IN PARTNERSHIP WITH:
CNRS
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Activity Report 2015

Project-Team ALICE

Geometry and Lighting

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

RESEARCH CENTER
Nancy - Grand Est

THEME
Interaction and visualization

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Creation of the Project-Team: 2006 January 09

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1. Members

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Samuel Hornus [Inria, Researcher]
Sylvain Lefebvre [Inria, Researcher, HdR]
Jonas Martínez Bayona [Inria, Research Position]
Jean-Claude Paul [Inria, Senior Researcher, HdR]
Nicolas Ray [Inria, Researcher]
Atsushi Suzuki [Inria, Research Position, until 05/15]
Rhaleb Zayer [Inria, Researcher, on leave at the Max Planck Institute for Informatics since March 2015]

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2. Overall Objectives

2.1. Overall Objectives

ALICE is a project-team in Computer Graphics. The fundamental aspects of this domain concern the interaction of *light* with the *geometry* of the objects. The lighting problem consists in designing accurate and efficient *numerical simulation* methods for the light transport equation. The geometrical problem consists in developing new solutions to *transform and optimize geometric representations*. Our original approach to both issues is to restate the problems in terms of *numerical optimization*. We try to develop solutions that are *provably correct*, *numerically stable* and *scalable*.

To reach these goals, our approach consists in transforming the physical or geometric problem into a numerical optimization problem, studying the properties of the objective function and designing efficient minimization algorithms. Besides Computer Graphics, our goal is to develop cooperations with researchers and people from the industry, who test applications of our general solutions to various domains, comprising CAD, industrial design, oil exploration, plasma physics... Our solutions are distributed in both open-source software (Graphite, OpenNL, CGAL) and industrial software (Gocad, DVIZ).

Since 2010, we started to develop techniques to model not only virtual objects, but also real ones. Our "modeling and rendering" research axis evolved, and we generalized our results on by-example texture synthesis to the production of real objects, using 3D printers. As compared to virtual objects, this setting defines higher requirements for the geometry processing techniques that we develop, that need to be adapted to both numerical simulation and computer-aided fabrication. We study how to include *computational physics* into the loop, and simulation methods for various phenomena (*e.g.*, fluid dynamics).

3. Research Program

3.1. Introduction

Computer Graphics is a quickly evolving domain of research. These last few years, both acquisition techniques (*e.g.*, range laser scanners) and computer graphics hardware (the so-called GPU's, for Graphics Processing Units) have made considerable advances. However, despite these advances, fundamental problems still remain open. For instance, a scanned mesh composed of hundred million triangles cannot be used directly in real-time visualization or complex numerical simulation. To design efficient solutions for these difficult problems, ALICE studies two fundamental issues in Computer Graphics:

- the representation of the objects, *i.e.*, their geometry and physical properties;
- the interaction between these objects and light.

Historically, these two issues have been studied by independent research communities. However, we think that they share a common theoretical basis. For instance, multi-resolution and wavelets were mathematical tools used by both communities [42]. We develop a new approach, which consists in studying the geometry and lighting from the *numerical analysis* point of view. In our approach, geometry processing and light simulation are systematically restated as a (possibly non-linear and/or constrained) functional optimization problem. This type of formulation leads to algorithms that are more efficient. Our long-term research goal is to find a formulation that permits a unified treatment of geometry and illumination over this geometry.

3.2. Geometry Processing for Engineering

Keywords: Mesh processing, parameterization, splines

Geometry processing recently emerged (in the middle of the 90's) as a promising strategy to solve the geometric modeling problems encountered when manipulating meshes composed of hundred millions of elements. Since a mesh may be considered to be a *sampling* of a surface - in other words a *signal* - the *digital signal processing* formalism was a natural theoretic background for this subdomain (see *e.g.*, [44]). Researchers of this domain then studied different aspects of this formalism applied to geometric modeling.

Although many advances have been made in the geometry processing area, important problems still remain open. Even if shape acquisition and filtering is much easier than 30 years ago, a scanned mesh composed of hundred million triangles cannot be used directly in real-time visualization or complex numerical simulation. For this reason, automatic methods to convert those large meshes into higher level representations are necessary. However, these automatic methods do not exist yet. For instance, the pioneer Henri Gouraud often mentions in his talks that the *data acquisition* problem is still open [32]. Malcolm Sabin, another pioneer of the "Computer Aided Geometric Design" and "Subdivision" approaches, mentioned during several conferences of the domain that constructing the optimum control-mesh of a subdivision surface so as to approximate a given surface is still an open problem [41]. More generally, converting a mesh model into a higher level representation, consisting of a set of equations, is a difficult problem for which no satisfying solutions have been proposed. This is one of the long-term goals of international initiatives, such as the AIMShape European network of excellence.

Motivated by gridding application for finite elements modeling for oil and gas exploration, in the frame of the Gocad project, we started studying geometry processing in the late 90's and contributed to this area at the early stages of its development. We developed the LSCM method (Least Squares Conformal Maps) in cooperation with Alias Wavefront [36]. This method has become the de-facto standard in automatic unwrapping, and was adopted by several 3D modeling packages (including Maya and Blender). We explored various applications of the method, including normal mapping, mesh completion and light simulation [2].

However, classical mesh parameterization requires to partition the considered object into a set of topological disks. For this reason, we designed a new method (Periodic Global Parameterization) that generates a continuous set of coordinates over the object [5]. We also showed the applicability of this method, by proposing the first algorithm that converts a scanned mesh into a Spline surface automatically [4].

