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Université Nice - Sophia Antipolis

# Activity Report 2015

# **Project-Team CASTOR**

# Control, Analysis and Simulations for TOkamak Research

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

RESEARCH CENTER Sophia Antipolis - Méditerranée

THEME Earth, Environmental and Energy Sciences

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#### **Project-Team CASTOR**

*Creation of the Team: 2012 July 01, updated into Project-Team: 2014 July 01* **Keywords:** 

#### **Computer Science and Digital Science:**

- 6. Modeling, simulation and control
- 6.1. Mathematical Modeling
- 6.1.1. Continuous Modeling (PDE, ODE)
- 6.1.4. Multiscale modeling
- 6.1.5. Multiphysics modeling
- 6.2. Scientific Computing, Numerical Analysis & Optimization
- 6.2.6. Optimization
- 6.2.7. High performance computing
- 6.2.8. Computational geometry and meshes
- 6.3. Computation-data interaction
- 6.3.1. Inverse problems
- 6.3.2. Data assimilation
- 6.3.4. Model reduction
- 6.4. Automatic control
- 6.4.1. Deterministic control
- 6.4.4. Stability and Stabilization

#### **Other Research Topics and Application Domains:**

- 4. Energy
- 4.1.3. Fusion

## 1. Members

#### **Research Scientists**

Herve Guillard [Inria, Senior Researcher, HdR] Holger Heumann [Inria, Starting Research position] Sebastian Minjeaud [CNRS, Researcher] Richard Pasquetti [CNRS, Senior Researcher, HdR]

#### **Faculty Members**

Jacques Blum [Team leader, Univ. Nice, Professor] Cedric Boulbe [Univ. Nice, Associate Professor] Boniface Nkonga [Univ. Nice, Professor] Francesca Rapetti [Univ. Nice, Associate Professor, HdR] Afeintou Sangam [Univ. Nice, Associate Professor]

#### Engineers

Blaise Faugeras [CNRS, HdR] Ayoub Iaagoubi [Inria, from Dec 2015]

#### **PhD Students**

Pierre Cargemel [Univ. Nice, until May 2015] Jose Costa [Univ. Nice] Elise Estibals [Inria]

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#### Others

Didier Auroux [Univ. Nice, Professor, HdR] Marco Bilanceri [CNRS, until Apr 2015] Jeaniffer Lissette Vides Higueros [until Oct 2015]

## 2. Overall Objectives

#### 2.1. Presentation

In order to fulfill the increasing demand, alternative energy sources have to be developed. Indeed, the current rate of fossil fuel usage and its serious adverse environmental impacts (pollution, greenhouse gas emissions, ...) lead to an energy crisis accompanied by potentially disastrous global climate changes.

Controlled fusion power is one of the most promising alternatives to the use of fossil resources, potentially with a unlimited source of fuel. France with the ITER (http://www.iter.org/default.aspx) and Laser Megajoule (http://www-lmj.cea.fr/) facilities is strongly involved in the development of these two parallel approaches to master fusion that are magnetic and inertial confinement. Although the principles of fusion reaction are well understood from nearly sixty years, (the design of tokamak dates back from studies done in the '50 by Igor Tamm and Andreï Sakharov in the former Soviet Union), the route to an industrial reactor is still long and the application of controlled fusion for energy production is beyond our present knowledge of related physical processes. In magnetic confinement, beside technological constraints involving for instance the design of plasma-facing component, one of the main difficulties in the building of a controlled fusion reactor is the poor confinement time reached so far. This confinement time is actually governed by turbulent transport that therefore determines the performance of fusion plasmas. The prediction of the level of turbulent transport in large machines such as ITER is therefore of paramount importance for the success of the researches on controlled magnetic fusion.

The other route for fusion plasma is inertial confinement. In this latter case, large scale hydrodynamical instabilities prevent a sufficient large energy deposit and lower the return of the target. Therefore, for both magnetic and inertial confinement technologies, the success of the projects is deeply linked to the theoretical understanding of plasma turbulence and flow instabilities as well as to mathematical and numerical improvements enabling the development of predictive simulation tools.

CASTOR gathers the activities in numerical simulation of fusion plasmas with the activities in control and optimisation done in the laboratory Jean-Alexandre Dieudonné of the University of Nice. The main objective of the CASTOR team is to contribute to the development of innovative numerical tools to improve the computer simulations of complex turbulent or unstable flows in plasma physics and to develop methods allowing the real-time control of these flows or the optimisation of scenarios of plasma discharges in tokamaks. CASTOR is a common project between Inria (http://www.inria.fr/centre/sophia) and the University of Nice Sophia-Antipolis and CNRS through the laboratory Jean-Alexandre Dieudonné, UMR UNS-CNRS 7351, (http://math.unice.fr).

## 3. Research Program

#### 3.1. Plasma Physics

**Participants:** Jacques Blum, Cédric Boulbe, Blaise Faugeras, Hervé Guillard, Holger Heumann, Sebastian Minjeaud, Boniface Nkonga, Richard Pasquetti, Afeintou Sangam.

The main reseach topics are:

- 1. Modelling and analysis
  - Fluid closure in plasma
  - Turbulence
  - Plasma anisotropy type instabilities
  - Coupling FBE Transport
  - Halo current
- 2. Numerical methods and simulations
  - High order methods
  - Curvilinear coordinate systems
  - Equilibrium simulation
  - Pressure correction scheme
  - Anisotropy
  - Solving methods and parallelism
- 3. Identification and control
  - Inverse problem: Equilibrium reconstruction
  - Open loop control
- 4. Applications
  - MHD instabilities : Edge-Localized Modes (ELMs)
  - Edge plasma turbulence
  - Optimization of scenarii
  - Ionospheric plasma

## 4. New Software and Platforms

#### 4.1. CEDRES++

#### FUNCTIONAL DESCRIPTION

In Tokamaks, at the slow resistive diffusion time scale, the magnetic configuration can be described by the MHD equilibrium equations inside the plasma and the Maxwell equations outside. Moreover, the magnetic field is often supposed not to depend on the azimutal angle.

