lévy-Véhel and Franklin Mendivil has received the Paul R. Halmos - Lester R. Ford award of the Mathematical Association of America.

6.2. Modelling the exchange of cultural goods on the Internet

Participant: Jacques Lévy Véhel.

In collaboration with Pierre Emmanuel Lévy Véhel and Victor Lévy Véhel.

Illegal sharing of cultural goods on the Internet has become a massive reality in today’s connected society. Numerous studies have been performed to try and evaluate the impact of these practices on the industry of cultural goods, and how much harm, if any, they have entailed. The effect of legal and technical responses to limit pirating has also been investigated, showing in general inconclusive effect. Instead of penalizing illegal actors - providers and/or consumers -, a totally different approach has been proposed recently by the french government agency Hadopi. The idea is to offer the possibility to sites that illegally share cultural goods to become legal in exchange of a retribution proportional to their activity. In the frame of a contract with the Hadopi, we have built a model that studies the economic feasibility of such a scheme under various assumptions on the behaviour of the different actors involved. Our main finding is that, supposing that more popular goods are more prone to pirating, a retribution of the order of the increase in benefit per user gained by legalized sites does indeed lead to a win-win situation for both producers/sellers of cultural goods and willing-to-be-legalized sites. This will be the case under two conditions: the proportion of pirates is large enough (which seems largely true) and the increase in the amount of money that forums will make from advertisement when becoming legal is sufficient [43].

An extension of our work is under way, that will consider further actors and refined modelling of the way illegal sharing takes place. Calibration issues will also be investigated more closely.

6.3. Financial risk analysis

Participant: Jacques Lévy Véhel.

Financial regulations have fundamentally changed since the Basel II Accords. Among other evolutions, Basel II and III explicitly impose that computations of capital requirements be model-based. This paradigm shift in risk management has been the source of strong debates among both practitioners and academics, who question whether such model-based regulations are indeed more efficient.

A common feeling in the industry is that regulations will sometimes give a false impression of security: risk manager tend to think that a financial company that would fulfil all the criteria of, say, the Basel III Accords on capital adequacy, is not necessarily on the safe side. This is so mainly because many risks, and most significantly systemic or system-wide risks, are not properly modelled, and also because it is easy to manipulate to some extent various risk measures, such as VaR.

In parallel, a fast growing body of academic research provides various arguments explaining why current regulations are not well fitted to address risk management in an adequate way, and may even, in certain cases, worsen the situation.

We use the term *regulation risk* to describe the fact that, in some situations, prudential rules are themselves the source of a systemic risk. We have shown how a combination of model risk and regulation risk leads to an effect which is exactly the opposite of what the regulator tries to enforce. More precisely, we explain how wrongly assuming a Gaussian dynamics (or, more generally, a left-light-tailed one) when the “true” one is pure jump (or, more generally, left-heavy-tailed), and imposing as a constraint *minimizing* VaR at constant volume results in effect in movements that will *maximize* VaR. This effect is related to the fact that regulations fail to consider that risk is endogenous. In a nutshell, the idea is simply that, by treating jumps in the evolution of prices as exceptional events and essentially ignoring them in model-based VaR computations, one misses an essential dimension of risk, and acts in a way that will in effect favour sudden large movements in the markets and ultimately increase VaR. Our simple setting predicts that VaR constraints result in an *increased* intensity of jumps and a *decrease* in volatility - a fact confirmed experimentally on certain datasets. This is a mathematical translation of the common feeling of practitioners that regulations give a false impression of security characterized by low volatility but increased risk of sudden large movements.

6.4. Functional central limit theorem for multistable Lévy motions

Participants: Xiequan Fan, Jacques Lévy Véhel.

We prove a functional central limit theorem (FCLT) for the independent-increments multistable Lévy motions (MsLM) $L_I(t), t \in [0,1]$, as well as of integrals with respect to these processes, using weighted sums of independent random variables. In particular, we prove that multistable Lévy motions are stochastic Hölder continuous and strongly localisable.