We are still not fully satisfied with these results, since the method remains quite complicated. We think that a deeper understanding of the underlying theory is likely to lead to both efficient and simple methods. For this reason, in 2012 we studied several ways of discretizing partial differential equations on meshes, including Finite Element Modeling and Discrete Exterior Calculus. In 2013, we also explored Spectral Geometry Processing and Sampling Theory (more on this below).

3.3. Computer Graphics

Keywords: texture synthesis, shape synthesis, texture mapping, visibility

Content creation is one of the major challenges in Computer Graphics. Modeling shapes and surface appearances which are visually appealing and at the same time enforce precise design constraints is a task only accessible to highly skilled and trained designers.

In this context the team focuses on methods for by-example content creation. Given an input example and a set of constraints, we design algorithms that can automatically generate a new shape (geometry+texture). We formulate the problem of content synthesis as the joint optimization of several objectives: Preserving the local appearance of the example, enforcing global objectives (size, symmetries, mechanical properties), reaching user defined constraints (locally specified geometry, contacts). This results in a wide range of optimization problems, from statistical approaches (Markov Random fields), to combinatorial and linear optimization techniques.

As a complement to the design of techniques for automatic content creation, we also work on the representation of the content, so as to allow for its efficient manipulation. In this context we develop data-structures and algorithms targeted at massively parallel architectures, such as GPUs. These are critical to reach the interactive rates expected from a content creation technique. We also propose novel ways to store and access content defined along surfaces [6] or inside volumes [1] [35].

The team also continues research in core topics of computer graphics at the heart of realistic rendering and realistic light simulation techniques; for example, mapping textures on surfaces, or devising visibility relationships between 3D objects populating space.

4. Application Domains

4.1. Geometric Tools for Simulating Physics with a Computer

Numerical simulation is the main targeted application domain for the geometry processing tools that we develop. Our mesh generation tools are tested and evaluated in the frame of our cooperation with the Gocad consortium, with applications in oil exploration and geomechanics, through co-advised Ph.D. thesis (Arnaud Botella, Julien Renaudeau). We think that the hex-dominant meshes that we generate have geometrical properties that make them suitable for some finite element analyses. We work on evaluating and measuring their impact with simple problems (heat equation, linear elasticity) and then practical applications (unfolding geological layer), with the Ph.D. thesis of Maxence Reberol.

In numerical simulation, developing discrete formulations that satisfy the conservation laws (conservation of mass, conservation of energy, conservation of momentum) is important to ensure that the numerical simulation faithfully reflects the behavior of the physics. There are interesting relations with optimal transport theory, as explained by Benamou and Brenier who developed a numerical algorithm for optimal transport that uses a fluid dynamics formulation [30]. Conversely, some dynamics can be approximated by a series of optimal transport problems, as in the Jordan-Kinderlehrer-Otto scheme [34] and in recent works by Mérigot. We started developing efficient geometric algorithms and optimisation methods that may serve as the basis for implementing these numerical methods in 3D. We started discussions / cooperation projects with Quentin Mérigot (MOKAPLAN project).

4.2. Fabrication

Our work around fabrication and additive manufacturing finds applications in different fields. Our algorithms for fast geometric computations on solids (boolean operations, morphological operations) are useful to model a variety of shapes, from mechanical engineering parts to prosthetics for medical applications.

Our by-example techniques allow for simpler modeling and processing of very intricate geometries and therefore also find applications in art and design, for unusual shapes that would be very difficult to obtain otherwise. Extensions of these techniques also find applications for reproducing naturally occurring micro-structures from a scanned sampled.

5. Highlights of the Year

5.1. Highlights of the Year

Geometry Processing: New Algorithms / New Software

This year we developed a set of geometric algorithms to robustly manipulate 3D data and generate volumetric meshes from them, with a special focus on usability, efficiency and robustness. The pipeline that we developed includes a simple and scalable surface reconstruction algorithms, a compiler for generating C++ code for robust geometric predicates, an efficient implementation of 3D Delaunay triangulation, the first algorithm to compute optimal transport in 3D, and an algorithm to generate hexahedral-dominant meshes.

As a result of the VORPALINE ERC Proof of Concept project, we distribute most of these algorithms in our open-source low-level Geogram library and Graphite graphics user interface. Some algorithms are distributed in the commercial VORPALINE software (hex-dominant meshing), proposed to the sponsors of the GOCAD consortium. The Proof of Concept project made it possible to set up tools for software quality (continuous integration, non-regression testing, systematic Doxygen documentation of all classes/functions/parameters).

Fabrication

This year has seen some important advances regarding the objectives of the ERC ShapeForge, with the publications of two novel techniques for the synthesis of structures from examples [9], [12]. We have proposed to formulate a shape synthesis problem as an appearance synthesis problem under minimal rigidity constraints. This affords for the automatic synthesis of structurally sound objects under specific boundary conditions (attachments and loads), while producing objects that visually resemble an example pattern.

We have continued to include the results of our research into our additive manufacturing software IceSL, which has been augmented with a new user interface to make it more accessible.

This year we also gave a half-day course at ACM SIGGRAPH on fused filament deposition software in collaboration with Makerbot, one of the major manufacturer of consumer level 3D printers. The course is available online at http://webloria.loria.fr/~slefebvr/sig15fdm/.

6. New Software and Platforms

6.1. Vorpaline

Participants: Dobrina Boltcheva, Bruno Lévy.