Under this assumption of axisymmetric configuration, the equilibrium in the whole space reduces to solving a 2D problem in which the magnetic field in the plasma is described by the well known Grad Shafranov equation. The unknown of this problem is the poloidal magnetic flux. The P1 finite element code CEDRES++ solves this free boundary equilibrium problem in direct, evolutive and inverse mode. The direct problem consists in the computation of the magnetic configuration and of the plasma boundary, given a plasma current density profile and the total current in each poloidal field coils (PF coils). In the evolutive mode, a time-dependent sequence of snapshots is obtained, being given a time evolution of the current density profiles in the plasmas and of the voltages in the power supplies of the poloidal field circuits. The aim of the inverse problem is to find currents in the PF coils in order to best fit a given plasma shape.

- Participants: Cédric Boulbe, Jacques Blum, Blaise Faugeras and Holger Heumann
- Partners: CNRS CEA Université de Nice Sophia Antipolis (UNS)
- Contact: Cédric Boulbe
- Reference: [16]

#### 4.2. Equinox

#### FUNCTIONAL DESCRIPTION

EQUINOX is a code dedicated to the numerical reconstruction of the equilibrium of the plasma in a Tokamak. The problem solved consists in the identification of the plasma current density, a non-linear source in the 2D Grad-Shafranov equation which governs the axisymmetric equilibrium of a plasma in a Tokamak. The experimental measurements that enable this identification are the magnetics on the vacuum vessel, but also polarimetric and interferometric measures on several chords, as well as motional Stark effect measurements. The reconstruction can be obtained in real-time and the numerical method implemented involves a finite element method, a fixed-point algorithm and a least-square optimization procedure.

- Participants: Jacques Blum, Cédric Boulbe and Blaise Faugeras
- Contact: Blaise Faugeras
- Reference: [1]

#### **4.3. FBGKI (Full Braginskii)**

FUNCTIONAL DESCRIPTION The Full Braginskii solver considers the equations proposed by Braginskii (1965), in order to describe the plasma turbulent transport in the edge part of tokamaks. These equations rely on a two fluid (ion - electron) description of the plasma and on the electroneutrality and electrostatic assumptions. One has then a set of 10 coupled non-linear and strongly anisotropic PDEs. FBGKI makes use in space of high order methods: Fourier in the toroidal periodic direction and spectral elements in the poloidal plane. The spectral vanishing viscosity (SVV) technique is implemented for stabilization. Static condensation is used to reduce the computational cost. In its sequential version, a matrix free solver is used to compute the potential. The parallel version of FBGKI presents two layers of parallelization: The first one corresponds to the poloidal plane and the second one to the toroidal direction. In the poloidal plane, the domain decomposition is achieved using the software METIS. For the parallel linear algebra, one uses the software PETSC (Portable Extensible Toolkit for Scientific Computation). The time discretization makes use of a Strang splitting, that decouples the explicit treatment of the advection and Braginskii terms, from the implicit treatment of the Lorentz forces and the computation of the electric potential. Whereas the explicit part is easily parallelized, the implicit one requires solving a strongly anisotropic elliptic problem for the potential. In the parallel version of FBGKI the system matrix is assembled in sparse manner, in order to allow using the multigrid HYPRE preconditionner implemented in PETSC. Till now results have only been obtained for computations done on a few tens of processors. Both the weak and strong scalings look satisfactory. Numerical experiments are still required to go up to hundreds or thousands of processors.

- Participants: Sébastian Minjeaud and Richard Pasquetti
- Contact: Sebastian Minjeaud
- Reference: [25]

#### **4.4. FEEQS.M**

#### FUNCTIONAL DESCRIPTION

FEEQS.M (Finite Element Equilibrium Solver in Matlab) is a MATLAB implementation of the numerical methods in [16] to solve equilibrium problems for toroidal plasmas. Direct and inverse problems for both the static and transient formulations of plasma equilibrium can be solved. FEEQS.M exploits MATLAB's evolved sparse matrix methods and uses heavily the vectorization programming paradigm, which results in running times comparable to C/C++ implementations. FEEQS.M complements the production code CEDRES++ in being considered as fast prototyping test bed for computational methods for equilibrium problems. This includes aspects of numerics such as improved robustness of the Newton iterations or optimization algorithms for inverse problems. The latest developments aim at incorporating the resistive diffusion equation.

- Participant: Holger Heumann
- Contact: Holger Heumann
- URL: https://scm.gforge.inria.fr/svn/holgerheumann/Matlab/FEEQS.M

#### 4.5. Fluidbox

#### FUNCTIONAL DESCRIPTION

FluidBox is a software dedicated to the simulation of inert or reactive flows. It is also able to simulate multiphase, multi-material and MDH flows. There exist 2D and 3D dimensional versions. The 2D version is used to test new ideas that are later implemented in 3D. Two classes of schemes are available : a classical finite volume scheme and the more recent residual distribution schemes. Several low Mach number preconditioning are also implemented. The code has been parallelized with and without domain overlapping.

- Participants: Rémi Abgrall, Boniface Nkonga, Michael Papin and Mario Ricchiuto
- Contact: Boniface Nkonga

#### 4.6. Jorek-Django

#### FUNCTIONAL DESCRIPTION

Jorek-Django is a non-production version of the JOREK software, for MHD modeling of plasma dynamic in tokamak geometries. The numerical approximation is derived in the context of finite elements where 3D basic functions are tensor products of 2D basis functions in the poloidal plane by 1D basis functions in the toroidal direction. More specifically, Jorek uses curved bicubic isoparametric elements in 2D and a spectral decomposition (sine, cosine) in the toroidal axis. Continuity of derivatives and mesh alignment to equilibrium surface fluxes are enforced. Resulting linear systems are solved by the PASTIX software developed at Inria-Bordeaux.