Theorem 0.1 Let $(\alpha_n(u))_n, \alpha(u), u \in [0,1]$, be a class of càdlàg functions ranging in $[a,b] \subset (0,2]$ such that the sequence $(\alpha)_n$ tends to α in the uniform metric. Let $(X(k,n))_{n\in\mathbb{N},\ k=1,\ldots,2^n}$ be a family of independent and symmetric $\alpha_n(\frac{k}{2^n})-$stable random variables with unit scale parameter, i.e., $X(k,n) \sim S_{\alpha_n(\frac{k}{2^n})}(1,0,0)$. Then the sequence of processes

$$L_I^{(n)}(u) = \sum_{k=1}^{\lfloor 2^n u \rfloor} \left(\frac{1}{2^n}\right)^{1/\alpha_n(\frac{k}{2^n})} X(k,n), \qquad u \in [0,1], \tag{6}$$

tends in distribution to $L_I(u)$ in $(D[0,1], d_S)$, where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. In particular, if α satisfies

$$(\alpha(x) - \alpha(x+t)) \ln t \to 0 \tag{7}$$

uniformly for all x as $t \searrow 0$, then $L_I(u)$ is localisable at all times.

We have defined integrals of MsLM, and given criteria for convergence,independence, stochastic Hölder continuity and strong localisability of such integrals.

6.5. Deviation inequalities for martingales with applications

Participant: Xiequan Fan.

In the papers [36], [37] we study some general exponential inequalities for supermartingales. The inequalities improve or generalize many exponential inequalities of Bennett (1962), Freedman (1975), van de Geer (1995), de la Peña (1999) and Pinelis (2006). Moreover, our concentration inequalities also improve some known inequalities for sums of independent random variables. Applications associated with linear regressions, autoregressive processes and branching processes are provided. In particular, an interesting application of de la Peña's inequality to self-normalized deviations is also provided.

We also considered an $\mathcal{X}$-valued Markov chain $X_1, X_2, ..., X_n$ belonging to a class of iterated random functions, which is "one-step contracting" with respect to some distance d on $\mathcal{X}$. If f is any separately Lipschitz function with respect to d, we use a well known decomposition of $S_n = f(X_1, ..., X_n) - \mathbb{E}[f(X_1, ..., X_n)]$ into a sum of martingale differences d_k with respect to the natural filtration $\mathcal{F}_k$. We show that each difference d_k is bounded by a random variable η_k independent of $\mathcal{F}_{k-1}$. Using this very strong property, we obtain a large variety of deviation inequalities for S_n, which are governed by the distribution of the η_k's. Finally, we give an application of these inequalities to the Wasserstein distance between the empirical measure and the invariant distribution of the chain.

6.6. Self-stabilizing Lévy motions

Participants: Xiequan Fan, Jacques Lévy Véhel.

Self-stabilizing processes have the property that the "local intensities of jumps" varies with amplitude. They are good models for, e.g., financial and temperature records.

The main aim of our work is to establish the existence of such processes and to give a simple construction. Formally, one says that a stochastic process $S(t), t \in [0,1]$, is a self-stabilizing process if, for almost surely all $t \in [0,1)$, S is localisable at t with tangent process S'_t an $g(S(t))-$stable process, with respect to the conditional probability measure $\mathbb{P}_{S(t)}$. In other words,

$$\lim_{r \searrow 0} \frac{S(t+ru) - S(t)}{r^{1/g(S(t))}} = S'_t(u), \tag{8}$$

where convergence is in finite dimensional distributions with respect to $\mathbb{P}_{S(t)}$. Heuristically, if $S'_t(u) = L_{g(S(t))}(u)$, equality (8) implies that

$$S(t+ru) - S(t) \approx r^{1/g(S(t))} L_{g(S(t))}(u) = (ru)^{1/g(S(t))} L_{g(S(t))}(1),$$

when r is small. Thus it is natural to define $S(t) = \lim_{n\to\infty} S_n(\frac{\lfloor nt \rfloor}{n})$, where

$$S_n\left(\frac{k+1}{n}\right) - S_n\left(\frac{k}{n}\right) = n^{-1/g(S_n(k/n))} L_{g(S_n(k/n))}(1).$$

This inspiration allows us to build Markov processes that converge to a self-stabilizing process. Note that, when $\alpha(x) \equiv 2$, this is simply Donsker's construction. The main difficult is to prove the weak convergence of S_n. To this aim, we make use of a generalization of the Arzelà-Ascoli theorem.