Vorpaline is a commercial software / programming library. The Vorpaline software takes a new approach to 3D mesh generation, based on the theory of numerical optimization. The optimal mesh generation algorithm, developed as part of the European Research Council GOODSHAPE project, globally and automatically optimizes the mesh elements with respect to geometric constraints. It is the subject of two patents. The mathematical foundations of this algorithm, *i.e.*, the minimization of a smooth energy function, result in practice in a faster algorithm, and–more importantly–in a higher flexibility. For instance, it will allow automatic generation of the aforementioned “hex-dominant” meshes. Vorpaline is based on Geogram (see below). It adds some specialized components targeted to specific industrial usage, such as 3D gridding for the oil and gas industry. It includes our latest research results in automatic meshing. Vorpaline is licensed under a proprietary license.

6.2. IceSL

Participants: Jérémie Dumas, Jean Hergel, Sylvain Lefebvre, Frédéric Claux, Jonas Martínez Bayona, Samuel Hornus.

IceSL exploits parallel algorithms running on the GPU to afford for interactive modeling of objects described by a Constructive Solid Geometry [1] scripting language. This also enables direct slicing for additive manufacturing by considering the printer bed as a screen onto which each object slice has to be drawn as quickly as possible. During display and slicing the CSG model is converted on the fly into an intermediate representation enabling fast processing on the GPU. Slices can be quickly extracted, and the tool path is prepared through image erosion. The interactive preview of the final geometry uses the exact same code path as the slicer, providing an immediate, accurate visual feedback.

IceSL allows practitioners to design and combine complex objects with unprecedented ease. Our latest version can combine meshes as well as analytical primitives (*i.e.*, shapes described by an equation), and outputs printer instructions for filament printers as well as stereolithography printers and laser cutters.

We also augmented IceSL with a modern UI that allows users to immediately visualize changes made to the script, as well as expose a set of parameters to non-expert users who are interested in customizing a model created with IceSL. https://youtu.be/I2y_yZ4VEgk.

IceSL is the recipient software for our ERC research project “ShapeForge”, led by Sylvain Lefebvre and includes several research results from the project.

6.3. Graphite

Participants: Dobrina Boltcheva, Samuel Hornus, Bruno Lévy, Nicolas Ray.

Graphite is an experimental 3D modeler, built in top of the Geogram programming library. It has data structures and efficient OpenGL visualization for pointsets, surfacic meshes (triangles and polygons), volumetric meshes (tetrahedra and hybrid meshes). It has state-of-the-art mesh repair, remeshing, reconstruction algorithms. It also has an interface to the Tetgen tetrahedral mesh generator (by Hang Si). This year, Graphite3 was released. It is a major rewrite, based on Geogram, with increased software quality standards (zero warnings on all platforms, systematic documentation of all classes / all functions / all parameters, dramatically improved performances). It embeds Geogram (and optionally Vorpaline) with an easy-to-use Graphic User Interface. Graphite is licensed under the GPLv3.

6.4. GraphiteLifeExplorer

Participant: Samuel Hornus.

[1] Boolean operations between solids: difference, union, intersection.

GLE is a 3D modeler, developed as a plugin of Graphite, dedicated to molecular biology. It is developed in cooperation with the Fourmentin Guilbert foundation and has recently been renamed "GraphiteLifeExplorer". Biologists need simple spatial modeling tools to help in understanding the role of the relative position of objects in the functioning of the cell. In this context, we develop a tool for easy DNA modeling. The tool generates DNA along any user-given curve, open or closed, allows fine-tuning of atomic positions and, most importantly, exports to PDB (Protein Daba Bank) file format.

The development of GLE is currently on hold, but it is still downloaded (freely) regularly. We plan to add some functionalities in 2016.

6.5. OpenNL - Open Numerical Library

Participants: Bruno Lévy, Nicolas Ray, Rhaleb Zayer.

OpenNL is a standalone library for numerical optimization, especially well-suited to mesh processing. The API is inspired by the graphics API OpenGL, this makes the learning curve easy for computer graphics practitioners. The included demo program implements our LSCM [36] mesh unwrapping method. It was integrated in Blender by Brecht Van Lommel and others to create automatic texture mapping methods. OpenNL is extended with two specialized modules :

- CGAL parameterization package: this software library, developed in cooperation with Pierre Alliez and Laurent Saboret, is a CGAL package for mesh parameterization.
- Concurrent Number Cruncher: this software library extends OpenNL with parallel computing on the GPU, implemented using the CUDA API.

6.6. Geogram

Participant: Bruno Lévy.

Stemming from project GOODSHAPE (ERC Starting Grant) and project VORPALINE (ERC Proof of Concept) Geogram is a programming library of geometric algorithms. It includes a simple yet efficient mesh data structure (for surface and volumetric meshes), exact computer arithmetics (a-la Shewchuck, implemented in GEO::expansion), a predicate code generator (PCK: Predicate Construction Kit), standard geometric predicates (orient/insphere), Delaunay triangulation, Voronoi diagram, spatial search data structures, spatial sorting, and less standard ones (more general geometric predicates, intersection between a Voronoi diagram and a triangular or tetrahedral mesh embedded in n dimensions). The latter is used by FWD/WarpDrive, the first algorithm that computes semi-discrete optimal transport in 3D that scales up to 1 million Dirac masses (see compute_OTM in the example programs). Geogram is licensed under the three-clauses BSD license.