- Participants: Boniface Nkonga, Hervé Guillard, Emmanuel Franck (EPI Tonus), Ayoub Iaagoubi and Ahmed Ratnani (IPP, Garching)
- Contact: Hervé Guillard
- URL: https://gforge.inria.fr/projects/jorek/

#### 4.7. Plato

A platform for Tokamak simulation FUNCTIONAL DESCRIPTION

PlaTo (A platform for Tokamak simulation) is a suite of data and software dedicated to the geometry and physics of Tokamaks. Plato offers interfaces for reading and handling distributed unstructured meshes, numerical templates for parallel discretizations, interfaces for distributed matrices and linear and non-linear equation solvers. Plato provides meshes and solutions corresponding to equilibrium solutions that can be used as initial data for more complex computations as well as tools for visualization using Visit or Paraview. Plato is no more developed and is in the process of being merged with Jorek-Django

- Participants: Boniface Nkonga, Hervé Guillard, Giorgio Giorgiani, Afeintou Sangam and Elise Estibals
- Contact: Hervé Guillard

#### **4.8. VacTH**

#### FUNCTIONAL DESCRIPTION

VacTH implements a method based on the use of toroidal harmonics and on a modelization of the poloidal field coils and divertor coils for the 2D interpolation and extrapolation of discrete magnetic measurements in a tokamak. The method is generic and can be used to provide the Cauchy boundary conditions needed as input by a fixed domain equilibrium reconstruction code like EQUINOX. It can also be used to extrapolate the magnetic measurements in order to compute the plasma boundary itself. The proposed method and algorithm are detailed in [4] and results from numerous numerical experiments are presented. The method is foreseen to be used in the real-time plasma control loop on the WEST tokamak.

• Contact: Blaise Faugeras

# 5. New Results

#### 5.1. Plasma boundary reconstruction

Participants: Jacques Blum, Cédric Boulbe, Blaise Faugeras.

A new fast and stable algorithm has been developped for the reconstruction of the plasma boundary from discrete magnetic measurements taken at several locations surrounding the vacuum vessel. The resolution of this inverse problem takes two steps. In the first one we transform the set of measurements into Cauchy conditions on a fixed contour  $\Gamma_O$  close to the measurement points. This is done by least square fitting a truncated series of toroidal harmonic functions to the measurements. The second step consists in solving a Cauchy problem for the elliptic equation satisfied by the flux in the vacuum and for the overdetermined boundary conditions on a fixed annular domain of external boundary  $\Gamma_O$  and fictitious inner boundary  $\Gamma_I$ . A regularized Kohn-Vogelius cost function depending on the value of the flux on  $\Gamma_I$  and measuring the discrepancy between the solution to the equation satisfied by the flux obtained using Dirichlet conditions on  $\Gamma_O$  and the one obtained using Neumann conditions is minimized. The method presented here has led to the development of a software, called VacTH-KV, which enables plasma boundary reconstruction in any Tokamak (see [14]).

#### 5.2. Free boundary - Transport Solver - Controller coupling

**Participants:** Cédric Boulbe, Blaise Faugeras, Jean François Artaud [IRFM CEA Cadarache], Vincent Basiuk [IRFM CEA Cadarache], Emiliano Fable [Max-Planck-Institut für Plasmaphysik, Garching], Philippe Huyn [IRFM CEA Cadarache], Eric Nardon [IRFM CEA Cadarache], Jakub Urban [IPP, Academy of Sciences of the Czech Republic, Prague].

Last year, a first version of the workflow coupling a free boundary equilibrium code, the European transport solver ETS and a plasma shape and position controller had been developped. In 2015, this new tool has been tested and improved. An experiment realized on the Tokamak TCV and called "yoyo" shot has been successfully simulated. This work has been realised in the framework of the Eurofusion Work Package: Code Development for integrated modelling project.

# **5.3.** A finite element method with overlapping meshes for free-boundary toroidal plasma equilibria in realistic geometry

Participants: Holger Heumann, Francesca Rappetti.

Existing finite element implementations for the computation of free-boundary toroidal plasma equilibria approximate the flux function by piecewise polynomial, globally continuous functions. Recent numerical results for the self-consistent coupling of equilibrium and resistive diffusion in the spirit of Grad-Hogan suggest the necessity of higher regularity. Enforcing continuity of the gradient in finite elements methods on triangular meshes, leads to a drastic increase in the number of unknowns, since the degree of the polynomial approximation needs to be increased beyond four. Therefore existing implementations for the fixed boundary problem resort to (curvilinear) quadrilateral meshes and approximation spaces based on cubic Hermite splines. Fine substructures in the realistic geometry of a tokamak, such as air-gaps, passive structures and the vacuum vessel prevent the use of quadrilateral meshes for the whole computational domain, as it would be necessary for the free-boundary problem.

In this work we propose a finite element method that employs two meshes, one of quadrilaterals in the vacuum domain and one of triangles outside, which *overlap* in a narrow region around the vacuum domain. This approach gives the flexibility to achieve easily and at low cost higher order regularity for the approximation of the flux function in the domain covered by the plasma, while preserving accurate meshing of the geometric details exterior to the vacuum. The continuity of the numerical solution in the region of overlap is weakly enforced by relying on the mortar projection. A publication is in preparation.

#### 5.4. Inverse transient plasma equilibrium problem

Participants: Holger Heumann, Jacques Blum.

The inverse transient plasma equilibrium problem aims at precomputing the trajectories of externally applied voltages in the poloidal field coils of a tokamak. A basic implementation of this problem in 2011/2012 in CEDRES++ during Holger Heumann's PostDoc at Inria, provided first insight into the capabilities and also difficulties of this approach. Application engineers are highly interested in this application, but realistic cases will require more evolved numerical methods to reduce the computational time and memory requirements. In 2014 we implemented the inverse transient plasma equilibrium problem in FEEQS.M to facilitate our search for better algorithms. In 2015 we started working on realistic test cases for the upcoming WEST tokamak. In order to make such problems accessible by the current version of our code, we had to split the time interval of interest into 5 subintervals, on which we solve 5 inverse problems. Only by the initial condition the problem on a subinterval is connected to its predecessor. Next we faced some serious convergence problem of the optimisation algorithms for some of these problems. These led us to do extensive benchmark runs with different optimisation algorithms and implementation, including both Gradient and SQP-type methods, either with handcoded or MATLAB-native implementations. As a result we envisage for 2016 the incorporation of the SQP implementation of Jean-Charles Gilbert, which seems to be perfectly adapted to optimal control problems such as ours. Another improvement was achieved in reducing the actual number of free control parameters and to replace piecewise linear control trajectories with high order polynomials.

# 5.5. High order for the axisymmetric magnetohydrodynamic equilibrium problem

Participants: Holger Heumann, Lukas Drescher [TU Berlin], Kersten Schmidt [TU Berlin].