Definition 0.1 We call the sequence $(f_n(\theta))_{n\geq 1}$ is sub-equicontinuous on $I \subset \mathbb{R}^d$, if for any $\varepsilon > 0$, there exist $\delta > 0$ and a sequence of nonnegative numbers $(\varepsilon_n)_{n\geq 1}$, $\varepsilon_n \to 0$ as $n \to \infty$, such that, for all functions f_n in the sequence,

$$|f_n(\theta_1) - f_n(\theta_2)| \ \leq \ \varepsilon + \varepsilon_n, \qquad \theta_1, \theta_2 \in I, \tag{9}$$

whenever $||\theta_1 - \theta_2|| < \delta$ (if $\varepsilon_n = 0$ for all n, then $(f_n(\theta))_{n\geq 1}$ is just equicontinuous).

The slightly generalized version of the Arzelà-Ascoli theorem reads:

Lemma 0.1 Assume that $(f_n)_{n\geq 1}$ be a sequence of real-valued continuous functions defined on a closed and bounded set $\Pi_{i=1}^{d}[a_i, b_i] \subset \mathbb{R}^d$. If this sequence is uniformly bounded and sub-equicontinuous, then there exists a subsequence $(f_{n_k})_{k\geq 1}$ that converges uniformly.

The following theorem states that self-stabilizing processes do exist.

Theorem 0.2 Let g be a Hölder function defined on $\mathbb{R}$ and ranging in $[a, b] \subset (0, 2]$. There exists a self-stabilizing process $S(t), t \in [0,\ 1]$, that it is tangent at all u to a $g(S(u))-$stable Lévy process under the conditional expectation with respect to $S(u)$. Moreover, the process $S(t), t \in [0,\ 1]$, satisfies, for all $(\theta_j, t_j) \in \mathbb{R} \times [0, 1]$, $j = 1, 2, ..., d$,

$$\mathbb{E}_{S(t_1)}\left[\exp\left\{i\sum_{j=2}^{d}\theta_j\left(S(t_j) - S(t_1)\right) + \int |\sum_{j=2}^{d}\theta_j \mathbf{1}_{[t_1,t_j]}(z)|^{g(S(z))}dz\right\}\right] = 1. \tag{10}$$

We are currently studying the main properties of self-stabilizing processes.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

- The Tandem Project is a consortium involving several industrial companies (e.g. Bull Amesys) and some research laboratories (e.g. CMAP). The aim is to detect landmines from 3D radar images.
- Hadopi contract on the economical feasibility of a way to reduce pirating of cultural goods on the Internet.

8. Partnerships and Cooperations

8.1. Regional Initiatives

Regularity has strong collaborations with Nantes University (Anne Philippe) [40] and Rennes University (Ronan Le Guével) [42].

8.2. International Initiatives

8.2.1. Inria International Partners

8.2.1.1. Informal International Partners

- Regularity collaborates with St Andrews University (Prof. Kenneth Falconer) on the study of multistable processes.
- Regularity collaborates with Acadia University (Prof. Franklin Mendivil) on the study of fractal strings, certain fractals sets, and the study of the regularization dimension.

8.3. International Research Visitors

8.3.1. Visits of International Scientists

Pr. Franklin Mendivil, from Acadia University was invited for one month in the team.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific events organisation

9.1.1.1. General chair, scientific chair

Regularity has organized and hosted a conference in honour of Pr. K. Falconer's 60th birthday in May 2014.

9.1.2. Journal

9.1.2.1. Member of the editorial board

Jacques Lévy Véhel is associate editor of the journal *Fractals*.

9.1.2.2. Reviewer

Xiequan Fan is a reviewer for Mathematical Reviews (AMS). Jacques Lévy Véhel reviewed papers for many journals and conferences.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master: Jacques Lévy Véhel, Wavelets and Fractals, M2, 8h, Ecole Centrale Nantes.

Master: Jacques Lévy Véhel, Wavelets and Fractals, M2, 18h, ESIEA.

9.2.2. Supervision

PhD : Benjamin Arras, Around some selfsimilar processes with stationary increments, Ecole Centrale Paris, December 2014, advisor : J. Lévy Véhel

PhD : Alexandre Richard, Local regularity of some fractional Brownian fields, Ecole Centrale Paris, September 2014, advisor : E. Merzbach

9.2.3. Juries

J. Lévy Véhel has been a member of the juries for recruiting two AER, one AS and one AF at Inria Saclay.

9.3. Institutional commitment

J. Lévy Véhel is a member of the Bureau du Comité des Projets, of the Commission Scientifique, and of the Comité de Centre at Inria Saclay. He is the animator of the Commission de Suivi Doctoral also at Inria Saclay. Finally, he was the head of the jury for the 2014 CR2 positions contest for Inria Saclay.