Geogram Pluggable Software Modules: Some users are interested in small subsets of Geogram. Following the principle that Geogram should be as easy to use/compile as possible, some subsets of functionalities are alternatively available as a standalone pair of (header,implementation) files, automatically extracted/assembled from Geogram sourcetree. This makes the functionality usable with 0 dependency: client code that uses a PSM just need to insert the header and the implementation file into the project (rather than linking with the entire Geogram library). The Pluggable Software Modules are licensed under the three-clause BSD license. These Pluggable Software Modules include:

- OpenNL: a library of easy-to-use numerical solvers for sparse matrices,
- MultiPrecision: a number-type that can be used for computations in arbitrary precision, based on Shewchuk's arithmetic expansion [43]. It is shipped with wrapper classes to use it as a number type for CGAL,
- Predicates: implementation of exact and symbolically perturbed predicates with arithmetic filters (based on a combination of Meyer and Pion's arithmetic filter generator [38] and our MultiPrecision library).

6.7. LibSL

Participant: Sylvain Lefebvre.

LibSL is a Simple library for graphics. Sylvain Lefebvre continued development of the LibSL graphics library (under CeCill-C licence, filed at the APP). LibSL is a toolbox for rapid prototyping of computer graphics algorithms, under both OpenGL, DirectX 9/10, Windows and Linux. The library is actively used in both the REVES / Inria Sophia-Antipolis Méditerrannée and the ALICE / Inria Nancy - Grand Est teams.

7. New Results

7.1. Dihedral Angle-Based Maps of Tetrahedral Meshes

Participants: Nicolas Ray, Bruno Lévy.

This work is a collaboration with Gilles-Philippe Paillé (visiting), Pierre Poulin (U. de Montréal) and Alla Sheffer (UBC).

Given a 2D triangulation, it is well known that it is reasonably easy to reconstruct the shape of all the triangles from the sole data of the angles at the triangle corners, provided that they satisfy some constraints. In this project, we studied how this idea can be generalized in the volumetric setting. In other words, we proposed a geometric representation of a tetrahedral mesh that is solely based on dihedral angles, and what are the constraints that these dihedral angles need to satisfy to make that possible. We first show that the shape of a tetrahedral mesh is completely defined by its dihedral angles. This proof leads to a set of angular constraints that must be satisfied for an immersion to exist in $\mathbb{R}^3$. This formulation lets us easily specify conditions to avoid inverted tetrahedra and multiply-covered vertices, thus leading to locally injective maps. We then present a constrained optimization method that modifies input angles when they do not satisfy constraints. Additionally, we develop a fast spectral reconstruction method to robustly recover positions from dihedral angles. We demonstrate the applicability of our representation with examples of volume parameterization, shape interpolation, mesh optimization, connectivity shapes, and mesh compression. This work has been published in Transactions on Graphics [17].

7.2. Hexahedral-dominant Remeshing

Participants: Dmitry Sokolov, Nicolas Ray, Bruno Lévy, Maxence Reberol.

Representing the geometry of complex objects in a computer is usually achieved by a mesh: the object is decomposed in cells that have a simple geometry. Each cell is defined by a set of facets. The simplest choice is to use meshes with tetrahedral cells that are relatively easy to produce and to work with. However, some applications involving numerical simulations better work with hexahedral cells. Such hexahedral meshes are very difficult to produce, even when it is completely done by a designer.

Our objective is to relax the intrinsic difficulties of full hexahedral remeshing by allowing the process to generate a few tetrahedra in the hexahedral mesh (hexahedral-dominant meshes). Our approach is a two steps procedure that defines the desired orientation of the hexahedra with a frame field, then integrates this frame field to generate a deformed 3D grid inside the object. Hexahedra are generated where the grid is non degenerated and not too distorted, and tetrahedra will fill the remaining volume.

7.2.1. Frame Field Generation

A frame field must define the orientation of a cube (the less deformed hexahedron) everywhere inside the object. This object is very difficult to manipulate because it has to be invariant by rotation of 90 degrees around each of its facet normal vector. In [26] we have designed a fast algorithm that is able to define a smooth frame field constrained to be aligned with the object boundary. We represent frames by spherical harmonics (as introduced in [33]) and greatly improve the state of the art thanks to an expression of the boundary constraints that keep the objective function of the optimisation problem very close to quadratic.

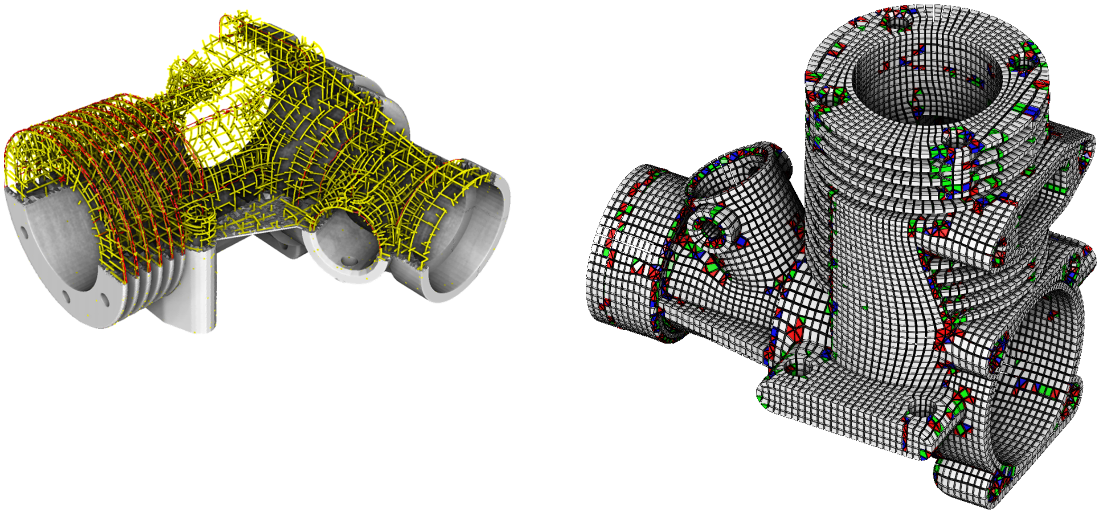

Figure 1. Our Hexahedral dominant meshes generation pipeline have two steps: the generation of a smooth frame field (yellow curves on the left image), and the construction of a mesh with hexahedron that aligns with the frame field.