We implemented a higher order finite element method (FEM) for solving numerically axisymmetric magnetohydrodynamic (MHD) equilibrium problems. The focus is on high accuracy and the capabilities of high-order FEM implementations for faster calculations. High order FEM for elliptic problems, such as the considered MHD equilibrium problem, is well established and understood. This work uses the hp-FEM software CON-CEPTS developed at ETH Zürich/TU Berlin. Further, we developed a novel method for computing accurately the so-called flux surface averages, that are important in transient MHD calculations. This new method circumvents the expensive and very technical computation of line-integrals and fits seamlessly into the high order finite element method. A publication is in preparation.

#### 5.6. Towards automated magnetic divertor design for optimal heat exhaust

**Participants:** Holger Heumann, Maarten Blommaert [FZ, Jülich (Germany)], Martine Baelmans [KU Leuven (Belgium)], Nicolas R. Gauger [TU, Kaiserslautern (Germany)], Detlev Reiter [FZ, Jülich (Germany)].

Avoiding excessive structure heat loads in future fusion tokamaks is regarded as one of the greatest design challenges. In this joint effort, we aim at developing a tool to study how the severe divertor heat loads can be mitigated by reconfiguring the magnetic confinement. For this purpose, the free boundary equilibrium code FEEQS.M was integrated with a plasma edge transport code to work in an automated fashion. A practical and efficient adjoint based sensitivity calculation was proposed to evaluate the sensitivities of the integrated code. The sensitivity calculation was applied to a realistic test case and compared with finite difference sensitivity calculations.

The integration of the free boundary equilibrium solver FEEQS.M allowed to assess the validity of a previous simplified model introduced by M. Bloomaert. It was found that the absence of plasma response currents significantly limits the accuracy of this simplified model.

The novel procedure was applied to obtain first results for the new WEST (Tungsten Environment in Steadystate Tokamak) divertor currently under construction in the Tore Supra tokamak at CEA. The sensitivities and the related divertor optimization paths are strongly affected by the extension of the magnetic model (see [24]).

#### 5.7. Bohm boundary conditions

Participants: Richard Pasqueti, Sebastian Minjeaud.

Focusing on a minimal model proposed in the late 2000's by the IRFM (Cadarache), an algorithm has been proposed to enforce at the plates the inequality  $M \ge 1$ , where M is the parallel Mach number. The algorithm is implemented in the FBGKI code, but still requires improvements to enhance the robustness of the numerical method (see [18]).

# 5.8. High order approximation of dispersive equations and conservation of invariants

Participants: Richard Pasqueti, Sebastian Minjeaud.

Focusing on the Korteweg-de Vries (KdV) equation, algorithms have been proposed to handle high order derivative terms (third order for KdV) with  $C^0$  elements and to preserve invariants (mass and momentum for KdV) through the time-scheme (see [33]).

#### 5.9. Taylor-Galerkin stabilized Finite Element

Participants: José Costa, Boniface Nkonga.

The theoretical part of Taylor-Galerkin/Variational multi-scales (TG/VMS) strategy applied to MHD and reduced MHD modeling has been achieved last year. The final method amounts to adding in the finite element formulation, a self-adjoint operator associated to the most critical hyperbolic component of the system to be solved. The design of the critical contours and the identification of associated waves to be stabilized is problem dependent and related to the Jacobian matrix. This year we have continued the investigations for the design and improvement of the stabilization started in 2015. For application to plasma configurations with X-point, we have designed a numerical strategy that preserved the initial equilibrium without flows. The Bohm boundary condition on open flux walls has been formulated and is now under validation.

#### 5.10. Toward full MHD numerical modeling with C1 finite element.

Participants: José Costa, Giorgio Giorgiani, Hervé Guillard, Boniface Nkonga.

In this context the single fluid full MHD model is considered and the divergence free constraint on the magnetic field is achieved by introduction of a vector potential. The use of the vector potential has the additional advantage that the toroidal component is the magnetic flux of the Grad-Shafranov equilibrium. However, using the vector potential as variable introduces higher order derivatives in the system and classical C0 finite elements cannot be directly applied. This is why our finite element strategies uses shape/test functions whose derivatives have global continuity in space (smooth finite elements). The global approach uses cross product shape/test functions between poloidal(2D) and the Toroidal(1D). In the 2D poloidal plane, discretization uses either quadrangular or triangular elements. In order to derive efficient strategies for the full MHD in the vector potential formulation, the Gauge condition on the vector potential and the boundary conditions have been enforced by penalization. For the Gauge condition it gives rise to element contributions but also boundary integrals that should be computed on curved surfaces that sometime fitted the magnetic surfaces. Equations are formulated in semi-conservative form such as to apply integration by parts. Therefore, boundary conditions can be viewed as evolution of fluxes or variables. Integral formulation on the boundary is very useful for higher order finite elements and also easier for the treatment of corners. Indeed in this context the boundary conditions are edge/surfaces oriented and boundary corners are driven by the neighborhood edge penalization. This strategy is the one that will be used for future developments.

2D Quadrangular Cubic Bezier Finite Elements:

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This finite element has been used for a while for reduced MHD models in the software Jorek. Reduced MHD uses the projection of the momentum equation in a space orthogonal to the equilibrium. When full MHD models are used, the momentum equation needs to be projected in the equilibrium space and this projection should be consistent with the Grad-Shafranov equilibrium that is used to compute the initial state. This has been achieved by a proper computation of the JxB contribution in the momentum equation, taking into account the poloidal variation of the toroidal component of the magnetic field. After a detailed analysis, we have performed this year some implementations and numerical validations. An Inria report is under preparation.