10. Bibliography

Major publications by the team in recent years

[1] P. BALANÇA. *A increment type set-indexed Markov property*, 2012, http://arxiv.org/abs/1207.6568

[2] J. BARRAL, J. LÉVY VÉHEL. *Multifractal Analysis of a Class of Additive Processes with Correlated Non-Stationary Increments*, in "Electronic Journal of Probability", 2004, vol. 9, pp. 508–543

[3] O. BARRIÈRE, J. LÉVY VÉHEL. *Application of the Self Regulating Multifractional Process to cardiac interbeats intervals*, in "J. Soc. Fr. Stat.", 2009, vol. 150, n^{o} 1, pp. 54–72

[4] O. BARRIÈRE, A. ECHELARD, J. LÉVY VÉHEL. *Self-Regulating Processes*, in "Electronic Journal of Probability", December 2012 [*DOI :* 10.1214/EJP.v17-2010], http://hal.inria.fr/hal-00749742

[5] F. CHALOT, Q. V. DINH, E. HERBIN, L. MARTIN, M. RAVACHOL, G. ROGÉ. *Estimation of the impact of geometrical uncertainties on aerodynamic coefficients using CFD*, in "10th AIAA Non-Deterministic Approaches", Schaumburg, USA, April 2008

[6] K. DAOUDI, J. LÉVY VÉHEL, Y. MEYER. *Construction of continuous functions with prescribed local regularity*, in "Journal of Constructive Approximation", 1998, vol. 014, n^{o} 03, pp. 349–385

[7] Y. DEREMAUX, J. NÉGRIER, N. PIÉTREMONT, E. HERBIN, M. RAVACHOL. *Environmental MDO and uncertainty hybrid approach applied to a supersonic business jet*, in "12th AIAA/ISSMO Multidisciplinary Analysis and Optimization conference", 2008, Victoria

[8] A. ECHELARD, O. BARRIÈRE, J. LÉVY VÉHEL. *Terrain modelling with multifractional Brownian motion and self-regulating processes*, in "ICCVG 2010", Warsaw, Poland, Lecture Notes in Computer Science, Springer, 2010, vol. 6374, pp. 342-351, http://hal.inria.fr/inria-00538907/en

[9] A. ECHELARD, J. LÉVY VÉHEL. *Self-regulating processes-based modeling for arrhythmia characterization*, in "Imaging and Signal Processing in Health Care and Technology", Baltimore, USA, May 2012, http://hal.inria.fr/hal-00670064

[10] K. FALCONER, R. LE GUÉVEL, J. LÉVY VÉHEL. *Localisable moving average stable and multistable processes*, in "Stoch. Models", 2009, vol. 25, pp. 648–672

[11] K. FALCONER, J. LÉVY VÉHEL. *Multifractional, multistable, and other processes with prescribed local form*, in "J. Theoret. Probab.", 2008, vol. 119, pp. 2277–2311, DOI 10.1007/s10959-008-0147-9

[12] L. J. FERMIN, J. LÉVY VÉHEL. *Modeling patient poor compliance in in the multi-IV administration case with Piecewise Deterministic Markov Models*, 2011, preprint

[13] L. J. FERMIN, J. LÉVY VÉHEL. *Variability and singularity arising from poor compliance in a pharmacodynamical model II: the multi-oral case*, 2011, preprint

[14] E. HERBIN, B. ARRAS, G. BARRUEL. *From almost sure local regularity to almost sure Hausdorff dimension for Gaussian fields*, 2010, preprint

[15] E. HERBIN. *From n parameter fractional brownian motions to n parameter multifractional brownian motions*, in "Rocky Mountain Journal of Mathematics", 2006, vol. 36, n^{o} 4, pp. 1249–1284

[16] E. HERBIN, J. JAKUBOWSKI, M. RAVACHOL, Q. V. DINH. *Management of uncertainties at the level of global design*, in "Symposium "Computational Uncertainties", RTO AVT-147", 2007, Athens

[17] E. HERBIN, J. LEBOVITS, J. LÉVY VÉHEL. *Stochastic integration with respect to multifractional Brownian motion via tangent fractional Brownian motion*, in "preprint", 2011

[18] E. HERBIN, J. LÉVY VÉHEL. *Stochastic 2-microlocal analysis*, in "Stochastic Proc. Appl.", 2009, vol. 119, n^{o} 7, pp. 2277–2311, http://arxiv.org/abs/math.PR/0504551