7.2.2. Generation of Hexahedral-dominant Meshes

The generation of the hex dominant meshes is performed in two steps: place a deformed 3D grid inside the object such that it is aligned with the frame field, then use it to produce hexahedra and fill the rest of the volume with tetrahedra. We developed two solutions for the first step: a 3D extension of PGP [40] and an adaptation of Cubecover [39]. Both solutions have pros and cons so we plan to make them cooperate in a near future. The conversion of this result in an hexahedral-dominant mesh is a very complex problem for which we have a fair solution: we extract a point set from the deformed 3D grid, generate a tetrahedral mesh of the object that is constrained to include the point set in its vertices. From this tetrahedral mesh, we merge sets of tetrahedra into hexahedra with an extension of [37]. We are now working on an alternative solution that will generate hexahedra directly from the deformed 3D grid, and extract the boundary of the rest of the volume as a 2D mesh. From this mesh, we will try to produce more hexahedra by adapting existing combinatorial methods [27].

7.2.3. Impact on FEM Performance

It is admitted by our scientific community that hexahedral meshes are better than tetrahedral meshes for some FEM simulation. We would like to demonstrate evidence of this belief, including fair comparisons with equal running time and/or result accuracy, but the best function basis for each case. For hexahedral dominant meshes, we want to determine if the benefit of using hexahedra deserves having specific function bases devoted to properly link tetrahedral and hexahedral elements. We are developing a new function basis, tailored to non-conformal mixed hexahedra-tetrahedra meshes. Using a combination of tri-linear and quadratic hexahedra, it is possible to construct a space of continuous functions even on a non-conformal mesh. We are now proceeding to analyse the properties of this function space, both theoretically and experimentally. This topic is addressed in the (ongoing) Ph.D. thesis of Maxence Reberol.

7.3. Semi-discrete Optimal Transport in 3D

Participant: Bruno Lévy.

This work introduces a practical algorithm to compute the optimal transport map between a piecewise linear density and a sum of Dirac masses in 3D. In this semi-discrete setting, Aurenhammer *et al.* showed that the optimal transport map is determined by the weights of a power diagram [28]. The optimal weights are computed by minimizing a convex objective function with a quasi-Newton method. To evaluate the value and gradient of this objective function, we propose an efficient and robust algorithm that computes at each iteration the intersection between a power diagram and the tetrahedral mesh that defines the measure. Like in the multilevel proposed by Mérigot, we use a hierarchical algorithm, that uses nested point sets to discretize the source measure.
We think this work may lead to interesting discretizations of the physics, that include the conservation laws (conservation of energy, conservation of momentum ...) deep in their definition, as explained by Jean-David Benamou and Yann Brenier in their fluid dynamics formulation of optimal transport [30].

This work was published in the journal *Mathematical Modeling and Analysis* [10].

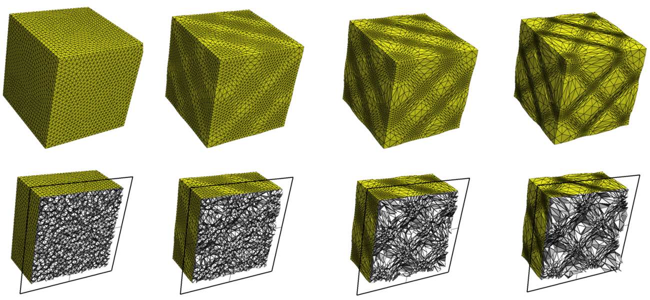

Figure 2. Semi-discrete optimal transport from a constant density to a varying one (product of sines).

7.4. By-example Synthesis of Structurally Sound Shapes

Participants: Jonas Martínez Bayona, Jérémie Dumas, Sylvain Lefebvre.

This is a collaboration with Li-Yi Wei (HKU) on a first project, An Lu (Inria/TU. Munich), Jun Wu (TU. Munich) and Christian Dick (TU. Munich) on a second project.

This work is at the heart of the ERC ShapeForge and considers the by-example synthesis of shapes under structural constraints. We considered two views of the problem that lead to different methodologies.

In a first approach, our goal is to cover a surface with a pattern – an operation akin to texturing in Computer Graphics. The pattern is however used to define the final shape, by determining which parts of the surface are solid or empty. The method operates on a thin voxel shell and does not require any parametrization of the input surface. The pattern is synthesized using a novel formulation for by-example pattern synthesis along surfaces. It is analyzed for structural weaknesses and this information is fed back to the pattern synthesizer, so that seamless reinforcements are added to the structure. We collaborated with researchers from T.U. Munich to analyze the structural behaviour of our structures, and developed a fast evaluation scheme that can be used within our optimization loop to guarantee structural soundness of the resulting design. The work was published in ACM Transactions on Graphics in 2015 [9] (proceedings of SIGGRAPH 2015).

In a second approach we considered the synthesis of shapes that are as rigid as possible under specific boundary conditions and using a prescribed amount of material, while resembling a given input example pattern, as illustrated in Figure 3. Our method is inspired by the field of topology optimization, where rigid shapes are optimized but without any appearance constraints. Our algorithm generates shapes that resemble the input exemplar while being within a user specified percentage of the most rigid shape obtained without the appearance objective. The work was published in ACM Transactions on Graphics in 2015 [12] (proceedings of SIGGRAPH Asia 2015).