#### 2D Triangular Powell-Sabin Finite Elements:

In order to avoid some mesh singularities when using quadrangular meshes for complex geometries and flux surfaces shapes, triangular elements are a possible option. It is not so easy to derive smooth finite elements on triangles with reduced number of degrees of freedom (ddl). The Bell reduced-quintic finite elements we have considered in the previous years have too many unknowns (6 per vertex). Powell-Sabin splines are piecewise quadratic polynomials with a global C1-continuity and 3 unknowns per vertex, they have a local support, they form a convex partition of unity, they are stable, and they have a geometrically intuitive interpretation involving control triangles. Construction of the Powell-Sabin splines needs some geometrical tools that have been developed: Minimum area enclosing triangle of a set of control points (sequential and parallel). This construction is applied to each vertex of the triangular mesh and used to derive the local shape/test functions. These Powell-Sabin splines have been used successfully in the area of computer aided geometric design for the modeling and fitting of surfaces. We have used the Powell-Sabin (PS) splines for the approximation of elliptic partial differential equations (including Grad-Shafranov) in a rectangular domain. In this context, the optimal rate of convergence (order 3) has been recovered. This year, validations have been performed for hyperbolic 2D Euler equations with VMS stabilization. The context of the 3D toroidal geometries has been considered and implemented. Preliminary validations are satisfactory. An Inria report is also under preparation.

#### 5.11. Genuinely multidimensional Riemann Solver

Participants: Jeaniffer Vides, Boniface Nkonga.

Multidimensional Riemann solvers were pioneered by Abgrall, Abgrall, Maire, Nkonga, Després and Loubere have extensively developed them especially as node-solvers for Lagrangian hydrodynamics. Another strain of work comes from explorations by Wendroff and Balsara who took a space-time approach. In this work, the resolved state is obtained via space-time integration over a wave model, just as was done by Wendroff and Balsara. However, an algebraic approach is used for the development of the fluxes. It is, therefore, shown that the multidimensional fluxes can be obtained by application of jump conditions at the boundaries of the wave model. The problem is of course over determined with the result that the shock jump conditions are only satisfied approximately in a least squares sense. Even so, this work gives us new perspective on multidimensional Riemann solvers. The literal satisfaction of the shock jump conditions (up to least squares approximation) makes it easier to understand multidimensional Riemann solvers as a natural extension of the one-dimensional Riemann solvers. Contributions have also been made on the development of a minimalist wave model, which might help in reducing dissipation. Further innovations are reported on the assembling of fluxes based on the structure of the wave model, and those innovations are potentially useful. For MHD the CT approach consists of constraining the transport of magnetic field so that the divergence is always kept zero. The method relies on exploiting the dualism between the flux components and the electric field. Since the electric field is needed at the edges of the mesh, the multidimensional Riemann solver can also provide the electric field. By running an extensive set of simulations, it is shown that the multidimensional Riemann solver is robust and can be used to obtain divergence-free formulations for MHD that perform well on several stringent calculations. The work performed this year was to improve this strategy by enriching of substructures the description of the strongly interaction of waves. These improvements were done in collaboration with the invited professor D. Balsara. This work has resulted in an article to be published in the Journal of Computational Physics in 2016.

#### 5.12. Multi scales approximations of "Shallow water" flows.

Participants: Jeaniffer Vides, Boniface Nkonga, Sergey Gavrilyuk, Kseniya Ivanova.

The terminology "Shallow water" is used to characterize thin flows on curved surfaces. It is customary for this type of flows; to use the incompressible Navier-Stokes equations to asymptotically derive reduced models for the evolution of the depth integrated speed and the thickness of the flow. Reduced models are mainly hyperbolic and finite volume method are often used for their numerical approximation. Approximation strategies are generally structured as follow:

- Construction of a global coordinate system associated with an assumed analytical surface.
- Reduction of the model relatively to the global coordinate system
- Approximation of the surface by a finite number of elements.
- Approximation of the reduced model using the discrete surface.

In the context of real applications, it is presumptuous to expect an analytical formulation of the surface. From the data provided by observation satellites, we can usually extract a discrete description of the surfaces that drives thin flow. Therefore, it is more practical to use the discrete description as the starting point of the resolution strategy. This is the angle of approach that we have considered. We locally define two mesh scales: the element scale and the cell scale. The discrete mapping and the reduced model are defined at the element scale and the average values that evolve in time are defined at the cell scale. First applications have been successfully performed. Our efforts have been extended to include relevant physics at each scale, including sheared flows. We have used a multi-dimensional formulation. An Inria report is under preparation.

#### 5.13. Asymptotic theory of reduced MHD models

#### Participant: Hervé Guillard.

In the study of fusion plasma, one of the fundamental model used for stability studies is the magnetohydrodynamic (MHD) model. Many theoretical and numerical works in this field use specific approximations of this model known as *reduced* MHD models. The derivation of these reduced MHD models has been formulated as a special instance of the theory of singular limit of hyperbolic system of partial differential equations with a large operator. This formulation allows to use the general results of this theory and to prove rigorously that reduced MHD models are valid approximations of the full MHD equations [29]. In particular, it is proven that the solutions of the full MHD system converge to the solutions of an appropriate reduced model. These results substantiate the intuitive physical idea that in the presence of a strong magnetic field, motion in the plane perpendicular to the plasma is nearly incompressible.

#### 5.14. Finite volume approximations for fusion plasma

Participants: Hervé Guillard, Afeintou Sangam, Elise Estibals.

The MHD model used for plasma studies in tokamak is very often based on the magnetic vector potential form of the equations where the vector potential satisfies  $\nabla \times \mathbf{A} = \mathbf{B}$  with  $\mathbf{B}$  the magnetic field and only a small number of numerical models use the conservative formulation based on  $\mathbf{B}$ . One of the shortcomings of this latter formulation is the necessity to enforce numerically the divergence free constraint on the magnetic field that can be difficult to achieve and/or computationally costly. Another difficulty is that the equilibrium solution of the MHD equation given by the Grad-Shafranov equation is not an exact solution of the discrete equation.

We have begun to investigate the use of the  $\mathbf{B}$  formulation for tokamak studies. The divergence free constraint is taken into account by a projection at each time step on a rotated gradient field. This step ensures a strict respect of the divergence free constraint while being extremely cheap since the scalar field is simply advected by the flow. Preliminary numerical experiments show that this approach can have some interest. The design of a well-balanced solver will be the next step of these studies.