[19] E. HERBIN, E. MERZBACH. *A characterization of the set-indexed fractional Brownian motion*, in "C. R. Acad. Sci. Paris", 2006, vol. Ser. I 343, pp. 767–772

[20] E. HERBIN, E. MERZBACH. *A set-indexed fractional brownian motion*, in "J. of theor. probab.", 2006, vol. 19, n^{o} 2, pp. 337–364

[21] E. HERBIN, E. MERZBACH. *The multiparameter fractional Brownian motion*, in "Math everywhere", Berlin, Springer, 2007, pp. 93–101, http://dx.doi.org/10.1007/978-3-540-44446-6_8

[22] E. HERBIN, E. MERZBACH. *Stationarity and self-similarity characterization of the set-indexed fractional Brownian motion*, in "J. of theor. probab.", 2009, vol. 22, n^{o} 4, pp. 1010–1029

[23] E. HERBIN, E. MERZBACH. *The set-indexed Lévy process: Stationarity, Markov and sample paths properties*, 2010, preprint

[24] E. HERBIN, A. RICHARD. *Hölder regularity for set-indexed processes*, 2011, forthcoming

[25] K. KOLWANKAR, J. LÉVY VÉHEL. *A time domain characterization of the fine local regularity of functions*, in "J. Fourier Anal. Appl.", 2002, vol. 8, n^{o} 4, pp. 319–334

[26] J. LEBOVITS, J. LÉVY VÉHEL. *Stochastic Calculus with respect to multifractional Brownian motion*, http://hal.inria.fr/inria-00580196/en

[27] J. LÉVY VÉHEL, C. TRICOT. *On various multifractal spectra*, in "Fractal Geometry and Stochastics III, Progress in Probability", Birkhäuser, ISBN 376437070X, 9783764370701, 2004, vol. 57, pp. 23-42, C. Bandt, U. Mosco and M. Zähle (Eds), Birkhäuser Verlag

[28] J. LÉVY VÉHEL, R. VOJAK. *Multifractal Analysis of Choquet Capacities: Preliminary Results*, in "Advances in Applied Mathematics", January 1998, vol. 20, pp. 1–43

[29] R. PELTIER, J. LÉVY VÉHEL. *Multifractional Brownian Motion*, Inria, 1995, n^{o} 2645, http://hal.inria.fr/inria-00074045

[30] M. RAVACHOL, Y. DEREMAUX, Q. V. DINH, E. HERBIN. *Uncertainties at the conceptual stage: Multilevel multidisciplinary design and optimization approach*, in "26th International Congress of the Aeronautical Sciences", 2008, Anchorage

[31] F. ROUEFF, J. LÉVY VÉHEL. *A Regularization Approach to Fractional Dimension Estimation*, in "Fractals'98", 1998, Malta

[32] S. SEURET, J. LÉVY VÉHEL. *A time domain characterization of of 2-microlocal Spaces*, in "J. Fourier Anal. Appl.", 2003, vol. 9, n^{o} 5, pp. 472–495

Publications of the year

Doctoral Dissertations and Habilitation Theses

[33] B. ARRAS. *Around some selfsimilar processes with stationary increments*, Ecole Centrale de Paris, December 2014, https://tel.archives-ouvertes.fr/tel-01108624

[34] A. RICHARD. *Local regularity of some fractional Brownian fields*, Ecole Centrale Paris, September 2014, https://tel.archives-ouvertes.fr/tel-01091243

Articles in International Peer-Reviewed Journals

[35] S. CORLAY, J. LEBOVITS, J. LÉVY VÉHEL. *Multifractional Stochastic volatility models*, in "Mathematical Finance", April 2014, vol. 24, n^{o} 2, pp. 364-402, https://hal.inria.fr/hal-00653150

[36] X. FAN, I. GRAMA, Q. LIU. *A generalization of Cramér large deviations for martingales*, in "C. R. Acad. Sci. Paris, Ser. I", September 2014, vol. 352, pp. 853-858, https://hal.archives-ouvertes.fr/hal-01069115

[37] X. FAN, I. GRAMA, Q. LIU. *Exponential inequalities for martingales with applications*, in "Electronic Journal of Probability", January 2015, vol. 20, pp. 1 - 22 [*DOI :* 10.1214/EJP.v20-3496], https://hal.inria.fr/hal-01108032