Figure 3. Left. A chair automatically synthesized from a load scenario and an example pattern. The rigidity of the chair is within controlled bounds of a shape optimized without appearance objective. Right. A table design automatically synthesized.

7.5. Modeling for Fabrication

We pursued our research regarding automatic modeling techniques for fabrication, where an algorithm takes into account fabrication constraints to simplify the modeling process. This year we have worked on three projects in this area: the modeling of mechanisms from incomplete 2D definitions, the modeling of self-supporting tight enclosures to assist the fabrication process, and the interactive sculpting of support-free objects.

7.5.1. 3D Fabrication of 2D Mechanisms

Participants: Jean Hergel, Sylvain Lefebvre.

This project considered the automatic modeling of 3D mechanisms from an under-specified 2D model of the mechanism. Our approach casts the synthesis problem as an edge orientation problem in a graph, where graph nodes represent parts of the mechanisms and edges capture their interactions as analyzed by the 2D simulation of the mechanism. The edge orientation determines which parts include which others. Once all inclusions have been determined, we formulate a CSP to solve for the layering problem: each part is assigned a depth 'layer' in 3D. We finally compute the final geometry through CSG (boolean combinations of shapes). This work has been published in Computer Graphics Forum (proceedings of Eurographics 2015) [8]. It received an honorable mention from the best paper committee.

7.5.2. Self-supporting Tight Enclosures

Participants: Samuel Hornus, Sylvain Lefebvre, Frédéric Claux, Jérémie Dumas.

The aim of this project was to develop a technique to automatically generated a tight enclosure in the free space around an object. The challenge was to ensure that the enclosure stays close to the object and be as thin as possible while still being printable without collapsing. Such an enclosure finds at least two important applications : 1. as a protective skin to avoid artifacts when 3D-printing a multi-material object. 2. for generating as-large-as-possible cavities inside the printed object in order to minimized material usage and print time. The work is available as an Inria technical report [22].

7.5.3. Interactive Sculpting of Support-free Objects

Participants: Tim-Christopher Reiner, Sylvain Lefebvre.

Tim Reiner, former PhD student at the Karlsruhe Institute of Technology, joined the team on a Post-Doc position to explore new ideas in the context of modeling, rendering, and fabrication. Starting in March 2015, he developed a voxel-based environment for interactive modeling. In a research project together with Sylvain Lefebvre, our team has derived novel techniques for sculpting support-free 3D shapes. These shapes have the property that they do not require support structures during fabrication on fused deposition modeling or resin-based printers. This work is currently under review.

7.6. Intersection Detection via Gauss Maps; a Review and New Techniques

Participant: Samuel Hornus.

We have revisited the problem of deciding whether two convex objects intersect or not. A systematic view of the problem for polyhedra led us to a unified view of several techniques developed in the computer graphics community and to a new and very fast technique specialized to pairs of tetrahedra. A novel view of the problem as a minimization problem over the 2-sphere led us to the description of new interesting links between the set of planes separating two objects and the silhouette edges of their Minkowski difference. From there, we devised a new algorithm for separating two arbitrary convex objects that is a little bit faster and much more robust than the state of the art technique of Gilbert, Johnson and Keerthi [31]. The work has been summarized in [21].

7.7. Fractal Geometry

Participant: Dmitry Sokolov.

This is a collaboration with Christian Gentil (LE2I), Gilles Gouatay (LSIS), Anton Mishkinis (LE2I).

Additive manufacturing enables for the first time the physical realization of objects having complex geometries. Good approximations of fractals, in particular, can now be manufactured in a variety of materials, including metals. The application domains of fabricated fractal geometries are vast, from the be design of “fractal” micro-strip antennas, to the creation of meta-materials.

The main challenge with traditional fractals is the control of the resulting geometry. For example, it is quite challenging to get the desired shape using the system of fractal homeomorphisms proposed by Barnsley [29]. We elaborate here a new type of modeling system, using the facilities of existing CAGD software, while extending their capabilities and their application areas. This new type of modeling system will offer designers (engineers in industry) and creators (visual artists, stylists, designers, architects, etc.) new opportunities to design and produce a quick mock-up, a prototype or a single object. Our approach is to expand the possibilities of a standard CAD system by including fractal shapes while preserving ease of use for end users.

This year we published two papers on the subject [20], [16].

8. Partnerships and Cooperations

8.1. Regional Initiatives

8.1.1. CPER (2014-2020)

50 k€. Sylvain Lefebvre coordinates a work package for the CPER 2014-2020. It involves several members of ALICE as well as laboratories within the Nancy area (Institut Jean Lamour, LRGP, ERPI). Our goal is to consider the interaction between software and material in the additive manufacturing process, with a focus on filament-based printers.

8.1.2. PIC (2015-2017)

150 k€. The PIC project (Polymères Innovants Composites) is a collaboration between Inria, Institut Jean Lamour and Ateliers Cini, funded by Région Lorraine. The goal is to develop a new additive manufaturing process using filament of composite materials with applications in mechanical engineering and the medical domain. Our goal in the project is to provide novel ways to deposit the filament that is better suited to the considered materials and improves the quality of the final parts.