# 6. Partnerships and Cooperations

#### **6.1. National Initiatives**

#### 6.1.1. ANR

• ANEMOS : ANR-11-MONU-002

ANEMOS : Advanced Numeric for Elms : Models and Optimized Strategies associates JAD Laboratory/Inria (Nice, Manager), IRFM-CEA (Cadarache), Maison de la Simulation (Saclay) and Inria EPI Bacchus (Bordeaux). Elms are disruptive instabilities occurring in the edge region (SOL) of a tokamak plasma. The development of Elms poses a major challenge in magnetic fusion research with tokamaks, as these instabilities can damage plasma-facing components, particularly divertor plates. The mitigation or suppression of large Elms is a critical issue for successful operation of ITER. Goal for ANEMOS is to develop and improve numerical tools in order to simulate physical mechanisms of Elms and to qualify some strategies for their control. We then need to design efficient numerical strategies on the most advanced computers available to contribute to the science base underlying of proposed burning plasma tokamak experiments such as ITER.

- LIVE-CAMS: Till September 30 th 2015, R. Pasquetti was involved in the ANR project LIVE-CAMS.
- MEDIMAX: In 2015 R. Pasquetti and F. Rappeti were involved in in the ANR project MEDIMAX.

#### 6.1.2. Inria Project Lab: FRATRES (Fusion Reactors Research and Simulation)

- Participants : Inria project-teams : CASTOR, IPSO, TONUS,
- Partners : IRFM-CEA, Max Planck Institute-IPP Garching, LJLL-Jussieu, IMT-Toulouse

The current rate of fossil fuel usage and its serious adverse environmental impacts (pollution, greenhouse gas emissions, ...) leads to an energy crisis accompanied by potentially disastrous global climate changes. The research of alternative energy sources is thus of crucial importance. Controlled fusion is one of the most promising alternatives to the use of fossil resources, potentially with a unlimited source of fuel. Controlled nuclear fusion can be considered as an example of grand challenge in many fields of computational sciences from physical modeling, mathematical and numerical analysis to algorithmics and software development and several Inria teams and their partners are developing mathematical and numerical tools in these areas.

Since january 2015, H. Guillard is coordinating the Inria Project Lab FRATRES (https://team.inria.fr/iplfratres/) to organize these developments on a collaborative basis in order to overcome the current limitations of today numerical methodologies. The ambition is to prepare the next generation of numerical modeling methodologies able to use in an optimal way the processing capabilities of modern massively parallel architectures. This objective requires close collaboration between a) applied mathematicians and physicists that develop and study mathematical models of PDE; b) numerical analysts developing approximation schemes; c) specialists of algorithmic proposing solvers and libraries using the many levels of parallelism offered by the modern architecture and d) computer scientists. This Inria Project Lab will contribute in close connection with National and European initiatives devoted to nuclear Fusion to the improvement and design of numerical simulation technologies applied to plasma physics and in particular to the ITER project for magnetic confinement fusion.

Contact : Hervé Guillard

#### 6.2. European Initiatives

#### 6.2.1. FP7 & H2020 Projects

- EUROfusion Grant agreement number 633053. Enabling Research program.
  - CfP-WP14-ER-01/CEA-01; JOREK, BOUT++ non-linear MHD modelling of MHD instabilities and their control in existing tokamaks and ITER (PI: Matthias Hoelzl, IPP)

- CfP-WP14-ER-01; Synergetic numerical-experimental approach to fundamental aspects of turbulent transport in the tokamak edge (PI: Paolo Ricci, École Polytechnique Fédérale de Lausanne).
- EUROfusion WPCD (Working Package Code Development)
  - ACT1: Extended equilibrium and stability chain (participation)
  - ACT2: Free boundary equilibrium and control (participation and coordination)
- The team also participates in the EoCoE European project. Grant Agreement number: 676629 EoCoE — H2020-EINFRA-2014-2015/H2020-EINFRA-2015-1.

#### **6.3. International Initiatives**

#### 6.3.1. Inria Associate Teams not involved in an Inria International Labs

#### 6.3.1.1. AMOSS

Title: Advanced Modeling on Shear Shallow Flows for Curved Topography : water and granular flows.

International Partner (Institution - Laboratory - Researcher):

NCKU (Taiwan)- Yih-Chin Tai

Start year: 2014

Our objective here is to generalize the promising modeling strategy proposed by S. Gavrilyuk (2012-2013) to genuinely 3D shear flows and also take into account the curvature effects related to topography. Special care will be exercised to ensure that the numerical methodology can take full advantage of massively parallel computational platforms and serve as a practical engineering tool. Cross validations will be achieved by experiments and numerical simulations with applications to landslides.

#### 6.3.1.2. Informal International Partners

The team collaborates with TUC technical University of Crete (Prof. Argyris Delis) on the subject of shallow water models. Part of this collaboration is common with the works done in the framework of the AMOSS associate team.

#### 6.4. International Research Visitors

#### 6.4.1. Visits of International Scientists

- D. Balsara of the Notre Dame University (USA), as invited professor for one month at the university of Nice (June/July 2015).
- Key-Ming Shyue of the National Taiwan University, as invited professor for one month at the university of Marseille (September 2015)
- Chih-Yu Kuo, Associate Research Fellow, Research Center for Applied Sciences, Academia Sinica, Taipei, Taiwan, and Yih-Chin Tai, Professor, National Cheng Kung University, Tainan, Taiwan. Visit at Inria Sophia in July 2015.

#### 6.4.1.1. Internships

- J. Llobell, March-June 2015, T. Goudon, S. Minjeaud, M. Ribot.
- L. Drescher, TU Berlin, September-October 2015, H. Heumann
- P. Wang, June-September 2015, J. Blum, C.Boulbe

# 7. Dissemination

#### 7.1. Promoting Scientific Activities

#### 7.1.1. Scientific events organisation

- 7.1.1.1. Member of the organizing committees
  - Journées CASTOR, Valberg, 26-27 janvier 2015.
  - Rencontre NTM (Nice-Toulon-Marseille), Porquerolles, may 2015. http://champion.univ-tln.fr/ NTM/NTM2015.html
  - Journées numériques, Décomposition de domaine, Librairie de calcul parallèle, Nice, april 2015. http://math.unice.fr/~minjeaud/Donnees/JourneesNumeriques\_15-1/index.php

#### 7.1.2. Journal

- 7.1.2.1. Member of the editorial boards
  - J. Blum is in the editorial board of Journal of Scientific Computing
  - J. Blum is in the scientific committee of mathematics and statistics of ISTE books