[38] J. LEBOVITS, J. LÉVY VÉHEL, E. HERBIN. *Stochastic integration with respect to multifractional Brownian motion via tangent fractional Brownian motions*, in "Stochastic Processes and their Applications", 2014, n^{o} 124, pp. 678-708, https://hal.inria.fr/hal-00653808

[39] J. LEBOVITS, J. LÉVY VÉHEL. *White noise-based stochastic calculus with respect to multifractional Brownian motion*, in "Stochastics An International Journal of Probability and Stochastic Processes", 2014, vol. 86, n^{o} 1, pp. 87-124, https://hal.inria.fr/inria-00580196

Other Publications

[40] A. ECHELARD, J. LÉVY VÉHEL, A. PHILIPPE. *Statistical estimation of a class of self-regulating processes*, August 2014, To appear in Scandinavian Journal of Statistics, https://hal.inria.fr/hal-00868604

[41] X. FAN, J. LÉVY VÉHEL. *Multistable Lévy motions and their continuous approximations*, January 2015, https://hal.archives-ouvertes.fr/hal-01110716

[42] R. LE GUÉVEL, J. LÉVY VÉHEL. *Hausdorff, Large Deviation and Legendre Multifractal Spectra of Lévy Multistable Processes*, December 2014, https://hal.inria.fr/hal-01089482

[43] J. LÉVY VÉHEL, P.-E. LÉVY VÉHEL, V. LÉVY VÉHEL. *A Model Investigating Incentives for Illegal Cultural Goods Sharing Sites to Become Legal*, January 2015, https://hal.inria.fr/hal-01103048

References in notes

[44] F. BACCELLI, D. HONG. *AIMD, Fairness and Fractal Scaling of TCP Traffic*, in "INFOCOM'02", June 2002

[45] A. BENASSI, S. JAFFARD, D. ROUX. *Elliptic Gaussian random processes*, in "Rev. Mathemàtica Iberoamericana", 1997, vol. 13, n^{o} 1, pp. 19–90

[46] J. BONY. *Second microlocalization and propagation of singularities for semilinear hyperbolic equations*, in "Conf. on Hyperbolic Equations and Related Topics", 1984, pp. 11–49, Kata/Kyoto,Academic Press, Boston

[47] G. BROWN, G. MICHON, J. PEYRIÈRE. *On the multifractal analysis of measures*, in "J. Statist. Phys.", 1992, vol. 66, n^{o} 3, pp. 775–790

[48] ESREDA. *Uncertainty in Industrial Practice, a Guide to Quantitative Uncertainty Management*, Wiley, 2009

[49] K. FALCONER. *The local structure of random processes*, in "J. London Math. Soc.", 2003, vol. 2, n^{o} 67, pp. 657–672

[50] K. FALCONER. *The multifractal spectrum of statistically self-similar measures*, in "J. Theor. Prob.", 1994, vol. 7, pp. 681–702

[51] A. GOLDBERGER, L. A. N. AMARAL, J. HAUSDORFF, P. IVANOV, C. PENG, H. STANLEY. *Fractal dynamics in physiology: Alterations with disease and aging*, in "PNAS", 2002, vol. 99, pp. 2466–2472

[52] P. IVANOV, L. A. N. AMARAL, A. GOLDBERGER, S. HAVLIN, M. ROSENBLUM, Z. STRUZIK, H. STANLEY. *Multifractality in human heartbeat dynamics*, in "Nature", June 1999, vol. 399

[53] S. JAFFARD. *Pointwise smoothness, two-microlocalization and wavelet coefficients*, in "Publ. Mat.", 1991, vol. 35, n^{o} 1, pp. 155–168

[54] H. KEMPKA. *2-Microlocal Besov and Triebel-Lizorkin Spaces of Variable Integrability*, in "Rev. Mat. Complut.", 2009, vol. 22, n^{o} 1, pp. 227–251

[55] M. B. MARCUS, J. ROSEN. *Markov Processes, Gaussian Processes and Local Times*, Cambridge University Press, 2006

[56] G. SAMORODNITSKY, M. TAQQU. *Stable Non-Gaussian Random Processes*, Chapman and Hall, 1994

[57] S. STOEV, M. TAQQU. *Stochastic properties of the linear multifractional stable motion*, in "Adv. Appl. Probab.", 2004, vol. 36, pp. 1085–1115