8.2. National Initiatives

8.2.1. ANR BECASIM (2013 – 2016)

890 k€. X. Antoine heads the second partner, which includes Bruno Lévy. Budget for Nancy: 170 k€ of which 100 k€ are for IECL (team CORIDA). This project is managed by Inria. Becasim is a thematic "Numerical Models" ANR project granted by the French Agence Nationale de la Recherche for years 2013-2016. The acronym Becasim is related to Bose-Einstein Condensates: Advanced SIMulation Deterministic and Stochastic Computational Models, HPC Implementation, Simulation of Experiments. The members of the ANR Project Becasim belong to 10 different laboratories.

8.3. European Initiatives

8.3.1. FP7 & H2020 Projects

The SHAPEFORGE project (ERC Starting Grant, FP7, 2012–2017) aims at developing new methods for creating objects from examples, with 3D printers. The main challenge with this project is combining approaches that are very different in nature: algorithms from computer graphics which are used to build forms and textures using examples are combined with digital optimization methods which make sure that the real object complies with the function it is assigned. Thus, to produce a Louis XV bench, on the basis of a Louis XV chair, you need to not only capture the appearance of the example but also formalize the characteristics of a bench as well as its mechanical properties to ensure that it is solid enough. You then need to find, from among all the shapes that can be produced from a single example, the one that best complies with the various criteria. The project is led by Sylvain Lefebvre.

8.4. International Initiatives

8.4.1. Inria International Partners

8.4.1.1. Informal International Partners

We have continued our informal collaboration with Wenping Wang and Li-Yi Wei from Hong Kong University, both on geometry processing and by-example techniques. We published two joint papers this year [7], [12].

Bruno Lévy and Nicolas Ray collaborated with Gilles-Philippe Paillé, Pierre Poulin (U. Montréal, Canada) and Alla Sheffer (UBC). The result of this collaboration was published in Transactions on Graphics [17].

We have on-going collaborations with Marc Alexa (TU Berlin) regarding slicing algorithms for additive manufacturing and Niloy Mitra (University College London) on minimal wastage design of furniture.

8.5. International Research Visitors

8.5.1. Visits of International Scientists

Connelly Barnes visited our team for four weeks in June 2015. We initiated a collaboration on 3D printing, which is ongoing.

Gilles-Philippe Paillé (U. Montréal, Canada) visited us (2 months) to develop "dihedral angle-based maps")

Wenping Wang (Hong-Kong U.) visited us (2 days) to discuss/launch new cooperation projects on Voronoi diagrams and on 3D printing.

8.5.1.1. Internships

Bolun Zhang is an undergraduate student from the mechanical engineering department of Hong Kong University. He visited us for three months as a summer intern, and worked on FEM simulation of infilling patterns within 3D printed parts. He was co-supervised by Jonas Martínez Bayona and Sylvain Lefebvre.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Selection

9.1.1.1. Member of Conference Program Committees

Bruno Lévy was a member of the conference committees of SGP2015 and ACM SIGGRAPH 2015.

Sylvain Lefebvre was a member of the technical papers committee of SIGGRAPH 2015.

9.1.1.2. Reviewer

Bruno Lévy was a reviewer for Eurographics.

Sylvain Lefebvre was a reviewer for SIGGRAPH Asia and Eurographics.

Nicolas Ray was a reviewer for SIGGRAPH, SIGGRAPH Asia and Eurographics.

Jérémie Dumas was a reviewer for UIST.

Laurent Alonso was a reviewer for LATIN 2016.

Samuel Hornus was a reviewer for Eurographics.

9.1.2. Journal

9.1.2.1. Member of Editorial Boards

Sylvain Lefebvre is associate editor for the journal ACM Transactions on Graphics.

Bruno Lévy is a member of the editorial boards of Graphical Models J. and the Visual Computer J.

9.1.2.2. Reviewer - Reviewing Activities

Sylvain Lefebvre reviewed articles for ACM Transactions on Graphics.

Bruno Lévy was a reviewer for ACM Transactions on Graphics, SIAM journal on scientific computing, IEEE TVCG, Computer Graphics Forum.

Jérémie Dumas was a reviewer for IEEE TVCG.

Samuel Hornus was a reviewer for IEEE TVCG, ACM Transactions on Graphics, REFIG.

Laurent Alonso reviewed articles for Discrete Applied Mathematics, Graphs and Combinatorics.

9.1.3. Invited Talks

- Bruno Lévy, 11/24/2015, Seminaire Equations aux Dérivées Partielles de l'IRMA - Strasbourg, invited talk
- Bruno Lévy, 19/10/2015: Intl. Conference on 3D Vision (3DV), invited tutorial
- Bruno Lévy, 07/10/2015: Journées Informatique et Géométrie (JIG2015), invited talk
- Bruno Lévy, 06/29/2015: Journée de l'ANR Geometrya - Summer School Geometric Measure Theory at Institut Fourier - invited talk, video available.
- Bruno Lévy, 06/24-26/2015: Shape Modeling International - keynote talk
- Bruno Lévy, 03/6/2015: New trends in Optimal Transport - Computational Optimal Transport and applications - Bonn, Germany
- Bruno Lévy, 02/15/2015-02/20/2015: BIRS workshop, Advances in Numerical Optimal Transportation - Banff, Canada. Video available.
- Bruno Lévy, 01/23/2015: seminar at the Jacques Louis Lions lab on Optimal Transport. Slides available.
- Jonas Martínez Bayona, gave a talk in CNR Pisa about offset surfaces for fabrication while he was visiting the Visual Computing Lab from 03/03/2015 to 06/03/2015.
- Jonas Martínez Bayona, 29/09/2015, seminar at UPC (Barcelona) to introduce IceSL and the research results it includes.
- Sylvain Lefebvre, 06/03/2015, seminar at Collège de France entitled *Balancing and supporting shapes for 3D printing*, video available.
- Sylvain Lefebvre, 01/07/2015, keynote talk at CEIG 2015 (Spain).