#### 7.1.2.2. Reviewer - Reviewing activities

R. Pasquetti is reviewer for several journals including

- Journal of Computational Physics
- Computers and Fluids
- J. Blum is reviewer for several journals including
  - Journal of Computational Physics
  - Fusion Science and Technology
  - Mathematics and Computers in Simulations

#### 7.1.3. Invited talks

- J. Blum, Journée Mathématiques X-UPS, Des problèmes à N corps aux Tokamaks, May 2015
- Holger Heumann, An enhanced approximate cloaking scheme for the conductivity problem, Mathematics for Imaging, ENS, Paris, France, October 20-24, 2015.
- Holger Heumann, Quasi-static Free-Boundary Equilibrium of Toroidal Plasma: Computational Methods and Applications, Modeling and Numerical Methods for Hot Plasmas II, Institut de Mathématiques de Bordeaux, France, October 12-14, 2015.
- Hervé Guillard, "Asymptotic theory of reduced MHD models for fusion plasmas" : Oberwolfach Workshop: Recent Developments in the Numerics of Nonlinear Hyperbolic Conservation Laws, 13 September 19 September 2015. Oberwolfach, Germany.
- Jacques Blum, Control methods for the optimization of plasma scenarios in a Tokamak, 27th IFIP TC7 Conference, Sophia Antipolis, June 29 July 3rd, 2015

#### 7.1.4. Leadership within the scientific community

- J. Blum is president of thematic committee 6 (Mathematics, algoritmic and computer science) of GENCI
- J. Blum represents the University of Nice Sophia Antipolis in Alliance Nationale de Coordination pour la Recherche pour l'Energie (ANCRE).

#### 7.2. Teaching - Supervision - Juries

#### 7.2.1. Teaching

Ecole d'ingénieur: D. Auroux, Optimisation, 66h, M1, Polytech Nice, Université de Nice Sophia Antipolis, France

Ecole d'ingénieur: D. Auroux, Méthodes numériques, 36h, M1, Polytech Nice Sophia, Université de Nice Sophia Antipolis, France

Master: B. Faugeras, Optimisation, 18h, M1, Université de Nice Sophia Antipolis, France

Master: J. Blum, Optimisation et contrôle, 20h, M2, Université de Nice Sophia Antipolis, France

Master: J. Blum, Optimisation, 18h, M1, Université de Nice Sophia Antipolis, France

Ecole d'ingénieur: J. Blum, Commande Optimale, 37.5h, M2, Polytech Nice Sophia, Université de Nice Sophia Antipolis, France

Ecole d'ingénieur: C. Boulbe, Analyse Numérique, 71.5h, L3, Polytech Nice Sophia Antipolis, France

Ecole d'ingenieur: C. Boulbe, Méthodes numérique - EDP, 66h, M1, Polytech Nice Sophia Antipolis, France

Licence: S. Minjeaud, Analyse Numérique, 18 h, L3, Université de Nice Sophia Antipolis, France.

Licence: S. Minjeaud, Eléments de calcul différentiel, 18 h, L3, Université de Nice Sophia Antipolis, France.

Master: S. Minjeaud, Méthodes numériques en EDP, 18 h, M1, Université de Nice Sophia Antipolis, France.

Master: B. Nkonga, Analyse Numérique, 40h, M1, Université de Nice Sophia Antipolis, France Ecole d'ingénieur/Master: B. Nkonga, Méthode des éléments finis, 24h, M2, Polytech Nice Sophia, France

Ecole d'ingénieur/Master: B. Nkonga, Calcul Parallèle, 48h, M2, Polytech Nice Sophia, France

Licence: F. Rapetti, Analyse Numérique, 18h, L3, Université de Nice Sophia Antipolis, France

Licence: F. Rapetti, Analyse, 70h, L1, Université de Nice Sophia Antipolis, France

Licence: A. Sangam, Analyse, 40h, L1, Université Nice Sophia Antipolis, France

Licence: A. Sangam, Mathématiques 2, 30h, L1, Université Nice Sophia Antipolis, France

Licence: A. Sangam, Modélisation, 10h, L1, Université Nice Sophia Antipolis, France

Licence: A. Sangam, Analyse, 50h, L2, Université Nice Sophia Antipolis, France

Licence: A. Sangam, Méthodes Numériques et Formelles, 40h, L2, Université Nice Sophia Antipolis, France

Licence: A. Sangam, Mathématiques Appliquées, 50h, L3, Université de Nice Sophia Antipolis, France

Master: R. Pasquetti, module "Modèles de turbulence", 20 h, Masters MSC & IMAG2E, Université de Nice Sophia Antipolis, France.

#### 7.2.2. Supervision

HdR : B. Faugeras, Modélisation, simulation numérique et problèmes inverses. Contributions en physique des plasmas de Tokamak, en écologie marine et autres travaux, Université de Nice Sophia Antipolis,12 Oct. 2016

PhD : C. Le Touze, "Etude du couplage entre modèles à phase séparée et modèles à phase dispersée pour la simulation de l'atomisation primaire en combustion cryotechnique", Université de Nice Sophia Antipolis, Décembre 2015, Hervé Guillard.

PhD in progress : J. Costa, Modeling of Elms, Sep 2012 - Sep. 2016, B. Nkonga

PhD in progress : E. Estibals, "MHD réduite: Modélisation et Simulation numérique utilisant des éléments finis stabilisés d'ordre élevés sur un maillage courbe non-structuré. Application à l'injection de glaçons et de masse dans ITER", 15th october 2013, Hervé Guillard, Afeintou Sangam.

#### 7.2.3. Juries

R. Pasquetti was referee for the thesis of:

- L. Cappanera, Nonlinear stabilization of magnetohydrodynamics equations and applications to multiphase flows, Université Paris-Saclay, Orsay (3-12-2015).
- R. Oguic, Une méthode multidomaine parallèlle pour les écoulements incompressibles en géométrie cylindrique: application aux écoulements turbulents soumis à rotation, Aix-Marseille Université, Marseille (19-10-2015).
- C. Mimeau, Conception and implementation of a hybrid vortex penalization method for solid-fluid porous media: application to the passive control of incompressible flows, Université de Grenoble, Grenoble (07-07-2015).