9.1.4. Research Administration

Bruno Lévy was Vice-president of the Inria Chargé de Recherche hiring commitee in Centre Nancy - Grand Est.

Samuel Hornus is a member of the CDT (Commission du Développement Technologique) of Inria Nancy - Grand Est. He is the « correspondant RAweb » to organize the writing of the activity reports of the Inria Nancy research teams.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

9.2.1.1. Master

Bruno Lévy, Pépites Algorithmiques, 20h ETD, École des Mines de Nancy, France

Bruno Lévy, Analyse Numérique, 20h ETD, 2ème année école d'Ingénieurs en Géologie de Nancy, France

Dmitry Sokolov, Infographie, 30h ETD, Université de Lorraine, France

Dmitry Sokolov, Algorithmique et programmation, 45h ETD, Université de Lorraine, France

Dmitry Sokolov, Algorithmique et Programmation Avancée, 60h ETD, Université de Lorraine, France

Dmitry Sokolov, Optimisation Combinatoire, 30h ETD, Université de Lorraine, France

Sylvain Lefebvre, Programmation OpenCL, 20h ETD, Université de Lorraine, France.

Sylvain Lefebvre, Programmation pour le jeux vidéo, 30h ETD, École des Mines de Nancy, France.

Sylvain Lefebvre, Introduction au parallélisme et au graphique, 9h ETD, ENSG Nancy, France.

Sylvain Lefebvre, Introduction à la fabrication additive, 9h ETD, ENSEM Nancy, France.

9.2.1.2. Licence

Dobrina Boltcheva, Image et son numériques, 100h ETD, IUT Saint-Dié-des-Vosges, France

Dobrina Boltcheva, 2A DUT INFO, Advanced programming, 100h ETD, IUT Saint-Dié-des-Vosges, France

Dobrina Boltcheva, 1A DUT INFO, Programming, 50h ETD, IUT Saint-Dié-des-Vosge, France

Dmitry Sokolov, Informatique graphique, 55h ETD, Université de Lorraine, France

Dmitry Sokolov, Programmation Avancée, 40h ETD, Université de Lorraine, France

Samuel Hornus, Mathématique pour l'informatique, 32h ETD, Télécom-Nancy, France

Samuel Hornus, Programmation Fonctionnelle en OCaml, 32h ETD, Épitech Nancy, France.

9.2.2. Internships

Tristan Ezequel, Université Jean Monnet (St Etienne), *méthodes d'apprentissage pour l'aide à la sélection de paramètres de modèles 3D*, March-July 2015, supervised by Sylvain Lefebvre and Amaury Habrard (Université Jean Monnet).

Charles Helbing, ENSEM (Nancy), *dépot de filament suivant une surface courbe*, March-August 2015, supervised by Dmitry Sokolov, Nicolas Ray and Sylvain Lefebvre.

Guillaume Devoille, Epitech, *OpenSCAD plugin for IceSL*, March-July 2015, supervised by Frédéric Claux.

Bolun Zhang, Hong Kong University, *Using the finite element method to determine infill parameters*, June-August 2015, supervised by Jonas Martínez Bayona and Sylvain Lefebvre.

Semigambi Andy-Saleh and Ouhdadi Yassine, Lorraine University, *Modeling and analysis of preferences in multicritera decisions*, Mars-June 2015, supervised by Dmitry Sokolov.

Thomas Pagelot and Thibaut Humbert, Lorraine University, *Measuring filament diameter on the fly*, Mars-June 2015, supervised by Dmitry Sokolov.

9.2.3. Supervision

PhD in progress : Maxence Reberol, *Finite elements for non-conformal mixed hexahedral-tetrahedral meshes*, Janvier 2015, co-directed by Bruno Lévy and Sylvain Lefebvre

PhD in progress : Jean Hergel, *Modeling under Fabrication Constraints*, Oct. 2013, directed by Sylvain Lefebvre

PhD in progress : Jérémie Dumas, *By-example shape synthesis for Fabrication*, Sept. 2013, directed by Sylvain Lefebvre

PhD in progress : Arnaud Botella, *Génération de maillages hex-dominants pour la simulation numérique appliquée à la géologie* , Oct. 2012, co-directed by Guillaume Caumon and Bruno Lévy

Julien Renaudeau, *Méthode de l'élément frontière et ses applications en géomécanique*, Nov. 2015, co-directed by Guillaume Caumon and Bruno Lévy

PhD : Kun Liu, *Multi-view oriented 3D data processing*, Université de Lorraine, defended on December 14, 2015, co-directed by Bruno Lévy and Rhaleb Zayer.

9.2.4. Juries

Bruno Lévy participated to the following Ph.D. and habilitation juries:

- Committee member for the Habilitation Thesis of Boris Thibert (LJK Grenoble)
- Committee member for the Habilitation Thesis of Franck Hetroy (Inria Grenoble)
- Committee member for the Ph.D. defense of Norbert Bus (ESIEE, Paris)

9.3. Popularization

Bruno Lévy gave a talk in front of the general public in the context of the conference cycle *Sciences et Société*, entitled *Simuler la physique avec un ordinateur* on 12/10/2015.

Sylvain Lefebvre demonstrated IceSL at the Science Festival using a custom designed 3D printer. He jointly organized with the Nancy Makerspace (fablab) an event to present IceSL to the fablab users on October 22, 2015 http://nybi.cc/2015/10/workshop-icesl/.

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Publications of the year

Articles in International Peer-Reviewed Journals

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