J. Blum and H. Guillard were in the HdR jury of B. Faugeras.

B. Nkonga war referee in in the PhD thesis jury of

- Jan Velechovsky (Prague),
- Fabien Rozard(Bordeaux)

B. Nkonga was in the PhD thesis jury of

- Xavier Lacoste (Bordeaux),
- Clément Le Touze(Nice),
- Matthias Mimault(Nice),
- Gauthier Brerhes(Nice).

## 8. Bibliography

#### Major publications by the team in recent years

- J. BLUM, C. BOULBE, B. FAUGERAS. Reconstruction of the equilibrium of the plasma in a Tokamak and identification of the current density profile in real time, in "Journal of Computational Physics", 2012, vol. 231, pp. 960-980, http://hal.archives-ouvertes.fr/hal-00419608
- [2] B. BRACONNIER, B. NKONGA. Relaxation method for low Mach number compressible multiphase flow, in "Journal of Computational Physics", 2009, vol. 228(16), pp. 5722-5739
- [3] P. DEGOND, F. DELUZET, A. SANGAM, M. H. VIGNAL. An Asymptotic Preserving scheme for the Euler equations in a strong magnetic field, in "J. Comput. Phys.", 2009, vol. 228, n<sup>o</sup> 10, pp. 3540–3558, http://dx. doi.org/10.1016/j.jcp.2008.12.040
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- [5] J. L. GUERMOND, R. PASQUETTI, B. POPOV. Entropy viscosity method for non-linear conservation laws, in "J. of Comput. Phys.", 2011, vol. 230, pp. 4248-4267
- [6] H. GUILLARD, F. DUVAL. A Darcy law for the drift velocity in a two-phase model, in "J. Comput. Phys.", 2007, vol. 224, pp. 288–313

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- [9] R. PASQUETTI. Spectral vanishing viscosity method for large-eddy simulation of turbulent flows, in "J. Sci. Comp.", 2006, vol. 27(1-3), pp. 365-375
- [10] A. SANGAM. An HLLC scheme for Ten-Moments approximation coupled with magnetic field, in "Int. J. Comput. Sci. Math.", 2008, vol. 2, n<sup>o</sup> 1/2, pp. 73–109, http://dx.doi.org/10.1504/IJCSM.2008.019724

#### **Publications of the year**

#### **Doctoral Dissertations and Habilitation Theses**

[11] B. FAUGERAS. Modélisation, simulation numérique et problèmes inverses. Contributions en physique des plasmas de Tokamak, en écologie marine et autres travaux, Université de Nice Sophia-Antipolis, October 2015, Habilitation à diriger des recherches, https://hal.archives-ouvertes.fr/tel-01227694

#### **Articles in International Peer-Reviewed Journals**

- [12] D. S. BALSARA, J. VIDES, K. GURSKI, B. NKONGA, M. DUMBSER, S. GARAIN, E. AU-DIT. A two-dimensional Riemann solver with self-similar sub-structure – Alternative formulation based on least squares projection, in "Journal of Computational Physics", January 2016, vol. 304 [DOI: 10.1016/J.JCP.2015.10.013], https://hal.archives-ouvertes.fr/hal-01254231
- [13] C. BERTHON, B. DUBROCA, A. SANGAM. An entropy preserving relaxation scheme for ten-moments equations with source terms, in "Communications in Mathematical Sciences", 2015, vol. 13, n<sup>o</sup> 8, pp. 2119-2154 [DOI: 10.4310/CMS.2015.v13.N8.A7], https://hal.inria.fr/hal-01255069
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- [15] T. GOUDON, B. NKONGA, M. RASCLE, M. RIBOT. Self-organized populations interacting under pursuitevasion dynamics, in "Physica D: Nonlinear Phenomena", 2015, 41 p., https://hal.archives-ouvertes.fr/hal-01070626
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#### **Invited Conferences**

- [21] J. BLUM, C. BOULBE, H. HEUMANN, B. FAUGERAS. Control methods for the optimization of plasma scenarios in a tokamak, in "27 th IFIP TC7 Conference 2015 on System Modelling and Optimization", Sophia Antipolis, France, June 2015, https://hal.archives-ouvertes.fr/hal-01246098
- [22] R. PASQUETTI. Entropy viscosity stabilized spectral element approximation of the shallow water equations with dry-wet transitions, in "ICCP 9", Singapour, Singapore, January 2015, https://hal-unice.archives-ouvertes. fr/hal-01144715

#### **Conferences without Proceedings**

- [23] M. BLOMMAERT, H. HEUMANN, M. BAELMANS, N. R. GAUGER, D. REITER. An automated approach to magnetic divertor configuration design, using an efficient optimization methodology, in "DPG Frühjahrstagung 2015 Plasmaphysik", Bochum, Germany, 2015, https://hal.archives-ouvertes.fr/hal-01248151
- [24] M. BLOMMAERT, H. HEUMANN, M. BAELMANS, Y. MARANDET, H. BUFFERAND, N. R. GAUGER, D. REITER. *Magnetic Field Models and their Application in Optimal Magnetic Divertor Design*, in "15th International Workshop on Plasma Edge Theory in Fusion Devices (PET15)", Nara, Japan, 2015, https://hal. archives-ouvertes.fr/hal-01248154
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#### Scientific Books (or Scientific Book chapters)

- [27] J. BLUM. Modélisation fluide des plasmas dans les Tokamak, in "Des problèmes à N corps aux Tokamaks", Edition de l'Ecole Polytechnique, 2015, https://hal.inria.fr/hal-01253955
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- [29] H. GUILLARD. The mathematical theory of reduced MHD models for fusion plasmas, Inria, April 2015, n<sup>o</sup> RR-8715, https://hal.inria.fr/hal-01145009
- [30] H. HEUMANN, R. HIPTMAIR, C. PAGLIANTINI. Stabilized Galerkin for Transient Advection of Differential Forms, SAM, ETH Zürich, January 2015, https://hal.archives-ouvertes.fr/hal-01119481

#### **Other Publications**

- [31] D. AREGBA-DRIOLLET, J. BREIL, S. BRULL, B. DUBROCA, E. ESTIBALS. *Modelling and Numerical Approximation for the Nonconservative Bitemperature Euler Model*, 2015, working paper or preprint, https:// hal.archives-ouvertes.fr/hal-01204631
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